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An Optimal Location-Allocation Model for Equipment Supporting System Based on Uncertainty Theory

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Abstract: Scientific support depot location and reasonable spare parts transportation are the keys to improving the support level of complex systems. The current equipment support system has the problems of chaotic warehouse layout and low efficiency of spare parts. The reliability and completeness of spare parts' historical data are hard to believe. In order to deal with the cognitive uncertainty caused by the asymmetry of data, this paper adopts the uncertainty theory to optimize the depot location and transportation volume. Under the constraints of shortage rate, supply availability, average logistic delay time, and inventory limit, the uncertain chance-constrained model of equipment supporting depot is established. The optimization model is transformed into a deterministic model by using the inverse uncertainty distribution. The genetic algorithm is used to optimize the solution of this model. Finally, the practicability and operability of the model method are verified through the example analysis.

Keywords: equipment support system; location optimization; uncertainty theory; uncertain chance-constrained programming

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1. Introduction

The equipment supporting depot is an important part of the supporting system and is responsible for supplying spare parts and other important tasks. The effect of spare parts supply directly affects the use and maintenance condition of the equipment. Under the current trend of emphasizing rapidity and high efficiency, it is of great significance to solve the problems of selecting the location of the equipment supporting depot and the distribution of spare parts transportation to improve the efficiency of the supporting system.

Early related studies focused on spare parts inventory strategies for multi-layer assurance sites, which were forecasted in a system perspective. Liu et al. [1] established a three-level assurance supply structure and spent another spare parts supply assurance strategy with the expected number of shortages as a performance parameter. Guo et al. [2] gave a spare parts demand simulation model by analyzing the multi-level multi-layer spare parts flow. Wang and Kang [3] constructed a multi-level inventory optimization model with the objective of spare parts security probability and predicted the multi-level inventory spare parts demand. Fan et al. [4] established an equipment availability model with the help of simulation methods, and optimized it through genetic algorithms. Cost is used as a constraint in its model to maximize availability. Wang et al. [5] built a three-level inventory model for valuable spare parts considering the effect of repair based on the analysis of the behavior of spare parts demand, inventory, and replenishment. Sun et al. [6] extended the classical METRIC model and studied the optimization of inventory strategy under different levels. Wang et al. [7] proposed a spare parts supply strategy based on the spare parts pool network and established a location selection model for the spare parts central warehouse. Dui et al. [8] gave a method of site importance measure based on horizontal supply time. These studies emphasized that multi-level inventory is beneficial to cost saving, but ignored the management chaos brought by multi-level inventory. It also brings some cost wastage due to the management delay of supply.

The location of the safeguarded site is the primary factor affecting transit time. Among the studies related to the siting problem, the median, coverage, and center problems were first proposed. The p-Median siting problem was proposed by Hakimi in 1964, who argued that this type of problem is to optimize the location of p facilities to make the effect optimal [9]. In 1971 Toregas et al. [10] first proposed the ability to perform facility establishment with minimum cost while covering all demand points. The p-center problem, also proposed by Hakimi, is the problem of minimizing the worst-case scenario by optimizing the location of P facilities [11]. The three types of classical site selection models described above treat the problem as deterministic. However, the problems faced in reality are often uncertain, which makes the classical models fail to meet the practical requirements. Therefore, some scholars conducted some research based on considering uncertainty. Ballou [12] pointed out the shortcomings of deterministic siting models and introduced dynamic planning methods in for the siting problem. Subsequently, Drezner [13] investigated the dynamic p-median problem. Weaver and Church [14] explored the stochastic p-median problem. Jamil et al. [15] studied the stochastic p-center problem. Some scholars applied queuing theory to the siting problem and developed some models [16]. Berman et al. [17] developed a relevant siting optimization model considering queuing waiting pairs. Some other scholars used fuzzy mathematical theory to solve the uncertainty in the siting problem. Canós et al. [18] studied the fuzzy median problem and the fuzzy center problem. Peng et al. [19] proposed an emergency resource center location-routing model based on the fuzzy demand.

In 2007, Liu created the uncertainty theory, which was soon applied to the study of various problems, including the siting problem [20]. Wen et al. [21] studied and modeled the siting problem in uncertain environments. He et al. [22] investigated the multi-level warehouse layout problem with indeterminate factors. Yu et al. [23] established an optimization model for the maximum coverage location of emergency facilities considering shared uncertainties. Li et al. [24] established a two-stage continuous stochastic programming model for uncertain customer demand. Recently so, many papers have been available to show the importance of the work; Sinha and Shende [25] introduced a feature selection method for stock marketing based on uncertainty optimization. Sinha et al. [26] proposed a novel approach to dealing with incomplete information systems for more effective dataset analysis. In this paper, we also apply the knowledge of uncertainty theory to solve the uncertainty in the problem to be studied.

In recent studies, researchers have started to focus on deeper factors such as reliability, time satisfaction, etc. Snyder and Daskin [27] developed a facility reliability siting model by considering the cost of site damage. Cui et al. [28] combined site damage probability into the siting problem for site selection design. Murali et al. [29] conducted a siting study after considering the possibility of mass destruction of the site. Ma et al. [30] defined a time satisfaction function in the site selection supply. Zhou and Shen [31] developed a time-satisfaction-based site selection model. Wen et al. [32] developed an optimization model of depot location with the ILS factors as constraints. Li and Yi [33] proposed a multi-objective location model based on reliability. However, the existing siting models have shortcomings, such as insufficient adaptability in solving the siting problem of equipment supporting depots. In equipment support and spare parts supply management, supportability indicator requirements play a very important role. However, the previous site selection models did not take these key supportability indicator parameters into consideration, making the model unable to effectively adapt to the development of equipment support, and the adaptability needs to be improved. It is easy to make the selection of the final site location and the arrangement of the transportation volume unreasonable, which makes the management efficiency of spare parts low and the level of guarantee not high. In the process of spare parts assurance supply, there are many other assurance indicators besides meeting the availability requirements. In this paper, optimization modeling is aimed at this kind of problem.

At the same time, due to the lack of historical data on spare parts demand for each maintenance depot, there is uncertainty in demand forecasting. This results in the asymmetry of equipment data that needs to be transported from the equipment supporting depot to each maintenance depot. Therefore, this paper will analyze the spare parts supply and modeling process to establish a site selection model that considers uncertain supply requirements and comprehensive assurance requirements.

The purpose of this paper is to establish an optimization model for location selection and allocation in the context of an integrated support system, with the aim of reducing transportation costs and improving equipment support levels. The structure of this paper is organized as follows: Section 2 introduces some definitions and theorems of uncertainty theory, which provide a mathematical foundation for the establishment of the model. Meanwhile, we adopt uncertainty theory to deal with the asymmetry problem caused by missing historical data. Section 3 describes the location problem to be addressed, illustrating model assumptions and notation descriptions. In Section 4, a location optimization model based on uncertain chance constraints is proposed considering supportability indicator requirements. Section 5 will use the genetic algorithm to solve the model according to the characteristics of the model and introduces the solution steps of the genetic algorithm. Then, we will verify the utility of the model with a numerical example in Section 6. Finally, in Section 7, the paper will discuss and draw conclusions.

2. Preliminaries

At present, the rapid development of equipment has resulted in a relatively fast replacement of equipment, and it is often impossible to accumulate enough historical data. Therefore, the forecast of spare parts demand for some new parts often relies on experts to make assessments on the basis of previous spare parts data information. This leads to assessments with cognitive uncertainty due to asymmetric information. In order to better avoid the influence caused by asymmetric information, we use uncertainty theory to deal with it.

This section mainly introduces some basic definitions and theorems of uncertainty theory and provides theoretical support for the supporting depot selection optimization model under uncertain demand.

The uncertain measure is a class of aggregate functions that satisfy the axioms of uncertainty theory. It is used to express the degree of belief that an uncertain event may occur [34]. Uncertain measure $\mathcal{M}$ on the σ-algebra $\mathcal{L}$. $\mathcal{M}\{\Lambda\}$ is assigned to the event Λ to indicate the belief degree with which we believe Λ will happen.

Definition 1. *(Uncertain Variable) (Liu [20]) An uncertain variable is a function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that $\{\xi \in B\}$ it is an event for any Borel set B of real numbers.*

Definition 2. *(Uncertainty distribution]) (Liu [20]) The uncertainty distribution ξ of an uncertain variable is defined by*

$$\Phi(x) = \mathcal{M}\{\xi \le x\}, \tag{1}$$

for any real number x.

Definition 3. *(Normal uncertainty distribution) (Liu [20]) An uncertain variable ξ is called normal if it has a normal uncertainty distribution*

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right)^{-1}, \; x \in \Re, \tag{2}$$

denoted by $N(e, \sigma)$ where e and σ are real numbers with $\sigma > 0$.

Definition 4. *(Inverse uncertainty distribution) (Liu [34]) Let ξ be an uncertain variable with regular uncertainty distribution $\Phi(x)$. Then the inverse function $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of ξ.*

Theorem 1. *(Liu [34]) A function Φ^{-1} is an inverse uncertainty distribution of an uncertain variable ξ is and only if*

$$\mathcal{M}\left\{\xi < \Phi^{-1}(\alpha)\right\} = \alpha, \tag{3}$$

for all $\alpha \in [0,1]$.

Theorem 2. *(Sufficient and necessary condition) (Liu [35]) A function $\Phi^{-1}(\alpha) : (0,1) \to \Re$ is an inverse uncertainty distribution if and only if it is a continuous and strictly increasing function with respect to α.*

Theorem 3. *(Liu [34]) Let $\xi_1, \xi_2, \cdots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. If the function $f(x_1, x_2, \cdots, x_n)$ is strictly increasing with respect to $x_1, x_2, \cdots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \cdots, x_n$, then the uncertain variable*

$$\xi = f(\xi_1, \xi_2, \cdots, \xi_n), \tag{4}$$

has an inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = f\left(\Phi_1^{-1}(\alpha), \cdots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \cdots \Phi_n^{-1}(1-\alpha)\right). \tag{5}$$

3. Problem Description

The supporting depots provide the safeguard resources needed for the equipment system in the use and maintenance activities and assist in the management to ensure that the equipment system can be used normally. One of the most important resources is the spare parts, which determines the use of security and maintenance security and other functions. In the event of equipment system failure, the timeliness of spare parts supply seriously affects the speed of equipment system restoration to normal status. This paper will address the problem of how to select the optimal location of the supporting depots and allocate the optimal amount of spare parts supply to ensure the timeliness of spare parts supply.

3.1. Model Assumption

In this paper, a model to solve the problem of equipment supporting depots is developed based on some assumptions given as follows:

(1) The demand of each maintenance site is an uncertain variable, and the demand between maintenance sites is independent of each other.

(2) The transportation paths of the safeguard sites and the maintenance sites are connected in a straight line, and the transportation costs are only related to the transportation distance and the transportation volume.

(3) The importance level of each maintenance site is the same.

(4) There may be a supply relationship between any safeguard site and the maintenance site.

(5) The number of safeguard sites is given.

3.2. Notation Description

Notations that will be used are first introduced as follows:

I: the total number of equipment supporting depots;
J: the total number of maintenance depots;
i: index of equipment supporting depots, $i = 1, 2, 3, \ldots, I$;
j: index of maintenance depots, $j = 1, 2, 3, \ldots, J$;
C: transportation cost;
(x_i, y_i): the coordinates of equipment supporting depot i, $i = 1, 2, 3, \ldots, I$;
(a_j, b_j): the coordinates of maintenance depot j, $j = 1, 2, 3, \ldots, J$;
ξ_j: uncertain spare parts demand of maintenance depot j, $j = 1, 2, 3, \ldots, J$;

z_{ij}: freight volume from equipment supporting depot i to maintenance depot j, $i = 1,2,3,\ldots,I$, $j = 1,2,3,\ldots,J$;

α_j: confidence level of the demand met at maintenance depot j, $j = 1,2,3,\ldots,J$;

M_j: number of equipment at maintenance depot j, $j = 1,2,3,\ldots,J$;

N: number of units installed in single equipment;

A_j: the equipment supply availability at maintenance depot j, $j = 1,2,3,\ldots,J$;

β_j: confidence level of equipment supply availability requirement at maintenance depot j, $j = 1,2,3,\ldots,J$;

V_{ij}: the velocity from supporting depot i to maintenance depot j; $i = 1,2,3,\ldots,I$, $j = 1,2,3,\ldots,J$;

T_{SR}: the average spare parts supply response time;

T_{LD_j}: the upper limit of the average logistic delay time requirement at maintenance depot j, $j = 1,2,3,\ldots,J$;

m_i: the inventory cap of each supporting depot i, $i = 1,2,3,\ldots,I$.

4. Uncertain Chance-Constrained Model of Equipment Supporting Depot

In the actual optimization problem process, the established chance-constrained programming model often contains uncertain variables caused by data asymmetry. For the situation that such constraints contain asymmetric data, we will use the uncertain chance-constraint model to carry out conditional constraints.

4.1. Objective Function

In the actual supply process, factors affecting transportation costs generally include transportation vehicles, fixed operating expenses, transportation distance, and transportation volume. Using different means of transportation, such as cars, ships, planes, etc., may result in different transportation costs. Considering the general principles of modeling, this paper does not consider the differences brought about by means of transportation. Fixed operating expenses generally refer to the expenses brought about by operating facilities, such as station maintenance, repairs, etc. The expenses are often relatively fixed and will not affect the site selection decision, so they are not considered. The objective function is the cost C generated by the transportation of spare parts between the supporting depot and the maintenance depot, which is mainly related to the transportation distance and the transportation volume z. The following expressions can be established.

$$C(x,y,z) = \sum_{i=1}^{I}\sum_{j=1}^{J} z_{ij}\sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}. \tag{6}$$

4.2. Constraint Functions

Constraint 1. *Shortage Rate Constraint.*

Due to the complexity of the supply network, the maintenance depot j may accept the supply of spare parts from any supporting depot. It will lead to a shortage when the sum of the spare parts transportation volume z_{ij} of each supporting depot to the maintenance depot j is less than the spare parts demand ξ_j. The following expression guarantees that the uncertain measure that the spare parts of the maintenance depot j can meet the requirements is greater than or equal to α_j, which is specifically expressed as:

$$\mathcal{M}\left\{\xi_j \le \sum_{i=1}^{I} z_{ij}\right\} \ge \alpha_j, j = 1,2,\ldots,J. \tag{7}$$

Constraint 2. *Supply Availability Constraint.*

Supply availability is a parameter index that directly reflects the guarantee effectiveness in the spare parts supply chain. It refers to the expected value of the percentage of the total number of equipment that is not shut down due to any spare parts shortage. Assume that the number of equipment is M_j, each equipment is installed with N spare parts, and the spare parts in the equipment system are in series with each other. According to the definition of supply availability, using the idea of opportunity constraint, it is necessary to ensure that the supply availability requirement A_j of maintenance site j can be met with a confidence level greater than or equal to β_j, specifically expressed as follows:

$$\mathcal{M}\left\{\left[1-\frac{\xi_j-\sum\limits_{i=1}^{I} z_{ij}}{M_j \times N}\right]^N \geq A_j\right\} \geq \beta_j, j=1,2,\ldots,J. \tag{8}$$

Constraint 3. *Average Logistic Delay Time Constraint.*

The guarantee delay time is the delay time caused by the resupply of spare parts due to the shortage of spare parts. When the planned transportation volume is less than the demand for spare parts at the maintenance depot due to the complexity of the equipment task, there will be a support delay. This necessitates the supply of spare parts again from the support site. Using the idea of opportunity constraint, the average guarantee delay time generated during the supply of spare parts is required, which is expressed as

$$\mathcal{M}\left\{\left[1-\frac{\xi_j-\sum\limits_{i=1}^{I} z_{ij}}{M_j \times N}\right]^N \geq A_j\right\} \geq \beta_j, j=1,2,\ldots,J. \tag{9}$$

where T_{SR} is the average spare parts supply response time, mainly related to the distance and transport speed V between the safeguard site and the maintenance site. The expression can be established as follows:

$$T_{SR}=\frac{\sqrt{(x-a)^2+(y-b)^2}}{V}. \tag{10}$$

Therefore, Equation (9) can be rewritten as:

$$\left(1-\mathcal{M}\left\{\xi_j \leq \sum_{i=1}^{I} z_{ij}\right\}\right)\sum_{i=1}^{I}\frac{\sqrt{(x_i-a_j)^2+(y_i-b_j)^2}}{I \times V_{ij}} \leq T_{LDj}, j=1,2,\ldots,J. \tag{11}$$

Constraint 4. *Inventory Limit Constraint.*

Safeguarding site i often has an inventory cap m_i, which is specified as follows:

$$\sum_{j=1}^{J} z_{ij} \leq m_i, i=1,2,\ldots,I. \tag{12}$$

Based on the above constraints, for the optimization of equipment supporting depot, the model established in this paper is as follows:

$$\begin{cases} \underset{x,y,z}{\text{Min}}\, C(x,y,z) = \sum\limits_{i=1}^{I}\sum\limits_{j=1}^{J} z_{ij}\sqrt{(x_i-a_j)^2+(y_i-b_j)^2}, \\ \text{s.t.} \\ \quad \mathcal{M}\left\{\xi_j \le \sum\limits_{i=1}^{I} z_{ij}\right\} \ge \alpha_j, j=1,2,\ldots,J \\ \quad \mathcal{M}\left\{\left[1-\frac{\xi_j-\sum_{i=1}^{I} z_{ij}}{M_j\times N}\right]^N \ge A_j\right\} \ge \beta_j, j=1,2,\ldots,J \\ \quad \left(1-\mathcal{M}\left\{\xi_j \le \sum\limits_{i=1}^{I} z_{ij}\right\}\right)\sum\limits_{i=1}^{I}\frac{\sqrt{(x_i-a_j)^2+(y_i-b_j)^2}}{I\times V_{ij}} \le T_{LDj}, j=1,2,\ldots,J \\ \quad \sum\limits_{j=1}^{J} z_{ij} \le m_i, i=1,2,\ldots,I \\ \quad z_j \in N, j=1,2,\ldots,J \end{cases} \tag{13}$$

The uncertain chance constraint conditions can be transformed into deterministic problems to solve. Additionally, the deterministic model is as follows:

$$\begin{cases} \underset{x,y,z}{\text{Min}}\, C(x,y,z) = \sum\limits_{i=1}^{I}\sum\limits_{j=1}^{J} z_{ij}\sqrt{(x_i-a_j)^2+(y_i-b_j)^2}, \\ \text{s.t.} \\ \quad \Phi_j^{-1}(\alpha_j) \le \sum\limits_{i=1}^{I} z_{ij}, j=1,2,\ldots,J \\ \quad \left[1-\frac{\Phi_j^{-1}(\beta_j)-\sum_{i=1}^{I} z_{ij}}{M_j\times N}\right]^N \ge A_j, j=1,2,\ldots,J \\ \quad \left(1-\Phi_j\left(\sum\limits_{i=1}^{I} z_{ij}\right)\right)\sum\limits_{i=1}^{I}\frac{\sqrt{(x_i-a_j)^2+(y_i-b_j)^2}}{I\times V_{ij}} \le T_{LDj}, j=1,2,\ldots,J \\ \quad \sum\limits_{j=1}^{J} z_{ij} \le m_i, i=1,2,\ldots,I \\ \quad z_j \in N.j=1,2,\ldots,J. \end{cases} \tag{14}$$

5. Solution Algorithm

In this section, we will use the genetic algorithm to find the solution that satisfies the constraints and achieves the numerical minimum in the objective function.

A genetic algorithm is a method to search for optimal solutions by simulating the evolution of natural selection and the genetic mechanism of Darwinian biological evolution. This heuristic algorithm is commonly used to generate solutions to optimize and search for problems. The genetic algorithm simulates the behavior of reproductive crossover and genetic mutation in the process of natural selection and heredity, keeps a set of candidate solutions in each iteration and selects the better individuals from the mass of candidate solutions according to some index, and then uses genetic operators to combine these individuals to produce a new generation of solution population, by repeating this process until some requirement is satisfied.

The algorithm is implemented in the following steps:

Step 1: Randomly generate initial chromosomes to form an evolutionary population as the initial solution;

Step 2: Perform chromosome screening to delete individuals in the population that do not satisfy the constraints of Equation (14);

Step 3: Use the evaluation function to evaluate the fitness of each chromosome. We define the fitness function as the objective function and update the record of the individual with the highest fitness;

Step 4: Genetic manipulation (reproduction, crossover, mutation) to form the next generation population;

Step 5: Determine if the maximum number of generations is reached. If not, return to the second step;

Step 6: Finally, retain the individual with the highest fitness as the optimal solution.

6. A Numerical Example

In this section, we use a spare parts supply assurance system to verify the practicality and operability of the above model approach.

The supply guarantee system in the example consists of 10 maintenance depots and 4 equipment supporting depots. The distribution of each maintenance depot is shown in the table below. Due to the complexity of the supply network, the maintenance site can accept the supply of spare parts from any support site. Now it is necessary to optimize the site selection of the four supporting depots and optimize the transportation volume allocation from the supporting depot to each maintenance depot so as to minimize the transportation cost. The demand for spare parts at each maintenance depot is set to obey a normal distribution. The upper limit of the inventory of the equipment supporting depot is set to 100, where the guarantee index requirements such as shortage rate, average guarantee delay time, uncertainty measure, supply availability, and related parameters are shown in Table 1.

Table 1. Relevant parameters of maintenance depots.

Parameters	**1**	**2**	**3**	**4**	**5**
(a,b)	(5,10)	(10,90)	(70,20)	(40,40)	(50,70)
$\mathcal{N}(e,\sigma)$	(20,5)	(40,6)	(20,5)	(30,6)	(20,5)
M	5	10	5	8	5
N	2	2	2	2	2
A	0.8	0.8	0.8	0.8	0.8
T_{LD}	10	10	10	10	10
α	0.8	0.8	0.8	0.8	0.8
β	0.7	0.7	0.7	0.7	0.7
V	60	60	60	60	60
Parameters	6	7	8	9	10
(a,b)	(70,70)	(90,20)	(80,60)	(50,5)	(60,100)
$\mathcal{N}(e,\sigma)$	(40,6)	(20,5)	(30,6)	(40,6)	(30,6)
M	10	5	8	10	8
N	2	2	2	2	2
A	0.8	0.8	0.8	0.8	0.8
T_{LD}	10	10	10	10	10
α	0.8	0.8	0.8	0.8	0.8
β	0.7	0.7	0.7	0.7	0.7
V	60	60	60	60	60

Bring the above parameters into the optimization model, and use the genetic algorithm to solve the result. Among them, the genetic population is set to 50, the crossover probability and the mutation probability is set to constraint dependent. Table 2 shows the results of the models after 800 generations of evolution:

Table 2. Optimal supply plan: spare parts supply of each depot.

Parameters	1	2	3	4
(x,y)	(28,76)	(67,68)	(36,35)	(71,29)
z_{i1}	2	0	22	0
z_{i2}	37	0	8	0
z_{i3}	0	0	2	22
z_{i4}	5	0	25	5
z_{i5}	19	3	3	0
z_{i6}	10	35	0	0
z_{i7}	0	0	4	20
z_{i8}	0	22	3	10
z_{i9}	0	0	17	28
z_{i10}	10	25	0	0

Table 2 shows the optimal site selection results and spare parts supply results of the equipment supporting depots. The supply volume planning of each support depot to the maintenance depots is shown in Figure 1. Under this freight volume and route planning, the optimized transportation cost is 7978.4.

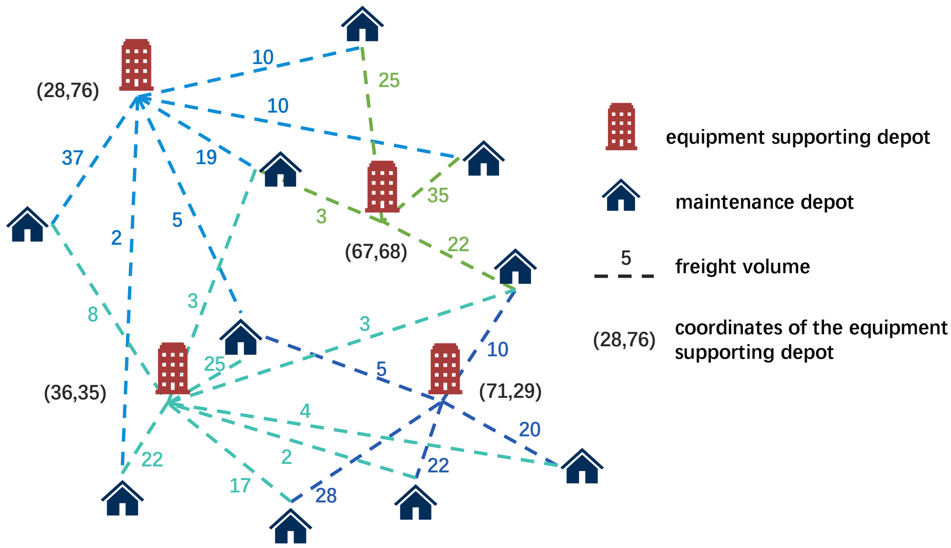

Figure 1. Optimal supply plan: Supply volume planning of each support depot.

7. Discussion and Conclusions

The effect of spare parts supply directly affects the use safety and maintenance status of equipment. In the current location selection research, the previous models did not fully consider the supportability indicator requirements in the process of spare parts guarantee supply. The adaptability of the previous model is poor in the location selection of supporting depots, and the adaptability needs to be further improved. Simultaneously, we recognize that there is a great deal of uncertainty in spare part requirements at maintenance depots. Therefore, we use uncertainty theory to reasonably quantify the need for maintenance depots. In this study, the uncertain spare parts demand prediction is combined with the site selection problem of support depots. An optimization model for site selection of support depots is established for coordinating large support supply areas. First, we make the assumption that the demand for spare parts at each repair site is an uncertain variable. Then, we analyze the constraints of the supply network. We choose shipping costs as the objective function for this model. The model is solved using a genetic algorithm. Finally, the validity and operability of the model method are verified by a numerical example.

When the support supply area is large, and there are many maintenance stations, it is necessary for several support stations to cooperate to jointly complete the task of supplying spare parts to the maintenance stations in the support area. In the early stages of operation, it often happens that historical reference data for spare parts requirements is insufficient. In this case, the existing spare parts demand forecasting models have a high probability of cognitive uncertainty. This paper presents a more reasonable site selection method that takes into account the uncertainty of spare parts demand for maintenance sites. In addition, the method takes into account the requirements of various guarantee indicators. To some extent, it provides a method to improve the efficiency of spare parts management, shorten the supply time of spare parts, and improve the guarantee level of equipment.

However, our work also has certain limitations, and there is still room for improvement. In the research process, we ignored some practical constraints. In the model construction process, only the influence of transportation distance and transportation volume of a single type of spare parts on transportation cost is considered. In fact, different kinds of spare parts transportation and inventory strategies exist in the operation of spare parts supply systems. In addition, the choice of transportation route will also affect the transportation cost.

As mentioned earlier, this paper provides a method of site selection based on uncertainties and supportability indicator requirements. Future research can be based on different inventory strategies and different transportation routes to optimize the location selection of the equipment supporting depots for the transportation of various spare parts.

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