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Editorial

Symmetry in Chaotic Systems and Circuits II

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Today, one of the most studied phenomena of nonlinear dynamical systems is Chaos. As it is known from the literature, for the case of deterministic chaos to exist, a nonlinear dynamical system must have a dense set of periodic orbits and be transitive and sensitive to initial conditions. Furthermore, during the last decade, the significance of this kind of dynamical systems has been increased because of their applications in diverse fields ranging from living systems, such as synchronization in neurobiology, chemical reactions, and social, economical, or political events to nonliving systems including low power high-speed data transceivers, robotics, encrypted systems, secure communications, and chaotic electrochemical oscillators.

In addition, symmetry, which is a traditional and highly developed area of Mathematics, seems to give special features in many systems, when it is reported. In this direction the last few years, the research community tries to find out the importance of symmetry in many physical, biological, chemical, and mechanical chaotic nonlinear systems. Especially, the design of chaotic systems and circuits with symmetric nonlinear terms or the study of system's equilibria with symmetry, in the case of self-excited attractors, or symmetric line of equilibria, in the case of hidden attractors, have gained a lot of attention in the last decade.

The overall purpose of the second volume of this Special Issue lies in presenting the latest scientific advances in nonlinear chaotic systems and circuits that introduce various kinds of symmetries. Therefore, the study of chaotic systems with symmetry as well as applications of chaotic systems and circuits with symmetries is presenting in this Special Issue. The volume has eight published papers, where the authors are from geographically distributed countries (Egypt, Canada, China, Greece, India, Iran, Iraq, Italy, Malta, Mexico, Nigeria, Qatar, Saudi Arabia, Taiwan, Turkey, the UK, United Arab Emirates, and Vietnam). This reflects the high impact of the proposed topic and the seniority in organization of this Special Issue.

In the first paper of this Special Issue entitled "A Chaotic Quadratic Oscillator with Only Squared Terms: Multistability, Impulsive Control, and Circuit Design," D. Veeman et al. propose a chaotic quadratic oscillator with only squared terms. The chaotic oscillator has eight equilibrium points, and none of them is stable. Various bifurcation diagrams of the oscillator are investigated, and its Lyapunov Exponents (LEs) are discussed. The multistability of the oscillator is also investigated by plotting the bifurcation diagrams with various initiation methods. The basin of attraction of the oscillator is presented in two planes. Furthermore, the impulsive control method is applied to the oscillator in order to control its chaotic dynamics. Additionally, the system's circuitry is implemented to reveal its feasibility [1].

In the next paper "Applicable Image Security Based on New Hyperchaotic System," Jingya Wang et al. present a two-dimensional Sine Coupling Logistic Modulated Sine (2D-SCLMS) system, which is based on Logistic and Sine map. By a series of analyses, including Lyapunov Ecponent (LE), 0–1 test, two complexity analysis methods, and two entropy analysis methods, it is concluded that the new 2D-SCLMS map is hyperchaotic with a wider range of chaos and more complex randomness. The new system combined

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with two-dimensional Logistic-Sine Coupling Mapping (2D-LSCM) is further applied to an image encryption application. SHA-384 is used to generate the initial values and parameters of the two chaotic systems. Symmetric keys are generated during this operation, which can be applied to the proposed image encryption and decryption algorithms. The encryption process and the decryption process of the new image encryption approaches mainly include pixel scrambling, exclusive NOR, and diffusion operations. Multiple experiments illustrate that this scheme has higher security and lower time complexity [2].

In the paper "Symmetric Oscillator: Special Features, Realization, and Combination Synchronization," Victor K. Tamba et al. present an oscillator with different special features and symmetry. The features and oscillator dynamics are discovered through different tools of nonlinear dynamics. An electronic circuit is designed to mimic the oscillator's dynamics. Moreover, the combined synchronization of two drives and one response oscillator is reported [3].

The next paper, entitled "The Dynamic Analysis of a Novel Reconfigurable Cubic Chaotic Map and Its Application in Finite Field," by Chanfu Wang et al. combines two cubic chaotic maps, which have a symmetrical and reconfigurable form in a digital circuit. The dynamic behavior of the cubic chaotic map and the corresponding digital cubic chaotic map are analyzed, and the reasons for the complex period and weak randomness of output sequences are studied. On this basis, the digital cubic chaotic map is optimized, and the complex periodic behavior is improved. In addition, a reconfigurable pseudorandom sequence generator based on the digital cubic chaotic map is constructed from the point of saving on consumption of logical resources. Through theoretical and numerical analysis, the pseudorandom sequence generator solves the complex period and weak randomness of the cubic chaotic map after digitization and give the output sequence better performance and less resource consumption, which lays the foundation for applying it to the field of secure communication [4].

In the next paper, "Control of a Symmetric Chaotic Supply Chain System Using a New Fixed-Time Super-Twisting Sliding Mode Technique Subject to Control Input Limitations," Bo Wang et al. study a new control scheme of supply chain systems. In the designed control input, limitations in control inputs, as well as robustness against uncertainties, are taken into account. The proposed scheme is equipped with a fixed time disturbance observer to eliminate the destructive effects of uncertainties and disturbances. Additionally, the super-twisting sliding mode technique guarantees the fixed-time convergence of the closed-loop system. After that, a symmetric supply chain system is presented, and its chaotic attractors are demonstrated. Finally, the proposed controller is applied to the symmetric supply chain system. Numerical simulations exhibit the proposed scheme's excellent performance even though the system is subjected to control input limitations and time-varying uncertainties [5].

The paper "FPGA Realization of the Parameter-Switching Method in the Chen Oscillator and Application in Image Transmission" by Vincent-Ademola Adeyemi et al. presents a parameter-switching technique, which is applied to control chaos in the Chen oscillator and as a decryption mechanism in a secure transmission system, in order to transmit RGB and grayscale images. The main contributions of this work are the FPGA realizations of the parameter-switching method and a secure image transmission system using a synchronized master and slave topology. The results of the parameter-switching technique and synchronization are verified using phase plots and time series. The chaos-encrypted image from the image transmission system is verified by using correlation, showing no relativity with the original image, while the recovery of the decrypted image has no loss of quality. The encryption and decryption system is symmetric, whereby the key is private. Furthermore, in this work, co-simulations are performed in Active-HDL with MATLAB/Simulink, while the target FPGA board is the Xilinx's Artix-7 AC701 [6].

In the next paper, "SIEA: Secure Image Encryption Algorithm Based on Chaotic Systems Optimization Algorithms and PUFs," Aina'u Shehu Muhammad and Fatih Özkaynak propose a new image encryption algorithm in order to address the security problems of

digital images. The aspect that differentiates the proposed algorithm from the thousands of image encryption algorithms in the literature is that it is designed within the framework of the provable security design principle. The provable security design approach has ensured that the proposed algorithm is theoretically secure with mathematical proof techniques. In addition to addressing the proposed architecture security concerns, the hybrid random number generator used as the key generator constitutes another unique aspect. This generator, which is designed using chaotic systems, physical unclonable functions, and optimization algorithms, stands out as the innovative aspect of the study. The statistical randomness properties of the proposed random number generator are tested using the NIST SP 800-22 Statistical Test Suite. Successful results are obtained for 15 tests in the test package. In addition, the success of these outputs is tested on a new image encryption algorithm. The security of the proposed algorithm is also tested from different angles using various experimental analyzes and a 12-step provable security analysis roadmap. Successful analysis results and performance measurements indicate that the proposed cryptographic components can be used in many information security applications and many future designs [7].

In the last paper, entitled "Fractional-Order Analysis of Modified Chua's Circuit System with the Smooth Degree of 3 and Its Microcontroller-Based Implementation with Analog Circuit Design," Junxia Wang et al. present a fractional-order system from a modified Chua's circuit system with the smooth degree of 3 proposed by Fu et al. Bifurcation analysis, multistability, and coexisting attractors in the fractional-order modified Chua's circuit are studied. In addition, a microcontroller-based circuit is implemented in real digital engineering applications by using the fractional-order Chua's circuit with the piecewise-smooth continuous system [8].

The Guest Editor hopes that the readers will delight in reading the second volume of this Special Issue focused on cutting-edge research on symmetry in chaotic systems and circuits. We expect that the collected works will motivate researchers to strive for further advances in this emerging area.

Finally, the Guest Editor would like to thank all the authors of their papers submitted to this Special Issue, as well as all the anonymous reviewers, some of whom helped with multiple review assignments. Additionally, the Guest Editor would like to thank the journal's Editorial Board for being very encouraging and accommodative regarding this Special Issue.

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