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An Extended TODIM Method and Applications for Multi-Attribute Group Decision-Making Based on Bonferroni Mean Operators under Probabilistic Linguistic Term Sets

Juxiang Wang [1,2,*], Xiangyu Zhou [1], Si Li [1] and Jianwei Hu [3]

1 School of Mathematics and Physics, Anhui Jianzhu University, Hefei 230601, China; zxy@stu.ahjzu.edu.cn (X.Z.); 20213307013@stu.ahjzu.edu.cn (S.L.)
2 School of Management, Hefei University of Technology, Hefei 230009, China
3 School of Mathematics and Statistics, Huangshan University, Huangshan 245041, China; jianweihu@hsu.edu.cn
* Correspondence: wjxahjzdx@ahjzu.edu.cn

Abstract: Due to the complexity and uncertainty of decision-making, probabilistic linguistic term sets (PLTSs) are currently important tools for qualitative evaluation of decision-makers. The asymmetry of evaluation information can easily lead to the loss of subjective preference information for decision-makers, and the existing operation of decision-maker evaluation information fusion operators is difficult to solve this problem. To solve such problems, this paper proposes some new operational methods for PLTSs based on Dombi T-conorm and T-norm. Considering the interrelationships between the input independent variables of PLTSs, the probabilistic linguistic weighted Dombi Bonferroni mean Power average (PLWDBMPA) operators are extended and the properties of these aggregation operators are proposed. Secondly, the PLWDBMPA operator is used to fuse the evaluation information of decision-makers, avoiding the loss of decision information as much as possible. This paper uses social media platforms and web crawler technology to obtain online comments from users on decision-making to obtain the public's attitude towards decision events. TF-IDF and Word2Vec are used to calculate the weight of alternatives on each attribute. Under traditional group decision-making methods and integrating the wisdom of the public, a novel multi-attribute group decision-making method based on TODIM method is proposed. Finally, the case study of Turkey earthquake shelter selection proves this method is scientific and effective. Meanwhile, the superiority of this method was further verified through comparisons with the PL-TOPSIS, PLWA, SPOTIS and PROMETHEE method.

Keywords: Bonferroni mean; MAGDM; probabilistic linguistic term set; TODIM

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1. Introduction

Multi-criteria decision making (MCDM) is employed to make decisions based on a variety of criteria determining whether or not each alternative is acceptable, considering the complexity of the decision making (DM) process, preferences of decision makers (DMs), conflicting criteria and sources of uncertainty. Multi-attribute group decision making (MAGDM) is an integral part of MCDM [1–7]. Ziemba [8] proposed the dynamic multi-criteria decision making (DMCDM) methodology for dynamic energy security assessment. Ahmad [9] proposed a multi-choice best–worst method (MCBWM), which takes various alternatives into account when comparing preferences between the criteria in pairs. Zavadskas [10] developed an integrated fuzzy MCDM model to select orders.

The current study on information aggregation is based on a variety of operational rules, including Einstein operation, Hamacher operation, and Frank operation. These research findings are connected to lots of application fields. Dombi operation [11], which is essential generalizations of existing operations, is more flexible than existing operations in terms of collecting information and making decisions because it includes a parameter reflecting the decision-makers' subject preference. To address DM problems, Liu [12] recently proposed some Dombi Bonferroni mean operators under intuitionistic fuzzy set. Xu [13] extended the Dombi operations to hesitant fuzzy set and devised a variety of aggregation operators. Nevertheless, the current state of research reveals a gap in the field of probabilistic linguistic term set under Dombi operations. Therefore, it is crucial and imperative to conduct research on probabilistic linguistic Dombi operations and employ them to solve MAGDM problems.

Within the process of aggregating information, the Bonferroni mean (BM) method, which was first developed by Bonferroni [14], is utilized to explore the relationships among the input arguments. This approach has been effectively employed in the MCDM. These research endeavors conducted by BM have seen significant development during the past few decades. Xu and Yager [15], for example, extended the BM to the intuitionistic fuzzy set.

Liang [16] combined BM with Grey Relational Analysis method to solve MAGDM problems. Zhu [17] extended BM to the hesitant fuzzy sets. The PLTSs has to demonstrate the relationship between the input arguments since it also confronts the interrelationship phenomenon. We therefore bring BM into the probabilistic linguistic environment while taking into account the special characters of BM [17].

Methods of MADM are crucial in making purchase decisions, which have been extended to several practical applications in a variety of fields, such as TOPSIS [18,19], VIKOR [20,21], ELECTRE [22,23], PROMETHEE [24–27], SPOTIS [28,29], COMET [30], SIMUS [31], and TOPSI-DARIA [32]. Nevertheless, a common limitation of these methods is their failure to account for the restricted rationality of decision makers. To address this limitation, Gomes and Lima [33] devised a TODIM method that could take into account the constrained rationality on the basis of prospect theory. Liu [34] conducted a hybrid fuzzy TODIM-ERA method under intuitionistic fuzzy sets. Liu [35] combined Analytic Network Process with TODIM method under the Z-information environment. Fan [36] extended the Exponential TODIM method to probabilistic uncertain linguistic term sets.

Around the world, there have been several significant natural disasters in recent years that have resulted in significant losses of people and property. Of all the natural disasters induced by hazards, earthquakes are the most horrific and destructive, and they could conceivably result in immeasurable environmental harm, construction damage, human casualties and population relocation [37]. Houses and infrastructure are destroyed or damaged by earthquakes, necessitating the construction of temporary structures to provide shelter for disaster-affected individuals [38]. Trivedi [39], for instance, proposed a MCDM model to assess determinants of shelter site selection on the basis of decision-making trial and evaluation laboratory (DEMATEL). Xu [40] devised a two-stage consensus reaching model and employed it to the selection of earthquake shelter. Song [41] devised a method that incorporates the advantage of the qualitative flexible multiple criteria (QUALIFLEX) method with the uncertainty inherent in sustainable shelter-site selection.

A novel approach for MAGDM based on new Dombi operational rules under PLTSs and the probabilistic linguistic weighted Dombi Bonferroni mean power average (PLWDBMPA) operator is proposed in this paper. The following are the innovations and advantages of this paper: (1) the online comments of large groups of users and the evaluation information of experts about alternatives are fused to improve the results of MAGDM to be more scientific; (2) the introduction of BM operator and Dombi operation, distinguished from other methods, which not only take into account the decision-makers' subject preference, but also considering the relationship between the input arguments; (3) the extension of the BM operators under PLTSs to utilize the definition of Dombi and develop some operations, which can be applied to other fuzzy sets like intuitionistic fuzzy set, q-rung ortho pair fuzzy set and spherical fuzzy set.

The rest of this paper is organized as follows. Section 2 presents preliminaries. Section 3 proposes some aggregation operators. Section 4 provides a novel extension model and design an approach for the application of PLMAGDM utilizing the PLWBMPA operator. Section 5 provides an illustrative example to verify the validity of proposed method and compares the proposed method in this paper with the PL-TOPSIS method and the PLWA method. Section 6 summarizes the conclusions of this study.

2. Preliminaries

2.1. Keyword Extraction Technique

TF-IDF (term frequency-inverse document frequency) is widely used to determine the keyword weight and evaluate the importance of the extracted words in the field of data mining [42].

The formula for the weight of the extracted words is as Equation (1):

$$TF - IDF(T) = td_d(T) \cdot \log(N/df(T)) \quad (1)$$

where $td_d(T)$ indicates the frequency of the word in the text; N is the number of all texts; $df(T)$ indicates the number of texts containing the word T in the text collection.

2.2. Probabilistic Linguistic Term Sets

Definition 1 [43]**.** Let $S = \{S_{-\tau}, S_{-\tau+1}, \cdots, S_{\tau-1}, S_\tau\}$ be a linguistic term set, then a PLTS can be defined as Equation (2):

$$L(P) = \left\{ L^{(k)}(P^{(k)}) | L^{(k)} \in S, P^{(k)} \geq 0, k = 1, 2, \ldots, \#L(P), \sum_{k=1}^{\#L(P)} P^{(k)} \leq 1 \right\} \quad (2)$$

where $L^{(k)}(P^{(k)})$ represents the linguistic term $L^{(k)}$ with regard to the probability $P^{(k)}$, $L^{(k)}$ is the kth linguistic terms of $L(P)$ and $\#L(P)$ is the number of all different linguistic terms in $L(P)$.

According to the definition in reference [43], all the $L(P)$ below are ordered and standardized in the normalization of PLTSs.

Definition 2 [43]**.** Let $L(P) = \left\{ L^{(k)}(P^{(k)}) | k = 1, 2, \ldots, \#L(P) \right\}$ and $r^{(k)}$ be the subscript of the linguistic term $L^{(k)}$, then the score of $L(P)$ is as Equation (3):

$$E(L(P)) = S_{\overline{\alpha}} \quad (3)$$

where $\overline{\alpha} = \sum_{k=1}^{\#L(P)} r^{(k)} P^{(k)} \Big/ \sum_{k=1}^{\#L(P)} P^{(k)}$. For two PLTSs $L_1(P_1) = \left\{ L^{(k)}(P_1{}^{(k)}) | k = 1, 2, \ldots, \#L_1(P_1) \right\}$ and $L_2(P_2) = \left\{ L^{(l)}(P_2{}^{(l)}) | l = 1, 2, \ldots, \#L_2(P_2) \right\}$, if $E(L_1(P_1)) > E(L_2(P_2))$, then $L_1(P_1)$ is superior to $L_2(P_2)$, denoted by $L_1(P_1) > L_2(P_2)$; if $E(L_1(P_1)) < E(L_2(P_2))$, then $L_2(P_2)$ is superior to $L_1(P_1)$, denoted by $L_1(P_1) < L_2(P_2)$. However, if $E(L_1(P_1)) = E(L_2(P_2))$, the degree of deviation needs to be further defined.

Definition 3 [43]**.** Let $L(P) = \left\{ L^{(k)}(P^{(k)}) | k = 1, 2, \ldots, \#L(P) \right\}$ and $r^{(k)}$ be the subscript of the linguistic term $L^{(k)}$, where $\overline{\alpha} = \sum_{k=1}^{\#L(P)} r^{(k)} P^{(k)} \Big/ \sum_{k=1}^{\#L(P)} P^{(k)}$. Then the deviation of degree $L(P)$ is as Equation (4):

$$\sigma(L(P)) = \left(\sum_{k=1}^{\#L(P)} \left(P^{(k)} \left(r^{(k)} - \overline{\alpha} \right) \right)^2 \right) \Big/ \sum_{k=1}^{\#L(P)} P^{(k)} \quad (4)$$

where $E(L_1(P_1)) = E(L_2(P_2))$, if $\sigma(L_1(P_1)) > \sigma(L_2(P_2))$, then $L_1(P_1) < L_2(P_2)$; if $\sigma(L_1(P_1)) < \sigma(L_2(P_2))$, then $L_1(P_1) > L_2(P_2)$; however, if $\sigma(L_1(P_1)) = \sigma(L_2(P_2))$, then $L_1(P_1)$ is indifferent to $L_2(P_2)$, denoted by $L_1(P_1) \sim L_2(P_2)$.

Definition 4 [44]**.** Let $S = \{S_{-\tau}, S_{-\tau+1}, \cdots, S_{\tau-1}, S_\tau\}$ be a linguistic term set, $L(P)$ is a PLTS. The equivalent transformation function of $L(P)$ is defined as Equation (5):

$$g(L(P)) = \left\{ \left[\frac{\alpha^{(k)} + \tau}{2\tau} \right] \left(P^{(k)} \right) \right\} = L_\varpi(P), \tag{5}$$

where $g : [-\tau, \tau] \to [0,1]$ and $\varpi \in [0,1]$. Additionally, we can obtain the transformation function of $L_\varpi(P)$ as Equation (6):

$$g^{-1}(L_\varpi(P)) = \left\{ s_{\varpi\tau}\left(P^{(k)}\right) | \varpi \in [0,1] \right\} = L(P). \tag{6}$$

where $g^{-1} : [0,1] \to [-\tau, \tau]$.

Definition 5 [43]**.** There are two PLTSs $L_1(P_1) = \left\{ L^{(k)}({P_1}^{(k)}) | k = 1,2,\ldots,\#L_1(P_1) \right\}$ and $L_2(P_2) = \left\{ L^{(l)}({P_2}^{(l)}) | l = 1,2,\ldots,\#L_2(P_2) \right\}$, $\#L_1(P_1) = \#L_2(P_2)$. The deviation degree between $L_1(P_1)$ and $L_2(P_2)$ is defined as Equation (7):

$$d(L_1(P_1), L_2(P_2)) = \left[\frac{1}{\#L_1(P_1)} \sum_{j=1}^{\#L_1(P_1)} \left(P_1^{(j)} g\left(L_1^{(j)}\right) - P_2^{(j)} g\left(L_2^{(j)}\right) \right)^2 \right]^{\frac{1}{2}} \tag{7}$$

where $g\left(L_1^{(j)}\right) = \dfrac{r_1^{(j)} + \tau}{2\tau}$, $g\left(L_2^{(j)}\right) = \dfrac{r_2^{(j)} + \tau}{2\tau}$.$r_1^{(j)}$ and $r_2^{(j)}$ are the subscripts of linguistic terms $L^{(k)}$ and $L^{(l)}$ respectively.

Definition 6 [11]**.** Let λ be a parameter, $\lambda > 0$ and $x, y \in [0,1]$, the Dombi T-norm and T-conorm are defined as Equations (8) and (9):

$$T_{D,\lambda}(x,y) = \frac{1}{1 + \left(\left(\frac{1-x}{x} \right)^\lambda + \left(\frac{1-y}{y} \right)^\lambda \right)^{1/\lambda}} \tag{8}$$

$$T^*_{D,\lambda}(x,y) = 1 - \frac{1}{1 + \left(\left(\frac{x}{1-x} \right)^\lambda + \left(\frac{y}{1-y} \right)^\lambda \right)^{1/\lambda}} \tag{9}$$

Recently, Liu [12] developed the Dombi operation for intuitionistic fuzzy numbers (IFNs). Under Dombi T-norm and T-conorm, Xu [13] extended the Dombi operations to hesitant fuzzy set. We give some Dombi operations for PLTSs based on the findings of the previous research.

Definition 7. Let $S = \{S_{-\tau}, S_{-\tau+1}, \cdots, S_{\tau-1}, S_\tau\}$ be a linguistic term set, $L(P)$, $L_1(P_1)$ and $L_2(P_2)$ be three PLTSs, and δ be a positive real numbers. $\eta^{(k)} \in g(L)$, $\eta_1^{(i)} \in g(L_1)$, $\eta_2^{(j)} \in g(L_2)$ and $k = 1,2,\cdots,\#L(P)$, $i = 1,2,\cdots,\#L_1(P_1)$, $j = 1,2,\cdots,\#L_2(P_2)$ where is the equivalent transformation function. The Dombi operations are defined as follows:

$$L_1(P_1) \oplus L_2(P_2) = g^{-1}\left(\bigcup_{\substack{\eta_1^{(i)} \in g(L_1), \\ \eta_2^{(j)} \in g(L_2)}} \left\{ \left(1 - \frac{1}{1 + \left(\left(\frac{\eta_1^{(i)}}{1-\eta_1^{(i)}} \right)^\lambda + \left(\frac{\eta_2^{(j)}}{1-\eta_2^{(j)}} \right)^\lambda \right)^{1/\lambda}} \right) \left(P_1^{(i)} P_2^{(j)} \right) \right\} \right) \tag{10}$$

$$L_1(P_1)\otimes L_2(P_2)=g^{-1}\left(\bigcup_{\substack{\eta_1^{(i)}\in g(L_1),\\ \eta_2^{(j)}\in g(L_2)}}\left\{\left(\frac{1}{1+\left(\left(\frac{1-\eta_1^{(i)}}{\eta_1^{(i)}}\right)^{\lambda}+\left(\frac{1-\eta_2^{(j)}}{\eta_2^{(j)}}\right)^{\lambda}\right)^{1/\lambda}}\right)\left(P_1^{(i)}P_2^{(j)}\right)\right\}\right) \tag{11}$$

$$\delta L(P)=g^{-1}\left(\bigcup_{\eta^{(k)}\in g(L)}\left\{\left(1-\frac{1}{1+\left(\delta\left(\frac{\eta^{(k)}}{1-\eta^{(k)}}\right)^{\lambda}\right)^{1/\lambda}}\right)\left(P^{(k)}\right)\right\}\right) \tag{12}$$

$$(L(P))^{\delta}=g^{-1}\left(\bigcup_{\eta^{(k)}\in g(L)}\left\{\left(\frac{1}{1+\left(\delta\left(\frac{1-\eta^{(k)}}{\eta^{(k)}}\right)^{\lambda}\right)^{1/\lambda}}\right)\left(P^{(k)}\right)\right\}\right) \tag{13}$$

Theorem 1. Let $S=\{S_{-\tau},S_{-\tau+1},\cdots,S_{\tau-1},S_{\tau}\}$ be a linguistic term set, $L(P)$,$L_1(P_1)$, and $L_2(P_2)$ be three PLTSs, and δ,δ_1,δ_2 be three positive real numbers. Then

(1) $L_1(P_1)\oplus L_2(P_2)=L_2(P_2)\oplus L_1(P_1)$
(2) $L_1(P_1)\otimes L_2(P_2)=L_2(P_2)\otimes L_1(P_1)$
(3) $\delta(L_1(P_1)\oplus L_2(P_2))=\delta L_1(P_1)\oplus\delta L_2(P_2)$
(4) $(L_1(P_1)\otimes L_2(P_2))^{\delta}=(L_1(P_1))^{\delta}\otimes(L_2(P_2))^{\delta}$
(5) $\delta_1 L(P)\oplus\delta_2 L(P)=(\delta_1+\delta_2)L(P)$
(6) $(L(P))^{\delta_1}\otimes(L(P))^{\delta_2}=(L(P))^{\delta_1+\delta_2}$.

The proof of Theorem 1 is given in Appendix A.

2.3. Bonferroni Mean (BM) and Power Average (PA)

Definition 8 [14]**.** Let $p,q\geq 0$ and $X=(x_1,x_2,\cdots x_n)$ be a collection of non-negative numbers. Then the Bonferroni Mean is defined as Equation (14):

$$B^{p,q}(X)=\left(\frac{1}{n(n-1)}\sum_{i,j=1,i\neq j}^{n}x_i^p x_j^q\right)^{\frac{1}{p+q}} \tag{14}$$

Definition 9 [45]**.** Let $A=\{a_1,a_2,\cdots a_n\}$ be a collection of non-negative numbers. Then the power aggregation is defined as Equation (15):

$$PA(a_1,a_2,\cdots,a_n)=\textstyle\sum_{i=1}^{n}\frac{1+T(a_i)}{\sum_{i=1}^{n}(1+T(a_i))}a_i \tag{15}$$

where $T(a_i)=\sum\limits_{j=1,j\neq i}^{n}\sup(a_i,a_j)$ and $\sup(a_i,a_j)$ is denoted as the support for a_i from a_j, which satisfies the following properties:

(1) $\sup(a_i,a_j)\in[0,1]$;
(2) $\sup(a_i,a_j)=\sup(a_j,a_i)$;
(3) $\sup(a_i,a_j)\geq\sup(a_i,a_k), if\ d(a_i,a_j)\leq d(a_i,a_k)$;

3. Probabilistic Linguistic Weighted Dombi Bonferroni Mean Power Average Operators

We present the PLDBMPA operator and PLWDBMPA operator under the new PLTSs operational rules, and then investigate their properties in this section.

3.1. PLDBMPA Operators

We explore the fact that the input arguments of GBM given in [14] are PLTSs and then present the PLDBMPA operator as follow.

Definition 10. Let $L_i(P_i) = \left\{ L_i^{(k)}(P_i^{(k)}) | k = 1,2,\cdots,\#L_i(P_i) \right\}$ $i = 1,2,\cdots,n$ be n PLTSs. The PLDBMPA operator is defined as Equation (16):

$$PLDBMPA^{p,q}(L_1(P_1), L_2(P_2), \cdots, L_n(P_n)) = \left(\frac{1}{n(n-1)} \bigoplus_{\substack{i,j=1 \\ i \neq j}}^{n} \left(\left(\frac{n(1+T(L_i(P_i)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))} L_i(P_i) \right)^p \otimes \left(\frac{n(1+T(L_j(P_j)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))} L_j(P_j) \right)^q \right) \right)^{\frac{1}{p+q}} \tag{16}$$

where $p,q \geq 0$, $T(L_i(P_i)) = \sum_{j=1, j\neq i}^{n} Sup\left(L_i(P_i), L_j(P_j)\right)$, the support degree of $L_1(P_1)$ and $L_2(P_2)$ is $Sup\left(L_i(P_i), L_j(P_j)\right) = 1 - d\left(L_i(P_i), L_j(P_j)\right)$, which satisfies the following properties:

(1) $Sup\left(L_i(P_i), L_j(P_j)\right) \in [0,1]$;
(2) $Sup\left(L_i(P_i), L_j(P_j)\right) = Sup\left(L_j(P_j), L_i(P_i)\right)$;
(3) If $d\left(L_i(P_i), L_j(P_j)\right) < d(L_i(P_i), L_l(P_l))$, then $Sup\left(L_i(P_i), L_j(P_j)\right) > Sup(L_i(P_i), L_l(P_l))$.

Theorem 2. Let $L_i(P_i) = \left\{ L_i^{(k)}(P_i^{(k)}) | k = 1,2,\cdots,\#L_i(P_i), i = 1,2,\cdots,n \right\}$ be n PLTSs and $p,q \geq 0$, then the aggregated value obtained by PLDBMPA operator is still a PLTS, and

$$\begin{aligned} &PLDBMPA^{p,q}(L_1(P_1), L_2(P_2), \cdots, L_n(P_n)) \\ &= \left(\frac{1}{n(n-1)} \bigoplus_{\substack{i,j=1 \\ i \neq j}}^{n} \left(\left(\frac{n(1+T(L_i(P_i)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))} L_i(P_i) \right)^p \otimes \left(\frac{n(1+T(L_j(P_j)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))} L_j(P_j) \right)^q \right) \right)^{\frac{1}{p+q}} \\ &= \bigcup_{\substack{\eta_i^{(k)} \in g(L_i), \\ \eta_j^{(d)} \in g(L_j)}} g^{-1} \left(\left\{ \left(1 + \left(\frac{n(n-1)}{(p+q)} \left(\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \left(\frac{p}{E_i} K_i^{\lambda} + \frac{q}{E_j} D_j^{\lambda} \right)^{-1} \right)^{-1} \right)^{1/\lambda} \right)^{-1} \left(\prod_{\substack{i,j=1 \\ i \neq j}}^{n} P_i^{(k)} P_j^{(d)} \right) \right\} \right) \end{aligned} \tag{17}$$

where $E_i = \dfrac{n(1+T(L_i(P_i)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}$, $E_j = \dfrac{n(1+T(L_j(P_j)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}$, $K_i = \dfrac{1-\eta_i^{(k)}}{\eta_i^{(k)}}$, $D_j = \dfrac{1-\eta_j^{(d)}}{\eta_j^{(d)}}$.

The proof of Theorem 2 is given in Appendix B.

The corollaries of $PLDBMPA^{p,q}(L_1(P_1), L_2(P_2), \cdots, L_n(P_n))$ for Theorem 1 based on the results of Reference [16] are as follows.

Corollary 1. Commutativity, If $L_i'(P_i')$ is any permutation of $L_i(P_i)$ $(i = 1,2,\cdots,n)$, we then obtain the relationship:

$$PLDBMPA^{p,q}(L_1(P_1), L_2(P_2), \cdots, L_n(P_n)) = PLDBMPA^{p,q}\left(L_1'(P_1'), L_2'(P_2'), \cdots, L_n'(P_n')\right)$$

Corollary 2 (Monotonicity). Let $\varepsilon_{ij} = \left(1 + \left(\frac{n(n-1)}{(p+q)}\left(\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \left(\frac{p}{E_i}K_i^{\lambda} + \frac{q}{E_j}D_j^{\lambda}\right)^{-1}\right)^{-1}\right)^{1/\lambda}\right)^{-1}$,

when the values of n, p, q are constant, with regard to the increase of $\eta_i^{(k)}$ and $\eta_j^{(d)}$, ε_{ij} is increasing monotonously.

3.2. PLWDBMPA Operators

We propose the Probabilistic Linguistic Weighted Dombi Bonferroni Mean Power Average (PLWDBMPA) operator in this section, with the consideration of importance of aggregated multi-input arguments.

Definition 11. Let $L_i(P_i) = \left\{L_i^{(k)}(P_i^{(k)}) | k = 1, 2, \cdots, \#L_i(P_i), i = 1, 2, \cdots, n\right\}$ be n PLTSs and $p, q \geq 0$, $w = (w_1, w_2, \cdots, w_n)^T$ is the weight vector of $L_i(P_i)$, where w_i indicates the importance degree of $L_i(P_i)$, $w_i \in [0,1]$ and $\sum_i^n w_i = 1$. Then, the PLWDBMPA operator is defined as Equation (18):

$$PLWDBMPA^{p,q}(L_1(P_1), L_2(P_2), \cdots, L_n(P_n)) = \left(\frac{1}{n(n-1)} \bigoplus_{\substack{i,j=1 \\ i \neq j}}^{n} \left(\left(\frac{nw_i(1+T(L_i(P_i)))}{\sum_{t=1}^{n} w_t(1+T(L_t(P_t)))}L_i(P_i)\right)^p \otimes \left(\frac{nw_j(1+T(L_j(P_j)))}{\sum_{t=1}^{n} w_t(1+T(L_t(P_t)))}L_j(P_j)\right)^q\right)\right)^{\frac{1}{p+q}} \quad (18)$$

where $T(L_i(P_i)) = \sum_{j=1, j\neq i}^{n} w_j Sup\big(L_i(P_i), L_j(P_j)\big)$, the support degree of $L_1(P_1)$ and $L_2(P_2)$ is $Sup\big(L_i(P_i), L_j(P_j)\big) = 1 - \frac{d\left(L_i(P_i), L_j(P_j)\right)}{\sum_{\substack{g=1 \\ g \neq i}}^{n} d\left(L_i(P_i), L_g(P_g)\right)}$.

Theorem 3. Let $L_i(P_i) = \left\{L_i^{(k)}(P_i^{(k)}) | k = 1, 2, \cdots, \#L_i(P_i), i = 1, 2, \cdots, n\right\}$ be n PLTSs and $p, q \geq 0$, $w = (w_1, w_2, \cdots, w_n)^T$ is the weight vector of $L_i(P_i)$, where w_i indicates the importance degree of $L_i(P_i)$, $w_i \in [0,1]$ and $\sum_i^n w_i = 1$. Then, the aggregated value obtained by the PLWDBMPA operator is still a PLTS, and

$$\begin{aligned} &PLWDBMPA^{p,q}(L_1(P_1), L_2(P_2), \cdots, L_n(P_n)) \\ &= \left(\frac{1}{n(n-1)} \bigoplus_{\substack{i,j=1 \\ i \neq j}}^{n} \left(\left(\frac{nw_i(1+T(L_i(P_i)))}{\sum_{t=1}^{n} w_t(1+T(L_t(P_t)))}L_i(P_i)\right)^p \otimes \left(\frac{nw_j(1+T(L_j(P_j)))}{\sum_{t=1}^{n} w_t(1+T(L_t(P_t)))}L_j(P_j)\right)^q\right)\right)^{\frac{1}{p+q}} \\ &= \bigcup_{\substack{\eta_i^{(k)} \in g(L_i), \\ \eta_j^{(d)} \in g(L_j)}} g^{-1}\left(\left\{\left(1 + \left(\frac{n(n-1)}{(p+q)}\left(\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \left(\frac{p}{v_i}K_i^{\lambda} + \frac{q}{v_j}D_j^{\lambda}\right)^{-1}\right)^{-1}\right)^{1/\lambda}\right)^{-1}\left(\prod_{\substack{i,j=1 \\ i \neq j}}^{n} P_i^{(k)}P_j^{(d)}\right)\right\}\right) \end{aligned} \quad (19)$$

where $v_i = \frac{nw_i(1+T(L_i(P_i)))}{\sum_{t=1}^{n} w_t(1+T(L_t(P_t)))}$, $v_j = \frac{nw_j(1+T(L_j(P_j)))}{\sum_{t=1}^{n} w_t(1+T(L_t(P_t)))}$, $K_i = \frac{1-\eta_i^{(k)}}{\eta_i^{(k)}}$, $D_j = \frac{1-\eta_j^{(d)}}{\eta_j^{(d)}}$.

4. Solving Multi-Attribute Group Decision-Making Problem with the PLWDBMPA Operator

4.1. The Problem Description of MAGDM

The PLMAGDM problem contains several decision matrices that provides assessments of all alternatives employing PLTSs for each attribute. Let $A = \{A_i | i = 1, 2, \cdots, m\}$ be a discrete set of alternatives and $C = \{C_j | j = 1, 2, \cdots, n\}$ be the set of attributes. $S = \{S_{-\tau}, S_{-\tau+1}, \cdots, S_{\tau-1}, S_\tau\}$ be the linguistic term set. Suppose that $E = \{e_z | z = 1, 2, \cdots, y\}$ is the set of experts with the corresponding weight information $W = (w_1, w_2, \cdots, w_y)^T$ where $0 \le w_z \le 1, \sum_{z=1}^{y} w_z = 1$. Let $L_{ij}^z\left(P_{ij}^z\right)$ be the PLTS converted from the evaluation of expert e_z for alternative A_i with the attribute C_j. Then, let $D^z = \left(L_{ij}^z\left(P_{ij}^z\right)\right)_{m\times n}$ be the probabilistic linguistic decision matrix provided by the expert $e_z (z = 1, 2, \cdots, y)$. Hence, the probabilistic linguistic decision matrix D^z can be written as:

$$D^z = \begin{bmatrix} L_{11}^z(P_{11}^z) & L_{12}^z(P_{12}^z) & \cdots & L_{1n}^z(P_{1n}^z) \\ L_{21}^z(P_{21}^z) & L_{22}^z(P_{22}^z) & \cdots & L_{2n}^z(P_{2n}^z) \\ \vdots & \vdots & \cdots & \vdots \\ L_{m1}^z(P_{m1}^z) & L_{m2}^z(P_{m2}^z) & \cdots & L_{mn}^z(P_{mn}^z) \end{bmatrix}$$

where the element $L_{ij}^z(P_{ij}^z)$ denotes the evaluation value of the alternative A_i according to the attribute C_j provided by the expert $e_z (i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$..

The flowchart of the decision procedure is as Figure 1.

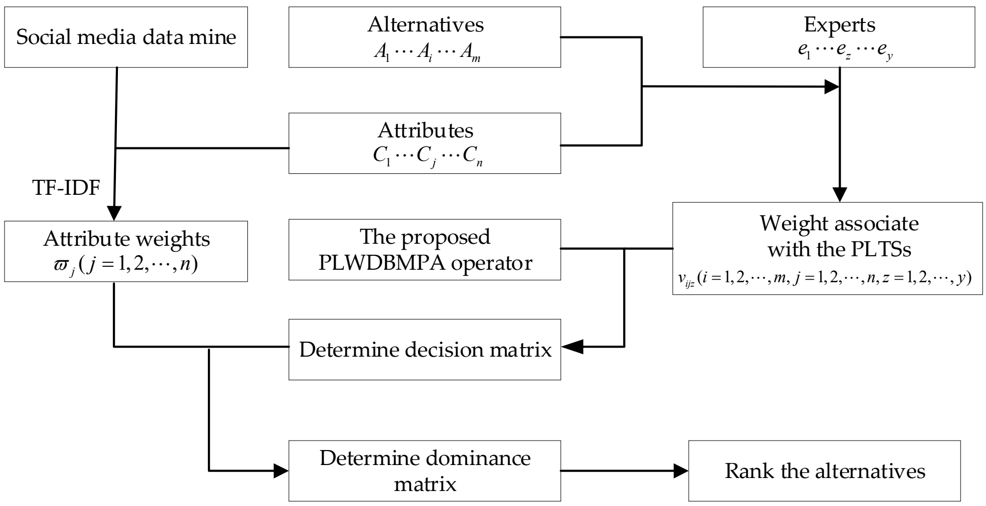

Figure 1. Flowchart of the decision procedure.

4.2. The Decision Procedure

We propose a novel approach of application for MAGDM with PLWDBMPA and TODIM method based on the results mentioned above. We apply the PLWDBMPA to integrate the information of MAGDM while the TODIM method also can assist us in making decisions. This novel approach is proposed as follows:

Step 1. When it comes to the DM problem, we define the discrete set of alternatives $A = \{A_i | i = 1, 2, \cdots, m\}$ and the set of attributes $C = \{C_j | j = 1, 2, \cdots, n\}$. Suppose that $E = \{e_z | z = 1, 2, \cdots, y\}$ is the set of experts, their corresponding weight is $W = (w_1, w_2, \cdots, w_y)^T$, the linguistic term set is $S = \{S_{-\tau}, S_{-\tau+1}, \cdots, S_{\tau-1}, S_\tau\}$ and the probabilistic linguistic decision matrix is constructed as $D^z = \left(L_{ij}^z\left(P_{ij}^z\right)\right)_{m\times n}$ $(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; z = 1, 2, \cdots, y)$ provided by the expert e_z.

Step 2. Determine the weight associated with the PLTSs.

(1) The deviation degree between $L_{ij}^{z}(P_{ij}^{z}) = \left\{ L_{ij}^{z(k)}(P_{ijz}{}^{(k)}) | k = 1, 2, \ldots, \#L_{ij}^{z}(P_{ij}^{z}) \right\}$ and $L_{ij}^{u}(P_{ij}^{u}) = \left\{ L_{ij}^{u(l)}(P_{iju}{}^{(l)}) | l = 1, 2, \ldots, \#L_{ij}^{u}(P_{ij}^{u}) \right\}$, which based on the matrix $D^{z}(z = 1, 2, \cdots, y)$ and Definition 5, $\#L_{ij}^{z}(P_{ij}^{z}) = \#L_{ij}^{u}(P_{ij}^{u})$ $(i = 1, 2, \cdots, m;$ $j = 1, 2, \cdots, n; z, u = 1, 2, \cdots, y)$ is calculated as follows:

$$d(L_{ij}^{z}(P_{ij}^{z}), L_{ij}^{u}(P_{ij}^{u})) = \left[\frac{1}{\#L_{ij}^{z}(P_{ij}^{z})} \sum_{j=1}^{\#L_{ij}^{z}(P_{ij}^{z})} \left(P_{ijz}^{(j)} g\left(L_{ij}^{z(j)}\right) - P_{iju}^{(j)} g\left(L_{ij}^{u(j)}\right) \right)^{2} \right]^{\frac{1}{2}} \tag{20}$$

where $g\left(L_{ij}^{z(j)}\right) = \dfrac{r_{ij}^{z(j)} + \tau}{2\tau}$, $g\left(L_{ij}^{u(j)}\right) = \dfrac{r_{ij}^{u(j)} + \tau}{2\tau}$. $r_{ij}^{z(j)}$ and $r_{ij}^{u(j)}$ are the subscripts of linguistic terms L_{ij}^{z} and L_{ij}^{u} respectively.

(2) Calculate the support of the alternative A_i on attribute C_j by the result of Definition 11 as follows:

$$\sup(L_{ij}^{z}(P_{ij}^{z}), L_{ij}^{u}(P_{ij}^{u})) = 1 - \frac{d(L_{ij}^{z}(P_{ij}^{z}), L_{ij}^{u}(P_{ij}^{u}))}{\sum_{g=1, g \neq z}^{y} d(L_{ij}^{z}(P_{ij}^{z}), L_{ij}^{g}(P_{ij}^{g}))} \tag{21}$$

(3) Calculate the support $T(L_{ij}^{z}(P_{ij}^{z}))$ of $L_{ij}^{z}(P_{ij}^{z})$ by all of other $L_{ij}^{u}(P_{ij}^{u})$ $(z, u = 1, 2, \cdots, y; u \neq z)$ based on the result of Definition 11 as follows:

$$T(L_{ij}^{z}(P_{ij}^{z})) = \sum_{u=1, u \neq z}^{y} w_u \sup(L_{ij}^{z}(P_{ij}^{z}), L_{ij}^{u}(P_{ij}^{u})) \tag{22}$$

(4) Then, the weight v_{ijz} associated with the PLTS $L_{ij}^{z}(P_{ij})$ is as follows:

$$v_{ijz} = \frac{w_z \left(1 + T(L_{ij}^{z}(P_{ij}^{z}))\right)}{\sum_{z=1}^{y} \left(1 + T(L_{ij}^{z}(P_{ij}^{z}))\right)} \tag{23}$$

Step 3. We employ the PLWDBMPA operator to combine the individual evaluations into the group opinion with the given the values of p and q. Based on Equation (18), the aggregate evaluation value of the alternative A_i with regard to the attribute C_j is as follows:

$$\begin{aligned} L_{ij}(P_{ij}) &= PLWDBMPA^{p,q}\left(L_{ij}^{1}(P_{ij}^{1}), L_{ij}^{2}(P_{ij}^{2}), \cdots, L_{ij}^{y}(P_{ij}^{y})\right) \\ &= \left(\frac{1}{y(y-1)} \bigoplus_{\substack{z, u = 1 \\ z \neq u}}^{y} \left(\left(\frac{n w_z (1 + T(L_{ij}^{z}(P_{ij}^{z})))}{\sum_{g=1}^{y} w_g (1 + T(L_{ij}^{g}(P_{ij}^{g})))} L_{ij}^{z}(P_{ij}^{z}) \right)^{p} \otimes \left(\frac{n w_u (1 + T(L_{ij}^{u}(P_{ij}^{u})))}{\sum_{g=1}^{y} w_g (1 + T(L_{ij}^{g}(P_{ij}^{g})))} L_{ij}^{u}(P_{ij}^{u}) \right)^{q} \right) \right)^{\frac{1}{p+q}} \\ &= \bigcup_{\substack{\eta_{ijz}^{(k)} \in g(L_{ij}^{z}), \\ \eta_{iju}^{(d)} \in g(L_{ij}^{u})}} g^{-1} \left(\left\{ \left(1 + \left(\frac{y(y-1)}{(p+q)} \left(\sum_{z=1}^{y} \sum_{\substack{u = 1 \\ u \neq z}}^{u} \left(\frac{p}{v_{ijz}} K_z^{\lambda} + \frac{q}{v_{iju}} D_u^{\lambda} \right)^{-1} \right)^{-1} \right)^{1/\lambda} \right)^{-1} \left(\prod_{\substack{z, u = 1 \\ z \neq u}}^{y} P_{iiz}^{(k)} P_{iju}^{(d)} \right) \right\} \right) \end{aligned} \tag{24}$$

where $v_{ijz} = \dfrac{w_z \left(1 + T(L_{ij}^{z}(P_{ij}^{z}))\right)}{\sum_{z=1}^{y} \left(1 + T(L_{ij}^{z}(P_{ij}^{z}))\right)}$, $v_{iju} = \dfrac{w_u \left(1 + T(L_{ij}^{u}(P_{ij}^{u}))\right)}{\sum_{u=1}^{y} \left(1 + T(L_{ij}^{u}(P_{ij}^{u}))\right)}$, $K_z = \dfrac{1 - \eta_{ijz}^{(k)}}{\eta_{ijz}^{(k)}}$, $D_u = \dfrac{1 - \eta_{iju}^{(d)}}{\eta_{iju}^{(d)}}$.

Thus, the integrated group decision matrix $D = \left(L_{ij}\left(P_{ij}\right)\right)_{m\times n}(i = 1,2,\cdots,m;$ $j = 1,2,\cdots,n)$ is as follows:

$$D = \begin{bmatrix} L_{11}(P_{11}) & L_{12}(P_{12}) & \cdots & L_{1n}(P_{1n}) \\ L_{21}(P_{21}) & L_{22}(P_{22}) & \cdots & L_{2n}(P_{2n}) \\ \vdots & \vdots & \cdots & \vdots \\ L_{m1}(P_{m1}) & L_{m2}(P_{m2}) & \cdots & L_{mn}(P_{mn}) \end{bmatrix} \tag{25}$$

Step 4. Integrate network behavior data for event keywords mining, get event evaluation attributes, then determine the weights of event attributes using the TF-IDF technique.

Based on the text data of users' comments on alternatives on social media platforms, crawl the data of users' comments by techniques such as crawlers in Python. The text data is pre-processed by Jieba word splitting and deactivation of thesaurus in Python. The keywords were extracted from the pre-processed text data by TF-IDF technique, and the similarity between the keywords was analyzed by Word2vec technique, then the initial clustering was done. The attributes $C = \left\{C_j | j = 1,2,\cdots,n\right\}$ and the number $N = \left\{n_j | j = 1,2,\cdots,n\right\}$ of keywords contained in each attribute were determined after discussion with experts.

By processing and analyzing the online comments to obtain attribute $C_j(j = 1,2,\cdots,n)$ and number $n_j(j = 1,2,\cdots,n)$ of keywords contained in them, then obtain the attributes weights ϖ_j as follows:

$$\varpi_j = \frac{n_j}{\sum_{j=1}^{n} n_j}, j = 1,2,\cdots,n \tag{26}$$

Step 5. Determine the relative weights of all attributes.

Based on the maximum deviation method, we get the weight vector and then the subsequent formula is used to determine the relevant weight of each attribute as follows:

$$\varpi_j' = \frac{\varpi_j}{\varpi_{\max}}, j = 1,2,\cdots,n. \tag{27}$$

where $\varpi_{\max} = \max\left\{\varpi_j | j = 1,2,\cdots,n\right\}$.

Step 6. Calculate the dominance of alternative A_i over A_e as follows:

$$\vartheta(A_i, A_e) = \sum_{j=1}^{n} \phi_j(A_i, A_e) \tag{28}$$

and

$$\phi_j(A_i, A_e) = \begin{cases} \sqrt{\varpi_j' d\left(L_{ij}(P_{ij}), L_{kj}(P_{kj})\right) \Big/ \sum_{j=1}^{n} \varpi_j'}, if\ L_{ij}(P_{ij}) \succ L_{kj}(P_{kj}) \\ 0, \qquad if\ L_{ij}(P_{ij}) \sim L_{kj}(P_{kj}) \\ -\frac{1}{\theta}\sqrt{\left(\sum_{j=1}^{n} \varpi_j'\right) d\left(L_{ij}(P_{ij}), L_{kj}(P_{kj})\right) \Big/ \varpi_j'}, if\ L_{ij}(P_{ij}) \prec L_{kj}(P_{kj}) \end{cases} \tag{29}$$

Step 7. Calculate the overall prospect value of alternative A_i as follows:

$$\delta(A_i) = \frac{\sum\limits_{e=1}^{m} \vartheta(A_i, A_e) - \min\limits_{i}\left\{\sum\limits_{e=1}^{m} \vartheta(A_i, A_e)\right\}}{\max\limits_{i}\left\{\sum\limits_{e=1}^{m} \vartheta(A_i, A_e)\right\} - \min\limits_{i}\left\{\sum\limits_{e=1}^{m} \vartheta(A_i, A_e)\right\}}, i = 1,2,\cdots,m. \tag{30}$$

Step 8. Determine the desirable alternative by $\delta(A_i)$. The bigger $\delta(A_i)$ is, the better A_i is.

5. An Illustrative Example

A 7.8-magnitude earthquake shook Turkey on 6 February 2023. The epicenter was monitored at 37.15 degrees north latitude and 36.95 degrees east longitude, up to a depth of 20 km, according to the China Earthquake Networks Center (https://www.cea.gov.cn/cea/xwzx/369242/5711147/index.html) (CENC). There are 33 large- and medium-sized cities within a 300-km radius of the epicenter. The nearest location is Gaziantep, which is located approximately 40 km from the epicenter.

On 6 March 2023, 45,968 people were killed in Turkey, more than 200,000 buildings had collapsed or been seriously damaged in the earthquake, and millions of people became homeless (https://baijiahao.baidu.com/s?id=1759645587075783185&wfr=baike).

The construction of emergency shelters was vitally important to alleviate the suffering of the disaster-affected population. Three experts (denoted as e_1, e_2, and e_3) were invited to consult and access the shelter sites alternative. All the experts were professionals with more than 10 years of research and work experience. These experts included emergency officials from the local governments, disaster relief experts from international organizations, and field experts in emergency management. A group discussion with the experts was held to evaluate all the information to make the assessment process more scientific and legitimate.

Let the weight vector of experts be $W = (w_1, w_2, w_3)^T = (0.3, 0.3, 0.4)^T$. Four sites were selected after the preliminary examination, which are here denoted as A_1, A_2, A_3, and A_4. Utilizing Python to crawl the data of microblog, keywords such as Turkey Earthquake information, aerial photography of Turkey Earthquake scene, and Turkey Earthquake latest progress can be used. Relevant data are available from the "Wei Bo" website (https://weibo.com/ (accessed on 15 June 2023)). Around 500 valid data pieces were kept and utilized to generate a word cloud map as Figure 2 with the process of data cleaning and filtering. The attributes for shelter sites are denoted as C_1, C_2, C_3, where C_1 is "scale and location", which indicates whether the address is appropriate for people to live in. C_2 is "accessibility", which describes the security of the sites. C_3 is "resource availability", which illustrates the shelter's support facilities.

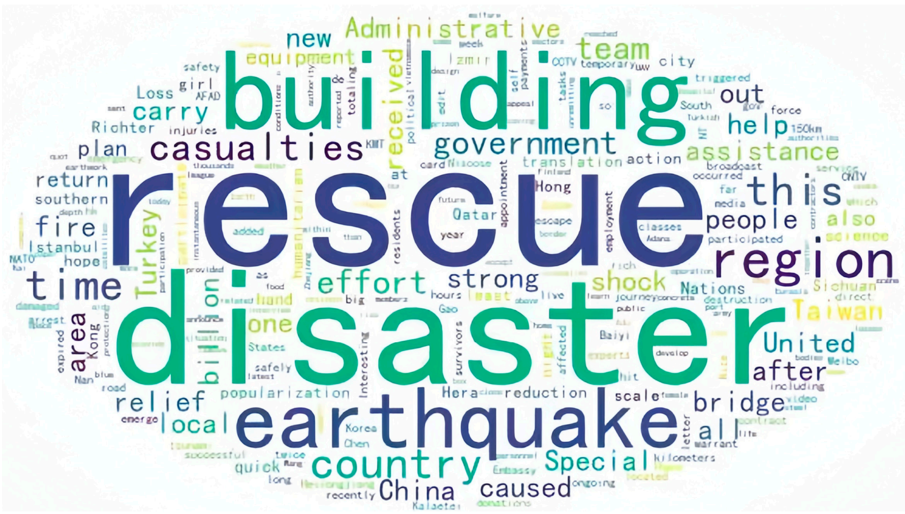

Figure 2. "2.6" Turkey Earthquake word cloud map.

Let $S = \{s_\tau | \tau = -3, -2, -1, 0, 1, 2, 3\}$ be the linguistic term set. The experts' judgements of the alternatives according to each attribute are expected to be portrayed by PLTSs. Results are presented in Tables 1–3.

Table 1. The probabilistic linguistic decision matrix D^1 provided by the expert e_1.

e_1	C_1	C_2	C_3
A_1	$\{s_1(1)\}$	$\{s_2(1)\}$	$\{s_0(1)\}$
A_2	$\{s_2(1)\}$	$\{s_1(0.5), s_2(0.5)\}$	$\{s_0(0.2), s_1(0.4), s_2(0.4)\}$
A_3	$\{s_1(1)\}$	$\{s_0(1)\}$	$\{s_1(0.6), s_2(0.4)\}$
A_4	$\{s_1(0.5), s_2(0.5)\}$	$\{s_0(0.4), s_1(0.6)\}$	$\{s_1(1)\}$

Table 2. The probabilistic linguistic decision matrix D^2 provided by the expert e_2.

e_2	C_1	C_2	C_3
A_1	$\{s_0(0.4), s_1(0.6)\}$	$\{s_1(0.5), s_2(0.5)\}$	$\{s_{-2}(0.2), s_{-1}(0.4), s_0(0.4)\}$
A_2	$\{s_0(0.3), s_1(0.3), s_2(0.4)\}$	$\{s_0(1)\}$	$\{s_2(1)\}$
A_3	$\{s_0(1)\}$	$\{s_1(1)\}$	$\{s_2(1)\}$
A_4	$\{s_1(1)\}$	$\{s_1(1)\}$	$\{s_0(0.1), s_1(0.9)\}$

Table 3. The probabilistic linguistic decision matrix D^3 provided by the expert e_3.

e_3	C_1	C_2	C_3
A_1	$\{s_1(1)\}$	$\{s_2(1)\}$	$\{s_0(1)\}$
A_2	$\{s_2(1)\}$	$\{s_0(1)\}$	$\{s_2(1)\}$
A_3	$\{s_1(0.8), s_2(0.2)\}$	$\{s_1(0.7), s_2(0.3)\}$	$\{s_2(1)\}$
A_4	$\{s_2(1)\}$	$\{s_1(1)\}$	$\{s_1(1)\}$

5.1. Decision Analysis with the Proposed Approach

Step 1. Integrate the information presented in the decision matrices D^1-D^3, utilizing the proposed approach of Section 4. Following the results of [16,44], let $p = 1$ and $q = 1$.

Step 2. To demonstrate the calculation procedure, we take the alternative A_i $(i = 1, 2, 3, 4)$ on the attribute C_1 as an example by Equations (20)–(23).

(1) By Equation (20), calculate the deviation degree between PLTSs based on matrices D^1, D^2, and D^3 on attribute C_1, C_2 and C_3, as presented in Tables 4–6.

Table 4. The deviation degree between PLTSs based on matrices D^1, D^2, and D^3 according to attribute C_1.

i	$d(L^1_{i1}(P^1_{i1}), L^2_{i1}(P^2_{i1}))$	$d(L^1_{i1}(P^1_{i1}), L^3_{i1}(P^3_{i1}))$	$d(L^2_{i1}(P^2_{i1}), L^3_{i1}(P^3_{i1}))$
1	0.1925	0	0.1925
2	0.3227	0	0.3227
3	0.0962	0.1232	0.0981
4	0.2406	0.3081	0.0962

Table 5. The deviation degree between PLTSs based on matrices D^1, D^2, and D^3 according to attribute C_2.

i	$d(L^1_{i2}(P^1_{i2}), L^2_{i2}(P^2_{i2}))$	$d(L^1_{i2}(P^1_{i2}), L^3_{i2}(P^3_{i2}))$	$d(L^2_{i2}(P^2_{i2}), L^3_{i2}(P^3_{i2}))$
1	0.3081	0	0.3081
2	0.1984	0.1984	0
3	0.0962	0.1456	0.1848
4	0.1925	0.1925	0

Table 6. The deviation degree between PLTSs based on matrices D^1, D^2, and D^3 according to attribute C_3.

i	$d(L_{i3}^1(P_{i3}^1),L_{i3}^2(P_{i3}^2))$	$d(L_{i3}^1(P_{i3}^1),L_{i3}^3(P_{i3}^3))$	$d(L_{i3}^2(P_{i3}^2),L_{i3}^3(P_{i3}^3))$
1	0.1905	0	0.1905
2	0.3322	0.3322	0
3	0.3156	0.3156	0
4	0.0481	0	0.0481

(2) By Equation (21), we have the support degree between PLTSs based on matrices D^1, D^2, and D^3 on attribute C_1, C_2 and C_3, the results are presented in Tables 7–9.

Table 7. The support degree between PLTSs based on matrices D^1, D^2, and D^3 according to attribute C_1.

i	$\sup(L_{i1}^1(P_{i1}^1),L_{i1}^2(P_{i1}^2))$	$\sup(L_{i1}^1(P_{i1}^1),L_{i1}^3(P_{i1}^3))$	$\sup(L_{i1}^2(P_{i1}^2),L_{i1}^3(P_{i1}^3))$
1	0.5	1	0.5
2	0.5	1	0.5
3	0.6970	0.6120	0.6910
4	0.6270	0.5223	0.8508

Table 8. The support degree between PLTSs based on matrices D^1, D^2, and D^3 according to attribute C_2.

i	$\sup(L_{i2}^1(P_{i2}^1),L_{i2}^2(P_{i2}^2))$	$\sup(L_{i2}^1(P_{i2}^1),L_{i2}^3(P_{i2}^3))$	$\sup(L_{i2}^2(P_{i2}^2),L_{i2}^3(P_{i2}^3))$
1	0.5	1	0.5
2	0.5	0.5	1
3	0.7745	0.6587	0.5668
4	0.5	0.5	1

Table 9. The support degree between PLTSs based on matrices D^1, D^2, and D^3 according to attribute C_3.

i	$\sup(L_{i3}^1(P_{i3}^1),L_{i3}^2(P_{i3}^2))$	$\sup(L_{i3}^1(P_{i3}^1),L_{i3}^3(P_{i3}^3))$	$\sup(L_{i3}^2(P_{i3}^2),L_{i3}^3(P_{i3}^3))$
1	0.5	1	0.5
2	0.5	0.5	1
3	0.5	0.5	1
4	0.5	1	0.5

(3) By Equation (22), we have the support $T(L_{ij}^z(P_{ij}^z))$ of $L_{ij}^z(P_{ij}^z)$ by all of other $L_{ij}^u(P_{ij}^u)$ $(j = 1, 2, 3)$ on the attributes C_1, C_2 and C_3, the results are as follows:

$$T\big(L_{i1}^z(P_{i1}^z)\big) = \begin{pmatrix} 0.55 & 0.35 & 0.45 \\ 0.55 & 0.35 & 0.45 \\ 0.4539 & 0.4855 & 0.3909 \\ 0.3970 & 0.5284 & 0.4119 \end{pmatrix}$$

$$,T\big(L_{i2}^z(P_{i2}^z)\big) = \begin{pmatrix} 0.55 & 0.35 & 0.45 \\ 0.35 & 0.55 & 0.45 \\ 0.4958 & 0.4591 & 0.3677 \\ 0.35 & 0.55 & 0.45 \end{pmatrix}$$

$$T\big(L_{i3}^z(P_{i3}^z)\big) = \begin{pmatrix} 0.55 & 0.35 & 0.45 \\ 0.35 & 0.55 & 0.45 \\ 0.35 & 0.55 & 0.45 \\ 0.55 & 0.35 & 0.45 \end{pmatrix}.$$

(4) By Equation (23), we have the weight v_{ijz} associated with the PLTS $L_{ij}^{z}(P_{ij})$ $(j = 1,2,3)$ on C_1, C_2 and C_3, the results are as follows:

$$v_{i1z} = \begin{pmatrix} 0.9621 & 0.8379 & 1.2 \\ 0.9621 & 0.8379 & 1.2 \\ 0.9099 & 0.9296 & 1.1605 \\ 0.8717 & 0.9537 & 1.1746 \end{pmatrix},$$

$$v_{i2z} = \begin{pmatrix} 0.9621 & 0.8379 & 1.2 \\ 0.8379 & 0.9621 & 1.2 \\ 0.9391 & 0.9160 & 1.1449 \\ 0.8379 & 0.9621 & 1.2 \end{pmatrix},$$

$$v_{i3z} = \begin{pmatrix} 0.9621 & 0.8379 & 1.2 \\ 0.8379 & 0.9621 & 1.2 \\ 0.8379 & 0.9621 & 1.2 \\ 0.9621 & 0.8379 & 1.2 \end{pmatrix}.$$

Step 3. By the PLWDBMPA operator, the aggregating evaluation value of the alternative A_i on $C_j (i = 1,2,3,4; j = 1,2,3)$ can be derived via Equation (24). To demonstrate the procedure of calculation, taking the alternative A_3 on C_1 as an example. According to the results of Tables 1–3 and Equation (24), the results are as follows and presented in Table 10:

$$\begin{aligned} L_{31}(P_{31}) &= PLWDBMPA^{1,1}(L_{31}^{1}(P_{31}^{1}), L_{31}^{2}(P_{31}^{2}), L_{31}^{3}(P_{31}^{3})) \\ &= \bigcup_{\substack{\eta_{31z}^{(k)} \in g(L_{31}^{z}), \\ \eta_{31u}^{(d)} \in g(L_{31}^{u})}} g^{-1}\left(\left\{\left(1+\left(\frac{y(y-1)}{(p+q)}\left(\sum_{z=1}^{y}\sum_{\substack{u=1 \\ u \neq z}}^{u}\left(\frac{p}{v_{31z}}K_z^{\lambda}+\frac{q}{v_{31u}}D_u^{\lambda}\right)^{-1}\right)^{-1}\right)^{1/\lambda}\right)^{-1}\left(\prod_{\substack{z,u=1 \\ z \neq u}}^{y} P_{31z}^{(k)}P_{31u}^{(d)}\right)\right\}\right) \\ &= \{s_{0.6373}(0.4096), s_{0.6623}(0.0256), s_{0.8412}(0.0256), s_{0.8593}(0.0016)\}, \end{aligned}$$

where $v_{311} = 0.9099,\ v_{312} = 0.9296,\ v_{313} = 1.1605$. $K_z = \dfrac{1-\eta_{31z}^{(k)}}{\eta_{31z}^{(k)}}$, $D_u = \dfrac{1-\eta_{31u}^{(d)}}{\eta_{31u}^{(d)}}$.

Table 10. The integrated probabilistic linguistic decision matrix D.

	C_1	C_2	C_3
A_1	$\{s_{0.6338}(0.0256), s_{0.8252}(0.0576), s_{0.8482}(0.0576), s_{0.9889}(0.1296)\}$	$\{s_{1.6921}(0.0625), s_{1.8650}(0.0625), s_{1.8824}(0.0625), s_{1.9930}(0.0625)\}$	$\{s_{-0.6767}(0.0016), s_{-0.5259}(0.0064), s_{-0.5199}(0.0064), s_{-0.3940}(0.0256), s_{-0.2924}(0.0064), s_{-0.2599}(0.0064), s_{-0.1924}(0.0256), s_{-0.1676}(0.0256), s_{-0.0125}(0.0256)\}$
A_2	$\{s_{1.5575}(0.0081), s_{1.6292}(0.0081), s_{1.6313}(0.0081), s_{1.6921}(0.0081), s_{1.8263}(0.0144), s_{1.8468}(0.0144), s_{1.8650}(0.0144), s_{1.8824}(0.0144), s_{1.9930}(0.0256)\}$	$\{s_{0.2745}(0.0625), s_{0.3313}(0.0625), s_{0.3560}(0.0625), s_{0.4068}(0.0625)\}$	$\{s_{1.5575}(0.0016), s_{1.6292}(0.0064), s_{1.6313}(0.0064), s_{1.6921}(0.0256), s_{1.8263}(0.0064), s_{1.8468}(0.0064), s_{1.8650}(0.0256), s_{1.8824}(0.0256), s_{1.9930}(0.0256)\}$
A_3	$\{s_{0.6373}(0.4096), s_{0.6623}(0.0256), s_{0.8412}(0.0256), s_{0.8593}(0.0016)\}$	$\{s_{0.6388}(0.2401), s_{0.6644}(0.0441), s_{0.8450}(0.0441), s_{0.8635}(0.0081)\}$	$\{s_{1.6921}(0.1296), s_{1.8650}(0.0576), s_{1.8824}(0.0576), s_{1.9930}(0.0256)\}$
A_4	$\{s_{1.2113}(0.0625), s_{1.3090}(0.0625), s_{1.6474}(0.0625), s_{1.6875}(0.0625)\}$	$\{s_{0.6338}(0.0256), s_{0.8252}(0.0576), s_{0.8482}(0.0576), s_{0.9889}(0.1296)\}$	$\{s_{0.6338}(0.0001), s_{0.8252}(0.0081), s_{0.8482}(0.0081), s_{0.9889}(0.6561)\}$

Step 4. Calculate the weight vector of attributes $C_j (j = 1,2,3)$ by Equation (26). The results are presented in Table 11.

Table 11. Attributes and their corresponding keywords.

Attribute	Keywords Included	Number of Terms	ϖ_j
Scale and location	Contractor, building, campsite, city, etc.	8	0.2581
Accessibility	Security, aftershock, safeguard, hospital, etc.	13	0.4193
Resource availability	Supplies, assistance, container, contribution, etc.	10	0.3226

Then, $\varpi_1 = 0.2581, \varpi_2 = 0.4193, \varpi_3 = 0.3226$.

Step 5. Determine the relative weights of all attributes by Equation (27). The results are as follows:

$$\varpi_1' = 0.6155,\ \varpi_2' = 1,\ \varpi_3' = 0.7694.$$

Step 6. Obtain the dominance of each alternative A_i over each alternative A_e according to attributes $C_j(j = 1, 2, 3)$ by Equations (28)–(30). Then, all the dominance degrees are as follows: $\theta = 2.5$.

$$\phi_1(A_i, A_e) = \begin{bmatrix} 0 & -0.2083 & 0.1516 & -0.1957 \\ 0.1344 & 0 & 0.1868 & 0.1329 \\ -0.2349 & -0.2896 & 0 & -0.3013 \\ 0.1263 & -0.2059 & 0.1944 & 0 \end{bmatrix},$$

$$\phi_2(A_i, A_e) = \begin{bmatrix} 0 & 0.1329 & 0.2164 & 0.1675 \\ -0.1268 & 0 & -0.2034 & -0.1569 \\ -0.2064 & 0.2132 & 0 & -0.1352 \\ -0.1598 & 0.1645 & 0.1418 & 0 \end{bmatrix},$$

$$\phi_3(A_i, A_e) = \begin{bmatrix} 0 & -0.1519 & -0.2234 & -0.3074 \\ 0.1225 & 0 & 0.1599 & 0.2449 \\ 0.1802 & -0.1982 & 0 & 0.1991 \\ 0.2479 & -0.3037 & -0.2469 & 0 \end{bmatrix}.$$

Then, the overall dominance degree matrix is:

$$\vartheta(A_i, A_e) = \begin{bmatrix} 0 & -0.2273 & 0.1446 & -0.3356 \\ 0.1301 & 0 & 0.1433 & 0.2209 \\ -0.2611 & -0.2746 & 0 & -0.2374 \\ 0.2144 & -0.3451 & 0.0893 & 0 \end{bmatrix}.$$

Step 7. Calculate the overall prospect value of each alternative $A_i(i = 1, 2, 3, 4)$, and get:

$$\delta(A_1) = 0.2799, \delta(A_2) = 1, \delta(A_3) = 0, \delta(A_4) = 0.5773.$$

Step 8. Rank the alternatives A_i by the values $\delta(A_i)(i = 1, 2, 3, 4)$, and obtain $A_2 \succ A_4 \succ A_1 \succ A_3$.

5.2. Comparative Analysis

5.2.1. Comparison with the PL-TOPSIS Method [43]

Step 1. By Tables 1–3, the probabilistic linguistic decision matrix is obtained and presented in Table 12.

Table 12. The probabilistic linguistic decision matrix.

	C_1	C_2	C_3
A_1	$\{s_1(0.87), s_0(0.13)\}$	$\{s_2(0.83), s_1(0.17)\}$	$\{s_0(0.8), s_{-1}(0.13), s_{-2}(0.07)\}$
A_2	$\{s_2(0.8), s_1(0.1), s_0(0.1)\}$	$\{s_1(0.5), s_2(0.5)\}$	$\{s_2(0.8), s_1(0.13), s_0(0.07)\}$
A_3	$\{s_1(0.6), s_0(0.4)\}$	$\{s_2(0.1), s_1(0.57), s_0(0.33)\}$	$\{s_1(0.2), s_2(0.8)\}$
A_4	$\{s_1(0.5), s_2(0.5)\}$	$\{s_1(0.87), s_0(0.13)\}$	$\{s_1(0.97), s_0(0.03)\}$

Step 2. Calculate the weight vector of attribute $C_j(j = 1, 2, 3)$ by Equation (26). The result is as follows:

$$w = (0.2581, 0.4193, 0.3226)^T$$

Step 3. Calculate PIS L^+ and NIS L^-. The results are as follows:

$$L^+ = (\{s_{0.667}, s_{0.333}, s_{0.050}\}, \{s_{0.692}, s_{0.333}, s_{0.083}\}, \{s_{0.667}, s_{0.133}, s_{0.035}\}),$$
$$L^- = (\{s_{0.400}, s_{0.065}, s_{0.000}\}, \{s_{0.380}, s_{0.065}, s_{0.000}\}, \{s_{0.400}, s_{0.015}, s_{0.000}\})$$

Step 4. Calculate the deviation degrees $d(A_i, L^+)$ and $d(A_i, L^-)$ by Equation (7), and determine $d_{\min}(A_i, L^+)$ and $d_{\max}(A_i, L^-)$. Then, the results are as follows:

$$d(A_1, L^+) = 0.152, d(A_2, L^+) = 0.118, d(A_3, L^+) = 0.137, d(A_4, L^+) = 0.134,$$
$$d(A_1, L^-) = 0.109, d(A_2, L^-) = 0.158, d(A_3, L^-) = 0.106, d(A_4, L^-) = 0.134,$$
$$d_{\min}(A_i, L^+) = 0.118, d_{\max}(A_i, L^-) = 0.158.$$

Step 5. Calculate the closeness coefficient $CI(A_i)(i = 1, 2, 3, 4)$. Then, the results are as follows:

$$CI(A_1) = -0.600, CI(A_2) = 0.000, CI(A_3) = -0.492, CI(A_4) = -0.288.$$

Step 6. Rank the alternatives A_i by $CI(A_i)(i = 1, 2, 3, 4)$, and obtain $A_2 \succ A_4 \succ A_3 \succ A_1$.

5.2.2. Comparison with the PLWA Method [43]

Step 1–2. The same steps are in Section 5.2.1. The results are omitted here.
Step 3. Obtain the overall attribute values $\widetilde{Z}_i(w), i = 1, 2, 3$. The results are as follows:

$$CI(A_1) = -0.600, CI(A_2) = 0.000, CI(A_3) = -0.492, CI(A_4) = -0.288.$$
$$\widetilde{Z}_3(w) = \{s_{0.4776}, s_{0.1638}, s_{0.0349}\}, \widetilde{Z}_4(w) = \{s_{0.5594}, s_{0.1181}, s_{0.0000}\}.$$

Step 4. Calculate the scores of all attribute values $E(\widetilde{Z}_i(w)), i = 1, 2, 3$. The results are as follows:

$$E(\widetilde{Z}_1(w)) = s_{0.2169}, E(\widetilde{Z}_2(w)) = s_{0.2570}, E(\widetilde{Z}_3(w)) = s_{0.2255}, E(\widetilde{Z}_4(w)) = s_{0.2258}$$

Step 5. Rank the alternatives A_i by $E(\widetilde{Z}_i(w)), i = 1, 2, 3$, and obtain $A_2 \succ A_4 \succ A_3 \succ A_1$.

5.2.3. Comparison with the PROMETHEE Method [26]

Step 1. The same step is in Section 5.2.1. The results are omitted here.
Step 2. Calculate the dominance degree of alternatives. The results are presented in Table 13.

Table 13. The dominance degree of the six cars.

	A_1	A_2	A_3	A_4
A_1	0.5000	0.2543	0.5482	0.4180
A_2	0.7457	0.5000	0.8460	0.7476
A_3	0.4518	0.1540	0.5000	0.3268
A_4	0.5820	0.2524	0.6732	0.5000

Step 3. Calculate the relative dominance degree among the alternatives. The results are presented in Table 14.

Table 14. The relative dominance degree among the six cars.

	$\Phi^+(A_i)$	$\Phi^-(A_i)$	$\Phi(A_i)$
A_1	0.4301	0.5699	−0.1398
A_2	0.7098	0.2902	0.4197
A_3	0.3582	0.6418	−0.2836
A_4	0.5019	0.4981	0.0038

Step 5. Rank the alternatives A_i by the relative dominance degree and obtain $A_2 \succ A_4 \succ A_1 \succ A_3$.

5.2.4. Comparison with the SPOTIS Method [28]

Step 1. Integrate the probabilistic linguistic decision matrix in Section 5.2.1 under Definition 2. The results are presented in Table 15.

Table 15. The probabilistic linguistic generated decision matrix.

	C_1	C_2	C_3
A_1	0.645	0.805	0.455
A_2	0.783	0.750	0.788
A_3	0.600	0.628	0.800
A_4	0.750	0.645	0.662
Weights	0.3	0.3	0.4
Types	Profit	Profit	Profit

Step 2. Consider the min and max bounds for each attribute, as follows:

$$\left[S_1^{\min}, S_1^{\max}\right] = [0.600, 0.783], \left[S_2^{\min}, S_2^{\max}\right] = [0.628, 0.805], \left[S_3^{\min}, S_3^{\max}\right] = [0.455, 0.800].$$

Step 3. Calculate the distances vector. The results are as follows:

$$d(A_1, s^*) = 0.6264, d(A_2, s^*) = 0.1069, d(A_3, s^*) = 0.6000, d(A_4, s^*) = 0.4866.$$

Step 4. Rank the alternatives A_i by $d(A_i, s^*)$ and obtain $A_2 \succ A_4 \succ A_3 \succ A_1$.

5.3. Visualization of Ranking Results

In order to better study the advantages of proposed algorithm, several algorithms have been compared and tested with it. The results are presented in Table 16 and Figure 3.

Table 16. Ranking results of different methods.

	A_1	A_2	A_3	A_4	Rank
Our Method	0.2799	1.0000	0.0000	0.5733	$A_2 \succ A_4 \succ A_1 \succ A_3$
PL-TOPSIS	−0.6000	0.0000	−0.4920	−0.2880	$A_2 \succ A_4 \succ A_3 \succ A_1$
PLWA	0.2169	0.2570	0.2255	0.2258	$A_2 \succ A_4 \succ A_3 \succ A_1$
PROMETHEE	−0.1398	0.4197	−0.2836	0.0038	$A_2 \succ A_4 \succ A_1 \succ A_3$
SPOTIS	0.6264	0.1069	0.6000	0.4866	$A_2 \succ A_4 \succ A_3 \succ A_1$

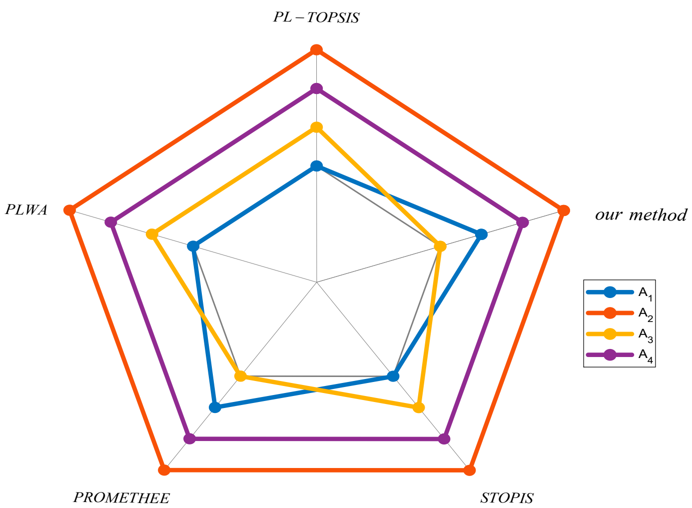

Figure 3. Radar chart of the ranking results.

The proposed method ranked similarly with the PROMETHEE method. The PL-TOPSIS, SPOTIS, and PLWA method had comparable outcomes, differing one or two positions in accordance with the proposed method. All the ranking results yield the best results for A_2 and the second best for A_4. WS Similarity Coefficient, which is sensitive to significant changes in ranking, is calculated by Equation (31). Additionally, RW Weighted Spearman's Rank Correlation Coefficient, which permits the comparation of two vectors, is calculated by Equation (32).

$$WS = 1 - \sum\left(2^{-x_i} \frac{|x_i - y_i|}{\max(|x_i - 1|, |x_i - n|)}\right) \tag{31}$$

where n is ranking size and x_i and y_i are the values in the comparing rankings.

$$RW = 1 - \frac{6\sum(x_i - y_i)^2((n - x_i - 1) + (n - y_i - 1))}{n(n^3 + n^2 - n - 1)} \tag{32}$$

where the same elements in the Equation (31) are mentioned.

The WS and RW coefficients determined by the proposed methods are shown in Table 17. The results demonstrate that the PROMETHEE method ranked the proposed method equally. PL-TOPSIS, SPOTIS, and PLWA demonstrated a lesser degree of resemblance than PROMETHEE. Despite the disparities between the ranks, the correlation between their results and the similarity of our proposed method is substantial, ensuring a high degree of stability in the outcomes.

Table 17. Correlations with reference ranking of proposed methods.

Coefficient	PL-TOPSIS	PLWA	PROMETHEE	SPOTIS
WS	0.917	0.917	1.000	0.917
RW	0.740	0.740	1.000	0.740

5.4. The Second Case Study

In order to illustrate the feasibility and validity of the proposed method better, a second case study is given. There are six attributes for shelter sites denoted as C_1, C_2, C_3, C_4, C_5, and C_6. Six sites are selected, denoted as A_1, A_2, A_3, A_4, A_5, and A_6. The computational and analytical processes are the same in Section 5.1.

Collect the experts' evaluation towards the shelter sites selection (Tables 18–20).

Table 18. The probabilistic linguistic decision matrix D^1 provided by the expert e_1.

e_1	C_1	C_2	C_3
A_1	$\{s_1(1)\}$	$\{s_2(1)\}$	$\{s_0(1)\}$
A_2	$\{s_2(1)\}$	$\{s_1(0.5), s_2(0.5)\}$	$\{s_0(0.2), s_1(0.4), s_2(0.4)\}$
A_3	$\{s_1(0.5), s_2(0.5)\}$	$\{s_1(1)\}$	$\{s_0(0.1), s_1(0.9)\}$
A_4	$\{s_1(1)\}$	$\{s_0(1)\}$	$\{s_1(0.6), s_2(0.4)\}$
A_5	$\{s_0(1)\}$	$\{s_0(1)\}$	$\{s_2(1)\}$
A_6	$\{s_1(0.5), s_2(0.5)\}$	$\{s_0(0.4), s_1(0.6)\}$	$\{s_1(1)\}$
e_1	C_4	C_5	C_6
A_1	$\{s_2(1)\}$	$\{s_0(1)\}$	$\{s_1(1)\}$
A_2	$\{s_1(1)\}$	$\{s_1(0.5), s_2(0.5)\}$	$\{s_0(1)\}$
A_3	$\{s_0(0.3), s_1(0.3), s_2(0.4)\}$	$\{s_1(1)\}$	$\{s_0(0.1), s_1(0.9)\}$
A_4	$\{s_0(1)\}$	$\{s_0(1)\}$	$\{s_1(0.6), s_2(0.4)\}$
A_5	$\{s_0(1)\}$	$\{s_0(1)\}$	$\{s_2(1)\}$
A_6	$\{s_1(0.5), s_2(0.5)\}$	$\{s_0(0.4), s_1(0.6)\}$	$\{s_1(1)\}$

Table 19. The probabilistic linguistic decision matrix D^2 provided by the expert e_2.

e_2	C_1	C_2	C_3
A_1	$\{s_0(0.4), s_1(0.6)\}$	$\{s_1(0.5), s_2(0.5)\}$	$\{s_{-2}(0.2), s_{-1}(0.4), s_0(0.4)\}$
A_2	$\{s_0(0.3), s_1(0.3), s_2(0.4)\}$	$\{s_0(1)\}$	$\{s_2(1)\}$
A_3	$\{s_2(1)\}$	$\{s_2(1)\}$	$\{s_1(1)\}$
A_4	$\{s_0(1)\}$	$\{s_1(1)\}$	$\{s_2(1)\}$
A_5	$\{s_0(0.3), s_2(0.7)\}$	$\{s_1(0.2), s_2(0.8)\}$	$\{s_2(1)\}$
A_6	$\{s_1(1)\}$	$\{s_1(1)\}$	$\{s_0(0.1), s_1(0.9)\}$
e_2	C_4	C_5	C_6
A_1	$\{s_1(0.6), s_2(0.4)\}$	$\{s_1(0.5), s_2(0.5)\}$	$\{s_1(0.6), s_2(0.4)\}$
A_2	$\{s_0(1)\}$	$\{s_0(1)\}$	$\{s_1(1)\}$
A_3	$\{s_1(1)\}$	$\{s_2(1)\}$	$\{s_1(1)\}$
A_4	$\{s_0(1)\}$	$\{s_1(1)\}$	$\{s_2(1)\}$
A_5	$\{s_0(0.3), s_2(0.7)\}$	$\{s_1(0.2), s_2(0.8)\}$	$\{s_2(1)\}$
A_6	$\{s_1(1)\}$	$\{s_1(1)\}$	$\{s_0(0.1), s_1(0.9)\}$

Table 20. The probabilistic linguistic decision matrix D^3 provided by the expert e_3.

e_3	C_1	C_2	C_3
A_1	$\{s_1(1)\}$	$\{s_2(1)\}$	$\{s_0(1)\}$
A_2	$\{s_2(1)\}$	$\{s_0(1)\}$	$\{s_2(1)\}$
A_3	$\{s_2(1)\}$	$\{s_1(0.5), s_2(0.5)\}$	$\{s_1(1)\}$
A_4	$\{s_1(0.8), s_2(0.2)\}$	$\{s_1(0.7), s_2(0.3)\}$	$\{s_2(1)\}$
A_5	$\{s_1(1)\}$	$\{s_0(1)\}$	$\{s_1(0.7), s_2(0.3)\}$
A_6	$\{s_2(1)\}$	$\{s_1(1)\}$	$\{s_1(1)\}$
e_3	C_4	C_5	C_6
A_1	$\{s_2(1)\}$	$\{s_2(1)\}$	$\{s_2(1)\}$
A_2	$\{s_0(0.3), s_2(0.7)\}$	$\{s_0(1)\}$	$\{s_2(1)\}$
A_3	$\{s_1(1)\}$	$\{s_1(0.5), s_2(0.5)\}$	$\{s_2(1)\}$
A_4	$\{s_1(0.2), s_2(0.8)\}$	$\{s_1(0.7), s_2(0.3)\}$	$\{s_2(1)\}$
A_5	$\{s_1(1)\}$	$\{s_0(1)\}$	$\{s_1(0.7), s_2(0.3)\}$
A_6	$\{s_2(1)\}$	$\{s_1(1)\}$	$\{s_1(1)\}$

According to the results of Tables 18–20 and Equation (24), the results are presented in Table 21.

Table 21. The integrated probabilistic linguistic decision matrix D.

	C_1	C_2	C_3
A_1	$\{s_{0.6338}(0.0256), s_{0.8252}(0.0576),$ $s_{0.8482}(0.0576), s_{0.9889}(0.1296)\}$	$\{s_{1.6921}(0.0625), s_{1.8650}(0.0625),$ $s_{1.8824}(0.0625), s_{1.9930}(0.0625)\}$	$\{s_{-0.6767}(0.0016), s_{-0.5259}(0.0064), s_{-0.5199}(0.0064),$ $s_{-0.3940}(0.0256), s_{-0.2924}(0.0064), s_{-0.2599}(0.0064),$ $s_{-0.1924}(0.0256), s_{-0.1676}(0.0256), s_{-0.0125}(0.0256)\}$
A_2	$\{s_{1.5575}(0.0081), s_{1.6292}(0.0081), s_{1.6313}(0.0081),$ $s_{1.6921}(0.0081), s_{1.8263}(0.0144), s_{1.8468}(0.0144),$ $s_{1.8650}(0.0144), s_{1.8824}(0.0144), s_{1.9930}(0.0256)\}$	$\{s_{0.2745}(0.0625), s_{0.3313}(0.0625),$ $s_{0.3560}(0.0625), s_{0.4068}(0.0625)\}$	$\{s_{1.5575}(0.0016), s_{1.6292}(0.0064), s_{1.6313}(0.0064),$ $s_{1.6921}(0.0256), s_{1.8263}(0.0064), s_{1.8468}(0.0064),$ $s_{1.8650}(0.0256), s_{1.8824}(0.0256), s_{1.9930}(0.0256)\}$
A_3	$\{s_{1.6921}(0.0625), s_{1.8650}(0.0625),$ $s_{1.8824}(0.0625), s_{1.9930}(0.0625)\}$	$\{s_{1.2113}(0.0625), s_{1.3090}(0.0625),$ $s_{1.6474}(0.0625), s_{1.6875}(0.0625)\}$	$\{s_{0.6338}(0.0001), s_{0.8252}(0.0081),$ $s_{0.8482}(0.0081), s_{0.9889}(0.6561)\}$
A_4	$\{s_{0.6373}(0.4096), s_{0.6623}(0.0256),$ $s_{0.8412}(0.0256), s_{0.8593}(0.0016)\}$	$\{s_{0.6388}(0.2401), s_{0.6644}(0.0441),$ $s_{0.8450}(0.0441), s_{0.8635}(0.0081)\}$	$\{s_{1.6921}(0.1296), s_{1.8650}(0.0576),$ $s_{1.8824}(0.0576), s_{1.9930}(0.0256)\}$
A_5	$\{s_{0.2117}(0.0081), s_{0.3553}(0.0441),$ $s_{0.8906}(0.0441), s_{0.9449}(0.2401)\}$	$\{s_{0.2745}(0.0016), s_{0.3313}(0.0256),$ $s_{0.3560}(0.0256), s_{0.4068}(0.4096)\}$	$\{s_{1.6921}(0.2401), s_{1.8650}(0.0441),$ $s_{1.8824}(0.0441), s_{1.9930}(0.0081)\}$
A_6	$\{s_{1.2113}(0.0625), s_{1.3090}(0.0625),$ $s_{1.6474}(0.0625), s_{1.6875}(0.0625)\}$	$\{s_{0.6338}(0.0256), s_{0.8252}(0.0576),$ $s_{0.8482}(0.0576), s_{0.9889}(0.1296)\}$	$\{s_{0.6338}(0.0001), s_{0.8252}(0.0081),$ $s_{0.8482}(0.0081), s_{0.9889}(0.6561)\}$

Table 21. *Cont.*

	C_4	C_5	C_6
A_1	$\{s_{1.6921}(0.1296), s_{1.8650}(0.0576), s_{1.8824}(0.0576), s_{1.9930}(0.0256)\}$	$\{s_{0.8577}(0.0625), s_{0.8796}(0.0625), s_{1.5330}(0.0625), s_{1.5390}(0.0625)\}$	$\{s_{1.2163}(0.1296), s_{1.3109}(0.0576), s_{1.6511}(0.0576), s_{1.6899}(0.0256)\}$
A_2	$\{s_{0.2117}(0.0081), s_{0.3553}(0.0441), s_{0.8906}(0.0441), s_{9449}(0.2401)\}$	$\{s_{0.2745}(0.0625), s_{0.3313}(0.0625), s_{0.3560}(0.0625), s_{0.4068}(0.0625)\}$	$\{s_{0.6388}(0.2401), s_{0.6644}(0.0441), s_{0.8450}(0.0441), s_{0.8635}(0.0081)\}$
A_3	$\{s_{0.6338}(0.0081), s_{0.8252}(0.0081), s_{0.8482}(0.0081), s_{0.9889}(0.0081), s_{1.0110}(0.0144), s_{1.0757}(0.0144), s_{1.1339}(0.0144), s_{1.1736}(0.0144), s_{1.2803}(0.0256)\}$	$\{s_{1.2113}(0.0625), s_{1.3090}(0.0625), s_{1.6474}(0.0625), s_{1.6875}(0.0625)\}$	$\{s_{0.8738}(0.0001), s_{1.0118}(0.0081), s_{1.1315}(0.0081), s_{1.2223}(0.6561)\}$
A_4	$\{s_{0.2745}(0.0016), s_{0.3313}(0.0256), s_{0.3560}(0.0256), s_{0.4068}(0.4096)\}$	$\{s_{0.6388}(0.2401), s_{0.6644}(0.0441), s_{0.8450}(0.0441), s_{0.8635}(0.0081)\}$	$\{s_{1.6921}(0.1296), s_{1.8650}(0.0576), s_{1.8824}(0.0576), s_{1.9930}(0.0256)\}$
A_5	$\{s_{0.2117}(0.0081), s_{0.3553}(0.0441), s_{0.8906}(0.0441), s_{9449}(0.2401)\}$	$\{s_{0.2745}(0.0016), s_{0.3313}(0.0256), s_{0.3560}(0.0256), s_{0.4068}(0.4096)\}$	$\{s_{1.6921}(0.2401), s_{1.8650}(0.0441), s_{1.8824}(0.0441), s_{1.9930}(0.0081)\}$
A_6	$\{s_{1.2113}(0.0625), s_{1.3090}(0.0625), s_{1.6474}(0.0625), s_{1.6875}(0.0625)\}$	$\{s_{0.6338}(0.0256), s_{0.8252}(0.0576), s_{0.8482}(0.0576), s_{0.9889}(0.1296)\}$	$\{s_{0.6338}(0.0001), s_{0.8252}(0.0081), s_{0.8482}(0.0081), s_{0.9889}(0.6561)\}$

Given the weights of all attributes $\varpi_1 = 0.1, \varpi_2 = 0.2, \varpi_3 = 0.1, \varpi_4 = 0.2, \varpi_5 = 0.2, \varpi_6 = 0.2$.

We get the dominance degree matrix, as follows:

$$\vartheta(A_i, A_e) = \begin{bmatrix} 0 & -0.2341 & -0.6094 & 0.0077 & -0.2329 & -0.3654 \\ -0.5949 & 0 & -1.1027 & -0.5144 & -0.6257 & -0.7954 \\ -0.4412 & -0.0131 & 0 & -0.1938 & 0.07 & 0.0689 \\ -1.0686 & -0.569 & -1.1216 & 0 & 0.0027 & -0.94 \\ -0.9401 & -0.4755 & -1.2215 & -0.7807 & 0 & -1.0698 \\ -0.7855 & -0.3692 & -0.5622 & -0.2742 & 0.0061 & 0 \end{bmatrix}.$$

Calculate the overall prospect value of each alternative $A_i (i = 1, 2, 3, 4, 5, 6)$, and get:

$$\delta(A_1) = 0.9322, \delta(A_2) = 0.4354, \delta(A_3) = 0 = 1, \delta(A_4) = 0.2378, \delta(A_5) = 0, \delta(A_6) = 0.5151.$$

The ranking results of different methods are as follows, see Table 22.

Table 22. Ranking results of different methods.

	A_1	A_2	A_3	A_4	A_5	A_6	Rank
Our Method	0.9332	0.4354	1.0000	0.2378	0.0000	0.5151	$A_3 \succ A_1 \succ A_6 \succ A_2 \succ A_4 \succ A_5$
PL-TOPSIS	−0.1223	−0.5892	0.0000	−0.5169	−0.5568	−0.1505	$A_3 \succ A_1 \succ A_6 \succ A_4 \succ A_5 \succ A_2$
PLWA	0.2397	0.2238	0.2468	0.2194	0.2156	0.2375	$A_3 \succ A_1 \succ A_6 \succ A_2 \succ A_4 \succ A_5$
PROMETHEE	0.0825	−0.0413	0.1897	−0.1096	−0.1057	−0.0155	$A_3 \succ A_1 \succ A_6 \succ A_2 \succ A_5 \succ A_4$
SPOTIS	0.3270	0.6749	0.3216	0.6865	0.6935	0.4410	$A_3 \succ A_1 \succ A_6 \succ A_2 \succ A_4 \succ A_5$

All the ranking results yield the best results for A_3, the second best for A_1, and the third best for A_6. The proposed method ranked similarly with other several methods when there are many attributes and alternatives.

6. Conclusions

The paper propsed a novel approach for MAGDM based on the new operational rules under PLTSs and the PLWDBMPA operator.

The innovations and advantages of this paper include the introduction of the TF-IDF keyword extraction technique as new weights, the definition of some new Dombi operations for PLTSs, and the PLDBMPA and PLWDBMPA operator on the basis of the proposed operational rules of PLTSs, which take into account the decision-makers' subject preference and the relationship between the input arguments. Compared with the PL-TOPSIS, PLWA, SPOTIS, and PROMETHEE methods, the proposed method is demonstrated to be more scientific and accurate for the decision results derived from the case study, providing a more reasonable reference for experts to select the alternative shelter sites.

The proposed method in this paper also has some limitations. The PLWDBMPA operator cannot independently rank the alternatives; it still needs to be combined with some ranking methods like TOPSIS, VIKOR, and PROMETHEE. If there are too many alternatives and attributes, it might be quite flexible for Section 5.1. In terms of future research, we will improve the PLDBMPA and PLWDBMPA operators, making it rank the alternatives independently. We can also use Python, MATLAB to deal with the process of decision-making information fusion utilizing the PLWDBMPA operator. The Dombi operator can be applied to different fuzzy sets such as the intuitionistic fuzzy set, the q-rung orthopair fuzzy set, and the spherical fuzzy set. In addition, we also plan to extend our method to more scenarios, such as liquor brand evaluation, medical service, and emergency decision making.

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Appendix A

Proof of Theorem 1.

$$
\begin{aligned}
L_1(P_1) \oplus L_2(P_2) &= g^{-1}\left(\bigcup_{\substack{\eta_1^{(i)} \in g(L_1), \\ \eta_2^{(j)} \in g(L_2)}} \left\{ \left(1 - \frac{1}{1+\left(\left(\frac{\eta_1^{(i)}}{1-\eta_1^{(i)}}\right)^{\lambda} + \left(\frac{\eta_2^{(j)}}{1-\eta_2^{(j)}}\right)^{\lambda}\right)^{1/\lambda}} \right) \left(P_1^{(i)} P_2^{(j)}\right) \right\} \right) \\
&= g^{-1}\left(\bigcup_{\substack{\eta_1^{(i)} \in g(L_1), \\ \eta_2^{(j)} \in g(L_2)}} \left\{ \left(1 - \frac{1}{1+\left(\left(\frac{\eta_2^{(j)}}{1-\eta_2^{(j)}}\right)^{\lambda} + \left(\frac{\eta_1^{(i)}}{1-\eta_1^{(i)}}\right)^{\lambda}\right)^{1/\lambda}} \right) \left(P_2^{(j)} P_1^{(i)}\right) \right\} \right) = L_2(P_2) \oplus L_1(P_1)
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
L_1(P_1) \otimes L_2(P_2) &= g^{-1}\left(\bigcup_{\substack{\eta_1^{(i)} \in g(L_1), \\ \eta_2^{(j)} \in g(L_2)}} \left\{ \left(\frac{1}{1+\left(\left(\frac{1-\eta_1^{(i)}}{\eta_1^{(i)}}\right)^{\lambda} + \left(\frac{1-\eta_2^{(j)}}{\eta_2^{(j)}}\right)^{\lambda}\right)^{1/\lambda}} \right) \left(P_1^{(i)} P_2^{(j)}\right) \right\} \right) \\
&= g^{-1}\left(\bigcup_{\substack{\eta_1^{(i)} \in g(L_1), \\ \eta_2^{(j)} \in g(L_2)}} \left\{ \left(\frac{1}{1+\left(\left(\frac{1-\eta_2^{(j)}}{\eta_2^{(j)}}\right)^{\lambda} + \left(\frac{1-\eta_1^{(i)}}{\eta_1^{(i)}}\right)^{\lambda}\right)^{1/\lambda}} \right) \left(P_2^{(j)} P_1^{(i)}\right) \right\} \right) = L_2(P_2) \otimes L_1(P_1).
\end{aligned}
\tag{A2}
$$

$$\begin{aligned}
\delta(L_1(P_1) \oplus L_2(P_2)) &= g^{-1}\left(\bigcup_{\substack{\eta_1^{(i)} \in g(L_1), \\ \eta_2^{(j)} \in g(L_2)}} \left\{ \left(1 - \frac{1}{1+\left(\delta\left(\left(\frac{\eta_1^{(i)}}{1-\eta_1^{(i)}}\right)^{\lambda}+\left(\frac{\eta_2^{(j)}}{1-\eta_2^{(j)}}\right)^{\lambda}\right)\right)^{1/\lambda}} \right) \left(P_1^{(i)} P_2^{(j)} \right) \right\} \right) \\
&= g^{-1}\left(\bigcup_{\substack{\eta_1^{(i)} \in g(L_1), \\ \eta_2^{(j)} \in g(L_2)}} \left\{ \left(1 - \frac{1}{1+\left(\delta\left(\frac{\eta_1^{(i)}}{1-\eta_1^{(i)}}\right)^{\lambda}+\delta\left(\frac{\eta_2^{(j)}}{1-\eta_2^{(j)}}\right)^{\lambda}\right)^{1/\lambda}} \right) \left(P_1^{(i)} P_2^{(j)} \right) \right\} \right) = \delta L_1(P_1) \oplus \delta L_2(P_2)
\end{aligned} \tag{A3}$$

$$\begin{aligned}
(L_1(P_1) \otimes L_2(P_2))^{\delta} &= g^{-1}\left(\bigcup_{\substack{\eta_1^{(i)} \in g(L_1), \\ \eta_2^{(j)} \in g(L_2)}} \left\{ \left(\frac{1}{1+\left(\delta\left(\left(\frac{1-\eta_1^{(i)}}{\eta_1^{(i)}}\right)^{\lambda}+\left(\frac{1-\eta_2^{(j)}}{\eta_2^{(j)}}\right)^{\lambda}\right)\right)^{1/\lambda}} \right) \left(P_1^{(i)} P_2^{(j)} \right) \right\} \right) \\
&= g^{-1}\left(\bigcup_{\substack{\eta_1^{(i)} \in g(L_1), \\ \eta_2^{(j)} \in g(L_2)}} \left\{ \left(\frac{1}{1+\left(\delta\left(\frac{1-\eta_1^{(i)}}{\eta_1^{(i)}}\right)^{\lambda}+\delta\left(\frac{1-\eta_2^{(j)}}{\eta_2^{(j)}}\right)^{\lambda}\right)^{1/\lambda}} \right) \left(P_1^{(i)} P_2^{(j)} \right) \right\} \right) = (L_1(P_1))^{\delta} \otimes (L_2(P_2))^{\delta}
\end{aligned} \tag{A4}$$

$$\begin{aligned}
(L(P))^{\delta_1} \otimes (L(P))^{\delta_2} &= g^{-1}\left(\bigcup_{\eta^{(k)} \in g(L)} \left\{ \left(\frac{1}{1+\left(\delta_1\left(\frac{1-\eta^{(k)}}{\eta^{(k)}}\right)^{\lambda}+\delta_2\left(\frac{1-\eta^{(k)}}{\eta^{(k)}}\right)^{\lambda}\right)^{1/\lambda}} \right) \left(P^{(k)} \right) \right\} \right) \\
&= g^{-1}\left(\bigcup_{\eta^{(k)} \in g(L)} \left\{ \left(\frac{1}{1+\left((\delta_1+\delta_2)(\frac{1-\eta^{(k)}}{\eta^{(k)}})^{\lambda}\right)^{1/\lambda}} \right) \left(P^{(k)} \right) \right\} \right) = (L(P))^{\delta_1+\delta_2}
\end{aligned} \tag{A5}$$

□

Appendix B

Proof of Theorem 2. According to the operational rules of the PLTSs, we have

$$\frac{n(1+T(L_i(P_i)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))} L_i(P_i) = g^{-1}\left(\bigcup_{\eta_i^{(k)} \in g(L_i)} \left\{ \left(1 - \frac{1}{1+\left(\frac{n(1+T(L_i(P_i)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}\left(\frac{\eta_i^{(k)}}{1-\eta_i^{(k)}}\right)^{\lambda}\right)^{1/\lambda}} \right) (P_i^{(k)}) \right\} \right)$$

$$\frac{n(1+T(L_j(P_j)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))} L_j(P_j) = g^{-1}\left(\bigcup_{\eta_j^{(d)} \in g(L_j)} \left\{ \left(1 - \frac{1}{1+\left(\frac{n(1+T(L_j(P_j)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}\left(\frac{\eta_j^{(d)}}{1-\eta_j^{(d)}}\right)^{\lambda}\right)^{1/\lambda}} \right) (P_j^{(d)}) \right\} \right)$$

Let $E_i = \frac{n(1+T(L_i(P_i)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}$, $E_j = \frac{n(1+T(L_j(P_j)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}$, $K_i = \frac{1-\eta_i^{(k)}}{\eta_i^{(k)}}$, $D_j = \frac{1-\eta_j^{(d)}}{\eta_j^{(d)}}$.

Then,

$$\frac{n(1+T(L_i(P_i)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}L_i(P_i)=g^{-1}\left(\bigcup_{\eta_i^{(k)}\in g(L_i)}\left\{\left(1-\left(1+\left(E_i\left(\frac{1}{K_i}\right)^{\lambda}\right)^{1/\lambda}\right)^{-1}\right)(P_i^{(k)})\right\}\right)$$

$$\frac{n(1+T(L_j(P_j)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}L_j(P_j)=g^{-1}\left(\bigcup_{\eta_j^{(d)}\in g(L_j)}\left\{\left(1-\left(1+\left(E_j\left(\frac{1}{D_j}\right)^{\lambda}\right)^{1/\lambda}\right)^{-1}\right)(P_j^{(d)})\right\}\right)$$

$$\left(\frac{n(1+T(L_i(P_i)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}L_i(P_i)\right)^{p}=g^{-1}\left(\bigcup_{\eta_i^{(k)}\in g(L_i)}\left\{\left(1+\left(p\left(\left(E_i\left(\frac{1}{K_i}\right)^{\lambda}\right)^{-1/\lambda}\right)^{\lambda}\right)^{1/\lambda}\right)^{-1}(P_i^{(k)})\right\}\right)=g^{-1}\left(\bigcup_{\eta_i^{(k)}\in g(L_i)}\left\{\left(1+\left(\frac{p}{E_i}K_i^{\lambda}\right)^{1/\lambda}\right)^{-1}(P_i^{(k)})\right\}\right)$$

$$\left(\frac{n(1+T(L_j(P_j)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}L_j(P_j)\right)^{q}=g^{-1}\left(\bigcup_{\eta_j^{(d)}\in g(L_j)}\left\{\left(1+\left(\frac{q}{E_j}D_j^{\lambda}\right)^{1/\lambda}\right)^{-1}(P_j^{(d)})\right\}\right)$$

$$\left(\frac{n(1+T(L_i(P_i)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}L_i(P_i)\right)^{p}\otimes\left(\frac{n(1+T(L_j(P_j)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}L_j(P_j)\right)^{q}$$

$$=\bigcup_{\substack{\eta_i^{(k)}\in g(L_i),\\ \eta_j^{(d)}\in g(L_j)}}g^{-1}\left(\left\{\left(\frac{1}{1+\left(\left(\left(\frac{p}{E_i}K_i^{\lambda}\right)^{1/\lambda}\right)^{\lambda}+\left(\left(\frac{q}{E_j}D_j^{\lambda}\right)^{1/\lambda}\right)^{\lambda}\right)^{1/\lambda}}\right)\left(P_i^{(k)}P_j^{(d)}\right)\right\}\right)=\bigcup_{\substack{\eta_i^{(k)}\in g(L_i),\\ \eta_j^{(d)}\in g(L_j)}}g^{-1}\left(\left\{\left(\frac{1}{1+\left(\frac{p}{E_i}K_i^{\lambda}+\frac{q}{E_j}D_j^{\lambda}\right)^{1/\lambda}}\right)\left(P_i^{(k)}P_j^{(d)}\right)\right\}\right)$$

$$\bigoplus_{\substack{i,j=1\\ i\neq j}}^{n}\left(\left(\frac{n(1+T(L_i(P_i)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}L_i(P_i)\right)^{p}\otimes\left(\frac{n(1+T(L_j(P_j)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}L_j(P_j)\right)^{q}\right)$$

$$=\bigcup_{\substack{\eta_i^{(k)}\in g(L_i),\\ \eta_j^{(d)}\in g(L_j)}}g^{-1}\left(\left\{\left(1-\left(1+\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{p}{E_i}K_i^{\lambda}+\frac{q}{E_j}D_j^{\lambda}\right)^{-1}\right)^{1/\lambda}\right)^{-1}\right)\left(\prod_{\substack{i,j=1\\ i\neq j}}^{n}P_i^{(k)}P_j^{(d)}\right)\right\}\right)$$

$$=\bigcup_{\substack{\eta_i^{(k)}\in g(L_i),\\ \eta_j^{(d)}\in g(L_j)}}g^{-1}\left(\left\{\left(1-\left(1+\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\left(\frac{p}{E_i}K_i^{\lambda}+\frac{q}{E_j}D_j^{\lambda}\right)^{-1/\lambda}\right)^{\lambda}\right)^{1/\lambda}\right)^{-1}\right)\left(\prod_{\substack{i,j=1\\ i\neq j}}^{n}P_i^{(k)}P_j^{(d)}\right)\right\}\right)$$

$$\frac{1}{n(n-1)}\bigoplus_{\substack{i,j=1\\ i\neq j}}^{n}\left(\left(\frac{n(1+T(L_i(P_i)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}L_i(P_i)\right)^{p}\otimes\left(\frac{n(1+T(L_j(P_j)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}L_j(P_j)\right)^{q}\right)$$

$$=\bigcup_{\substack{\eta_i^{(k)}\in g(L_i),\\ \eta_j^{(d)}\in g(L_j)}}g^{-1}\left(\left\{\left(1-\left(1+\left(\frac{1}{n(n-1)}\left(\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{p}{E_i}K_i^{\lambda}+\frac{q}{E_j}D_j^{\lambda}\right)^{-1}\right)^{1/\lambda}\right)^{\lambda}\right)^{1/\lambda}\right)^{-1}\right)\left(\prod_{\substack{i,j=1\\ i\neq j}}^{n}P_i^{(k)}P_j^{(d)}\right)\right\}\right)$$

$$= \bigcup_{\substack{\eta_i^{(k)} \in g(L_i), \\ \eta_j^{(d)} \in g(L_j)}} g^{-1}\left(\left\{\left(1-\left(1+\left(\frac{1}{n(n-1)}\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \left(\frac{p}{E_i}K_i^{\lambda} + \frac{q}{E_j}D_j^{\lambda}\right)^{-1}\right)^{1/\lambda}\right)^{-1}\right)\left(\prod_{\substack{i,j=1 \\ i\neq j}}^{n} P_i^{(k)}P_j^{(d)}\right)\right\}\right)$$

$$\left(\frac{1}{n(n-1)} \bigoplus_{\substack{i,j=1 \\ i\neq j}}^{n} \left(\left(\frac{n(1+T(L_i(P_i)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}L_i(P_i)\right)^{p} \otimes \left(\frac{n(1+T(L_j(P_j)))}{\sum_{t=1}^{n}(1+T(L_t(P_t)))}L_j(P_j)\right)^{q}\right)\right)^{\frac{1}{p+q}}$$

$$= \bigcup_{\substack{\eta_i^{(k)} \in g(L_i), \\ \eta_j^{(d)} \in g(L_j)}} g^{-1}\left(\left\{\left(1+\left(\frac{1}{p+q}\left(\left(\frac{1}{n(n-1)}\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \left(\frac{p}{E_i}K_i^{\lambda} + \frac{q}{E_j}D_j^{\lambda}\right)^{-1}\right)^{-1/\lambda}\right)^{\lambda}\right)^{1/\lambda}\right)^{-1}\left(\prod_{\substack{i,j=1 \\ i\neq j}}^{n} P_i^{(k)}P_j^{(d)}\right)\right\}\right)$$

$$= \bigcup_{\substack{\eta_i^{(k)} \in g(L_i), \\ \eta_j^{(d)} \in g(L_j)}} g^{-1}\left(\left\{\left(1+\left(\frac{n(n-1)}{(p+q)}\left(\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \left(\frac{p}{E_i}K_i^{\lambda} + \frac{q}{E_j}D_j^{\lambda}\right)^{-1}\right)^{-1}\right)^{1/\lambda}\right)^{-1}\left(\prod_{\substack{i,j=1 \\ i\neq j}}^{n} P_i^{(k)}P_j^{(d)}\right)\right\}\right)$$

□

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