symmetry

MDPI

Article

Deflecting an Asteroid on a Collision Course with Earth Using a Powered Swing-By Maneuver

Bruno S. Chagas [1,*], Antonio Fernando Bertachini de Almeida Prado [2,3] and Othon Cabo Winter [1]

[1] Grupo de Dinâmica Orbital e Planetologia, São Paulo State University (UNESP), Guaratinguetá 12516-410, SP, Brazil
[2] Academy Engineering, Peoples' Friendship University of Russia (RUDN University), Miklukho-Maklaya Street 6, 117198 Moscow, Russia
[3] Postgraduate Division, National Institute for Space Research (INPE), São José dos Campos 12227-010, SP, Brazil
* Correspondence: bruno.chagas@unesp.br or bruno.ba.987@gmail.com; Tel.: +55-(12)99618-9401

Abstract: The deflection of potentially dangerous asteroids has been treated with great intensity and has gained more and more attention in scientific research. Different techniques are developed over the years. Among these techniques, we found the kinetic impact deflection technique to be the most viable at the moment. In this work we address the kinetic impact deflection technique, but in a scenario where we have a short time to deflect an asteroid that will collide with Earth. For this, we also use a maneuver similar to a powered gravity-assisted maneuver with Earth in a previous passage to change the trajectory of the asteroid to avoid the collision. We apply this technique in three scenarios: (i) impulse before the close encounter, (ii) impulse during the close encounter, and (iii) impulse after the close encounter. We observe that some trajectories are symmetric with respect to the line Sun–Earth, and others are asymmetric. We show that, using this technique, it is possible to change the trajectory of the asteroid, even in a short period, to avoid the collision without using a large variation of velocity in the orbit of the asteroid.

Keywords: deflect asteroids; planetary defense; asteroid collision; swing by; astrodynamics

Citation: Chagas, B.S.; Prado, A.F.B.d.A.; Winter, O.C. Deflecting an Asteroid on a Collision Course with Earth Using a Powered Swing-By Maneuver. *Symmetry* **2022**, *14*, 1658. https://doi.org/10.3390/sym14081658

Academic Editors: Charalampos Moustakidis and Grant J. Mathews

Received: 30 May 2022
Accepted: 8 August 2022
Published: 11 August 2022

1. Introduction

Asteroids are small bodies in the Solar System, but they are very important for scientific exploration, since they must have material from the formation of the Solar System. Besides being very important for scientific research, they also represent a danger to Earth, since the possibility of a collision exists [1–3]. This type of problem has been addressed in many studies, and ideas to avoid a collision like that are under study [4,5]. One of the most popular methods is sending a spacecraft to collide with the asteroid to change its route. Several studies have considered this possibility, such as [1–4,6–15]. In general, the magnitude of the impulse required depends on the warning time, which is the period of time that is available to take some action, in other words, the time between the possible collision and the discovery of the risk.

Gravitational tractors have also been considered, where a spacecraft is placed near the asteroid to slowly changes its course [16–18]. This is possible only when there is a long warning time.

The present paper uses the idea of the application of a small impulse in the asteroid using a collision of a spacecraft, but this impulse is applied near a previous passage by the Earth, before the one that may end in collision. The importance of a previous passage like this for deflecting the trajectory of the asteroid to avoid a collision with Earth was noted in [19], when performing general simulations. Therefore, the present paper is a sequence of this study and focus in the situation where the impulse is applied close to this previous passage, instead of well before the collision. The idea follows the concept of "powered swing-by" used to maneuver a spacecraft [20–22], which is considered an alternative to the pure gravity swing-by [23–25]. The main idea is to change the trajectory of the spacecraft a little bit to get a more adequate geometry in this passage of the Earth such that it makes a swing-by that generates larger changes in the orbit of the asteroid around the Sun, so making larger deviations in the asteroid–Earth distance in the next passage. It will avoid the possibility of the collision without using very large variations of velocity and/or long warning times.

In that sense, a passage by the Earth is used to change the orbit of the spacecraft such that it avoids a future collision with Earth. The gravity of the Earth is used to protect itself, with the help of smaller impulses. The main advantage, when compared to the standard kinetic impact technique, is that the impact required is much smaller, and impulses in the order of mm/s may make large variations in the Earth-asteroid distances in the dangerous passage in about two years. It happens because the impulse does not change the orbit itself too much, but it slight guides the trajectory to make the asteroid to have a more efficient swing-by. Thus, this technique allows the possibility of using small impulses with short warning times.

Another interesting point to be mentioned is that the first real asteroid deflection mission is already scheduled. It is the DART mission, whose main objective is to measure the deflection capacity of an asteroid during a collision [6,10,11,13]. The spacecraft will collide with Dimorphos, which is a moon of the main asteroid Didymos. The main goal is to study the effects of such a collision.

2. Mathematical Modeling

In this work, we will analyze the gravitational effects involved in the kinetic impact asteroid deflection technique over a relatively short period. In this process we will solve the three-body problem using the Mercury integrator package [26], considering the Burlish–Stoer integrator.

The equations of motion of the restricted three-body problem using the rotating system of reference and the canonical system of units can be written as follows [27]:

$$\ddot{x} - 2\dot{y} = \frac{\partial\Omega}{\partial x} \tag{1}$$

$$\ddot{y} - 2\dot{x} = \frac{\partial\Omega}{\partial y} \tag{2}$$

where,

$$\Omega = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \tag{3}$$

where x and y are the positions of the asteroid, μ is the mass of the smaller body in canonical units, r_1 is the distance between the Sun and the asteroid and r_2 is the distance between the Earth and the asteroid. Those equations depend on the potential Ω.

The equation below gives the two-body energy of the asteroid (E) as a function of the semi-major axis. This equation is important for us, as we can get an idea of what physically happens to the orbit of the asteroid over the period during which it is undergoing gravitational perturbation.

$$E = -\frac{\mu}{2a} \tag{4}$$

where $\mu = GM$ and "a" is the semi-major axis of the asteroid.

Our work elaborated based in the swing-by concept, where the asteroid will suffer the gravitational perturbation from a swing-by with the Earth in a previous close encounter, and thus avoid the collision two years later. The maneuver is better described in [20].

We can formulate the maneuver mathematically as

$$\Delta E = -2V_z V_\infty sin(\delta) sin(\psi) \tag{5}$$

where ΔE is the variation of the energy of the smaller body (the asteroid in our situation), V_z is the velocity of M2 in relation to M1, V_∞ is the particle velocity modulus at infinity before or after passing close to a body of mass M2, δ is the deflection angle and represents half the rotation of the velocity vector due to the "swing-by" and ψ is the angle between the periapsis line and the line M1 − M2, where

$$sin(\delta) = \frac{1}{1 + \frac{r_p V_\infty^2}{\mu_2}} \tag{6}$$

since r_p is the maximum approximation distance between M2 and M3 and $\mu_2 = GM2$ [28].

3. Materials and Methods

In this work, we present a proposal for a planetary defense scenario that may eventuality occur at some point in the future. An asteroid will have a close encounter with Earth, and after this close encounter, it will return in a period of about 2 years to collide with Earth. To study the intended effects, we are simplifying our problem, considering it as a three-body problem (Sun, Earth and asteroid) and we present numerical simulations using the Mercury integrator package; we opted for the Bulirsch–Stoer numerical integrator [26]. The Sun is considered the center of our system and the Earth is orbiting it in a circular orbit with zero inclination. The asteroid is considered to have negligible mass and its orbit has zero inclination.

Of course, the perturbations coming from the other planets of the Solar System influence the results. It is difficult to predict those influences, because they depend a lot on each particular trajectory. To have an idea, when we consider the planets Jupiter and Saturn, the largest ones, when applying the velocity variation before the first encounter, we have differences in the minimum asteroid–Earth distances in the order of 40% to 50% of the values estimated considering the model with no planets. Regarding solar radiation pressure, due to the short period simulated, and the very small area/mass ratio of the asteroid, this effect is very small and can be neglected at this time.

Initially, we define a distance between the asteroid and Earth with the intention that, from this position, the asteroid can return close to Earth after 2 years of what we will call the first encounter. We determined a period of 90 days from this encounter and divided these 90 days into 30 parts with the intention of varying the orbit of the asteroid little by little to find the desired scenario; this division gave us ranges of average asteroid anomaly between 1° and 2°. What we did is: From the first position, we started to apply variations in the velocity of the asteroid from −50 to 50 mm/s with an inclement of 10 mm/s, and we simulated forward in time. After performing all the velocity variations in this position, we went back in time in our simulations, to the positions of the bodies referring to the value found in the first division, that is, we went back between 1° and 2°, and performed the same procedure of applying variations of velocity but in this new position. The procedure was carried out successively until we found a scenario in which the asteroid, when passing through the first close encounter with Earth, returned 2 years later and collided with it. It is worth mentioning that, when going back in time in our simulations and when performing the simulations in forward time again, the asteroid returned with the same characteristics found before going back in time. That is, our simulations were not affected when performing this action.

The smallest distances found between asteroid and Earth within this period can be found in Figure 1, which work as Maps for Impulses in the Orbit of Asteroids (M.I.O.A.) [19]. In this figure we can see that by applying the 50 mm/s (blue) impulse to the mean asteroid anomaly of 352°, we get a collision with Earth. The work presented below refers to the initial conditions found at this position where, by measuring the days between the initial position of the asteroid that allowed the collision and the first encounter between the asteroid and Earth, we obtained an interval of 63 days. It is important to remember here that we divided an interval of 90 days into 30 parts, so we always tested a number of days that was multiple of 3. By searching those days, we found that 63 days was the time that allowed the collision. The Cartesian coordinates that we used as input can be found in Tables 1 and 2.

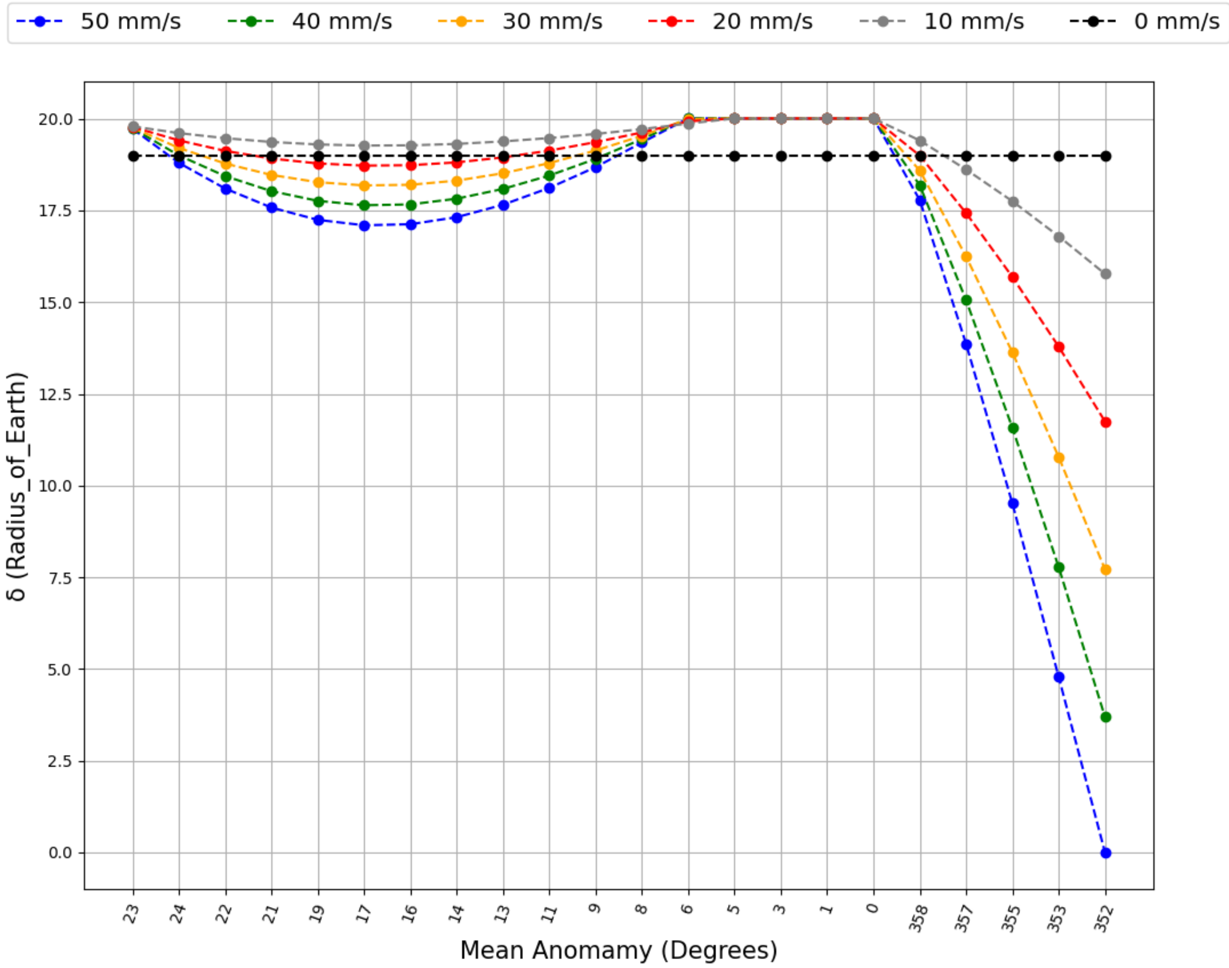

Figure 1. Shortest distances between the asteroid and Earth (δ) over a period of approximately 2 years from an initial first encounter distance; the simulations were going back in time by a 1° to 2° space in the Mean anomaly of the asteroid, until finding a scenario collision with Earth 2 years after the first encounter. This scenario occurred by applying a velocity variation of −50 mm/s to the mean anomaly of 352°.

Table 1. Initial positions of Earth and the asteroid.

Bodies	X (au)	Y (au)
Earth	0.995881559395000	0.090645867243292
Asteroid	0.710575343723782	−0.378599263365500

Table 2. Initial velocity of Earth and asteroid.

Bodies	VX (au/Day)	VY (au/Day)
Earth	−0.001559258543264	0.017131312901762
Asteroid	0.007992408896389	0.021802505831201

We did not consider the composition and dimensions of the asteroid and its rotation properties. We choose initial conditions that ended in a collision of the asteroid with Earth, so it was a fictitious scenario made with the goal of showing our proposed technique.

We applied velocity variations of -1 and 1 m/s in the orbit of the asteroid, with the impulses applied opposite and in the same direction of the velocity vector of the asteroid, respectively, simulating the kinetic impact technique. Initially, the impulses were applied 63 days before the first close encounter with Earth, the one that changed its orbit and made it possible to avoid the collision with Earth in the next encounter. Next, the same impulse was applied at this first close encounter and then 63 days after the first close encounter. These velocity variations of -1 and 1 m/s are addressed in the literature as magnitudes that can be used in cases where we have to deviate the trajectory of an asteroid in a short period [1,29], as is our case here. The first close encounter between the asteroid and Earth took place at a distance of 20 times the radius of Earth from Earth.

To understand the effects, we are looking at the relative distance of the asteroid from Earth, but also at the behavior of the semi-major axis and eccentricity of the asteroid in the moment of the collision with Earth. These elements were obtained directly from the numerical integrator. Figure 2 shows an example of behavior of the relative distance between asteroid and Earth over time of simulation. We present only the moment of collision and how changes occur with variations in velocity of the asteroid.

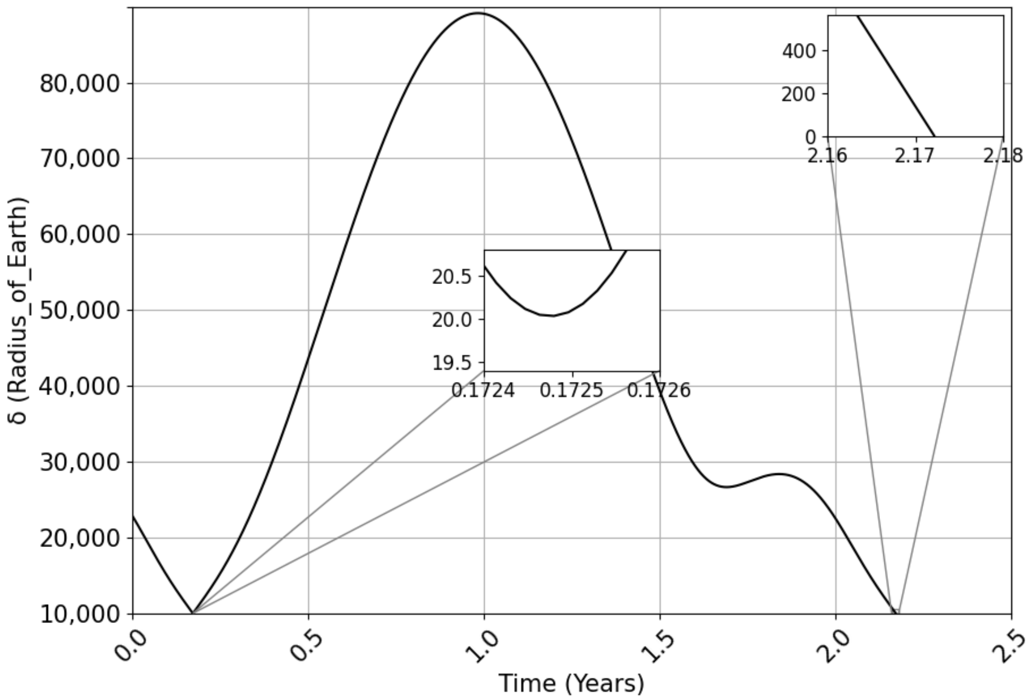

Figure 2. Relative distances between the asteroid and Earth (δ) in relation to the time for the scenario where the collision occurs after the first encounter in a period of 2 years. The first zoom shows the first encounter between asteroid and Earth, and the second zoom shows the moment of the collision. This scenario occurs by applying a velocity variation of -50 mm/s to the mean anomaly of 352°.

4. Results

To understand the scenario, we will show the relative distances between the asteroid and Earth at the moment of impact with Earth, that is, two years after the first encounter, for three scenarios where we applied the impulses: (i) the impulse was applied 63 days before the first close encounter (Figure 3a), or (ii) the impulse was applied upon the first close encounter, which means during the minimum Earth–asteroid distance (Figure 3b), or (iii) the impulse was applied 63 days after the first close encounter (Figure 3c).

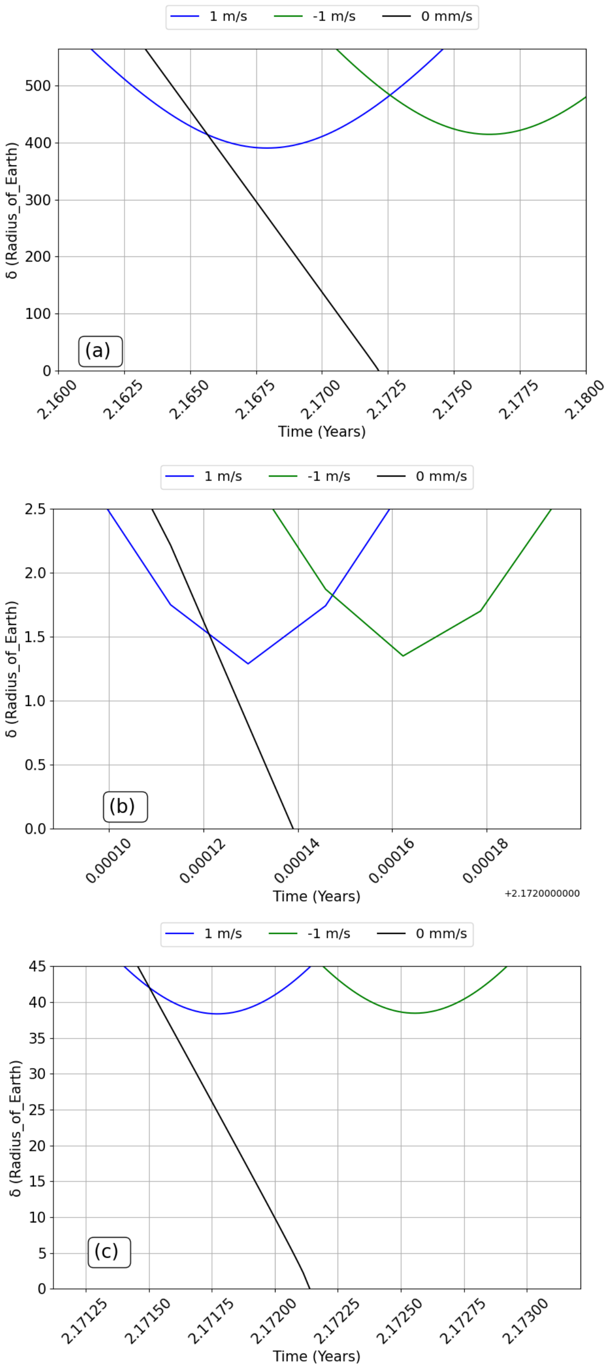

Figure 3. Distances (in proportion to the radius of the Earth) between asteroid and Earth for a moment of collision. (**a**) Impulse applied 63 days before the first encounter, (**b**) impulse applied at the first encounter and (**c**) impulse applied 63 days after the first encounter. Those figures show how much the distance between the asteroid and Earth varied after the impulses were applied.

We found that, by applying the impulses before the first close encounter, Figure 3a, we obtained a considerable distance of the asteroid from Earth: about 414 times the radius of Earth for the negative impulse and about 390 times the radius of Earth for the positive impulse. If we compare these results with the literature, we find variations of around 1 times the radius of Earth. It means that the use of a previous close approach to the Earth is a very effective technique to change the trajectory of the asteroid. Other considerable results were also found when we applied the impulse after the first close encounter, as shown in Figure 3c. However, looking at Figure 3b, we can see that if the impulse is applied when the asteroid passes at the closest approach to the Earth, the deviations are much smaller compared to the scenarios presented in Figure 3a,c. It is only about 1.35 times the radius of Earth for the negative impulse and 1.29 times the radius of Earth for the positive impulse.

These distances can be better observed in Figure 4, where we have the smallest distance in the second passage, the dangerous one, when the collision would occur. Figure 4 shows the larger differences in the final results with respect to the application of the impulse. It is evident that applying the impulses before the first close encounter helps in the deviation of the orbit of the asteroid. This is due to the fact that an impulse applied before the passage can adjust the geometry of the closest approach to maximize the variations in the trajectory of the asteroid, so making the next passage be a much larger distance from the Earth.

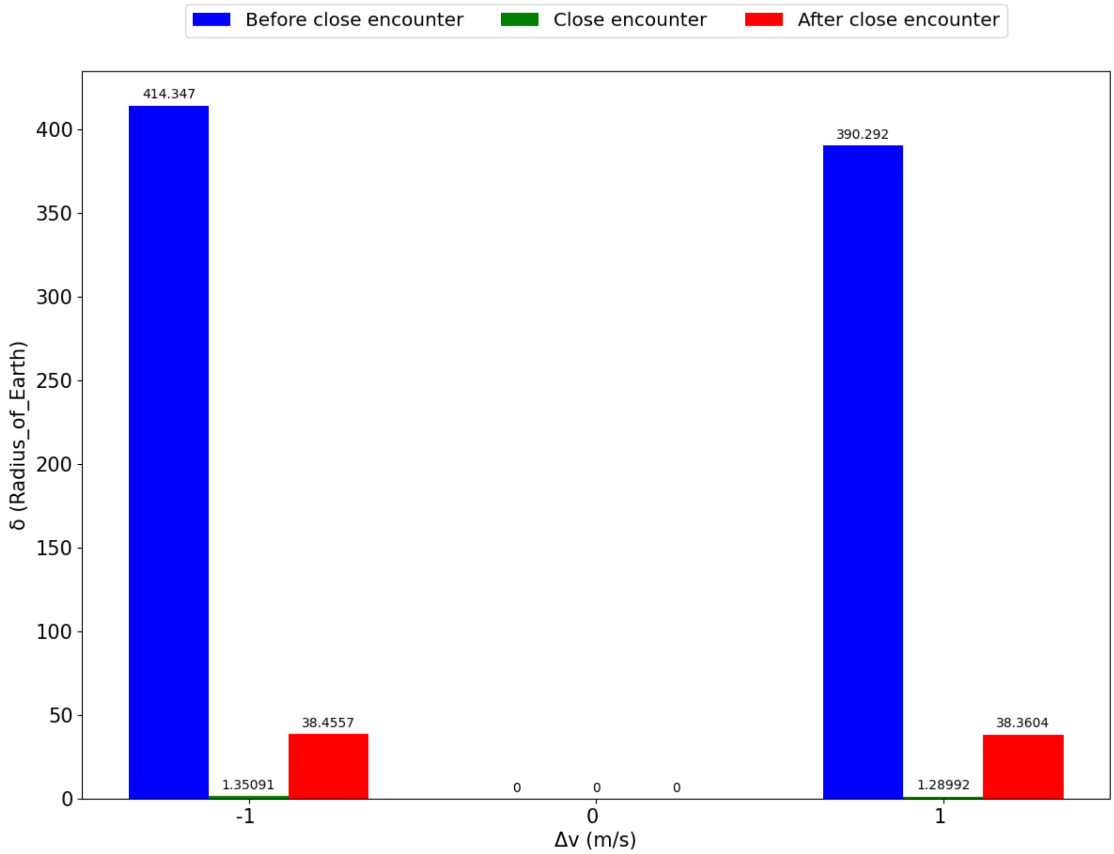

Figure 4. Distance between asteroid and Earth on the “potential collision” passage after 2 years of the maneuver. In blue we have the situation where the impulse is applied 63 days before the first encounter, in green we have an impulse applied exactly in the first encounter and in red we have the situation where the impulse is applied 63 days after the first encounter.

Our proposal is then directly related to the gravity-assisted maneuver [20–25], where the asteroid will make a swing-by with the Earth that changes its orbit and avoid the collision to occur.

Therefore, a new question arises: if we apply the impulse before the encounter, can we move the asteroid away by a satisfactory magnitude in the next passage? What is the smallest impulse we need to apply to the asteroid to avoid a collision with Earth?

This question is important due to the size of the impactor that generates this variation in velocity, which is highly questioned due to the impossibility of launching an impactor with a mass large enough to generate large impulses.

We performed simulations starting from 63 days before the first close encounter between asteroid and Earth, and we varied the velocity of the asteroid with impulse magnitudes starting from 1 m/s. After that, we decreased the scale, and we found that the minimum magnitude that can deflect the asteroid using an impulse before the close encounter to take advantage of the gravitational effects is 4 mm/s, which is contrary to the velocity vector of the asteroid or in the same direction. The minimum asteroid–Earth distances found were 1.30 and 1.25 times the radius of Earth, for the respective impulses. Those are small deviations, but larger ones can be obtained with very small impulses. Figure 5 shows those results. The minimum Earth–asteroid distance increases almost linearly with the magnitude of the impulse. It is visible that a deviation of 7.32637 times the radius of the Earth can be obtained with an impulse of just 19 mm/s, a very good result for only about 2 years of warning time. Of course, this technique requires an earlier passage of the asteroid by the Earth before the passage ending in collision, but it is a good alternative for those cases. These are small variations, and they should be studied in greater detail, but it shows that we found a situation where, taking advantage of the swing-by, we do not need large impulses, even for short warning times. The point is now to know that, if we apply the impulse 20 days or 30 days before the close encounter, what would be the smallest impulse to avoid the collision? Our simulations show that the smallest impulse to avoid a collision when applying the impulse 20 days before the close encounter is -3 cm/s, achieving a deviation of 1.3 times the radius of Earth. The smallest impulse required to avoid the collision when applying the impulse 30 days before the close encounter is -6 cm/s, reaching a deviation of 1.05 times the radius of Earth. As we can see, applying the impulse at the moment that we detect that the asteroid would collide with Earth, in 63 days before the first close encounter, gives a much smaller impulse.

It is worth mentioning that we are dealing here with a large gravitational perturbation, since the close encounter between the asteroid and Earth is considerably large, and that, under these conditions, deviations are larger. However, within the dynamic context, this may be a possible event. However, our main objective here is to understand how the powered swing-by can influence the asteroid deflection technique, so that we can use this technique for greater effectiveness with short warning periods.

To date, we have looked specifically at the distances between the asteroid and Earth. Figure 6 shows the behavior of the semi-major axis at the moment of the collision between the asteroid and the Earth, where the behavior it would have without the application of the impulse and with the applied impulses is presented. The semi-major axis of the orbit of the asteroid when applying the impulses opposite to the velocity vector of the asteroid (negative variations in velocity) are found in Figure 6a, and in the same direction of the velocity vector of the asteroid (positive variations in velocity), are found in Figure 6b. Figure 6a shows that the semi-major axis of the asteroid has large increases after the second passage, where, without the impulse, the semi-major axis would go from 1.6 to 1.9 au naturally and the collision would occur. However, when applying the impulses before and after the first encounter, the semi-major axis has a slight modification that can avoid the collision. Still, in Figure 6a, we can see an explicit increase in the semi-major axis for when we applied the impulse during the close encounter, where the semi-major axis has a peak that reaches 5.6 au and then has a reduction until it stabilizes again at 3.5 au. We know that in this passage, the distance between the asteroid and the Earth is about 1.35 au for the case where the impulse is applied exactly in the first encounter, and that the disturbance of the Earth in this second passage is very large. We will look more carefully at this result later.

Using Equation (5), we found that the energy variation for this case is $-229{,}767{,}429.8$ m^2/s^2 for the -1 m/s impulse and $-232{,}882{,}902.7$ m^2/s^2 for the impulse of 1m/s, considering that the approach angle (ψ) is 89.5°. Using Equation (4), we found that the analytical variations in the evolution of the semi-major axis are 1.93 au and 1.9 au, respectively. Looking at Figure 6a, we noticed that just before the peak in semi major axis occurs, the semi-major axis of the orbit of the asteroid is equal to the semi-major axis without the application of the impulse at 1.9 au, and then it continues to increase. Our numerical results are consistent with the analytical results.

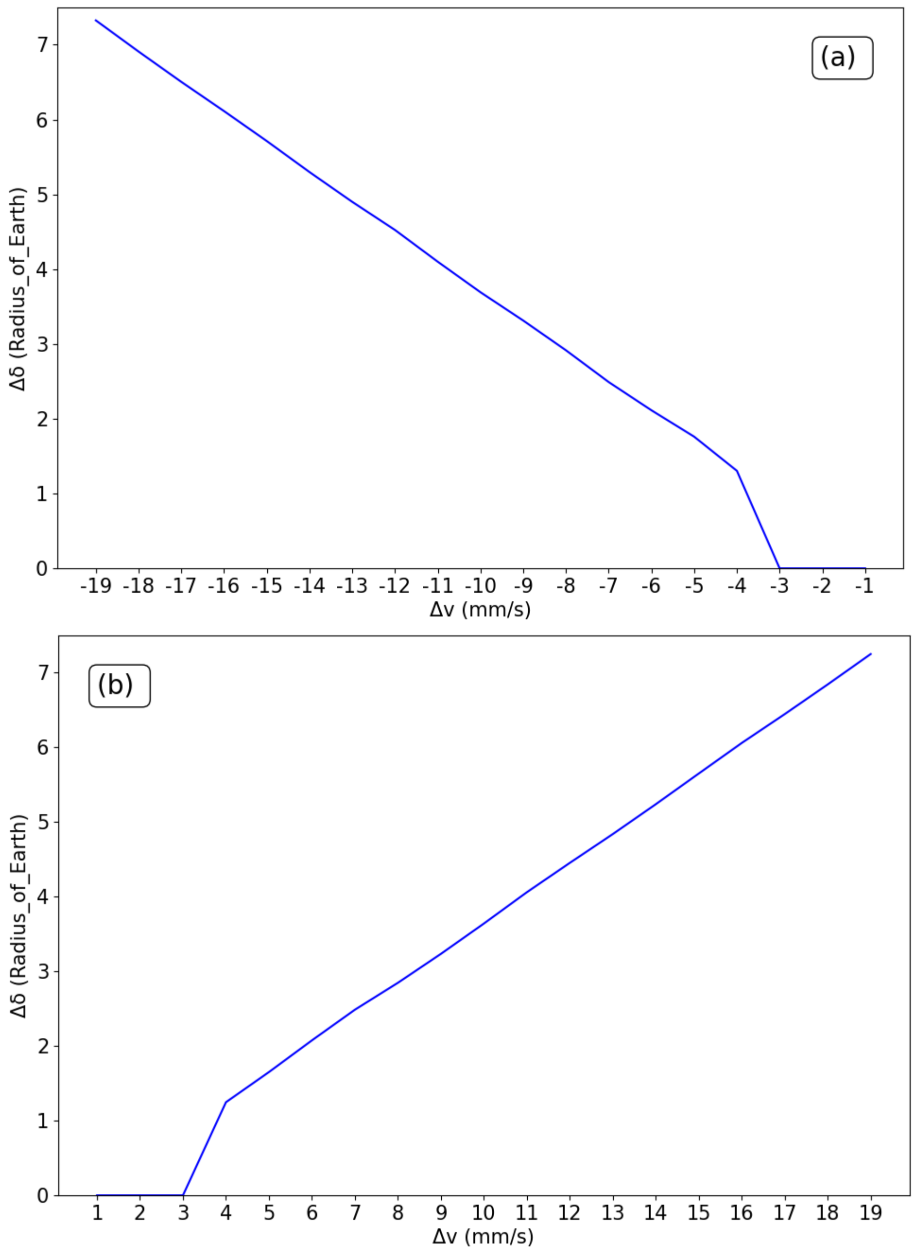

Figure 5. Minimum asteroid–Earth distance on the "potential collision" passage after 2 years of the maneuver, with the impulse applied 63 days before the first encounter. (**a**) Negative velocity variations. (**b**) Positive velocity variations.

Figure 6b also shows a large variation in the semi-major axis of the asteroid when we apply the impulse. However, we see that the semi-major axis has a small peak and starts to decrease after that, until it stabilizes around 1 au. Comparing the results for Figure 6a,b, we see that these results are expected, since the asteroid passed through different sides of the planet, a situation that can be seen in Figure 7b. In this figure, we can see that, when applying the impulse of 1 m/s in the orbit of the asteroid, it will return 2 years later and pass the by left of the planet, and its orbit will be directed to the right, after the encounter. For the -1 m/s impulse, the asteroid will pass by the right of Earth, and the swing-by will direct it to the left. This explains that the semi-major axis increases in one case and decreases in the other case. The high peak of the semi-major axis can be explained by the close approach between the asteroid and the Earth. When the asteroid gets close to Earth, it increase its velocity. Thus, its velocity with respect to the Sun also increases, so the semi-major axis gets larger. When the asteroid leaves the Earth, the opposite occurs and the semi-major axis decreases again.

Figure 7 shows the motion of the asteroid relative to the Earth. In this figure, we can see the moment of the collision of the asteroid with the Earth. We can also see the movement of the asteroid after the impulses in all situations, and observe that the trajectories are symmetric with respect to the Sun–Earth line. Figure 7a shows the movement of the asteroid when the impulse is applied 63 days before the first close encounter. We also see that the asteroid passes at a distance that we can consider safe, and we can assume that the disturbance that the asteroid will suffer at this time of approach to the Earth is not large enough to make a big change in its orbit. Figure 7b shows the movement of the asteroid in relation to Earth when the impulses are applied exactly in the first close encounter. In this case, we can see a great influence of the planet when the asteroid passes through it, completely changing the orbit of the asteroid, redirecting it to new trajectories. Figure 7c shows the motion of the asteroid relative to Earth when the impulses are applied 63 days after the first close encounter, and again, we see that the asteroid passes at a slightly larger distance from Earth and it does not suffer too much from the perturbation, making only small changes in its orbit after the encounter.

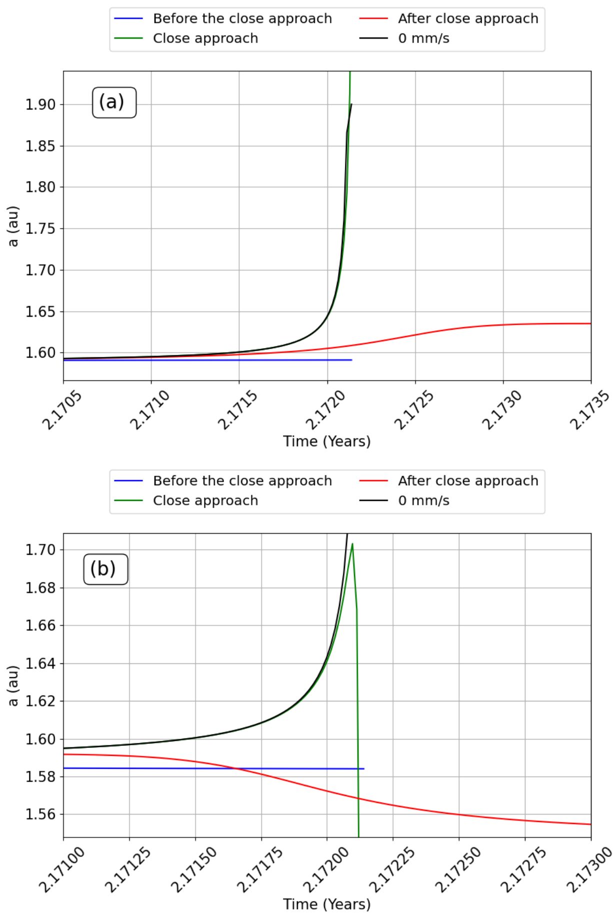

Figure 6. Semi-major axis of the asteroid (au) in the moment of collision. (**a**) Scenarios: -1 m/s impulse applied 63 days before the first close encounter, upon the first close encounter and 63 days after the first close encounter. (**b**) Scenarios: 1 m/s impulse applied 63 days before the first close encounter, upon the first close encounter and 63 days after the first close encounter.

We talked about the increase in velocity that the asteroid has in the second passage after the impulses, which generates a significant variation in the semi-major axis of the asteroid. This effect is even more evident in Figure 8a,b. It shows the behavior of the eccentricity of the asteroid in the moment that the collision occurs, first for the negative impulse, for the three points of application, and then for the positive impulse, also for the three points of application. In these figures, we can see that the same phenomenon occurs as that explained in Figure 6. When applying the impulses in the first close encounter, there is a large variation in the eccentricity in the second passage, at the moment that the collision would occur. However, when deflecting the asteroid, but not so significantly, the large variation in eccentricity changes the velocity of the asteroid and its orbital period. In Figure 3, we see that the asteroid has a maximum approach to Earth that is not at the same time as the collision. Figure 9a–c shows the time behavior of the orbital period of the asteroid in the moment that occurs the collision, for the respective positions where the impulses are applied.

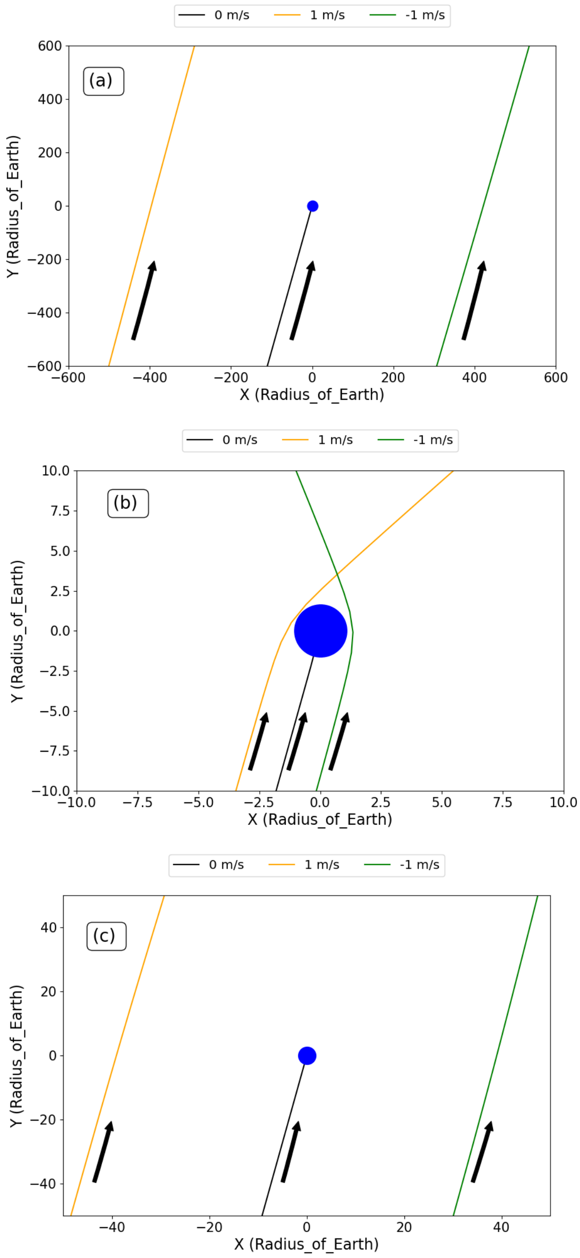

Figure 7. Trajectory of the asteroid relative to Earth at the second encounter, where the x and y axes are in multiples of the radius of Earth. The arrows indicate the direction of the trajectory of the asteroid and the blue circle represents the Earth. (**a**) Impulses applied 63 days before the first encounter, (**b**) impulses applied exactly during the first encounter and (**c**) impulses applied 63 days after the first encounter.

Looking at Figure 9a, we can see that without the impulse, there is a significant increase in the orbital period of the asteroid before the collision, which is an effect generated by the singularity. However, when we apply the impulses, we can see that the orbital period has small variations in the scenario before the second encounter. Figure 9b shows that there is a large increase in the orbital period of the asteroid for the negative impulse, and it decreases for the positive impulse. In Figure 9c, we can see that the orbital period of the asteroid decreases for the negative impulse and decreases even more for the positive impulse. These results were expected due to the velocity variation.

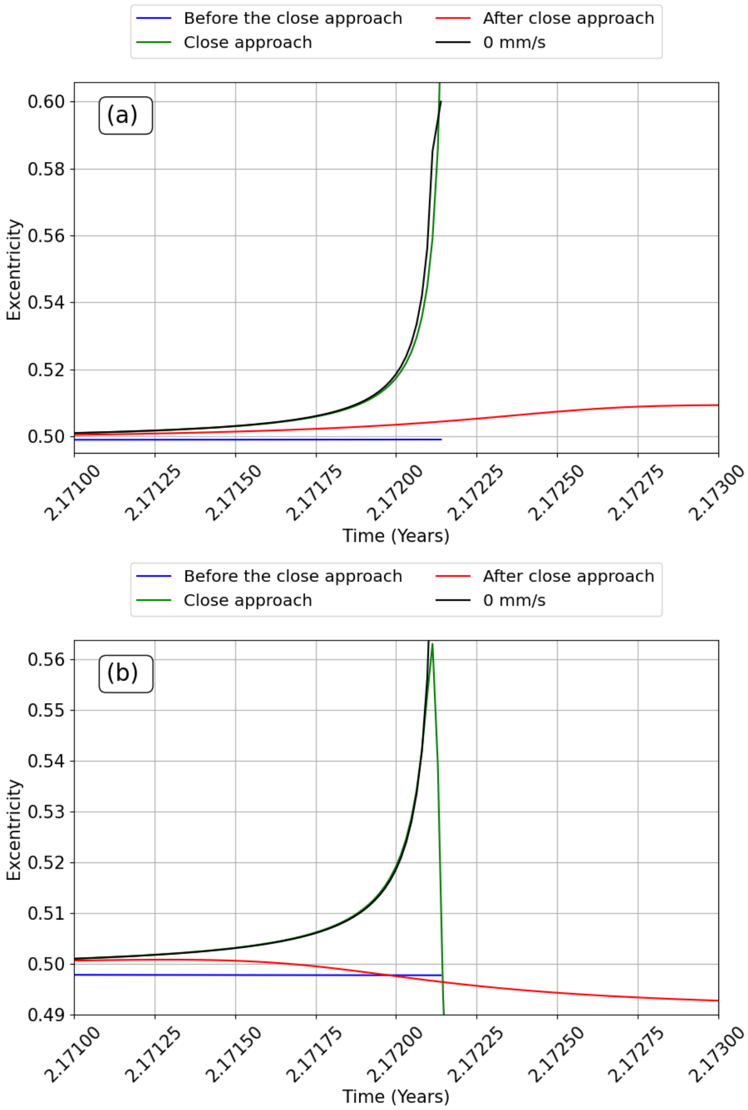

Figure 8. Eccentricity of the asteroid in the moment of collision. (**a**) Scenarios: −1 m/s impulse applied 63 days before the first close encounter, upon the first close encounter and 63 days after the first close encounter. (**b**) Scenarios: 1 m/s impulse applied 63 days before the first close encounter, upon the first close encounter and 63 days after the first close encounter.

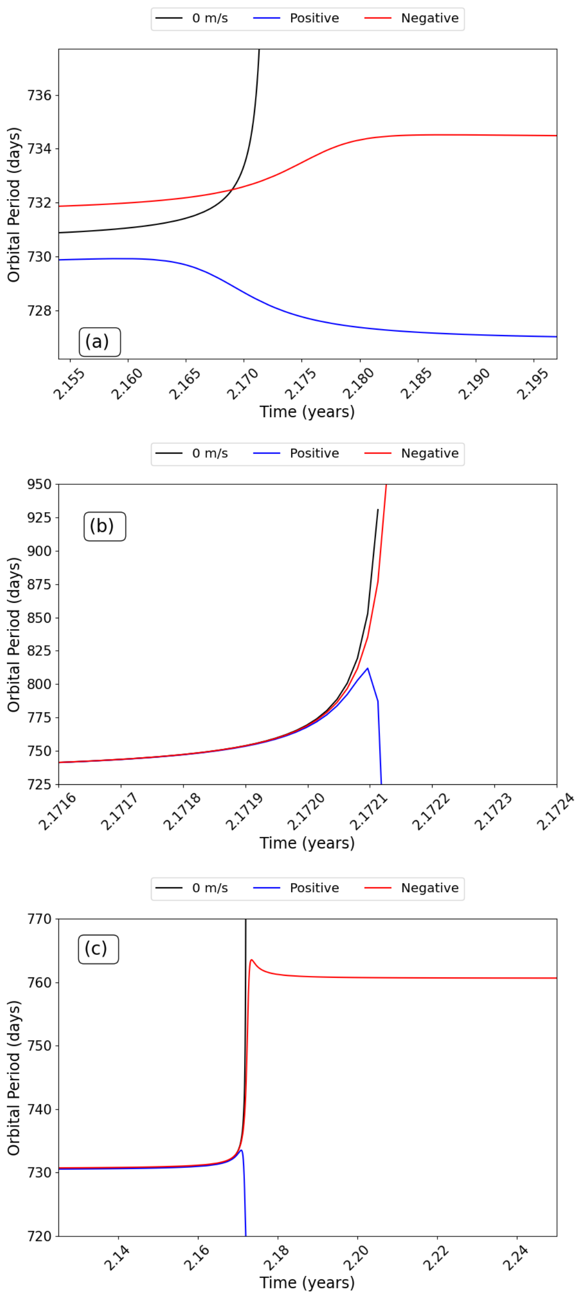

Figure 9. Orbital period of the asteroid as a function of time in the moment of collision. (**a**) Impulse applied 63 days before the first encounter, (**b**) impulse applied on the first encounter and (**c**) impulse applied 63 days after the first encounter.

5. Discussion

Taking advantage of a swing-by with Earth, it becomes feasible to use the kinetic impact deflection technique to deflect an asteroid on a collision course with Earth, even for a short warning period using small velocity variations. Remember that, for our study, the asteroid had its orbit altered by the swing-by and that the collision will occur due to this disturbance, which also shows that we must be prepared for all situations involving the PHAs where, in a possible close encounter, the asteroid may return again on another date and collide with Earth. Perhaps this return time is not long enough for an entire study if we know the scenario only at the time of the event.

Figure 3 allowed us to verify that it is possible to apply impulses to the orbit of the asteroid and changes its trajectory enough to avoid the collision. It can also be observed that, if we apply the impulse before the close encounter (Figure 3a), where the swing-by will occur, the asteroid can be deflected by considerable distances, even for a warning period of only 2 years. This possibility becomes real from the perspective that, by applying the impulse before the swing-by, we will have little changes in its energy, which causes that, when passing through the minimum distance, it amplifies the gravitational disturbances, causing the asteroid to deviate. We have to point out that, due to the difference in mass between the asteroid and the Earth and knowing that the minimum distance that we are dealing with is 20 times the radius of Earth, it is expected that this swing-by has a magnitude high enough to change completely the orbit of the asteroid, and that, when applying the impulses before the close encounter, the variation in the perturbation is sufficient to change the trajectory of the asteroid and to avoid the collision.

The scenario where the impulse is applied exactly upon the close encounter shows that the asteroid did not have a large deflection. It was enough to avoid the collision, but the asteroid passed very close to Earth (Figure 3b). When comparing the results obtained here with the powered swing-by applied to a spacecraft, we see that the effects of applying the impulse at the closest approach in the spacecraft gives more energy in many cases. This is not the case for asteroids. The reason is that the transfer of energy of the impulse is given by $\vec{v}.\ \Delta\vec{v}$, where $\vec{v}$ is the velocity of the spacecraft or the asteroid, and $\Delta\vec{v}$ is the impulse. For the asteroid, $\Delta\vec{v}$ is much smaller than $\vec{v}$, compared to the spacecraft situation. Thus, it is much more efficient to use the $\Delta\vec{v}$ to adjust the geometry of the close approach than to give energy directly to the asteroid. Therefore, it is better to apply the impulse before the closest approach.

One can still question why the asteroid had its orbit more disturbed when applying the impulse after the close encounter (Figure 3c) in relation to the impulse applied in the close encounter? At this time, the asteroid has already passed the close encounter and starts to move away from Earth. However, it is still possible to change the angle of curvature of the maneuver, which is a key parameter for the variation of energy, and so the semi-major axis of the orbit of the asteroid.

It is clear that smaller impulses are required when they are applied earlier. However, even weak disturbances applied for longer times can dominate the dynamics. Thus, an accurate model for the orbital motion can be critical for the proposed Earth defense method. This is dependent on the period of integration, where, when we make integrations for a period of 100 years, for example, it is critical to have a more accurate model, but if we are integration for shorter times such as that in the present paper, we do not need an accurate model.

6. Conclusions

Based on the results presented here, we were able to show the influence of a swing-by of the Earth on a passage before a collision on the orbit of an asteroid. It can be used to avoid a collision. By applying the impulse and taking advantage of this gravitational effect, we can avoid the collision in a period of 2 years with a velocity variation that can be considered small. We also showed that, if we apply the impulse before the close encounter, it is possible use a smaller $\Delta\vec{v}$ compared to other locations, and still avoid the collision. This is very interesting, because we can reduce the size of the impactor used to generate the $\Delta\vec{v}$. With this, we achieved our goal of avoiding an asteroid collision with the Earth in a period of 2 years using $\Delta\vec{v}$ in the order of 4 mm/s. It is impossible to use a $\Delta\vec{v}$ that is very low without the technique of using a previous swing-by around the Earth in short warning times. We tried to apply the impulse 20 and 30 days before the passage, but the $\Delta\vec{v}$s required were much higher, in the order of cm/s.

Author Contributions: B.S.C. wrote the main version of the manuscript, generated data and prepared the figures. B.S.C., A.F.B.d.A.P. and O.C.W. contributed on its conception and analysis, and interpretation of data and revisions. All authors have read and agreed to the published version of the manuscript.

Funding: The publication has been prepared with the support of the RUDN University Strategic Academic Leadership Program. We would like to thank FAPESP proc. 2016/024561-0 and 2018/17864-1; CNPq proc. 305210/2018-1 and 309089/2021-2 and CAPES for funding and contributing to this work.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: All data generated or analyzed during this study are included in this published article in the form of figures. For more information, contact Bruno Chagas Santos (bruno.ba.987@gmail.com or bruno.chagas@unesp.br).

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Ahrens, T.J.; Harris, A.W. Deflection and fragmentation of near-Earth asteroids. *Nature* **1992**, *360*, 429–433. [CrossRef]
2. Cheng, A.F.; Atchison, J.; Kantsiper, B.; Rivkin, A.S.; Stickle, A.; Reed, C.; Galvez, A.; Carnelli, I.; Michel, P.; Ulamec, S. Asteroid impact and deflection assessment mission. *Acta Astronaut.* **2015**, *115*, 262–269. [CrossRef]
3. Carusi, A. Early neo deflections: A viable, lower-energy option. *Earth Moon Planets* **2005**, *96*, 81–94. [CrossRef]
4. Maccone, C. Planetary defense from the nearest 4 lagrangian points plus rfi-free radioastronomy from the farside of the moon: A unified vision. *Acta Astronaut.* **2002**, *50*, 185–199. [CrossRef]
5. Gonzaga, E. A method of controlling asteroid collision with the Earth. *Cosm. Res.* **2010**, *48*, 459–466. [CrossRef]
6. Brophy, J.R.; Muirhead, B. Near-earth asteroid retrieval mission (ARM) study. In Proceedings of the 33rd International Electric Propulsion Conference, Washington, DC, USA, 6–10 October 2013.
7. Carusi, A.; Valsecchi, G.B.; D'Abramo, G.; Boattini, A. Deflecting NEOs in route of collision with the Earth. *Icarus* **2002**, *159*, 417–422. [CrossRef]
8. Carusi, A.; D'Abramo, G.; Valsecchi, G.B. Orbital and mission planning constraints for the deflection of NEOs impacting on Earth. *Icarus* **2008**, *194*, 450–462. [CrossRef]
9. Cheng, A.; Michel, P.; Jutzi, M.; Rivkin, A.; Stickle, A.; Barnouin, O.; Ernst, C.; Atchison, J.; Pravec, P.; Richardson, D.; et al. Asteroid impact & deflection assessment mission: Kinetic impactor. *Planet. Space Sci.* **2016**, *121*, 27–35.
10. Cheng, A.F.; Rivkin, A.S.; Michel, P.; Atchison, J.; Barnouin, O.; Benner, L.; Chabot, N.L.; Ernst, C.; Fahnestock, E.G.; Kueppers, M.; et al. AIDA DART asteroid deflection test: Planetary defense and science objectives. *Planet. Space Sci.* **2018**, *157*, 104–115. [CrossRef]
11. Cheng, A.F.; Stickle, A.M.; Fahnestock, E.G.; Dotto, E.; Della Corte, V.; Chabot, N.L.; Rivkin, A.S. DART mission determination of momentum transfer: Model of ejecta plume observations. *Icarus* **2020**, *352*, 113989. [CrossRef]
12. Gibbings, A.; Vasile, M. Kinematic Impactors–Improved Modelling of Asteroid Deflection. In Proceedings of the European Planetary Science Congress 2010, Rome, Italy, 20–23 September 2010; p. 116.
13. Hirabayashi, M.; Davis, A.B.; Fahnestock, E.G.; Richardson, D.C.; Michel, P.; Cheng, A.F.; Rivkin, A.S.; Scheeres, D.J.; Chesley, S.R.; Yu, Y.; et al. Assessing possible mutual orbit period change by shape deformation of Didymos after a kinetic impact in the NASA-led Double Asteroid Redirection Test. *Adv. Space Res.* **2019**, *63*, 2515–2534. [CrossRef]
14. Sokolov, L.; Kuteeva, G.; Petrov, N.; Eskin, B.; Balyaev, I.; Vasil'ev, A. On the Characteristics of Singular Trajectories of the Asteroid Apophis and the Possibility of Deflecting It to Avoid Collisions with the Earth. *Sol. Syst. Res.* **2021**, *55*, 259–265. [CrossRef]
15. Wlodarczyk, I. Possible Impact Solutions of the Dangerous Asteroid 29075 (1950 DA). *LPI Contrib.* **2022**, *2678*, 2023.
16. Fahnestock, E.G.; Scheeres, D.J. Dynamical characterization and stabilization of large gravity-tractor designs. *J. Guid. Control. Dyn.* **2008**, *31*, 501–521. [CrossRef]
17. Ledkov, A.; Eysmont, N.; Boyarsky, M.; Dunham, D.W.; Nazirov, R.; Fedyaev, K. Small near Earth asteroids and gravity assist maneuvers as basic constituents of planetary defense against hazardous sky objects. In Proceedings of the SpaceOps 2014 Conference, Pasadena, CA, USA, 5–9 May 2014; p. 1834.
18. Ketema, Y. A mass-optimized gravity tractor for asteroid deflection. *arXiv* **2021**, arXiv:2110.04631.
19. Chagas, B.S.; Prado, A.F.B.A.; Winter, O.C. Gravitational perturbations correlated with the asteroid kinetic impact deflection technique. *Sci. Rep.* **2022**, *12*, 1–17. [CrossRef]
20. Ferreira, A.F.; Prado, A.F.; Winter, O.C. A numerical mapping of energy gains in a powered Swing-By maneuver. *Nonlinear Dyn.* **2017**, *89*, 791–818. [CrossRef]
21. Ferreira, A.F.; Prado, A.F.; Winter, O.C.; Santos, D.P. Effects of the eccentricity of the primaries in powered Swing-By maneuvers. *Adv. Space Res.* **2017**, *59*, 2071–2087. [CrossRef]
22. Silva, A.; Prado, A.; Winter, O. Powered swing-by maneuvers around the Moon. *J. Phys. Conf. Ser.* **2013**, *465*, 012001. [CrossRef]
23. Prado, A.F.B.d.A.; Broucke, R. Classification of swing-by trajectories using the Moon. *ASME. Appl. Mech. Rev.* **1995**, *48*, S138–S142. [CrossRef]
24. Negri, R.B.; Prado, A.F.B.d.A.; Sukhanov, A. Studying the errors in the estimation of the variation of energy by the "patched-conics" model in the three-dimensional swing-by. *Celest. Mech. Dyn. Astron.* **2017**, *129*, 269–284. [CrossRef]

25. Gomes, V.M.; Prado, A. A study of the impact of the initial energy in a close approach of a cloud of particles. *WSEAS Trans. Math.* **2010**, *9*, 811–820.
26. Chambers, J.E. A hybrid symplectic integrator that permits close encounters between massive bodies. *Mon. Not. R. Astron. Soc.* **1999**, *304*, 793–799. [CrossRef]
27. Szebehely, V. *Theory of Orbits*; Academic Press: New York, NY, USA, 1967.
28. Broucke, R. The celestial mechanics of gravity assist. In Proceedings of the Astrodynamics Conference, Minneapolis, MN, USA, 15–17 August 1988; p. 4220.
29. Conway, B.A. Near-optimal deflection of earth-approaching asteroids. *J. Guid. Control. Dyn.* **2001**, *24*, 1035–1037. [CrossRef]