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Computational Analysis on the Influence of Normal Force in a Homogeneous Isotropic Microstretch Thermoelastic Diffusive Solid

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Abstract: In this study, the identification of thermoelastic mass diffusion was examined on a homogeneous isotropic microstretch thermoelastic diffusion (HIMTD) solid due to normal force on the surface of half space. In the framework of Cartesian symmetry, the components of displacement, stresses, temperature change, and microstretch as well as couple stress were investigated with and without microstretch and diffusion. The expression of the field functions was obtained using the Laplace and Fourier transforms. So as to estimate the nature of the components of displacement, stresses, temperature change, and microstretch as well as couple stress in the physical domain, an efficient approximate numerical inverse Laplace and Fourier transform technique and Romberg's integration technique was adopted. It was meticulously considered and graphically illustrated how mass diffusion and microstretch affect thermoelastic deformation. Our objective was to address the inquiry regarding the impact of thermoelastic mass diffusion and microstretch on the field functions in the presence of a mass concentration source within the medium. Specifically, we aimed to investigate how these phenomena amplify the aforementioned effect.

Keywords: isotropic; microstretch thermoelastic solid; mass concentration source; Laplace and Fourier transform; diffusion

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1. Introduction

New synthetic materials with microdeformations and microrotations cannot be represented via the classical elastic theory. A micropolar thermoelasticity theory is suitable for describing these types of materials. A micropolar continuum consists of an array of integrated particles that are capable of undergoing rotational and translational motion as well as supporting body and surface couplings. Furthermore, the micropolar elastic thermoelasticity theory provides a more accurate explanation of a material's response to external stimulation than the classical elastic theory.

According to the micropolar continuum theory, a continuous material's displacement and rotation vectors are equal in magnitude. In 1966, Eringen and Suhubi [1,2] developed the microelastic nonlinear theory. For materials that undergo microdeformations and microrotations and are not covered via the classical thermoelasticity theory, Eringen [3,4] developed and introduced the *"Theory of micropolar elasticity"*. The thermo-microstretch elastic materials with contractions and expansions of their microstructure were established by Eringen [5]. Using vibrations to study thermo-diffusion, Nowacki [6] examined the effects of vibrations. A micropolar thermoelasticity theory based on heat flux was developed by Chandrasekharaiah [7]. In his book, Eringen [8] provides a complete explanation of the microelastic theory.

Microrotations can occur without macrodisplacements. Singh and Kumar [9] deliberated the propagation of waves in thermo-microstretch elastic (TME) material. Kumar and Partap [10] exhibited the Rayleigh–Lamb waves in a TME plate. Kumar and Kansal [11] developed a method for the theory of TME diffusive solids using differential equations. Using Green and Lindsay's thermoelasticity theory, Boschi and Ieşan [12] developed the micropolar theory. According to Mindlin [13], high-frequency vibrating crystal plates are coupled mechanically, electrically, and thermally via a two-dimensional problem. A micropolar thermoelasticity theory based on the GN theory was developed by Ciarletta [14]. Micropolar piezoelectric coupling was studied by Aouadi [15], and micropolar thermoelastic coupling was studied by Aouadi [16,17]. Using the hyperbolic heat equation, El-Karamany and Ezzat [18] established the micropolar electromagnetic thermoelastic theory for modeling issues involving extreme heat fluxes or extremely short time intervals. Among their numerous accomplishments, Marin and Baleanu [19] solved a time-harmonic problem for load dissipation in a cylinder by developing a theory of micropolar thermoelasticity without energy dissipation. Chirila et al. [20] obtained solutions to boundary value problems based on properties of homogeneous and anisotropic micropolar thermoelasticity. In addition, several other researchers have formulated diverse theories pertaining to thermoelasticity, including, Marin [21–23], Alzahrani et al. [24], Malik et al. [25], Trivedi et al. [26], Kumar et al. [27], Gupta et al. [28], Zhu et al. [29], Marin and Florea [30], Chen et al. [31], Kaur and Singh [32,33], Jafari et al. [34], Marin et al. [35,36], and Kuang et al. [37].

In this study, the identification of thermoelastic mass diffusion was examined on a HIMTD solid due to normal force on the surface of half space. Our objective was to address the inquiry regarding the impact of thermoelastic mass diffusion and microstretch on the field functions (components of displacement, stresses, temperature change, and microstretch as well as couple stress components, etc.) in the presence of a mass concentration source within the medium. Displacement refers to the change in position or shape of material under various forces, while stresses are the internal resistance that materials exhibit against deformation. Temperature change plays a crucial role in affecting the material's properties and its ability to withstand external stresses. Microstretch is a measure of the minute deformation within the microstructure of a material and contributes significantly to its overall behavior and performance. Couple stresses represent the moment per unit area in a material and have a significant impact on its mechanical response under complex loading conditions. To evaluate the nature of the components of displacement, stresses, temperature change, and microstretch as well as couple stress in the physical domain, the Laplace and Fourier inverse transform technique and Romberg's integration technique were adopted. The influence of mass diffusion and microstretch on the thermoelastic deformation was meticulously analyzed and has been depicted graphically in various quantities.

2. Basic Equations

The equations governing the motion and constitutive relations in an isotropic microstretch thermoelastic diffusion solid without external heat sources, body forces, stretch force, and body couples, can be expressed as follows [5,11]:

$$(\lambda+2\mu+k)\nabla(\nabla.\boldsymbol{u})-(\mu+k)\nabla\times\nabla\times\boldsymbol{u}+k\nabla\times\boldsymbol{\varphi}+\lambda_0\nabla\varphi^*-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T\ -\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla C=\rho\frac{\partial^2\boldsymbol{u}}{\partial t^2},\quad(1)$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\boldsymbol{\varphi})-\gamma\,\nabla\times(\nabla\times\boldsymbol{\varphi})+k\,\nabla\times\boldsymbol{u}-2k\boldsymbol{\varphi}=\rho j\frac{\partial^2\boldsymbol{\varphi}}{\partial t^2},\quad(2)$$

$$\left(\alpha_0\nabla^2-\lambda_1\right)\varphi^*-\lambda_0\nabla.\boldsymbol{u}+v_1\left(1+\tau_1\frac{\partial}{\partial t}\right)T+v_2\left(1+\tau^1\frac{\partial}{\partial t}\right)C=\frac{\rho j_0}{2}\frac{\partial^2\varphi*}{\partial t^2},\quad(3)$$

$$\beta_1 T_0\left(1+\frac{\partial}{\partial t}\right)\nabla.\boldsymbol{u}+v_1 T_0\left(1+\frac{\partial}{\partial t}\right)\dot{\varphi}*+\rho C*\left(1+\tau_0\frac{\partial}{\partial t}\right)\dot{T}+aT_0\left(1+\tau^0\frac{\partial}{\partial t}\right)\dot{C}=K^*\nabla^2 T, \tag{4}$$

$$D\beta_2\nabla^2(\nabla.\boldsymbol{u})+Dv_2\nabla^2\varphi*+Da\nabla^2\left(T+\tau_1\dot{\mathrm{T}}\right)+\dot{\mathrm{C}}-Db\nabla^2\left(C+\tau^1\dot{\mathrm{C}}\right)=0. \tag{5}$$

The constitutive relations are:

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)+K\left(u_{j,i}-\varepsilon_{ijr}\varphi_r\right)+\lambda_0\delta_{ij}\varphi*-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)T\delta_{ij}-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)C\delta_{ij}, \tag{6}$$

$$m_{ij}=\alpha\varphi_{r,r}\delta_{i,j}+\beta\varphi_{i,j}+\gamma\varphi_{j,i}+b_0\varepsilon_{mji}\varphi^*_{,m}, \tag{7}$$

$$\lambda^*_i=\alpha_0\varphi^*_{,i}+b_0\varepsilon_{ijm}\varphi_{j,m} \tag{8}$$

where $\lambda,\mu,\alpha,\beta,\gamma,K,\lambda_0,\alpha_0,$ and β_0 denote material constants, and T_0 was chosen such that $\left|\frac{T}{T_0}\right|$ is ≤ 1.

$$\beta_1=(3\lambda+2\mu+K)\alpha_{t1,}v_1=(3\lambda+2\mu+K)\alpha_{t2,}$$

$$\beta_2=(3\lambda+2\mu+K)\alpha_{c1,}v_2=(3\lambda+2\mu+K)\alpha_{c2,}$$

The variable u denotes displacement, while φ is a local dilatation variable that represents a change in volume fraction. It is important to note that φ, as a scalar, is not related to the local rotation vector, φ_k, in Cosserat elasticity. In terms of elastic constants, λ and μ are considered traditional elastic moduli, while α, β, and γ are Cosserat elastic constants that offer sensitivity to rotation gradient. Additionally, K is a Cosserat elastic constant that measures the coupling between macro rotation fields and micro rotation fields. The constants α_0 and β_0 provide sensitivity to the gradient of local dilatation. This means that they determine how sensitive the model is to changes in the amount of local expansion or contraction. The other constant λ_0 denotes a micro-stretch modulus that couples dilatation variable change to stress and dilatation to equilibrated body force. In other words, it relates how changes in the amount of stretch or compression on a microscopic level impact the stress within a material, and how that stretch or compression balances out the internal forces within the material.

3. Formulation of the Problem

We considered a HIMTD half space $x_1\geq 0$ initially at uniform temperature, T_0. It is important to note that the various functions under consideration are contingent upon both the temporal variable t and the coordinates x_1 and x_3. Thus, we assumed the displacement vector, microrotation vector, temperature change, mass concentration, and microstretch function were of the form:

$$\boldsymbol{u}(x_1,x_3,t)=(u_1,0,u_3),\boldsymbol{\varphi}=(0,\varphi_2,0),\varphi*=\varphi*(x_1,x_3,\mathrm{t}),\mathrm{T}=\mathrm{T}(x_1,x_3,\mathrm{t}),\ \mathrm{C}=\mathrm{C}(x_1,x_3,\mathrm{t}). \tag{9}$$

Using Equation (9) in the system of Equations (1)–(5) yields the following:

$$(\lambda+\mu)\frac{\partial e}{\partial x_1}+(\mu+K)\nabla^2u_1-K\frac{\partial\varphi_2}{\partial x_3}+\lambda_0\frac{\partial\varphi^*}{\partial x_1}-\beta_1\tau^1_t\frac{\partial T}{\partial x_1}-\beta_2\tau^1_c\frac{\partial C}{\partial x_1}=\rho\frac{\partial^2u_1}{\partial t^2}, \tag{10}$$

$$(\lambda+\mu)\frac{\partial e}{\partial x_3}+(\mu+K)\nabla^2u_3+K\frac{\partial\varphi_2}{\partial x_1}+\lambda_0\frac{\partial\varphi^*}{\partial x_3}-\beta_1\tau^1_t\frac{\partial T}{\partial x_3}-\beta_2a_3\tau^1_c\frac{\partial C}{\partial x_3}=\rho\frac{\partial^2u_3}{\partial t^2}, \tag{11}$$

$$\gamma\nabla^2\varphi_2+K\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)-2K\varphi_2=j\rho\frac{\partial^2\varphi_2}{\partial t^2}, \tag{12}$$

$$\left(\alpha_0\nabla^2-\lambda_1\right)\varphi^*+v_1\tau^1_tT+v_2\tau^1_cC-\lambda_0e=\rho j_0\frac{\partial^2\varphi^*}{\partial t^2}, \tag{13}$$

$$\beta_1 T_0 e + v_1 T_0 \frac{\partial \varphi^*}{\partial t} + \rho C^* \tau_t^0 \frac{\partial T}{\partial t} + a T_0 \tau_c^0 \frac{\partial C}{\partial t} = K^* \nabla^2 T, \tag{14}$$

$$D\beta_2 \nabla^2 e + Dv_2 \nabla^2 \varphi^* + Da\tau_t^1 \nabla^2 T + \frac{\partial C}{\partial t} - Db\tau_c^1 \nabla^2 C = 0, \tag{15}$$

where

$$\tau_t^1 = 1 + \tau_1 \frac{\partial}{\partial t},\ \tau_c^1 = 1 + \tau^1 \frac{\partial}{\partial t}, \tau_t^0 = 1 + \tau_0 \frac{\partial}{\partial t}, \tau_c^0 = 1 + \tau^0 \frac{\partial}{\partial t}, e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}.$$

The dimensionless quantities we considered were:

$$\begin{gathered}(x_1', x_3') = \tfrac{\omega^*}{c_1}(x_1,x_3), (u_1', u_3') = \tfrac{\rho c_1 \omega^*}{\beta_1 T_0}(u_1,u_3), t_{ij}' = \tfrac{t_{ij}}{\beta_1 T_0}, t' = \omega^* t, \tau_0' = \omega^* \tau_0, \tau^{0\prime} = \omega^* \tau^0, \tau_1' = \omega^* \tau_1, \\ \tau^{1\prime} = \omega^* \tau^1, \varphi^{*\prime} = \tfrac{\rho c_1^2}{\beta_1 T_0} \varphi^*, \lambda_i^{*\prime} = \tfrac{\lambda_i^* \omega^*}{c_1 \beta_1 T_0}, \varphi_2' = \tfrac{\rho c_1^2}{\beta_1 T_0} \varphi_2, C' = \tfrac{C\beta_2}{\rho c_1^2}, m_{ij}' = \tfrac{\omega^*}{c_1 \beta_1 T_0} m_{ij},\end{gathered} \tag{16}$$

where $\omega^* = \frac{\rho C^* c_1^2}{K^*}$, $c_1^2 = \frac{\lambda + 2\mu + K}{\rho}$.

Utilizing the given quantities provided by Equation (16) in Equations (10)–(15) and eliminating the primes, we obtained the following:

$$\delta^2 \frac{\partial e}{\partial x_1} + \left(1 - \delta^2\right) \nabla^2 u_1 - a_1 \frac{\partial \varphi_2}{\partial x_3} + a_2 \frac{\partial \varphi^*}{\partial x_1} - \tau_t^1 \frac{\partial T}{\partial x_1} - a_3 \tau_c^1 \frac{\partial C}{\partial x_1} = \frac{\partial^2 u_1}{\partial t^2}, \tag{17}$$

$$\delta^2 \frac{\partial e}{\partial x_3} + \left(1 - \delta^2\right) \nabla^2 u_3 + a_1 \frac{\partial \varphi_2}{\partial x_1} + a_2 \frac{\partial \varphi^*}{\partial x_3} - \tau_t^1 \frac{\partial T}{\partial x_3} - a_3 \tau_c^1 \frac{\partial C}{\partial x_3} = \frac{\partial^2 u_3}{\partial t^2}, \tag{18}$$

$$a_4 \nabla^2 \varphi_2 + a_5 \left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \right) - a_6 \varphi_2 = \frac{\partial^2 \varphi_2}{\partial t^2}, \tag{19}$$

$$(\delta_1^2 \nabla^2 - a_7)\varphi^* - a_8 e + a_9 \tau_t^1 T + a_{10} \tau_c^1 C = \frac{\partial^2 \varphi^*}{\partial t^2}, \tag{20}$$

$$a_{11} e + a_{12} \frac{\partial \varphi^*}{\partial t} + \tau_t^0 \frac{\partial T}{\partial t} + a_{13} \tau_c^0 \frac{\partial C}{\partial t} = \nabla^2 T, \tag{21}$$

$$a_{14} \nabla^2 e + a_{21} \nabla^2 \varphi^* + a_{15} \tau_t^1 \nabla^2 T + \frac{\partial C}{\partial t} - a_{16} \tau_c^1 \nabla^2 C = 0, \tag{22}$$

where

$$\begin{gathered}(a_1, a_2) = \tfrac{1}{\rho c_1^2}(K, \lambda_0), a_3 = \tfrac{\rho c_1^2}{\beta_1 T_0}, (a_4, a_5, a_6) = \tfrac{1}{j\rho}\left(\tfrac{\gamma}{c_1^2}, \tfrac{K}{\omega^{*2}}, \tfrac{2K}{\omega^{*2}} \right), \\ (a_7, a_8, a_9, a_{10}) = \tfrac{2}{j_0 \omega^{*2}}\left(\tfrac{\lambda_1}{\rho}, \tfrac{\lambda_0}{\rho}, \tfrac{v_1 c_1^2}{\beta_1}, \tfrac{v_2 \rho c_1^4}{\beta_1 \beta_2 T_0} \right), (a_{11}, a_{12}, a_{13}) = \tfrac{1}{K^* \omega^*}\left(\tfrac{T_0 \beta_1^2}{\rho}, \tfrac{T_0 \beta_1 v_1}{\rho}, \tfrac{\rho c_1^4 a}{\beta_2} \right), \\ (a_{14}, a_{15}, a_{16}) = \tfrac{D\omega^*}{c_0^2}\left(\tfrac{\beta_2^2}{\rho c_1^2}, \tfrac{\beta_2 a}{\beta_1}, b \right),\ a_{21} = \tfrac{Dv_2 \beta_2 \omega^*}{\rho c_1^4}, \delta_1^2 = \tfrac{c_2^2}{c_1^2},\ c_2^2 = \tfrac{2\alpha_0}{j_0}.\end{gathered}$$

Introducing potential functions defined by:

$$u_1 = \frac{\partial \Phi}{\partial x_1} + \frac{\partial \psi}{\partial x_3} \text{ and } u_3 = \frac{\partial \Phi}{\partial x_3} - \frac{\partial \psi}{\partial x_1} \tag{23}$$

In Equations (17)–(22), where $\Phi(x_1, x_3, \mathrm{t})$, and $\psi(x_1, x_3, \mathrm{t})$, are scalar potential functions, we obtained:

$$\nabla^2 \Phi + a_2 \varphi^* - \tau_t^1 T - a_3 \tau_c^1 C = \frac{\partial^2 \Phi}{\partial t^2}, \tag{24}$$

$$\left(1 - \delta^2\right) \nabla^2 \psi - a_1 \varphi_2 = \frac{\partial^2 \psi}{\partial t^2}, \tag{25}$$

$$a_4\left(\frac{\partial^2\varphi_2}{\partial x_1^2}+\frac{\partial^2\varphi_2}{\partial x_3^2}\right)+a_5\nabla^2\psi-a_6\varphi_2=\frac{\partial^2\varphi_2}{\partial t^2}, \tag{26}$$

$$\delta_1^2\left(\frac{\partial^2\varphi^*}{\partial x_1^2}+\frac{\partial^2\varphi^*}{\partial x_3^2}\right)-a_7\varphi^*-a_8\frac{\partial^2\Phi}{\partial x_1^2}-a_8\frac{\partial^2\Phi}{\partial x_3^2}+a_9\tau_t^1T+a_{10}\tau_c^1C=\frac{\partial^2\varphi^*}{\partial t^2}, \tag{27}$$

$$a_{11}\left(\frac{\partial^2\Phi}{\partial x_1^2}+\frac{\partial^2\Phi}{\partial x_3^2}\right)+a_{12}\frac{\partial\varphi^*}{\partial t}+\tau_t^0\frac{\partial T}{\partial t}+a_{13}\tau_c^0\frac{\partial C}{\partial t}=\frac{\partial^2 \mathrm{T}}{\partial x_1^2}+\frac{\partial^2 \mathrm{T}}{\partial x_3^2}, \tag{28}$$

$$a_{14}\nabla^2\left(\frac{\partial^2\Phi}{\partial x_1^2}+\frac{\partial^2\Phi}{\partial x_3^2}\right)+a_{21}\left(\frac{\partial^2\varphi^*}{\partial x_1^2}+\frac{\partial^2\varphi^*}{\partial x_3^2}\right)+a_{15}\tau_t^1\left(\frac{\partial^2 T}{\partial x_1^2}+\frac{\partial^2 T}{\partial x_3^2}\right)+\frac{\partial C}{\partial t}-a_{16}\tau_c^1\left(\frac{\partial^2 C}{\partial x_1^2}+\frac{\partial^2 C}{\partial x_3^2}\right)=0. \tag{29}$$

We utilized the Laplace and Fourier transforms, as defined by:

$$\overline{f}(x_1,x_3,s)=\int_0^\infty f(x_1,x_3,t)e^{-st}dt, \tag{30}$$

$$\hat{f}(\xi,x_3,s)=\int_{-\infty}^\infty f(x_1,x_3,\boldsymbol{s})e^{i\xi x_1}\boldsymbol{d}x_1. \tag{31}$$

From Equations (24)–(29), we obtained a set of homogeneous equations in terms of $\overline{\Phi},\overline{\varphi^*},\overline{T},\overline{C}$ and $\overline{\varphi_2},\overline{\psi}$. We eliminated these variables to find a non-trivial solution as:

$$\left(D^8+\zeta_1D^6+\zeta_2D^4+\zeta_3D^2+\zeta_4\right)\left(\overline{\Phi},\overline{\varphi^*},\overline{T},\overline{C}\right)=0, \tag{32}$$

$$\left(D^4+\zeta_5D^2+\zeta_6\right)\left(\overline{\varphi_2},\overline{\psi}\right)=0, \tag{33}$$

where

$$\zeta_1=\frac{\left(-4a^*\xi^2+b^*\right)}{a^*},\zeta_2=\frac{\left(6a^*\xi^4-3\xi^2b^*+c\right)}{a^*},\zeta_3=\frac{-\left(4a^*\xi^6+3b^*\xi^4-2\xi^2c+d\right)}{a^*},\zeta_4=\frac{\left(\xi^8a^*-\xi^6b^*+\xi^4c-\xi^2d+e\prime\right)}{a^*},$$
$$\zeta_5=\frac{\left(-2a'\xi^2+b'\right)}{a'},\zeta_6=\frac{\left(a\prime\xi^4-b'\xi^2+c'\right)}{a'},\frac{d}{dx_3}\equiv D,$$
$$a'=\left(1-\delta^2\right)a_4$$
$$b'=-\left(1-\delta^2\right)\left(\boldsymbol{s}^2a_4+a_6+s^2\right)+a_1a_5,c'=\left(1-\delta^2\right)s^2\left(a_6+s^2\right),$$
$$a^*=\delta_1^2s^*$$
$$b^*=a_{16}s^*(-\delta_1^2s_0s-a_7-s^2-s^2\delta_1^2+a_2a_8-s\delta_1^2\Big)-s\delta_1^2\left(1+a_{13}a_{15}s_*\gamma_1'-a_{15}ss^*s_*\right)+s^*\left(a_{10}-a_8-\delta_1^2a_{14}\right),$$
$$c=ss_*s^*\left\{-a_{16}a_2a_9+\left(a_{15}+a_{16}\right)\left(a_7+\mathrm{s}^2\right)+a_9-a_{10}-a_{15}a_{10}a_2\right\}+ss^*\gamma_1'(a_{13}a_9-a_{13}a_8+$$
$$a_{13}a_{15}s^3\delta_1^2-a_{13}sa_{14}\delta_1^2+a_{13}a_{15}s\left(a_7+\mathrm{s}^2\right))+s^*s_*\left\{\left(a_9-a_8\right)a_{12}a_{16}+a_{12}a_5\left(a_8-a_{10}\right)\right\}+$$
$$ss^*s_0\left\{a_8-a_{10}+\delta_1^2a_{14}+a_{16}\left(a_7+\mathrm{s}^2\right)+\delta_1^2s^2a_{16}-a_8a_{16}a_2\right\}+s^*s^2\left\{\delta_1^2-a_{10}+a_{16}\left(a_7+\mathrm{s}^2\right)\right\}+$$
$$s^*\left\{a_4\left(a_7+\mathrm{s}^2\right)-\delta_1^2a_{10}a_2\right\}-s\left\{a_2a_8-\left(a_7+\mathrm{s}^2\right)-\delta_1^2s^2-\delta_1^2ss_0\right\},$$
$$d=-s_*s^*\left\{a_{12}s^2a_{16}+a_{10}a_{12}a_{15}s^2+a_{12}a_{14}\left(a_{10}-a_9\right)\right\}+s^*s_0\left(s^3a_{10}+a_{10}a_2s\delta_1^2-s^3a_{16}\right)+$$
$$ss_*\gamma_1'\left\{-a_2a_9a_{14}a_{13}+a_{13}\left(a_7+\mathrm{s}^2\right)a_{14}-a_{13}a_9s^2-a_{13}a_{15}s^2\left(a_7+\mathrm{s}^2\right)\right\}+ss_*\left\{a_{12}\left(a_8-a_9\right)-s\left(a_7+\mathrm{s}^2\right)\right\}$$
$$+s^*\left(a_9a_2s^2+\left(a_7+\mathrm{s}^2\right)a_2a_{10}\right)+\left(a_7+\mathrm{s}^2\right)\left(-s_0s^2-s^3-s_0s^4\delta_1^2+s_0s^2a_2a_8\right),$$
$$e'=\left(a_7+\mathrm{s}^2\right)s_0\left(s^4-sa_2a_{10}s^*\right)+a_{12}s^3a_2s^*,$$
$$s^*=1+\tau^1s,s_*=1+\tau_1s,\gamma_1'=1+\tau^0s,s_0=1+\tau_0s.$$

The roots of Equation (32) are $\pm m_i$ $(i=1,2,3,4)$, and the roots of Equation (33) are $\pm m_i$ $(i=5,6)$.

With the use of the radiation conditions that $\overline{\Phi}, \overline{\varphi^*}, \overline{T}, \overline{C} \to 0$ and $\overline{\psi}, \overline{\varphi_2} \to 0$ as $x_3 \to \infty$, the solutions to Equations (37) and (38) can be expressed as:

$$\overline{\Phi} = A_1 e^{-m_1 x_3} + A_2 e^{-m_2 x_3} + A_3 e^{-m_3 x_3} + A_4 e^{-m_4 x_3}, \tag{34}$$

$$\overline{\varphi^*} = d_1 A_1 e^{-m_1 x_3} + d_2 A_2 e^{-m_2 x_3} + d_3 A_3 e^{-m_3 x_3} + d_4 A_4 e^{-m_4 x_3}, \tag{35}$$

$$\overline{T} = d_1{}^* A_1 e^{-m_1 x_3} + d_2{}^* A_2 e^{-m_2 x_3} + d_3{}^* A_3 e^{-m_3 x_3} + d_4{}^* A_4 e^{-m_4 x_3}, \tag{36}$$

$$\overline{C} = d_1{}^{**} A_1 e^{-m_1 x_3} + d_2{}^{**} A_2 e^{-m_2 x_3} + d_3{}^{**} A_3 e^{-m_3 x_3} + d_4{}^{**} A_4 e^{-m_4 x_3}, \tag{37}$$

$$\overline{\psi} = A_5 e^{-m_5 x_3} + A_6 e^{-m_6 x_3}, \tag{38}$$

$$\overline{\varphi_2} = d_5 A_5 e^{-m_5 x_3} + d_6 A_6 e^{-m_6 x_3}, \tag{39}$$

where

$$d_i = \frac{a_2' m_i^6 + \left(-3a_2'\xi^2 + b_2'\right)m_i^4 - \left(3a_2'\xi^4 - 2b_2'\xi^2 + c_2'\right)m_i^2 + \left(-a_2'\xi^6 + \xi^4 b_2' - \xi^2 c_2'\right)}{a_1' m_i^6 + \left(-3a_1'\xi^2 + b_1'\right)m_i^4 - \left(3a_1'\xi^4 - 2\xi^2 b_1' + c_1'\right)m_i^2 + \left(-a_1'\xi^6 + \xi^4 b_1' - \xi^2 c_1' + e_1'\right)} \mathrm{i} = 1,2,3,4,$$

$$d_i^* = \frac{a_3' m_i^6 + \left(-3a_3'\xi^2 + b_3'\right)m_i^4 + \left(3a_3'\xi^4 - 2\xi^2 b_3' + c_3'\right)m_i^2 + \left(-a_3'\xi^6 + \xi^4 b_3' + \xi^2 c_3'\right)}{a_1' m_i^6 + \left(-3a_1'\xi^2 + b_1'\right)m_i^4 - \left(3a_1'\xi^4 - 2\xi^2 b_1' + c_1'\right)m_i^2 + \left(-a_1'\xi^6 + \xi^4 b_1' - \xi^2 c_1' + e_1'\right)} \mathrm{i} = 1,2,3,4,$$

$$d_i^{**} = \frac{a_4' m_i^6 + \left(-3a_4'\xi^2 + b_4'\right)m_i^4 + \left(3a_4'\xi^4 - 2\xi^2 b_4' + c_4'\right)m_i^2 + \left(-a_4'\xi^6 + \xi^4 b_4' + \xi^2 c_4'\right)}{a_1' m_i^6 + \left(-3a_1'\xi^2 + b_1'\right)m_i^4 - \left(3a_1'\xi^4 - 2\xi^2 b_1' + c_1'\right)m_i^2 + \left(-a_1'\xi^6 + \xi^4 b_1' - \xi^2 c_1' + e_1'\right)} \mathrm{i} = 1,2,3,4,$$

$$d_i = \frac{a_4 m_i^2 - \left(a_4\xi^2 + a_6 + s^2\right)}{a_5 m_i^2 - a_5\xi^2} \quad i = 5,6,$$

$$a_2' = -a_{16} a_8 s^*,$$

$$b_2' = s^* s_* s(a_8 a_{16} + a_{16} a_9 + a_{10} a_{15}) + a_{13} a_{15} a_8 s \gamma_1' s_* + a_{14} a_{10} s^* + a_8 s,$$

$$c_2' = s_*\left(-a_8 s^2 - a_9 s^2 + a_{13} s \gamma_1' a_{14} a_9\right) - s_0 s a_{14} a_{10} s^*,$$

$$a_1' = s^* a_{16} \delta_1^2,$$

$$b_1' = -\left(a_{16} s^* s s_0 + a_{13} s \gamma_1'\right)\delta_1^2 - a_{16} s^*\left(a_7 + s^2\right) + a_{10} s^* a_{14},$$

$$c_1' = \left(a_7 + s^2\right)\left(s^* s_0 a_{16} s + s\right) + \delta_1^2 s_0 s^2 + a_{16} a_{12} s^* s_* a_9 + a_{14} a_{13} s \gamma_1' s_* a_9 + a_{10} a_{12} s^* - a_{10} s^* s_* a_{14},$$

$$e_1' = -a_9 s_* s a_{12} - \left(a_7 + s^2\right)s^2 s_0,\ s^* = 1 + \tau^1 s, s_* = 1 + \tau_1 s,\ s_0 = 1 + \tau_0 s$$

$$a_3' = a_{16} s a_{16} s^* \delta_1^2,$$

$$b_3' = a_{16} s^* a_{12} a_8 + a_{13} s \gamma_1' a_8 + \left(a_7 + s^2\right) a_{16} s s^* - \delta_1^2\left(s^2 - a_{13} s \gamma_1' a_{14}\right) + a_{10} s^* s,$$

$$c_3' = -a_8 a_{12} s + \left(a_7 + s^2\right)\left(s^2 - a_{13} s \gamma_1' a_{14}\right) - a_{10} a_{12} a_{14} s^*,$$

$$a_4' = -a_8 - \delta_1^2 s\left(s_* a_{15} + s_0 a_{14}\right),$$

$$b_4' = a_{12}a_8a_{15}(s_0 s - s_*) - s_0 s a_{14}\left(\delta_1^2 - 1\right) + s_* \mathrm{s}\left\{a_{15}\left(a_7 + \mathrm{s}^2\right) + a_9\right\},$$
$$c_4' = -\left(a_7 + \mathrm{s}^2\right)s_0 s a_{14} - a_9 a_{12} a_{14} s_*.$$

4. Boundary Conditions

On the half space $x_3 = 0$, normal force was applied. The boundary conditions in this particular scenario were as follows:

$$\tau_{33}(x_1, x_3, \mathrm{t}) = -F_1\psi_1(x)\delta(t), \tag{40}$$

$$\tau_{31}(x_1, x_3, \mathrm{t}) = 0, \tag{41}$$

$$m_{32} = 0, \tag{42}$$

$$\lambda_3^* = 0, \tag{43}$$

$$\frac{\partial T}{\partial x_3} + h_1 T = 0, \tag{44}$$

$$\frac{\partial C}{\partial x_3} + h_2 C = 0. \tag{45}$$

where $h_1 \to 0$ for an insulated boundary, $h_1 \to \infty$ for an isothermal boundary, $h_2 \to 0$ for an impermeable boundary, and $h_2 \to \infty$ for a concentrated boundary.

5. Applications

To synthesize the Green function, i.e., as a result of concentrated normal force on the half space, the solution can be obtained by setting:

$$\psi_1(x) = \delta(x).$$

Using the Fourier transform defined via Equation (31), we obtain:

$$\psi_1(\hat{\xi}) = 1. \tag{46}$$

Making use of the quantities defined via Equation (16) on the boundary conditions (40)–(45), along with $F_1' = \frac{F_1}{\beta_1 T_0}$, the boundary conditions were obtained in a non-dimensional form using primes. After eliminating the primes and applying the transformation defined by Equations (30) and (31), substituting the values from radiation conditions (34)–(39), and making use of Equations (6)–(8), (16), and (46), we derived the expressions for displacement components, stresses, microrotation, temperature change, microstretch, and mass concentration as follows:

$$\begin{aligned}\hat{u_1} = F_1\psi_1(\hat{\xi})\Big\{& i\xi\left(\tfrac{\Delta_1'}{\Delta}\right)e^{-m_1x_3} + i\xi\left(\tfrac{\Delta_2'}{\Delta}\right)e^{-m_2x_3} + i\xi\left(\tfrac{\Delta_3'}{\Delta}\right)e^{-m_3x_3} + i\xi\left(\tfrac{\Delta_4'}{\Delta}\right)e^{-m_4x_3} - m_5\left(\tfrac{\Delta_5'}{\Delta}\right)e^{-m_5x_3} \\ & - m_6\left(\tfrac{\Delta'_6}{\Delta}\right)e^{-m_6x_3}\Big\}\end{aligned} \tag{47}$$

$$\begin{aligned}\hat{u_3} = F_1\psi_1(\hat{\xi})\Big\{& -m_1\left(\tfrac{\Delta_1'}{\Delta}\right)e^{-m_1x_3} - m_2\left(\tfrac{\Delta_2'}{\Delta}\right)e^{-m_2x_3} - m_3\left(\tfrac{\Delta_3'}{\Delta}\right)e^{-m_3x_3} - m_4\left(\tfrac{\Delta_4'}{\Delta}\right)e^{-m_4x_3} + i\xi\left(\tfrac{\Delta_5'}{\Delta}\right)e^{-m_5x_3} \\ & + i\xi\left(\tfrac{\Delta'_6}{\Delta}\right)e^{-m_6x_3}\Big\}\end{aligned} \tag{48}$$

$$\begin{aligned}\hat{\tau_{33}} = F_1\psi_1(\hat{\xi})\Big[& g_1\tfrac{\Delta_1'}{\Delta}e^{-m_1x_3} + g_2\left(\tfrac{\Delta_2'}{\Delta}\right)e^{-m_2x_3} + g_3\left(\tfrac{\Delta_3'}{\Delta}\right)e^{-m_3x_3} + g_4\left(\tfrac{\Delta_4'}{\Delta}\right)e^{-m_4x_3} + g_5\left(\tfrac{\Delta_5'}{\Delta}\right)e^{-m_5x_3} \\ & + g_6\left(\tfrac{\Delta'_6}{\Delta}\right)e^{-m_6x_3}\Big]\end{aligned} \tag{49}$$

$$\hat{\tau_{31}} = F_1\psi_1(\hat{\xi})\left\{g'_1\frac{\Delta'_1}{\Delta}e^{-m_1x_3} + g'_2\left(\frac{\Delta'_2}{\Delta}\right)e^{-m_2x_3} + g'_3\left(\frac{\Delta'_3}{\Delta}\right)e^{-m_3x_3} + g'_4\left(\frac{\Delta'_4}{\Delta}\right)e^{-m_4x_3} + g'_5\left(\frac{\Delta'_5}{\Delta}\right)e^{-m_5x_3} + g'_6\left(\frac{\Delta'_6}{\Delta}\right)e^{-m_6x_3}\right\} \tag{50}$$

$$\hat{m_{32}} = F_1\psi_1(\hat{\xi})\left\{b_0 i\xi \mathrm{d}_1\left(\frac{\Delta'_1}{\Delta}\right)e^{-m_1x_3} + b_0 i\xi d_2\left(\frac{\Delta'_2}{\Delta}\right)e^{-m_2x_3} + b_0 i\xi \mathrm{d}_3\left(\frac{\Delta'_3}{\Delta}\right)e^{-m_3x_3} + b_0 i\xi \mathrm{d}_4\left(\frac{\Delta'_4}{\Delta}\right)e^{-m_4x_3} - \beta d_5 m_5\left(\frac{\Delta'_5}{\Delta}\right)e^{-m_5x_3} - \beta d_6 m_6\left(\frac{\Delta'_6}{\Delta}\right)e^{-m_6x_3}\right\} \tag{51}$$

$$\hat{\lambda_3^*} = F_1\psi_1(\hat{\xi})\left[-\alpha_0 d_1 m_1\left(\frac{\Delta'_1}{\Delta}\right)e^{-m_1x_3} - \alpha_0 d_2 m_2\left(\frac{\Delta'_2}{\Delta}\right)e^{-m_2x_3} - \alpha_0 d_3 m_3\left(\frac{\Delta'_3}{\Delta}\right)e^{-m_3x_3} - \alpha_0 d_4 m_4\left(\frac{\Delta'_4}{\Delta}\right)e^{-m_4x_3} - b_0 i\xi d_5\left(\frac{\Delta'_5}{\Delta}\right)e^{-m_5x_3} - b_0 i\xi d_6\left(\frac{\Delta'_6}{\Delta}\right)e^{-m_6x_3}\right] \tag{52}$$

$$\hat{T} = F_1\psi_1(\hat{\xi})\left[d_1^*\left(\frac{\Delta'_1}{\Delta}\right)e^{-m_1x_3} + d_2^*\left(\frac{\Delta'_2}{\Delta}\right)e^{-m_2x_3} + d_3^*\left(\frac{\Delta'_3}{\Delta}\right)e^{-m_3x_3} + d_4^*\left(\frac{\Delta'_4}{\Delta}\right)e^{-m_4x_3}\right] \tag{53}$$

$$\hat{C} = F_1\psi_1(\hat{\xi})\left\{d_1^{**}\left(\frac{\Delta'_1}{\Delta}\right)e^{-m_1x_3} + d_2^{**}\left(\frac{\Delta'_2}{\Delta}\right)e^{-m_2x_3} + d_3^{**}\left(\frac{\Delta'_3}{\Delta}\right)e^{-m_3x_3} + d_4^{**}\left(\frac{\Delta'_4}{\Delta}\right)e^{-m_4x_3}\right\} \tag{54}$$

where

$$g_i = \left\{-\lambda\xi^2 + (\lambda + 2\mu + k)m_i^2 + \lambda_0 d_i - \beta_1 s_* d_i^* - \beta_2 s^* d_i^{**}\right\} i = 1,2,3,4$$

$$g_i = -\lambda i\xi m_i + (\lambda + 2\mu + k)\lambda i\xi m_i, i = 5,6$$

$$g'_i = -\mu i\xi m_i - (\mu + k)i\xi m_i,\ \mathrm{i} = 1,2,3,4\ g'_i = \mu\xi^2 + (\mu + k)m_i^2 - kd_i,\ i = 5,6$$

$$\Delta'_1 = [s_6^*\{-s_1 g_{22} + s_6(g_{25}) + s_8(-g_{26})\} + s_{10}^*\{s_1(-g_{24}) + s_2(g_{25}) + s_3(-g_{26})\} + s_9^*\{s_1(-g_{23}) + s_4(g_{25}) + s_5(-g_{26})\}$$

$$\Delta'_2 = [s_{14}^*\{s_1(-g_{24}) + s_2(g_{25}) + s_3(-g_{26})\} + s_{13}^*\{s_1(-g_{23}) + s_4(g_{25}) + s_5(-g_{26})\} + s_6^*\{s_1 g_{21} + s_6 s'_{12} + s_{11} g_{25} + s_{12} g_{26}\}]$$

$$\Delta'_3 = [s_{15}^*\{s_1(-g_{24}) + s_2(g_{25}) + s_3(-g_{13}))\} + s_{13}^*\{s_7(-g_{25}) + s_8(g_{26})\} + s_9^*\{s_1 g_{21}) + s_{11}(-g_{25}) + s_{12}(g_{26})\}] + s_{13}^* s_1 s'_9$$

$$\Delta'_4 = [s_{15}^*\{s_1(g_{23}) + s_4(-g_{25}) + s_5(g_{26})\} + s_{10}^*\{s_1(g_{21}) + s_{11}(g_{25}) + s_{12}(-g_{26}))\}] + s_{14}^*$$

$$\Delta'_5 = [(g_{24})(s_4 s_{15}^* + s_7 s_{14}^* + s_{11} s_{10}^*) + (-g_{22})(s_4 s_{13}^* + s_2 s_{14}^* + s_{11} s_6^*) + (g_{26})(s_9 s_{14}^* + s_{10} s_{15}^* + s_{13} s_{10}^* - s_{15} s_6^*) + (-g_{21})(s_4 s_9^* + s_2 s_{10}^* + s_7 s_6^*)]$$

$$\Delta'_6 = [(g_{24})(s_4 s_{15}^* + s_7 s_{14}^* + s_{11} s_{10}^*) + (-g_{22})(s_4 s_{13}^* + s_2 s_{14}^* + s_{11} s_6^*) + (g_{26})(s_9 s_{14}^* + s_{10} s_{15}^* + s_{13} s_{10}^* - s_{15} s_6^*) + (-g_{21})(s_4 s_9^* + s_2 s_{10}^* + s_7 s_6^*)]$$

$$\Delta = s^*_{15}\{s_1s'_6 + s_2s'_5 + s_3s'_4 + s_4s'_3 + s_5s'_2 + s_6s'_1\}$$
$$+s^*_{14}\{s_3s'_7 + s_1s'_9 + s_2s'_3 + +s_7s'_3 + s_8s'_2 + s_9s'_1\}$$
$$+s^*_{13}\{s_1s'_{10} + s_4s'_8 + s_5s'_7 + s_7s'_5 + s_8s'_4 + s_{10}s'_1\}$$
$$+s^*_6\{s_1s'_{15} + s_6s'_{12} + s_8s'_{11} + s_{11}s'_8 + s_{12}s'_7 + s_{15}s'_1\}$$
$$+s^*_{10}\{s_1s'_{13} + s_2s'_{12} + s_3s'_{11} + s_{11}s'_3 + s_{12}s'_2 + s_{13}s'_1\}$$
$$+s^*_9\{s_1s'_{14} + s_4s'_{12} + s_5s'_{11} + s_{11}s'_5 + s_{12}s'_4 + s_{14}s'_1\}$$

Where $s_1 = g_{35}g_{46} - g_{36}g_{45}, s_2 = g_{34}g_{46} - g_{44}g_{36}, s_3 = g_{34}g_{45} - g_{44}g_{35},$

$$s_4 = g_{33}g_{46} - g_{36}g_{43},\ s_5 = g_{33}g_{45} - g_{35}g_{43},\ s_6 = g_{33}g_{44} - g_{34}g_{43}$$

$$s_7 = g_{32}g_{46} - g_{42}g_{36},\ s_8 = g_{32}g_{45} - g_{35}g_{42},\ s_9 = g_{32}g_{44} - g_{42}g_{34},$$
$$s_{10} = g_{32}g_{43} - g_{42}g_{33},\ s_{11} = g_{31}g_{46} - g_{41}g_{36},\ s_{12} = g_{31}g_{45} - g_{41}g_{35}$$
$$s_{13} = g_{31}g_{44} - g_{41}g_{34}, s_{14} = g_{31}g_{43} - g_{41}g_{33}, s_{15} = g_{31}g_{42} - g_{32}g_{41}$$
$$s'_1 = g_{15}g_{26} - g_{16}g_{25}, s'_2 = g_{14}g_{26} - g_{24}g_{16}, s'_3 = g_{14}g_{25} - g_{24}g_{15},$$
$$s'_4 = g_{13}g_{26} - g_{16}g_{23},\ s'_5 = g_{13}g_{25} - g_{23}g_{15},\ s'_6 = g_{13}g_{24} - g_{14}g_{23}$$
$$s'_7 = g_{12}g_{26} - g_{22}g_{16},\ s'_8 = g_{12}g_{25} - g_{15}g_{22},\ s'_9 = g_{12}g_{24} - g_{22}g_{14},$$
$$s'_{10} = g_{12}g_{23} - g_{22}g_{13},\ s'_{11} = g_{11}g_{26} - g_{21}g_{16},\ s'_{12} = g_{11}g_{25} - g_{21}g_{15}$$
$$s'_{13} = g_{11}g_{24} - g_{21}g_{14}, s'_{14} = g_{11}g_{23} - g_{21}g_{13}, s'_{15} = g_{11}g_{22} - g_{12}g_{21}$$
$$s^*_1 = g_{55}g_{66} - g_{56}g_{65}, s^*_2 = g_{54}g_{66} - g_{64}g_{56}, s^*_3 = g_{54}g_{65} - g_{64}g_{55},$$
$$s^*_4 = g_{53}g_{66} - g_{56}g_{63},\ s^*_5 = g_{53}g_{65} - g_{63}g_{55},\ s^*_6 = g_{53}g_{64} - g_{54}g_{63}$$
$$s^*_7 = g_{52}g_{66} - g_{62}g_{56},\ s^*_8 = g_{52}g_{65} - g_{55}g_{62},\ s^*_9 = g_{52}g_{64} - g_{62}g_{54},$$
$$s^*_{10} = g_{52}g_{63} - g_{62}g_{53},\ s^*_{11} = g_{51}g_{66} - g_{61}g_{56},\ s^*_{12} = g_{51}g_{65} - g_{61}g_{55}$$
$$s^*_{13} = g_{51}g_{64} - g_{61}g_{54}, s^*_{14} = g_{51}g_{63} - g_{61}g_{53}, s^*_{15} = g_{51}g_{62} - g_{52}g_{61}$$

where

$$g_{1i} = -b_1\xi^2 + m_i^2 + a_2 - a_3 s^* d_i^{**} - s_* d_i^{**}\mathrm{i} = 1,2,3,4\ and = -(b_1+1)i\xi m_i,\ i = 5,6$$
$$g_{2i} = -m_i b_3 i\xi - i m_i, i = 1,2,3,4\ and = b_3\xi^2 + i\xi m_i^2 - a_1 d_i, i = 5,6$$
$$g_{3i} = -b_6 m_i d_i, i = 1,2,3,4\ and = -b_5 i\xi d_i, i = 5,6$$
$$g_{4i} = -b_5 i\xi m_i d_i, i = 1,2,3,4\ and = -b_4 m_i d_i, i = 5,6$$
$$g_{5i} = -d_i^* m_i, i = 1,2,3,4\ and = 0, i = 5,6$$
$$g_{6i} = -d_i^{**} m_i, i = 1,2,3,4\ and = 0, i = 5,6$$

6. Particular Cases

If we remove the diffusion effect using $\beta_2, a,$ and $b = 0$ in Equations (47)–(54), the expressions for the displacement components, temperature distribution, stress components, and mass concentration in the generalized thermoelastic microstretch half space can be derived.

If the stretch effect (i.e., $\alpha_0, \lambda_0, \lambda_1,\ and\ K = 0$) is disregarded in Equations (47)–(54), the expressions for the displacement components, temperature distribution, stress components, and mass concentration in a generalized thermoelastic diffusive half space can be derived.

7. Inversion of the Transformation

The transformed stresses and temperature distribution are dependent upon ξ, x_3, and s, which serve as the parameters for the Laplace and Fourier transforms. To derive the expressions of (47)–(54) in the physical domain, we must first perform an inversion of the Fourier transform:

$$f(x_1, x_3, s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x_1}\hat{f}(\xi, x_3, s)d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\cos(\xi x_1) f_e - isin(\xi x_1) f_o| d\xi \quad (55)$$

where f_o andf_e denote the odd and even parts of $\hat{f}(\xi, x_3, s)$, respectively. Hence, Equation (55) provides the Laplace transform $f(x_1,\ x_3, s)$ of the function $f(x_1, x_3, t)$. According to the findings of Honig and Hirdes [38], it is possible to inverse the Laplace transform $f(x_1,\ x_3, s)$ to obtain $f(x_1, x_3, t)$. The final stage in the computation of Equation (55) entails a method for assessing the integral, as expanded upon in Press et al. [39]. This approach employs Romberg's integration technique, which incorporates an adaptive step size. Additionally, it leverages the outcomes derived from successive enhancements of the extended trapezoidal rule, culminating in the extrapolation of the outcome to the point where the step size approaches zero.

8. Numerical Results and Discussion

To demonstrate our theoretical results and the effect of diffusion and micro-stretching, physical data for magnesium material wad chosen from Eringen [40] as follows:

$$\begin{gathered}
\lambda = 9.4 \times 10^{10} Nm^{-2}, \mu = 4.0 \times 10^{10} Nm^{-2},\ K = 1.0 \times 10^{10} Nm^{-2},\ \rho = 1.74 \times 10^{3} Kgm^{-3}, \\
\gamma = 0.779 \times 10^{-9} N,\ j = 0.2 \times 10^{-19} m^2,\ C^* = 1.04 \times 10^{3} JKg^{-1}K^{-1},\ K^* = 1.7 \times 10^{6} Jm^{-1}s^{-1}K^{-1}, \\
\alpha_{t1} = 2.33 \times 10^{-5} K^{-1},\ \alpha_{t2} = 2.48 \times 10^{-5} K^{-1},\ \alpha_{c1} = 2.65 \times 10^{-4} m^3 Kg^{-1},\ \alpha_{c2} = 2.83 \times 10^{-4} m^3 Kg^{-1}, \\
T_0 = 0.298 \times 10^{3} K,\ a = 2.9 \times 10^{4} m^2 s^{-2} K^{-1},\ b = 32 \times 10^{5} Kg^{-1} m^5 s^{-2},\ D = 0.85 \times 10^{-8} Kgm^{-3}s, \\
\tau_0 = 0.02s, \tau_1 = 0.01s, \tau^0 = 0.03s, \tau^1 = 0.04s,\ j_0 = 0.19 \times 10^{-19} m^2, \alpha_0 = 0.779 \times 10^{-9} N, b_0 = 0.5 \times 10^{-9} N, \\
\lambda_0 = 0.5 \times 10^{10} Nm^{-2}, \text{and } \lambda_1 = 0.5 \times 10^{10} Nm^{-2}.
\end{gathered}$$

In Figures 1–8,

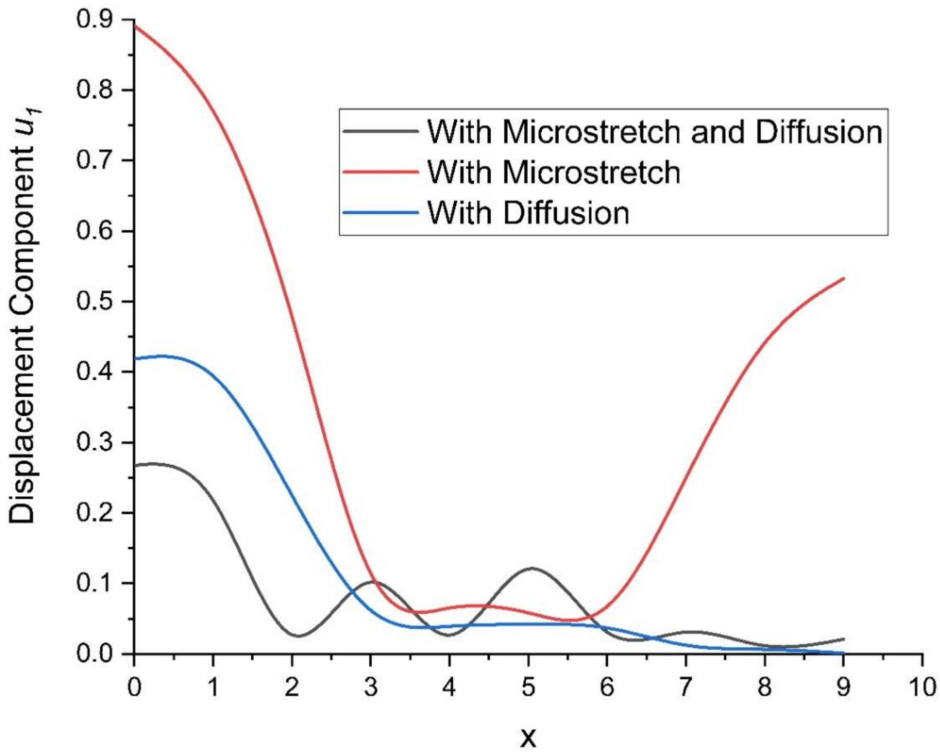

Figure 1. The variation in the displacement component u_1 with respect to distance x.

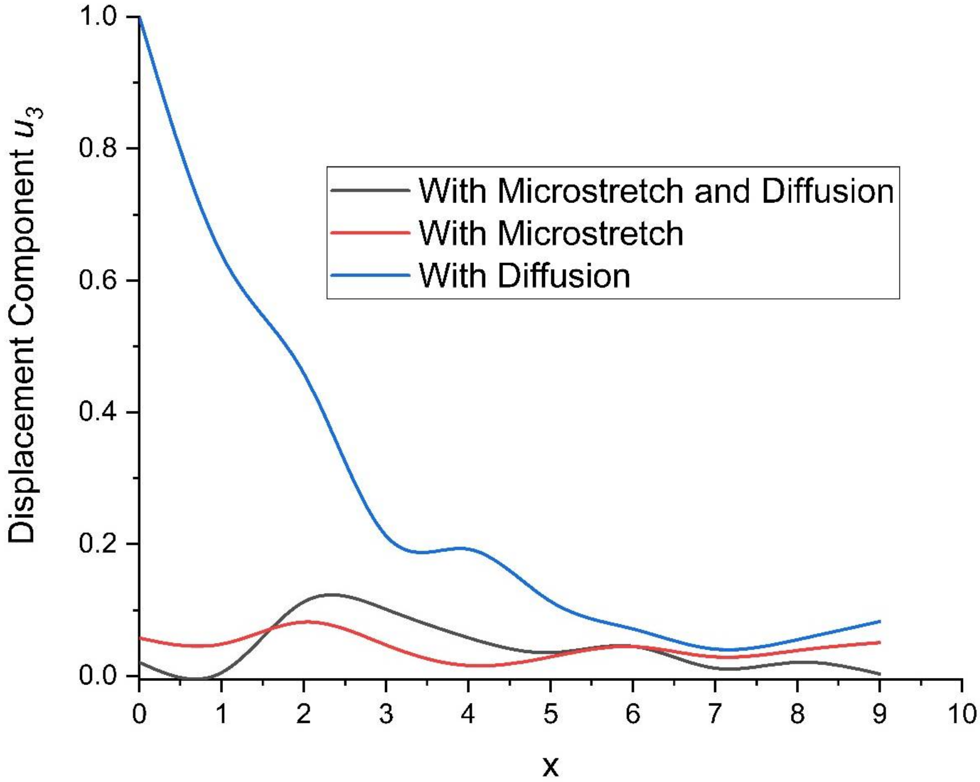

Figure 2. The variation in the displacement component u_3 with respect to distance x.

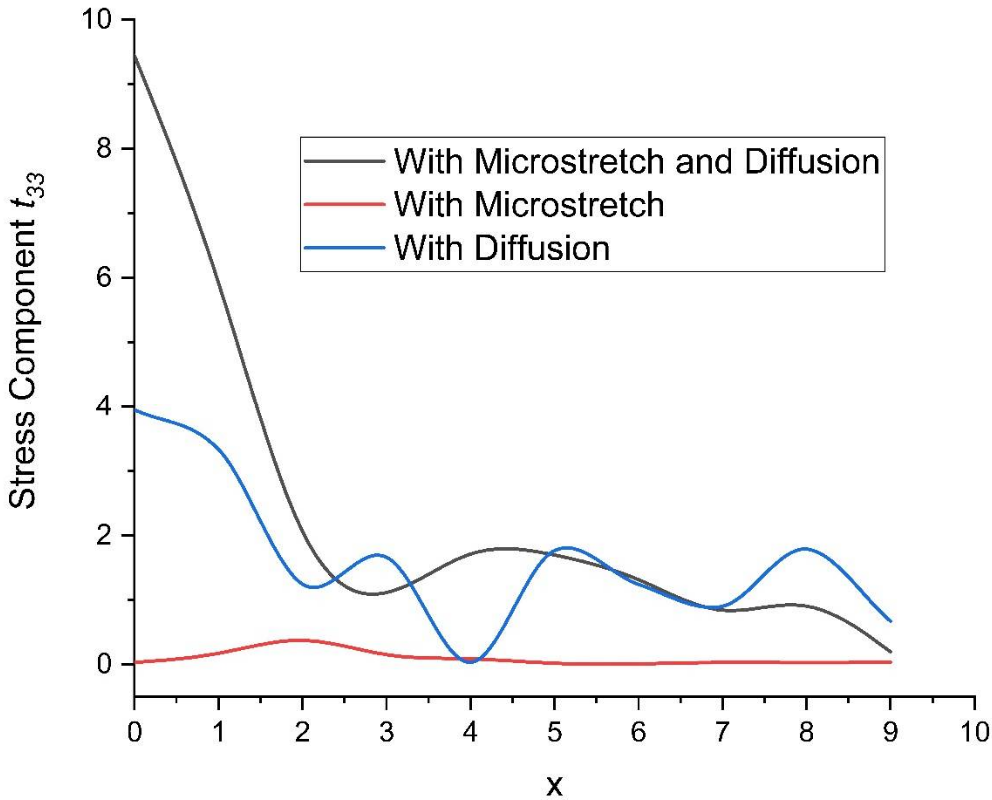

Figure 3. The variation in the stress component t_{33} with respect to distance x.

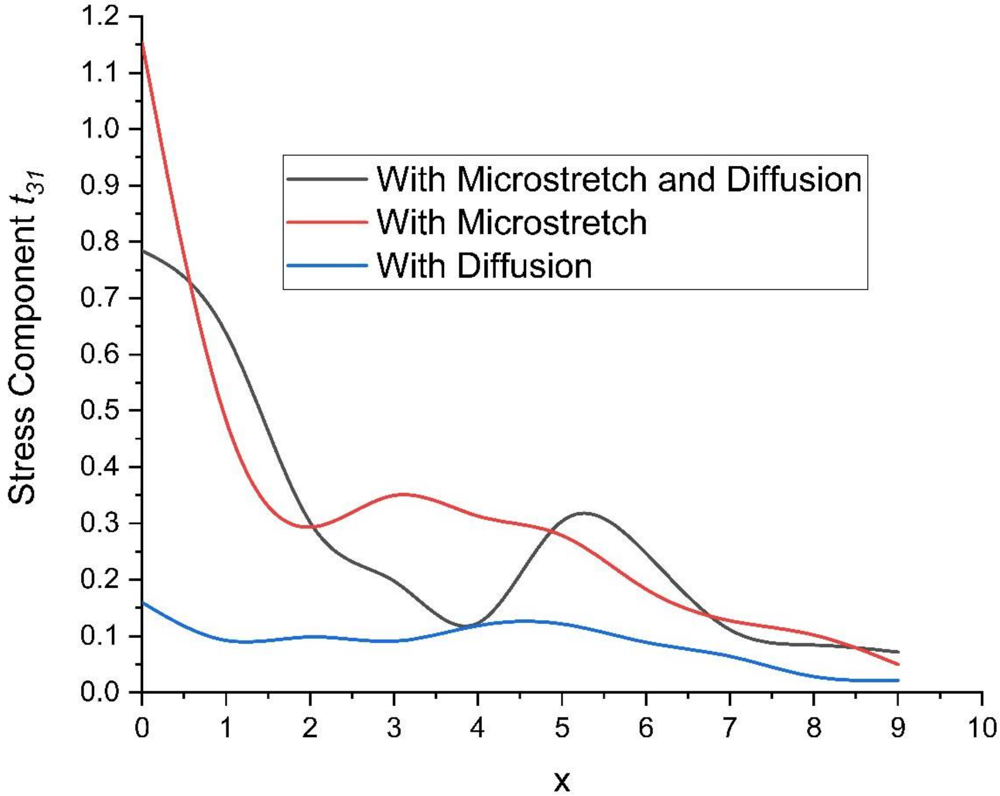

Figure 4. The variation in the stress component t_{31} with respect to distance x.

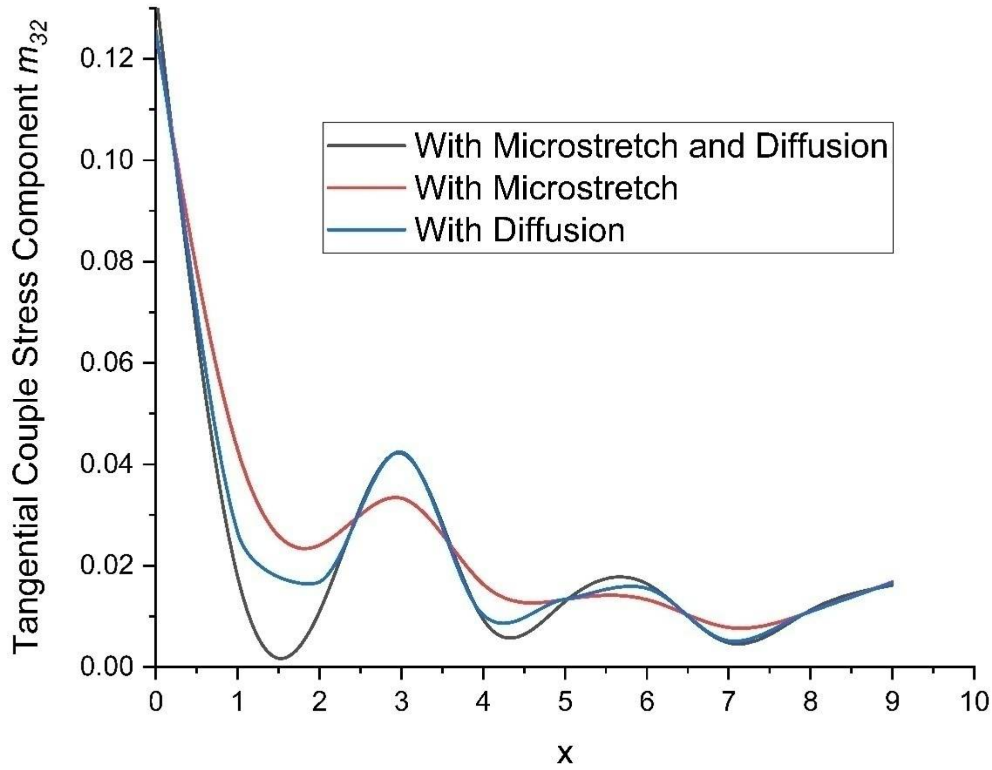

Figure 5. The variation in the tangential couple stress component m_{32} with respect to distance x.

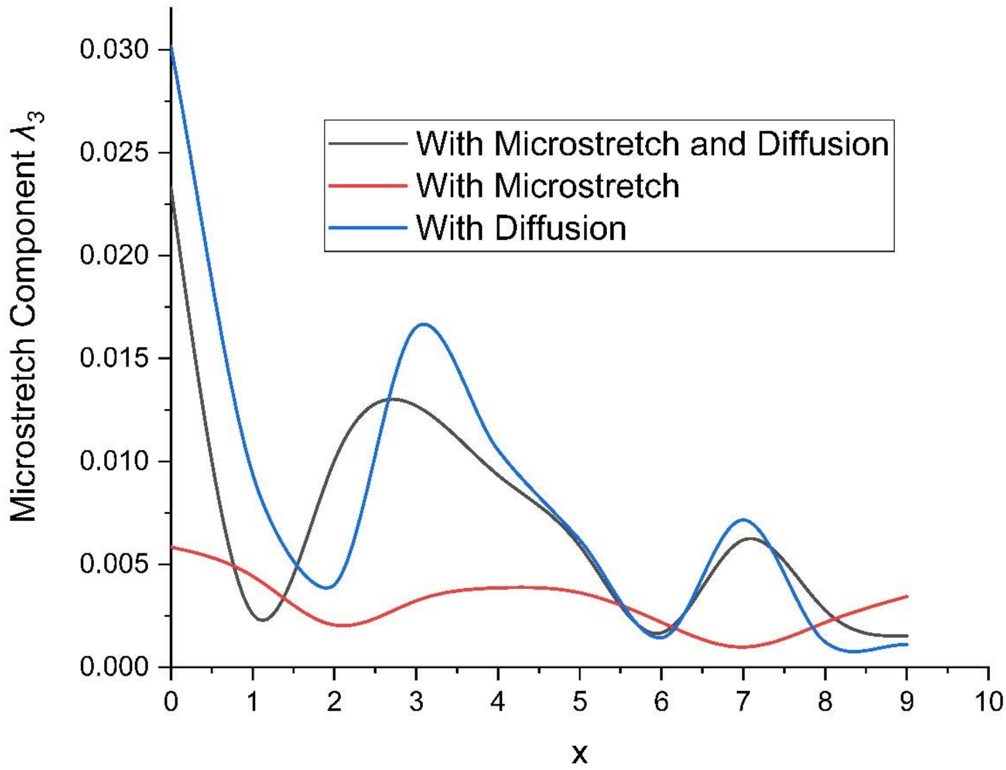

Figure 6. The variations in the microstretch component with respect to distance x.

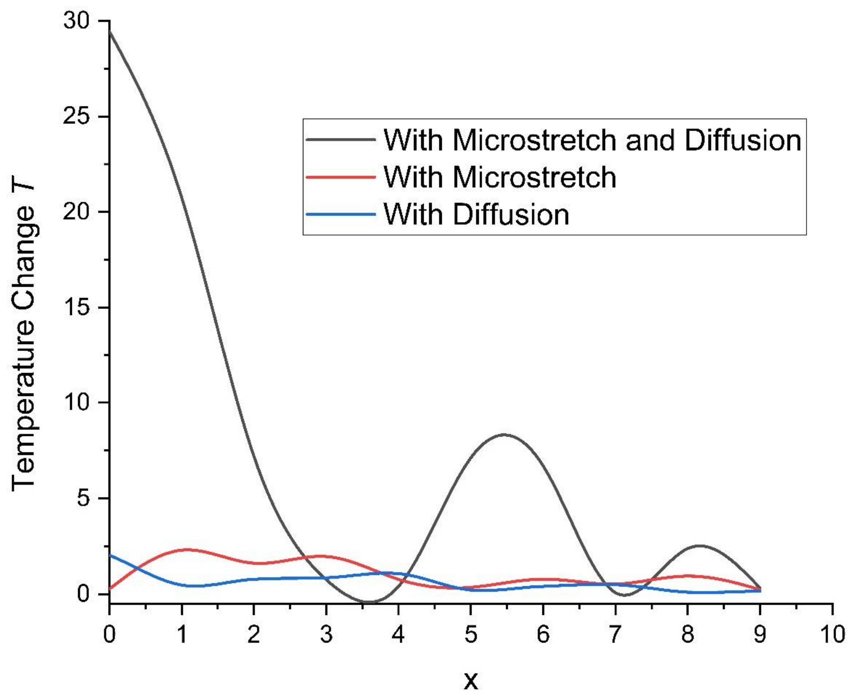

Figure 7. The variations in temperature change, *T*, with respect to distance x.

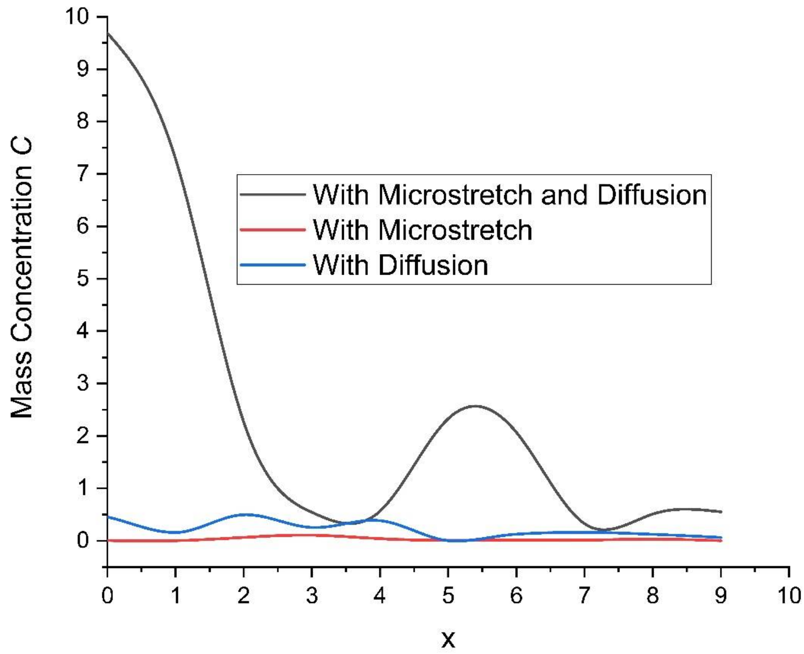

Figure 8. The variations in mass concentration, *C*, with respect to distance x.

- The black line represents the variations with microstretch and diffusion.
- The red line represents the variations with microstretch neglecting diffusion.
- The blue line represents the variations with diffusion neglecting microstretch.

Figure 1 shows the variation in the displacement component u_1 with respect to distance x from the applied source with and without micro-stretching and mass diffusion. It was found that the displacement component u_1 decreases as the distance from the applied source increases. In the case of microstretch elastic solids with microstretch and diffusion, the variation of u_1 was less pronounced. This means that the displacement of particles or elements in the material is relatively uniform, indicating a higher level of homogeneity. On the other hand, in generalized thermoelastic microstretch half spaces without diffusion, the variation in u_1 was at its maximum. This implies that the displacement can vary significantly within the material, showing a lower level of uniformity. Understanding these variations in displacement is crucial for analyzing the behavior of different materials under various conditions and for designing efficient structures and systems.

Figure 2 demonstrates the variation in the displacement component u_3 with respect to distance x from the applied source with and without micro-stretching and mass diffusion. The displacement component u_3 decreases as the distance from the applied source increases in microstretch elastic solids with microstretch and diffusion. On the other hand, the variation in the displacement component u_3 is at its maximum in generalized thermoelastic half spaces without microstretch. This suggests that the behavior of u_3 is influenced by the material properties and conditions present. Further exploration and analyses are required to understand the underlying mechanisms and implications of these findings. Figures 3 and 4 exhibit the variation in the stress components t_{33} and t_{31} with respect to distance x from the applied source with and without micro-stretching and mass diffusion. The study of stress components in microstretch elastic solids reveals a noteworthy relationship between the distribution of these components and the distance from an applied source. As observed, both t_{33} and t_{31} experience a reduction in their values as they move further away from this source. This fascinating behavior accentuates the crucial roles played by the microstretch and diffusion mechanisms in influencing the variation of the stress component t_{33}. In addition, the analysis of t_{31} demonstrated that its maximum fluctuation occurs in a generalized thermoelastic half space without microstretch involvement.

Figure 5 exhibits the variation in the tangential couple stress component m_{32} with respect to distance x from the applied source with and without micro-stretching and mass diffusion. It was found that the stress components t_{33} and t_{31} decrease as the distance from the applied source increases and exhibits an oscillatory pattern. These results indicate that variation in the tangential couple stress component m_{32} arises more under microstretch elastic solids with microstretch and diffusion, and variation in the tangential couple stress component m_{32} occurs less under generalized thermoelastic half spaces without microstretch.

Figure 6 exhibits the variations in the microstretch component with respect to distance x from the applied source with and without micro-stretching and mass diffusion. It was found that the microstretch component decreases as the distance from the applied source increases and exhibits an oscillatory pattern. These results indicate that variation in the microstretch component arises more under microstretch elastic solids with diffusion, and that variation in the microstretch component occurs less under generalized thermoelastic half spaces without microstretch.

Figures 7 and 8 exhibit the variations in temperature change, T, and mass concentration, C, respectively, with respect to distance x from the applied source with and without micro-stretching and mass diffusion. It was found that the temperature change, T, and mass concentration, C, decrease as the distance from the applied source increases and exhibits an oscillatory pattern. These results indicate that variations in temperature change, T, and mass concentration, C, arise more in microstretch elastic solids with microstretch and diffusion, and that variations in temperature change, T, and mass concentration, C, occur less in generalized thermoelastic half spaces without microstretch.

9. Conclusions

The present research provides a complete investigation of mass diffusion and microstretch impact on the thermoelastic disturbances in a homogeneous isotropic microstretch thermoelastic solid. In this framework, the components of displacement, stresses, temperature change, and microstretch as well as couple stress components have been distinguished with and without microstretch and diffusion.

- To estimate the nature of the components of displacement, stresses, temperature change, and microstretch as well as couple stress in the physical domain, an efficient approximate numerical inverse Laplace and Fourier transform technique and Romberg's integration technique were adopted.
- A comprehensive graphical representation has been provided for a range of variables, detailing the precise effects of mass diffusion and microstretch on thermoelastic deformation through meticulous analysis.
- In the thermo-microstretch theory, the combined effect of microstretch and diffusion is the dominating factor over a single parameter, i.e., microstretch or diffusion.
- It was observed that stress components increase in the microstretch elastic solid with the combined effect of microstretch and diffusion.
- Theoretical analysis and computational findings have substantiated that the impact of mass diffusion and microstretch can amplify the perturbations in the thermoelastic domain.
- The outcome of this problem holds significant value in the realm of two-dimensional dynamic responses, particularly with diverse sources of thermo-diffusion. This phenomenon has numerous applications in both geophysical and industrial domains. The exploration of thermoelasticity is instrumental in enhancing the efficacy of oil extraction processes.

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Nomenclature

δ_{ij}	Kronecker delta	$C*$	Specific heat at constant strain
T_0	Reference temperature	ρ	Medium density
c_{ijkl}	Elastic parameters	$K*$	Thermal conductivity
$\boldsymbol{\varphi}$	Microrotation vector	C	Concentration of the diffusion material
e_{ij}	Strain tensors	a	Coefficients of measure of thermo-diffusion effect
$\boldsymbol{\varphi}*$	Scalar microstretch function	b	Coefficients of mass-diffusion effect
β_{ij}	Thermal elastic coupling tensor	j	Micro-inertia
D	Thermoelastic diffusion constant	m_{ij}	Couple stress tensors
C_E	Specific heat	j_0	Micro-inertia of micro elements
α_{ij}	Linear thermal expansion coefficient	$\alpha_{t1},$ α_{t2}	Coefficients of linear thermal expansion
α_{c1} α_{c2}	The coefficients of linear diffusion expansion	σ_{ij}	Components of stress
a_{ij}	Two-temperature parameter	T	Temperature change
τ^0, τ^1	Diffusion relaxation times	u_i	Displacement components
T	Time	λ	Microstress tensor
F_1	Force	$\delta(t)$	Dirac delta function
h_2	Mass transfer coefficient	e_{kk}	Dilatation
e_{ij}	Components of strain	w	Lateral deflection of the beam
$\omega*$	The characteristic frequency of the medium	$\psi_1(x)$	source distribution function along the x-axis
h_1	Heat transfer coefficient	λ_i^*	Microstress tensor

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