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Between Soft Complete Continuity and Soft Somewhat-Continuity

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Abstract: This paper introduces two novel concepts of mappings over soft topological spaces: "soft somewhat-r-continuity" and "soft somewhat-r-openness". We provide characterizations and discuss soft composition and soft subspaces. With the use of examples, we offer numerous connections between these two notions and their accompanying concepts. We also offer extension theorems for them. Finally, we investigated a symmetry between our new concepts with their topological analogs.

Keywords: somewhat-continuity; somewhat-r-continuity; soft somewhat-continuity; soft somewhat-openness; generated soft topologies

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1. Introduction

In 1999, Molodtsov [1] introduced soft set theory as a novel technique for dealing with partial information problems. This concept has been applied in a variety of fields, including the "smoothness of function", "Riemann integration", "measurement theory", "probability theory", "game theory", and others. The nature of parameter sets, which give a broad framework for modeling uncertain data, is critical to soft set theory. This has made a significant contribution to the development of soft set theory in a relatively short period of time. Maji et al. [2] thoroughly investigated the theoretical foundations of soft set theory. They created operators and operations between soft sets, particularly for this purpose. Then, other mathematicians rebuilt and proposed new kinds of the operators and operations between soft sets provided by Maji et al.'s work; for a list of recent contributions employing soft operators and operations, see [3].

Shabir and Naz [4] established the structure of the soft topology in 2011 as an extension of the general topology. Following that, several generic topological ideas were expanded to incorporate the soft topology. The soft continuity of functions was described by Nazmul and Samanta [5] in 2013. Then, in the literature, many types of the soft continuity and soft openness of functions were developed in [6–16], and others.

Different types of generalized continuity are explored in various branches of mathematics, particularly in the theory of real functions. Our paper's goal is to introduce soft somewhat-r-continuous functions as a new type of generalized continuity in soft topological spaces, as well as soft somewhat-r-open functions as a new type of soft open functions in soft topological spaces.

This article is organized as follows:

Section 2 provides some basic ideas and results that will be utilized in the next sections.

Section 3 defines soft somewhat-r-continuous functions. We show that this class of soft functions lies strictly between the classes soft complete continuity and soft somewhat-continuity and independent of the class of soft δ-continuous functions. Moreover, regarding soft somewhat-r-continuity, we introduce several characterizations, soft subspaces, soft composition, and soft preservation theorems. In addition, we investigated the links between this class of soft functions and its analogs in the general topology.

Section 4 defines soft somewhat-r-open functions. We show that this class of soft functions is a subclass of soft somewhat-open functions. With the help of examples, we introduce various properties of this new class of soft functions.

Section 5 contains some findings and potential future studies.

2. Preliminaries

In this section, we introduce some essential concepts and outcomes that will be used in the sequel.

Let L be an initial universe and S be a set of parameters. A soft set over L relative to S is a function $K : S \longrightarrow \mathcal{P}(L)$. $SS(L, S)$ denotes the collection of all soft sets over L relative to S. Let $K \in SS(L, S)$. If $K(s) = \emptyset$ for all $s \in S$, then K is called the null soft set over L relative to S and denoted by 0_S. If $K(s) = L$ for all $s \in S$, then K is called the absolute soft set over L relative to S and denoted by 1_S. K is called a soft point over L relative to S and denoted by a_x if there exist $a \in S$ and $x \in L$ such that $K(a) = \{x\}$ and $K(s) = \emptyset$ for all $s \in S - \{a\}$. $SP(L, S)$ denotes the collection of all soft points over L relative to S. If for some $a \in S$ and $Y \subseteq L$, we have $K(a) = Y$ and $K(s) = \emptyset$ for all $s \in S - \{a\}$, then K will be denoted by a_Y. If for some $Y \subseteq L$, $K(s) = Y$ for all $s \in S$, then K will be denoted by C_Y. If $K(s)$ is a countable subset of L for all $s \in S$, then K is called a countable soft set. If $K \in SS(L, S)$ and $a_x \in SP(L, S)$, then a_x is said to belong to K (notation: $a_x \widetilde{\in} K$) if $x \in K(a)$.

For the sake of clarity, we employed the concepts and terminology from [17,18] throughout this study.

Definition 1 ([4])**.** *Let L be an initial universe and S be a set of parameters. Let $\Gamma \subseteq SS(L, S)$. Then, Γ is called a soft topology on L relative to S if:*

(1) $0_S, 1_S \in \Gamma$;
(2) *Γ is closed under an arbitrary soft union;*
(3) *Γ is closed under a finite soft intersection.*

The triplet (L, Γ, S) is called a soft topological space. The members of Γ are called soft open sets in (L, Γ, S), and their complements are called soft closed sets in (L, Γ, S).

From now on, the topological space and the soft topological space are abbreviated as TS and STS, respectively.

Let (L, α) be a TS, (L, Γ, S) be an STS, $W \subseteq L$, and $T \in SS(L, S)$. Then, the closure of W in (L, α), the interior of W in (L, α), the soft closure of T in (L, Γ, S), and the soft interior of T in (L, Γ, S) will be denoted by $Cl_\alpha(W)$, $Int_\alpha(W)$, $Cl_\Gamma(T)$, and $Int_\Gamma(T)$, respectively, and the family of all closed sets in (L, α) (respectively, soft closed sets in (L, Γ, S)) will be denoted by α^c (respectively, Γ^c).

Definition 2 ([18])**.** *Let $\{\alpha_s : s \in S\}$ be an indexed family of topologies on L. Then, $\{K \in SS(L, S) : K(s) \in \alpha_s$ for all $s \in S\}$ defines a soft topology on L relative to S. This soft topology is denoted by $\oplus_{s \in S} \alpha_s$.*

Definition 3 ([18])**.** *For any topological space (L, α) and any set of parameters S, the family $\{K \in SS(L, S) : K(s) \in \alpha$ for all $s \in S\}$ defines a soft topology on L relative to S. This soft topology is denoted by $\tau(\alpha)$.*

Definition 4 ([19])**.** *Let (L, α) be a TS, and let $W \subseteq L$. Then, W is called a:*

(a) *"Regular open set in (L, α)" if $W = Int_\alpha(Cl_\alpha(W))$. $RO(\alpha)$ denotes the collection that includes all regular open sets in (L, α).*
(b) *"Regular closed set in (L, α)" if $L - W$ is a regular open set in (L, α). $RC(\alpha)$ denotes the collection that includes all regular closed sets in (L, α).*

Definition 5. *A function $q : (L, \alpha) \longrightarrow (M, \phi)$ between the TSs (L, α) and (M, ϕ) is called:*

(a) [20] "Somewhat-continuous" (s-c) if, for every $W \in \phi$ such that $q^{-1}(W) \neq \emptyset$, we find $V \in \alpha - \{\emptyset\}$ such that $V \subseteq q^{-1}(W)$.
(b) [20] "Somewhat-open" (s-o) if, for every $V \in \alpha - \{\emptyset\}$, we find $W \in \phi - \{\emptyset\}$ such that $W \subseteq q(V)$.
(c) [21] "Completely continuous" if $q^{-1}(W) \in RO(\alpha)$ for every $W \in \phi$.
(d) [22] "δ-continuous" if for every $x \in L$ and $W \in RO(\phi)$ such that $q(x) \in W$, we find $U \in RO(\alpha)$ such that $x \in U$ and $q(U) \subseteq W$.
(e) [23] "Somewhat-r-continuous" (s-r-c) if, for every $W \in \phi$ such that $q^{-1}(W) \neq \emptyset$, we find $V \in RO(\alpha) - \{\emptyset\}$ such that $V \subseteq q^{-1}(W)$.
(f) [23] "Somewhat-r-open" (s-r-o) if, for every $V \in \alpha - \{\emptyset\}$, we find $W \in RO(\phi) - \{\emptyset\}$ such that $W \subseteq q(V)$.

Definition 6 ([24]). *Let (L, Γ, S) be an STS, and let $H \in SS(L, S)$. Then, H is called a:*

(a) "Soft regular open set in (L, Γ, S)" if $H = Int_\Gamma(Cl_\Gamma(H))$. $RO(\Gamma)$ denotes the collection that includes all soft regular open sets in (L, Γ, S).
(b) "Soft regular closed set in (L, Γ, S)" if $1_S - H \in RO(\Gamma)$. $RC(\Gamma)$ denotes the collection that includes all soft regular closed sets in (L, Γ, S).

Definition 7. *A soft function $f_{qv} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ between the STSs (L, Γ, S) and $(M, \digamma, T)$ is called:*

(a) [25] "Soft δ-continuous" if, for each $a_x \in SP(L, S)$ and $G \in RO(\digamma)$ such that $f_{qv}(a_x)\widetilde{\in}G$, there exists $H \in RO(\Gamma)$ such that $a_x\widetilde{\in}H$ and $f_{qv}(H)\widetilde{\subseteq}G$.
(b) [26] "Soft somewhat-continuous" (soft s-c) if, for each $K \in \digamma$ such that $f_{qv}^{-1}(K) \neq 0_S$, there exists $N \in \Gamma - \{0_S\}$ such that $N\widetilde{\subseteq} f_{qv}^{-1}(K)$.
(c) [26] "Soft somewhat-open" (soft s-o) if, for each $K \in \Gamma - \{0_S\}$, there exists $H \in \digamma - \{0_T\}$ such that $H\widetilde{\subseteq} f_{qv}(K)$.
(d) [27] "Soft completely continuous" if $f_{qv}^{-1}(K) \in RO(\Gamma)$ for each $K \in \digamma$.

Definition 8. *An STS (L, Γ, S) is called:*

(a) [28] "Soft locally indiscrete" if $\Gamma \subseteq \Gamma^c$.
(b) [29] A "soft D-space" if, for every $G, H \in \Gamma - \{0_S\}$, $G\widetilde{\cap}H \neq 0_S$.

Definition 9 ([11]). *Let (L, Γ, S) and (L, Υ, S) be two STSs. Then, we say that "Γ is soft weakly equivalent to Υ" if, for each $A \in \Gamma - \{0_S\}$, we find $B \in \Upsilon - \{0_S\}$ such that $B\widetilde{\subseteq}A$ and, for each $A \in \Upsilon - \{0_S\}$, we find $B \in \Gamma - \{0_S\}$ such that $B\widetilde{\subseteq}A$.*

3. Soft Somewhat-r-Continuous Functions

In this section, we define soft somewhat-r-continuous functions. We show the soft somewhat-r-continuity between the soft complete continuity and soft somewhat-continuity and independent of the class of soft δ-continuity. Moreover, regarding soft somewhat-r-continuity, we introduce several characterizations, soft subspaces, soft composition, and soft preservation theorems. In addition, we investigated the links between this class of soft functions and its analogs in general topology.

The basic concept of this section is defined as follows:

Definition 10. *A soft function $f_{qv} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ is called soft somewhat-r-continuous (soft s-r-c) if, for each $K \in \digamma$ such that $f_{qv}^{-1}(K) \neq 0_S$, there exists $N \in RO(\Gamma) - \{0_S\}$ such that $N\widetilde{\subseteq} f_{qv}^{-1}(K)$.*

In Theorems 1 and 2 and Corollaries 1 and 2, we investigate the correspondence between the concepts soft somewhat-r-continuity and soft somewhat-continuity with their analogous topological concepts:

Theorem 1. *Let $\{(L, \Gamma_s) : s \in S\}$ and $\{(M, F_t) : t \in T\}$ be two collections of TSs. Let $q : L \longrightarrow M$ and $v : S \longrightarrow T$ be functions where v is bijective. Then, $f_{qv} : (L, \oplus_{s\in S}\Gamma_s, S) \longrightarrow (M, \oplus_{t\in T}F_t, T)$ is soft s-r-c if and only if $q : (L, \Gamma_s) \longrightarrow \left(M, F_{v(s)}\right)$ is s-r-c for all $s \in S$.*

Proof. *Necessity*: Assume that $f_{qv} : (L, \oplus_{s\in S}\Gamma_s, S) \longrightarrow (M, \oplus_{t\in T}F_t, T)$ is soft s-*r*-c. Let $s \in S$. Let $W \in F_{v(s)}$ such that $q^{-1}(W) \neq \emptyset$. Then, $(v(s))_W \in \oplus_{t\in T}F_t$ and $f_{qv}^{-1}((v(s))_W) = s_{q^{-1}(W)} \neq 0_S$. Then, we find $N \in RO(\oplus_{s\in S}\Gamma_s) - \{0_S\}$ such that $N \widetilde{\subseteq} s_{q^{-1}(W)}$. Thus, $N(s) \subseteq \left(s_{q^{-1}(W)}\right)(s) = q^{-1}(W)$. Since for all $i \in S - \{s\}$, $N(i) \subseteq \left(s_{q^{-1}(W)}\right)(i) = \emptyset$, then $N(s) \neq \emptyset$. Also, by Theorem 14 of [30], $N(s) \in RO(\Gamma_s)$. This shows that $q : (L, \Gamma_s) \longrightarrow \left(M, F_{v(s)}\right)$ is s-*r*-c.

Sufficiency: Assume that $q : (L, \Gamma_s) \longrightarrow \left(M, F_{v(s)}\right)$ is s-*r*-c for all $s \in S$. Let $K \in \oplus_{t\in T}F_t$ such that $f_{qv}^{-1}(K) \neq 0_S$. Choose $a \in S$ such that $\left(f_{qv}^{-1}(K)\right)(a) = q^{-1}(K(v(a))) \neq \emptyset$. Since $K \in \oplus_{t\in T}F_t$, then $K(v(a)) \in F_{v(a)}$. Since $q : (L, \Gamma_a) \longrightarrow \left(M, F_{v(a)}\right)$ is s-*r*-c, then there exists $X \in RO(\Gamma_a) - \{\emptyset\}$ such that $X \subseteq \left(f_{qv}^{-1}(K)\right)(a)$. Then, we have $a_X \widetilde{\subseteq} f_{qv}^{-1}(K)$ and $a_X \neq 0_S$. Also, by Theorem 14 of [30], $a_X \in RO(\oplus_{s\in S}\Gamma_s)$. This shows that $f_{qv} : (L, \oplus_{s\in S}\Gamma_s, S) \longrightarrow (M, \oplus_{t\in T}F_t, T)$ is soft s-*r*-c. □

Corollary 1. *Let $q : (L, \alpha) \longrightarrow (M, \phi)$ and $v : S \longrightarrow T$ be two functions where v is a bijection. Then, $q : (L, \alpha) \longrightarrow (M, \phi)$ is s-r-c if and only if $f_{qv} : (L, \tau(\alpha), S) \longrightarrow (M, \tau(\phi), T)$ is soft s-r-c.*

Proof. For each $s \in S$ and $t \in T$, put $\Gamma_s = \alpha$ and $F_t = \phi$. Then, $\tau(\alpha) = \oplus_{s\in S}\Gamma_s$ and $\tau(\phi) = \oplus_{t\in T}F_t$. We obtain the result by using Theorem 1. □

Theorem 2. *Let $\{(L, \Gamma_s) : s \in S\}$ and $\{(M, F_t) : t \in T\}$ be two collections of TSs. Let $q : L \longrightarrow M$ and $v : S \longrightarrow T$ be functions where v is bijective. Then, $f_{qv} : (L, \oplus_{s\in S}\Gamma_s, S) \longrightarrow (M, \oplus_{t\in T}F_t, T)$ is soft s-c if and only if $q : (L, \Gamma_s) \longrightarrow \left(M, F_{v(s)}\right)$ is s-c for all $s \in S$.*

Proof. *Necessity:* Assume that $f_{qv} : (L, \oplus_{s\in S}\Gamma_s, S) \longrightarrow (M, \oplus_{t\in T}F_t, T)$ is soft s-c. Let $s \in S$. Let $W \in F_{v(s)}$ such that $q^{-1}(W) \neq \emptyset$. Then, $(v(s))_W \in \oplus_{t\in T}F_t$ and $f_{qv}^{-1}((v(s))_W) = s_{q^{-1}(W)} \neq 0_S$. So, we find $N \in \oplus_{s\in S}\Gamma_s - \{0_S\}$ such that $N \widetilde{\subseteq} s_{q^{-1}(W)}$. Then, $N(s) \in \Gamma_s$ and $N(s) \subseteq \left(s_{q^{-1}(W)}\right)(s) = q^{-1}(W)$. Since, for all $i \in S - \{s\}$, $N(i) \subseteq \left(s_{q^{-1}(W)}\right)(i) = \emptyset$, then $N(s) \neq \emptyset$. This shows that $q : (L, \Gamma_s) \longrightarrow \left(M, F_{v(s)}\right)$ is s-c.

Sufficiency: Assume that $q : (L, \Gamma_s) \longrightarrow \left(M, F_{v(s)}\right)$ is s-c for all $s \in S$. Let $K \in \oplus_{t\in T}F_t$ such that $f_{qv}^{-1}(K) \neq 0_S$. Choose $a \in S$ such that $\left(f_{qv}^{-1}(K)\right)(a) = q^{-1}(K(v(a))) \neq \emptyset$. Since $K \in \oplus_{t\in T}F_t$, then $K(v(a)) \in F_{v(a)}$. Since $q : (L, \Gamma_a) \longrightarrow \left(M, F_{v(a)}\right)$ is s-c, then there exists $X \in \Gamma_a - \{\emptyset\}$ such that $X \subseteq \left(f_{qv}^{-1}(K)\right)(a)$. Then, we have $a_X \in \oplus_{s\in S}\Gamma_s$, $a_X \widetilde{\subseteq} f_{qv}^{-1}(K)$, and $a_X \neq 0_S$. This is shown to be soft s-c. □

Corollary 2. *Let $q : (L, \alpha) \longrightarrow (M, \phi)$ and $v : S \longrightarrow T$ be two functions where v is a bijection. Then, $q : (L, \alpha) \longrightarrow (M, \phi)$ is s-c if and only if $f_{qv} : (L, \tau(\alpha), S) \longrightarrow (M, \tau(\phi), T)$ is soft s-c.*

Proof. For each $s \in S$ and $t \in T$, put $\Gamma_s = \alpha$ and $F_t = \phi$. Then, $\tau(\alpha) = \oplus_{s\in S}\Gamma_s$ and $\tau(\phi) = \oplus_{t\in T}F_t$. We obtain the result by using Theorem 2. □

In Theorem 3 and Example 1, we discuss the relationships between the classes of soft completely continuous functions and soft somewhat-*r*-continuous functions:

Theorem 3. *Every soft completely continuous function is soft s-r-c.*

Proof. Let $f_{qv} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ be soft complete continuous. Let $K \in \digamma$ and $f_{qv}^{-1}(K) \neq 0_S$. Since f_{qv} is soft complete continuous, then $f_{qv}^{-1}(K) \in RO(\Gamma)$. Put $N = f_{qv}^{-1}(K)$. Then, $N \in RO(\Gamma) - \{0_S\}$ and $N = f_{qv}^{-1}(K) \widetilde{\subseteq} f_{qv}^{-1}(K)$. As a result, f_{qv} is soft s-r-c. □

In general, Theorem 3's inverse does not have to be true.

Example 1. *Let $L = \mathbb{R}$, $M = \{c, d\}$, α be the standard topology on L, and ϕ be the discrete topology on M. Define $q : (L, \alpha) \longrightarrow (M, \phi)$ and $v : \mathbb{Z} \longrightarrow \mathbb{Z}$ as follows:*

$$q(x) = \begin{cases} c & \text{if } x < 3 \\ d & \text{if } x \geq 3 \end{cases} \text{ and } v(z) = z \text{ for all } z \in \mathbb{Z}.$$

To see that q is s-r-c, let $U \in \phi$ such that $q^{-1}(U) \neq \emptyset$. If $U = \{c\}$, then $(-\infty, 3) \in RO(\alpha) - \{\emptyset\}$ and $(-\infty, 3) \subseteq (-\infty, 3) = q^{-1}(U)$. If $U = \{d\}$, then $(3, \infty) \in RO(\alpha) - \{\emptyset\}$ and $(3, \infty) \subseteq [3, \infty) = q^{-1}(U)$. If $U = M$, then $\mathbb{R} \in RO(\alpha) - \{\emptyset\}$ and $\mathbb{R} \subseteq \mathbb{R} = q^{-1}(U)$.

Since $\{d\} \in \phi$ while $q^{-1}(\{d\}) = [3, \infty) \notin RO(\alpha)$, then q is not completely continuous.

Therefore, by Corollary 1 and Corollary 1 of [27], $f_{qv} : (L, \tau(\alpha), \mathbb{Z}) \longrightarrow (M, \tau(\phi), \mathbb{Z})$ is soft s-r-c, but not soft completely continuous.

In Theorems 4 and 5 and Example 2, we discuss the relationships between the classes of somewhat-r-continuous functions and soft somewhat-continuous functions:

Theorem 4. *Soft s-r-c functions are soft s-c.*

Proof. Let $f_{qv} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ be soft s-r-c. Let $K \in \digamma$ and $f_{qv}^{-1}(K) \neq 0_S$. Then, there exists $N \in RO(\Gamma) - \{0_S\}$ such that $N \widetilde{\subseteq} f_{qv}^{-1}(K)$. Since $RO(\Gamma) \subseteq \Gamma$, then $N \in \Gamma$. This ends the proof. □

The opposite of Theorem 4 does not have to be true.

Example 2. *Let $L = \{1, 2, 3, 4\}$, $M = \{5, 6, 7\}$, $\alpha = \{\emptyset, L, \{1, 3\}, \{4\}, \{3\}, \{3, 4\}, \{1, 3, 4\}\}$, and $\phi = \{\emptyset, M, \{6\}, \{7\}, \{6, 7\}\}$. Define $q : L \longrightarrow M$ and $v : \mathbb{N} \longrightarrow \mathbb{N}$ by $q(1) = q(4) = 5$, $q(2) = q(3) = 7$, and $v(n) = n$ for all $n \in \mathbb{N}$.*

To see that $q : (L, \alpha) \longrightarrow (M, \phi)$ is s-c, let $U \in \phi$ such that $q^{-1}(U) \neq \emptyset$. Then, $\{7\} \subseteq U$ and $q^{-1}(\{7\}) = \{2, 3\} \subseteq q^{-1}(U)$, and thus, we can choose $\{3\} \in \alpha - \{\emptyset\}$ such that $\{3\} \subseteq q^{-1}(U)$.

Since $\{7\} \in \phi - \{\emptyset\}$ such that $q^{-1}(\{7\}) = \{2, 3\} \neq \emptyset$ while there is no $W \in RO(\alpha) - \{\emptyset\}$ such that $W \subseteq \{2, 3\}$, then q is not s-r-c.

Therefore, by Corollaries 1 and 2, $f_{qv} : (L, \tau(\alpha), \mathbb{N}) \longrightarrow (M, \tau(\phi), \mathbb{N})$ is soft s-c, but not soft s-r-c.

Theorem 5. *If $f_{qv} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ is soft s-c and (L, Γ, S) is soft locally indiscrete, then f_{qv} is soft s-r-c.*

Proof. Let $K \in \digamma$ such that $f_{qv}^{-1}(K) \neq 0_S$. Then, we find $N \in \Gamma - \{0_S\}$ such that $N \widetilde{\subseteq} f_{qv}^{-1}(K)$. Since (L, Γ, S) is soft locally indiscrete, then $\Gamma = RO(\Gamma)$, and so, $N \in RO(\Gamma)$. This shows that f_{qv} is soft s-r-c. □

From the above theorems, we have the following implications. However, Examples 1 and 2 show that the converses of these implications are not true.

Soft complete continuity $\longrightarrow$ soft somewhat-r-continuity $\longrightarrow$ soft somewhat-continuity.

In Theorem 6 and Corollary 3, we investigate the correspondence between the concept of soft δ-continuity with its analogous topological concept:

Theorem 6. *Let $\{(L,\Gamma_s) : s \in S\}$ and $\{(M,\digamma_t) : t \in T\}$ be two collections of TSs. Let $q : L \longrightarrow M$ and $v : S \longrightarrow T$ be functions. Then, $f_{qv} : (L, \oplus_{s\in S}\Gamma_s, S) \longrightarrow (M, \oplus_{t\in T}\digamma_t, T)$ is soft δ-continuous if and only if $q : (L,\Gamma_s) \longrightarrow \left(M, \digamma_{v(s)}\right)$ is δ-continuous for all $s \in S$.*

Proof. *Necessity:* Assume that $f_{qv} : (L, \oplus_{s\in S}\Gamma_s, S) \longrightarrow (M, \oplus_{t\in T}\digamma_t, T)$ is soft δ-continuous. Let $s \in S$. Let $x \in L$, and let $W \in RO\left(\digamma_{v(s)}\right)$ such that $q(x) \in W$. Then, we have $f_{qv}(s_x) = (v(s))_{q(x)} \tilde{\in} (v(s))_W$, and by Theorem 14 of [30], $(v(s))_W \in RO(\oplus_{t\in T}\digamma_t)$. So, we find $N \in RO(\oplus_{s\in S}\Gamma_s)$ such that $f_{qv}(N) \tilde{\subseteq} (v(s))_W$ and, thus, $q(N(s)) \subseteq \left(f_{qv}(N)\right)(v(s)) \subseteq W$. Moreover, by Theorem 14 of [30], $N(s) \in RO(\Gamma_s)$. This shows that $q : (L,\Gamma_s) \longrightarrow \left(M, \digamma_{v(s)}\right)$ is δ-continuous.

Sufficiency: Assume that $q : (L,\Gamma_s) \longrightarrow \left(M, \digamma_{v(s)}\right)$ is δ-continuous for all $s \in S$. Let $a_x \in SP(L,S)$, and let $K \in RO(\oplus_{t\in T}\digamma_t)$ such that $f_{qv}(a_x) = v(a)_{q(x)} \tilde{\in} K$. Then, we have $q(x) \in K(v(a))$, and by Theorem 14 of [30], $K(v(a)) \in RO\left(\digamma_{v(a)}\right)$. Since $q : (L,\Gamma_a) \longrightarrow \left(M, \digamma_{v(a)}\right)$ is δ-continuous, then we find $W \in RO(\Gamma_a)$ such that $x \in W$ and $q(W) \subseteq K(v(a))$. Now, we have $a_x \tilde{\in} a_W$, and by Theorem 14 of [30], $a_W \in RO(\Gamma_a)$. Also, it is not difficult to check that $f_{qv}(a_W) \tilde{\subseteq} K$. Therefore, $f_{qv} : (L, \oplus_{s\in S}\Gamma_s, S) \longrightarrow (M, \oplus_{t\in T}\digamma_t, T)$ is soft δ-continuous. □

Corollary 3. *Let $q : (L,\alpha) \longrightarrow (M,\phi)$ and $v : S \longrightarrow T$ be two functions. Then, $q : (L,\alpha) \longrightarrow (M,\phi)$ is δ-continuous if and only if $f_{qv} : (L, \tau(\alpha), S) \longrightarrow (M, \tau(\phi), T)$ is soft δ-continuous.*

Proof. For each $s \in S$ and $t \in T$, put $\Gamma_s = \alpha$ and $\digamma_t = \phi$. Then, $\tau(\alpha) = \oplus_{s\in S}\Gamma_s$ and $\tau(\phi) = \oplus_{t\in T}\digamma_t$. We obtain the result by using Theorem 6. □

The following two examples demonstrate the independence of the concepts of soft s-*r*-c and soft δ-continuous:

Example 3. *Let $f_{qv} : (L, \tau(\alpha), \mathbb{Z}) \longrightarrow (M, \tau(\phi), \mathbb{Z})$ be as in Example 1. Then, f_{qv} is soft s-r-c. Since $\{d\} \in RO(\phi)$ while $q^{-1}(\{d\}) = [3,\infty) \notin \alpha$, then q is not δ-continuous. Thus, by Corollary 3, f_{qv} is not soft δ-continuous.*

Example 4. *Let $L = \{1,2,3\}$, $\alpha = \{\emptyset, L, \{1,2\}, \{3\}\}$, and $\phi = \{\emptyset, L, \{3\}\}$. Define $q : (L,\alpha) \longrightarrow (L,\phi)$ and $v : \mathbb{N} \longrightarrow \mathbb{N}$ by $q(1) = 2$, $q(2) = 3$, $q(3) = 1$, and $v(n) = n$ for all $n \in \mathbb{N}$.*

If $U \in RO(\phi) = \{\emptyset, L\}$, then $q^{-1}(U) \in \{\emptyset, L\}$. Hence, q is δ-continuous. Since $\{3\} \in \phi - \{\emptyset\}$ such that $q^{-1}(\{3\}) = \{2\} \neq \emptyset$ while there is no $W \in RO(\alpha) - \{\emptyset\}$ such that $W \subseteq \{2\}$, then q is not s-r-c.

As a result of Corollaries 1 and 3, $f_{qv} : (L, \tau(\alpha), \mathbb{N}) \longrightarrow (L, \tau(\phi), \mathbb{N})$ is soft δ-continuous, yet not soft s-r-c.

In the following result, we give a sufficient condition for soft δ-continuous functions to be soft somewhat-*r*-continuous:

Theorem 7. *If $f_{qv} : (L,\Gamma,S) \longrightarrow (M,\digamma,T)$ is soft δ-continuous and $(M,\digamma,T)$ is soft locally indiscrete, then f_{qv} is soft s-r-c.*

Proof. Let $K \in \digamma$ such that $f_{qv}^{-1}(K) \neq 0_S$. Since $(M,\digamma,T)$ is soft locally indiscrete, then $K \in RO(\digamma)$. Choose $a_x \tilde{\in} f_{qv}^{-1}(K)$. Then, $f_{qv}(a_x) \tilde{\in} K$. Since f_{qv} is soft δ-continuous, we find $N \in RO(\Gamma)$ such that $a_x \tilde{\in} N$ and $f_{qv}(N) \tilde{\subseteq} K$. Thus, we have $N \in RO(\Gamma) - \{0_S\}$ such that $N \tilde{\subseteq} f_{qv}^{-1}\left(f_{qv}(N)\right) \tilde{\subseteq} f_{qv}^{-1}(K)$. This shows that f_{qv} is soft s-*r*-c. □

Definition 11. *Let (L, Γ, S) be an STS, and let $H \in SS(L, S)$. Then, H is called soft r-dense in (L, Γ, S) if there is no $K \in RC(\Gamma) - \{1_S\}$ such that $M \widetilde{\subseteq} K$.*

In Theorem 8 and Example 5, we discuss the relationships between the soft dense sets and soft r-dense sets:

Theorem 8. *In any STS (L, Γ, S), soft dense sets are soft r-dense sets.*

Proof. Assume, on the other hand, that a soft dense set H exists in (L, Γ, S) that is not soft r-dense in (L, Γ, S). Then, we find $K \in RC(\Gamma) - \{1_S\}$ such that $H \widetilde{\subseteq} K$. Since $RC(\Gamma) \subseteq \Gamma^c$, then $K \in \Gamma^c$. Hence, H is not soft dense in (L, Γ, S), which is a contradiction. □

The following example demonstrates that the inverse of Theorem 8 is not true:

Example 5. *Let $L = \mathbb{R}$, $S = \mathbb{R}$, and $\Gamma = \{0_S, 1_S, K\}$, where $K(s) = \{s+1\}$ for every $s \in S$. Let $H = 1_S - K$. Since $RC(\Gamma) = \{0_S, 1_S\}$, then H is soft r-dense in (L, Γ, S). On the other hand, since $H \in \Gamma^c$, then H is not soft dense in (L, Γ, S).*

The following result characterizes soft r-dense sets in terms of soft regular open sets:

Theorem 9. *Let (L, Γ, S) be an STS, and let $H \in SS(L, S)$. Then, H is soft r-dense in (L, Γ, S) if and only if, for any $G \in RO(\Gamma) - \{0_S\}$, $G \widetilde{\cap} H \neq 0_S$.*

Proof. *Necessity:* Assume that H is soft r-dense in (L, Γ, S) and, on the contrary, that there exists $G \in RO(\Gamma) - \{0_S\}$ such that $G \widetilde{\cap} H = 0_S$. Then, we have $1_S - G \in RC(\Gamma) - \{1_S\}$ and $H \widetilde{\subseteq} 1_S - G$. Hence, H is not soft r-dense in (L, Γ, S), which is a contradiction.

Sufficiency: Assume that $G \widetilde{\cap} H \neq 0_S$ for each $G \in RO(\Gamma) - \{0_S\}$. Assume, on the other hand, there exists $K \in RC(\Gamma) - \{1_S\}$ such that $H \widetilde{\subseteq} K$. Then, we have $1_S - K \in RO(\Gamma) - \{0_S\}$ and $(1_S - K) \widetilde{\cap} H = 0_S$, which is a contradiction. □

In Theorems 10 and 11, we give sufficient conditions for the soft composition of two soft somewhat-r-continuous functions to be soft somewhat-r-continuous:

Theorem 10. *If $f_{q_1 v_1} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ and $f_{q_2 v_2} : (M, \digamma, T) \longrightarrow (R, \Upsilon, B)$ are soft s-r-c functions and $f_{q_1 v_1}(1_S)$ is soft r-dense in $(M, \digamma, T)$, then $f_{(q_2 \circ q_1)(v_2 \circ v_1)} : (L, \Gamma, S) \longrightarrow (R, \Upsilon, B)$ is soft s-r-c.*

Proof. Let $H \in \Upsilon$ such that $f^{-1}_{(q_2 \circ q_1)(v_2 \circ v_1)}(H) \neq 0_S$. Then, $f^{-1}_{q_1 v_1}\left(f^{-1}_{q_2 v_2}(H)\right) = f^{-1}_{(q_2 \circ q_1)(v_2 \circ v_1)}(H) \neq 0_S$, and thus, $f^{-1}_{q_2 v_2}(H) \neq 0_T$. Since $f_{q_2 v_2}$ is soft s-r-c, we find $G \in RO(\digamma) - \{0_T\}$ such that $G \widetilde{\subseteq} f^{-1}_{q_2 v_2}(H)$. Since $f_{q_1 v_1}(1_S)$ is soft r-dense in $(M, \digamma, T)$, then by Theorem 9, $G \widetilde{\cap} f_{q_1 v_1}(1_S) \neq 0_T$, and thus, $f^{-1}_{q_1 v_1}(G) \neq 0_S$. Since $f_{q_1 v_1}$ is soft s-r-c, we find $K \in RO(\Gamma) - \{0_S\}$ such that $K \widetilde{\subseteq} f^{-1}_{q_1 v_1}(G) \widetilde{\subseteq} f^{-1}_{q_1 v_1}\left(f^{-1}_{q_2 v_2}(H)\right) = f^{-1}_{(p_2 \circ p_1)(u_2 \circ u_1)}(H)$. This shows that $f_{(q_2 \circ q_1)(v_2 \circ v_1)}$ is soft s-r-c. □

Theorem 11. *If $f_{q_1 v_1} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ is soft s-r-c and $f_{q_2 v_2} : (M, \digamma, T) \longrightarrow (R, \Upsilon, B)$ is soft continuous, then $f_{(q_2 \circ q_1)(v_2 \circ v_1)} : (L, \Gamma, S) \longrightarrow (R, \Upsilon, B)$ is soft s-r-c.*

Proof. Let $H \in \Upsilon$ such that $f^{-1}_{(q_2 \circ q_1)(v_2 \circ v_1)}(H) \neq 0_S$. Since $f_{q_2 v_2}$ is soft continuous, then $f^{-1}_{q_2 v_2}(H) \in \digamma$. Since $f_{q_1 v_1}$ is soft s-r-c and $f^{-1}_{q_1 v_1}\left(f^{-1}_{q_2 v_2}(H)\right) = f^{-1}_{(q_2 \circ q_1)(v_2 \circ v_1)}(H) \neq 0_S$, then we find $G \in RO(\Gamma) - \{0_S\}$ such that $G \widetilde{\subseteq} f^{-1}_{q_1 v_1}\left(f^{-1}_{q_2 v_2}(H)\right) = f^{-1}_{(q_2 \circ q_1)(v_2 \circ v_1)}(H)$. □

The soft composite of two soft s-r-c functions is not necessarily soft s-r-c:

Example 6. *Let $L = \{1,2,3\}$, $\alpha = \{\emptyset, L\}$, $\phi = \{\emptyset, L, \{2\}, \{1,3\}\}$, and $\rho = \{\emptyset, L, \{1,2\}\}$. Define $q_1 : (L,\alpha) \longrightarrow (L,\phi)$, $q_2 : (L,\phi) \longrightarrow (L,\rho)$, and $v : \mathbb{N} \longrightarrow \mathbb{N}$ by $q_1(1) = 1$, $q_1(2) = 3$, $q_1(3) = 3$, $q_2(l) = l$ for all $l \in L$ and $v(n) = n$ for all $n \in \mathbb{N}$. Then, clearly, q_1 is continuous and q_2 is s-r-c while $q_2 \circ q_1 : (L,\alpha) \longrightarrow (L,\rho)$ is not s-r-c. So, by Theorem 5.31 of [18], $f_{q_1 v_1} : (L, \tau(\alpha), \mathbb{N}) \longrightarrow (L, \tau(\phi), \mathbb{N})$ is soft continuous, and by Corollary 1, $f_{q_2 v_2} : (L, \tau(\phi), \mathbb{N}) \longrightarrow (L, \tau(\rho), \mathbb{N})$ is soft s-r-c, but $f_{(q_2 \circ q_1)(v_2 \circ v_1)} : (L, \tau(\alpha), \mathbb{N}) \longrightarrow (L, \tau(\rho), \mathbb{N})$ is not soft s-r-c.*

The following result gives two characterizations of soft somewhat-r-continuous surjective functions:

Theorem 12. *Let $f_{qv} : (L, \Gamma, S) \longrightarrow (M, F, T)$ be surjective. Then, the following are equivalent:*

(a) f_{qv} is soft s-r-c.

(b) For each $A \in F^c$ such that $f_{qv}^{-1}(A) \neq 1_S$, we find $B \in RC(\Gamma) - \{1_S\}$ such that $f_{qv}^{-1}(A) \widetilde{\subseteq} B$.

(c) For each soft r-dense set H in (L, Γ, S), $f_{qv}(H)$ is a soft dense set in (M, F, T).

Proof. (a) $\longrightarrow$ (b): Let $A \in F^c$ such that $f_{qv}^{-1}(A) \neq 1_S$. Then, $1_T - A \in F$ and $f_{qv}^{-1}(1_T - A) = 1_S - f_{pu}^{-1}(A) \neq 0_S$. Based on (a), we find $R \in RO(\Gamma) - \{0_S\}$ such that $R \widetilde{\subseteq} f_{qv}^{-1}(1_T - A) = 1_S - f_{pu}^{-1}(A)$. Let $B = 1_S - R$. Then, $B \in RC(\Gamma) - \{1_S\}$ such that $f_{qv}^{-1}(A) \widetilde{\subseteq} B$.

(b) $\longrightarrow$ (c): On the contrary, assume a soft r-dense set H exists in (L, Γ, S) such that $f_{qv}(H)$ is not soft dense in (M, F, T). Then, there exists $A \in F^c - \{1_T\}$ such that $f_{qv}(H) \widetilde{\subseteq} A$. If $f_{qv}^{-1}(A) = 1_S$, then $1_T = f_{qv}(1_S) = f_{qv}\left(f_{qv}^{-1}(A)\right) \widetilde{\subseteq} A$, and hence, $A = 1_T$. Therefore, $f_{pu}^{-1}(A) \neq 1_S$. So, by (b), we find $B \in RC(\Gamma) - \{1_S\}$ such that $f_{qv}^{-1}(A) \widetilde{\subseteq} B$, and so, $H \widetilde{\subseteq} f_{qv}^{-1}\left(f_{qv}(H)\right) \widetilde{\subseteq} f_{qv}^{-1}(A) \widetilde{\subseteq} B$. This conflicts with the statement that H is a soft r-dense set in (L, Γ, S).

(c) $\longrightarrow$ (a): On the contrary, assume that f_{qv} is not soft s-r-c. Then, we find $H \in F$ such that $f_{qv}^{-1}(H) \neq 0_S$, but there is no $G \in RO(\Gamma) - \{0_S\}$ such that $G \widetilde{\subseteq} f_{qv}^{-1}(H)$. □

Claim 1. *$1_S - f_{qv}^{-1}(H)$ is soft r-dense in (L, Γ, S).*

Proof of Claim 1. Assume, however, that $1_S - f_{qv}^{-1}(H)$ is not soft r-dense in (L, Γ, S). Then, by Theorem 9, there exists $G \in RO(\Gamma) - \{0_S\}$ such that $G \widetilde{\cap} \left(1_S - f_{qv}^{-1}(H)\right) = 0_S$, and hence, $G \widetilde{\subseteq} f_{qv}^{-1}(H)$, a contradiction. □

Thus, by the above Claim 1 and (c), $f_{qv}(1_S - f_{qv}^{-1}(H)) = f_{qv}(f_{qv}^{-1}(1_T - H))$ is soft dense in (M, F, T). Since $f_{qv}(f_{qv}^{-1}(1_T - H)) \widetilde{\subseteq} 1_T - H$, then $1_T - H$ is soft dense in (M, F, T), and thus, $1_T - H = Cl_F(1_T - H) = 1_T$. Therefore, $H = 0_T$, and hence, $f_{qv}^{-1}(H) = 0_S$. This is a contradiction.

Theorems 13 and 14 discuss the behavior of soft somewhat-r-continuous functions under soft subspaces:

Theorem 13. *Let (L, Γ, S) and (M, F, T) be any two STSs. Let $U \subseteq L$ such that $C_U \in RO(\Gamma)$. If $f_{qv} : (U, \Gamma_U, S) \longrightarrow (M, F, T)$ is soft s-r-c such that $f_{qv}(C_U)$ is soft dense in (M, F, T), then for each extension $q_1 : L \longrightarrow M$, $f_{q_1 v} : (L, \Gamma, S) \longrightarrow (M, F, T)$ is soft s-r-c.*

Proof. Let $G \in F$ such that $f_{q_1 v}^{-1}(G) \neq 0_S$. Since $f_{qv}(C_U)$ is soft dense in (M, F, T), $f_{qv}(C_U) \widetilde{\cap} G \neq 0_T$. Then, $C_U \widetilde{\cap} f_{qv}^{-1}(G) \neq 0_S$, and so, $C_U \widetilde{\cap} f_{qv}^{-1}(G) \neq 0_S$. Since $f_{qv} : (U, \Gamma_U, S) \longrightarrow (M, F, T)$ is soft s-r-c, then there exists $H \in RO(\Gamma_U) - \{0_S\}$ such that $H \widetilde{\subseteq} f_{qv}^{-1}(G)$. Since $H \in RO(\Gamma_U)$ and $C_U \in RO(\Gamma)$, then $H \in RO(\Gamma)$. This shows that $f_{q_1 v}$ is soft s-r-c. □

Theorem 14. *Let (L, Γ, S) and (M, F, T) be any two STSs, and let $L = A \cup B$, where $C_A, C_B \in RO(\Gamma)$. If $f_{qv} : (L, \Gamma, S) \longrightarrow (M, F, T)$ is a soft function such that the soft restrictions $\left(f_{qv}\right)_{|C_A}$:*

$(A, \Gamma_A, S) \longrightarrow (M, \digamma, T)$ and $(f_{qv})_{|C_B} : (B, \Gamma_B, S) \longrightarrow (M, \digamma, T)$ are soft s-r-c, then f_{qv} is soft s-r-c.

Proof. Let $G \in \digamma$ such that $f_{qv}^{-1}(G) \neq 0_S$, then $\left((f_{qv})_{|C_A}\right)^{-1}(G) \neq 0_S$ or $\left((f_{qv})_{|C_B}\right)^{-1}(G) \neq 0_S$. We can assume, without loss of generality, that $\left((f_{qv})_{|C_A}\right)^{-1}(G) \neq 0_S$. Then, we find $K \in RO(\Gamma_A) - \{0_S\}$ such that $K \widetilde{\subseteq} \left((f_{qv})_{|C_A}\right)^{-1}(G) \widetilde{\subseteq} f_{qv}^{-1}(G)$. Since $K \in RO(\Gamma_A)$ and $C_A \in RO(\Gamma)$, then $K \in RO(\Gamma)$. This completes the proof. □

Definition 12. *An STS (L, Γ, S) is soft r-separable if there exists a countable soft set $H \in SS(L, S)$ such that H is soft r-dense in (L, Γ, S).*

By Theorem 8, soft separable STSs are soft r-separable. The following is an example to show that soft *r*-separability does not imply soft separability in general:

Example 7. *Let $L = \mathbb{R}$, $S = \{a\}$, and $\Gamma = \{0_S\} \cup \{a_U : U \subseteq \mathbb{R}$ and $\mathbb{R} - U$ is countable$\}$. Then, $RC(\Gamma) = \{0_S, 1_S\}$. So, any $H \in RC(\Gamma) - \{0_S\}$ is soft r-dense. In particular, $a_{\{1\}}$ is a countable soft set and soft r-dense in (L, Γ, S), and hence, (L, Γ, S) is soft r-separable. On the other hand, it is not difficult to show that (L, Γ, S) is not soft separable.*

The following result shows that the soft somewhat-*r*-continuous image of a soft *r*-separable space is soft separable:

Theorem 15. *If $f_{qv} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ is soft s-r-c and (L, Γ, S) is soft r-separable, then $(M, \digamma, T)$ is soft separable.*

Proof. Let $f_{qv} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ be soft s-*r*-c such that (L, Γ, S) is soft *r*-separable. Choose a countable soft set $H \in SS(L, S)$ such that H is soft *r*-dense in (L, Γ, S). Then, $f_{qv}(H)$ is a countable soft set, and by Theorem 12 (c), $f_{qv}(H)$ is soft dense in $(M, \digamma, T)$. Therefore, $(M, \digamma, T)$ is soft separable. □

Definition 13. *Let (L, Γ, S) and (L, Υ, S) be two STSs. Then, Γ is called soft r-weakly equivalent to Υ if, for each $A \in RO(\Gamma) - \{0_S\}$, we find $B \in RO(\Upsilon) - \{0_S\}$ such that $B \widetilde{\subseteq} A$ and, for each $A \in RO(\Upsilon) - \{0_S\}$, we find $B \in RO(\Gamma) - \{0_S\}$ such that $B \widetilde{\subseteq} A$.*

Theorem 16. *Let (L, Γ, S) and (L, Υ, S) be two STSs. Let $q : L \longrightarrow L$ and $v : S \longrightarrow S$ denote the identities. Then, the following are equivalent:*

(a) *Γ is soft r-weakly equivalent to Υ.*
(b) *The soft functions $f_{qv} : (L, \Gamma, S) \longrightarrow (L, \Upsilon, S)$ and $f_{pu} : (L, \Upsilon, S) \longrightarrow (L, \Gamma, S)$ are both soft s-r-c.*

Proof. Obvious. □

Theorem 17. *Let $f_{qv} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ be soft s-r-c. If (L, Υ, S) is an STS such that Γ is soft r-weakly equivalent to Υ, then $f_{qv} : (L, \Upsilon, S) \longrightarrow (M, \digamma, T)$ is soft s-r-c.*

Proof. Let $H \in \digamma$ such that $f_{qv}^{-1}(H) \neq 0_S$. Since $f_{qv} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ is soft s-*r*-c, then we find $K \in RO(\Gamma) - \{0_S\}$ such that $K \widetilde{\subseteq} f_{pu}^{-1}(H)$. Since Γ is soft *r*-weakly equivalent to Υ, then we find $G \in RO(\Upsilon) - \{0_S\}$ such that $G \widetilde{\subseteq} K \widetilde{\subseteq} f_{pu}^{-1}(K)$. This shows that $f_{qv} : (L, \Upsilon, S) \longrightarrow (M, \digamma, T)$ is soft s-*r*-c. □

Theorem 18. *Let $f_{qv} : (L, \Gamma, S) \longrightarrow (M, F, T)$ be soft s-r-c and surjective. If (L, Υ, S) and (M, Ψ, T) are STSs such that Γ is soft r-weakly equivalent to Υ and F is soft weakly equivalent to Ψ, then $f_{qv} : (L, \Upsilon, S) \longrightarrow (M, \Psi, T)$ is soft s-r-c.*

Proof. Let $G \in \Psi$ such that $f_{pu}^{-1}(G) \neq 0_S$. Since F is soft weakly equivalent to Ψ, then we find $R \in F - \{0_T\}$ such that $R \widetilde{\subseteq} G$. Since f_{qv} is surjective, then $f_{pu}^{-1}(R) \neq 0_S$. Since $f_{qv} : (L, \Gamma, S) \longrightarrow (M, F, T)$ is soft s-r-c, then we find $H \in RO(\Gamma) - \{0_S\}$ such that $H \widetilde{\subseteq} f_{pu}^{-1}(R)$. Since Γ is soft r-weakly equivalent to Υ, then we find $K \in RO(\Upsilon) - \{0_S\}$ such that $K \widetilde{\subseteq} H \widetilde{\subseteq} f_{pu}^{-1}(R) \widetilde{\subseteq} f_{pu}^{-1}(G)$. This completes the proof. □

Definition 14. *An STS (L, Γ, S) is called a soft r-D-space if, for every $G, H \in RO(\Gamma) - \{0_S\}$, $G \widetilde{\cap} H \neq 0_S$.*

Soft D-spaces are soft r-D-spaces. However, we raise the following question about the converse:

Question 1. *Is it true that every soft r-D-space is a soft D-space?*

The following result shows that the soft somewhat-r-continuous image of a soft r-D-space is a soft D-space:

Theorem 19. *Let $f_{qv} : (L, \Gamma, S) \longrightarrow (M, F, T)$ be soft s-r-c and surjective. If (L, Γ, S) is a soft r-D-space, then (M, F, T) is a soft D-space.*

Proof. Let $f_{qv} : (L, \Gamma, S) \longrightarrow (M, F, T)$ be soft s-r-c and surjective such that (L, Γ, S) is a soft r-D-space. To the contrary, assume that (L, Γ, S) is not a soft D-space. Then, we find $K, H \in F - \{0_T\}$ such that $K \widetilde{\cap} H = 0_T$. Since f_{qv} is surjective, then $f_{qv}^{-1}(K) \neq 0_S$ and $f_{qv}^{-1}(H) \neq 0_S$. Since $f_{qv} : (L, \Gamma, S) \longrightarrow (M, F, T)$ is soft s-r-c, then we find $G, R \in RO(\Gamma) - \{0_S\}$ such that $G \widetilde{\subseteq} f_{qv}^{-1}(K)$ and $R \widetilde{\subseteq} f_{qv}^{-1}(H)$. Therefore, $G \widetilde{\cap} R \widetilde{\subseteq} f_{qv}^{-1}(K) \widetilde{\cap} f_{qv}^{-1}(H) = f_{qv}^{-1}(K \widetilde{\cap} H) = f_{qv}^{-1}(0_T) = 0_S$. This shows that (L, Γ, S) is not a soft r-D-space, a contradiction. □

4. Soft Somewhat-*r*-Open Functions

In this section, we define soft somewhat-r-open functions. We show that this class of soft functions is a subclass of soft somewhat open functions. With the help of examples, we introduce various properties of this new class of soft functions.

Definition 15. *A soft function $f_{qv} : (L, \Gamma, S) \longrightarrow (M, F, T)$ is called soft somewhat-r-open (soft s-r-o) if, for each $K \in \Gamma - \{0_S\}$, there exists $H \in RO(F) - \{0_T\}$ such that $H \widetilde{\subseteq} f_{qv}(K)$.*

In Theorems 20 and 21 and Corollaries 4 and 5, we investigate the correspondence between the concepts of soft somewhat-r-openness and soft somewhat-openness with their analogous topological concepts:

Theorem 20. *Let $\{(L, \Gamma_s) : s \in S\}$ and $\{(M, F_t) : t \in T\}$ be two collections of TSs. Let $q : L \longrightarrow M$ and $v : S \longrightarrow T$ be functions where v is bijective. Then $f_{qv} : (L, \oplus_{s \in S} \Gamma_s, S) \longrightarrow (M, \oplus_{t \in T} F_t, T)$ is soft s-r-o if and only if $q : (L, \Gamma_s) \longrightarrow \left(M, F_{v(s)}\right)$ is s-r-o for all $s \in S$.*

Proof. *Necessity:* Assume that $f_{qv} : (L, \oplus_{s \in S} \Gamma_s, S) \longrightarrow (M, \oplus_{t \in T} F_t, T)$ is soft s-r-o. Let $s \in S$. Let $U \in \Gamma_s - \{\emptyset\}$. Then, $s_U \in \oplus_{s \in S} \Gamma_s - \{0_S\}$. So, we find $H \in RO(\oplus_{t \in T} F_t) - \{0_T\}$ such that $H \widetilde{\subseteq} f_{qv}(s_U) = (v(s))_{q(U)}$. Thus, we have $H(v(s)) \subseteq \left((v(s))_{q(U)}\right)(v(s)) = q(U)$, and by Theorem 14 of [30], $H(v(s)) \in RO\left(F_{v(s)}\right)$. This shows that $q : (L, \Gamma_s) \longrightarrow \left(M, F_{v(s)}\right)$ is s-r-o.

Sufficiency: Assume that $q : (L, \Gamma_s) \longrightarrow \left(M, \digamma_{v(s)}\right)$ is s-r-o for all $s \in S$. Let $K \in \oplus_{s \in S} \Gamma_s - \{0_S\}$. Choose $a \in S$ such that $K(a) \neq \emptyset$. Since $q : (L, \Gamma_a) \longrightarrow \left(M, \digamma_{v(a)}\right)$ is s-r-o, then we find $W \in RO\left(\digamma_{v(a)}\right) - \{\emptyset\}$ such that $W \subseteq q(K(a))$. Then, $(v(a))_W \neq 0_T$, and by Theorem 14 of [30], $(v(a))_W \in RO(\oplus_{t \in T} \digamma_t)$. Moreover, it is not difficult to check that $a_W \widetilde{\subseteq} f_{qv}(K)$. This shows that $f_{qv} : (L, \oplus_{s \in S} \Gamma_s, S) \longrightarrow (M, \oplus_{t \in T} \digamma_t, T)$ is soft s-r-o. □

Corollary 4. *Let $q : (L, \alpha) \longrightarrow (M, \phi)$ and $v : S \longrightarrow T$ be two functions where v is a bijection. Then, $q : (L, \alpha) \longrightarrow (M, \phi)$ is s-r-o if and only if $f_{qv} : (L, \tau(\alpha), S) \longrightarrow (M, \tau(\phi), T)$ is soft s-r-o.*

Proof. For each $s \in S$ and $t \in T$, put $\Gamma_s = \alpha$ and $\digamma_t = \phi$. Then, $\tau(\alpha) = \oplus_{s \in S} \Gamma_s$ and $\tau(\phi) = \oplus_{t \in T} \digamma_t$. We obtain the result by using Theorem 20. □

Example 8. *Let $L = \{1, 2, 3\}$, $\alpha = \{\emptyset, L, \{1\}\}$, and $\phi = \{\emptyset, L, \{2\}, \{3\}, \{2, 3\}\}$. Define $q : L \longrightarrow L$ and $v : \mathbb{Z} \longrightarrow \mathbb{Z}$ by $q(1) = 3$, $q(2) = 1$, $q(3) = 2$, and $v(x) = x$ for every $x \in \mathbb{Z}$. Then, $q : (L, \alpha) \longrightarrow (L, \phi)$ is s-r-o, and by Corollary 4, $f_{qv} : (L, \tau(\alpha), \mathbb{Z}) \longrightarrow (L, \tau(\phi), \mathbb{Z})$ is soft s-r-o.*

Theorem 21. *Let $\{(L, \Gamma_s) : s \in S\}$ and $\{(M, \digamma_t) : t \in T\}$ be two collections of TSs. Let $q : L \longrightarrow M$ and $v : S \longrightarrow T$ be functions where v is bijective. Then, $f_{qv} : (L, \oplus_{s \in S} \Gamma_s, S) \longrightarrow (M, \oplus_{t \in T} \digamma_t, T)$ is soft s-o if and only if $q : (L, \Gamma_s) \longrightarrow \left(M, \digamma_{v(s)}\right)$ is s-o for all $s \in S$.*

Proof. *Necessity:* Assume that $f_{qv} : (L, \oplus_{s \in S} \Gamma_s, S) \longrightarrow (M, \oplus_{t \in T} \digamma_t, T)$ is soft s-o. Let $s \in S$. Let $U \in \Gamma_s - \{\emptyset\}$. Then, $s_U \in \oplus_{s \in S} \Gamma_s - \{0_S\}$. So, we find $H \in \oplus_{t \in T} \digamma_t - \{0_T\}$ such that $H \widetilde{\subseteq} f_{qv}(s_U) = (v(s))_{q(U)}$. Thus, we have $H(v(s)) \subseteq \left((v(s))_{q(U)}\right)(v(s)) = q(U)$ and $H(v(s)) \in RO\left(\digamma_{v(s)}\right)$. This shows that $q : (L, \Gamma_s) \longrightarrow \left(M, \digamma_{v(s)}\right)$ is s-o.

Sufficiency: Assume that $q : (L, \Gamma_s) \longrightarrow \left(M, \digamma_{v(s)}\right)$ is s-o for all $s \in S$. Let $K \in \oplus_{s \in S} \Gamma_s - \{0_S\}$. Choose $a \in S$ such that $K(a) \neq \emptyset$. Since $q : (L, \Gamma_a) \longrightarrow \left(M, \digamma_{v(a)}\right)$ is s-o, then we find $W \in \digamma_{v(a)} - \{\emptyset\}$ such that $W \subseteq q(K(a))$. Then, $(v(a))_W \in \oplus_{t \in T} \digamma_t - \{0_T\}$. Moreover, it is not difficult to check that $(v(a))_W \widetilde{\subseteq} f_{qv}(K)$. This shows that $f_{qv} : (L, \oplus_{s \in S} \Gamma_s, S) \longrightarrow (M, \oplus_{t \in T} \digamma_t, T)$ is soft s-o. □

Corollary 5. *Let $q : (L, \alpha) \longrightarrow (M, \phi)$ and $v : S \longrightarrow T$ be two functions where v is a bijection. Then, $q : (L, \alpha) \longrightarrow (M, \phi)$ is s-o if and only if $f_{qv} : (L, \tau(\alpha), S) \longrightarrow (M, \tau(\phi), T)$ is soft s-o.*

Proof. For each $s \in S$ and $t \in T$, put $\Gamma_s = \alpha$ and $\digamma_t = \phi$. Then, $\tau(\alpha) = \oplus_{s \in S} \Gamma_s$ and $\tau(\phi) = \oplus_{t \in T} \digamma_t$. We obtain the result by using Theorem 21. □

In Theorems 22 and 23 and Example 9, we discuss the relationships between the classes of somewhat-r-open functions and soft somewhat-open functions:

Theorem 22. *Every soft s-r-o function is soft s-o.*

Proof. Let $f_{qv} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ be soft s-r-o. Let $K \in \Gamma - \{0_S\}$. Since f_{qv} is soft s-r-o, then we find $H \in RO(\digamma) - \{0_T\}$ such that $H \widetilde{\subseteq} f_{pu}(K)$. Since $RO(\digamma) \subseteq \digamma$, then $H \in \digamma - \{0_T\}$. Therefore, f_{qv} is soft s-o. □

The converse of Theorem 22 does not have to be true in all cases.

Example 9. *Let α be the cofinite topology on $\mathbb{R}$. Consider the identities $q : (\mathbb{R}, \alpha) \longrightarrow (\mathbb{R}, \alpha)$ and $v : [0, 1] \longrightarrow [0, 1]$. Consider $f_{qv} : (\mathbb{R}, \tau(\alpha), [0, 1]) \longrightarrow (\mathbb{R}, \tau(\alpha), [0, 1])$. Then, f_{qv} is soft* *s-o. Since* $C_{\mathbb{R}-\{0\}} \in \tau(\alpha)$ *while there is no* $H \in RO(\tau(\alpha)) - \left\{0_{[0,1]}\right\} = \left\{1_{[0,1]}\right\}$ *such that* $H \widetilde{\subseteq} f_{qv}(C_{\mathbb{R}-\{0\}}) = C_{\mathbb{R}-\{0\}}$*, then* f_{qv} *is not soft s-r-o.*

Theorem 23. *If* $f_{qv} : (L, \Gamma, S) \longrightarrow (M, F, T)$ *is soft s-o and* (L, Γ, S) *is soft locally indiscrete, then* f_{qv} *is soft s-r-o.*

Proof. Let $K \in \Gamma - \{0_S\}$. Since f_{qv} is soft s-o, we find $H \in F - \{0_T\}$ such that $H \widetilde{\subseteq} f_{qv}(K)$. Since (M, F, T) is soft locally indiscrete, then $H \in RO(F)$. This shows that f_{qv} is soft s-r-o. □

In Theorem 24, we give a sufficient condition for the soft composition of two soft somewhat-r-open functions to be soft somewhat-r-open:

Theorem 24. *If* $f_{q_1v_1} : (L, \Gamma, S) \longrightarrow (M, F, T)$ *is soft open and* $f_{q_2v_2} : (M, F, T) \longrightarrow (R, \Upsilon, B)$ *is soft s-r-o, then* $f_{(q_2 \circ q_1)(v_2 \circ v_1)} : (L, \Gamma, S) \longrightarrow (R, \Upsilon, B)$ *is soft s-r-o.*

Proof. Let $K \in \Gamma - \{0_S\}$. Since $f_{q_1v_1}$ is soft open, then $f_{q_1v_1}(K) \in F - \{0_T\}$. Since $f_{q_2v_2}$ is soft s-r-o, then we find $H \in \Upsilon - \{0_B\}$ such that $H \widetilde{\subseteq} f_{q_2v_2}(f_{q_1v_1}(K)) = f_{(q_2 \circ q_1)(v_2 \circ v_1)}(K)$. This shows that $f_{(q_2 \circ q_1)(v_2 \circ v_1)}$ is soft s-r-o. □

In Theorems 25 and 26, we give characterizations of soft somewhat-r-open functions:

Theorem 25. *Let* $f_{qv} : (L, \Gamma, S) \longrightarrow (M, F, T)$ *be a soft function. Then, the following are equivalent:*

(a) f_{qv} *is soft s-r-o.*

(b) *If H is soft r-dense in* (M, F, T)*, then* $f_{qv}^{-1}(H)$ *is soft dense in* (L, Γ, S)*.*

Proof. (a) $\Longrightarrow$ (b): Assume, on the other hand, that we find a soft r-dense set H in (M, F, T) such that $f_{qv}^{-1}(H)$ is not soft dense in (L, Γ, S). Then, $1_S - Cl_\Gamma\left(f_{qv}^{-1}(H)\right) \in \Gamma - \{0_S\}$. So, by (a), we find $K \in RO(F) - \{0_T\}$ such that

$$\begin{array}{rcl} K & \widetilde{\subseteq} & f_{qv}(1_S - Cl_\Gamma\left(f_{qv}^{-1}(H)\right)) \\ & \widetilde{\subseteq} & f_{qv}(1_S - f_{qv}^{-1}(H)) \\ & = & f_{qv}(f_{qv}^{-1}(1_T - H)) \\ & \widetilde{\subseteq} & 1_T - H. \end{array}$$

Thus, $H \widetilde{\cap} K = 0_T$. Therefore, by Theorem 9, H is not soft r-dense in (M, F, T), which is a contradiction.

(b) $\Longrightarrow$ (a): Assume, on the other hand, that there exists $H \in \Gamma - \{0_S\}$ such that, if $K \in RO(F)$ such that $K \widetilde{\subseteq} f_{qv}(H)$, then $K = 0_T$. □

Claim 2. $1_T - f_{qv}(H)$ *is soft r-dense in* (M, F, T)*.*

Proof of Claim 2. Suppose to the contrary that $1_T - f_{qv}(H)$ is not soft r-dense in (M, F, T). Then, by Theorem 5, there exists $K \in RO(F) - \{0_T\}$ such that $K \widetilde{\cap} (1_T - f_{qv}(H)) = 0_T$, and hence, $K \widetilde{\subseteq} f_{qv}(H)$, a contradiction.

Thus, by the above Claim 2 and (b), $f_{qv}^{-1}(1_T - f_{qv}(H))$ is soft dense in (L, Γ, S). Since $f_{qv}^{-1}(1_T - f_{qv}(H)) = 1_S - f_{qv}^{-1}(f_{qv}(H)) \widetilde{\subseteq} 1_S - H$, then $H \widetilde{\cap} f_{qv}^{-1}(1_T - f_{qv}(H)) = 0_S$, which implies that $f_{qv}^{-1}(1_T - f_{qv}(H))$ is not soft dense in (L, Γ, S), which is a contradiction. □

Theorem 26. *Let* $f_{qv} : (L, \Gamma, S) \longrightarrow (M, F, T)$ *be bijective. Then, the following are equivalent:*

(a) f_{qv} *is soft s-r-o.*

(b) *If* $A \in \Gamma^c$ *such that* $f_{qv}(A) \neq 0_T$*, then there exists* $B \in RC(F) - \{0_T\}$ *such that* $f_{qv}(A) \widetilde{\subseteq} B$*.*

Proof. Since f_{qv} is bijective, then f_{qv} is soft s-r-o if and only if $f_{q^{-1}v^{-1}}$ is soft s-r-c. So, we obtain the result by using Theorem 12. □

Theorems 27 and 28 discuss the behavior of soft somewhat-r-open functions under soft subspaces:

Theorem 27. *If $f_{qv} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ is soft s-r-o and $W \subseteq L$ such that $C_W \in \Gamma$, then the soft restriction $\left(f_{qv}\right)_{|C_W} : (W, \Gamma_W, S) \longrightarrow (M, \digamma, T)$ is soft s-r-o.*

Proof. Assume that $f_{qv} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ is soft s-r-o, and let $W \subseteq L$ such that $C_W \in \Gamma$. Let $H \in \Gamma_W - \{0_S\}$. Since $C_W \in \Gamma$, then $\Gamma_W \subseteq \Gamma$, and thus, $H \in \Gamma - \{0_S\}$. Since $f_{qv} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ is soft s-r-o, then we find $K \in RO(\digamma) - \{0_T\}$ such that $K \widetilde{\subseteq} f_{qv}(H) = \left(f_{qv}\right)_{|C_W}(H)$. □

Theorem 28. *Let (L, Γ, S) and $(M, \digamma, T)$ be any two STSs. Let $U \subseteq L$ such that C_U is soft dense in (L, Γ, S). If $f_{qv} : (U, \Gamma_U, S) \longrightarrow (M, \digamma, T)$ is soft s-r-o, then for any extension $q_1 : L \longrightarrow M$, $f_{q_1 v} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ is soft s-r-o.*

Proof. Let $H \in \Gamma - \{0_S\}$. Since C_U is soft dense in (L, Γ, S), then $H \widetilde{\cap} C_U \neq 0_S$. Since $f_{qv} : (U, \Gamma_U, S) \longrightarrow (M, \digamma, T)$ is soft s-r-o and $H \widetilde{\cap} C_U \in \Gamma_U - \{0_S\}$, then we find $K \in RO(\digamma) - \{0_T\}$ such that $K \widetilde{\subseteq} f_{qu}(H \widetilde{\cap} C_U) \widetilde{\subseteq} f_{q_1 v}(H)$. This completes the proof. □

Theorem 29. *Let $f_{qv} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ be soft s-r-o. If (L, Υ, S) and (M, Ψ, T) are STSs such that Γ is soft weakly equivalent to Υ and $\digamma$ is soft r-weakly equivalent to Ψ, then $f_{qv} : (L, \Upsilon, S) \longrightarrow (M, \Psi, T)$ is soft s-r-o.*

Proof. Let $H \in \Upsilon - \{0_S\}$. Since Γ is soft weakly equivalent to Υ, then we find $G \in \Gamma - \{0_S\}$ such that $G \widetilde{\subseteq} H$. Since $f_{qv} : (L, \Gamma, S) \longrightarrow (M, \digamma, T)$ is soft s-r-o, then we find $R \in RO(\digamma) - \{0_T\}$ such that $R \widetilde{\subseteq} f_{qv}(G)$. Since $\digamma$ is soft r-weakly equivalent to Ψ, then we find $K \in RO(\Psi) - \{0_T\}$ such that $K \widetilde{\subseteq} R \widetilde{\subseteq} f_{qv}(G) \widetilde{\subseteq} f_{pu}(H)$. This shows that $f_{qv} : (L, \Upsilon, S) \longrightarrow (M, \Psi, T)$ is soft s-r-o. □

5. Conclusions

The concepts of "soft somewhat-r-continuity" and "soft somewhat-r-openness" for mappings over soft topological spaces were introduced in this research. We examined soft composition (Theorems 10, 11, and 24) and soft subspaces (Theorems 13, 14, 27, and 28) and presented characterizations (Theorems 12, 25, and 26). We provided several linkages between these two concepts and their associated concepts in soft topology (Theorems 3, 4, and 22). We also provided soft preservation results (Theorems 15 and 19). Finally, we looked at the correspondence between them and their topological analogs (Theorems 1, 2, 20, and 21 and Corollaries 1, 2, 4, and 5). Future research might look into the following topics: (1) defining soft semi-continuity; (2) defining soft pre-continuity; (3) finding a use for our new soft topological principles in a "decision-making problem".

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