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Prediction Model of a Generative Adversarial Network Using the Concept of Complex Picture Fuzzy Soft Information

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Abstract: A computer vision model known as a generative adversarial network (GAN) creates all the visuals, including images, movies, and sounds. One of the most well-known subfields of deep learning and machine learning is generative adversarial networks. It is employed for text-to-image translations, as well as image-to-image and conceptual image-to-image translations. Different techniques are used in the processing and generation of visual data, which can lead to confusion and uncertainty. With this in mind, we define some solid mathematical concepts to model and solve the aforementioned problem. Complex picture fuzzy soft relations are defined in this study by taking the Cartesian product of two complex picture fuzzy soft sets. Furthermore, the types of complex picture fuzzy soft relations are explained, and their results are also discussed. The complex picture fuzzy soft relation has an extensive structure comprising membership, abstinence, and non-membership degrees with multidimensional variables. Therefore, this paper provides modeling methodologies based on complex picture fuzzy soft relations, which are used for the analysis of generative adversarial networks. In the process, the score functions are also formulated. Finally, a comparative analysis of existing techniques was performed to show the validity of the proposed work.

Keywords: generative adversarial networks; uncertainty; deep learning; complex picture fuzzy soft set; complex picture fuzzy soft relations

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1. Introduction

During the conceptual design phase, mechanical design methods are presented based on the data and information that are currently available, which can occasionally be unclear, imprecise, and unpredictable. The decision-making process in conceptual designs is one of these frequent occurrences, which frequently relies on the strategy of dealing with ambiguous facts and information. Based on their knowledge and experience, designers often offer several primary design strategies in conceptual design. However, the subjective qualities of the schemes are frequently unknown and must be assessed based on the decision maker's knowledge and judgments. When subjective judgments are used in decision making, the nature of the confusion and ambiguity becomes fuzzy rather than random. Zadeh's [1] fuzzy theory provides a useful tool that deals with ambiguous and unclear data and information, as well as the subjective features of human nature, in the decision-making process. A fuzzy set enables the ambiguity of a set with a membership degree between 0 and 1. Zadeh [2] expanded fuzzy sets and proposed the concept of interval-valued fuzzy sets in 1975. The single value of the degree of membership in a fuzzy set is replaced by an interval, the extremes of which are part of the [0, 1] interval, i.e., the degree of membership is a subinterval of [0, 1]. Mendel [3] coined the term "fuzzy relations" to describe the concepts of relations for fuzzy sets. Maiers and Sherif [4] introduced the application of fuzzy sets theory, which is applicable to a wide range of problems and fuzzy control algorithms. Goguen [5] created a set of axioms for a relativity simple kind of fuzzy set theory, which he used to investigate the accuracy of various fuzzy set representations. Román et al. [6] proposed a note on Zadeh's extensions. Ramot et al. [7] proposed the concept of complex fuzzy set in which the membership degree is defined by a complex number. The complex fuzzy sets discus both amplitude and phase terms. Li and Tu [8] investigated complex fuzzy sets and their applications in multiclass prediction. Zhang et al. [9] explored complex fuzzy sets in various operation features and δ-equalities. Pekaslan et al. [10] proposed the concept of Adonis-adaptive online non-singleton fuzzy logic systems. Hu et al. [11] established the application of orthogonality between complex fuzzy sets, which was used to identify signals. Nasir et al. [12] defined the interval-valued complex fuzzy relations and applied them to life expectancy and medical diagnosis. Chen et al. [13] proposed the concept of a neuro-fuzzy architecture employing complex fuzzy sets. Tamir et al. [14] proposed some applications of complex sets.

Later, Atanassov [15] developed the concept of an intuitionistic fuzzy set, which is broader than fuzzy sets. A fuzzy set only discusses the membership degree, whereas intuitionistic fuzzy sets discuss both the membership degree and non-membership degree. Both of these degrees attain values between the unit interval of [0, 1], and their sum also lies within this interval. De et al. [16] applied intuitionistic fuzzy sets to medical diagnosis. Tang et al. [17] investigated the symmetric implicational algorithm derived from intuitionistic fuzzy entropy. Alkouri and Saleh [18] expressed the notion of a complex intuitionistic fuzzy set. Complex intuitionistic fuzzy sets determine both membership and non-membership degrees with a complex number. They solve a multidimensional problem. Cuong and Kreinovich [19] later proposed picture fuzzy sets by including a degree of abstinence in the intuitionistic fuzzy set structure. The levels of membership, abstinence, and non-membership in a picture fuzzy set all accept values from the unit interval as long as their sum is between 0 and 1. The correlation coefficients for picture fuzzy sets were first established by Singh [20]. New procedures for picture fuzzy relations and fuzzy comprehensive assessment were presented by Bo and Zhang [21]. The idea of a decision-making model under a complex picture fuzzy set was introduced by Akram et al. [22].

Existing theories have limitations because of their inadequacy relative to parameterization tools. To overcome these drawbacks, Molodtsov [23] corroborated the soft set theory in which rating values are expressed over certain parameters. Ali et al. [24] defined some new soft set operations. Bozena Kostek [25] attempted to analyze sound quality using a soft-set-based technique. Mushrif et al. [26] proposed a new soft-set-theory-based technique for classification of natural textures. Cagman et al. [27] constructed an application of $uni - int$ decision making in the structure of soft set theory. Ibrahim and Yusuf [28] provided a crisp and critical overview of the evaluation of soft set theory. Maji et al. [29] used a soft set theory to solve the problem of decision making. Babitha and Sunil [30] proposed the idea of soft relations between the Cartesian product of soft sets. Mahmood [31] attempted to analyze a novel approach to bipolar soft sets and their applications. Maji et al. [32] proposed the idea of fuzzy soft sets by integrating soft sets and fuzzy sets. Fuzzy soft set theory uses different parameters to select the best option by providing a value of membership degree. Alcantud et al. [33] proposed a new generalization method for fuzzy soft sets. Kong et al. [34] used fuzzy soft sets in decision-making problems. Gogoi et al. [35] investigated the applications of fuzzy soft set theory to solve different problems. Borah et al. [36] proposed the novel idea of fuzzy soft relations by studying the Cartesian product of fuzzy soft sets. Sut [37] discussed the use of fuzzy soft relations in decision making. Thirunavukarasu et al. [38] investigated the theory of complex fuzzy soft sets, which also explains periodicity. Complex fuzzy soft sets discuss the membership degree with a phase term. The concepts of hybrid integrated decision-making algorithms for clustering analysis based on a bipolar complex fuzzy and soft sets, soft representation of soft groups, and comparison of the social justice leadership behaviors of school administrators according to teacher perceptions using classical and fuzzy logic were proposed in [39–41]. Xu et al. [42] proposed an intuitionistic fuzzy soft set, which combines soft sets and intuitionistic fuzzy soft sets. The intuitionistic fuzzy soft set is the generalization form of fuzzy soft set. Agarwal et al. [43] introduced generalized intuitionistic fuzzy soft sets with applications in decision making. Bashir et al. [44] investigated the applications of intuitionistic fuzzy soft sets. Dinda and Samanta [45] introduced the intuitionistic fuzzy soft relations. Kumar and Bajaj [46] proposed the concept of complex intuitionistic fuzzy soft sets, which are parametric in nature. On the other hand, complex fuzzy set theory and complex intuitionistic fuzzy set theory are independent of the parameterization technique. Complex intuitionistic fuzzy soft sets are used to solve the multicriteria decision-making problem with parameterization tools. Yang et al. [47] proposed the concept of a picture fuzzy soft set. A picture fuzzy soft set is a hybrid model of a picture fuzzy set and a soft set. Khan et al. [48] investigated the applications of generalized picture fuzzy soft sets. Jan et al. [49] introduced the multivalued picture fuzzy soft sets and their applications in group decision-making problems. Močkoř and Hurtik [50] introduced the concept of fuzzy soft relations with image processing applications. Shanthi et al. [51] investigated the concept of a complex picture fuzzy soft set. Complex picture fuzzy soft sets are used to solve the multicriteria decision-making problem with parameterization tools.

In fuzzy soft set theory, the concept of a complex picture fuzzy soft set is a powerful tool for dealing with ambiguity and uncertainty. For complex picture fuzzy soft sets, however, the concept of relations has yet to be determined. The concept of complex picture fuzzy soft relations is introduced in this paper. Complex picture fuzzy soft relations are defined using a novel definition of the Cartesian product of two complex picture fuzzy soft sets. In addition, the types of complex picture fuzzy soft relations are explained, including complex picture fuzzy soft reflexive relations, complex picture fuzzy soft irreflexive relations, complex picture fuzzy soft symmetric relations, complex picture fuzzy soft antisymmetric relations, complex picture fuzzy soft asymmetric relations, complex picture fuzzy soft complete relations, complex picture fuzzy soft transitive relations, complex picture fuzzy soft equivalence relations, complex picture fuzzy soft partial-order relations, complex picture fuzzy soft strict-order relations, complex picture fuzzy soft preorder relations, and complex picture fuzzy soft equivalence classes. Each definition of complex picture fuzzy soft relations is defined with examples. Furthermore, several outcomes are proven for the types of complex picture fuzzy soft relations. The introduced complex picture fuzzy soft relations are preferable to predefined structures of soft sets, fuzzy soft sets, complex fuzzy soft sets, intuitionistic fuzzy soft sets, complex intuitionistic fuzzy soft sets, and picture fuzzy soft sets. Complex picture fuzzy soft sets discuss membership, abstinence, and non-membership degrees. The real term of each of the complex-valued functions is called the amplitude, and the imaginary term is called the phase term. This structure has the ability to solve the multidimensional problems of uncertain natures. Since the importance of generative adversarial networks and modern graphics is well known, as argued earlier, the primary goal of this article is to use complex picture fuzzy soft relations to select the best generative adversarial networks. Experts have suggested a number of parameters, and in this study, we use those suggestions and parameters to select the best generative adversarial networks.

The remainder of this paper is organized as follows. Section 1 consists of an Introduction. Section 2 discusses some previously defined structures. Section 3 introduces the novel concepts of complex picture fuzzy soft relations, the Cartesian product between two complex picture fuzzy soft sets, the types of complex picture fuzzy soft relations, and some related results. Section 4 proposes an application of complex picture fuzzy soft relations for the study and review of generative adversarial networks. Section 5 presents a comparative analysis of the proposed structure with existing frameworks. In Section 6, the article is finished with concluding remarks.

2. Preliminaries

In this section, we explain some predefined concepts of fuzzy algebra such as complex fuzzy set, soft set, soft relation, fuzzy soft set, complex fuzzy soft set, complex intuitionistic fuzzy soft set, intuitionistic fuzzy soft set, complex intuitionistic fuzzy soft set, picture fuzzy soft set, and complex picture fuzzy soft set.

Definition 1 ([7]). *Let $\dot{X}$ be a universal set; then, a complex fuzzy set ($\mathcal{A}$ on $\dot{X}$) with mappings ($\mathscr{r}_{\mathfrak{m}}, \mathfrak{s}_{\mathfrak{m}} : \dot{X} \to [0,1]$) is expressed as:*

$$\mathcal{A} = \left\{ \left(\underline{k}, \mathscr{r}_{\mathfrak{m}}(\underline{k}) e^{(\mathfrak{s}_{\mathfrak{m}}(\underline{k}))2(2\pi i)} \right) : \underline{k} \in \dot{X} \right\}$$

where $\mathscr{r}_{\mathfrak{m}}$ and $\mathfrak{s}_{\mathfrak{m}}$ are the amplitude term and phase term of the degree of membership, respectively.

Definition 2 ([14]). *Let $\dot{X}$ be a universal set and $\dot{\grave{E}}$ be a set of parameters. Let $\mathcal{P}(\dot{X})$ denote the power set of $\dot{X}$ and $\mathcal{A} \subseteq \dot{\grave{E}}$. Then, a soft set $(\mathcal{F}, \dot{\grave{E}})$ with a mapping ($\mathcal{F} : \mathcal{A} \to \mathcal{P}(\dot{X})$) represented by a set of ordered pairs is expressed as:*

$$\mathcal{F} = \left\{ (\underline{k}, \mathcal{F}(\underline{k})) : \underline{k} \in \dot{\grave{E}}, \mathcal{F}(\underline{k}) \in \mathcal{P}(\dot{X}) \right\}$$

Example 1. *Assume that $\dot{X} = \{b_1, b_2, b_3, b_4, b_5\}$ is a universal set consisting of a set of five bags under consideration, and $\dot{\grave{E}} = \{\underline{k}_1, \underline{k}_2, \underline{k}_3, \underline{k}_4, \underline{k}_5\}$ is a set of parameters for $\dot{X}$, where each parameter ($\underline{k}_i, i = 1,2,3,4,5$) for beautiful, modern, expensive, very beautiful, or cheap, respectively. Suppose a soft set $(\mathcal{F}, \dot{\grave{E}})$ that describes the attractiveness of the bags, such that*

$$\mathcal{F}(\underline{k}_1) = \{b_1, b_5\} \qquad \mathcal{F}(\underline{k}_2) = \{b_2, b_3, b_5\} \qquad \mathcal{F}(\underline{k}_3) = \{b_1, b_4\}$$
$$\mathcal{F}(\underline{k}_4) = \{b_3, b_4, b_5\} \qquad \mathcal{F}(\underline{k}_5) = \{b_1, b_2, b_5\}$$

Then, the soft set $(\mathcal{F}, \dot{\grave{E}})$ is a parameterized family for $\{\underline{k}_i, i = 1,2,3,4,5\}$.

Definition 3 ([21]). *Let $(\mathcal{F}, \mathcal{A})$ and $(\mathcal{G}, \mathcal{B})$ be two soft sets on $\dot{X}$ and $\mathcal{A}, \mathcal{B} \subseteq \dot{\grave{E}}$. Let $(\mathcal{F}, \mathcal{A}) \times (\mathcal{G}, \mathcal{B}) = (\mathcal{H}, \mathcal{C})$ with a mapping ($\mathcal{H} : \mathcal{C} \to \mathcal{P}(\dot{X})$); then, the Cartesian product of the soft set is denoted and defined as*

$$\mathcal{H}(\ddot{u}, \ddot{v}) = \left\{ \left(\underline{k}_{\ddot{u}}, \dot{t}_{\ddot{v}} \right) : \underline{k}_{\ddot{u}}(\mathcal{F}, \mathcal{A}), \in \dot{t}_{\ddot{v}} \in (\mathcal{G}, \mathcal{B}) \right\}.$$

Definition 4 ([21]). *Let $(\mathcal{F}, \mathcal{A})$ and $(\mathcal{G}, \mathcal{B})$ be two soft sets on $\dot{X}$ and $\mathcal{A}, \mathcal{B} \subseteq \dot{\grave{E}}$. Then, a soft relation ($\underline{R}$) is any subset of the Cartesian product of $(\mathcal{F}, \mathcal{A}) \times (\mathcal{G}, \mathcal{B})$. It is denoted and defined as*

$$\underline{R} = \left\{ \left(\underline{k}_{\ddot{u}}, \dot{t}_{\ddot{v}} \right) : \left(\underline{k}_{\ddot{u}}, \dot{t}_{\ddot{v}} \right) \in (\mathcal{F}, \mathcal{A}) \times (\mathcal{G}, \mathcal{B}) \right\}.$$

Definition 5 ([23]). *Let $\dot{X}$ be a universal set and $\dot{\grave{E}}$ be a set of parameters. Let $P^{\dot{X}}$ denote the set of all fuzzy subsets of $\dot{X}$ and $\mathcal{A} \subseteq \dot{\grave{E}}$. Then, a fuzzy soft set $\left(\mathcal{F}, \dot{\grave{E}}\right)$ with a mapping ($\mathcal{F} : \mathcal{A} \to P^{\dot{X}}$) is represented by the set of ordered pair as*

$$\mathcal{F} = \left\{ (\underline{k}, \mathfrak{m}(\underline{k})) : \underline{k} \in \dot{\grave{E}}, \mathfrak{m}(\underline{k}) \in P^{\dot{X}} \right\}.$$

where $\mathfrak{m}(\underline{k})$ is the degree of membership.

Example 2. *Let $\dot{X}$ be a set of refrigerator companies and $\dot{\grave{E}}$ be a set of parameters. Assume that a fuzzy soft set $\left(\mathcal{F}, \dot{\grave{E}}\right)$ depicts the refrigerator characteristics in relation to some parameter and each membership degree assigned by experts. $\dot{X} = \left\{ \mathring{w}_1, \mathring{w}_2, \mathring{w}_3, \mathring{w}_4 \right\}$, i.e., $\mathring{w}_1 = LG$, $\mathring{w}_2 = Samsung$,*

$\overset{\circ}{w}_3$ = *Orient*, *and* $\overset{\circ}{w}_4$ = *Dawlance*. $\grave{\dot{E}} = \{\underset{\sim}{k}_1, \underset{\sim}{k}_2, \underset{\sim}{k}_3\}$, *i.e.*, $\underset{\sim}{k}_1$ = *design*, $\underset{\sim}{k}_2$ = *beautiful*, *and* $\underset{\sim}{k}_3$ = *digital*.

$$\begin{aligned}\mathcal{F}(\underset{\sim}{k}_1) &= \left\{\overset{\circ}{w}_1 = 0.3, \overset{\circ}{w}_2 = 0.6, \overset{\circ}{w}_3 = 0.7, \overset{\circ}{w}_4 = 0.2\right\}\\ \mathcal{F}(\underset{\sim}{k}_2) &= \left\{\overset{\circ}{w}_1 = 0.2, \overset{\circ}{w}_2 = 0.7, \overset{\circ}{w}_3 = 0.5, \overset{\circ}{w}_4 = 0.6\right\}\\ \mathcal{F}(\underset{\sim}{k}_3) &= \left\{\overset{\circ}{w}_1 = 0.5, \overset{\circ}{w}_2 = 0.3, \overset{\circ}{w}_3 = 0.2, \overset{\circ}{w}_4 = 0.7\right\}\end{aligned}$$

The $\left(\mathcal{F}, \grave{\dot{E}}\right)$ *is a parameterized family of* $\{\mathcal{F}(\underset{\sim}{k}_i), i = 1, 2, 3\}$.

Definition 6 ([28]). ***Let*** $\dot{X}$ ***be a universal set and*** $\grave{\dot{E}}$ ***be a set of parameters.*** $C(P^{\dot{X}})$ *denotes a set of all complex fuzzy subsets of* $\dot{X}$ *and* $\mathcal{A} \subseteq \grave{\dot{E}}$. *Then, a complex fuzzy soft set* $\left(\mathcal{F}, \grave{\dot{E}}\right)$ *with a mapping (* $\mathcal{F} : \mathcal{A} \to C(P^{\dot{X}})$ *) represented by the set of ordered pairs is expressed as*

$$\mathcal{F} = \left\{(\underset{\sim}{k}, \mathfrak{m}_c(\underset{\sim}{k})) : \underset{\sim}{k} \in \grave{\dot{E}}, \mathfrak{m}(\underset{\sim}{k}) \in C(P^{\dot{X}})\right\} = \left\{\left(\underset{\sim}{k}, r_{\mathfrak{m}}(\underset{\sim}{k})e^{\mathfrak{s}_{\mathfrak{m}}(\underset{\sim}{k})2(2\pi i)}\right) : \underset{\sim}{k} \in \grave{\dot{E}}\right\}$$

where $\mathfrak{m}_c(\underset{\sim}{k}) = r_{\mathfrak{m}}(\underset{\sim}{k})e^{(\mathfrak{s}_{\mathfrak{m}}(\underset{\sim}{k}))2(2\pi i)}$, *and* $r_{\mathfrak{m}}(\underset{\sim}{k}), \mathfrak{s}_{\mathfrak{m}}(\underset{\sim}{k}) : \grave{\dot{E}} \to [0, 1]$, *where* $r_{\mathfrak{m}}$ *and* $\mathfrak{s}_{\mathfrak{m}}$ *are the amplitude term and phase term of the degree of membership, respectively.*

Definition 7 ([31]). *Let* $\dot{X}$ *be a universal set. Then, a complex intuitionistic fuzzy soft set (*$\mathcal{F}$ *on* $\dot{X}$*) with a mapping (* $r_{\mathfrak{m}}, r_{\mathfrak{n}}\mathfrak{s}_{\mathfrak{m}}, \mathfrak{s}_{\mathfrak{n}} : \dot{X} \to [0, 1]$ *) is expressed as*

$$\mathcal{F} = \left\{\left(\underset{\sim}{k}, r_{\mathfrak{m}}(\underset{\sim}{k})e^{\mathfrak{s}_{\mathfrak{m}}(\underset{\sim}{k})2(2\pi i)}, r_{\mathfrak{n}}(\underset{\sim}{k})e^{\mathfrak{s}_{\mathfrak{n}}(\underset{\sim}{k})2(2\pi i)}\right) : \underset{\sim}{k} \in \dot{X}\right\}$$

$r_{\mathfrak{m}}(\underset{\sim}{k}) + r_{\mathfrak{n}}(\underset{\sim}{k}) \in [0, 1]$ *and* $\mathfrak{s}_{\mathfrak{m}}(\underset{\sim}{k}) + \mathfrak{s}_{\mathfrak{n}}(\underset{\sim}{k}) \in [0, 1]$, *where* $r_{\mathfrak{m}}, r_{\mathfrak{n}}$ *are the amplitude terms of membership and non-membership degrees, respectively, and* $\mathfrak{s}_{\mathfrak{m}}, \mathfrak{s}_{\mathfrak{n}}$ *are the phase terms of membership and non-membership degrees, respectively.*

Definition 8 ([32]). *Let* $\dot{X}$ *be a universal set and* $\grave{\dot{E}}$ *be a set of parameters.* $P\mathcal{F}^{\dot{X}}$ *denotes the set of all intuitionistic fuzzy subsets of* $\dot{X}$ *and* $\mathcal{A} \subseteq \grave{\dot{E}}$. *Then, an intuitionistic fuzzy soft set* $\left(\mathcal{F}, \grave{\dot{E}}\right)$ *with a mapping (* $\mathcal{F} : \mathcal{A} \to P\mathcal{F}^{\dot{X}}$ *) represented by the set of ordered pair is expressed as*

$$\mathcal{F} = \left\{(\underset{\sim}{k}, \mathfrak{m}(\underset{\sim}{k}), \mathfrak{n}(\underset{\sim}{k})) : \underset{\sim}{k} \in \grave{\dot{E}}, \mathfrak{m}(\underset{\sim}{k}), \mathfrak{n}(\underset{\sim}{k}) \in P\mathcal{F}^{\dot{X}}\right\}$$

where $\mathfrak{m}(\underset{\sim}{k}), \mathfrak{n}(\underset{\sim}{k})$ *are the membership and non-membership degree, respectively.*

Example 3. *From Example 2, assume an intuitionistic fuzzy soft set* $\left(\mathcal{F}, \grave{\dot{E}}\right)$ *describing the characteristics of the refrigerator with respect to some parameter and each membership and non-membership degree assigned by experts.*

$$\begin{aligned}\mathcal{F}(\underset{\sim}{k}_1) &= \left\{\overset{\circ}{w}_1 = (0.3, 0.5), \overset{\circ}{w}_2 = (0.2, 0.7), \overset{\circ}{w}_3 = (0.3, 0.6), \overset{\circ}{w}_4 = (0.2, 0.5)\right\}\\ \mathcal{F}(\underset{\sim}{k}_2) &= \left\{\overset{\circ}{w}_1 = (0.2, 0.3), \overset{\circ}{w}_2 = (0.1, 0.4), \overset{\circ}{w}_3 = (0.3, 0.5), \overset{\circ}{w}_4 = (0.4, 0.6)\right\}\\ \mathcal{F}(\underset{\sim}{k}_3) &= \left\{\overset{\circ}{w}_1 = (0.1, 0.3), \overset{\circ}{w}_2 = (0.2, 0.4), \overset{\circ}{w}_3 = (0.2, 0.5), \overset{\circ}{w}_4 = (0.3, 0.5)\right\}\end{aligned}$$

Then, the intuitionistic fuzzy soft set $\left(\mathcal{F}, \grave{\dot{E}}\right)$ *is a parameterized family, i.e.,*$\{\mathcal{F}(\underset{\sim}{k}_i), i = 1, 2, 3\}$.

Definition 9 ([36]). *Let $\dot{\mathrm{X}}$ be a universal set and $\grave{\dot{\mathrm{E}}}$ be a set of parameters. $C(P\mathcal{F}^{\dot{\mathrm{X}}})$ denotes a set of all complex intuitionistic fuzzy subsets of $\dot{\mathrm{X}}$ and $\mathcal{A} \subseteq \grave{\dot{\mathrm{E}}}$. Then, a complex intuitionistic fuzzy soft set $\left(\mathcal{F}, \grave{\dot{\mathrm{E}}}\right)$ with a mapping ($\mathcal{F} : \grave{\dot{\mathrm{E}}} \to C(P\mathcal{F}^{\dot{\mathrm{X}}})$) represented by the set of order pairs is expressed as*

$$\mathcal{F} = \left\{ (\underline{k}, \mathfrak{m}_c(\underline{k}), \mathfrak{n}_c(\underline{k})) : \underline{k} \in \grave{\dot{\mathrm{E}}}, \mathfrak{m}(\underline{k}), \mathfrak{n}(\underline{k}) \in C(P\mathcal{F}^{\dot{\mathrm{X}}}) \right\} = \left\{ \left(\underline{k}, \mathscr{r}_{\mathfrak{m}}(\underline{k}) e^{\mathfrak{s}_{\mathfrak{m}}(\underline{k}) 2(2\pi i)}, \mathscr{r}_{\mathfrak{n}}(\underline{k}) e^{\mathfrak{s}_{\mathfrak{n}}(\underline{k}) 2(2\pi i)} \right) : \underline{k} \in \grave{\dot{\mathrm{E}}} \right\}$$

where $\mathscr{r}_{\mathfrak{m}}, \mathscr{r}_{\mathfrak{n}}$ are the amplitude terms of membership and non-membership degree, respectively, and $\mathfrak{s}_{\mathfrak{m}}, \mathfrak{s}_{\mathfrak{n}}$ are the phase terms of membership and non-membership degree, respectively.

Definition 10 ([41]). *Let $\dot{\mathrm{X}}$ be a universal set and $\grave{\dot{\mathrm{E}}}$ be the set of parameters. $P\mathcal{F}^{\dot{\mathrm{X}}}$ denotes the set of all picture fuzzy subsets of $\dot{\mathrm{X}}$ and $\mathcal{A} \subseteq \grave{\dot{\mathrm{E}}}$. Then, a picture fuzzy soft set $(\mathcal{F}, \grave{\dot{\mathrm{E}}})$ with a mapping ($\mathcal{F} : \mathcal{A} \to P\mathcal{F}^{\dot{\mathrm{X}}}$) represented by the set of ordered pairs is expressed as*

$$\mathcal{F} = \left\{ (\underline{k}, \mathfrak{m}(\underline{k}), \mathfrak{n}(\underline{k}), \breve{\alpha}(\underline{k})) : \underline{k} \in \grave{\dot{\mathrm{E}}}, \mathfrak{m}(\underline{k}), \mathfrak{n}(\underline{k}), \breve{\alpha}(\underline{k}) \in P\mathcal{F}^{\dot{\mathrm{X}}} \right\}$$

where $\mathfrak{m}(\underline{k}), \mathfrak{n}(\underline{k}), \breve{\alpha}(\underline{k})$ are the membership, abstinence, and non-membership degree, respectively.

Example 4. *From Example 2, assume a picture fuzzy soft set $\left(\mathcal{F}, \grave{\dot{\mathrm{E}}}\right)$ that describes the characteristics of a refrigerator with respect to some parameter, and each membership, abstinence, and non-membership degree assigned by experts.*

$$\mathcal{F}(\underline{k}_1) = \left\{ \mathring{w}_1 = (0.1, 0.3, 0.5), \mathring{w}_2 = (0.2, 0.3, 0.4,), \mathring{w}_3 = (0.3, 0.2, 0.5), \mathring{w}_4 = (0.2, 0.4, 0.1) \right\}$$
$$\mathcal{F}(\underline{k}_2) = \left\{ \mathring{w}_1 = (0.2, 0.3, 0.2), \mathring{w}_2 = (0.1, 0.3, 0.4), \mathring{w}_3 = (0.3, 0.4, 0.3), \mathring{w}_4 = (0.1, 0.3, 0.4) \right\}$$
$$\mathcal{F}(\underline{k}_3) = \left\{ \mathring{w}_1 = (0.1, 0.3, 0.2), \mathring{w}_2 = (0.2, 0.4, 0.1), \mathring{w}_3 = (0.2, 0.1, 0.4), \mathring{w}_4 = (0.3, 0.1, 0.5) \right\}$$

Then, the picture fuzzy soft set $\left(\mathcal{F}, \grave{\dot{\mathrm{E}}}\right)$ is a parameterized family, i.e., $\{\mathcal{F}(\underline{k}_i), i = 1, 2, 3\}$.

Definition 11 ([45]). *Let $\dot{\mathrm{X}}$ be a universal set and $\grave{\dot{\mathrm{E}}}$ be a set of parameters. $C(P\mathcal{F}^{\dot{\mathrm{X}}})$ denotes the set of all complex picture fuzzy subsets of $\dot{\mathrm{X}}$ and $\mathcal{A} \subseteq \grave{\dot{\mathrm{E}}}$. Then, a complex picture fuzzy soft set $\left(\mathcal{F}, \grave{\dot{\mathrm{E}}}\right)$ with a mapping ($\mathcal{F} : \grave{\dot{\mathrm{E}}} \to C(P\mathcal{F}^{\dot{\mathrm{X}}})$) represented by the set of order pairs is expressed as*

$$\mathcal{F} = \left\{ (\underline{k}, \mathfrak{m}_c(\underline{k}), \breve{\alpha}(\underline{k}), \mathfrak{n}_c(\underline{k})) : \underline{k} \in \grave{\dot{\mathrm{E}}}, \mathfrak{m}(\underline{k}), \breve{\alpha}(\underline{k}), \mathfrak{n}(\underline{k}) \in C(P\mathcal{F}^{\dot{\mathrm{X}}}) \right\}$$
$$= \left\{ \left(\underline{k}, \mathscr{r}_{\mathfrak{m}}(\underline{k}) e^{\mathfrak{s}_{\mathfrak{m}}(\underline{k}) 2(2\pi i)}, \mathscr{r}_{\breve{\alpha}}(\underline{k}) e^{\mathfrak{s}_{\breve{\alpha}}(\underline{k}) 2(2\pi i)}, \mathscr{r}_{\mathfrak{n}}(\underline{k}) e^{\mathfrak{s}_{\mathfrak{n}}(\underline{k}) 2(2\pi i)} \right) : \underline{k} \in \grave{\dot{\mathrm{E}}} \right\}$$

where $\mathscr{r}_{\mathfrak{m}}, \mathscr{r}_{\breve{\alpha}}, \mathscr{r}_{\mathfrak{n}}$ are amplitude terms of membership, abstinence, and non-membership degree, respectively, and $\mathfrak{s}_{\mathfrak{m}}, \mathfrak{s}_{\breve{\alpha}}, \mathfrak{s}_{\mathfrak{n}}$ are the phase terms of membership, abstinence, and non-membership degree, respectively.

3. Main Results

In this section, we explain the ideas of the Cartesian product of two complex picture fuzzy soft sets, complex picture fuzzy soft relations, and different types of these relations. Moreover, several examples and useful results are also described.

Definition 12. *Suppose that* $(\mathcal{F},\mathcal{A})$ *and* $(\mathcal{G},\mathcal{B})$ *are two complex picture fuzzy soft sets on* $\dot{X}$ *and* $\mathcal{A},\mathcal{B} \subseteq \grave{\dot{E}}$. *Let* $(\mathcal{F},\mathcal{A}) \times (\mathcal{G},\mathcal{B}) = (\mathcal{H},\mathcal{C})$ *with a mapping* $(\mathcal{H}:\mathcal{C} \to \mathcal{C}(P\mathcal{F}^{\dot{X}}))$ *and* $\mathcal{H}(\ddot{u},\ddot{v}) = \mathcal{F}(\ddot{u}) \times \mathcal{G}(\ddot{v})$. *Then, the Cartesian product of complex picture fuzzy soft sets is denoted by*

$$\mathcal{A} = \left\{ \left(\underline{k}, r_{\mathfrak{m}}^{\mathcal{A}}(\underline{k}) e^{\mathfrak{s}_{\mathfrak{m}}^{\mathcal{A}}(\underline{k})2(2\pi i)}, r_{\breve{\alpha}}^{\mathcal{A}}(\underline{k}) e^{\mathfrak{s}_{\breve{\alpha}}^{\mathcal{A}}(\underline{k})2(2\pi i)}, r_{\mathfrak{n}}^{\mathcal{A}}(\underline{k}) e^{\mathfrak{s}_{\mathfrak{n}}^{\mathcal{A}}(\underline{k})2(2\pi i)} \right) : \underline{k} \in \grave{\dot{E}} \right\} \text{ and}$$
$$\mathcal{B} = \left\{ \left(f, r_{\mathfrak{m}}^{\mathcal{B}}(f) e^{\mathfrak{s}_{\mathfrak{m}}^{\mathcal{B}}(f)2(2\pi i)}, r_{\breve{\alpha}}^{\mathcal{B}}(f) e^{\mathfrak{s}_{\breve{\alpha}}^{\mathcal{B}}(f)2(2\pi i)}, r_{\mathfrak{n}}^{\mathcal{B}}(f) e^{\mathfrak{s}_{\mathfrak{n}}^{\mathcal{B}}(f)2(2\pi i)} \right) : f \in \grave{\dot{E}} \right\}$$

and defined as,

$$(\mathcal{H},\mathcal{C}) = \mathcal{A} \times \mathcal{B} = \left\{ \begin{pmatrix} (\underline{k},f), r_{\mathfrak{m}}^{\mathcal{A}\times\mathcal{B}}(\underline{k},f) e^{\mathfrak{s}_{\mathfrak{m}}^{\mathcal{A}\times\mathcal{B}}(\underline{k},f)2(2\pi i)}, r_{\breve{\alpha}}^{\mathcal{A}\times\mathcal{B}}(\underline{k},f) e^{\mathfrak{s}_{\breve{\alpha}}^{\mathcal{A}\times\mathcal{B}}(\underline{k},f)2(2\pi i)}, \\ r_{\mathfrak{n}}^{\mathcal{A}\times\mathcal{B}}(\underline{k},f) e^{\mathfrak{s}_{\mathfrak{n}}^{\mathcal{A}\times\mathcal{B}}(\underline{k},f)2(2\pi i)} \end{pmatrix} : \underline{k} \in \mathcal{A}, f \in \mathcal{B} \right\}$$
$$\text{Where} \left\{ \begin{matrix} r_{\mathfrak{m}}^{\mathcal{A}\times\mathcal{B}}(\underline{k},f) = min\{ r_{\mathfrak{m}}^{\mathcal{A}}(\underline{k}), r_{\mathfrak{m}}^{\mathcal{B}}(f)\}, r_{\breve{\alpha}}^{\mathcal{A}\times\mathcal{B}}(\underline{k},f) = min\{ r_{\breve{\alpha}}^{\mathcal{A}}(\underline{k}), r_{\breve{\alpha}}^{\mathcal{B}}(f)\}, \\ \text{and } r_{\mathfrak{n}}^{\mathcal{A}\times\mathcal{B}}(\underline{k},f) = max\{ r_{\mathfrak{n}}^{\mathcal{A}}(\underline{k}), r_{\mathfrak{n}}^{\mathcal{B}}(f)\}, \end{matrix} \right\}$$
$$\left\{ \begin{matrix} \mathfrak{s}_{\mathfrak{m}}^{\mathcal{A}\times\mathcal{B}}(\underline{k},f) = min\{ \mathfrak{s}_{\mathfrak{m}}^{\mathcal{A}}(\underline{k}), \mathfrak{s}_{\mathfrak{m}}^{\mathcal{B}}(f)\}, \mathfrak{s}_{\breve{\alpha}}^{\mathcal{A}\times\mathcal{B}}(\underline{k},f) = min\{ \mathfrak{s}_{\breve{\alpha}}^{\mathcal{A}}(\underline{k}), \mathfrak{s}_{\breve{\alpha}}^{\mathcal{B}}(f)\}, \\ \text{and } \mathfrak{s}_{\mathfrak{n}}^{\mathcal{A}\times\mathcal{B}}(\underline{k},f) = max\{ \mathfrak{s}_{\mathfrak{n}}^{\mathcal{A}}(\underline{k}), \mathfrak{s}_{\mathfrak{n}}^{\mathcal{B}}(f)\} \end{matrix} \right\}.$$

Example 5. *Suppose the universal set* $\dot{X} = \{\hbar_1, \hbar_2, \hbar_3\}$ *consists of three laptop companies, i.e.,* $\hbar_1 =$ *Dell,* $\hbar_2 =$ *Toshiba, and* $\hbar_3 =$ *Lenovo, and there are three parameters* ($\grave{\dot{E}} = \{\underline{k}_1, \underline{k}_2, \underline{k}_3\}$)*, i.e.,* $\underline{k}_1 =$ *expensive,* $\underline{k}_2 =$ *fast, and* $\underline{k}_3 =$ *cheap. Let* $(\mathcal{F},\mathcal{A})$ *and* $(\mathcal{G},\mathcal{B})$ *be two complex picture fuzzy soft sets on* $\dot{X}$*, as shown below;*

$$(\mathcal{F},\mathcal{A}) = \left\{ \begin{matrix} \left(\underline{k}_1, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.4(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left(\underline{k}_2, \begin{pmatrix} 0.3e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right), \\ \left(\underline{k}_3, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.4e^{0.6(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.4e^{0.5(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.2e^{0.4(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.3(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right) \end{matrix} \right\}$$

and

$$(\mathcal{G},\mathcal{B}) = \left\{ \begin{matrix} \left(\underline{k}_1, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.4e^{0.5(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.6e^{0.2(2\pi i)}, \\ 0.2e^{0.4(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.2e^{0.4(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left(\underline{k}_2, \begin{pmatrix} 0.4e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.1e^{0.5(2\pi i)} \end{pmatrix} \right), \\ \left(\underline{k}_3, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.4(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.2e^{0.4(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.4e^{0.6(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix} \right) \end{matrix} \right\}$$

In the above observations, the first three values of each parameter show the membership, abstinence, and non-membership degree of each company, and the fourth value shows the general value, which is known as the degree of belongingness. Each row represents the parametric observations.

Then, their Cartesian product of $(\mathcal{F},\mathcal{A})$ and $(\mathcal{G},\mathcal{B})$ is defined as

$$
(\mathcal{H},\mathcal{C}) = \left\{ \begin{array}{l}
\left((\underline{k}_1,\underline{k}_1), \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.2e^{0.4(2\pi i)}, 0.4e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\
\left((\underline{k}_1,\underline{k}_2), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.5(2\pi i)} \end{pmatrix} \right), \\
\left((\underline{k}_1,\underline{k}_3), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.4(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\
\left((\underline{k}_2,\underline{k}_1), \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\
\left((\underline{k}_2,\underline{k}_2), \begin{pmatrix} 0.3e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.5(2\pi i)} \end{pmatrix} \right), \\
\left((\underline{k}_2,\underline{k}_3), \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.2(2\pi i)} \end{pmatrix} \right), \\
\left((\underline{k}_3,\underline{k}_1), \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.4e^{0.5(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.4(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.2e^{0.4(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right), \\
\left((\underline{k}_3,\underline{k}_2), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.2e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.5(2\pi i)} \end{pmatrix} \right), \\
\left((\underline{k}_3,\underline{k}_3), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.4(2\pi i)}, 0.4e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.2e^{0.4(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right)
\end{array} \right\}
$$

Definition 13. *The complex picture fuzzy soft relations ($\underline{R}$) and complex picture fuzzy soft relations are a subset of any Cartesian product of two complex picture fuzzy soft sets.*

Example 6. *Consider two complex picture fuzzy soft sets in Example 5 (($\mathcal{F},\mathcal{A}$) and ($\mathcal{G},\mathcal{B}$)) in the same universe ($\dot{\mathrm{X}}$). Their Cartesian product is $(\mathcal{H},\mathcal{C})$, as calculated in the previous example. Then, the subset ($\underline{R}$) of $(\mathcal{F},\mathcal{A}) \times (\mathcal{G},\mathcal{B})$ is a complex picture fuzzy soft relation expressed as*

$$
\underline{R} = \left\{ \begin{array}{l}
\left((\underline{k}_1,\underline{k}_1), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.4(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\
\left((\underline{k}_1,\underline{k}_3), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.4(2\pi i)}, 0.4e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right), \\
\left((\underline{k}_2,\underline{k}_1), \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\
\left((\underline{k}_3,\underline{k}_2), \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right), \\
\left((\underline{k}_3,\underline{k}_3), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.4e^{0.6(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.4e^{0.5(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.2e^{0.4(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.3(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right)
\end{array} \right\}
$$

Definition 14. *Suppose that $(\mathcal{F},\mathcal{A})$ is a complex picture fuzzy soft set on $\dot{\mathrm{X}}$, and*

$$\underline{R} = \left(\left((\underline{k},f)[r_{\mathfrak{m}}(\underline{k},f)]e^{\mathfrak{s}_{\mathfrak{m}}(\underline{k},f)(2\pi i)}, (\underline{k},f)[r_{\check{\alpha}}(\underline{k},f)]e^{\mathfrak{s}_{\check{\alpha}}(\underline{k},f)(2\pi i)}, (\underline{k},f)[r_{\mathfrak{n}}(\underline{k},f)]e^{\mathfrak{s}_{\mathfrak{n}}(\underline{k},f)(2\pi i)} \right) (\underline{k},f) \in \underline{R} \right)$$

is a complex picture fuzzy soft relation on $(\mathcal{F},\mathcal{A})$. Then the inverse of the complex picture fuzzy soft relation is denoted by $\underline{R}^{-1}$ and defined as

$$\underline{R}^{-1} = \left(\left((f,\underline{k})[r_{\mathfrak{m}}(f,\underline{k})]e^{\mathfrak{s}_{\mathfrak{m}}(f,\underline{k})(2\pi i)}, (f,\underline{k})[r_{\check{\alpha}}(f,\underline{k})]e^{\mathfrak{s}_{\check{\alpha}}(f,\underline{k})(2\pi i)}, (f,\underline{k})[r_{\mathfrak{n}}(f,\underline{k})]e^{\mathfrak{s}_{\mathfrak{n}}(f,\underline{k})(2\pi i)} \right) (f,\underline{k}) \in \underline{R}^{-1} \right)$$

Example 7. *Consider the complex picture fuzzy soft relation ($\underline{R}$) in Example 6. Then, its inverse is calculated as*

$$\underset{\sim}{R}^{-1} = \left\{ \begin{array}{l} \left((\underset{\sim}{k}_1, \underset{\sim}{k}_1), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.4(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left((\underset{\sim}{k}_3, \underset{\sim}{k}_1), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.4(2\pi i)}, 0.4e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right), \\ \left((\underset{\sim}{k}_1, \underset{\sim}{k}_2), \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1(2\pi i)} \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)} \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left((\underset{\sim}{k}_2, \underset{\sim}{k}_3), \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right), \\ \left((\underset{\sim}{k}_3, \underset{\sim}{k}_3), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.4e^{0.6(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.4e^{0.5(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.2e^{0.4(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.3(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right) \end{array} \right\}$$

NOTE: For convenience, throughout this article, $\underset{\sim}{k}$ and $(\underset{\sim}{k}, f)$ are used to denote,

$$\left(\left(\underset{\sim}{k}, \left[\mathscr{r}_{(\mathcal{F},\mathcal{A})\mathfrak{m}}(\underset{\sim}{k})\right] e^{[\mathfrak{s}_{(\mathcal{F},\mathcal{A})\mathfrak{m}}(\underset{\sim}{k})(2\pi i)}, \left[\mathscr{r}_{(\mathcal{F},\mathcal{A})\breve{\mathfrak{a}}}(\underset{\sim}{k})\right] e^{[\mathfrak{s}_{(\mathcal{F},\mathcal{A})\breve{\mathfrak{a}}}(\underset{\sim}{k})(2\pi i)}, \left[\mathscr{r}_{(\mathcal{F},\mathcal{A})\mathfrak{n}}(\underset{\sim}{k})\right] e^{[\mathfrak{s}_{(\mathcal{F},\mathcal{A})\mathfrak{n}}(\underset{\sim}{k})(2\pi i)} \right) \right)$$

and

$$\begin{pmatrix} (\underset{\sim}{k}, f), \left[\mathscr{r}_{((\mathcal{F},\mathcal{A})\times(\mathcal{F},\mathcal{A}))\mathfrak{m}}(\underset{\sim}{k}, f)\right] e^{[\mathfrak{s}_{((\mathcal{F},\mathcal{A})\times(\mathcal{F},\mathcal{A}))\mathfrak{m}}(\underset{\sim}{k}, f)](2\pi i)}, \\ \left[\mathscr{r}_{((\mathcal{F},\mathcal{A})\times(\mathcal{F},\mathcal{A}))\breve{\mathfrak{a}}}(\underset{\sim}{k}, f)\right] e^{[\mathfrak{s}_{((\mathcal{F},\mathcal{A})\times(\mathcal{F},\mathcal{A}))\breve{\mathfrak{a}}}(\underset{\sim}{k}, f)](2\pi i)}, \left[\mathscr{r}_{((\mathcal{F},\mathcal{A})\times(\mathcal{F},\mathcal{A}))\mathfrak{n}}(\underset{\sim}{k}, f)\right] e^{[\mathfrak{s}_{((\mathcal{F},\mathcal{A})\times(\mathcal{F},\mathcal{A}))\mathfrak{n}}(\underset{\sim}{k}, f)](2\pi i)} \end{pmatrix},$$

respectively, unless otherwise specified.

Definition 15. *For an initial universal set ($\dot{X}$), a set $(\mathcal{F}, \mathcal{A})$ is a complex picture fuzzy soft set, and $\underset{\sim}{R}_1$ is a complex picture fuzzy soft relation on $(\mathcal{F}, \mathcal{A})$; then,*

- ❖ *$\underset{\sim}{R}_1$ is said to be complex picture fuzzy soft reflexive relation on $(\mathcal{F}, \mathcal{A})$ if $(\underset{\sim}{k}, \underset{\sim}{k}) \in \underset{\sim}{R}_1$ for all $\underset{\sim}{k} \in (\mathcal{F}, \mathcal{A})$;*
- ❖ *$\underset{\sim}{R}_1$ is said to be complex picture fuzzy soft irreflexive relation on $(\mathcal{F}, \mathcal{A})$ if $(\underset{\sim}{k}, \underset{\sim}{k}) \notin \underset{\sim}{R}_1$ for all $\underset{\sim}{k} \in (\mathcal{F}, \mathcal{A})$;*
- ❖ *$\underset{\sim}{R}_1$ is said to be complex picture fuzzy soft symmetric relation on $(\mathcal{F}, \mathcal{A})$ if for all $\underset{\sim}{k}, f \in (\mathcal{F}, \mathcal{A}), (\underset{\sim}{k}, f) \in \underset{\sim}{R}_1$ then $f, \underset{\sim}{k} \in \underset{\sim}{R}_1$;*
- ❖ *$\underset{\sim}{R}_1$ is said to be complex picture fuzzy soft antisymmetric relation on $(\mathcal{F}, \mathcal{A})$ if for all $\underset{\sim}{k}, f \in (\mathcal{F}, \mathcal{A}), (\underset{\sim}{k}, f) \in \underset{\sim}{R}_1$ and $(f, \underset{\sim}{k}) \in \underset{\sim}{R}_1$; then $\underset{\sim}{k} = f$;*
- ❖ *$\underset{\sim}{R}_1$ is said to be complex picture fuzzy soft asymmetric relation on $(\mathcal{F}, \mathcal{A})$ if for all $\underset{\sim}{k}, f \in (\mathcal{F}, \mathcal{A}), (\underset{\sim}{k}, f) \in \underset{\sim}{R}_1$, and $(f, \underset{\sim}{k}) \in \underset{\sim}{R}_1$; then, $(\underset{\sim}{k}, f) \notin \underset{\sim}{R}_1$;*
- ❖ *$\underset{\sim}{R}_1$ is said to be complex picture fuzzy soft complete relation on $(\mathcal{F}, \mathcal{A})$ if for all $\underset{\sim}{k}, f \in \underset{\sim}{R}_1, (\underset{\sim}{k}, f) \in \underset{\sim}{R}_1$, or $(f, \underset{\sim}{k}) \in \underset{\sim}{R}_1$;*
- ❖ *$\underset{\sim}{R}_1$ is said to be complex picture fuzzy soft transitive relation on $(\mathcal{F}, \mathcal{A})$ if for all $\underset{\sim}{k}, f, g \in (\mathcal{F}, \mathcal{A}), (\underset{\sim}{k}, f) \in \underset{\sim}{R}_1$, and $(f, g) \in \underset{\sim}{R}_1$; then, $(\underset{\sim}{k}, g) \in \underset{\sim}{R}_1$;*
- ❖ *$\underset{\sim}{R}_1$ is said to be a complex picture fuzzy soft equivalence relation on $(\mathcal{F}, \mathcal{A})$ if $\underset{\sim}{R}_1$ is a complex picture fuzzy soft reflexive relation, complex picture fuzzy soft symmetric relation, or complex picture fuzzy soft transitive relation on $(\mathcal{F}, \mathcal{A})$;*
- ❖ *$\underset{\sim}{R}_1$ is said to be a complex picture fuzzy soft preorder relation on $(\mathcal{F}, \mathcal{A})$ if $\underset{\sim}{R}_1$ is a complex picture fuzzy soft reflexive relation and complex picture fuzzy soft transitive relation on $(\mathcal{F}, \mathcal{A})$;*
- ❖ *$\underset{\sim}{R}_1$ is said to be a complex picture fuzzy soft strict-order relation on $(\mathcal{F}, \mathcal{A})$ if $\underset{\sim}{R}_1$ is a complex picture fuzzy soft irreflexive relation and complex picture fuzzy soft transitive relation on $(\mathcal{F}, \mathcal{A})$;*
- ❖ *$\underset{\sim}{R}_1$ is said to be a complex picture fuzzy soft partial-order relation on $(\mathcal{F}, \mathcal{A})$ if $\underset{\sim}{R}_1$ is a complex picture fuzzy soft preorder relation and a complex picture fuzzy soft antisymmetric relation on $(\mathcal{F}, \mathcal{A})$;*

- ❖ $\underline{R}_1$ *is said to be a complex picture fuzzy soft linear-order relation on* $(\mathcal{F},\mathcal{A})$ *if* $\underline{R}_1$ *is a complex picture fuzzy soft partial-order relation and complex picture fuzzy soft complete relation on* $(\mathcal{F},\mathcal{A})$.

Example 8. *Assume that* $(\mathcal{F},\mathcal{A})$ *is a complex picture fuzzy soft set on* $\dot{X}$ *defined as*

$$(\mathcal{F},\mathcal{A})=\left\{\begin{array}{l}\left(\underline{k}_1,\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.4(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.3e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix}\right),\\\left(\underline{k}_2,\begin{pmatrix}0.3e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.5e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\\left(\underline{k}_3,\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.4e^{0.6(2\pi i)},0.1e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.4e^{0.5(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.2e^{0.4(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.3(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right)\end{array}\right\}$$

Then, the Cartesian product of $(\mathcal{F},\mathcal{A})\times(\mathcal{F},\mathcal{A})$ *is defined as*

$$(\mathcal{F},\mathcal{A})\times(\mathcal{F},\mathcal{A})=\left\{\begin{array}{l}\left((\underline{k}_1,\underline{k}_1),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.4(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.3e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix}\right),\\\left((\underline{k}_1,\underline{k}_2),\begin{pmatrix}0.3e^{0.2(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.1(2\pi i)}\\0.3e^{0.2(2\pi i)},0.3e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)}\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix}\right),\\\left((\underline{k}_1,\underline{k}_3),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.1(2\pi i)},\\0.3e^{0.4(2\pi i)},0.4e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.2(2\pi i)},\\0.2e^{0.2(2\pi i)},0.3e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\\left((\underline{k}_2,\underline{k}_1),\begin{pmatrix}0.3e^{0.2(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.3e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix}\right),\\\left((\underline{k}_2,\underline{k}_2),\begin{pmatrix}0.3e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.5e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\\left((\underline{k}_2,\underline{k}_3),\begin{pmatrix}0.3e^{0.2(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.2e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\\left((\underline{k}_3,\underline{k}_1),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.1(2\pi i)},\\0.3e^{0.4(2\pi i)},0.4e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.2(2\pi i)},\\0.2e^{0.2(2\pi i)},0.3e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\\left((\underline{k}_3,\underline{k}_2),\begin{pmatrix}0.3e^{0.2(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.2e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\\left((\underline{k}_3,\underline{k}_3),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.4e^{0.6(2\pi i)},0.1e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.4e^{0.5(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.2e^{0.4(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.3(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right)\end{array}\right\}$$

i. *The complex picture fuzzy soft reflexive relation* ($\underline{R}_1$) *is*

$$\underline{R}_1=\left\{\begin{array}{l}\left((\underline{k}_1,\underline{k}_1),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.4(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.3e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix}\right),\\\left((\underline{k}_1,\underline{k}_3),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.1(2\pi i)},\\0.3e^{0.4(2\pi i)},0.4e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.2(2\pi i)},\\0.2e^{0.2(2\pi i)},0.3e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\\left((\underline{k}_2,\underline{k}_2),\begin{pmatrix}0.3e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.5e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\\left((\underline{k}_3,\underline{k}_3),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.4e^{0.6(2\pi i)},0.1e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.4e^{0.5(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.2e^{0.4(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.3(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\\left((\underline{k}_3,\underline{k}_2),\begin{pmatrix}0.3e^{0.2(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.2e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right)\end{array}\right\}$$

ii. *The complex picture fuzzy soft irreflexive relation* ($\underline{R}_2$) *is*

$$\underline{R}_2=\left\{\begin{array}{l}\left((\underline{k}_1,\underline{k}_2),\begin{pmatrix}0.3e^{0.2(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.1(2\pi i)}\\0.3e^{0.2(2\pi i)},0.3e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)}\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix}\right),\\\left((\underline{k}_2,\underline{k}_3),\begin{pmatrix}0.3e^{0.2(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.2e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\\left((\underline{k}_3,\underline{k}_2),\begin{pmatrix}0.3e^{0.2(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.2e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right)\end{array}\right\}$$

Example 9. *Suppose the universal set $\dot{X} = \{\hbar_1, \hbar_2, \hbar_3\}$ consists of three mobile companies, i.e., $\hbar_1 =$ Samsung, $\hbar_2 =$ Apple, and $\hbar_3 =$ Nokia, and there are three parameters ($\grave{\bar{E}} = \{f_1, f_2, f_3\}$), i.e., $f_1 =$ expensive, $f_2 =$ fast, and $f_3 =$ cheap. Let $(\mathcal{G}, \mathcal{B})$ be a complex picture fuzzy soft set on $\dot{X}$, and let their corresponding membership, abstinence, and non-membership be defined as follows:*

$$(\mathcal{G},\mathcal{B}) = \left\{ \begin{array}{l} \left(f_1, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.4e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.4e^{0.2(2\pi i)}, 0.1e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.4e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left(f_2, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3(2\pi i)}, \\ 0.2e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)} \end{pmatrix} \right), \\ \left(f_3, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.4e^{0.3(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix} \right) \end{array} \right\}$$

In the above observations, the first three values of each parameter show the membership, abstinence, and non-membership of each company, and the fourth value shows the general value, which is known as the degree of belongingness. Each row represents the parametric observations.

Then, their Cartesian product is $(\mathcal{G},\mathcal{B}) \times (\mathcal{G},\mathcal{B}) = (\mathcal{H},\mathcal{C})$, defined as

$$(\mathcal{H},\mathcal{C}) = \left\{ \begin{array}{l} \left((f_1,f_1), \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.4e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.4e^{0.2(2\pi i)}, 0.1e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.4e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left((f_1,f_2), \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)} \\ 0.2e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)} \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left((f_1,f_3), \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.4e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left((f_2,f_1), \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)} \\ 0.2e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)} \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left((f_2,f_2), \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3(2\pi i)}, \\ 0.2e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)} \end{pmatrix} \right), \\ \left((f_2,f_3), \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left((f_3,f_1), \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.4e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left((f_3,f_2), \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left((f_3,f_3), \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.4e^{0.3(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix} \right) \end{array} \right\}$$

i. The complex picture fuzzy soft symmetric relation ($\underset{\sim}{R}_1$) on $(\mathcal{G},\mathcal{B})$ is

$$\underset{\sim}{R}_1 = \left\{ \begin{array}{l} \left(\left(f_1,f_2 \right), \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)} \\ 0.2e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)} \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left(\left(f_2,f_1 \right), \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)} \\ 0.2e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)} \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left(\left(f_2,f_3 \right), \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left(\left(f_3,f_2 \right), \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right) \end{array} \right\}$$

ii. The complex picture fuzzy soft antisymmetric relation ($\underset{\sim}{R}_2$) on $(\mathcal{G},\mathcal{B})$ is

$$\underline{R}_2 = \left\{ \begin{array}{l} \left(\left(f_1, f_1 \right), \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.4e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.4e^{0.2(2\pi i)}, 0.1e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.4e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left(\left(f_2, f_2 \right), \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3(2\pi i)}, \\ 0.2e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)} \end{pmatrix} \right), \\ \left(\left(f_3, f_3 \right), \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.4e^{0.3(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix} \right) \end{array} \right\}$$

iii. The complex picture fuzzy soft asymmetric relation $(\underline{R}_3)$ on $(\mathcal{G}, \mathcal{B})$ is

$$\underline{R}_3 = \left\{ \begin{array}{l} \left(\left(f_1, f_2 \right), \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)} \\ , 0.2e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)} \\ , 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left(\left(f_1, f_3 \right), \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.4e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left(\left(f_2, f_3 \right), \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right) \end{array} \right\}$$

iv. The complex picture fuzzy soft transitive relation $(\underline{R}_1)$ on $(\mathcal{G}, \mathcal{B})$ is

$$\underline{R}_1 = \left\{ \begin{array}{l} \left(\left(f_1, f_2 \right), \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)} \\ 0.2e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)} \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left(\left(f_2, f_3 \right), \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left(\left(f_3, f_1 \right), \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.4e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right) \end{array} \right\}$$

v. The complex picture fuzzy soft equivalence relation $(\underline{R}_2)$ on $(\mathcal{G}, \mathcal{B})$ is

$$\underline{R}_2 = \left\{ \begin{array}{l} \left(\left(f_1, f_1 \right), \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.4e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.4e^{0.2(2\pi i)}, 0.1e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.4e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left(\left(f_2, f_3 \right), \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left(\left(f_2, f_2 \right), \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3(2\pi i)}, \\ 0.2e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)} \end{pmatrix} \right), \\ \left(\left(f_3, f_2 \right), \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left(\left(f_3, f_3 \right), \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.4e^{0.3(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix} \right) \end{array} \right\}$$

vi. The complex picture fuzzy soft complete relation $(\underline{R}_3)$ on $(\mathcal{G}, \mathcal{B})$ is

$$\underline{R}_3 = \left\{ \begin{array}{l} \left(\left(f_1, f_2 \right), \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)} \\ 0.2e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)} \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left(\left(f_2, f_3 \right), \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.1e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.1(2\pi i)}, 0.3e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix} \right) \end{array} \right\}$$

Example 10. *Considering the $(\mathcal{F}, \mathcal{A})$ and $(\mathcal{F}, \mathcal{A}) \times (\mathcal{F}, \mathcal{A})$ in Example 8, we have:*

i. *The complex picture fuzzy soft preorder relation $(\underline{R}_1)$ on $(\mathcal{F}, \mathcal{A})$ is expressed as*

$$\underline{R}_1 = \left\{ \begin{array}{l} \left((\underline{k}_1, \underline{k}_1), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.4(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left((\underline{k}_2, \underline{k}_1), \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left((\underline{k}_2, \underline{k}_2), \begin{pmatrix} 0.3e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right), \\ \left((\underline{k}_3, \underline{k}_3), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.4e^{0.6(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.4e^{0.5(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.2e^{0.4(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.3(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right) \end{array} \right\}$$

ii. *The complex picture fuzzy soft strict-order relation ($\underline{R}_2$) on $(\mathcal{F},\mathcal{A})$is expressed as*

$$\underline{R}_2=\left\{\begin{array}{l}\left((\underline{k}_2,\underline{k}_1),\begin{pmatrix}0.3e^{0.2(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.3e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix}\right),\\ \left((\underline{k}_3,\underline{k}_1),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.1(2\pi i)},\\0.3e^{0.4(2\pi i)},0.4e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.2(2\pi i)},\\0.2e^{0.2(2\pi i)},0.3e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\ \left((\underline{k}_3,\underline{k}_2),\begin{pmatrix}0.3e^{0.2(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.2e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right)\end{array}\right\}$$

iii. *The complex picture fuzzy soft partial-order relation ($\underline{R}_3$) on $(\mathcal{F},\mathcal{A})$ is expressed as*

$$\underline{R}_3=\left\{\begin{array}{l}\left((\underline{k}_1,\underline{k}_1),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.4(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.3e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix}\right),\\ \left((\underline{k}_2,\underline{k}_1),\begin{pmatrix}0.3e^{0.2(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.3e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix}\right),\\ \left((\underline{k}_2,\underline{k}_2),\begin{pmatrix}0.3e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.5e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\ \left((\underline{k}_3,\underline{k}_1),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.1(2\pi i)},\\0.3e^{0.4(2\pi i)},0.4e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.2(2\pi i)},\\0.2e^{0.2(2\pi i)},0.3e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\ \left((\underline{k}_3,\underline{k}_3),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.4e^{0.6(2\pi i)},0.1e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.4e^{0.5(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.2e^{0.4(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.3(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right)\end{array}\right\}$$

iv. *The complex picture fuzzy soft linear-order relation ($\underline{R}_3$) on $(\mathcal{F},\mathcal{A})$ is expressed as*

$$\underline{R}_4=\left\{\begin{array}{l}\left((\underline{k}_1,\underline{k}_1),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.4(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.3e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix}\right),\\ \left((\underline{k}_2,\underline{k}_1),\begin{pmatrix}0.3e^{0.2(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.3e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix}\right),\\ \left((\underline{k}_2,\underline{k}_2),\begin{pmatrix}0.3e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.5e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\ \left((\underline{k}_3,\underline{k}_1),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.1(2\pi i)},\\0.3e^{0.4(2\pi i)},0.4e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.2(2\pi i)},\\0.2e^{0.2(2\pi i)},0.3e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\ \left((\underline{k}_3,\underline{k}_2),\begin{pmatrix}0.3e^{0.2(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.2e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\ \left((\underline{k}_3,\underline{k}_3),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.4e^{0.6(2\pi i)},0.1e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.4e^{0.5(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.2e^{0.4(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.3(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right)\end{array}\right\}$$

Definition 16. *Suppose that $\underline{R}$ is a complex picture fuzzy soft relation; then, the Cartesian product of fuzzy soft equivalence class $\underline{k}$ is defined as $\underline{R}^{\underline{k}}=\{f\,|\,(f,\underline{k})\in\underline{R}\}$.*

Example 11. *If*

$$\underline{R}=\left\{\begin{array}{l}\left((\underline{k}_1,\underline{k}_1),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.4(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.3e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix}\right),\\ \left((\underline{k}_2,\underline{k}_3),\begin{pmatrix}0.3e^{0.2(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.2e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\ \left((\underline{k}_2,\underline{k}_2),\begin{pmatrix}0.3e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.5e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\ \left((\underline{k}_3,\underline{k}_2),\begin{pmatrix}0.3e^{0.2(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.2e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\ \left((\underline{k}_3,\underline{k}_3),\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.4e^{0.6(2\pi i)},0.1e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.4e^{0.5(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.2e^{0.4(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.3(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right)\end{array}\right\}$$

is a complex picture fuzzy soft relation on a complex picture fuzzy soft set $(\mathcal{F},\mathcal{A})$,

$$(\mathcal{F},\mathcal{A})=\left\{\begin{array}{l}\left(\underline{k}_1,\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.4(2\pi i)},0.4e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.2e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.3e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix}\right),\\ \left(\underline{k}_2,\begin{pmatrix}0.3e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.4e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.3(2\pi i)},\\0.3e^{0.2(2\pi i)},0.2e^{0.1(2\pi i)}\end{pmatrix},\begin{pmatrix}0.5e^{0.1(2\pi i)},\\0.3e^{0.2(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right),\\ \left(\underline{k}_3,\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.4e^{0.6(2\pi i)},0.1e^{0.2(2\pi i)}\end{pmatrix},\begin{pmatrix}0.3e^{0.1(2\pi i)},\\0.4e^{0.5(2\pi i)},0.2e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.2(2\pi i)},\\0.2e^{0.4(2\pi i)},0.1e^{0.3(2\pi i)}\end{pmatrix},\begin{pmatrix}0.4e^{0.3(2\pi i)},\\0.3e^{0.3(2\pi i)},0.2e^{0.4(2\pi i)}\end{pmatrix}\right)\end{array}\right\};$$

then, the complex picture fuzzy soft equivalence class of:

i. $\underline{k}_1$ *modulo* $\underline{R}$ *is expressed as*

$$\underline{R}^{\underline{k}} = \left\{ \left(\underline{k}_1, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.4(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right) \right\}$$

ii. $\underline{k}_2$ *modulo* $\underline{R}$ *is expressed as*

$$\underline{R}^{\underline{k}_2} = \left\{ \begin{array}{l} \left(\underline{k}_3, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right), \\ \left(\underline{k}_2, \begin{pmatrix} 0.3e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.5e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right) \end{array} \right\}$$

iii. $\underline{g}$ *modulo* $\underline{R}$ *is expressed as*

$$\underline{R}^{\underline{k}_3} = \left\{ \begin{array}{l} \left(\underline{k}_2, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right), \\ \left(\underline{k}_3, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.4e^{0.6(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.4e^{0.5(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.2e^{0.4(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.3(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right) \end{array} \right\}$$

Definition 17. *Assume a complex picture fuzzy soft relation ($\underline{R}_1$) on a complex picture fuzzy soft set; then, the complex picture fuzzy soft composite relation is denoted by $\overline{\underset{\cdot}{R}}_1 \circ \overline{\underset{\cdot}{R}}_1$ and defined as: for each $(\underline{k}, f) \in \overline{\underset{\cdot}{R}}_1$ and $(f, \underline{g}) \in \overline{\underset{\cdot}{R}}_1 \Rightarrow (\underline{k}, \underline{g}) \in \overline{\underset{\cdot}{R}}_1 \circ \overline{\underset{\cdot}{R}}_1, \forall \underline{k}, f, \underline{g} \in \dot{X}$.*

Example 12. *Assume complex picture fuzzy soft relations:*

$$\underline{R}_1 = \left\{ \begin{array}{l} \left((\underline{k}, f), \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1(2\pi i)} \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)} \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left((f, \underline{k}), \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left((\underline{g}, f), \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right) \end{array} \right\}$$

$$\underline{R}_2 = \left\{ \begin{array}{l} \left((\underline{k}, \underline{g}), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.4(2\pi i)}, 0.4e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right), \\ \left((f, \underline{k}), \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left((\underline{g}, f), \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right), \\ \left((\underline{g}, \underline{g}), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.4e^{0.6(2\pi i)}, 0.1e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.4e^{0.5(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.2e^{0.4(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.3(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right) \end{array} \right\};$$

then, the complex picture fuzzy soft composite relation is expressed as

$$\overline{\underset{\cdot}{R}}_1 \circ \overline{\underset{\cdot}{R}}_2 = \left\{ \begin{array}{l} \left((\underline{k}, \underline{k}), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.4(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\ \left((f, \underline{g}), \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right), \\ \left((\underline{g}, \underline{k}), \begin{pmatrix} 0.4e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.4(2\pi i)}, 0.4e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.4(2\pi i)} \end{pmatrix} \right) \end{array} \right\}$$

Theorem 1. *A complex picture fuzzy soft relation ($\overline{\underset{\cdot}{R}}$) is a complex picture fuzzy soft symmetric relation on a complex picture fuzzy soft set ($\mathcal{F}$) iff $\overline{\underset{\cdot}{R}} = \overline{\underset{\cdot}{R}}^c$.*

Proof. Suppose that $\overline{\underset{\cdot}{R}} = \overline{\underset{\cdot}{R}}^c$; then, $((\underline{k}, f), (\mathfrak{m}_c(\underline{k}, f)), (\breve{\alpha}_c(\underline{k}, f)), (\mathfrak{n}_c(\underline{k}, f))) \in \overline{\underset{\cdot}{R}}$

$$\Longrightarrow ((f, \underline{k}), (\mathfrak{m}_c(f, \underline{k})), (\breve{\alpha}_c(f, \underline{k})), (\mathfrak{n}_c(f, \underline{k}))) \in \overline{\underset{\cdot}{R}}^c$$
$$\Longrightarrow ((f, \underline{k}), (\mathfrak{m}_c(f, \underline{k})), (\breve{\alpha}_c(f, \underline{k})), (\mathfrak{n}_c(f, \underline{k}))) \in \overline{\underset{\cdot}{R}}.$$

Thus, $\overline{\underset{\cdot}{R}}$ is a complex picture fuzzy soft symmetric relation on a complex picture fuzzy soft set ($\mathcal{F}$). □

Conversely, assume that $\overline{\underset{\cdot}{R}}$ is a complex picture fuzzy soft symmetric relation on a complex picture fuzzy soft set ($\mathcal{F}$); then

$$((\underline{k}, f), (\mathfrak{m}_c(\underline{k}, f)), (\breve{\alpha}_c(\underline{k}, f)), (\mathfrak{n}_c(\underline{k}, f))) \in \overline{\underset{\cdot}{R}} \Longrightarrow ((f, \underline{k}), (\mathfrak{m}_c(f, \underline{k})), (\breve{\alpha}_c(f, \underline{k})), (\mathfrak{n}_c(f, \underline{k}))) \in \overline{\underset{\cdot}{R}}.$$
$$\mathit{However,}\ ((f, \underline{k}), (\mathfrak{m}_c(f, \underline{k})), (\breve{\alpha}_c(f, \underline{k})), (\mathfrak{n}_c(f, \underline{k}))) \in \overline{\underset{\cdot}{R}}^c$$
$$\Longrightarrow \overline{\underset{\cdot}{R}} = \overline{\underset{\cdot}{R}}^c.$$

Theorem 2. *A complex picture fuzzy soft relation ($\overline{\underset{\cdot}{R}}$) is a complex picture fuzzy soft transitive relation on a complex picture fuzzy soft set ($\mathcal{F}$) iff $\overline{\underset{\cdot}{R}} \circ \overline{\underset{\cdot}{R}} \subseteq \overline{\underset{\cdot}{R}}^c$.*

Proof. Suppose that $\overline{\underset{\cdot}{R}}$ is a complex picture fuzzy soft transitive relation on a complex picture fuzzy soft set ($\mathcal{F}$).

$$\text{Let } ((\underline{k}, g), (\mathfrak{m}_c(\underline{k}, g)), (\breve{\alpha}_c(\underline{k}, g)), (\mathfrak{n}_c(\underline{k}, g))) \in \overline{\underset{\cdot}{R}} \circ \overline{\underset{\cdot}{R}},$$

Then, according to the definition of a complex picture fuzzy soft transitive relation,

$$((\underline{k}, f), (\mathfrak{m}_c(\underline{k}, f)), (\breve{\alpha}_c(\underline{k}, f)), (\mathfrak{n}_c(\underline{k}, f))) \in \overline{\underset{\cdot}{R}} \text{ And } ((f, g), (\mathfrak{m}_c(f, g)), (\breve{\alpha}_c(f, g)), (\mathfrak{n}_c(f, g))) \in \overline{\underset{\cdot}{R}}$$
$$((\underline{k}, g), (\mathfrak{m}_c(\underline{k}, g)), (\breve{\alpha}_c(\underline{k}, g)), (\mathfrak{n}_c(\underline{k}, g))) \in \overline{\underset{\cdot}{R}}$$
$$\Longrightarrow \overline{\underset{\cdot}{R}} \circ \overline{\underset{\cdot}{R}} \subseteq \overline{\underset{\cdot}{R}}.$$

Conversely, assume that $\overline{\underset{\cdot}{R}} \circ \overline{\underset{\cdot}{R}} \subseteq \overline{\underset{\cdot}{R}}$; then,

$$\text{for } ((\underline{k}, f), (\mathfrak{m}_c(\underline{k}, f)), (\breve{\alpha}_c(\underline{k}, f)), (\mathfrak{n}_c(\underline{k}, f))) \in \overline{\underset{\cdot}{R}} \text{ and } ((f, g), (\mathfrak{m}_c(f, g)), (\breve{\alpha}_c(f, g)), (\mathfrak{n}_c(f, g))) \in \overline{\underset{\cdot}{R}},$$
$$((\underline{k}, g), (\mathfrak{m}_c(\underline{k}, g)), (\breve{\alpha}_c(\underline{k}, g)), (\mathfrak{n}_c(\underline{k}, g))) \in \overline{\underset{\cdot}{R}} \circ \overline{\underset{\cdot}{R}} \subseteq \overline{\underset{\cdot}{R}}.$$
$$((\underline{k}, g), (\mathfrak{m}_c(\underline{k}, g)), (\breve{\alpha}_c(\underline{k}, g)), (\mathfrak{n}_c(\underline{k}, g))) \in \overline{\underset{\cdot}{R}}.$$

Thus, $\overline{\underset{\cdot}{R}}$ is a complex picture fuzzy soft transitive relation on a complex picture fuzzy soft set ($\mathcal{F}$). □

Theorem 3. *If $\overline{\underset{\cdot}{R}}_1$ is a complex picture fuzzy soft equivalence relation on a complex picture fuzzy soft set (F), then $(x, y) \in \overline{\underset{\cdot}{R}}_1$, iff $\overline{\underset{\cdot}{R}}_1[f] = \overline{\underset{\cdot}{R}}_1[g]$.*

Proof. Suppose that $(f, g) \in \overline{\underset{\cdot}{R}}_1$ and $\underline{k} \in \overline{\underset{\cdot}{R}}_1[f]$, $\overline{\underset{\cdot}{R}}_1(\underline{k}, f) \in \overline{\underset{\cdot}{R}}_1$.

Given that a complex picture fuzzy soft equivalence relation is also a complex picture fuzzy soft transitive relation, $(\underline{k}, g) \in \overline{\underset{\cdot}{R}}_1 \Longrightarrow \underline{k} \in \overline{\underset{\cdot}{R}}_1[g]$.

Thus,

$$\overline{\underset{\cdot}{R}}_1[f] \subseteq \overline{\underset{\cdot}{R}}_1[g] \tag{1}$$

As $(f, g) \in \overline{\underset{\cdot}{R}}_1$, given that a complex picture fuzzy soft equivalence relation is also a complex picture fuzzy soft symmetric relation,

$$(g, f) \in \overline{\underset{\cdot}{R}}_1.$$

Additionally, assume that $\underline{k} \in \overline{\underset{\cdot}{R}}_1[g] \Longrightarrow (\underline{k}, g) \in \overline{\underset{\cdot}{R}}_1$.

Now, given that a complex picture fuzzy soft equivalence relation is also a complex picture fuzzy soft transitive relation,

$$(\underline{k}, f) \in \overline{\underset{\cdot}{R}}_1 \Longrightarrow \underline{k} \in \overline{\underset{\cdot}{R}}_1[f].$$

Thus,

$$\overline{\underset{\cdot}{R}}_1[g] = \overline{\underset{\cdot}{R}}_1[f] \tag{2}$$

Therefore, (1) and (2) afford $\overline{\underset{\cdot}{R}}_1[g] = \overline{\underset{\cdot}{R}}_1[f]$.

Conversely, assume that $\overline{\underset{\cdot}{R}}_1[g] = \overline{\underset{\cdot}{R}}_1[f], \underline{k} \in \overline{\underset{\cdot}{R}}_1[f]$, $\underline{k} \in \overline{\underset{\cdot}{R}}_1[g] \Longrightarrow (\underline{k}, g) \in \overline{\underset{\cdot}{R}}_1$, and $(\underline{k}, f) \in \overline{\underset{\cdot}{R}}_1$.

Again, given that a complex picture fuzzy soft equivalence relation is also a complex picture fuzzy soft symmetric relation, $(\underline{k}, f) \in \overline{\underset{\cdot}{R}}_1 \Longrightarrow (f, \underline{k}) \in \overline{\underset{\cdot}{R}}_1$.

According to the definition of a complex picture fuzzy soft transitive relation,

$$(f, \underline{k}) \in \overline{\underset{\cdot}{R}}_1 \text{ and } (\underline{k}, g) \in \overline{\underset{\cdot}{R}}_1 \Longrightarrow (f, g) \in \overline{\underset{\cdot}{R}}_1,$$

which completes the proof. □

4. Applications

In this section, an application of the proposed ideas is discussed with the aim of selecting the best generative adversarial networks.

Generative Adversarial Networks

In June 2014, a family of machine learning frameworks called generative adversarial networks was developed. Given a training set, this strategy learns to produce fresh data with the same characteristics as the training set. Generative adversarial networks represent a new method and concepts for computer vision that have appeared recently. The concepts of generative adversarial networks using the competition training method are superior to those of traditional machine learning methods based on feature learning and image generation. In this section, we use the proposed conceptions to study and analyze this novel deep-learning-based image processing technique. The procedure of this application is explained in Figure 1.

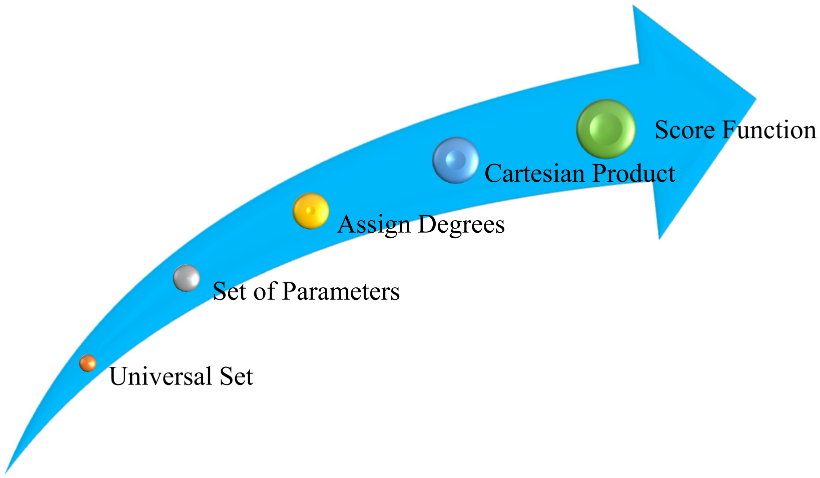

Figure 1. Generative adversarial network application procedure.

First of all, we define a universal set, which includes some fundamental generative adversarial networks. The universal set $\dot{X} = \{\hbar_1, \hbar_2, \hbar_3\}$ consists of three types of generative adversarial networks, i.e., $\hbar_1 = Vanilla\ GAN$, $\hbar_2 = Super\ Resolution\ GAN$, and $\hbar_3 = Conditional\ GAN$. The types of generative adversarial networks are discussed in Figure 2.

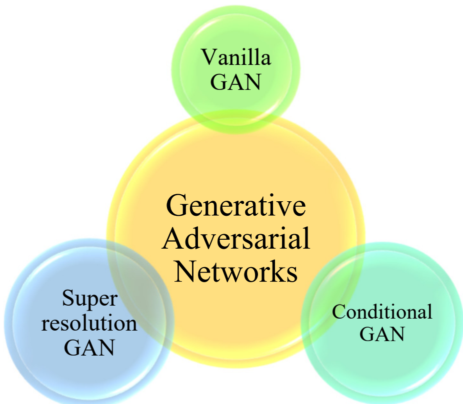

Figure 2. Summary of generative adversarial networks.

❖ Vanilla Generative Adversarial Networks

These the most basic version of generative adversarial networks. Simple multilayer artificial neurons are used as the generator and discriminator in this experiment. The algorithm of vanilla generative adversarial networks is really simple; it uses stochastic gradient descent to try to optimize mathematical equations.

❖ Super-Resolution Generative Adversarial Networks

Super-resolution generative adversarial networks represent a method of creating a generative adversarial network in which a deep neural network is combined with an adversarial network to produce higher-resolution images, as the title suggests. This sort of generative adversarial network is particularly good for upscaling native low-resolution photos to improve their details while minimizing mistakes.

❖ Conditional Generative Adversarial Networks

Conditional generative adversarial networks represent a version of generative adversarial networks in which the generator and discriminator are dependent on ancillary data such as a classifier during training. The conditional generative adversarial networks approach can be described as a deep learning model with conditional parameters.

The parameters of a generative adversarial network are defined as $\grave{\dot{E}} = \{\underline{k}_1, \underline{k}_2, \underline{k}_3, \underline{k}_4, \underline{k}_5\}$, i.e., $\underline{k}_1 = popular$, $\underline{k}_2 = successful$, $\underline{k}_3 = efficient$, $\underline{k}_4 = reliable$, and $\underline{k}_5 = simple$. A summary of generative adversarial network parameters is shown in Figure 3.

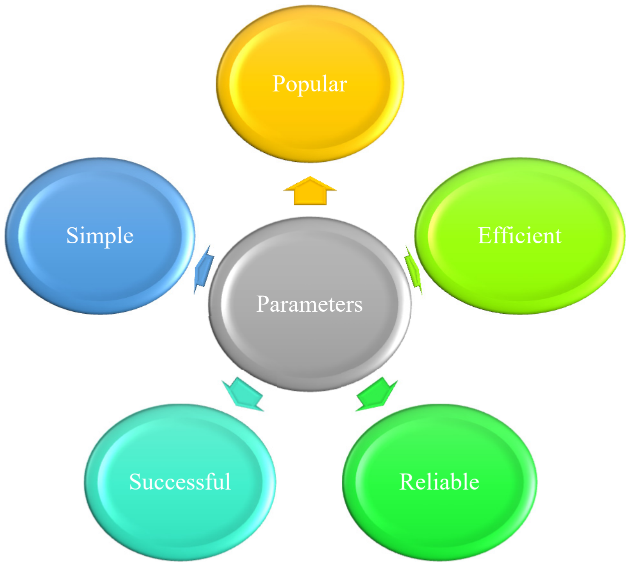

Figure 3. Summary of generative adversarial network parameters.

An expert examines the generative adversarial networks in which all parameters are considered. Let $(\mathcal{F}, \mathcal{A})$ represent observations by an expert, who assigns values of membership, abstinence, and non-membership in the base of parameters.

Suppose that the corresponding membership, abstinence, and non-membership matrices are as follows:

$$(\mathcal{F}, \mathcal{A}) = \left(\begin{array}{c}
\left(\underline{k}_1, \begin{pmatrix} 0.3e^{0.2}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.1e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\
\left(\underline{k}_2, \begin{pmatrix} 0.2e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.1(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\
\left(\underline{k}_3, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.2e^{0.3(2\pi i)}, 0.4e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.2e^{0.3(2\pi i)}, 0.3e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.2e^{0.3(2\pi i)}, 0.3e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.3(2\pi i)}, \\ 0.4e^{0.3(2\pi i)}, 0.2e^{0.2(2\pi i)} \end{pmatrix} \right), \\
\left(\underline{k}_4, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.2e^{0.3(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.4e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.3(2\pi i)}, \\ 0.1e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.2e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix} \right), \\
\left(\underline{k}_5, \begin{pmatrix} 0.3e^{0.3(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.2e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.4e^{0.1(2\pi i)}, \\ 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)} \end{pmatrix}, \begin{pmatrix} 0.3e^{0.2(2\pi i)}, \\ 0.3e^{0.3(2\pi i)}, 0.2e^{0.2(2\pi i)} \end{pmatrix} \right)
\end{array} \right)$$

Suppose that the first three values of matrices $\varkappa_1, \varkappa_2$, and $\varkappa_3$ of each parameter correspond to the values of membership, abstinence, and non-membership assigned by an expert. The value (λ) of each parameter corresponds to the value of membership, abstinence, and non-membership assigned by an expert, and the fourth indicates the general belongingness value in the generative adversarial network. Then, the self-Cartesian product of $(\mathcal{F}, \mathcal{A})$ is expressed as shown in Table 1.

Table 1. Cartesian product of generative adversarial networks.

Ordered Pair	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	λ
$(\underline{k}_1, \underline{k}_1)$	$\left(0.3e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}, 0.2e^{0.1(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.3(2\pi i)}, 0.1e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_1, \underline{k}_2)$	$\left(0.2e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_1, \underline{k}_3)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}, 0.4e^{0.2(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.2e^{0.3(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_1, \underline{k}_4)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}, 0.2e^{0.1(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.2(2\pi i)}, 0.1e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.1(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_1, \underline{k}_5)$	$\left(0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_2, \underline{k}_1)$	$\left(0.2e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_2, \underline{k}_2)$	$\left(0.2e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)}\right)$	$\left(0.4e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_2, \underline{k}_3)$	$\left(0.2e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_2, \underline{k}_4)$	$\left(0.2e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.1e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.1(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_2, \underline{k}_5)$	$\left(0.2e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)}\right)$	$\left(0.3e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_3, \underline{k}_1)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}, 0.4e^{0.2(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.2e^{0.3(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_3, \underline{k}_2)$	$\left(0.2e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_3, \underline{k}_3)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}, 0.4e^{0.2(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.2e^{0.3(2\pi i)}, 0.3e^{0.2(2\pi i)}\right)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.3(2\pi i)}, 0.4e^{0.3(2\pi i)}, 0.2e^{0.2(2\pi i)}\right)$
$(\underline{k}_3, \underline{k}_4)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}, 0.4e^{0.2(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}\right)$	$\left(0.2e^{0.2(2\pi i)}, 0.1e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.1(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_3, \underline{k}_5)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}\right)$	$\left(0.3e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}, 0.2e^{0.2(2\pi i)}\right)$
$(\underline{k}_4, \underline{k}_1)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}, 0.2e^{0.1(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.2(2\pi i)}, 0.1e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.1(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_4, \underline{k}_2)$	$\left(0.2e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.1e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.1(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_4, \underline{k}_3)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}, 0.4e^{0.2(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}\right)$	$\left(0.2e^{0.2(2\pi i)}, 0.1e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.1(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_4, \underline{k}_4)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}, 0.2e^{0.1(2\pi i)}\right)$	$\left(0.2e^{0.3(2\pi i)}, 0.4e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}\right)$	$\left(0.2e^{0.3(2\pi i)}, 0.1e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$

Table 1. *Cont.*

Ordered Pair	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	λ
$(\underline{k}_4, \underline{k}_5)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)}\right)$	$\left(0.2e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.1e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.1(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_5, \underline{k}_1)$	$\left(0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_5, \underline{k}_2)$	$\left(0.2e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.3e^{0.1(2\pi i)}\right)$	$\left(0.3e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_5, \underline{k}_3)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.4e^{0.2(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.2(2\pi i)}\right)$	$\left(0.3e^{0.1(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}, 0.2e^{0.2(2\pi i)}\right)$
$(\underline{k}_5, \underline{k}_4)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)}\right)$	$\left(0.2e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.1e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)}\right)$	$\left(0.2e^{0.1(2\pi i)}, 0.3e^{0.1(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$
$(\underline{k}_5, \underline{k}_5)$	$\left(0.3e^{0.3(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)}\right)$	$\left(0.3e^{0.2(2\pi i)}, 0.2e^{0.2(2\pi i)}, 0.2e^{0.1(2\pi i)}\right)$	$\left(0.4e^{0.1(2\pi i)}, 0.3e^{0.2(2\pi i)}, 0.2e^{0.3(2\pi i)}\right)$	$\left(0.3e^{0.2(2\pi i)}, 0.3e^{0.3(2\pi i)}, 0.2e^{0.2(2\pi i)}\right)$

The Cartesian product of two complex picture fuzzy soft sets is shown in the table above. The complex values are converted to real numbers to compute the score values. First of all, to convert all exponential values to the form of $a + ib$. i.e., $a + ib = re^{(2\pi i)\theta}$, as $r = \sqrt{a^2 + b^2}$ and $e^{(2\pi i)\theta} = cos\ \pi(\theta) + i\ sin\ \pi(\theta)$. Then $a = rcos\ \pi(\theta), b = sin\ \pi(\theta)$. π represents a circular cycle. Take the modulus after converting the polar form to a standard form. These procedures apply to the membership, abstinence, and non-membership values. Then, this process is applied to membership, abstinence, and non-membership scores according to the formula $\mathfrak{m}^2 + \breve{\alpha}^2 - \mathfrak{n}^2$, as shown in Table 2.

Table 2. Score values of generative adversarial networks.

Ordered Pair	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	λ
$(\underline{k}_1, \underline{k}_1)$	0.140	0.120	0.039	0.090
$(\underline{k}_1, \underline{k}_2)$	0.039	0.089	0.040	0.089
$(\underline{k}_1, \underline{k}_3)$	−0.018	−0.009	0.039	0.090
$(\underline{k}_1, \underline{k}_4)$	0.089	0.090	−0.040	0.089
$(\underline{k}_1, \underline{k}_5)$	0.139	0.039	0.040	0.090
$(\underline{k}_2, \underline{k}_1)$	0.039	0.090	0.040	0.090
$(\underline{k}_2, \underline{k}_2)$	0.039	0.209	0.040	0.140
$(\underline{k}_2, \underline{k}_3)$	−0.068	−0.010	0.039	0.139
$(\underline{k}_2, \underline{k}_4)$	−0.010	−0.089	−0.048	0.089
$(\underline{k}_2, \underline{k}_5)$	0.040	0.090	0.089	0.039
$(\underline{k}_3, \underline{k}_1)$	−0.019	−0.009	0.040	0.090
$(\underline{k}_3, \underline{k}_2)$	−0.068	−0.010	0.039	0.139
$(\underline{k}_3, \underline{k}_3)$	−0.018	−0.009	0.040	0.209
$(\underline{k}_3, \underline{k}_4)$	−0.019	−0.010	−0.042	0.089
$(\underline{k}_3, \underline{k}_5)$	−0.018	−0.010	0.040	0.139
$(\underline{k}_4, \underline{k}_1)$	0.090	0.089	−0.048	0.090
$(\underline{k}_4, \underline{k}_2)$	−0.010	0.089	−0.048	0.089
$(\underline{k}_4, \underline{k}_3)$	−0.018	−0.010	−0.048	0.090
$(\underline{k}_4, \underline{k}_4)$	0.090	0.148	−0.048	0.089
$(\underline{k}_4, \underline{k}_5)$	0.089	0.039	0.001	0.090
$(\underline{k}_5, \underline{k}_1)$	0.139	0.039	0.039	0.089
$(\underline{k}_5, \underline{k}_2)$	0.089	0.090	0.089	0.139
$(\underline{k}_5, \underline{k}_3)$	0.040	−0.010	0.039	0.139
$(\underline{k}_5, \underline{k}_4)$	0.089	0.039	0.001	0.089
$(\underline{k}_5, \underline{k}_5)$	0.049	0.089	0.209	0.139

To find the best generative adversarial network, we must first determine the highest numerical degree in each row while ignoring the last column. The final column represents each generative adversarial network parameter's general belongingness. Each generative adversarial network score is computed by multiplying the product of these numerical degrees by the desired value of λ. The highest-scoring generative adversarial network is the best. We do not examine the numerical degree of the identical parametric ordered pair's generative adversarial network because it is not a unique effort to compare.

The score functions are calculated in Table 3.

$$\begin{gathered}
\mathrm{S}(\varkappa_1) = (0.139 \times 0.090) + (-0.010 \times 0.089) + (0.090 \times 0.090) + (0.089 \times 0.090) \\
+(0.139 \times 0.089) + (0.040 \times 0.139) + (0.089 \times 0.089) = 0.148 \\
\mathrm{S}(\varkappa_2) = (0.089 \times 0.089) + (0.090 \times 0.089) + (0.090 \times 0.090) \times (0.039 \times 0.139) + (0.090 \times 0.039) + (-0.010 \times 0.089) + \\
(0.089 \times 0.089) + (-0.010 \times 0.090) + (0.090 \times 0.139) = 0.051 \\
\mathrm{S}(\varkappa_3) = (0.039 \times 0.090) + (0.040 \times 0.090) + (0.039 \times 0.139) + (0.040 \times 0.139) = 0.018
\end{gathered}$$

Table 3. Grade table of the score function.

$\overline{\underset{\cdot}{R}}$	(k_1, k_1)	(k_1, k_2)	(k_1, k_3)	(k_1, k_4)	(k_1, k_5)	(k_2, k_1)	(k_2, k_2)	(k_2, k_3)	(k_2, k_4)
$\varkappa_i$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_2$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_2$	$\varkappa_1$
Highest degree	$\times$	0.089	0.039	0.090	0.139	0.090	$\times$	0.039	−0.010
$\boldsymbol{\lambda}$	$\times$	0.089	0.090	0.089	0.090	0.090	$\times$	0.139	0.089
$\overline{\underset{\cdot}{R}}$	$(\underline{k}_2, \underline{k}_5)$	$(\underline{k}_3, \underline{k}_1)$	$(\underline{k}_3, \underline{k}_2)$	$(\underline{k}_3, \underline{k}_3)$	$(\underline{k}_3, \underline{k}_4)$	$(\underline{k}_3, \underline{k}_5)$	$(\underline{k}_4, \underline{k}_1)$	$(\underline{k}_4, \underline{k}_2)$	$(\underline{k}_4, \underline{k}_3)$
$\varkappa_i$	$\varkappa_2$	$\varkappa_3$	$\varkappa_3$	$\varkappa_3$	$\varkappa_2$	$\varkappa_3$	$\varkappa_1$	$\varkappa_2$	$\varkappa_2$
Highest degree	0.090	0.040	0.039	$\times$	−0.010	0.040	0.090	0.089	−0.010
$\boldsymbol{\lambda}$	0.039	0.090	0.139	$\times$	0.089	0.139	0.090	0.089	0.090
$\overline{\underset{\cdot}{R}}$	$(\underline{k}_4, \underline{k}_4)$	$(\underline{k}_4, \underline{k}_5)$	$(\underline{k}_5, \underline{k}_1)$	$(\underline{k}_5, \underline{k}_2)$	$(\underline{k}_5, \underline{k}_3)$	$(\underline{k}_5, \underline{k}_4)$	$(\underline{k}_5, \underline{k}_5)$		
$\varkappa_i$	$\varkappa_2$	$\varkappa_1$	$\varkappa_1$	$\varkappa_2$	$\varkappa_1$	$\varkappa_1$	$\varkappa_3$		
Highest degree	$\times$	0.089	0.139	0.090	0.040	0.089	$\times$		
$\boldsymbol{\lambda}$	$\times$	0.090	0.089	0.139	0.139	0.089	$\times$		

Thus, the vanilla generative adversarial network is the best generative adversarial network.

5. Comparative Analysis

In this section, the concepts of complex picture fuzzy soft relations are compared with some preexisting structures in the theory of fuzzy soft sets, such as soft relations, fuzzy soft relations, complex fuzzy soft relations, intuitionistic fuzzy soft relations, and complex intuitionistic fuzzy soft relations. A soft relation is a mapping from a parameterized family to a crisp subset. The crisp relation can only be defined as 1 or 0 rather than yes or no. Thus, a crisp relation indicates restricted knowledge. A fuzzy soft set is a set defined by a degree of membership, which is a fuzzy number. The related relations are called fuzzy soft relations. Fuzzy soft relations discuss only the membership degree. In an ordered pair, fuzzy soft relations indicate only the effectiveness of the first parameter relative to the that of the second parameter. Fuzzy soft relations are single-dimension parameters and provide limited information. Complex fuzzy soft sets are described by a complex fuzzy number, and corresponding relations are called complex fuzzy soft relations. Complex fuzzy soft relations discuss only the membership degree with complex number. Complex fuzzy soft relations comprise two main parts, i.e., an amplitude term and a phase term. An amplitude term characterizes the strength of different generative adversarial networks, and the phase term is used to describe the time period under given conditions. Intuitionistic fuzzy soft sets are characterized the membership and non-membership degrees. The corresponding relations are known as intuitionistic fuzzy soft relations. In an ordered pair, intuitionistic fuzzy soft relations show the effectiveness and ineffectiveness of the first parameter relative to that of the second parameter. Intuitionistic fuzzy soft relations lack the ability to represent problems that include time and provide incomplete information. Picture fuzzy soft sets are characterized by the membership, abstinence, and non-membership degrees. The corresponding relations are known as picture fuzzy soft relations. Picture fuzzy soft relations show the effectiveness, ineffectiveness, and lack of effects of each parameter. Complex picture fuzzy soft relations discuss the membership, abstinence, and non-membership with a complex number. They discuss both the amplitude term and phase term, providing complete information for any problem. Table 4 shows a summary of the comparative study of complex picture fuzzy soft relations and a predefined structure.

Table 4. Comparative analysis on the basis of structure.

Structure	Membership	Abstinence	Non-Membership	Multidimension
Soft relation	No	No	No	No
Fuzzy soft relation	Yes	No	No	No
Complex fuzzy soft relation	Yes	No	No	Yes
Intuitionistic fuzzy soft relation	Yes	No	Yes	No
Complex intuitionistic fuzzy soft relation	Yes	No	Yes	Yes
Picture fuzzy soft relation	Yes	Yes	Yes	No
Complex picture fuzzy soft relation	Yes	Yes	Yes	Yes

6. Conclusions

In this paper, the Cartesian product of two complex picture fuzzy soft sets is defined. Moreover, new notions of complex picture fuzzy soft relations and their types are introduced, such as complex picture fuzzy soft converse relation, complex picture fuzzy soft reflexive relation, complex picture fuzzy soft irreflexive relation, complex picture fuzzy soft symmetric relation, complex picture fuzzy soft antisymmetric relation, complex picture fuzzy soft asymmetric relation, complex picture fuzzy soft complete relation, complex picture fuzzy soft transitive relation, complex picture fuzzy soft equivalence relation, complex picture fuzzy soft partial-order relation, complex picture fuzzy soft strict-order relation, complex picture fuzzy soft preorder relation, and complex picture fuzzy soft equivalence classes. Furthermore, these novel ideas of complex picture fuzzy soft relations are utilized in an application to study generative adversarial networks in which the best generative adversarial networks are chosen based on various parameters and characteristics. Experts' comments cloud these parameters by assigning membership, abstinence, and non-membership values. The score function for the novel structures was also defined during the decision-making process. Finally, complex picture fuzzy soft relations were proven to be superior to the predefined structures. The most noticeable advantage of complex picture fuzzy soft relations is that they are capable of solving periodicity. In the future, these notions can be extended to the further generalization of fuzzy soft sets, which will create innovative structures that may be used in a variety of fields.

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