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Construction of Solitary Wave Solutions to the (3 + 1)-Dimensional Nonlinear Extended and Modified Quantum Zakharov–Kuznetsov Equations Arising in Quantum Plasma Physics

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Abstract: Several types of solitary wave solutions of (3 + 1)-dimensional nonlinear extended and modified quantum Zakharov–Kuznetsov equations are established successfully via the implantation of three mathematical methods. The concerned models have many fruitful applications to describe the waves in quantum electron–positron–ion magnetoplasmas and weakly nonlinear ion-acoustic waves in plasma. The derived results via the MEAEM method, ESE method, and modified F-expansion have been retrieved and will be expedient in the future to illuminate the collaboration between lower nonlinear ion-acoustic waves. For the physical behavior of the models, some solutions are plotted graphically in 2D and 3D by imparting particular values to the parameters under the given condition at each solution. Hence explored solutions have profitable rewards in the field of mathematical physics.

Keywords: nonlinear extended quantum Zakharov–Kuznetsov equation; nonlinear modified quantum Zakharov–Kuznetsov equation; mathematical methods; soliton solutions

Citation: Areshi, M.; Seadawy, A.R.; Ali, A.; AlJohani, A.F.; Alharbi, W.; Alharbi, A.F. Construction of Solitary Wave Solutions to the (3 + 1)-Dimensional Nonlinear Extended and Modified Quantum Zakharov–Kuznetsov Equations Arising in Quantum Plasma Physics. *Symmetry* **2023**, *15*, 248. https://doi.org/10.3390/sym15010248

Academic Editors: Haci Mehmet Baskonus, Yolanda Guerrero, Carlo Cattani and Sergei D. Odintsov

Received: 26 November 2022
Revised: 6 January 2023
Accepted: 13 January 2023
Published: 16 January 2023

1. Introduction

A developing concentration has been engrossed in the research of analytical and numerical solutions of nonlinear evolutions equations (NLEEs) during the previous eras [1–6]. NLEEs are used to demonstrate phenomena in dissimilar fields of science and engineering such as optic fibers, plasmas, biology, fluid mechanics, acoustics, and numerous others [7–22]. Exact and solitary wave solutions of NLEEs were made possible with the initiation of the selection of mathematical tools [8–31]. Currently, diverse categories of nonlinear evolution equations were developed using a powerful reductive perturbation technique or a multiscale analysis [32–35]. Further specifically, the exploration of exact solutions called soliton-like solutions has advanced quickly today, which is one of the significant topics of nonlinear science. Solitons have enormous features because of their stuff (stability, robustness, and the ability to preserve their velocity and shape after interaction) [18–22], and they occur in various forms such as bright, dark, kink, pulses, breather, and so on. Furthermore, lately, novel forms of bright and dark solitons known as W-shape and M-shape have been exposed. A similar number of works have been approved to show the relevance of these results [36–40]. However, searching for the exact traveling wave solutions still poses a problem at times due to not all the known methods can be applied to NLEEs. In this current research, we explore solitary wave solutions by applying three mathematical methods, modified extended auxiliary equation mapping method, extended simple equation method, and modified F-expansion method [41,42]. The derived solutions have great potential to handle nonlinear problems in mathematical physics.

The article is arranged as follows: Section 2 is a survey of the proposed schemes. Section 3 is an implementation of the presented methods to concern models. The obtained solitary wave solutions to the (3 + 1)-Dimensional nonlinear extended and modified quantum Zakharov–Kuznetsov equations are given in Section 4. Section 5 gives the results and summary of the work.

2. Description of the Methods

Let the NPDE has a form as

$$E(U, U_t, U_x, U_{xx}, U_{xt}, \ldots) = 0. \tag{1}$$

Let

$$U = U(\xi), \qquad \xi = kx - \omega t. \tag{2}$$

Substitute (2) in (1),

$$F(U, U', U'', \ldots) = 0. \tag{3}$$

2.1. Modified Extended Auxiliary Equation Mapping Method

Suppose the solution of (3) is,

$$U = \sum_{i=0}^{N} A_i \Psi^i + \sum_{i=-1}^{-N} B_{-i} \Psi^i + \sum_{i=2}^{N} C_i \Psi^{i-2} \Psi' + \sum_{i=1}^{N} D_i \left(\frac{\Psi'}{\Psi}\right)^i. \tag{4}$$

Let Ψ satisfy,

$$\Psi' = \sqrt{\beta_1 \Psi^2 + \beta_2 \Psi^3 + \beta_3 \Psi^4}. \tag{5}$$

Put (4) with (5) in (3), we found the solution of (1).

2.2. Extended Simple Equation Method

Let (3) has solution,

$$U(\xi) = \sum_{i=-N}^{N} A_i \Psi^i(\xi). \tag{6}$$

Let Ψ satisfy,

$$\Psi' = c_0 + c_1 \Psi + c_2 \Psi^2 + c_3 \Psi^3. \tag{7}$$

Put (6) with (7) in (3). We get the solution of (1).

2.3. Modified F-Expansion Method

Suppose the solution of (3) is:

$$U = a_0 + \sum_{i=1}^{N} a_i F^i(\xi) + \sum_{i=1}^{N} b_i F^{-i}(\xi). \tag{8}$$

Let F obliges,

$$F' = A + BF + CF^2. \tag{9}$$

Put (8) with (9) in (3). Solve obtained system to establish the required solution of (1).

3. (3 + 1)-Dimensional Nonlinear Extended Quantum Zakharov–Kuznetsov (NLEQZK) Equation

Let NLEQZK equation [36,38,39].

$$\frac{\partial U}{\partial t} + \left(\mathrm{P}U + \mathrm{Q}U^2\right)\frac{\partial U}{\partial z} + R\frac{\partial^3 U}{\partial z^3} + S\frac{\partial}{\partial z}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)U = 0. \tag{10}$$

The (3 + 1)-dimensional NLEQZK model has fruitful applications to handle the quantum electron-positron-ion magneto-plasmas, warm ions, and hot isothermal electrons in the existence of a uniform magnetic field.

Let wave transformations,

$$U(x,y,z,t) = U(\xi), \quad \xi = \alpha x + \beta y + \gamma z - \omega t. \tag{11}$$

Put (11) in (10), after integrating and omitting the integral constant, we have

$$\frac{1}{2}\gamma P U^2 + \frac{1}{3}\gamma Q U^3 + U''\left(\gamma^3 R + \gamma S\left(\alpha^2 + \beta^2\right)\right) - U\omega = 0. \tag{12}$$

3.1. Application of Modified Extended Auxiliary Equation Mapping Method

Let solution of (12) is,

$$U = A_1\Psi + A_0 + \frac{B_1}{\Psi} + \frac{D_1\Psi'}{\Psi}. \tag{13}$$

Put (13) with (5) in (12), we obtained the coefficients of solutions as following

$$\begin{aligned} &A_0 = -\frac{P}{2Q},\ A_1 = \frac{\sqrt{\beta_3}P}{2\sqrt{\beta_1}Q},\ B_1 = 0,\ D_1 = \frac{P}{2\sqrt{\beta_1}Q},\ \omega = -\frac{\gamma P^2}{6Q}, \\ &\alpha = \frac{\sqrt{-P^2 - 6\beta_1\gamma^2 QR - 6\beta^2\beta_1 QS}}{\sqrt{6}\sqrt{\beta_1}\sqrt{Q}\sqrt{S}}. \end{aligned} \tag{14}$$

Substitute (14) in (13), we found the solutions of equation (10)

CASE I:

$$\begin{aligned} U_1 = &-\left(\frac{(\sqrt{\beta_3}P)\left(\beta_1\left(\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)+1\right)\right)}{\beta_2(2\sqrt{\beta_1}Q)}\right) + \left(\frac{P\left(\beta_1^{3/2}\epsilon\operatorname{csch}^2\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)\right)}{\frac{(2\sqrt{\beta_1}Q)\left((2\beta_2)\left(-\beta_1\left(\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)+1\right)\right)\right)}{\beta_2}}\right) \\ &-\left(\frac{P}{2Q}\right),\ \beta_1 > 0,\ \beta_2^2 - 4\beta_1\beta_3 = 0. \end{aligned} \tag{15}$$

CASE II:

$$\begin{aligned} U_2 = &-\left(\frac{P\left(\sqrt{\frac{\beta_1}{\beta_3}}\left(\frac{\sqrt{\beta_1}\epsilon\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta} - \frac{\sqrt{\beta_1}\epsilon\sinh^2\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\left(\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\right)^2}\right)\right)}{(2\sqrt{\beta_1}Q)\left(2\left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\epsilon\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta}+1\right)\right)\right)}\right) - \left(\frac{(\sqrt{\beta_3}P)\left(\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\epsilon\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta}+1\right)\right)}{2\sqrt{\beta_1}Q}\right) \\ &-\left(\frac{P}{2Q}\right),\ \beta_1 > 0,\ \beta_3 > 0,\ \beta_2 = (4\beta_1\beta_3)^{1/2}. \end{aligned} \tag{16}$$

CASE III:

$$U_3 = \left(\frac{P\left(-\left(\beta_1\left(\frac{\sqrt{\beta_1}\epsilon\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}} - \frac{\sqrt{\beta_1}\epsilon\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)\left(\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+P\right)}{\left(\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}\right)^2}\right)\right)\right)}{\frac{\left(2\sqrt{\beta_1}Q\right)\left(\beta_2\left(-\beta_1\left(\frac{\epsilon\left(\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+P\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}}+1\right)\right)\right)}{\beta_2}} \right) - \left(\frac{P}{2Q}\right) + \left(\frac{\frac{\sqrt{\beta_3}P}{2\sqrt{\beta_1}Q}\left(-\beta_1\left(\frac{\epsilon\left(\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+P\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}}+1\right)\right)}{\beta_2} \right), \quad \beta_1 > 0. \tag{17}$$

Some solutions are plotted graphically in 2-dimensional and 3-dimensional by imparting particular value to the parameters under the constrain condition on each disquiet solution (Figures 1–11).

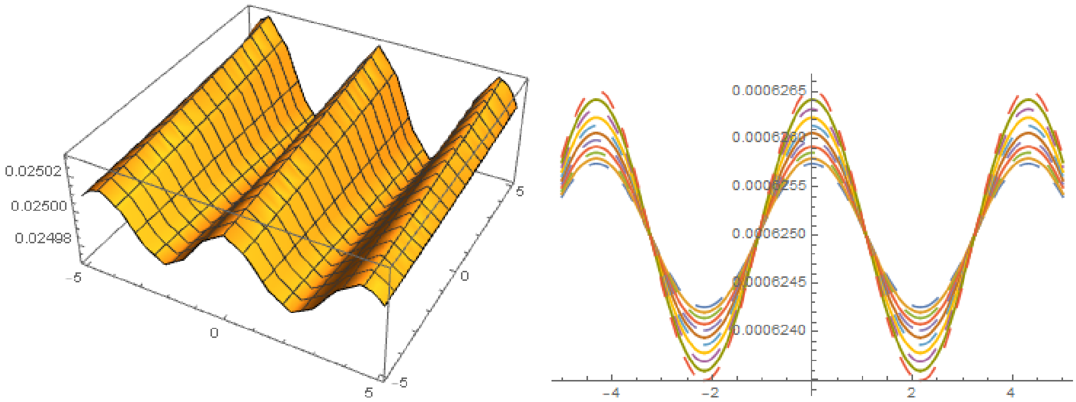

Figure 1. The profile of solution U_1 with $\beta_1 = 2,\ \beta_2 = -4,\ \beta_3 = 2,\ \beta = -0.5,\ \gamma = 4.5,\ \xi_0 = -4,\ P = 0.05,\ Q = 2,\ R = 0.1,\ S = 2.5,\ \omega = 0.05,\ y = 1,\ z = 1,\ \epsilon = -1$.

3.2. *Application of Extended Simple Equation Method*

Let solution of (12) is

$$U = A_1\Psi + \frac{A_{-1}}{\Psi} + A_0. \tag{18}$$

Put (18) with (7) in (12), we derived the coefficients as following

CASE 1: $c_3 = 0$,

FAMILY-I

$$A_{-1} = 0,\ \ A_1 = -\frac{c_2 P}{\sqrt{c_1^2Q^2 - 4c_0c_2Q^2}},\ \ A_0 = \frac{-\frac{c_1PQ}{\sqrt{(c_1^2-4c_0c_2)Q^2}} - P}{2Q},\ \ \omega = -\frac{\gamma P^2}{6Q},$$
$$\alpha = -\frac{\sqrt{-6\gamma^2c_1^2QR + 24\gamma^2c_0c_2QR - 6\beta^2c_1^2QS + 24\beta^2c_0c_2QS - P^2}}{\sqrt{6c_1^2QS - 24c_0c_2QS}}. \tag{19}$$

Put (19) in (18),

$$U_4 = \left(\frac{(c_2P)\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)\right)}{(2c_2)\sqrt{c_1^2Q^2 - 4c_0c_2Q^2}} \right) + \left(\frac{P\left(-\frac{c_1Q}{\sqrt{(c_1^2 - 4c_0c_2)Q^2}} - 1\right)}{2Q} \right),$$
$$4c_0c_2 > c_1^2. \tag{20}$$

FAMILY-II

$$A_{-1} = -\frac{c_0P}{\sqrt{c_1^2Q^2 - 4c_0c_2Q^2}},\ A_1 = 0,\ A_0 = \frac{-\frac{c_1PQ}{\sqrt{(c_1^2 - 4c_0c_2)Q^2}} - P}{2Q},\ \omega = -\frac{\gamma P^2}{6Q},$$
$$\alpha = -\frac{\sqrt{-6\gamma^2c_1^2QR + 24\gamma^2c_0c_2QR - 6\beta^2c_1^2QS + 24\beta^2c_0c_2QS - P^2}}{\sqrt{6c_1^2QS - 24c_0c_2QS}}. \tag{21}$$

Substitute (21) in (18),

$$U_5 = \left(\frac{-\frac{c_1PQ}{\sqrt{(c_1^2 - 4c_0c_2)Q^2}} - P}{2Q} \right) - \left(\frac{c_0P}{\frac{\sqrt{c_1^2Q^2 - 4c_0c_2Q^2}\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \xi_0)\right)\right)}{2c_2}} \right),$$
$$4c_0c_2 > c_1^2. \tag{22}$$

CASE 2: $c_0 = 0,\ c_3 = 0,$

$$A_{-1} = 0,\ A_1 = -\frac{c_2P}{c_1Q},\ A_0 = -\frac{P}{Q},\ \omega = -\frac{\gamma P^2}{6Q},\ \alpha = \frac{\sqrt{-6\gamma^2c_1^2QR - 6\beta^2c_1^2QS - P^2}}{\sqrt{6}c_1\sqrt{Q}\sqrt{S}}. \tag{23}$$

Put (23) in (18),

$$U_6 = -\left(\frac{(c_2P)(c_1\exp(c_1(\xi + \xi_0)))}{(c_1Q)(1 - c_2\exp(c_1(\xi + \xi_0)))} \right) - \left(\frac{P}{Q} \right),\ c_1 > 0, \tag{24}$$

$$U_7 = \left(\frac{(c_2P)(c_1\exp(c_1(\xi + \xi_0)))}{(c_1Q)(c_2\exp(c_1(\xi + \xi_0)) + 1)} \right) - \left(\frac{P}{Q} \right),\ c_1 < 0. \tag{25}$$

CASE 3: $c_1 = 0,\ c_3 = 0,$

FAMILY-I

$$A_{-1} = 0,\ A_1 = \frac{i\sqrt{c_2}P}{2\sqrt{c_0}Q},\ A_0 = -\frac{P}{2Q},\ \alpha = \frac{\sqrt{-24\gamma^2c_0c_2QR - 24\beta^2c_0c_2QS + P^2}}{2\sqrt{6}\sqrt{c_0}\sqrt{c_2}\sqrt{Q}\sqrt{S}},\ \omega = -\frac{\gamma P^2}{6Q}. \tag{26}$$

Put (26) in (18),

$$U_8 = -\left(\frac{P}{2Q} \right) + \left(\frac{(i\sqrt{c_2}P)(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi + \xi_0)))}{c_2(2\sqrt{c_0}Q)} \right),\ c_0c_2 > 0, \tag{27}$$

$$U_9 = -\left(\frac{P}{2Q}\right) + \left(\frac{(i\sqrt{c_2}P)(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0)))}{c_2(2\sqrt{c_0}Q)}\right),\ c_0c_2 < 0. \tag{28}$$

FAMILY-II

$$A_{-1} = \frac{i\sqrt{c_0}P}{2\sqrt{c_2}Q},\ A_1 = 0,\ A_0 = -\frac{P}{2Q},\ \alpha = -\frac{\sqrt{-24\gamma^2c_0c_2QR - 24\beta^2c_0c_2QS + P^2}}{2\sqrt{6}\sqrt{c_0}\sqrt{c_2}\sqrt{Q}\sqrt{S}},\ \omega = -\frac{\gamma P^2}{6Q}. \tag{29}$$

Put (29) in (18),

$$U_{10} = -\left(\frac{P}{2Q}\right) + \left(\frac{i\sqrt{c_0}P}{\frac{(2\sqrt{c_2}Q)\left(\sqrt{c_0c_2}\tan\left(\sqrt{c_0c_2}(\xi+\xi_0)\right)\right)}{c_2}}\right),\ c_0c_2 > 0, \tag{30}$$

$$U_{11} = -\left(\frac{P}{2Q}\right) + \left(\frac{i\sqrt{c_0}P}{\frac{(2\sqrt{c_2}Q)\left((i\sqrt{c_2}P)(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0)))\right)}{c_2(2\sqrt{c_0}Q)}})\right),\ c_0c_2 < 0. \tag{31}$$

FAMILY-III

$$\begin{aligned} &A_{-1} = \frac{\sqrt{c_0}P}{2\sqrt{2}\sqrt{c_2}Q},\ A_1 = \frac{\sqrt{c_2}P}{2\sqrt{2}\sqrt{c_0}Q},\ A_0 = -\frac{P}{2Q},\\ &\alpha = \frac{\sqrt{-48\gamma^2c_0c_2QR - 48\beta^2c_0c_2QS - P^2}}{4\sqrt{3}\sqrt{c_0}\sqrt{c_2}\sqrt{Q}\sqrt{S}},\ \omega = -\frac{\gamma P^2}{6Q}. \end{aligned} \tag{32}$$

Put (32) in (18),

$$\begin{aligned} U_{12} = &\left(\frac{(\sqrt{c_2}P)(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi+\xi_0)))}{c_2\left(2\sqrt{2}\sqrt{c_0}Q\right)}\right) + \left(\frac{\sqrt{c_0}P}{\frac{(2\sqrt{2}\sqrt{c_2}Q)\left(\sqrt{c_0c_2}\tan\left(\sqrt{c_0c_2}(\xi+\xi_0)\right)\right)}{c_2}}\right) - \left(\frac{P}{2Q}\right),\\ &c_0c_2 > 0, \end{aligned} \tag{33}$$

$$\begin{aligned} U_{13} = &\left(\frac{(\sqrt{c_2}P)(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0)))}{c_2\left(2\sqrt{2}\sqrt{c_0}Q\right)}\right) + \left(\frac{\sqrt{c_0}P}{\frac{(2\sqrt{2}\sqrt{c_2}Q)(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0)))}{c_2}}\right) - \left(\frac{P}{2Q}\right),\\ &c_0c_2 < 0. \end{aligned} \tag{34}$$

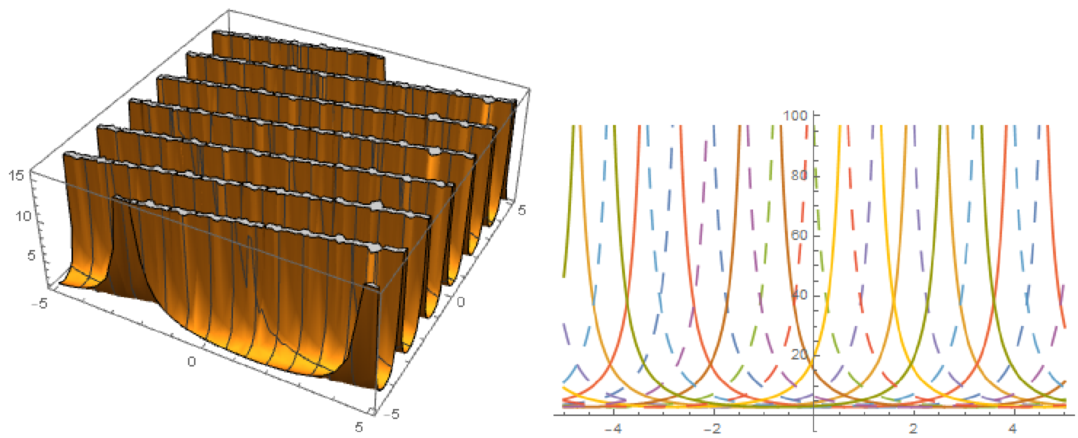

Figure 2. Solution U_5 with $\beta = 0.01$, $c_0 = 1$, $c_2 = 1$, $c_1 = 0.01$, $\gamma = -0.05$, $\xi_0 = -3.5$, $P = -4.5$, $Q = -1.3$, $R = -2.1$, $S = -3.5$, $\omega = -2$, $y = 1$, $z = 1$.

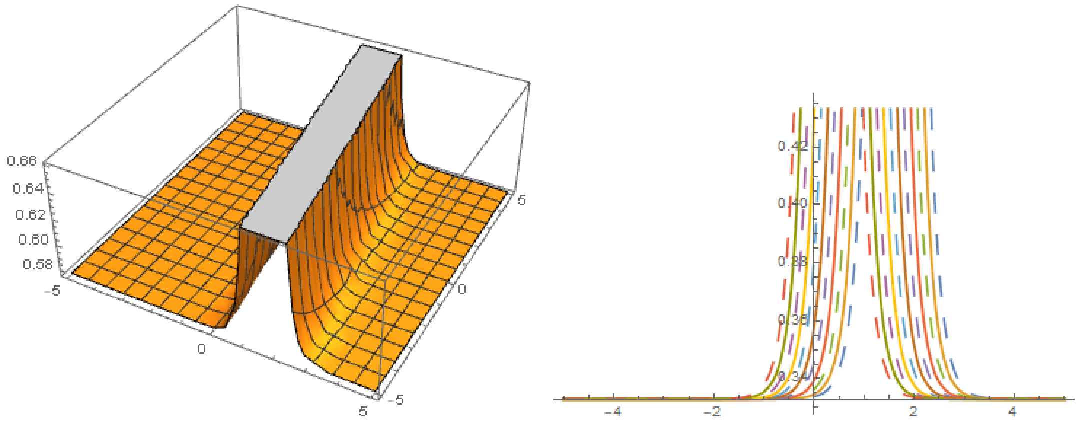

Figure 3. Solution U_{13} with $\beta = 0.3,\ c_0 = 1,\ c_2 = -0.4,\ \gamma = 0.9,\ \xi_0 = 0.05,\ P = 1.5,\ Q = -1.3,\ R = -1.5,\ S = 0.3,\ y = 1,\ z = 1.$

3.3. Application of Modified F-Expansion Method

Let (12) has solution,

$$U = a_0 + a_1 F + \frac{b_1}{F} \tag{35}$$

Put (35) with (9) in (12).

A = 0, B = 1, C = −1,

$$a_1 = \frac{P}{Q},\ a_0 = -\frac{P}{Q},\ b_1 = 0,\ \omega = -\frac{\gamma P^2}{6Q},\ \alpha = \frac{\sqrt{-P^2 - 6\gamma^2 QR - 6\beta^2 QS}}{\sqrt{6}\sqrt{Q}\sqrt{S}}. \tag{36}$$

Put (36) in (35),

$$U_{14} = \left(\frac{P\left(\frac{1}{2}\tanh\left(\frac{\xi}{2}\right) + \frac{1}{2}\right)}{Q}\right) - \left(\frac{P}{Q}\right). \tag{37}$$

A = 0, C = 1, B = −1,

$$a_1 = -\frac{P}{Q},\ a_0 = 0,\ b_1 = 0,\ \omega = -\frac{\gamma P^2}{6Q},\ \alpha = -\frac{\sqrt{-P^2 - 6\gamma^2 QR - 6\beta^2 QS}}{\sqrt{6}\sqrt{Q}\sqrt{S}}. \tag{38}$$

Put (38) into (35),

$$U_{15} = -\left(\frac{P\left(\frac{1}{2} - \frac{1}{2}\coth\left(\frac{\xi}{2}\right)\right)}{Q}\right). \tag{39}$$

A = 1/2, B = 0, C = −1/2

FAMILY-I

$$a_0 = a_1 = -\frac{P}{2Q} = -\frac{P}{2Q},\ b_1 = 0,\ \omega = -\frac{\gamma P^2}{6Q},\ \alpha = -\frac{\sqrt{-P^2 - 6\gamma^2 QR - 6\beta^2 QS}}{\sqrt{6}\sqrt{Q}\sqrt{S}}. \tag{40}$$

Substitute (40) into (35),

$$U_{16,1} = -\left(\frac{P(\cot(\xi) + \csc(\xi))}{2Q}\right) - \left(\frac{P}{2Q}\right). \tag{41}$$

FAMILY-II

$$a_1 = 0,\ a_0 = -\frac{P}{2Q},\ b_1 = -\frac{P}{2Q},\ \omega = -\frac{\gamma P^2}{6Q},\ \alpha = \frac{\sqrt{-P^2 - 6\gamma^2 QR - 6\beta^2 QS}}{\sqrt{6}\sqrt{Q}\sqrt{S}}. \tag{42}$$

Put (42) in (35),

$$U_{16,2} = -\left(\frac{P}{(2Q)(\cot(\xi) + \csc(\xi))}\right) - \left(\frac{P}{2Q}\right). \tag{43}$$

FAMILY-III

$$a_1 = -\frac{P}{4Q},\ a_0 = -\frac{P}{2Q},\ b_1 = -\frac{P}{4Q},\ \omega = -\frac{\gamma P^2}{6Q},\ \alpha = -\frac{\sqrt{-P^2 - 24\gamma^2 QR - 24\beta^2 QS}}{2\sqrt{6}\sqrt{Q}\sqrt{S}} \tag{44}$$

Put (44) in (35),

$$U_{16,3} = -\left(\frac{P(\cot(\xi) + \csc(\xi))}{4Q}\right) - \left(\frac{P}{(4Q)(\cot(\xi) + \csc(\xi))}\right) - \left(\frac{P}{2Q}\right). \tag{45}$$

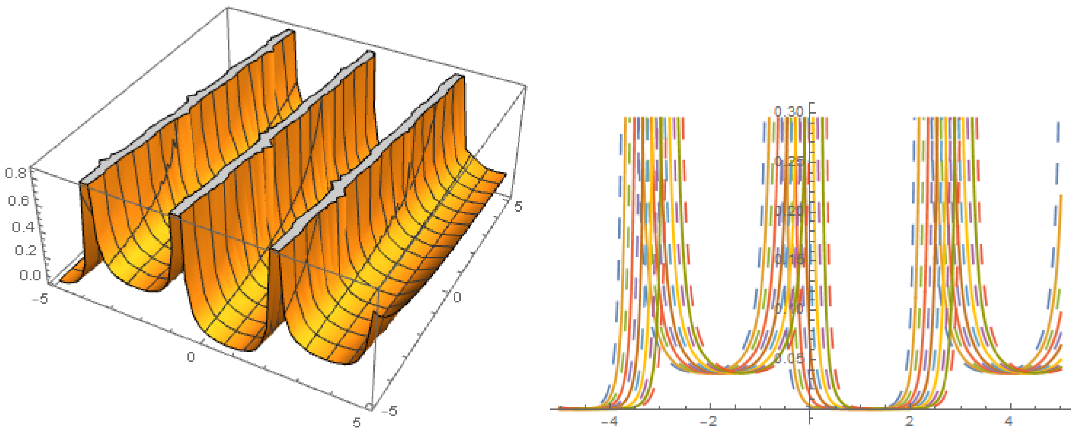

Figure 4. The profile of solution $U_{16,3}$ with $\beta = 1$, $\gamma = 3$, $P = -1.01$, $Q = 5.3$, $R = 0.5$, $S = -2.1$, $y = 1$, $z = 1$.

A = 1, B = 0, C = −1

FAMILY-I

$$a_1 = \frac{P}{2Q},\ \ a_0 = -\frac{P}{2Q},\ b_1 = 0,\ \omega = -\frac{\gamma P^2}{6Q},\ \alpha = \frac{\sqrt{-P^2 - 24\gamma^2 QR - 24\beta^2 QS}}{2\sqrt{6}\sqrt{Q}\sqrt{S}}. \tag{46}$$

Put (46) in (35),

$$U_{17,1} = \left(\frac{P\tanh(\xi)}{2Q}\right) - \left(\frac{P}{2Q}\right). \tag{47}$$

FAMILY-II

$$a_1 = 0,\ a_0 = -\frac{P}{2Q},\ b_1 = \frac{P}{2Q},\ \omega = -\frac{\gamma P^2}{6Q},\ \alpha = \frac{\sqrt{-P^2 - 24\gamma^2 QR - 24\beta^2 QS}}{2\sqrt{6}\sqrt{Q}\sqrt{S}}. \tag{48}$$

Put (48) in (35),

$$U_{17,2} = \left(\frac{P}{(2Q)\tanh(\xi)}\right) - \left(\frac{P}{2Q}\right). \tag{49}$$

FAMILY-III

$$a_1 = \frac{P}{4Q},\ a_0 = -\frac{P}{2Q},\ b_1 = \frac{P}{4Q},\ \omega = -\frac{\gamma P^2}{6Q},\ \alpha = \frac{\sqrt{-P^2 - 96\gamma^2 QR - 96\beta^2 QS}}{4\sqrt{6}\sqrt{Q}\sqrt{S}}. \tag{50}$$

Put (50) in (35),

$$U_{17,3} = \left(\frac{P\tanh(\xi)}{4Q} + \frac{P}{(4Q)\tanh(\xi)}\right) - \left(\frac{P}{2Q}\right). \tag{51}$$

A = C = 1/2, B = 0,

FAMILY-I

$$a_1 = -\frac{iP}{2Q},\ a_0 = -\frac{P}{2Q},\ b_1 = 0,\ \omega = -\frac{\gamma P^2}{6Q},\ \alpha = -\frac{\sqrt{P^2 - 6\gamma^2 QR - 6\beta^2 QS}}{\sqrt{6}\sqrt{Q}\sqrt{S}}. \tag{52}$$

Put (52) in (35),

$$U_{18,1} = -\left(\frac{P}{2Q}\right) - \left(\frac{(iP)(\tan(\xi) + \sec(\xi))}{2Q}\right). \tag{53}$$

FAMILY-II

$$a_1 = 0,\ a_0 = -\frac{P}{2Q},\ b_1 = \frac{iP}{2Q},\ \omega = -\frac{\gamma P^2}{6Q},\ \alpha = -\frac{\sqrt{P^2 - 6\gamma^2 QR - 6\beta^2 QS}}{\sqrt{6}\sqrt{Q}\sqrt{S}}. \tag{54}$$

Put (54) in (35),

$$U_{18,2} = -\left(\frac{P}{2Q}\right) + \left(\frac{iP}{(2Q)(\tan(\xi) + \sec(\xi))}\right). \tag{55}$$

FAMILY-III

$$a_1 = \frac{P}{2\sqrt{2}Q},\ a_0 = -\frac{P}{2Q},\ b_1 = \frac{P}{2\sqrt{2}Q},\ \omega = -\frac{\gamma P^2}{6Q},\ \alpha = \frac{\sqrt{-P^2 - 12\gamma^2 QR - 12\beta^2 QS}}{2\sqrt{3}\sqrt{Q}\sqrt{S}}. \tag{56}$$

Put (56) in (35),

$$U_{18,3} = \left(\frac{P(\tan(\xi) + \sec(\xi))}{2\sqrt{2}Q}\right) + \left(\frac{P}{\left(2\sqrt{2}Q\right)(\tan(\xi) + \sec(\xi))}\right) - \left(\frac{P}{2Q}\right). \tag{57}$$

A = C = −1/2, B = 0,

FAMILY-I

$$a_1 = -\frac{iP}{2Q},\ a_0 = -\frac{P}{2Q},\ b_1 = 0,\ \omega = -\frac{\gamma P^2}{6Q},\ \alpha = \frac{\sqrt{P^2 - 6\gamma^2 QR - 6\beta^2 QS}}{\sqrt{6}\sqrt{Q}\sqrt{S}}. \tag{58}$$

Put (58) in (35),

$$U_{19,1} = -\left(\frac{P}{2Q}\right) - \left(\frac{(iP)(\sec(\xi) - \tan(\xi))}{2Q}\right). \tag{59}$$

FAMILY-II

$$a_1 = 0,\ a_0 = -\frac{P}{2Q},\ b_1 = -\frac{iP}{2Q},\ \omega = -\frac{\gamma P^2}{6Q},\ \alpha = \frac{\sqrt{P^2 - 6\gamma^2 QR - 6\beta^2 QS}}{\sqrt{6}\sqrt{Q}\sqrt{S}}. \tag{60}$$

Put (60) in (35),

$$U_{19,2} = -\left(\frac{P}{2Q}\right) - \left(\frac{iP}{(2Q)(\sec(\xi) - \tan(\xi))}\right). \tag{61}$$

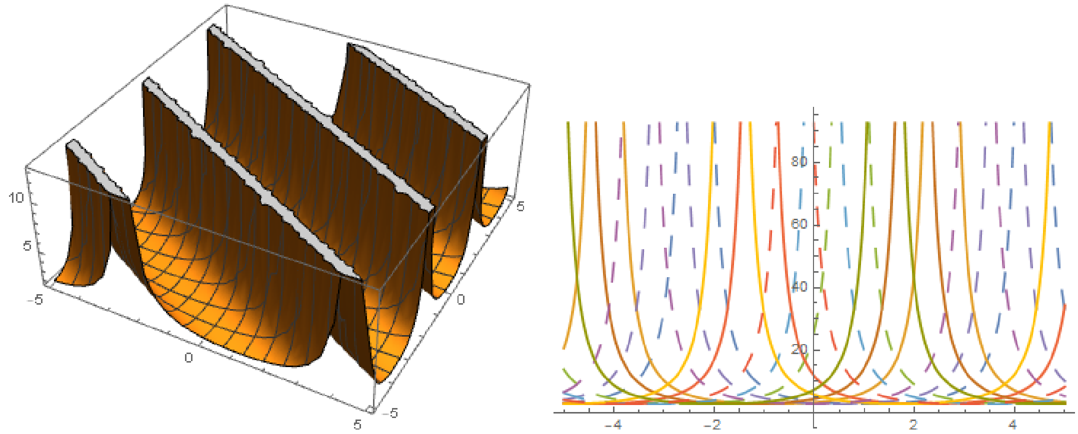

Figure 5. The profile of solution $U_{19,2}$ with $\beta = 1$, $\gamma = 3$, $P = -1.01$, $Q = 0.3$, $R = 0.5$, $S = -2.1$, $y = 1$, $z = 1$.

FAMILY-III

$$a_1 = -\frac{P}{2\sqrt{2}Q},\ a_0 = -\frac{P}{2Q},\ b_1 = -\frac{P}{2\sqrt{2}Q}, \omega = -\frac{\gamma P^2}{6Q},\ \alpha = \frac{\sqrt{-P^2 - 12\gamma^2 QR - 12\beta^2 QS}}{2\sqrt{3}\sqrt{Q}\sqrt{S}}. \tag{62}$$

Put (62) in (35),

$$U_{19,3} = -\left(\frac{P(\sec(\xi) - \tan(\xi))}{2\sqrt{2}Q}\right) - \left(\frac{P}{\left(2\sqrt{2}Q\right)(\sec(\xi) - \tan(\xi))}\right) - \left(\frac{P}{2Q}\right). \tag{63}$$

A = C = −1, B = 0,

FAMILY-I

$$a_1 = -\frac{iP}{2Q},\ a_0 = -\frac{P}{2Q},\ b_1 = 0,\ \alpha = -\frac{\sqrt{P^2 - 24\gamma^2 QR - 24\beta^2 QS}}{2\sqrt{6}\sqrt{Q}\sqrt{S}},\ \omega = -\frac{\gamma P^2}{6Q}. \tag{64}$$

Put (64) in (35),

$$U_{20,1} = -\left(\frac{P}{2Q}\right) - \left(\frac{(iP)\tan(\xi)}{2Q}\right). \tag{65}$$

FAMILY-II

$$a_1 = 0,\ a_0 = -\frac{P}{2Q},\ b_1 = -\frac{iP}{2Q},\ \alpha = \frac{\sqrt{P^2 - 24\gamma^2 QR - 24\beta^2 QS}}{2\sqrt{6}\sqrt{Q}\sqrt{S}},\ \omega = -\frac{\gamma P^2}{6Q}. \tag{66}$$

Put (66) in (35),

$$U_{20,2} = -\left(\frac{P}{2Q}\right) - \left(\frac{iP}{(2Q)\tan(\xi)}\right). \tag{67}$$

FAMILY-III

$$a_1 = \frac{P}{2\sqrt{2}Q}, \; a_0 = -\frac{P}{2Q}, \; b_1 = \frac{P}{2\sqrt{2}Q}, \; \alpha = \frac{\sqrt{-P^2 - 48\gamma^2 QR - 48\beta^2 QS}}{4\sqrt{3}\sqrt{Q}\sqrt{S}}, \; \omega = -\frac{\gamma P^2}{6Q}. \tag{68}$$

Put (68) in (35),

$$U_{20,3} = \left(\frac{P\tan(\xi)}{2\sqrt{2}Q}\right) + \left(\frac{P}{\left(2\sqrt{2}Q\right)\tan(\xi)}\right) - \left(\frac{P}{2Q}\right). \tag{69}$$

C = 0

$$a_1 = 0, \; a_0 = 0, \; b_1 = \frac{AP}{BQ}, \; \alpha = -\frac{\sqrt{-6B^2\gamma^2 QR - 6\beta^2 B^2 QS - P^2}}{\sqrt{6}B\sqrt{Q}\sqrt{S}}, \; \omega = -\frac{\gamma P^2}{6Q}. \tag{70}$$

Put (70) in (35),

$$U_{21} = \left(\frac{B(AP)}{(BQ)(\exp(B\xi) - A)}\right). \tag{71}$$

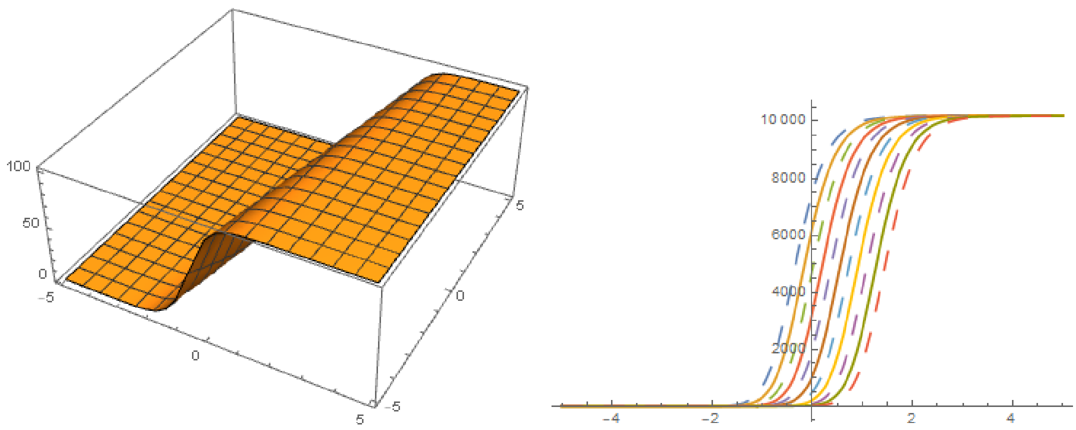

Figure 6. Solution U_{21} with $A = -0.5$, $\beta = 1$, $B = 0.01$, $\gamma = 3$, $P = -1.01$, $Q = 0.01$, $R = 5.5$, $S = -2.1$, $y = 1$, $z = 1$.

4. (3 + 1)-Dimensional Nonlinear Modified Quantum Zakharov–Kuznetsov (NLmQZK) Equation

Let NLmQZK equation [40,43].

$$16\left(\frac{\partial U}{\partial t} - \mu\frac{\partial U}{\partial x}\right) + 30\sqrt{U}\frac{\partial U}{\partial x} + \frac{\partial^3 U}{\partial x^3} + \frac{\partial}{\partial x}\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)U = 0. \tag{72}$$

The above model is an adequate NLEE which is used to point out the behavior of the electrons associated with the temperature on the latter [40].

Let wave transformations,

$$U(x,y,z,t) = U(\xi), \quad \xi = k_1 x + k_2 y + k_3 z - \omega t. \tag{73}$$

Put (73) in (72), after integrating twice with omitting the integral constant, we have

$$30k_1 U^{3/2} + k_1\left(k_1^2 + k_2^2 + k_3^2\right)U'' - 16U(\mu k_1 + \omega) = 0. \tag{74}$$

Let

$$V = \sqrt{U}. \tag{75}$$

Put (75) in (74),

$$20k_1V^3 - 16V^2(k_1\mu + \omega) + 2k_1\left(k_1^2 + k_2^2 + k_3^2\right)\left(\left(V'\right)^2 + VV''\right) = 0. \tag{76}$$

4.1. Application of Modified Extended Auxiliary Equation Mapping Method

Let solution of (76) is,

$$V = A_2\Psi^2 + A_1\Psi + A_0 + \frac{B_1}{\Psi} + \frac{B_2}{\Psi^2} + C_2\Psi' + D_2\left(\frac{\Psi'}{\Psi}\right)^2 + \frac{D_1\Psi'}{\Psi}. \tag{77}$$

Put (77) with (5) in (76),

$$\begin{aligned}
&A_0 = \beta_1(-D_2),\ A_1 = -\frac{1}{4}\beta_2\left(4D_2 + k_1^2 + k_2^2 + k_3^2\right),\ A_2 = -\frac{1}{2}\beta_3\left(2D_2 + k_1^2 + k_2^2 + k_3^2\right),\\
&C_2 = \frac{1}{2}\sqrt{\beta_3}\left(k_1^2 + k_2^2 + k_3^2\right),\ D_1 = 0, B_1 = 0, B_2 = 0,\ \omega = \frac{1}{4}\left(\beta_1k_1^3 + \beta_1k_2^2k_1 + \beta_1k_3^2k_1 - 4k_1\mu\right).
\end{aligned} \tag{78}$$

Subsitute (78) in (77),

CASE I:

$$\begin{aligned}
V_1 =& \beta_1(-D_2) - \left(\frac{\beta_2\left(4D_2 + k_1^2 + k_2^2 + k_3^2\right)\left(-\beta_1\left(\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi + \xi_0)\right) + 1\right)\right)}{4\beta_2}\right)\\
&- \frac{1}{2}\beta_3\left(2D_2 + k_1^2 + k_2^2 + k_3^2\right)\left(-\frac{\beta_1\left(\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi + \xi_0)\right) + 1\right)}{\beta_2}\right)^2 + \frac{1}{2}\sqrt{\beta_3}\left(k_1^2 + k_2^2 + k_3^2\right)\\
&\left(\frac{\beta_1^{3/2}\epsilon\operatorname{csch}^2\left(\frac{1}{2}\sqrt{\beta_1}(\xi + \xi_0)\right)}{2\beta_2}\right) + D_2\left(\frac{\beta_1^{3/2}\epsilon\operatorname{csch}^2\left(\frac{1}{2}\sqrt{\beta_1}(\xi + \xi_0)\right)}{\frac{(2\beta_2)\left(-\beta_1\left(\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)+1\right)\right)}{\beta_2}}\right)^2,\ \beta_1 > 0,\ \beta_2^2 - 4\beta_1\beta_3 = 0.
\end{aligned} \tag{79}$$

From (75), the solution of (79) can be written as,

$$U_{22} = (V_1)^2,\ \beta_1 > 0,\ \beta_2^2 - 4\beta_1\beta_3 = 0. \tag{80}$$

CASE II:

$$\begin{aligned}
V_2 =& -\beta_1D_2 - \frac{1}{4}\beta_2\left(4D_2 + k_1^2 + k_2^2 + k_3^2\right)\left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\epsilon\sinh\left(\sqrt{\beta_1}(\xi + \xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi + \xi_0)\right) + \eta} + 1\right)\right)\\
&- \frac{1}{2}\beta_3\left(2D_2 + k_1^2 + k_2^2 + k_3^2\right)\left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\epsilon\sinh\left(\sqrt{\beta_1}(\xi + \xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi + \xi_0)\right) + \eta} + 1\right)\right)^2 + \frac{1}{2}\sqrt{\beta_3}\left(k_1^2 + k_2^2 + k_3^2\right)\\
&\left(-\frac{1}{2}\sqrt{\frac{\beta_1}{\beta_3}}\left(\frac{\sqrt{\beta_1}\epsilon\cosh\left(\sqrt{\beta_1}(\xi + \xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi + \xi_0)\right) + \eta} - \frac{\sqrt{\beta_1}\epsilon\sinh^2\left(\sqrt{\beta_1}(\xi + \xi_0)\right)}{\left(\cosh\left(\sqrt{\beta_1}(\xi + \xi_0)\right) + \eta\right)^2}\right)\right) +\\
&\left(D_2\left(-\frac{\sqrt{\frac{\beta_1}{\beta_3}}\left(\frac{\sqrt{\beta_1}\epsilon\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta} - \frac{\sqrt{\beta_1}\epsilon\sinh^2\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\left(\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\right)^2}\right)}{2\left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\epsilon\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta} + 1\right)\right)}\right)^2\right),\ \beta_1 > 0,\ \beta_3 > 0,\ \beta_2 = (4\beta_1\beta_3)^{1/2}.
\end{aligned} \tag{81}$$

$$U_{23} = (V_2)^2,\ \beta_1 > 0,\ \beta_3 > 0,\ \beta_2 = (4\beta_1\beta_3)^{1/2}. \tag{82}$$

CASE III:

$$
\begin{aligned}
V_3 = {} & \left(\beta_1(-D_2) - \frac{\beta_2\left(4D_2 + k_1^2 + k_2^2 + k_3^2\right)\left(-\beta_1\left(\frac{\epsilon\left(\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+P\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}} + 1\right)\right)}{4\beta_2} \right) \\
& - \frac{1}{2}\beta_3\left(2D_2 + k_1^2 + k_2^2 + k_3^2\right)\left(- \frac{\beta_1\left(\frac{\epsilon\left(\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+P\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}} + 1\right)}{\beta_2} \right)^2 + \frac{1}{2}\sqrt{\beta_3}\left(k_1^2 + k_2^2 + k_3^2\right) \\
& \left(- \frac{\beta_1\left(\frac{\sqrt{\beta_1}\epsilon\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}} - \frac{\sqrt{\beta_1}\epsilon\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)\left(\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+P\right)}{\left(\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}\right)^2} \right)}{\beta_2} \right) \\
& + D_2\left(- \frac{\beta_1\left(\frac{\sqrt{\beta_1}\epsilon\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}} - \frac{\sqrt{\beta_1}\epsilon\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)\left(\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+P\right)}{\left(\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}\right)^2} \right)}{\frac{\beta_2\left(-\beta_1\left(\frac{\epsilon\left(\sinh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+P\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\xi_0)\right)+\eta\sqrt{P^2+1}}+1\right)\right)}{\beta_2}} \right)^2, \ \beta_1 > 0.
\end{aligned} \tag{83}
$$

$$
U_{24} = (V_3)^2, \ \beta_1 > 0, \ \beta_3 > 0, \ \beta_2 = (4\beta_1\beta_3)^{1/2}. \tag{84}
$$

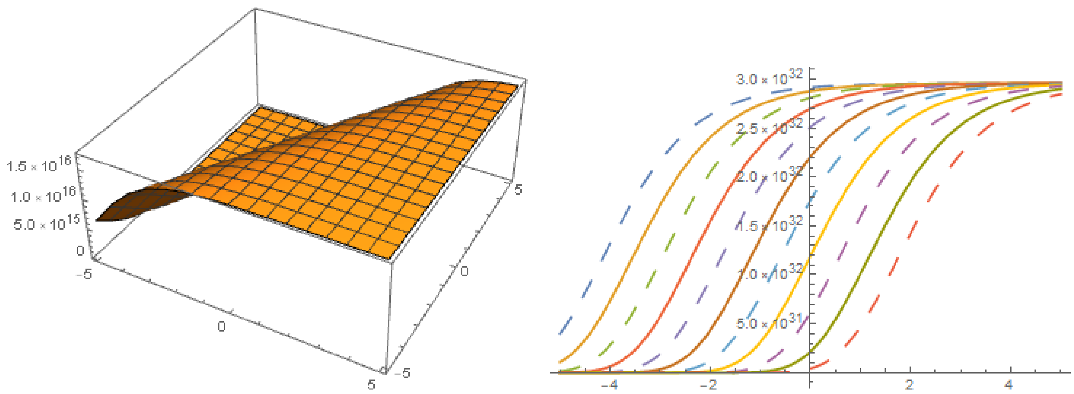

Figure 7. Solution U_{24} with $\beta_1 = 2$, $\beta_2 = 0.001$, $\beta_3 = 2$, $D_2 = 3.01$, $\eta = 1$, $\xi_0 = 0.4$, $k_1 = 0.7$, $k_2 = -0.0001$, $k_3 = 1.3$, $\mu = 0.5$, $P = 1$, $y = 1$, $z = 1$, $\epsilon = 1$.

4.2. Application of Extended Simple Equation Method

Let solution of (76) is

$$
V = A_2\Psi^2 + A_1\Psi + \frac{A_{-2}}{\Psi^2} + \frac{A_{-1}}{\Psi} + A_0. \tag{85}
$$

Put (85) with (7) in (76),

CASE 1: $c_3 = 0$,

FAMILY-I

$$
\begin{aligned}
& A_0 = -c_0c_2\left(k_1^2 + k_2^2 + k_3^2\right), \ A_{-2} = 0, \ A_{-1} = 0, \ A_2 = -c_2^2\left(k_1^2 + k_2^2 + k_3^2\right), \\
& A_1 = -c_1c_2\left(k_1^2 + k_2^2 + k_3^2\right), \ \omega = \frac{1}{4}k_1\left(\left(c_1^2 - 4c_0c_2\right)\left(k_1^2 + k_2^2 + k_3^2\right) - 4\mu\right).
\end{aligned} \tag{86}
$$

Put (86) in (85),

$$V_4 = -c_0c_2\left(k_1^2+k_2^2+k_3^2\right) + \left(c_1c_2\left(k_1^2+k_2^2+k_3^2\right)\right)\left(\frac{c_1-\sqrt{4c_2c_0-c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0-c_1^2}(\xi+\xi_0)\right)}{2c_2}\right)$$
$$\left(c_2^2\left(k_1^2+k_2^2+k_3^2\right)\right)\left(\left(\frac{c_1-\sqrt{4c_2c_0-c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0-c_1^2}(\xi+\xi_0)\right)}{2c_2}\right)^2\right),\ 4c_0c_2 > c_1^2. \tag{87}$$

$$U_{25} = (V_4)^2,\ 4c_0c_2 > c_1^2. \tag{88}$$

FAMILY-II

$$A_0 = -c_0c_2\left(k_1^2+k_2^2+k_3^2\right),\ A_{-2} = -c_0^2\left(k_1^2+k_2^2+k_3^2\right),\ A_{-1} = -c_0c_1\left(k_1^2+k_2^2+k_3^2\right),$$
$$A_2 = 0,\ A_1 = 0,\ \omega = \frac{1}{4}k_1\left(\left(c_1^2-4c_0c_2\right)\left(k_1^2+k_2^2+k_3^2\right)-4\mu\right). \tag{89}$$

Substitute (89) in (85),

$$V_5 = \left(\frac{4c_2^2}{\left(c_1-\sqrt{4c_2c_0-c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0-c_1^2}(\xi+\xi_0)\right)\right)^2}\right)\left(-c_0^2\left(k_1^2+k_2^2+k_3^2\right)\right)-$$
$$\left(-c_0c_1\left(k_1^2+k_2^2+k_3^2\right)\right)\left(\frac{2c_2}{c_1-\sqrt{4c_2c_0-c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0-c_1^2}(\xi+\xi_0)\right)}\right)-\left(c_0c_2\left(k_1^2+k_2^2+k_3^2\right)\right),$$
$$4c_0c_2 > c_1^2. \tag{90}$$

$$U_{26} = (V_5)^2,\ 4c_0c_2 > c_1^2. \tag{91}$$

CASE 2: $c_0 = 0,\ \ c_3 = 0,$

$$A_0 = 0,\ A_{-2} = 0,\ A_{-1} = 0,\ A_2 = -c_2^2\left(k_1^2+k_2^2+k_3^2\right),$$
$$A_1 = -c_1c_2\left(k_1^2+k_2^2+k_3^2\right),\ \omega = \frac{1}{4}\left(c_1^2k_1^3+c_1^2k_2^2k_1+c_1^2k_3^2k_1-4k_1\mu\right). \tag{92}$$

Put (92) in (85),

$$V_6 = \left(-\frac{\left(c_1c_2(k_1^2+k_2^2+k_3^2)\right)\left(c_1\exp(c_1(\xi+\xi_0))\right)}{1-c_2\exp(c_1(\xi+\xi_0))}\right)-\left(c_2^2\left(k_1^2+k_2^2+k_3^2\right)\right)$$
$$\left(\frac{c_1\exp(c_1(\xi+\xi_0))}{1-c_2\exp(c_1(\xi+\xi_0))}\right)^2,\ c_1 > 0, \tag{93}$$

$$U_{27} = (V_6)^2,\ c_1 > 0. \tag{94}$$

$$V_7 = \left(c_2^2\left(-\left(k_1^2+k_2^2+k_3^2\right)\right)\left(-\frac{c_1\exp(c_1(\xi+\xi_0))}{c_2\exp(c_1(\xi+\xi_0))+1}\right)^2\right)-\left(\frac{\left(-c_1c_2(k_1^2+k_2^2+k_3^2)\right)\left(c_1\exp(c_1(\xi+\xi_0))\right)}{c_2\exp(c_1(\xi+\xi_0))+1}\right),$$
$$c_1 < 0. \tag{95}$$

$$U_{28} = (V_6)^2,\ c_1 < 0. \tag{96}$$

CASE 3: $c_1 = 0,\ \ c_3 = 0,$

FAMILY-I

$$\begin{aligned} &A_0 = -c_0c_2\left(k_1^2 + k_2^2 + k_3^2\right),\ A_{-2} = 0,\ A_{-1} = 0,\ A_2 = -c_2^2\left(k_1^2 + k_2^2 + k_3^2\right), \\ &A_1 = 0,\ \omega = -k_1\left(c_0c_2k_1^2 + c_0c_2k_2^2 + c_0c_2k_3^2 + \mu\right). \end{aligned} \tag{97}$$

Put (97) in (85),

$$V_8 = -c_2^2\left(k_1^2 + k_2^2 + k_3^2\right)\left(\frac{\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi + \xi_0))}{c_2}\right)^2 - c_0c_2\left(k_1^2 + k_2^2 + k_3^2\right),\ \ c_0c_2 > 0, \tag{98}$$

$$U_{29} = (V_8)^2,\ c_0c_2 > 0. \tag{99}$$

$$V_9 = -c_2^2\left(k_1^2 + k_2^2 + k_3^2\right)\left(\frac{\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi + \xi_0))}{c_2}\right)^2 - c_0c_2\left(k_1^2 + k_2^2 + k_3^2\right),\ c_0c_2 < 0. \tag{100}$$

$$U_{30} = (V_9)^2,\ c_0c_2 < 0. \tag{101}$$

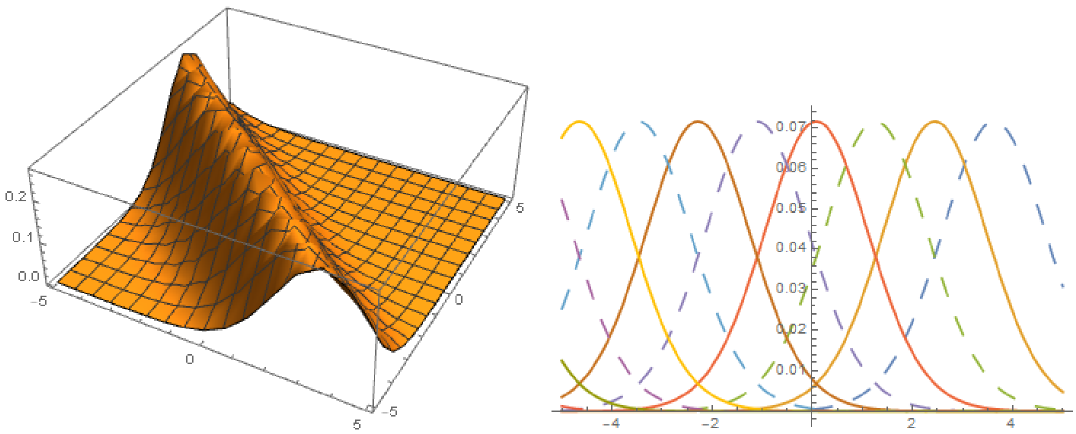

Figure 8. Solution U_{26} with $c_0 = 0.05,\ c_1 = 1.07,\ c_2 = 1,\ D_2 = 3.01,\ \xi_0 = 0.4,\ k_1 = 0.7, k_2 = -0.1,\ k_3 = 1.3,\ \mu = 1.7,\ y = 1,\ z = 1.$

FAMILY-II

$$\begin{aligned} &A_0 = -c_0c_2\left(k_1^2 + k_2^2 + k_3^2\right),\ A_{-2} = -c_0^2\left(k_1^2 + k_2^2 + k_3^2\right), \\ &A_{-1} = 0, A_2 = 0,\ A_1 = 0,\ \omega = -k_1\left(c_0c_2k_1^2 + c_0c_2k_2^2 + c_0c_2k_3^2 + \mu\right). \end{aligned} \tag{102}$$

Put (102) in (85),

$$V_{10} = \left(-\frac{c_0^2\left(k_1^2 + k_2^2 + k_3^2\right)}{\left(\frac{\sqrt{c_0c_2}\tan\left(\sqrt{c_0c_2}(\xi+\xi_0)\right)}{c_2}\right)^2}\right) - c_2c_0\left(k_1^2 + k_2^2 + k_3^2\right),\ c_0c_2 > 0, \tag{103}$$

$$U_{31} = (V_{10})^2,\ \ c_0c_2 > 0,. \tag{104}$$

$$V_{11} = \left(-\frac{c_0^2(k_1^2+k_2^2+k_3^2)}{\left(\frac{\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0))}{c_2}\right)^2} \right) - c_2c_0\left(k_1^2+k_2^2+k_3^2\right),\ c_0c_2 < 0. \tag{105}$$

$$U_{32} = (V_{11})^2,\ c_0c_2 < 0. \tag{106}$$

FAMILY-III

$$\begin{aligned} &A_0 = -2c_0c_2\left(k_1^2+k_2^2+k_3^2\right),\ A_{-2} = -c_0^2\left(k_1^2+k_2^2+k_3^2\right),\ A_{-1} = 0, \\ &A_2 = -c_2^2\left(k_1^2+k_2^2+k_3^2\right),\ A_1 = 0,\ \omega = -4c_0c_2k_1^3 - 4c_0c_2k_2^2k_1 - 4c_0c_2k_3^2k_1 - k_1\mu. \end{aligned} \tag{107}$$

Put (107) in (85),

$$\begin{aligned} V_{12} =& -c_2^2\left(k_1^2+k_2^2+k_3^2\right)\left(\frac{\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi+\xi_0))}{c_2}\right)^2 - \left(\frac{c_0^2(k_1^2+k_2^2+k_3^2)}{\left(\frac{\sqrt{c_0c_2}\tan\left(\sqrt{c_0c_2}(\xi+\xi_0)\right)}{c_2}\right)^2}\right) \\ & -2c_2c_0\left(k_1^2+k_2^2+k_3^2\right),\ c_0c_2 > 0, \end{aligned} \tag{108}$$

$$U_{33} = (V_{12})^2,\ c_0c_2 > 0, \tag{109}$$

$$\begin{aligned} V_{13} =& -c_2^2\left(k_1^2+k_2^2+k_3^2\right)\left(-\frac{\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0))}{c_2}\right)^2 - \left(\frac{c_0^2(k_1^2+k_2^2+k_3^2)}{\left(-\frac{\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0))}{c_2}\right)^2}\right) \\ & -2c_2c_0\left(k_1^2+k_2^2+k_3^2\right),\ c_0c_2 < 0. \end{aligned} \tag{110}$$

$$U_{34} = (V_{13})^2,\ c_0c_2 < 0., \tag{111}$$

4.3. Application of Modified F-Expansion Method

Let (76) has solution,

$$V = a_2F^2 + a_1F + a_0 + \frac{b_2}{F^2} + \frac{b_1}{F}. \tag{112}$$

Put (112) with (9) in (76).

A = 0, B = 1, C = −1,

$$\begin{aligned} &a_0 = 0,\ a_2 = -k_1^2 - k_2^2 - k_3^2,\ a_1 = k_1^2 + k_2^2 + k_3^2, \\ &b_1 = 0,\ b_2 = 0,\ \omega = \frac{1}{4}\left(-4k_1\mu + k_1^3 + k_2^2k_1 + k_3^2k_1\right). \end{aligned} \tag{113}$$

Put (113) in (112),

$$V_{14} = k_3^2\left(\frac{1}{2}\tanh\left(\frac{\xi}{2}\right) + \frac{1}{2}\right) + k_1^2 + k_2^2 + (-k_1^2 - k_2^2 - k_3^2)\left(\frac{1}{2}\tanh\left(\frac{\xi}{2}\right) + \frac{1}{2}\right)^2. \tag{114}$$

$$U_{35} = (V_{14})^2. \tag{115}$$

A = 0, C = 1, B = −1,

$$a_0 = 0,\ a_2 = -\left(k_1^2 + k_2^2 + k_3^2\right),\ a_1 = k_1^2 + k_2^2 + k_3^2,$$
$$b_1 = 0,\ b_2 = 0,\ \omega = \frac{1}{4}\left(-4k_1\mu + k_1^3 + k_2^2 k_1 + k_3^2 k_1\right). \tag{116}$$

Put (116) into (112),

$$V_{15} = k_3^2\left(\frac{1}{2} - \frac{1}{2}\coth\left(\frac{\xi}{2}\right)\right) + k_1^2 + k_2^2 + -\left(k_1^2 + k_2^2 + k_3^2\right)\left(\frac{1}{2} - \frac{1}{2}\coth\left(\frac{\xi}{2}\right)\right)^2. \tag{117}$$

$$U_{36} = (V_{15})^2. \tag{118}$$

A = 1/2, B = 0, C = −1/2

$$a_0 = \frac{1}{4}\left(k_1^2 + k_2^2 + k_3^2\right),\ a_2 = 0,\ a_1 = 0,\ b_1 = 0,$$
$$b_2 = \frac{1}{4}\left(-k_1^2 - k_2^2 - k_3^2\right), \omega = \frac{1}{4}\left(-4k_1\mu + k_1^3 + k_2^2 k_1 + k_3^2 k_1\right). \tag{119}$$

Substitute (119) into (112),

$$V_{16} = \frac{1}{4}\left(-k_1^2 - k_2^2 - k_3^2\right)\left(\frac{1}{(\cot(\xi) + \csc(\xi))^2}\right). \tag{120}$$

$$U_{37} = (V_{16})^2. \tag{121}$$

A = 1, B = 0, C = −1

$$a_0 = k_1^2 + k_2^2 + k_3^2,\ a_2 = 0,\ a_1 = 0,\ b_1 = 0,$$
$$b_2 = -k_1^2 - k_2^2 - k_3^2,\ \omega = -k_1\mu + k_1^3 + k_2^2 k_1 + k_3^2 k_1. \tag{122}$$

Put (122) in (112),

$$V_{17} = \left(-k_1^2 - k_2^2 - k_3^2\right)\left(\frac{1}{\tanh^2(\xi)}\right). \tag{123}$$

$$U_{38} = (V_{17})^2. \tag{124}$$

A = C = 1/2, B = 0,

$$a_0 = \frac{1}{4}\left(-k_1^2 - k_2^2 - k_3^2\right),\ a_2 = 0,\ a_1 = 0,\ b_1 = 0,$$
$$b_2 = \frac{1}{4}\left(-k_1^2 - k_2^2 - k_3^2\right),\ \omega = -\frac{1}{4}k_1\left(k_1^2 + k_2^2 + k_3^2 + 4\mu\right). \tag{125}$$

Put (125) in (112),

$$V_{18} = \frac{1}{4}\left(-k_1^2 - k_2^2 - k_3^2\right) + \frac{1}{4}\left(-k_1^2 - k_2^2 - k_3^2\right)\left(\frac{1}{(\tan(\xi) + \sec(\xi))^2}\right). \tag{126}$$

$$U_{39} = (V_{18})^2. \tag{127}$$

A = C = −1/2, B = 0,

$$a_0 = \frac{1}{4}\left(-k_1^2 - k_2^2 - k_3^2\right),\ a_2 = 0,\ a_1 = 0,\ b_1 = 0,$$
$$b_2 = \frac{1}{4}\left(-k_1^2 - k_2^2 - k_3^2\right),\ \omega = -\frac{1}{4}k_1\left(k_1^2 + k_2^2 + k_3^2 + 4\mu\right). \tag{128}$$

Put (128) in (112),

$$V_{19} = \frac{1}{4}\left(-k_1^2 - k_2^2 - k_3^2\right) + \frac{1}{4}\left(-k_1^2 - k_2^2 - k_3^2\right)\left(\frac{1}{(\sec(\xi) - \tan(\xi))^2}\right). \tag{129}$$

$$U_{40} = (V_{19})^2. \tag{130}$$

A = C = −1, B = 0,

$$a_0 = -k_1^2 - k_2^2 - k_3^2,\ a_2 = 0,\ a_1 = 0,$$
$$b_1 = 0,\ b_2 = -k_1^2 - k_2^2 - k_3^2,\ \omega = -k_1\left(k_1^2 + k_2^2 + k_3^2 + \mu\right). \tag{131}$$

Put (131) in (112),

$$V_{20} = -k_1^2 - k_2^2 - k_3^2 + \left(k_1^2 + k_2^2 + k_3^2\right)\left(\frac{1}{\tan^2(\xi)}\right). \tag{132}$$

$$U_{41} = (V_{20})^2. \tag{133}$$

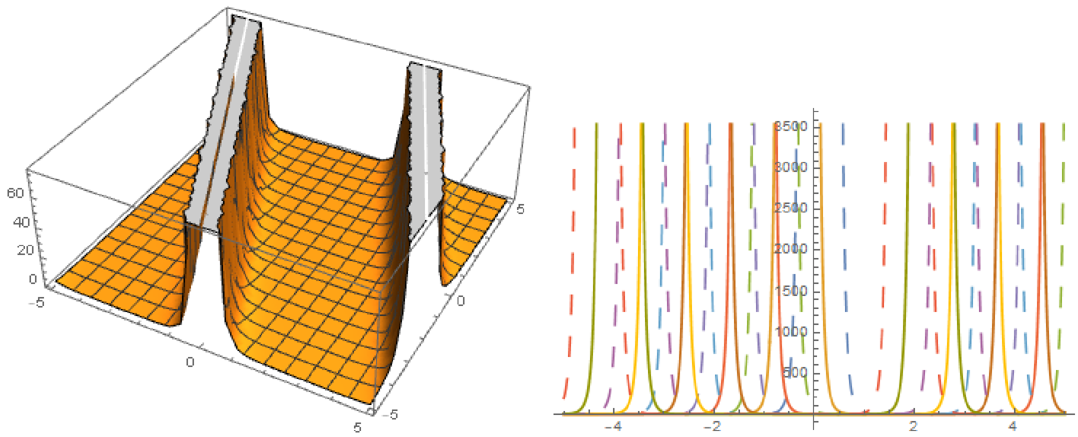

Figure 9. Solution U_{29} with $c_0 = 1.3,\ c_2 = 0.1,\ D_2 = -0.01,\ \lambda = -0.4,\ k_1 = 1.4,\ k_2 = -1.1,\ k_3 = 0.1,\ \mu = 0.03,\ y = 1,\ z = 1.$

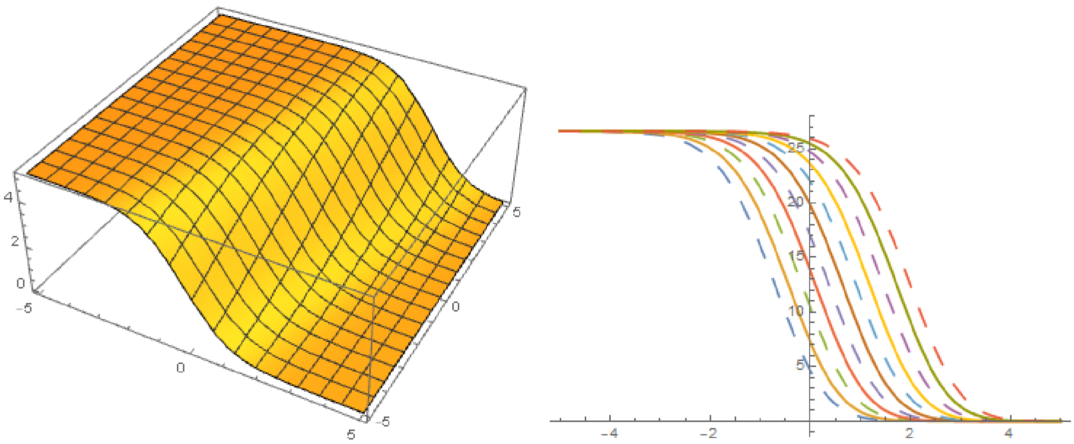

Figure 10. Solution U_{35} with $D_2 = -0.1,\ k_1 = 1.03,\ k_2 = -1.1,\ k_3 = 0.1,\ \mu = 0.3,\ y = 1,\ z = 1.$

A = B = 0

$$a_0 = 0,\ a_2 = -C^2\left(k_1^2 + k_2^2 + k_3^2\right),\ a_1 = 0,\ b_1 = 0,\ b_2 = 0,\ \omega = k_1(-\mu). \tag{134}$$

Put (134) in (112),

$$V_{21} = -C^2\left(k_1^2 + k_2^2 + k_3^2\right)\left(\frac{1}{C\xi + \eta}\right)^2. \tag{135}$$

$$U_{42} = (V_{21})^2. \tag{136}$$

B = C = 0

$$a_0 = 0,\ a_2 = 0,\ a_1 = 0,\ b_1 = 0,\ b_2 = -A^2\left(k_1^2 + k_2^2 + k_3^2\right),\ \omega = k_1(-\mu). \tag{137}$$

Put (137) in (112),

$$V_{22} = -A^2\left(k_1^2 + k_2^2 + k_3^2\right)\frac{1}{\left(\frac{1}{A\xi}\right)^2}. \tag{138}$$

$$U_{43} = (V_{22})^2. \tag{139}$$

C = 0

$$\begin{aligned} &a_0 = 0,\ a_2 = 0,\ a_1 = 0,\ b_1 = -AB\left(k_1^2 + k_2^2 + k_3^2\right), \\ &b_2 = -A^2\left(k_1^2 + k_2^2 + k_3^2\right),\ \omega = \frac{1}{4}k_1\left(B^2k_1^2 + B^2k_2^2 + B^2k_3^2 - 4.\mu\right) \end{aligned} \tag{140}$$

Put (140) in (112),

$$V_{23} = -AB\left(k_1^2 + k_2^2 + k_3^2\right)\left(\frac{1}{\frac{B}{e^{B\xi} - A}}\right) - A^2\left(k_1^2 + k_2^2 + k_3^2\right)\left(\frac{1}{\left(\frac{B}{e^{B\xi} - A}\right)^2}\right). \tag{141}$$

$$U_{44} = (V_{23})^2. \tag{142}$$

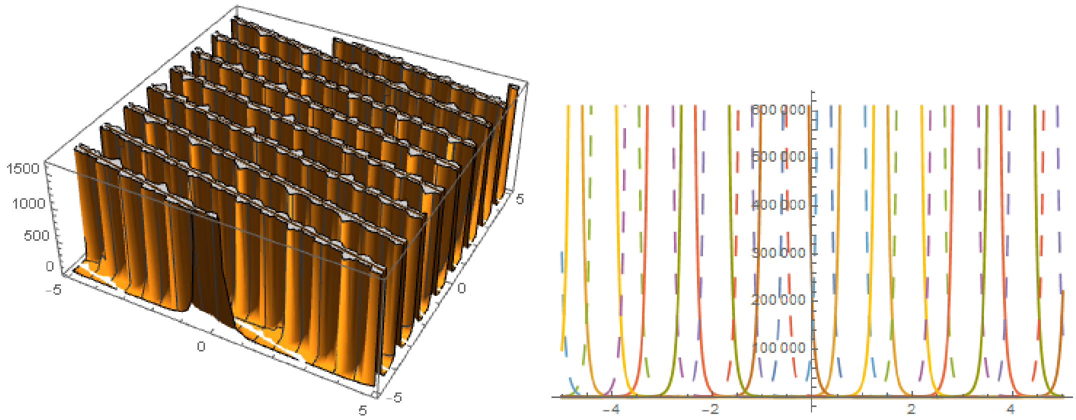

Figure 11. Solution U_{40} with $D_2 = -2.1$, $k_1 = -1.03$, $k_2 = -1.1$, $k_3 = 2.1$, $\mu = 3.3$, $y = 1$, $z = 1$.

5. Conclusions

Three mathematical schemes have employed to investigate solutions of NLEQZ and NLmQZK models. The derived solutions are in diverse types like exponential, hyperbolic, trigonometric and rational functions. Some solutions are plotted graphically in 2-dimensional and 3-dimensional by imparting particular value to the parameters under the constrain condition on each disquiet solution. Hence, it shows that our proposed mathematical methods are powerful, melodious and capacity be used in supplementary works to originate novel results for NPDEs ascending in physical science.

Author Contributions: Conceptualization, M.A., A.R.S., A.A., A.F.A. (Abdulrahman F. AlJohani), W.A. and A.F.A. (Amal F. Alharbi); methodology, M.A., A.R.S., A.A., A.F.A. (Abdulrahman F. AlJohani), W.A. and A.F.A. (Amal F. Alharbi); software, M.A., A.R.S., A.A., A.F.A. (Abdulrahman F. AlJohani), W.A. and A.F.A. (Amal F. Alharbi); validation, M.A., A.R.S., A.A., A.F.A. (Abdulrahman F. AlJohani), W.A. and A.F.A. (Amal F. Alharbi); formal analysis, M.A., A.R.S., A.A., A.F.A. (Abdulrahman F. AlJohani), W.A. and A.F.A. (Amal F. Alharbi); investigation, M.A., A.R.S., A.A., A.F.A. (Abdulrahman F. AlJohani), W.A. and A.F.A. (Amal F. Alharbi); resources, M.A., A.R.S., A.A., A.F.A. (Abdulrahman F. AlJohani), W.A. and A.F.A. (Amal F. Alharbi); data curation, M.A., A.R.S., A.A., A.F.A. (Abdulrahman F. AlJohani), W.A. and A.F.A. (Amal F. Alharbi); writing—original draft preparation, M.A., A.R.S., A.A., A.F.A. (Abdulrahman F. AlJohani), W.A. and A.F.A. (Amal F. Alharbi); writing—review and editing, M.A., A.R.S., A.A., A.F.A. (Abdulrahman F. AlJohani), W.A. and A.F.A. (Amal F. Alharbi); visualization, M.A., A.R.S., A.A., A.F.A. (Abdulrahman F. AlJohani), W.A. and A.F.A. (Amal F. Alharbi). All authors have read and agreed to the published version of the manuscript.

Funding: Deanship of Scientific Research (DSR), University of Tabuk, Tabuk, Saudi Arabia.

Data Availability Statement: Not applicable.

Acknowledgments: The authors would like to acknowledge support for this work from the Deanship of Scientific Research (DSR), University of Tabuk, Tabuk, Saudi Arabia, under Grant no. S-1442-0159.

Conflicts of Interest: The authors declare no conflict of interest.

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