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Dynamic Chaotic Multi-Attribute Group Decision Making under Weighted T-Spherical Fuzzy Soft Rough Sets

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Abstract: In this article, the parameter of the decision maker's familiarity with the attributes of the alternatives is introduced for the first time in dynamic multi-attribute group decision making to avoid the disadvantages arising from the inappropriate grouping of decision makers. We combine it with fuzzy soft rough set theory and dynamic multi-attribute-grouping decision making to obtain a new decision model, i.e., dynamic chaotic multiple-attribute group decision making. Second, we provide an algorithm for solving this model under a weighted T-spherical fuzzy soft rough set, which can not only achieve symmetry between decision evaluation and fuzzy information but also establish a good symmetrical balance between decision makers and attributes (evaluation indexes). Finally, a specific numerical computation case is proposed to illustrate the convenience and effectiveness of our constructed algorithm. Our contributions to the literature are: (1) We introduced familiarity for the first time in dynamic multi-attribute group decision making. This makes our given dynamic chaotic multi-attribute group decision-making (DCMAGDM) model more general and closer to the actual situation; (2) we combined dynamic chaotic multi-attribute group decision making with T-spherical fuzzy soft rough set theory to make the model more realistic and reflect the actual situation. In addition, our choice of T-spherical fuzzy soft rough set allows the decision maker to engage in a sensible evaluation rather than sticking to numerical size choices; and (3) we constructed a new and more convenient sorting/ranking algorithm based on weighted T-spherical fuzzy soft rough sets.

Keywords: multi-attribute group decision making; dynamic multi-attribute group decision making; chaotic multi-attribute group decision making; fuzzy set; fuzzy soft rough set; weighted T-spherical fuzzy soft rough set

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1. Introduction

As human society and the world at large become more complex, making the right or acceptable decisions becomes increasingly difficult. In reality, all decision problems themselves evolve over time, so none of the problems can be summarized in a static framework. At the same time, the process of human decision making is dynamic, evolving, interactive, and adaptive. Therefore, real decisions are usually dynamic and group-based, while involving multiple attributes in a changeable space [1,2].

Given this dynamic nature of the real world and the continuous self-learning of decision makers (DMs) and their interactions, relying on dynamic multi-attribute group decision making (DMAGDM) is a very obvious and sensible choice in practice. At the same time, in specific practical decisions, usually all information is uncertain and fuzzy. Therefore, dynamic multi-attribute group decision making (DMAGDM) in fuzzy environments has been a popular topic of research among scholars in recent years.

Xu [3] proposed a dynamic hybrid multi-attribute group decision-making method based on the hybrid geometric aggregation (HGA) operator and the DWGA operator, and he also first introduced a new operator called a dynamic linguistic weighted geometric (DLWG) operator [4]. Xu [5] firstly proposed a Poisson distribution-based method to determine the weight vector associated with a time-weighted averaging (TWA) operator and used this method to successfully solve the multi-stage multi-attribute group decision-making (MS-MAGDM) problem. A new model for dynamic multi-attribute group decision making based on a vague set and its TOPSIS solution was developed by Yang [6] in response to the problem of four-dimensional decision making, which consists of an alternative set, attribute set, time set, and valuator set. An interactive approach was devised by Su et al. [7] to solve dynamic intuitionistic fuzzy multi-attribute group decision-making (DIF-MAGDM) problems, in which all attribute values provided by decision makers (DMs) take the form of intuitionistic fuzzy numbers (IFNs) at periods. For hesitant fuzzy multi-criteria decision making (MCDM), which collects preference data on characteristics over time, Liao et al. [8] proposed various weight determination methods. In linguistic contexts, Dutta and Guha [9] proposed a flexible consensus technique that adopts a fresh advice-generating scheme by taking decision makers' attitudes into account in order to reach agreement at each round of the consensus process. In regards to dynamic multi-attribute group decision-making (DMAGDM) problems, where the weights (including expert, attribute, and time weights) are unknown and the attribute values take the form of intuitionistic fuzzy values (IFVs), the dynamic intuitionistic fuzzy power geometric weighted average (DIFPGWA) operator and the improved prediction model were developed by Yin et al. [10]. Wang et al. [11] proposed a novel multi-attribute group emergency decision-making (MAGEDM) method that deals with not only the dynamic evolution of MAGEDM problems but also takes into account experts' hesitation. For complex multi-attribute large-group decision making (CMALGDM), Chen et al. [12] proposed a random intuitionistic fuzzy factor analysis model, which reduces the dimensionality of the original data and takes into account the underlying random environmental factors that may affect the performances of alternatives. In order to be more consistent with the advantages of both operators and deal with the attitude of DMs more objectively, Aydemir and Gunduz [13] proposed a general score function for q-rung orthopair fuzzy sets (q-ROFSs) and the power neutrality aggregation operator. Li et al. [14] introduced a new MAGDM method that considers the dynamics of DM's opinions, where the opinions are represented using a 2-tuple linguistic model. Furthermore, the technique of sequential preference with similarity to the ideal solution (TOPSIS) was adopted. Ding et al. [15] introduced a interval-valued hesitant fuzzy TODIM method to tackle the dynamic emergency decision-making problem. In dealing with the intricate group decision problem of competitive strategies in a q-rung orthogonal fuzzy environment, Yang and Ding [16] proposed a two-person noncooperative matrix game approach based on a hybrid dynamic expert weight-determination model.Deb et al. [17] gave a study on MAGDM by means of the double hesitant Pythagorean fuzzy Dombi Bonferroni mean and the associated weighting operators. Some experts have also tried to use other methods to study this problem, and they have also achieved success, such as Zolfani et al. [18], who presented a new concept and a new approach in the MADM field, which is called the prospective multiple attribute decision making (PMADM). The PMADM model can very well cover the DMADM concept but instead chooses to focus on future topics. Baykasoğlu and Gölcük [19] developed a data-driven DMAGDM model in fuzzy conditions and proposed a dynamic MADM model by learning fuzzy cognitive maps [20]. More relevant research results can be found in the following two review papers (given by He and Xu [21] and Wang et al. [22]). All these researches not only enrich the theory of DMAGDM but also play an important role in practical applications.

2. Motivation and Comparison

The existing literature gives a large number of methods that have been well applied in solving DMAGDM problems, and most of these studies focus on aggregating or fitting fuzzy information and then modeling the DMAGDM problem according to a specific fuzzy set. However, there are still several aspects that need to be considered in DMAGDM.

- How to group DMs more reasonably.

  Although much of the literature provides some grouping methods for DMs, there are still many problems, such as: if DMs are grouped, and each DM only evaluates the attributes corresponding to their professional knowledge, then who should evaluate the composite attributes that contain different professional knowledge? That is to say, in practice, it is difficult to group DMs according to a certain condition because professional knowledge is also ambiguous. Since most DMs have relatively comprehensive knowledge and a lot of practical experience, they will have good insights even when evaluating attributes in non-professional fields. Therefore, simply grouping them will inevitably lose many very valuable evaluations.

- The relationship between DMs and decision-making attributes.

  Specifically, most studies weighted DMs and decision attributes or determined the weights of DMs and decision attributes separately by certain methods, and then aggregated evaluation scores. However, the weight of DMs' evaluations of each decision attribute is not considered, that is, the relationship between DMs and the decision attribute is not considered.

Here is a good example to illustrate these two points. In hospitals, it is common to encounter patients with serious conditions that require specialists from different medical fields to consult and then give a surgical plan. Because specialists do not necessarily specialize in the same field, their assessment of the same metric has a different impact on the overall treatment plan. For example, the assessment of a specialist specializing in surgery is more important in designing a procedure than that of a specialist specializing in cardiovascular treatment.Conversely, even the best surgical specialist needs to consult with the bedside nurse about the patient's physical status because the nurse is more aware of the patient's true physical condition compared with the surgical specialist. This means that the weight of the expert's evaluations is related to the corresponding attribute and not only to the popularity of the expert. Therefore, in MAGDM, we should consider not only the weights of DMs and the weights of attributes but also the relationship between DMs and decision attributes.

Based on these considerations, to avoid unreasonable groupings of DMs, and to consider the relationship between DMs and decision-making attributes, we use "familiarity" to describe the relationship between DMs and decision attributes. Since there will be a lot of uncertainty and fuzziness in the actual DMAGDM problem, and also different scenarios of DMADGMs will have different DMs and different decision attributes, in the absence of a clear way to identify the relationship between DMs and decision attributes, it is a good choice to use fuzzy theory to describe the familiarity of DMs with attributes.Therefore, we chose to combine the T-spherical fuzzy soft rough set (T-SFSRS) [23] given by Muahmmad and Martino with familiarity to obtain a new weighted T-SFSRS to describe all evaluation scores in DMAGDM. Accordingly, a new ranking function is also given based on the new weighted T-SFSRS. Thus, a new method for DMAGDM is given, and finally the good practicality of the method is analyzed by numerical calculations. Our contributions to the literature are:

- We introduced familiarity in DMAGDM for the first time to portray the relationship between DMs and attributes. By introducing familiarity, the annoyance and unreasonableness of DMs from groupings can be avoided;
- We obtained a more general new weighted T-SFSRS by considering the familiarity. By giving the weighted T-SFSRS concept, familiarity can be better taken into account;
- We provided a model for the DMAGDM considering familiarity, which we call dynamic chaotic multi-attribute group decision making (DCMAGDM). This model can better describe the real decision problem compared with the existing models;
- We provide an algorithm for DCMAGDM in complex fuzzy scenarios. A basic framework structure is also provided for the algorithm of the DMAGDM problem.

The remainder of this paper is structured as follows: The theoretical background on the methodologies used within the scope of this paper is given in Section 3. The proposed model and algorithm is provided in Section 4. The numerical analysis of the proposed model is illustrated in Section 5. Conclusions are given in Section 6. The data we used in the numerical case analysis can be found in Appendix A.

3. Preliminaries

In this section, to continuously extend the prevailing works, we recall several existing theories, called fuzzy soft rough sets (FSRSs) and dynamic multiple-attribute group decision making (DMAGDM), as well as their basic rules and properties.

3.1. Fuzzy Soft Rough Sets Theorem

In this subsection, according to the development order of fuzzy set, rough set, and soft set theory, we give the basic preliminary concepts related to a fuzzy set (FS), soft set (SS), rough set (RS), fuzzy soft set (FSS), and fuzzy soft rough set (FSRS), and their basic rules and properties are also described.

3.1.1. Fuzzy Set (FS)

To deal with uncertainties and vagueness. Zadeh [24] explored a concept which may be of use in dealing with "class" with a continuum of grades of membership, namely the famous Fuzzy Set (FS). It is defined as follows:

Definition 1 (Zadeh [24])**.** *A fuzzy set A on a universe X is an object of the form*

$$A = \{(x, \mu_A(x)) | x \in X\} \tag{1}$$

where $\mu_A(x) \in [0,1]$ is called the "degree of membership of x in A". The variable $\mu_A(x)$ is called an ordinary fuzzy number (FN).

For situations in which there is uncertainty about the degree of membership, an intuitionistic fuzzy set (IFS) was developed by Atanassov [25].

Definition 2 (Atanassov [26])**.** *An intuitionistic fuzzy set A on a universe X is an object of the form*

$$A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\} \tag{2}$$

where $\mu_A(x) \in [0,1]$ is called the "degree of membership of x in A", $\nu_A(x) \in [0,1]$ is called the "degree of non-membership of x in A", and where $\mu_A(x)$ and $\nu_A(x)$ satisfy the following condition:

$$(\forall x \in X) \quad (\mu_A(x) + \nu_A(x) \leq 1).$$

The pair $(\mu_A(x),\ \nu_A(x))$ is called an intuitionistic fuzzy number (IFN).

Smarandache [27] generalized intuitionistic fuzzy sets (IFSs) to neutrosophic sets (NSs). Neutrosophic sets are characterized by truth membership function (T), indeterminacy membership function (I), and falsity membership function (F).

Definition 3 (Smarandache [28])**.** *A neutrosophic set A on a universe X is an object of the form*

$$A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x)) | x \in X\} \tag{3}$$

where $\mu_A(x) \in [0,1]$ is called the "truth membership function of x in A", $\eta_A(x) \in [0,1]$ is called the "indeterminacy membership function of x in A", $\nu_A(x) \in [0,1]$ is called the "falsity membership function of x in A", and where $\mu_A(x)$, $\eta_A(x)$ and $\nu_A(x)$ satisfy the following condition:

$$(\forall x \in X) \quad (\mu_A(x) + \eta_A(x) + \nu_A(x) \leq 3).$$

Cuong and Kreinovich [29] introduced the concept of a picture fuzzy set (PFS). PFS is more practical for situations such as elections, where there are not only yes and no votes but also abstentions. Obviously, PFS is a direct extension of the FS and the IFS.

Definition 4 (Cuong and Kreinovich [29])**.** *A picture fuzzy set A on a universe X is an object of the form*

$$A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x)) | x \in X\} \tag{4}$$

where $\mu_A(x) \in [0,1]$ is called the "degree of positive membership of x in A", $\eta_A(x) \in [0,1]$ is called the "degree of neutral membership of x in A", $\nu_A(x) \in [0,1]$ is called the "degree of negative membership of x in A", and where $\mu_A(x)$, $\eta_A(x)$ and $\nu_A(x)$ satisfy the following condition:

$$(\forall x \in X) \quad (\mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1),$$

then, for $x \in X$, let $\pi_A(x) = 1 - (\mu_A(x) + \eta_A(x) + \nu_A(x))$, and $\pi_A(x)$ could be called the "degree of refusal membership of x in A". Let $PFS(X)$ denote the set of all the picture fuzzy sets on a universe X. A triplet $(\mu_A(x), \eta_A(x), \nu_A(x))$ can be identified as a picture fuzzy number (PFN).

Human opinion cannot be restricted to yes or no, as depicted by an conventional fuzzy set (FS) or intuitionistic fuzzy set (IFS), but it can be yes, abstain, no, and refusal, as explained by picture fuzzy set (PFS). In order for DMs to focus their attention more on scoring the decision attributes rather than on whether the scored values satisfy the requirements of condition $(\forall x \in X) \quad (\mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1)$ in the PFS. Mahmood et al. [30] proposed the concept of a spherical fuzzy set (SFS) and T-spherical fuzzy set (T-SFS) as a generalization of FS, IFS, and PFS.

Definition 5 (Mahmood et al. [30])**.** *A T-spherical fuzzy set A on a universe X is an object of the form*

$$A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x)) | x \in X\} \tag{5}$$

where $\mu_A(x) \in [0,1]$ is called the "degree of positive membership of x in A", $\eta_A(x) \in [0,1]$ is called the "degree of neutral membership of x in A", $\nu_A(x) \in [0,1]$ is called the "degree of negative membership of x in A", and where $\mu_A(x)$, $\eta_A(x)$ and $\nu_A(x)$ satisfy the following condition:

$$(\forall x \in X) \quad (\mu^t_A(x) + \eta^t_A(x) + \nu^t_A(x) \leq 1).$$

Then, for $x \in X$, let $\pi_A(x) = \sqrt[t]{1 - (\mu^t_A(x) + \eta^t_A(x) + \nu^t_A(x))}$, and $\pi_A(x)$ could be called the "degree of refusal membership of x in A". Let $T\text{-}SFS(X)$ denote the set of all the T-spherical fuzzy sets (T-SFS) on a universe X. A triplet $(\mu_A(x), \eta_A(x), \nu_A(x))$ can be identified as a spherical fuzzy number ($T\text{-}SFN$). If $t = 2$, the T-spherical fuzzy set is called a spherical fuzzy set (SFS), and the corresponding $SFS(X)$ denotes the set of all the spherical fuzzy sets on a universe X. A triplet $(\mu_A(x), \eta_A(x), \nu_A(x))$ can be identified as a spherical fuzzy number (SFN).

Although there are many other fuzzy sets, such as the interval-valued fuzzy set (IVFS) Zadeh [31], fuzzy multi-set (FMS) Yager [32], neutrosophic set (NS) Smarandache [28], hesitant fuzzy set (HFS) Torra and Narukawa [33], Pythagorean fuzzy set (PyFS) Yager and Abbasov [34], q-rung orthopair fuzzy sets (q-ROFS) Yager [35], Fermatean fuzzy set (FFS) Senapati and Yager [36], hybrid fuzzy set, etc. These theories play a very important role in practice; however, due to limited space and the focus of our article, we will not repeat them here.

3.1.2. Rough Set (RS)

The concept of rough sets (RS) was proposed in 1982 by Pawlak as a mathematical way to handle vagueness, uncertainty, and imprecision in data.

Definition 6 (Pawlak [37])**.** *Let R be an equivalence relation on the universe X ($X \neq \emptyset$) and (X, R) be a Pawlak approximation space. A subset $A \subseteq X$ is called definable if $\underline{R}(A) = \overline{R}(A)$; in the opposite case, i.e., if $\overline{R}(A) - \underline{R}(A) \neq \emptyset$, A is said to be a rough set, where the two operations are defined as:*

$$\underline{R}(A) = \{x \in X | [x]_R \subseteq A\} \tag{6}$$

$$\overline{R}(A) = \{x \in X | [x]_R \cap A \neq \emptyset\} \tag{7}$$

As an illustration, let us consider the following Example 1.

Example 1. *Suppose Table 1 is an information system of RS, the universe $X = \{a_i | i = 1, 2, \cdots, 8\}$, and $A = \{a_1, a_3, a_5, a_7\}$. Suppose the equivalence relation R is that the attributes c_1, c_3, and c_5 have the same value, then $[X]_R = \{\{a_1, a_2\}, \{a_3, a_7\}, \{a_4, a_6, a_8\}, \{a_5\}\}$. Thus, we can obtain the following results: $\underline{R}(A) = \{a_3, a_5, a_7\}$, $\overline{R}(A) = \{a_1, a_2, a_3, a_5, a_7\}$. Since $\overline{R}(A) - \underline{R}(A) \neq \emptyset$, A is a rough set.*

Table 1. An information system of rough set.

	c_1	c_2	c_3	c_4	c_5
a_1	1	2	0	0	1
a_2	1	0	0	2	1
a_3	2	0	0	1	0
a_4	0	0	1	2	1
a_5	2	1	0	2	1
a_6	0	0	1	2	1
a_7	2	0	0	1	0
a_8	0	1	1	1	1

3.1.3. Soft Set (SS)

Molodtsov [38] proposed a mathematical theory to deal with uncertain information, emphasizing the study of uncertainty information from the perspective of parameters—this is the famous soft set (SS) theory.

Let X be the non-empty finite set of objects called the universe. C is a set of parameters (attributes) about objects in X, and attributes are used to describe the characteristics or properties of the object X. The (X, C) is called a soft space, and the mathematical concept of a soft set is defined as follows:

Definition 7 (Molodtsov [38])**.** *Let X be the universe. C is a set of parameters (attributes) about objects in X, and $\mathcal{C} \subseteq C$, φ is a mapping given by $\varphi : \mathcal{C} \to 2^X$; 2^X is the power set of X, then a pair $(\varphi, \mathcal{C})$ is named a soft set (SS) over the universe X.*

As an illustration, let us consider the following Example 2.

Example 2. *Suppose Table 2 is an information system of soft set. $A = \{a_1, a_2, a_3, a_4\}$ is a universe of soft set, while $C = \{c_1, c_2, c_3, c_4, c_5\}$ is the set of parameters. According to Definition 7 of the soft set, we could easily come to the results as follows:*

$$\varphi(c_1) = \{a_1, a_2\}; \varphi(c_2) = \{a_2, a_3, a_4\}, \cdots,$$
$$\varphi(c_1, c_2) = \{a_2\}; \varphi(c_1, c_4) = \{a_1, a_2\}, \cdots,$$
$$\varphi(c_2, c_3, c_5) = \{a_3\}, \varphi(c_2, c_4, c_5) = \{a_4\}, \cdots.$$

Take $\varphi(c_1, c_4)$ as an example, meaning that the objects with attributes c_1 and c_4 are a_1 and a_2, respectively.

Table 2. An information system of the soft set.

	c_1	c_2	c_3	c_4	c_5
a_1	1	0	0	1	0
a_2	1	1	0	1	0
a_3	0	1	1	0	1
a_4	0	1	0	1	1

3.1.4. Fuzzy Soft Set (FSS)

Maji et al. [39] proposed the definition of fuzzy soft sets by combining fuzzy set theory and soft set theory. In fact, a fuzzy soft set can be thought of as a parameterized fuzzy set of a given universe, which is an object expression model with parameters and fuzzy information. It is now more versatile and can be applied in a wide range of uncertainty-related fields because it is no longer restricted to the soft set's object parameters of 0 and 1 [40].

Definition 8 (Maji et al. [39])**.** *Let X be a universal set , C be a collection of parameters regarding X, and $FS(X)$ represent the collection of all FSs over the universe X . A pair $(\varphi, \mathcal{C})$ is said to be a fuzzy soft set (FSS) over X, where $\mathcal{C} \subseteq C$ and $\varphi : \mathcal{C} \to FS(X)$, and where $FS(X)$ represents the collection of all the fuzzy sets on a universe X. For every $x \in X$, the FSS can be defined as follows:*

$$S = \{(x, \varphi(x))|x \in \mathcal{C}, \varphi(x) \in FS(X)\} \tag{8}$$

Particularly, when $|\mathcal{C}| = 1$, the fuzzy soft set degenerates to a fuzzy set.

Example 3. *Suppose Table 3 is an information system of the soft set and using IFS to score. $A = \{a_1, a_2, a_3, a_4\}$ is a universe of the soft set, $C = \{c_1, c_2, c_3, c_4, c_5\}$ is the set of parameters, and $\mathcal{C} = \{c_1, c_3\}$. According to Definition 8 of the FSS, we could easily obtain:*

$$S = \{(c_1, \{(a_1, 0.6, 0.1), (a_2, 0.8, 0.1)\}), (c_3, \{(a_3, 0.4, 0.4)\})\}$$

Table 3. An information system of fuzzy soft set.

	c_1	c_2	c_3	c_4	c_5
a_1	(0.6, 0.1)	(0.0, 0.0)	(0.0, 0.0)	(0.6, 0.2)	(0.0, 0.0)
a_2	(0.8, 0.1)	(0.7, 0.2)	(0.0, 0.0)	(0.8, 0.1)	(0.0, 0.0)
a_3	(0.0, 0.0)	(0.5, 0.3)	(0.4, 0.4)	(0.0, 0.0)	(0.8, 0.1)
a_4	(0.0, 0.0)	(0.9, 0.1)	(0.0, 0.0)	(0.7, 0.2)	(0.6, 0.4)

If the fuzzy set is a spherical fuzzy set, then we have the spherical fuzzy soft set (SFSS), and the definition is as follows:

Definition 9 (Ahmmad et al. [41])**.** *Let X be a universal set , C be a collection of parameters regarding X, and $SFS(X)$ represent the collection of all SFSs over the universe X . A pair $(\varphi, \mathcal{C})$ is said to be a spherical fuzzy soft set (SFSS) over X, where $\mathcal{C} \subseteq C$ and $\varphi : \mathcal{C} \to SFS(X)$, and where $SFS(X)$ represents the collection of all the fuzzy sets on a universe X. For every $x \in X$, the SFSS can be defined as follows:*

$$S = \{(x, \varphi(x))|x \in \mathcal{C}, \varphi(x) \in SFS(X)\}. \tag{9}$$

Particularly, when $|\mathcal{C}| = 1$, the fuzzy soft set degenerates to a spherical fuzzy set.

3.1.5. Fuzzy Soft Rough Set (FSRS)

By combining the FSS with the RS, we can obtain the FSRS, which is defined as follows:

Definition 10 (Muahmmad and Martino [23]). *Let X be the universe, C be a set of parameters (attributes) about objects in X, T-SFS$\{X\}$ be the collection of all T-spherical fuzzy soft sets over the universe X, $\mathbb{R}$ be a T-spherical fuzzy soft set relation from universe X to $\mathcal{C}$ (that is, $\forall c \in \mathcal{C} \subseteq C, \mathbb{R}(c) \in$T-SFSS$(X)$), and ψ be a mapping given by $\psi : \mathcal{C} \rightarrow$ T-SFSS$\{X\}$. Then, $(\psi, \mathcal{C}, \mathbb{R})$ is known as a T-spherical fuzzy soft rough approximation space. For every $\mathcal{F} \in$T-SFS$(\mathcal{C})$, the lower and upper approximation of $\mathcal{F}$ can be defined as follows:*

$$\underline{\mathbb{R}}(\mathcal{F}) = \{(x, \underline{\mu}(x), \underline{\eta}(x), \underline{\nu}(x))|x \in X\} \tag{10}$$
$$\overline{\mathbb{R}}(\mathcal{F}) = \{(x, \overline{\mu}(x), \overline{\eta}(x), \overline{\nu}(x))|x \in X\} \tag{11}$$

where

$$\underline{\mu}(x) = \wedge_{c \in \mathcal{C}}(\mu_{\mathbb{R}}(x, c) \wedge \mu_{\mathcal{F}}(c)), \tag{12}$$
$$\underline{\eta}(x) = \vee_{c \in \mathcal{C}}(\eta_{\mathbb{R}}(x, c) \vee \eta_{\mathcal{F}}(c)), \tag{13}$$
$$\underline{\nu}(x) = \vee_{c \in \mathcal{C}}(\nu_{\mathbb{R}}(x, c) \vee \nu_{\mathcal{F}}(c)). \tag{14}$$

and

$$\overline{\mu}(x) = \vee_{c \in \mathcal{C}}(\mu_{\mathbb{R}}(x, c) \vee \mu_{\mathcal{F}}(c)), \tag{15}$$
$$\overline{\eta}(x) = \wedge_{c \in \mathcal{C}}(\eta_{\mathbb{R}}(x, c) \wedge \eta_{\mathcal{F}}(c)), \tag{16}$$
$$\overline{\nu}(x) = \wedge_{c \in \mathcal{C}}(\nu_{\mathbb{R}}(x, c) \wedge \nu_{\mathcal{F}}(c)). \tag{17}$$

Here, $0 \leq \underline{\mu}^t(x) + \underline{\eta}^t(x) + \underline{\nu}^t(x) \leq 1$, $0 \leq \overline{\mu^t}(x) + \overline{\eta^t}(x) + \overline{\nu^t}(x) \leq 1$, *then,*

$$\mathbb{R}(\mathcal{F}) = (\underline{\mathbb{R}}(\mathcal{F}), \overline{\mathbb{R}}(\mathcal{F})) = \{(x, (\underline{\mu}(x), \overline{\mu}(x)), (\underline{\eta}(x), \overline{\eta}(x)), (\underline{\nu}(x), \overline{\nu}(x)))|x \in X\}. \tag{18}$$

when $t = 2$, the T-SFSRS is a SFSRS, and when $t = 1$, the T-SFSRS is a PFSRS.

Definition 11 (Muahmmad and Martino [23]). *Let*

$$\mathbb{R}(\mathcal{F}) = (\underline{\mathbb{R}}(\mathcal{F}), \overline{\mathbb{R}}(\mathcal{F})) = \{(x, (\underline{\mu}(x), \overline{\mu}(x)), (\underline{\eta}(x), \overline{\eta}(x)), (\underline{\nu}(x), \overline{\nu}(x)))|x \in X\}.$$

be a T-SFSRS on universe X, then the score function can be defined as:

$$\mathcal{S}(x) = \underline{\mu}^t(x) + \overline{\mu}^t(x) - \underline{\eta}^t(x) - \overline{\eta}^t(x) - \underline{\nu}^t(x) - \overline{\nu}^t(x), (x \in X). \tag{19}$$

Example 4. *In a multi-attribute decision-making (MADM) problem, suppose the set of alternatives is $A = \{a_1, a_2, a_3\}$ and the set of attributes is $C = \{c_1, c_2, c_3, c_4\}$, which are presented in Table 4 . Let $\mathcal{F}$ be a T-SFS over C, such that:*

$$\mathcal{F} = \{(c_1, 0.83, 0.12, 0.03), (c_2, 0.76, 0.12, 0.04), (c_3, 0.56, 0.21, 0.12), (c_4, 0.16, 0.81, 0.03)\}$$

Table 4. An information system of PFSS.

	c_1	c_2	c_3	c_4
	(0.83, 0.12, 0.03)	**(0.76, 0.12, 0.04)**	**(0.56, 0.21, 0.12)**	**(0.16, 0.81, 0.03)**
a_1	(0.96, 0.02, 0.01)	(0.82, 0.13, 0.02)	(0.73, 0.12, 0.13)	(0.43, 0.34, 0.15)
a_2	(0.97, 0.01, 0.01)	(0.91, 0.05, 0.04)	(0.77, 0.14, 0.18)	(0.76, 0.07, 0.02)
a_3	(0.92, 0.05, 0.02)	(0.56, 0.26, 0.02)	(0.78, 0.23, 0.01)	(0.99, 0.01, 0.00)

Then, we can obtain the following results (here t = 1).

$$\underline{\mathbb{R}}(\mathcal{F}) = \{(a_1, 0.16, 0.81, 0.15), (a_2, 0.16, 0.81, 0.18), (a_3, 0.16, 0.81, 0.12)\};$$
$$\overline{\mathbb{R}}(\mathcal{F}) = \{(a_1, 0.96, 0.02, 0.01), (a_2, 0.97, 0.01, 0.01), (a_3, 0.99, 0.01, 0.00)\}.$$

Additionally, $\mathcal{S}(a_1) = 0.13, \mathcal{S}(a_2) = 0.12, \mathcal{S}(a_3) = 0.21$, then $\mathcal{S}(a_3) > \mathcal{S}(a_1) > \mathcal{S}(a_2)$. So, we can obtain: $a_3 \succ a_1 \succ a_2$.

3.2. Dynamic Multiple-Attribute Group Decision Making (DMAGDM)

In this subsection, descriptions of general MAGDM and DMAGDM are given first, followed by the definition of chaotic MAGDM, and finally the concept of dynamic chaotic MAGDM.

3.2.1. Multi-Attribute Group Decision Making (MAGDM)

A decision-making problem can be defined simply as a problem of choosing the best answer from a set of possible answers based on a set of attributes or criteria. Let $A = \{a_1, a_2, \ldots, a_m\}$ be the set of alternatives (the set of possible answers) that are evaluated from different aspects, named attributes, criteria, or objects ($C = \{c_1, c_2, \ldots, c_n\}$ is the set of criteria). That is, for attribute c_j, for instance, the alternative A_i is described by the evaluation value x_{ij}, named evaluation value, or the preference of the ith alternative against the jth attribute. Thus, for each alternative A_i, we have an evaluation vector $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T$. Such problems, including a set of alternatives and a set of attributes, are called multi-attribute decision-making (MADM) problems. In these problems, making a decision or choosing the best answer is based on a set of attributes. A MADM method shows how attribute information should be processed in order to obtain a choice. To decide, a decision maker (DM) must assess the alternatives. These assessments are performed in two basic ways. The first, named the ordinal method, arranges alternatives from the worst to the best; however, the superiority of one over the other is ignored. The second, named the cardinal method, not only ranks the alternatives but also specifies the priority value [42].

In many real problems, we have a group of decision makers (DMs). The corresponding MADM becomes multi-attribute group decision making (MAGDM). The following notations are used to depict the MAGDM problems:

- $A = \{a_1, a_2, \ldots, a_m\}$ is the set of considered alternatives;
- $C = \{c_1, c_2, \ldots, c_n\}$ is the set of attributes, which are used for evaluating alternatives;
- $E = \{e_1, e_2, \ldots, e_l\}$ is the set of the DMs involved in the decision-making process;
- $w = (w_1, w_2, \cdots, w_n)^T$ is the weight vector of the attribute ($w \geqslant 0, \sum_{j=1}^{n} w_j = 1$);
- $\tau = (\tau_1, \tau_2, \cdots, \tau_l)^T$ is the weight vector of the DMs ($\tau \geqslant 0, \sum_{k=1}^{l} \tau_k = 1$);
- $X = \{X^k | e_k \in E\}$ is the decision matrix set, while X^k is the decision matrix of the kth DM:

$$X^k = \begin{array}{c c} & \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_m \end{array} & \begin{pmatrix} x_{11}^k & x_{12}^k & \cdots & x_{1n}^k \\ x_{21}^k & x_{22}^k & \cdots & x_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1}^k & x_{m2}^k & \cdots & x_{mn}^k \end{pmatrix} \end{array} \quad k = 1, 2, \cdots, l; \tag{20}$$

- x_{ij}^k is the kth DM evaluation of the ith alternative against the jth attribute ($k = 1, 2, \cdots, l,\ i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$).

In order to describe MAGDM more clearly and concisely, a MAGDM problem can usually be represented by a 6-tuple $\langle A, C, E, w, \tau, X\rangle$, that is:

$$MAGDM = \langle A, C, E, w, \tau, X\rangle. \tag{21}$$

3.2.2. Dynamic Multi-Attribute Group Decision Making (DMAGDM)

In practice, for the $MAGDM = \langle A, C, E, w, \tau, X \rangle$ problem, all its parameters will change according to the period κ ($\kappa \in Z^+$). In this case, MAGDM is called dynamic multi-attribute group decision making (DMAGDM). That is,

$$DMAGDM(\kappa) = \langle A(\kappa), C(\kappa), E(\kappa), w(\kappa), \tau(\kappa), X(\kappa) \rangle. \tag{22}$$

The following notations are used to depict the DMAGDM problems:

- $A(\kappa) = \{a_1^\kappa, a_2^\kappa, \ldots, a_{m(\kappa)}^\kappa\}$ is the set of considered alternatives and $m(\kappa)$ is the number of alternatives in the period κ;
- $C(\kappa) = \{c_1^\kappa, c_2^\kappa, \ldots, c_{n(\kappa)}^\kappa\}$ is the set of attributes, which are used for evaluating of alternatives, and $n(\kappa)$ is the number of attributes in the period κ;
- $E(\kappa) = \{e_1^\kappa, e_2^\kappa, \ldots, e_{l(\kappa)}^\kappa\}$ is the set of the DMs involved in the decision-making process and $l(\kappa)$ is the number of DMs in the period κ;
- $w(\kappa) = (w_1^\kappa, w_2^\kappa, \cdots, w_{n(\kappa)}^\kappa)^T$ is the weight vector of the attribute ($w_j^\kappa \geqslant 0, \sum_{j=1}^{n(\kappa)} w_j^\kappa = 1$);
- $\tau^\kappa = (\tau_1^\kappa, \tau_2^\kappa, \cdots, \tau_{l(\kappa)}^\kappa)^T$ is the weight vector of the DMs ($\tau_k^\kappa \geqslant 0, \sum_{k=1}^{l(\kappa)} \tau_k^\kappa = 1$);
- $X(\kappa) = \{X^k(\kappa) | k = 1, 2, \cdots, l(\kappa)\}$ is the decision matrix set, while $X^k(\kappa)$ is the decision matrix of the kth DM in the period κ;
- $x_{ij}^k(\kappa)$ is the kth DM evaluation of the ith alternative against the jth attribute in the period κ ($k = 1, 2, \cdots, l(\kappa);\ i = 1, 2, \cdots, m(\kappa);\ j = 1, 2, \cdots, n(\kappa)$).

4. A Novel New Method for DCMAGDM under Weighted T-SFSRS

In this section, first, we construct a DCMAGDM model that is more consistent with the actual decision; second, we define the concept of weighted T-SFSRS; third, we give the dynamic transfer equation of DCMAGDM; and finally, we design a more accurate and flexible algorithm for DCMAGDM.

4.1. Dynamic Chaotic Multi-Attribute Group Decision Making (DCMAGDM)

In the actual multi-attribute group decision-making process, it is often found that DMs have different levels of familiarity with the attributes; in other words, different DMs may have different levels of familiarity with the same attribute or the same DM has different levels of familiarity with different decision attributes.

There is a very pertinent example to illustrate this situation. When a doctor encounters a patient with a serious illness or a very complex condition, a group of specialists from different departments will usually be consulted. Different specialists have different areas of expertise, and different specialists are not necessarily familiar with the same areas, for example, some specialists specialize in the treatment of heart and brain diseases, while others are more familiar with the analysis of blood test results. In this case, the utility of each expert's evaluation of each indicator is related not only to that expert's weight but also to that expert's familiarity with that indicator.

Therefore, how to take familiarity into account in dynamic multi-attribute group decision making (DMAGDM) is very meaningful and challenging research. In this subsection, we propose a MAGDM with a new scenario or new condition in which not only the weights of the DMs and the weights of the decision attributes are considered but also the familiarity of the DMs with the attributes. That is, DMs must consider the influence of the DM's familiarity with the attributes in the decision-making process. Since the influence of the DM's familiarity with the attributes on the final decision is similar to the influence of the initial values on a deterministic dynamic system, we similarly refer to this complex MAGDM as "chaotic MAGDM".

Definition 12. *Let $MAGDM = \langle A, C, E, w, \tau, X \rangle$ be a multi-attribute group decision-making process and Φ be a mapping given by $\Phi : (E, C) \rightarrow \mathcal{F}$ (i.e., $\mathcal{F} = \Phi(E, C)$). Where $\mathcal{F}$ (the element in $\mathcal{F}$ can be a definite real number or a fuzzy number) is known as the DM's familiarity with the attributes. Then 7-tuple $\langle A, C, E, w, \tau, X, \mathcal{F} \rangle$ is named as chaotic multi-attribute group decision making and denoted as:*

$$CMAGDM = \langle A, C, E, w, \tau, X, \mathcal{F} \rangle \tag{23}$$

Theorem 1. *If $\forall e \in E$, $\forall c \in C$, $\mathcal{F}(e, c) \equiv \alpha$ (α is any constant), then*

$$\langle A, C, E, w, \tau, X, \alpha \rangle = \langle A, C, E, w, \tau, X \rangle.$$

Proof. Obviously, if $\mathcal{F}$ is a constant, it means that all familiarities are the same, and that there is no need to analyze the familiarity anymore. So, at this point, CMAGDM becomes MAGDM. □

Remark 1. *A MAGDM is called CMAGDM if there exists at least one decision attribute such that at least two DMs have the different familiarity with it, or the same DM has different levels of familiarity with at least two different decision attributes.*

To describe CMAGDM more intuitively, we give the CMAGDM information form, as shown in Table 5.

Table 5. Chaotic MAGDM information form.

		c_1	c_2	$\cdots$	c_n
		w_1	w_2	$\cdots$	w_n
$e_1 \vert \tau_1$		f_1^1	f_2^1	$\cdots$	f_n^1
	a_1	x_{11}^1	x_{12}^1	$\cdots$	x_{1n}^1
	a_2	x_{21}^1	x_{22}^1	$\cdots$	x_{2n}^1
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	a_m	x_{m1}^1	x_{m2}^1	$\cdots$	x_{mn}^1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$e_i \vert \tau_i$		f_1^i	f_2^i	$\cdots$	f_n^i
	a_1	x_{11}^i	x_{12}^i	$\cdots$	x_{1n}^i
	a_2	x_{21}^i	x_{22}^i	$\cdots$	x_{2n}^i
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	a_m	x_{m1}^i	x_{m2}^i	$\cdots$	x_{mn}^i
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$e_l \vert \tau_l$		f_1^l	f_2^l	$\cdots$	f_n^l
	a_1	x_{11}^l	x_{12}^l	$\cdots$	x_{1n}^l
	a_2	x_{21}^l	x_{22}^l	$\cdots$	x_{2n}^l
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	a_m	x_{m1}^l	x_{m2}^l	$\cdots$	x_{mn}^l

Similarly, for the $CMAGDM = \langle A, C, E, w, \tau, X, \mathcal{F} \rangle$ problem, all its parameters will change accordingly with the period κ ($\kappa \in Z^{+}$). In this case, CMAGDM is called dynamic chaotic multi-attribute group decision making (DCMAGDM). That is

$$DCMAGDM(\kappa) = \langle A(\kappa), C(\kappa), E(\kappa), w(\kappa), \tau(\kappa), X(\kappa), \mathcal{F}(\kappa) \rangle, \tag{24}$$

where $A(\kappa), C(\kappa), E(\kappa), w(\kappa), \tau(\kappa), X(\kappa)$ have exactly the same meaning as previously given, and the meaning of $\mathcal{F}(\kappa)$ is as follows:

- $\mathcal{F}(\kappa) = \{\mathcal{F}^{1}(\kappa), \mathcal{F}^{2}(\kappa), \cdots, \mathcal{F}^{l(\kappa)}(\kappa)\}$ is the set of the familiarity of DMs with attributes in the period κ;
- $\mathcal{F}^{k}(\kappa) = (f_1^k(\kappa), f_2^k(\kappa), \cdots, f_{n(\kappa)}^k)^T$ is the familiarity vector of the kth DM in the period κ $(k = 1, 2, \cdots, l(\kappa))$;
- $f_j^k(\kappa)$ is the familiarity of the kth DM against the jth attribute in the period κ. $(k = 1, 2, \cdots, l(\kappa);\ j = 1, 2, \cdots, n(\kappa))$

4.2. Weighted T-Spherical Fuzzy Soft Rough Sets

In the existing literature, many scholars have given different aggregation operators based on the spherical fuzzy set [41,43,44]. There are also many results on aggregation operators given on the basis of T-spherical fuzzy sets [45–48]. These results have been very inspiring to us. In order to allow DMs to both accurately describe the fuzzy information in DCMAGDM and to have a broader choice without having to be bound to the constraints of the evaluation score values, we chose to use T-SFSRS to express all the fuzzy information in the decision process. In addition, in order to better integrate familiarity $\mathcal{F}$ into the T-SFSRS, based on Definition 10 proposed by Muahmmad and Martino, we redefined the lower and upper approximation of $\mathcal{F}$. We named it as the weighted t-spherical fuzzy soft rough set. The definition is as follows:

Definition 13. *Let X be the universe, C be a set of attributes with the weight w_{C} about objects in X, $T\text{-}SFS\{X\}$ be the collection of all T-spherical fuzzy soft sets over the universe X, $\mathbb{R}$ be a T-spherical fuzzy soft set relation from universe X to $\mathcal{C}$ (that is, $\forall c \in \mathcal{C} \subseteq C, \mathbb{R}(c) \in T\text{-}SFSS(X)$), and ψ be a mapping given by $\psi : \mathcal{C} \to T\text{-}SFSS\{X\}$. Then, $(\psi, \mathcal{C}, \mathbb{R}, w_{C})$ is known as a weighted t-spherical fuzzy soft rough approximation space. For every $\mathcal{F} \in T\text{-}SFS(\mathcal{C})$, the lower and upper approximation of $\mathcal{F}$ can be defined as follows:*

$$\underline{\mathbb{R}}(\mathcal{F}) = \{(x, \underline{\mu}(x), \underline{\eta}(x), \underline{\nu}(x)) | x \in X\} \tag{25}$$

$$\overline{\mathbb{R}}(\mathcal{F}) = \{(x, \overline{\mu}(x), \overline{\eta}(x), \overline{\nu}(x)) | x \in X\} \tag{26}$$

where

$$\underline{\mu}(x) = \min_{c \in \mathcal{C}}(w_c \cdot \mu_{\mathbb{R}}(x, c) \cdot \min(\mu_{\mathcal{F}}(c), \sqrt[t]{(1 - \eta_{\mathcal{F}}^t(c) - \nu_{\mathcal{F}}^t(c))})), \tag{27}$$

$$\underline{\eta}(x) = \max_{c \in \mathcal{C}}(w_c \cdot \eta_{\mathbb{R}}(x, c) \cdot \max(\eta_{\mathcal{F}}(c), \sqrt[t]{(1 - \mu_{\mathcal{F}}^t(c) - \nu_{\mathcal{F}}^t(c))})), \tag{28}$$

$$\underline{\nu}(x) = \max_{c \in \mathcal{C}}(w_c \cdot \nu_{\mathbb{R}}(x, c) \cdot \max(\nu_{\mathcal{F}}(c), \sqrt[t]{(1 - \mu_{\mathcal{F}}^t(c) - \eta_{\mathcal{F}}^t(c))})). \tag{29}$$

and

$$\overline{\mu}(x) = \max_{c \in \mathcal{C}}(w_c \cdot \mu_{\mathbb{R}}(x,c) \cdot \max(\mu_{\mathcal{F}}(c), \sqrt[t]{(1 - \eta^t_{\mathcal{F}}(c) - \nu^t_{\mathcal{F}}(c))})), \tag{30}$$

$$\overline{\eta}(x) = \min_{c \in \mathcal{C}}(w_c \cdot \eta_{\mathbb{R}}(x,c) \cdot \min(\eta_{\mathcal{F}}(c), \sqrt[t]{(1 - \mu^t_{\mathcal{F}}(c) - \nu^t_{\mathcal{F}}(c))})), \tag{31}$$

$$\overline{\nu}(x) = \min_{c \in \mathcal{C}}(w_c \cdot \nu_{\mathbb{R}}(x,c) \cdot \min(\nu_{\mathcal{F}}(c), \sqrt[t]{(1 - \mu^t_{\mathcal{F}}(c) - \eta^t_{\mathcal{F}}(c))})). \tag{32}$$

where, $0 \leq \underline{\mu}^t(x) + \underline{\eta}^t(x) + \underline{\nu}^t(x) \leq 1$, $0 \leq \overline{\mu}^t(x) + \overline{\eta}^t(x) + \overline{\nu}^t(x) \leq 1$, *then,*

$$\mathbb{R}(\mathcal{F}) = (\underline{\mathbb{R}}(\mathcal{F}), \overline{\mathbb{R}}(\mathcal{F})) = \{(x, (\underline{\mu}(x), \overline{\mu}(x)), (\underline{\eta}(x), \overline{\eta}(x)), (\underline{\nu}(x), \overline{\nu}(x))) | x \in X\}. \tag{33}$$

The evaluation function in Definition 11 given by Muahmmad and Martino is:

$$\mathcal{S}(x) = \underline{\mu}^t(x) + \overline{\mu}^t(x) - \underline{\eta}^t(x) - \overline{\eta}^t(x) - \underline{\nu}^t(x) - \overline{\nu}^t(x), (x \in X).$$

Given that the evaluation function's value decreases as the parameter t rises, making it difficult to calculate and compare numerical results, the evaluation function must be optimized. Considering that the evaluation function value may be negative, it is natural to think of selecting an odd number $(2t - 1)$ for root operations in order to make sense of the root operation. The new evaluation function is as follows.

Definition 14. *The score function is as follows:*

$$\mathcal{S}(x) = (\underline{\mu}^t(x) + \overline{\mu}^t(x) - \underline{\eta}^t(x) - \overline{\eta}^t(x) - \underline{\nu}^t(x) - \overline{\nu}^t(x))^{\frac{1}{2t-1}}, (x \in X). \tag{34}$$

Example 5. *Still analyzing the data in Example 4 with the weight vector* $w = \{0.25, 0.25, 0.25, 0.25\}^T$, *under new Definitions 13 and 14, the corresponding results (here t = 1) are:*

$$\underline{\mathbb{R}}(\mathcal{F}) = \{(a_1, 0.0172, 0.0689, 0.0075), (a_2, 0.0304, 0.0142, 0.0104), (a_3, 0.0396, 0.0184, 0.0006)\};$$
$$\overline{\mathbb{R}}(\mathcal{F}) = \{(a_1, 0.2040, 0.0006, 0.0001), (a_2, 0.2061, 0.0003, 0.0001), (a_3, 0.1955, 0.0015, 0.0000)\}.$$

Additionally, $\mathcal{S}(a_1) = 0.1442, \mathcal{S}(a_2) = 0.2116, \mathcal{S}(a_3) = 0.2146$*; then,* $\mathcal{S}(a_3) > \mathcal{S}(a_2) > \mathcal{S}(a_1)$*. So, we can obtain:* $a_3 \succ a_2 \succ a_1$.

Considering that different DMs have different decision weights, we give the corresponding total evaluation function for the CMAGDM as follows:

Definition 15. *In CMAGDM, the total evaluation score function is* $\mathbb{S}(a_i)$*:*

$$\mathbb{S}(a_i) = \sum_{k=1}^{l} \tau_k \mathcal{S}_k(a_i) \tag{35}$$

$\mathcal{S}_k(a_i)$ *is the kth DM's score for the ith alternative (*$i = 1, \ldots, m$*;* $k = 1, 2, \ldots, l$*);* τ_k *is the weight of the kth DM (*$\tau_k \geqslant 0, \sum_{k=1}^{l} \tau_k = 1$*).*

In order to allow DMs to focus their attention on evaluation scoring without considering the limitations of scoring values, let DMs chose T-SFN (T-Spherical Fuzzy Number) for evaluation scoring. Therefore, after the DMs give the evaluation score, we need to determine the value of the corresponding parameter t. The value of the parameter t can be obtained by solving a nonlinear programming problem as follows:

$$\begin{aligned} &\min \quad t \\ &s.t. \quad \begin{cases} \mu^t_{\mathbb{R}}(a_i, c_j) + \eta^t_{\mathbb{R}}(a_i, c_j) + \nu^t_{\mathbb{R}}(a_i, c_j) \leq 1 \\ \mu^t_{\mathcal{F}}(a_i, c_j) + \eta^t_{\mathcal{F}}(a_i, c_j) + \nu^t_{\mathcal{F}}(a_i, c_j) \leq 1 \\ i = 1, 2, 3 \ldots, m; j = 1, 2, 3 \ldots, n; t \in Z^+ \end{cases} \end{aligned} \tag{36}$$

4.3. Dynamic Transfer Equation of DCMAGDM

Considering the dynamic nature of the decision process, in DCMAGDM, a central issue is how to transfer past evaluations of alternatives to the next period. In other words, how to establish a reasonable transfer function is the key to solving DCMAGDM.

In this subsection, we first give the general form of the dynamic transfer equation for DCMAGDM; then, we provide the definition of the dynamic transfer equation chosen for this paper

4.3.1. General Dynamic Transfer Equation in DCMAGDM

Definition 16. *In period κ ($\kappa \in Z^+$), let DCMAGDM be as follows.*

$$DCMAGDM(\kappa) = \langle A(\kappa), C(\kappa), E(\kappa), w(\kappa), \tau(\kappa), X(\kappa), \mathcal{F}(\kappa) \rangle, \tag{37}$$

with the corresponding evaluation function for the ith alternative in the κth period by the kth DM as $\mathcal{S}^k_\kappa(a_i)$.

The total evaluation score function in the κth period is

$$\mathbb{S}_\kappa(a_i) = \begin{cases} 0, & a_i \notin A(\kappa) \\ \xi(w(\kappa), \tau(\kappa), X(\kappa), \mathcal{F}(\kappa)), & a_i \in A(\kappa) \end{cases} \tag{38}$$

where $\xi(\cdot)$ is called the information aggregation function (it can be any function with practical meaning, and $0 \leq \xi(\cdot) \leq 1$).

The total evaluation value up to the κ period is as follows:

$$\hat{\mathbb{S}}_\kappa(a_i) = \begin{cases} \mathbb{S}_1(a_i), & \kappa = 1 \\ \mathbb{S}_\kappa(a_i) + \psi(\hat{\mathbb{S}}_{\kappa-1}(a_i)), & \kappa \geq 2 \end{cases} \tag{39}$$

where $\psi(\cdot)$ is called the evaluation cumulative function (it can be any function with practical meaning, and $0 \leq \psi(\cdot) \leq 1$), and Equation (39) is called a general dynamic transfer equation.

Usually, $\xi(\cdot)$ and $\psi(\cdot)$ can be chosen according to the specific reality of the decision problem, and in most studies the weighted arithmetic average operator or the weighted geometric average operator becomes the frequently chosen method.

4.3.2. Weighted Dynamic Transfer Equation Based on T-SFSRS

Our study is based on the following hypotheses:

- All objects in $A(\kappa)$ and $C(\kappa)$ may change with the period κ;
- All elements in $w(\kappa)$ and $\tau(\kappa)$ are the real number;
- All elements in $X(\kappa)$ and $\mathcal{F}(\kappa)$ are presented in the form of the T-Spherical Fuzzy Number (T-SFN);
- For the information aggregation function $\xi(\cdot)$, we choose the weighted arithmetic mean operator:

$$\xi(w(\kappa), \tau(\kappa), X(\kappa), \mathcal{F}(\kappa)) = \sum_{a_i \in A(\kappa)} \tau_k \mathcal{S}^k_\kappa(a_i);$$

- For the evaluation cumulative function $\psi(\cdot)$, considering that the evaluation score of the previous period will weaken as the period κ keeps increasing, we choose the following processing means to reflect this point,

$$\psi(\hat{\mathbb{S}}_{\kappa-1}(a_i)) = \theta\, \hat{\mathbb{S}}_{\kappa-1}(a_i),$$

where $0 \leqslant \theta < 1$, which we name as the discount coefficient.

Based on these assumptions, we can obtain the following definition:

Definition 17. *Let* $DCMAGDM(\kappa) = \langle A(\kappa), C(\kappa), E(\kappa), w(\kappa), \tau(\kappa), X(\kappa), \mathcal{F}(\kappa)\rangle, (\kappa \in Z^+)$, *while the corresponding evaluation function for the ith alternative in the κth period by the kth DM is* $\mathcal{S}_\kappa^k(a_i)$*:*

$$\mathcal{S}_\kappa^k(a_i) = (\underline{\mu}^t(a_i) + \overline{\mu}^t(a_i) - \underline{\eta}^t(a_i) - \overline{\eta}^t(a_i) - \underline{\nu}^t(a_i) - \overline{\nu}^t(a_i))^{\frac{1}{2t-1}}, a_i \in A(\kappa) \tag{40}$$

The total evaluation score function in the κth period is

$$\mathbb{S}_\kappa(a_i) = \begin{cases} 0, & a_i \notin A(\kappa) \\ \sum\limits_{e_k \in E(\kappa)} \tau_k \mathcal{S}_\kappa^k(a_i), & a_i \in A(\kappa) \end{cases} \tag{41}$$

The total evaluation value up to the κ period is as follows:

$$\hat{\mathbb{S}}_\kappa(a_i) = \begin{cases} \mathbb{S}_1(a_i), & \kappa = 1 \\ \mathbb{S}_\kappa(a_i) + \theta\, \hat{\mathbb{S}}_{\kappa-1}(a_i) & \kappa \geq 2 \end{cases} \tag{42}$$

we call Equation (42) the weighted dynamic transfer equation based on T-SFSRS, where θ $(0 \leqslant \theta < 1)$ *is a real number, which we call the discount coefficient.*

Theorem 2. *For* $\forall \kappa \in \{1, 2, \cdots, \mathcal{N}\}$ *and* $\forall a_i \in \bigcup\limits_{\kappa=1}^{\mathcal{N}} A(\kappa)$, *the value of* $\hat{\mathbb{S}}_\kappa(a_i)$ *must be bounded.*

Proof. Since the elements in the T-SFN are all greater than or equal to zero and less than or equal to 1, from Equations (40) and (41), we can derive:

$$|\mathbb{S}_\kappa(a_i)| \leqslant 6,$$

from Equation (42), we have:

$$\begin{aligned} \left|\hat{\mathbb{S}}_\kappa(a_i)\right| &= \left|\sum_{r=1}^{\kappa} \theta^{(\kappa-r)} \hat{\mathbb{S}}_r(a_i)\right| \\ &\leqslant \sum_{r=1}^{\kappa} \theta^{(\kappa-r)} \left|\hat{\mathbb{S}}_r(a_i)\right| \\ &\leqslant 6 \sum_{r=1}^{\kappa} \theta^{(\kappa-r)} \\ &\leqslant \frac{6}{1-\theta}. \end{aligned}$$

□

The boundedness analysis helps us to better choose the value of the discount coefficient θ when making specific decisions.

4.4. The Algorithm for DCMAGDM

Through the analysis in the previous subsection, we can then design an algorithm that can reflect the symmetry between decision evaluations and fuzzy information, while also achieving a symmetric balance between decision makers and attributes (evaluation metrics) as well. We summarize the Algorithm 1 for solving DCMAGDM as follows:

Algorithm 1 The Algorithm for DCMAGDM.

Input:
  Total number of periods: $\mathcal{N}$
  Discount Coefficient: θ
  for $\kappa = 1$ to $\mathcal{N}$ **do**
    Input: $A(\kappa), C(\kappa), E(\kappa), w(\kappa), \tau(\kappa), X(\kappa), \mathcal{F}(\kappa)$.
  end for
Output:
  Sorting/Ranking of alternatives: $a_1^* \succ a_2^* \succ \cdots \succ a_i^* \succ \cdots;$
  Step 1:
  Calculate the evaluation score.
  for $\kappa = 1$ to $\mathcal{N}$ **do**
    for $a_i \in A(\kappa)$ **do**
      for $e_k \in E(\kappa)$ **do**
        Calculate $\mathcal{S}_\kappa^k(a_i)$ according to the Formula (40)
      end for
      Calculate $\mathbb{S}_\kappa(a_i)$ according to the Formula (41)
    end for
  end for
  Calculate $\hat{\mathbb{S}}_\kappa(a_i)$ according to the Formula (42)
  Step 2:
  Sorting by the value of $\mathbb{S}_\kappa(a_i)$ and $\hat{\mathbb{S}}_\kappa(a_i)$
  return $a_1^* \succ a_2^* \succ \cdots \succ a_i^* \succ \cdots;$ and $\hat{a}_1^* \succ \hat{a}_2^* \succ \cdots \succ \hat{a}_i^* \succ \cdots;$

5. Numerical Analysis

To better illustrate our model, let us introduce a simple numerical example. It is assumed that this case has the following characteristics:

- DM has different weights in different periods, and there are three DMs in the whole decision-making process, $E = \{e_1, e_2, e_3\}$;
- Each attribute may have different weights in different periods, with a total of five attributes to be considered, $C = \{c_1, c_2, c_3, c_4, c_5\}$;
- There are nine alternatives, each of which may or may not appear in a given period, $A = \{a_1, a_2, \cdots, a_9\}$;
- DMs select T-SFN for scoring evaluation.

All the evaluation information is shown in Tables A1–A3 in Appendix A.

5.1. First Iteration

In the first period, we consider the inclusion of only eight alternatives,

$$A(1) = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\},$$

and a reasonable explanation is that a_9 is not found by the DMs in this period.

Step 1:

By Equation (36), we can obtain $t = 5$.

Step 2:

According to Definition 13, we can obtain the lower and upper bounds of T-SFSRS and the score in the first period, as shown in Table 6.

Table 6. The lower and upper bounds of T-SFSRS for the first period.

		$(\underline{\mu}, \underline{\eta}, \underline{\nu})$	$(\overline{\mu}, \overline{\eta}, \overline{\nu})$	$\mathcal{S}$
e_1	a_1	(0.04485, 0.02494, 0.01918)	(0.24750, 0.00013, 0.00000)	0.46037
	a_2	(0.04485, 0.03093, 0.01353)	(0.24500, 0.00050, 0.00000)	0.45778
	a_3	(0.04347, 0.02355, 0.01740)	(0.23750, 0.00013, 0.00000)	0.44994
	a_4	(0.04623, 0.03866, 0.02030)	(0.24000, 0.00000, 0.00000)	0.45256
	a_5	(0.04347, 0.03480, 0.01535)	(0.23750, 0.00000, 0.00000)	0.44994
	a_6	(0.04140, 0.02513, 0.03286)	(0.24750, 0.00025, 0.00000)	0.46036
	a_7	(0.04416, 0.03383, 0.03286)	(0.24250, 0.00000, 0.00000)	0.45517
	a_8	(0.04623, 0.02632, 0.02610)	(0.24500, 0.00013, 0.00000)	0.45778
e_2	a_1	(0.08742, 0.04534, 0.02625)	(0.22000, 0.00006, 0.00018)	0.43165
	a_2	(0.08554, 0.07266, 0.03873)	(0.22000, 0.00003, 0.00012)	0.43143
	a_3	(0.08460, 0.03580, 0.04295)	(0.21250, 0.00005, 0.00021)	0.42342
	a_4	(0.08930, 0.04844, 0.03147)	(0.21750, 0.00002, 0.00021)	0.42899
	a_5	(0.08836, 0.04117, 0.03873)	(0.21500, 0.00001, 0.00000)	0.42626
	a_6	(0.08836, 0.05570, 0.06293)	(0.21750, 0.00002, 0.00007)	0.42885
	a_7	(0.08742, 0.03580, 0.09440)	(0.22500, 0.00000, 0.00012)	0.43641
	a_8	(0.08930, 0.04773, 0.02662)	(0.21750, 0.00001, 0.00021)	0.42900
e_3	a_1	(0.08010, 0.09444, 0.02802)	(0.23750, -0.00020, -0.00013)	0.44965
	a_2	(0.07740, 0.04843, 0.04670)	(0.24750, -0.00020, -0.00003)	0.46049
	a_3	(0.07920, 0.03970, 0.01935)	(0.24250, -0.00050, -0.00008)	0.45535
	a_4	(0.08100, 0.08475, 0.05322)	(0.24250, -0.00050, -0.00003)	0.45509
	a_5	(0.07650, 0.08233, 0.08467)	(0.24250, -0.00010, -0.00003)	0.45484
	a_6	(0.07650, 0.06296, 0.03387)	(0.25000, 0.00000, -0.00005)	0.46302
	a_7	(0.08010, 0.06054, 0.09193)	(0.23750, -0.00030, 0.00000)	0.44966
	a_8	(0.07830, 0.04671, 0.06290)	(0.24500, -0.00010, -0.00003)	0.45787

Step 3:

According to Equation (41), we can calculate the total evaluation score in this period, as shown in Table 7. So, the final ranking obtained is

$$a_6 \succ a_2 \succ a_1 \succ a_8 \succ a_7 \succ a_4 \succ a_5 \succ a_3 \quad (43)$$

Table 7. Total score and best ranking for the first period.

	$\hat{a}_{-}^{1*} = a_{-}^{1*}$	$A(1)$	$\hat{\mathbb{S}}_1 = \mathbb{S}_1$
$\kappa = 1$	a_1^{1*}	a_6	0.450521
	a_2^{1*}	a_2	0.449652
	a_3^{1*}	a_1	0.448642
	a_4^{1*}	a_8	0.448302
	a_5^{1*}	a_7	0.447825
	a_6^{1*}	a_4	0.445317
	a_7^{1*}	a_5	0.443153
	a_8^{1*}	a_3	0.442324

5.2. Second Iteration

In the second iteration, we consider seven alternatives: $A(2) = \{a_3, a_4, a_5, a_6, a_7, a_8, a_9\}$, for the following reasons:

- We remove alternatives a_1 and a_2 from $A(1)$ because at the end of an iteration, an alternative will be included in the history set, which means that it may or may not appear in the next iteration. For example, in the new period, the historical alternative no longer exists;
- A new alternative a_9 appears at beginning of the second period.

In addition, the weights of DMs and the weights of attributes also changed in the new period. A reasonable interpretation is as follows:

- The weights of DMs have changed because of the addition of completely new DMs or the reallocation of the weights of DMs in the new period.
- The weights of the attributes have changed because of the DM's perspective in the new period or because new attribute requirements have been introduced.

Next, we analyze and solve for the second period of decision making. The analysis and solution process is exactly the same as in the previous Section 5.1, and we end up with the following results, as shown in Table 8.

Table 8. Total score and best ranking for the second period.

	a_{-}^{2*}	$A(2)$	$\mathbb{S}_2$
$\kappa = 2$	a_1^{2*}	a_6	0.498813
	a_2^{2*}	a_3	0.497476
	a_3^{2*}	a_5	0.493123
	a_4^{2*}	a_9	0.491535
	a_5^{2*}	a_8	0.490639
	a_6^{2*}	a_4	0.489837
	a_7^{2*}	a_7	0.486844

In this article, we choose the discount coefficient: $\theta = 0.25$ using Equation (42), and the total evaluation score of the attributes up to the end of the second period is calculated. Take a_6 as an example:

$$\hat{\mathbb{S}}_2(a_6) = \mathbb{S}_2(a_6) + \theta\, \hat{\mathbb{S}}_1(a_6) = 0.498813 + 0.25 \times 0.450521 = 0.611443 \quad (44)$$

Until the end of the second period, the final scores and ranking results are shown in Table 9.

Table 9. Total score and best ranking until the end of the second period.

	$\hat{a}_{-}^{2*}$	$A(1) \cup A(2)$	$\hat{\mathbb{S}}_2$
$\kappa = 2$	$\hat{a}_1^{2*}$	a_6	0.611443
	$\hat{a}_2^{2*}$	a_3	0.608057
	$\hat{a}_3^{2*}$	a_5	0.603911
	$\hat{a}_4^{2*}$	a_8	0.602714
	$\hat{a}_5^{2*}$	a_4	0.601166
	$\hat{a}_6^{2*}$	a_7	0.598800
	$\hat{a}_7^{2*}$	a_9	0.491535
	$\hat{a}_8^{2*}$	a_2	0.112413
	$\hat{a}_9^{2*}$	a_1	0.112161

So, the final ranking is obtained:

$$a_6 \succ a_3 \succ a_5 \succ a_8 \succ a_4 \succ a_7 \succ a_9 \succ a_2 \succ a_1 \quad (45)$$

5.3. Third Iteration

Similarly, we can obtain the results in the third period. They are shown in Tables 10 and 11.

Table 10. Total score and best ranking for the third period.

	a_{-}^{3*}	$A(3)$	$\mathbb{S}_3$
$\kappa = 3$	a_1^{3*}	a_7	0.598936
	a_2^{3*}	a_8	0.594738
	a_3^{3*}	a_2	0.594109
	a_4^{3*}	a_9	0.593653
	a_5^{3*}	a_6	0.592304

Table 11. Total score and best ranking until the end of the third period.

	$\hat{a}_{-}^{3*}$	$A(1) \cup A(2) \cup A(3)$	$\hat{\mathbb{S}}_3$
$\kappa = 3$	$\hat{a}_1^{3*}$	a_7	0.748636
	$\hat{a}_2^{3*}$	a_8	0.745416
	$\hat{a}_3^{3*}$	a_6	0.745164
	$\hat{a}_4^{3*}$	a_9	0.716537
	$\hat{a}_5^{3*}$	a_2	0.622212
	$\hat{a}_6^{3*}$	a_3	0.152014
	$\hat{a}_7^{3*}$	a_5	0.150978
	$\hat{a}_8^{3*}$	a_4	0.150292
	$\hat{a}_9^{3*}$	a_1	0.028040

Take a_6 as an example:

$$\begin{aligned}
\hat{\mathbb{S}}_3(a_6) &= \mathbb{S}_3(a_6) + \theta\, \hat{\mathbb{S}}_2(a_6) \\
&= \mathbb{S}_3(a_6) + \theta\, \mathbb{S}_2(a_6) + \theta^2\, \mathbb{S}_1(a_6) \\
&= 0.592304 + 0.25 \times 0.611443 + 0.25^2 \times 0.450521 \\
&= 0.745164
\end{aligned}$$

So, the final ranking at the end of third period is obtained:

$$a_7 \succ a_8 \succ a_6 \succ a_9 \succ a_2 \succ a_3 \succ a_5 \succ a_4 \succ a_1 \tag{46}$$

5.4. General Analysis of the Three Periods

We summarize the results corresponding to the three periods as follows (Table 12), and it is easy to see the following conclusions:

- Since the weight of DMs and the weight of attributes may change from period to period, the same alternative has different rankings, e.g., a_7 is ranked fifth in the first period, seventh in the second period, and first in the third period;
- Since each evaluation score can be transferred to subsequent periods, an alternative's ranking in the current period may be different from its overall ranking until current period. For example, the a_6 was ranked fifth in the third period, while it was ranked third overall at end of third period.

Table 12. Ranking of evaluation scores for this period compared with the ranking of total evaluation scores up to this period.

A	$\kappa = 1$		$\kappa = 2$		$\kappa = 3$	
	$\mathbb{S}_1$-Ranking	$\hat{\mathbb{S}}_1$-Ranking	$\mathbb{S}_2$-Ranking	$\hat{\mathbb{S}}_2$-Ranking	$\mathbb{S}_3$-Ranking	$\hat{\mathbb{S}}_3$-Ranking
a_1	3	3	$\times$	9	$\times$	9
a_2	2	2	$\times$	8	3	5
a_3	8	8	2	2	$\times$	6
a_4	6	6	6	5	$\times$	8
a_5	7	7	3	3	$\times$	7
a_6	1	1	1	1	5	3
a_7	5	5	7	6	1	1
a_8	4	4	5	4	2	2
a_9	$\times$	$\times$	4	7	4	4

$\times$ indicates that alternative a_i was not available in this corresponding period.

Information from the change from the 2nd period ($\kappa = 2$) to the new 3rd period ($\kappa' = 3$) can be found in Table A4. The difference between the new 3rd period ($\kappa' = 3$) and the previous 3rd period ($\kappa = 3$) is that the familiarity has changed while no other parameters have changed. We can also obtain the sorting results in the new third period, as shown in Tables 13 and 14.

Table 13. Total score and best ranking for the new third period under new familiarity.

	a_{-}^{3*}	$A(3)$	$\mathbb{S}_3$
$\kappa' = 3$	a_1^{3*}	a_2	0.596897
	a_2^{3*}	a_9	0.591764
	a_3^{3*}	a_6	0.591054
	a_4^{3*}	a_7	0.590738
	a_5^{3*}	a_8	0.585572

Table 14. Total score and best ranking until the end of the new third period under new familiarity.

	$\hat{a}_{-}^{3*}$	$A(1) \cup A(2) \cup A(3)$	$\hat{\mathbb{S}}_3$
$\kappa' = 3$	$\hat{a}_1^{3*}$	a_6	0.743915
	$\hat{a}_2^{3*}$	a_7	0.740438
	$\hat{a}_3^{3*}$	a_8	0.736250
	$\hat{a}_4^{3*}$	a_9	0.714648
	$\hat{a}_5^{3*}$	a_2	0.625000
	$\hat{a}_6^{3*}$	a_3	0.152014
	$\hat{a}_7^{3*}$	a_5	0.150978
	$\hat{a}_8^{3*}$	a_4	0.150292
	$\hat{a}_9^{3*}$	a_1	0.028040

We show the results of the new third period ($\kappa' = 3$) compared with the results of the previous third period ($\kappa = 3$) in Table 15. Obviously, we can obtain the following result:

- For familiarity, even a slight change will have a very significant impact on the sorting. This is exactly why we propose familiarity in the DMAGDM problem to give dynamic chaotic multi-attribute group decisions (DCMAGDM).

Table 15. Different familiarity produces different rankings.

A	$\kappa = 3$		$\kappa' = 3$	
	$\mathbb{S}_3$**-Ranking**	$\hat{\mathbb{S}}_3$**-Ranking**	$\mathbb{S}'_3$**-Ranking**	$\hat{\mathbb{S}}'_3$**-Ranking**
a_1	×	9	×	6
a_2	3	5	1	7
a_3	×	6	×	8
a_4	×	8	×	9
a_5	×	7	×	2
a_6	5	3	3	3
a_7	1	1	4	5
a_8	2	2	5	4
a_9	4	4	2	1

6. Conclusions

In this paper, we introduce a general model for decision making in a dynamic environment, in which we consider not only the dynamic changes of alternatives, attributes, and DMs but also the dynamic changes of their corresponding weights during the decision-making process; in addition, the familiarity between DMs and attributes should also be considered—the "dynamic chaotic multi-attribute group decision" that we first proposed. In order to better solve this problem, we proposed the concept of the familiarity of DMs with attributes in the dynamic multi-attribute group decision problem, which not only can efficiently avoid the information loss caused by grouping experts but also largely avoid the irrationality caused by grouping. Based on this, we apply T-SFSRS to our proposed dynamic chaotic multi-attribute group decision model and give an algorithm for dynamic chaotic multi-attribute group decision making under fuzzy scenarios. The algorithm not only yields the optimal ranking of the alternatives for each period but also the cumulative total ranking of the alternatives for all historical periods, which makes our proposed algorithm flexible enough to be applied in different contexts.

Compared with the models [1,2,4,5,14–16] and algorithm [6–11,13,17–20] in the existing literature, our proposed model is more general or closer to the real decision-making process. The algorithm we give takes into account not only the current optimal decision but also the need for the cumulative analysis of prior evaluations in decision making (e.g., in the evaluation ranking of personnel title promotion, all evaluation results of recent years are often needed instead of just considering only one year's evaluation scores).

Although we try our best to try to solve more realistic decision problems, there are still many problems for which we do not give reasonable solutions. For example, the problem of how to determine the weight of each expert, which is also originally a dynamic multi-attribute decision-making problem. Although a lot of research results have been given in the literature, there is little research on considering the weight of experts in a dynamic context, which is a very challenging and meaningful direction for future research. From another point of view, in our paper, the fuzzy sets we consider are T-spherical fuzzy sets (TSFSs), which means that the evaluation scores must satisfy the condition: $\mu^t_A(x) + \eta^t_A(x) + \nu^t_A(x) \leq 1$. What would be the result if the scoring is performed in a broader definition domain, such as under a neutrosophic set (NS) (which requires the condition: $\mu^t_A(x) + \eta^t_A(x) + \nu^t_A(x) \leq 3$ to be satisfied)? This is also a direction for future research.

In addition, applying our provided models and algorithms to particular practices in various settings, such as decision making in emergency management, personnel management, company administration, etc., is also a very good future study direction.

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Abbreviations

The following abbreviations are used in this manuscript:

DM	Decision Maker
MAGDM	Multi-attribute Group Decision Making
CMAGDM	Chaoti multi-attribute Group Decision Making
DMAGDM	Dynamic multi-attribute Group Decision Making
DCMAGDM	Dynamic chaoti multi-attribute Group Decision Making
FS	Fuzzy set
SFS	Spherical Fuzzy Set
T-SFS	T-spherical Fuzzy Set
T-SFN	T-spherical Fuzzy Number
RS	Rough Set
SS	Soft Set
FSS	Fuzzy Soft Set
FSRS	Fuzzy Soft Rough Set
SFSRS	Spherical Fuzzy Soft Rough Set
T-SFSRS	T-spherical Fuzzy Soft Rough Set

Appendix A. DCMAGDM Information Form

Table A1. DCMAGDM information form for the first period ($\kappa = 1$).

$\kappa = 1$	c_1	c_2	c_3	c_4	c_5
	0.25	**0.25**	**0.25**	**0.15**	**0.10**
e_1 ∣ 0.46	(0.97, 0.05, 0.00)	(0.94, 0.07, 0.01)	(0.88, 0.05, 0.09)	(0.80, 0.16, 0.07)	(0.69, 0.12, 0.15)
a_1	(0.99, 0.01, 0.03)	(0.90, 0.04, 0.10)	(0.86, 0.11, 0.00)	(0.71, 0.18, 0.12)	(0.65, 0.15, 0.15)
a_2	(0.98, 0.04, 0.03)	(0.90, 0.04, 0.02)	(0.89, 0.09, 0.02)	(0.75, 0.16, 0.05)	(0.65, 0.32, 0.14)
a_3	(0.95, 0.01, 0.03)	(0.90, 0.07, 0.00)	(0.89, 0.09, 0.05)	(0.80, 0.17, 0.02)	(0.63, 0.10, 0.18)
a_4	(0.96, 0.00, 0.03)	(0.94, 0.04, 0.00)	(0.88, 0.11, 0.07)	(0.79, 0.16, 0.04)	(0.67, 0.40, 0.21)
a_5	(0.95, 0.00, 0.03)	(0.90, 0.07, 0.08)	(0.89, 0.14, 0.04)	(0.80, 0.20, 0.01)	(0.63, 0.36, 0.06)
a_6	(0.99, 0.02, 0.00)	(0.91, 0.10, 0.01)	(0.88, 0.08, 0.10)	(0.78, 0.17, 0.02)	(0.60, 0.26, 0.34)
a_7	(0.97, 0.00, 0.02)	(0.95, 0.05, 0.04)	(0.88, 0.12, 0.05)	(0.79, 0.17, 0.10)	(0.64, 0.35, 0.34)
a_8	(0.98, 0.01, 0.01)	(0.95, 0.08, 0.02)	(0.87, 0.08, 0.02)	(0.77, 0.19, 0.15)	(0.67, 0.10, 0.27)
e_2 ∣ 0.33	(0.87, 0.05, 0.09)	(0.73, 0.18, 0.19)	(0.68, 0.33, 0.27)	(0.96, 0.01, 0.04)	(0.94, 0.01, 0.07)
a_1	(0.88, 0.09, 0.03)	(0.74, 0.19, 0.11)	(0.68, 0.17, 0.02)	(0.99, 0.04, 0.03)	(0.93, 0.08, 0.03)
a_2	(0.88, 0.07, 0.02)	(0.80, 0.19, 0.07)	(0.65, 0.30, 0.16)	(0.98, 0.02, 0.02)	(0.91, 0.06, 0.09)
a_3	(0.85, 0.06, 0.09)	(0.70, 0.15, 0.18)	(0.60, 0.13, 0.03)	(0.97, 0.04, 0.04)	(0.90, 0.05, 0.03)
a_4	(0.87, 0.15, 0.08)	(0.77, 0.16, 0.07)	(0.64, 0.20, 0.13)	(0.98, 0.01, 0.05)	(0.95, 0.07, 0.03)
a_5	(0.86, 0.12, 0.05)	(0.80, 0.17, 0.08)	(0.69, 0.17, 0.16)	(0.99, 0.01, 0.00)	(0.94, 0.01, 0.07)
a_6	(0.87, 0.14, 0.02)	(0.76, 0.19, 0.07)	(0.63, 0.23, 0.26)	(0.95, 0.01, 0.03)	(0.94, 0.02, 0.01)
a_7	(0.90, 0.08, 0.05)	(0.71, 0.15, 0.03)	(0.66, 0.12, 0.39)	(0.99, 0.00, 0.02)	(0.93, 0.04, 0.03)
a_8	(0.87, 0.13, 0.10)	(0.79, 0.20, 0.03)	(0.70, 0.13, 0.11)	(0.97, 0.02, 0.05)	(0.95, 0.01, 0.03)

Table A1. *Cont.*

$\kappa = 1$	c_1	c_2	c_3	c_4	c_5
	0.25	**0.25**	**0.25**	**0.15**	**0.10**
e_3 ǀ 0.21	(0.78, 0.19, 0.02)	(0.68, 0.36, 0.29)	(0.99, 0.01, 0.04)	(0.90, 0.10, 0.05)	(0.90, 0.05, 0.09)
a_1	(0.72, 0.15, 0.12)	(0.69, 0.39, 0.07)	(0.95, 0.02, 0.05)	(0.95, 0.10, 0.05)	(0.89, 0.10, 0.05)
a_2	(0.74, 0.16, 0.20)	(0.60, 0.20, 0.15)	(0.99, 0.02, 0.01)	(0.92, 0.02, 0.05)	(0.86, 0.13, 0.04)
a_3	(0.74, 0.17, 0.04)	(0.69, 0.10, 0.08)	(0.97, 0.05, 0.03)	(0.91, 0.03, 0.04)	(0.88, 0.09, 0.01)
a_4	(0.80, 0.18, 0.00)	(0.68, 0.35, 0.22)	(0.97, 0.05, 0.01)	(0.92, 0.09, 0.04)	(0.90, 0.08, 0.02)
a_5	(0.77, 0.18, 0.05)	(0.66, 0.34, 0.35)	(0.97, 0.01, 0.01)	(0.91, 0.03, 0.07)	(0.85, 0.12, 0.06)
a_6	(0.72, 0.15, 0.11)	(0.61, 0.26, 0.14)	(0.99, 0.00, 0.02)	(0.92, 0.10, 0.04)	(0.85, 0.06, 0.05)
a_7	(0.74, 0.16, 0.02)	(0.60, 0.25, 0.38)	(0.95, 0.03, 0.00)	(0.94, 0.09, 0.09)	(0.89, 0.09, 0.06)
a_8	(0.76, 0.20, 0.12)	(0.65, 0.10, 0.26)	(0.98, 0.01, 0.01)	(0.93, 0.05, 0.02)	(0.87, 0.13, 0.03)

Note: In the first iteration we consider a set with seven alternatives $A = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$.

Table A2. DCMAGDM information form for the second period ($\kappa = 2$).

$\kappa = 2$	c_1	c_2	c_3	c_4	c_5
	0.32	**0.10**	**0.15**	**0.25**	**0.18**
e_1 ǀ 0.40	(0.97, 0.03, 0.03)	(0.91, 0.03, 0.07)	(0.90, 0.08, 0.05)	(0.76, 0.18, 0.17)	(0.61, 0.38, 0.00)
a_3	(0.99, 0.02, 0.02)	(0.93, 0.02, 0.02)	(0.87, 0.10, 0.08)	(0.80, 0.15, 0.03)	(0.67, 0.24, 0.26)
a_4	(0.96, 0.01, 0.00)	(0.94, 0.00, 0.09)	(0.85, 0.07, 0.02)	(0.77, 0.16, 0.11)	(0.67, 0.27, 0.00)
a_5	(0.97, 0.01, 0.02)	(0.92, 0.03, 0.00)	(0.85, 0.10, 0.06)	(0.73, 0.20, 0.14)	(0.63, 0.30, 0.30)
a_6	(0.97, 0.03, 0.03)	(0.91, 0.02, 0.04)	(0.85, 0.10, 0.06)	(0.72, 0.15, 0.10)	(0.63, 0.35, 0.28)
a_7	(0.96, 0.00, 0.05)	(0.90, 0.03, 0.02)	(0.89, 0.09, 0.07)	(0.77, 0.18, 0.03)	(0.67, 0.31, 0.22)
a_8	(0.95, 0.04, 0.04)	(0.95, 0.08, 0.04)	(0.85, 0.11, 0.02)	(0.72, 0.19, 0.02)	(0.62, 0.21, 0.17)
a_9	(0.97, 0.00, 0.03)	(0.90, 0.05, 0.03)	(0.88, 0.09, 0.02)	(0.78, 0.19, 0.01)	(0.67, 0.10, 0.30)
e_2 ǀ 0.30	(0.90, 0.09, 0.01)	(0.72, 0.18, 0.19)	(0.61, 0.32, 0.00)	(0.98, 0.05, 0.00)	(0.95, 0.08, 0.01)
a_3	(0.85, 0.08, 0.05)	(0.72, 0.15, 0.12)	(0.62, 0.13, 0.39)	(0.97, 0.00, 0.00)	(0.95, 0.05, 0.10)
a_4	(0.87, 0.06, 0.04)	(0.80, 0.20, 0.10)	(0.69, 0.26, 0.15)	(0.96, 0.03, 0.05)	(0.92, 0.00, 0.01)
a_5	(0.89, 0.11, 0.03)	(0.77, 0.20, 0.09)	(0.63, 0.35, 0.05)	(0.97, 0.05, 0.00)	(0.91, 0.00, 0.04)
a_6	(0.90, 0.07, 0.00)	(0.76, 0.18, 0.03)	(0.66, 0.28, 0.35)	(0.95, 0.01, 0.03)	(0.91, 0.04, 0.10)
a_7	(0.87, 0.13, 0.01)	(0.79, 0.19, 0.02)	(0.67, 0.18, 0.38)	(0.97, 0.00, 0.05)	(0.91, 0.09, 0.10)
a_8	(0.87, 0.08, 0.09)	(0.78, 0.17, 0.16)	(0.60, 0.32, 0.37)	(0.99, 0.02, 0.05)	(0.93, 0.01, 0.03)
a_9	(0.89, 0.12, 0.09)	(0.77, 0.16, 0.09)	(0.67, 0.29, 0.27)	(0.95, 0.01, 0.02)	(0.90, 0.06, 0.08)
e_3 ǀ 0.30	(0.70, 0.16, 0.04)	(0.63, 0.11, 0.04)	(0.99, 0.04, 0.04)	(0.93, 0.08, 0.00)	(0.86, 0.13, 0.03)
a_3	(0.80, 0.20, 0.05)	(0.66, 0.26, 0.36)	(0.96, 0.05, 0.05)	(0.90, 0.08, 0.08)	(0.90, 0.08, 0.00)
a_4	(0.74, 0.16, 0.16)	(0.67, 0.32, 0.38)	(0.96, 0.01, 0.00)	(0.93, 0.07, 0.07)	(0.90, 0.11, 0.10)
a_5	(0.74, 0.16, 0.02)	(0.68, 0.34, 0.39)	(0.97, 0.00, 0.03)	(0.95, 0.00, 0.01)	(0.86, 0.13, 0.05)
a_6	(0.79, 0.16, 0.03)	(0.69, 0.25, 0.11)	(0.97, 0.00, 0.01)	(0.95, 0.07, 0.03)	(0.88, 0.05, 0.02)
a_7	(0.70, 0.17, 0.20)	(0.60, 0.20, 0.04)	(0.95, 0.03, 0.03)	(0.91, 0.04, 0.01)	(0.86, 0.15, 0.09)
a_8	(0.76, 0.15, 0.09)	(0.64, 0.30, 0.22)	(0.95, 0.03, 0.03)	(0.90, 0.06, 0.05)	(0.87, 0.06, 0.10)
a_9	(0.70, 0.15, 0.18)	(0.63, 0.26, 0.01)	(0.98, 0.05, 0.01)	(0.93, 0.03, 0.04)	(0.85, 0.14, 0.02)

Note: In the second iteration we consider a set with seven alternatives $A = \{a_3, a_4, a_5, a_6, a_7, a_8, a_9\}$.

Table A3. DCMAGDM information form for the third period ($\kappa = 3$).

$\kappa = 3$	c_1	c_2	c_3	c_4	c_5
	0.45	**0.15**	**0.15**	**0.15**	**0.10**
e_1 ǀ 0.30	(0.99, 0.03, 0.00)	(0.93, 0.03, 0.04)	(0.89, 0.11, 0.08)	(0.71, 0.17, 0.20)	(0.68, 0.24, 0.25)
a_2	(0.97, 0.05, 0.02)	(0.92, 0.04, 0.03)	(0.87, 0.13, 0.01)	(0.71, 0.18, 0.06)	(0.61, 0.30, 0.25)
a_6	(0.96, 0.05, 0.01)	(0.91, 0.08, 0.04)	(0.88, 0.12, 0.00)	(0.71, 0.16, 0.13)	(0.65, 0.11, 0.26)
a_7	(0.96, 0.00, 0.05)	(0.90, 0.04, 0.02)	(0.85, 0.08, 0.01)	(0.80, 0.16, 0.04)	(0.60, 0.15, 0.02)
a_8	(0.96, 0.03, 0.03)	(0.93, 0.02, 0.04)	(0.87, 0.10, 0.01)	(0.80, 0.16, 0.19)	(0.67, 0.40, 0.25)
a_9	(0.97, 0.03, 0.04)	(0.91, 0.09, 0.04)	(0.89, 0.07, 0.05)	(0.74, 0.15, 0.19)	(0.63, 0.16, 0.05)

Table A3. *Cont.*

$\kappa = 3$	c_1	c_2	c_3	c_4	c_5
	0.45	**0.15**	**0.15**	**0.15**	**0.10**
e_2 ∣ 0.40	(0.88, 0.11, 0.09)	(0.80, 0.18, 0.20)	(0.63, 0.16, 0.34)	(0.97, 0.00, 0.01)	(0.91, 0.08, 0.03)
a_2	(0.87, 0.05, 0.08)	(0.75, 0.16, 0.00)	(0.66, 0.35, 0.33)	(0.99, 0.01, 0.05)	(0.93, 0.05, 0.06)
a_6	(0.87, 0.11, 0.10)	(0.80, 0.16, 0.00)	(0.69, 0.33, 0.19)	(0.99, 0.03, 0.01)	(0.90, 0.09, 0.09)
a_7	(0.86, 0.13, 0.05)	(0.72, 0.17, 0.08)	(0.68, 0.29, 0.08)	(0.99, 0.01, 0.04)	(0.91, 0.07, 0.09)
a_8	(0.85, 0.14, 0.04)	(0.78, 0.20, 0.06)	(0.70, 0.13, 0.15)	(0.96, 0.05, 0.05)	(0.93, 0.06, 0.08)
a_9	(0.90, 0.10, 0.10)	(0.80, 0.16, 0.04)	(0.63, 0.31, 0.25)	(0.98, 0.00, 0.04)	(0.94, 0.00, 0.05)
e_3 ∣ 0.30	(0.80, 0.16, 0.01)	(0.66, 0.21, 0.03)	(0.98, 0.02, 0.04)	(0.91, 0.02, 0.02)	(0.90, 0.13, 0.00)
a_2	(0.80, 0.20, 0.20)	(0.63, 0.32, 0.31)	(0.97, 0.00, 0.03)	(0.90, 0.03, 0.05)	(0.87, 0.12, 0.04)
a_6	(0.76, 0.17, 0.05)	(0.60, 0.30, 0.03)	(0.96, 0.03, 0.05)	(0.94, 0.05, 0.04)	(0.88, 0.13, 0.07)
a_7	(0.77, 0.15, 0.16)	(0.66, 0.30, 0.12)	(0.95, 0.02, 0.02)	(0.90, 0.02, 0.08)	(0.85, 0.12, 0.10)
a_8	(0.74, 0.18, 0.05)	(0.67, 0.25, 0.24)	(0.95, 0.02, 0.04)	(0.91, 0.00, 0.05)	(0.86, 0.15, 0.02)
a_9	(0.72, 0.17, 0.02)	(0.63, 0.23, 0.31)	(0.97, 0.00, 0.02)	(0.92, 0.03, 0.08)	(0.87, 0.06, 0.04)

Note: In the third iteration we consider a set with seven alternatives $A = \{a_2, a_6, a_7, a_8, a_9\}$.

Table A4. DCMAGDM information form for the third period with different familiarity ($\kappa' = 3$).

$\kappa' = 3$	c_1	c_2	c_3	c_4	c_5
	0.45	**0.15**	**0.15**	**0.15**	**0.10**
e_1 ∣ 0.30	(0.95, 0.02, 0.03)	(0.91, 0.04, 0.00)	(0.90, 0.15, 0.04)	(0.74, 0.15, 0.08)	(0.61, 0.14, 0.38)
a_2	(0.97, 0.05, 0.02)	(0.92, 0.04, 0.03)	(0.87, 0.13, 0.01)	(0.71, 0.18, 0.06)	(0.61, 0.30, 0.25)
a_6	(0.96, 0.05, 0.01)	(0.91, 0.08, 0.04)	(0.88, 0.12, 0.00)	(0.71, 0.16, 0.13)	(0.65, 0.11, 0.26)
a_7	(0.96, 0.00, 0.05)	(0.90, 0.04, 0.02)	(0.85, 0.08, 0.01)	(0.80, 0.16, 0.04)	(0.60, 0.15, 0.02)
a_8	(0.96, 0.03, 0.03)	(0.93, 0.02, 0.04)	(0.87, 0.10, 0.01)	(0.80, 0.16, 0.19)	(0.67, 0.40, 0.25)
a_9	(0.97, 0.03, 0.04)	(0.91, 0.09, 0.04)	(0.89, 0.07, 0.05)	(0.74, 0.15, 0.19)	(0.63, 0.16, 0.05)
e_2 ∣ 0.40	(0.85, 0.10, 0.08)	(0.77, 0.19, 0.02)	(0.62, 0.35, 0.17)	(0.96, 0.04, 0.03)	(0.95, 0.06, 0.06)
a_2	(0.87, 0.05, 0.08)	(0.75, 0.16, 0.00)	(0.66, 0.35, 0.33)	(0.99, 0.01, 0.05)	(0.93, 0.05, 0.06)
a_6	(0.87, 0.11, 0.10)	(0.80, 0.16, 0.00)	(0.69, 0.33, 0.19)	(0.99, 0.03, 0.01)	(0.90, 0.09, 0.09)
a_7	(0.86, 0.13, 0.05)	(0.72, 0.17, 0.08)	(0.68, 0.29, 0.08)	(0.99, 0.01, 0.04)	(0.91, 0.07, 0.09)
a_8	(0.85, 0.14, 0.04)	(0.78, 0.20, 0.06)	(0.70, 0.13, 0.15)	(0.96, 0.05, 0.05)	(0.93, 0.06, 0.08)
a_9	(0.90, 0.10, 0.10)	(0.80, 0.16, 0.04)	(0.63, 0.31, 0.25)	(0.98, 0.00, 0.04)	(0.94, 0.00, 0.05)
e_3 ∣ 0.30	(0.74, 0.17, 0.14)	(0.65, 0.19, 0.23)	(0.99, 0.03, 0.01)	(0.92, 0.05, 0.00)	(0.85, 0.06, 0.00)
a_2	(0.80, 0.20, 0.20)	(0.63, 0.32, 0.31)	(0.97, 0.00, 0.03)	(0.90, 0.03, 0.05)	(0.87, 0.12, 0.04)
a_6	(0.76, 0.17, 0.05)	(0.60, 0.30, 0.03)	(0.96, 0.03, 0.05)	(0.94, 0.05, 0.04)	(0.88, 0.13, 0.07)
a_7	(0.77, 0.15, 0.16)	(0.66, 0.30, 0.12)	(0.95, 0.02, 0.02)	(0.90, 0.02, 0.08)	(0.85, 0.12, 0.10)
a_8	(0.74, 0.18, 0.05)	(0.67, 0.25, 0.24)	(0.95, 0.02, 0.04)	(0.91, 0.00, 0.05)	(0.86, 0.15, 0.02)
a_9	(0.72, 0.17, 0.02)	(0.63, 0.23, 0.31)	(0.97, 0.00, 0.02)	(0.92, 0.03, 0.08)	(0.87, 0.06, 0.04)

Note: Only the familiarity differs compared to the $\kappa = 3$ period. All other parameters are unchanged.

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