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Editorial

Review of Contributions to the Special Edition: New Applications of Symmetry in Lattice Field Theory

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Symmetry has been at the heart of lattice field theory since its inception. A spacetime lattice furnishes a robust and rigorous means of formulating the gauge symmetries lying at the heart of particle physics, at the cost of breaking another cherished symmetry, Poincaré invariance. One of the greatest challenges in lattice field theory has been to identify faithful implementations of the chiral symmetry protecting fermions from acquiring mass. Symmetry considerations, sometimes in idealised limits, also determine conceptually important strong interaction issues such as confinement of quarks.

This Special Issue highlights symmetry applications and consequences in several fast-developing directions in both gauged and non-gauged lattice field theories, including long-standing issues of the nature of colour confinement and the role of topological excitations; new quantum critical points in lower-dimensional fermionic theories with relevance to layered condensed matter systems; symmetry-protected topological phases yielding edge states; robust quantum computation and quantum algorithms for field theory simulations; and dynamical mass generation without symmetry breaking.

The quantum field theories underlying the standard model of particle physics are weakly coupled, and hence are well-understood and tractable, in at least one physically relevant limit energy regime; however, conceptually this is not the only possibility. Generically, continuum theories exist whenever a lattice model exhibits a continuous phase transition where some correlation length diverges. The resulting theory at the critical point may be strongly interacting, in the sense that no approximation relying on a small dimensionless parameter is applicable, and is characterised in general by features such as the nature and number of light degrees of freedom and the global symmetries broken at the transition, which correspond via universality and the machinery of the renormalisation group to a well-defined set of critical exponents and anomalous scaling dimensions.

One example currently receiving attention is the Thirring model; namely, relativistic fermions with a contact interaction between conserved currents in 2 + 1 dimensions [1]. This model has potential applications in layered condensed matter systems exhibiting relativistic dispersions, such as graphene. The paper by Wipf and Lenz reviews the model and its symmetries, together with recent attempts to formulate it as a lattice field theory and identify the critical number of flavours N_c such that for $N < N_c$ and sufficently strong self-coupling there is a $U(2N)$-breaking transition leading to dynamical fermion mass generation. Due to the strongly interacting nature of the problem, the result is sensitive to the employed properties of lattice discretisation and the issue of the most appropriate formulation is yet to be resolved; Wipf and Lenz pay particular attention to a formulation using a $U(2N)$-invariant SLAC derivative in the definition of the fermion kinetic operator. The paper by Hands revisits the formulation of the model using staggered lattice fermions and exploits the long-known relation between staggered fermions and the Kähler form of the Dirac equation to construct a distinct continuum field theory where the global symmetry is $U(N) \otimes U(N)$ [2]. An essential feature of the putative fixed point is that the different

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fermion "tastes", an inherent feature of the staggered approach, can no longer be regarded as autonomous flavours, as in QCD, but rather are entangled by the strong dynamics.

Two of the contributed papers are devoted to the topic of symmetric mass generation (SMG). Traditional mechanisms for generating fermion masses break some of the symmetries of the massless theory, whether through explicit mass terms, anomalies or spontaneous symmetry breaking. Typically these broken symmetries are chiral in nature. Symmetric mass generation is a proposed non-perturbative mechanism that is capable of gapping fermions without breaking symmetries. In the context of lattice gauge theory it offers the hope that it may be possible to construct lattice mirror models that yield chiral gauge theories in the continuum limit.

The paper by Wang and You gives a comprehensive survey of these ideas [3]. In fact, some of the earliest results originate within the condensed matter community in the context of symmetry protected topological (SPT) phases of matter and topological insulators and superconductors. A more formal understanding of these findings arose in mathematical physics with the realisation that new types of non-perturbative 't Hooft anomalies associated with discrete symmetries played a crucial role in understanding the IR behaviour of certain strongly coupled field theories and could support SMG. A necessary condition for SMG is that all such 't Hooft anomalies must vanish in the UV theory. This has been complemented by work in the lattice gauge theory community where certain concrete models involving staggered fermions have been studied, which appear capable of symmetric mass generation even in four dimensional Euclidean spacetime. There is now even an understanding of how the cancellation of certain exact lattice anomalies determines the fermion content of such models. The review covers all aspects of this physics and gives a snapshot of the field at this time. The possible kinematic constraints coming from anomalies are mostly now understood but many questions of dynamics remain, including whether these new results allow new solutions to the problem of constructing a chiral lattice gauge theory.

Butt et al. [4] describe a lattice model which appears capable of using strong gauge dynamics to gap fermions without breaking symmetries. They employ a reduced staggered fermion that transforms in a bifundamental representation of an $SU(2) \otimes SU(2)$ gauge symmetry. The pseudoreal nature of $SU(2)$ ensures that there are no sign problems. The model targets a theory whose UV degrees of freedom correspond to 16 Majorana spinors in the naive continuum limit. A small four fermion term is used to break the $U(1)$ fermion number symmetry down to Z_4. This prohibits bilinear fermion terms and, because of the fermion content, there are no 't Hooft anomalies and hence no spontaneous breaking of the Z_4. This is consistent with observations which show only the presence of a four fermion condensate and no Goldstone bosons. In the IR, the theory should confine fermions into colourless diquark and meson states.

Two further papers are devoted to topics in confinement. Poppitz reviews efforts to understand confinement in four dimensions using models in which one spatial direction is taken as small and periodic [5]. This allows for weak coupling and semi-classical expansions to yield information on non-perturbative phenomena in four dimensions. The stability of the approach relies on the existence of unbroken centre symmetry in the model. Hence, the models studied in this review contain Weyl fermions in the adjoint of the gauge group—taken mostly to be $SU(2)$. An analysis reveals the crucial importance of new topological field configurations called bions which play a crucial role in the IR dynamics for such theories. The presentation also reveals connections to the theory of resurgence and the importance of generalised anomalies. This latter observation makes a nice connection to some of the theoretical work described earlier by Wang in the context of SMG.

In contrast, Greensite and Matsuyama [6] take a new look at an old problem—how to distinguish the confining and Higgs phases in theories containing dynamical fields in the fundamental representation of the group. The authors are particularly interested in untangling these issues on the lattice where, because of Etitzur's theorem forbidding the spontaneous breaking of gauge symmetry, it is problematic to find a suitable gauge invariant order parameter that signals a Higgs phase. Nevertheless, inspired by the Edwards–Anderson order parameter for spin glasses, the authors argue that indeed the two phases can be distinguished using a gauge invariant non-local order parameter which is formally similar to the spin glass order parameter. They also introduce the idea of confinement as "separation of charge" and argue that this can be used to distinguish the confining and Higgs regimes, with the former corresponding to a phase with "separation of charge" confinement and the latter to the more conventional colour confinement.

One of the most exciting areas of contemporary lattice field theory is the advent of quantum computing, which offers the prospect of inherently new, powerful tools to tackle important problems such as systems with a non-vanishing density of a conserved charge or those evolving in real time, where progress with conventional Monte Carlo approaches is inhibited by a sign problem. To exploit this, possibility field theories need to be reformulated in terms of two-level quantum degrees of freedom known as *qubits*; for the finite systems within reach in the NISQ era such "qubit regularisations" inevitably require the truncation of the infinite-dimensional Hilbert space which must preserve the symmetries of the target continuum theory. The paper by Liu and Chandrasekharan examines the consequences of such a regularisation on the resulting quantum fields in terms of qubit embedding algebra; in particular, a systematic construction of the QEA starting from the target theory is presented, with examples including both spin models and gauge theories [7].

Many-body systems of quantum degrees of freedom also support SPT phases of matter characterised by gapless edge states, which can be realised synthetically using cold atoms trapped on optical lattices, and are potentially exploitable in robust quantum devices. Remarkably, the simplest and best-known model of such a topological insulator, the *Chern insulator*, is nothing more than the Hamiltonian for free Wilson fermions on a square lattice. The paper by Tirrito et al. continues a recent series of investigations of interacting versions of this paradigm, derived from a Hubbard model of fermionic spins located on a ladder (spatial dimension $d = 1$) or bilayer ($d = 2$), showing the resulting field theory to be a Gross–Neveu–Wilson model, where the dimensionless Wilson parameter r in general differs from its conventional unit value as a natural consequence of cold-atom realisations based on a synthetic spin–orbit coupling [8]. In the strong-coupling limit, it turns out that relaxing the constraint $r = 1$ modifies the effective spin model from Ising to Heisenberg–Ising (XXZ) for $d = 1$, or to a Compass model with distinct directional couplings for $d = 2$. Strong correlation effects are modelled using both the large S limit, where S is the spin quantum number, and numerically using tensor network techniques, and the resulting phase structure elucidated.

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References

1. Wipf, A.W.; Lenz, J.J. Symmetries of Thirring Models on 3D Lattices. *Symmetry* **2022**, *14*, 333. [CrossRef]
2. Hands, S. The Planar Thirring Model with KÃ€hler-Dirac Fermions. *Symmetry* **2021**, *13*, 1523. [CrossRef]
3. Wang, J.; You, Y.Z. Symmetric Mass Generation. *Symmetry* **2022**, *14*, 1475. [CrossRef]
4. Butt, N.; Catterall, S.; Toga, G.C. Symmetric Mass Generation in Lattice Gauge Theory. *Symmetry* **2021**, *13*, 2276. [CrossRef]
5. Poppitz, E. Notes on Confinement on R3 × S1: From Yang–Mills, Super Yang–Mills, and QCD (adj) to QCD(F). *Symmetry* **2022**, *14*, 180. [CrossRef]
6. Greensite, J.; Matsuyama, K. Symmetry, Confinement, and the Higgs Phase. *Symmetry* **2022**, *14*, 177. [CrossRef]

7. Liu, H.; Chandrasekharan, S. Qubit Regularization and Qubit Embedding Algebras. *Symmetry* **2022**, *14*, 305. [CrossRef]
8. Tirrito, E.; Hands, S.; Bermudez, A. Large-S and Tensor-Network Methods for Strongly-Interacting Topological Insulators. *Symmetry* **2022**, *14*, 799. [CrossRef]