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Editorial

Special Issue "Advanced Symmetry Methods for Dynamics, Control, Optimization and Applications"

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Nonlinear systems described by differential equations are of great theoretical significance and do frequently arise in practice. This class of nonlinear systems possess the properties of symmetry under certain conditions and can be widely applied to many different fields of science and engineering, such as economics, neural networks, mechanics, biology, electrical engineering, physics, automatic control, and medicine. Because of the influence of time delay, impulse effect, discontinuity, and randomness, among other factors, nonlinear systems possess very rich and complex dynamics, including periodicity, limit cycles, bifurcation, sliding mode orbit, and chaos. Due to the many practical needs in cybernetics, the method of symmetry should be developed, and various suitable control protocols should also be designed. Moreover, it is highly necessary to further study the control and optimization of nonlinear systems, including their stability, synchronization, and convergence. This Special Issue brings together articles concerning the dynamics, control, optimization, and applications of nonlinear systems via the use of symmetry methods. It includes 12 original research papers, either proposing advanced symmetry methods to investigate the dynamics of nonlinear systems or presenting new results on the control problems of nonlinear systems. Finally, one paper is a review on the geometric structures generated by the same dynamics in differential equations.

Khan et al. [1] provided several upper bounds for the sum of k largest reciprocal distance Laplacian eigenvalues of G in terms of various graph parameters, where a real symmetric matrix was used. The extremal cases corresponding to these bounds were also characterized. As a consequence, the upper bounds for the reciprocal distance Laplacian spectral radius were obtained and the extremal graphs were determined.

Shammakh et al. [2] investigated a hybrid-type boundary value problem with fractional-order discrete time. The existence and uniqueness of the solution of the nonlinear discrete fractional equation were established by selecting a suitable fixed-point theorem and defining the symmetric structure of the operators. The stability in the sense of Hyers and Ulam using the Mittag-Leffler function was illustrated for the boundary value problem. The obtained theoretical results were applied to deal with the heat transfer problem between surfaces using fins.

Al-Tahan et al. [3] studied ordered semigroups via several new substructures. The quasi-filters and (m, n)-quasi-filters of the ordered semigroups were discussed by exploring their properties. The relations between the quasi-filters and quasi-ideals, as well as between the (m, n)-quasi-filters and (m, n)-quasi-ideals, were also analyzed.

Lan et al. [4] considered a generalized delayed stage-structured prey–predator model possessing fear effect and prey refuge. Based on the implicit function derivative rule and the function monotonicity theory, the existence of a coexistence equilibrium point of the prey–predator model was analyzed. The Hopf bifurcation (i.e., an unstable state of

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symmetrical periodic oscillation) of the prey–predator system and the control of bifurcation of the controlled system were explored.

Zhou and Gao [5] studied a synchronous genetic regulatory network model with time delay, where the drive-response system possesses symmetry. By utilizing the fixed-time stability theory and the Lyapunov method, the fixed-time synchronization control of the genetic regulatory network was realized via two types of different switching controllers. Moreover, the settling times of the fixed-time synchronization were estimated.

Zeng et al. [6] analyzed the mechanism of mass imbalance and sensor runout to produce the flywheel vibration force by establishing an interference model for a magnetic suspension flywheel. The obtained analytical results show that the x–y direction and the flywheel structure have symmetry. A compound repetitive controller was designed and the same frequency–displacement stiffness force compensation was proposed. Furthermore, the parameter design and stability analysis were carried out.

Hu [7] established several novel stochastic finite-time stability criteria for stochastic nonlinear systems possessing stochastic impulse effects by using the probability theory and Lyapunov approach. The deterministic model considered in this paper can be used to deal with symmetric Markov process with Brownian motion.

Li et al. [8] proposed a three-dimensional fractional predator–prey model with two nonidentical delays. The stability switching curves in the delay plane was calculated by choosing two delays as the bifurcation parameter, and the stability was analyzed. Moreover, the existence of a Hopf bifurcation in the predator–prey system was discussed, in which the system's trajectories have axial symmetry.

Khan et al. [9] developed several new integral inequalities for left and right convex interval-valued function on coordinates via pseudo-order relation and double integral. In particular, the Hermite–Hadamard-type inequalities and Hadamard–Fejér-type inequalities were derived for the left and right convex interval-valued functions. These inequalities can be widely used in optimization research.

Staš [10] aimed to determine the crossing numbers of the joint products of six symmetric graphs on six vertices with paths and cycles on n vertices. Some of the proofs are supported by several well-known auxiliary statements.

Wang et al. [11] were interested in the convergence of neutral functional differential equations arising from population dynamics and high-dimensional Haddock conjecture. The subtle relations of both the ω-limited set and the special point were analyzed by using monotonicity techniques and functional methods. In addition, each bounded solution was proved to converge to a constant vector.

Zhang et al. [12] investigated the periodicity of neutral-type inertial neural networks with multiple delays. The existence and global exponential stability of periodic solutions were proved by exploring the Lyapunov theory and inequality techniques.

Finally, Ayala et al. [13] presented some relevant relationships between three geometric structures (i.e., Linear Control Systems, Almost Riemannian Structures, and Degenerate Dynamical Systems) generated by the same type of dynamics on several Lie groups. Because the dynamics comes from the algebraic structures of the Lie groups, the symmetry is fully embodied.

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