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A Study on k-Hyperideals in Ordered Semihyperrings

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Abstract: In this study, we propose the concept of left extension of a hyperideal by generalizing the concept of k-hyperideals in ordered semihyperrings with symmetrical hyper-operation $\oplus$. By using the notion of extension of a k-hyperideal, we prove some results in ordered semihyperrings. The results of this paper can be viewed as a generalization for k-ideals of semirings.

Keywords: ordered hyperstructure; k-hyperideal; left extension

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1. Introduction

The notion of ordered semihypergroup was pioneered by Heidari and Davvaz [1] in 2011. In Ref. [2], Shi et al. attempted to study factorizable ordered hypergroupoids. In Ref. [3], Davvaz et al. initiated the study of pseudoorders in ordered semihypergroups. Gu and Tang in Ref. [4] and Tang et al. in Ref. [5] constructed the ordered semihypergroup from an ordered semihypergroup by using ordered regular relations.

The concept of hyperstructure was introduced by Marty [6] in 1934. In 1990, Vougiouklis [7] defined the notion of semihyperrings and discussed some of its properties. The theory of hyperideals in LA-hyperrings was studied by Rehman et al. in Ref. [8]. Many notions of algebraic geometry were extended to hyperrings in Ref. [9].

Some recent studies on ordered semihyperrings are on left k-bi-quasi hyperideals and right pure (bi-quasi-)hyperideals done by Rao et al. in Ref. [10] and Shao et al. in Ref. [11]. A study on w-pseudo-orders in ordered (semi)hyperrings was done in Ref. [12]. In Ref. [13], Kou et al. discussed the relationship between ordered semihyperrings by using homomorphisms and homo-derivations. Moreover, the connection between the ordered semihyperrings is explained by Omidi and Davvaz in Ref. [14].

In Ref. [15], Hedayati investigated some results in semihyperrings using k-hyperideals. In 2007, Ameri and Hedayati [16] introduced the notion of k-hyperideals in ordered semihyperrings. In this paper, we first define the left extension of a left hyperideal in an ordered semihyperring. The concept of extension of a k-ideal on a semiring R was introduced and studied by Chaudhari et al. in Refs. [17,18]. In the results of Chaudhari et al. [18], we replace the condition of extension of a k-ideal in semirings by extension of a k-hyperideal in ordered semihyperrings. By using the notion of extension of a k-hyperideal instead of k-hyperideal, we prove some results in ordered semihyperrings. Left extension of hyperideals are discovered to be a generalization of k-hyperideals. Let Q, W be hyperideals of an ordered semihyperring $(R, \oplus, \odot, \leq)$ such that $Q \subseteq W$. Then

$$\overline{W_Q} = \{r \in R \mid r \oplus P \subseteq W,\ \exists P \subseteq Q, 0 \in P\}$$

is the smallest left extension of Q containing W. Moreover, we proved that $\overline{W_Q} = W$ if and only if W is a left extension of Q. Some conclusions on extension of a k-hyperideal are gathered in the last section of the study.

2. Preliminaries

A mapping $\odot : R \times R \to \mathcal{P}^*(R)$ is called a *hyperoperation* on R. If $\emptyset \neq L, L' \subseteq R$ and $x \in R$, then

$$L \odot L' = \bigcup_{\substack{l \in L \\ l' \in L'}} l \odot l', x \odot L = \{x\} \odot L \text{ and } L' \odot x = L' \odot \{x\}.$$

$(R, \odot)$ is called a *semihypergroup* if for every l, l', x in R,

$$l \odot (l' \odot x) = (l \odot l') \odot x.$$

Definition 1. *[7] A semihyperring is a triple $(R, \oplus, \odot)$ such that for each $x, y, z \in R$,*

(1) $(R, \oplus)$ is a commutative semihypergroup;
(2) $(R, \odot)$ is a semihypergroup;
(3) $x \odot (y \oplus z) = x \odot y \oplus x \odot z$ and $(x \oplus y) \odot z = x \odot z \oplus y \odot z$;
(4) There exists an element $0 \in R$ such that $x \oplus 0 = 0 \oplus x = \{x\}$ and $x \odot 0 = 0 \odot x = \{0\}$ for all x in R.

Definition 2. *[10] Take a semihyperring $(R, \oplus, \odot)$ and a partial order relation $\leq$. Then $(R, \oplus, \odot, \leq)$ is called an ordered semihyperring if for any $q, q', x \in R$,*

(1) $q \leq q' \Rightarrow q \oplus x \preceq q' \oplus x$;
(2)

$$q \leq q' \Rightarrow \begin{cases} q \odot x \preceq q' \odot x, \\ \\ x \odot q \preceq x \odot q'. \end{cases}$$

For every $\emptyset \neq L, L' \subseteq R$, $L \preceq L'$ is defined by $\forall l \in L, \exists l' \in L'$ such that $l \leq l'$. Clearly, $L \subseteq L'$ implies $L \preceq L'$, but the converse is not valid in general. In this definition, two types of relation are defined, one is between elements of R, which is denoted by $\leq$, and second one between subsets of R, which is $\preceq$.

Example 1. *Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. Consider the semiring $(\mathbb{N}_0, +, \cdot)$ where $+$ and $\cdot$ are usual addition and multiplication. Define*

$$l \oplus l' = \{l, l'\} \text{ and } l \odot l' = \{ll', cll'\}, \text{ where } c \in \mathbb{N}_0.$$

If $\leq$ is the natural ordering on $\mathbb{N}_0$, then $(\mathbb{N}_0, \oplus, \odot, \leq)$ is an ordered semihyperring.

Definition 3. *We will say that $\emptyset \neq K \subseteq R$ is a left (resp. right) hyperideal of R if*

(1) for all $a, b \in K$, $a \oplus b \subseteq K$;
(2) $R \odot K \subseteq K$ (resp. $K \odot R \subseteq K$);
(3) $(K] \subseteq K$.

The set $(K]$ is given by

$$(K] := \{r \in R \mid r \leq x \text{ for some } x \in K\}.$$

Definition 4. *We will say that a left hyperideal $\emptyset \neq W \subseteq R$ is a left k-hyperideal of R, if*

$$\forall w \in W, \forall q \in R, (w \oplus q) \cap W \neq \emptyset \Rightarrow q \in W.$$

Remark 1. *Clearly, every left k-hyperideal of R is a left hyperideal of R. The converse is not true, in general, that is, a left hyperideal may not be a left k-hyperideal of R (see Example 2).*

3. Main Results

Now, we study the extension of a k-hyperideal in an ordered semihyperring.

Definition 5. *Assume that K, L are left hyperideals of an ordered semihyperring $(R, \oplus, \odot, \leq)$ and $L \subseteq K$. Then K is said to be a left extension of L if*

$$\forall l \in L, \forall q \in R, l \oplus q \subseteq K \Rightarrow q \in K,$$

or

$$\forall l \in L, \forall q \in R, (l \oplus q) \cap K \neq \emptyset \Rightarrow q \in K.$$

Remark 2. *Every k-hyperideal K of $(R, \oplus, \odot, \leq)$ with $K \supseteq L$ is a left extension of L, where L is a hyperideal of R.*

Example 2. *Let $R = \{0, p, q\}$ and define the symmetrical hyper-operations $\oplus$ and $\odot$ as follows:*

$\oplus$	0	p	q
0	$\{0\}$	$\{p\}$	$\{q\}$
p	$\{p\}$	$\{0, p\}$	$\{0, p, q\}$
q	$\{q\}$	$\{0, p, q\}$	$\{0, p\}$

$\odot$	0	p	q
0	$\{0\}$	$\{0\}$	$\{0\}$
p	$\{0\}$	$\{0\}$	$\{0\}$
q	$\{0\}$	$\{0\}$	$\{0, p\}$

$$\leq := \{(0,0), (p,p), (q,q), (0,p), (0,q), (p,q)\}.$$

Then, $(R, \oplus, \odot, \leq)$ is an ordered semihyperring. Clearly, $L = \{0, p\}$ is a hyperideal of R, but it is not a k-hyperideal. Indeed:

$$R = (p \oplus q) \cap L \neq \emptyset \text{ and } p \in L \text{ but } q \notin L.$$

Obviously, L is a k-extension of $L' = \{0\}$,

Example 3. *Consider the ordered semihyperring $(R, \oplus, \odot, \leq)$ with the symmetrical hyper-operation $\oplus$ and hyper-operation $\odot$:*

$\oplus$	0	p	q	r
0	$\{0\}$	$\{p\}$	$\{q\}$	$\{r\}$
p	$\{p\}$	$\{p\}$	$\{p\}$	$\{p\}$
q	$\{q\}$	$\{p\}$	$\{0, q\}$	$\{0, q, r\}$
r	$\{r\}$	$\{p\}$	$\{0, q, r\}$	$\{0, r\}$

$\odot$	0	p	q	r
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
p	$\{0\}$	$\{p\}$	$\{0, q\}$	$\{0\}$
q	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
r	$\{0\}$	$\{0, r\}$	$\{0\}$	$\{0\}$

$$\leq \quad := \{(0,0), (0,p), (0,q), (0,r), (p,p), (q,p), (q,q), (r,p), (r,r)\}.$$

Clearly, $K = \{0, q, r\}$ is a left extension of $L = \{0, q\}$. In addition, L is a left extension of $\{0\}$, but it is not a k-hyperideal of R. Indeed:

$$(r \oplus q) \cap L \neq \emptyset \text{ and } q \in L \text{ but } r \notin L.$$

Example 4. *Let $R = \{0, p, q, r\}$ be a set with the symmetrical hyper-addition $\oplus$ and the multiplication $\odot$ defined as follows:*

$$
\begin{array}{ccccc}
\oplus & 0 & p & q & r \\
0 & \{0\} & \{p\} & \{q\} & \{r\} \\
p & \{p\} & \{p,q\} & \{q\} & \{r\} \\
q & \{q\} & \{q\} & \{0,q\} & \{r\} \\
r & \{r\} & \{r\} & \{r\} & \{0,r\}
\end{array}
$$

$$
\begin{array}{ccccc}
\odot & 0 & p & q & r \\
0 & \{0\} & \{0\} & \{0\} & \{0\} \\
p & \{0\} & \{p\} & \{p\} & \{p\} \\
q & \{0\} & \{q\} & \{q\} & \{q\} \\
r & \{0\} & \{r\} & \{r\} & \{r\}
\end{array}
$$

$$\leq := \{(x, x) \mid x \in R\}.$$

Then, $(R, \oplus, \odot, \leq)$ is an ordered semihyperring. Clearly, $K = \{0, r\}$ is a right hyperideal of R, but it is not a right k-hyperideal of R. Indeed:

$$r \oplus p = r \in K \text{ and } r \in K \text{ but } p \notin K.$$

Let $L = \{0\}$. Then, K is a right k-extension of L, but it is not a right k-hyperideal of R.

Remark 3. *In the following, we consider the following condition:*

$$\forall l \in L, \forall q \in R, l \oplus q \subseteq K \Rightarrow q \in K.$$

Definition 6. *Assume that Q, W are hyperideals of an ordered semihyperring $(R, \oplus, \odot, \leq)$ such that $Q \subseteq W$. Then, we denote*

$$\overline{Q} = \{r \in R \mid r \oplus P \subseteq Q,\ \exists P \subseteq Q, 0 \in P\},$$

$$\overline{W_Q} = \{r \in R \mid r \oplus P \subseteq W,\ \exists P \subseteq Q, 0 \in P\}.$$

$\overline{W_Q}$ will be called the k-closure of W with respect to Q.

Remark 4. *We have*

(1) $Q \subseteq \overline{Q} \subseteq \overline{W_Q} \subseteq \overline{W}$;

(2) $\overline{W_W} = \overline{W}$.

Lemma 1. *Assume that Q, W, Y are hyperideals of an ordered semihyperring $(R, \oplus, \odot, \leq)$ such that $Q \subseteq W \subseteq Y$. Then, $\overline{Y_Q} \subseteq \overline{Y_W}$.*

Proof. Let W be a hyperideal of R such that $Q \subseteq W \subseteq Y$ and $x \in \overline{Y_Q}$. Then, there exists $P \subseteq Q \subseteq W$ such that $x \oplus P \subseteq Y$. So, $x \in \overline{Y_W}$. Therefore, $\overline{Y_Q} \subseteq \overline{Y_W}$. □

Proposition 1. *$\overline{W_Q}$ is the smallest left extension of Q containing W.*

Proof. Clearly, $\overline{W_Q}$ is a hyperideal of R.
Indeed: Let $q_1, q_2 \in \overline{W_Q}$. By definition of $\overline{W_Q}$, there exist $P_1, P_2 \subseteq Q$ such that $q_1 \oplus P_1 \subseteq W$ and $q_2 \oplus P_2 \subseteq W$. Now,

$$(q_1 \oplus q_2) \oplus (P_1 \oplus P_2) = q_1 \oplus P_1 \oplus q_2 \oplus P_2 \subseteq W \oplus W \subseteq W.$$

It means that $q_1 \oplus q_2 \subseteq \overline{W_Q}$.
Now, let $u \in \overline{W_Q}$ and $x \in R$. Then, there exists $P \subseteq Q$ such that $u \oplus P \subseteq W$. So,

$$x \odot u \oplus x \odot P = x \odot (u \oplus P) \subseteq R \odot W \subseteq W.$$

Since $x \odot P \subseteq Q$, we get $x \odot u \subseteq \overline{W_Q}$. Similarly, $u \odot x \subseteq \overline{W_Q}$.
Now, let $u \in \overline{W_Q}$ and $(v, u) \in \leq$, where $v \in R$. By assumption, there exists $P \subseteq Q$ such that $u \oplus P \subseteq W$. Since R is an ordered semihyperring, we get $v \oplus p \preceq u \oplus p$ for any $p \in P$. So, for any $x \in v \oplus p$, $x \leq y$ for some $y \in u \oplus p \subseteq u \oplus P \subseteq W$. Since $(W] \subseteq W$, we obtain $x \in W$. So, $v \oplus p \subseteq W$ for each $p \in P$. Thus $v \oplus P \subseteq W$ and hence $v \in \overline{W_Q}$. Therefore, $\overline{W_Q}$ is a hyperideal of R.

Now, we prove that $\overline{W_Q}$ is a extension of Q. Let $q \in Q$ and $q \oplus r \subseteq \overline{W_Q}$, where $r \in R$. By assumption, $u \in \overline{W_Q}$ for all $u \in q \oplus r$. Hence, for any $u \in q \oplus r$, there exists $P_u \subseteq Q$ such that $u \oplus P_u \subseteq W$. Thus,

$$q \oplus r \oplus \bigcup_{u \in q \oplus r} P_u \subseteq \bigcup_{u \in q \oplus r} (u \oplus P_u) \oplus \bigcup_{u \in q \oplus r} P_u \subseteq W.$$

Since $q \oplus \bigcup_{u \in q \oplus r} P_u \subseteq Q$, it follows that $r \in \overline{W_Q}$. Therefore, $\overline{W_Q}$ is a left extension of Q.

Clearly, $W \subseteq \overline{W_Q}$. Now, let Y be a left extension of Q containing W and $q \in \overline{W_Q}$. Then, there exist $P \subseteq Q$ such that $q \oplus P \subseteq W \subseteq Y$. Since Y is a left extension of Q, we get $q \in Y$. Hence, $\overline{W_Q} \subseteq Y$. □

Theorem 1. *Assume that Q, W are hyperideals of an ordered semihyperring $(R, \oplus, \odot, \leq)$ such that $Q \subseteq W$. Then, W is a left extension of Q if and only if $\overline{W_Q} = W$.*

Proof. Necessity: Let W be a left extension of Q. By Proposition 1, $\overline{W_Q}$ is the smallest left extension of Q and $W \subseteq \overline{W_Q}$. Since W is a left extension of Q, we get $\overline{W_Q} \subseteq W$. So, $W \subseteq \overline{W_Q} \subseteq W$ and hence $\overline{W_Q} = W$.

Sufficiency: If $\overline{W_Q} = W$, then, since by Proposition 1, $\overline{W_Q}$ is a left extension of Q, it follows that W is a left extension of Q. □

Corollary 1. *Assume that Q, W are hyperideals of an ordered semihyperring $(R, \oplus, \odot, \leq)$ such that $Q \subseteq W$. Then, $\overline{(\overline{W_Q})_Q} = \overline{W_Q}$.*

Proof. The proof obtains from Proposition 1 and Theorem 1. □

Theorem 2. *Assume that Q, W, Y are hyperideals of an ordered semihyperring $(R, \oplus, \odot, \leq)$ such that $Q \subseteq W, Y$. Then,*

$$\overline{(W \cap Y)_Q} = \overline{W_Q} \cap \overline{Y_Q}.$$

Proof. Let $a \in \overline{(W \cap Y)_Q}$. Then, there exists $P \subseteq Q$ such that

$$a \oplus P \subseteq W \cap Y \subseteq W.$$

So, $a \in \overline{W_Q}$. Therefore, $\overline{(W \cap Y)_Q} \subseteq \overline{W_Q}$. Similarly,

$$\overline{(W \cap Y)_Q} \subseteq \overline{Y_Q}.$$

Hence,

$$\overline{(W \cap Y)_Q} \subseteq \overline{W_Q} \cap \overline{Y_Q}.$$

Now, let $x \in \overline{W_Q} \cap \overline{Y_Q}$. Then, there exist $P, P' \subseteq Q$ such that $x \oplus P \subseteq W$ and $x \oplus P' \subseteq Y$. Since $P' \subseteq Q \subseteq W$ and W is a hyperideal of R, we have

$$x \oplus P \oplus P' \subseteq W \oplus W \subseteq W.$$

Similarly, $x \oplus P \oplus P' \subseteq Y$. So, $x \oplus P \oplus P' \subseteq W \cap Y$. This implies that $x \in \overline{(W \cap Y)_Q}$. Therefore, $\overline{W_Q} \cap \overline{Y_Q} \subseteq \overline{(W \cap Y)_Q}$. □

Theorem 3. *Assume that Q, W, Y are hyperideals of an ordered semihyperring $(R, \oplus, \odot, \leq)$ such that $Q \subseteq W, Y$. If W, Y are left extensions of Q, then $W \cap Y$ is a left extension of Q.*

Proof. By Theorem 2, we have

$$\overline{(W \cap Y)_Q} = \overline{W_Q} \cap \overline{Y_Q}.$$

Since W, Y are left extensions of Q, then by Theorem 1, we get

$$\overline{W_Q} \cap \overline{Y_Q} = W \cap Y.$$

Hence,

$$\overline{(W \cap Y)_Q} = W \cap Y.$$

Now, by Theorem 1, $W \cap Y$ is a left extension of Q. □

Definition 7. *Assume that K, L are left hyperideals of an ordered semihyperring $(R, \oplus, \odot, \leq)$ and $L \subseteq K$. Then K is said to be a left m-extension of L if*

$$\forall l \in L, \forall q \in R, l \odot q \subseteq K \Rightarrow q \in K.$$

Theorem 4. *Assume that K, L are hyperideals of an ordered semihyperring $(R, \oplus, \odot, \leq)$ and $L \subseteq K$ such that $L \oplus R \subseteq L$. If K is a m-extension of L, then K is an extension of L.*

Proof. Let K be a m-extension of L. Consider $l \oplus q \subseteq K$, $l \in L$ and $q \in R$. Since K is a hyperideal of R, we get

$$(l \oplus q) \odot q \subseteq K \odot R \subseteq K.$$

So, for any $p \in l \oplus q$, $p \odot q \subseteq K$. Since K is a m-extension of L, we have $q \in K$. Thus, K is an extension of L. □

4. Conclusions

The concept of left extension of hyperideals in ordered semihyperrings is introduced in this study. Left extension of hyperideals are discovered to be a generalization of k-hyperideals. Let Q, W be hyperideals of an ordered semihyperring $(R, \oplus, \odot, \leq)$ such that $Q \subseteq W$. Then

$$\overline{W_Q} = \{r \in R \mid r \oplus P \subseteq W, \ \exists P \subseteq Q, 0 \in P\}$$

is the smallest left extension of Q containing W. In addition, we proved that $\overline{W_Q} = W$ if and only if W is a left extension of Q. By using the concept of extension of a k-hyperideal, we discussed some results in ordered semihyperrings. Some further works can be done on left extension of a fuzzy hyperideal in ordered semihyperrings.

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References

1. Heidari, D.; Davvaz, B. On ordered hyperstructures. *Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys.* **2011**, *73*, 85–96.
2. Shi, X.; Guan, H.; Akhoundi, M.; Omidi, S. Factorizable ordered hypergroupoids with applications. *Math. Probl. Eng.* **2021**, *2021*, 5. [CrossRef]
3. Davvaz, B.; Corsini, P.; Changphas, T. Relationship between ordered semihypergroups and ordered semigroups by using pseudoorder. *Eur. J. Combin.* **2015**, *44*, 208–217. [CrossRef]
4. Gu, Z.; Tang, X. Ordered regular equivalence relations on ordered semihypergroups. *J. Algebra* **2016**, *450*, 384–397. [CrossRef]
5. Tang, J.; Feng, X.; Davvaz, B.; Xie, X. A further study on ordered regular equivalence relations in ordered semihypergroups. *Open Math.* **2018**, *16*, 168–184. [CrossRef]
6. Marty, F. Sur une generalization de la notion de groupe. In Proceedings of the 8th Congress of Scandinavian Mathematicians, Stockholm, Sweden, 14–18 August 1934; pp. 45–49.
7. Vougiouklis, T. On some representations of hypergroups. *Ann. Sci. Univ. Clermont-Ferrand II Math.* **1990**, *26*, 21–29.
8. Rehman, I.; Yaqoob, N.; Nawaz, S. Hyperideals and hypersystems in LA-hyperrings. *Songklanakarin J. Sci. Technol.* **2017**, *39*, 651–657.
9. Jun, J. Algebraic geometry over hyperrings. *Adv. Math.* **2018**, *323*, 142–192. [CrossRef]
10. Rao, Y.; Xu, P.; Shao, Z.; Kosari, S. Left *k*-bi-quasi hyperideals in ordered semihyperrings. *Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys.* **2021**, *83*, 125–134.
11. Shao, Z.; Chen, X.; Kosari, S.; Omidi, S. On some properties of right pure (bi-quasi-)hyperideals in ordered semihyperrings. *Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys.* **2021**, *83*, 95–104.
12. Qiang, X.; Guan, H.; Rashmanlou, H. A note on the w-pseudo-orders in ordered (semi)hyperrings. *Symmetry* **2021** *13*, 2371. [CrossRef]
13. Kou, Z.; Kosari, S.; Monemrad, M.; Akhoundi, M.; Omidi, S. A note on the connection between ordered semihyperrings. *Symmetry* **2021**, *13*, 2035. [CrossRef]
14. Omidi, S.; Davvaz, B. Construction of ordered regular equivalence relations on ordered semihyperrings. *Honam Math. J.* **2018**, *40*, 601–610.
15. Hedayati, H. Closure of *k*-hyperideals in multiplicative semihyperrings. *Southeast Asian Bull. Math.* **2011**, *35*, 81–89.
16. Ameri, R.; Hedayati, H. On *k*-hyperideals of semihyperrings. *J. Discrete Math. Sci. Cryptogr.* **2007**, *10*, 41–54. [CrossRef]
17. Chaudhari, J.; Bonde, D. Ideal theory in quotient semirings. *Thai J. Math.* **2014**, *12*, 95–101.
18. Chaudhari, J.; Davvaz, B.; Ingale, K. Subtractive extension of ideals in semirings. *Sarajevo J. Math.* **2014**, *10*, 13–20. [CrossRef]