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A Differential Operator Associated with q-Raina Function

Adel A. Attiya [1,2,*], Rabha W. Ibrahim [3], Abeer M. Albalahi [1], Ekram E. Ali [1,4] and Teodor Bulboacă [5]

1 Department of Mathematics, College of Science, University of Ha'il, Ha'il 81451, Saudi Arabia; a.albalahi@uoh.edu.sa (A.M.A.); ekram_008eg@yahoo.com (E.E.A.)
2 Department of Mathematics Faculty of Science, Mansoura University, Mansoura 35516, Egypt
3 The Institute of Electrical and Electronics Engineers (IEEE): 94086547, Portland, OR 97005, USA; rabhaibrahim@yahoo.com
4 Department of Mathematics and Computer Science, Faculty of Science, Port Said University, Port Said 42521, Egypt
5 Faculty of Mathematics and Computer Science, Babeş-Bolyai University, 400084 Cluj-Napoca, Romania; bulboaca@math.ubbcluj.ro
* Correspondence: aattiy@mans.edu.eg

Abstract: The topics studied in the geometric function theory of one variable functions are connected with the concept of Symmetry because for some special cases the analytic functions map the open unit disk onto a symmetric domain. Thus, if all the coefficients of the Taylor expansion at the origin are real numbers, then the image of the open unit disk is a symmetric domain with respect to the real axis. In this paper, we formulate the q-differential operator associated with the q-Raina function using quantum calculus, that is the so-called *Jacksons' calculus*. We establish a new subclass of analytic functions in the unit disk by using this newly developed operator. The theory of differential subordination inspired our approach; therefore, we geometrically explore the most popular properties of this new operator: subordination properties, coefficient bounds, and the Fekete-Szegő problem. As special cases, we highlight certain well-known corollaries of our primary findings.

Keywords: quantum calculus; analytic function; subordination and superordination; differential subordination; univalent function; differential operator; convolution (Hadamard) product; fractional calculus; Fekete-Szegő functional; Mittag–Leffler function; Gaussian hypergeometric function

JEL Classification: Primary 30C45; 30C80; Secondary 30C50; 05A30; 33E12; 33C05

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1. Introduction and Preliminaries

Quantum calculus (QC) is a subject of mathematical analysis and its applications are relevant in mathematics and physics. The functions of q-differentiation and q-integration were first defined and enhanced by Jackson [1,2]. Then, Ismail et al. [3] adopted the concept of q-calculus ($0 < q < 1$) into geometric function theory. Researchers are now using the QC to introduce and develop new Ma-Minda type of subclasses of functions. Based on the concept of q-derivatives, Seoudy and Aouf [4] defined one type of quantum starlike function subclass. Zainab et al. [5] used recently a unique method to introduce and develop useful q-stalikeness criteria, and Samir et al. [6] investigated many types of q-starlike functions that are dominated by 2D-Julia set.

QC is also used to generalize a variety of differential and integral operators, including special functions (see [7–11]). Noor and Razzaque, for example, defined a q-differential operator based on the q-Mittag-Leffler function [11]. Tang et al. studied significant properties of the q-starlike functions [12], Karthikeyan et al. [13] investigated the q-higher order derivatives, and Riaz et al. [14] formulated interesting results for the q-starlike functions of negative order.

Many other studies are introduced in the field of geometric function theory including the Mittag–Leffler function and its generalizations (see [15–19]).

The topics studied in the geometric function theory of one variable functions are connected with the concept of Symmetry because for some special cases, the analytic functions map the open unit disk onto a symmetric domain. Thus, if all the coefficients of the Taylor expansion at the origin are real numbers, then the image of the open unit disk is a symmetric domain with respect to the real axis. Moreover, if the function is an odd one, then the image of $\mathbb{D}$ is a symmetric domain with respect to the origin. In this paper, we investigate how the q-Raina's function can be utilized to expand a differential operator in the open unit disk.

For two functions h and g analytic in $\mathbb{D}$, we say that the function h is *subordinate* to g, written $h(z) \prec g(z)$, if there exists a *Schwarz function* ω, which is analytic in $\mathbb{D}$ with $\omega(0) = 0$ and $|\omega(z)| < 1$, $z \in \mathbb{D}$, such that $h(z) = g(\omega(z))$ for all $z \in \mathbb{D}$. In particular, if the function g is univalent in $\mathbb{D}$, then we have the following equivalence relation (cf., e.g., [20], see also [21])

$$h(z) \prec g(z) \Leftrightarrow h(0) \prec g(0) \quad \text{and} \quad h(\mathbb{D}) \subset g(\mathbb{D}).$$

Let us define the normalized class Λ of analytic functions as follows:

$$h(z) = z + \sum_{n=2}^{\infty} a_n z^n, \; z \in \mathbb{D}, \tag{1}$$

where the set $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ represents the open unit disk in the complex plane $\mathbb{C}$.

The *convolution (or Hadamard) product* of the functions $h(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and $v(z) = z + \sum_{n=2}^{\infty} b_n z^n$ of Λ is defined by (see [22])

$$(h * v)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n, \; z \in \mathbb{D}.$$

Moreover, denote by $\mathcal{S}^*$ the class of starlike functions, and by $\mathcal{C}$ the class of convex functions in $\mathbb{D}$ is normalized with the conditions given by (1), that are $h(0) = h'(0) - 1 = 0$. Moreover, let

$$\mathcal{P} := \left\{ l : l(z) = 1 + l_1 z + l_2 z^2 + \ldots, \; \operatorname{Re} l(z) > 0, \; z \in \mathbb{D} \right\}$$

denote the well-known class of Carathéodory functions (see [23,24]).

Definition 1 ([1])**.** *The* Jackson derivative *of a function h is defined by*

$$(\eth_q) h(z) := \frac{h(z) - h(qz)}{z(1-q)}, \; q \in (0,1).$$

Therefore, since

$$\eth_q (z^k) = \frac{1-q^k}{1-q} z^{k-1}, \; k \in \mathbb{N}_0 := \mathbb{N} \cup \{0\},$$

if the function h has the form (1) it follows that

$$(\eth_q) h(z) = 1 + \sum_{n=2}^{\infty} a_n [n]_q z^{n-1},$$

where

$$[n]_q := \frac{1-q^n}{1-q}.$$

Moreover, note that

$$\eth_q \kappa = 0, \quad \lim_{q \to 1^-} (\eth_q)\, h(z) = h'(z),$$

if $\kappa \in \mathbb{C}$ is a constant.

If $t \in \mathbb{C}$, then the *q-shifted factorial* (see [1]) is given by the formula

$$(t;q)_\tau := \prod_{j=0}^{\tau-1} \left(1 - q^j t\right),\ \tau \in \mathbb{N} := \{1, 2, \dots\}, \quad (t;q)_0 = 1. \tag{2}$$

From (2), the *q-shifted gamma function* could be formulated as follows:

$$\left(q^t; q\right)_\tau = \frac{\Gamma(t+\tau)(1-q)^\tau}{\Gamma_q(t)}, \quad \Gamma_q(t) = \frac{(q;q)_\infty (1-q)^{1-t}}{(q;q)_\infty},$$

where

$$\Gamma_q(t+1) = \frac{\Gamma_q(t)(1-q)}{1-q},\ q \in (0,1),$$

and

$$(t;q)_\infty = \prod_{j=0}^{\infty} \left(1 - q^j t\right).$$

Special functions include some improper integrals and the outputs of many different types of differential equations. As a result, most integral sets include descriptions of special functions, and these special functions include the most basic integrals and the integral representation of special functions. Because differential operators are important in both physics and mathematics, the theory of special functions is tightly linked to various mathematical physics topics [25]. To begin, we'll look at the Mittag–Leffler function, which is a well-known special function.

Definition 2 ([26])**.** *The power series that determines the* Raina's function *is defined by*

$$_{\mathcal{M}}\mathcal{F}_{a,b}(z) = \sum_{n=0}^{\infty} \frac{\mathcal{M}(n)}{\Gamma(an+b)}\, z^n,\ z \in \mathbb{D},$$

where $a, b \in \mathbb{C}$ with $\operatorname{Re} a \geq 0$, $\operatorname{Re} b > 0$, and $\{\mathcal{M}(n)\}_{n \in \mathbb{N}_0}$ is a bounded sequence of arbitrary real or complex numbers.

Remark 1. *1. If we take $\mathcal{M}(n) \equiv 1$ for all $n \geq 0$, then the above definition leads us to the Mittag–Leffler function*

$$\mathcal{F}_{a,b}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(an+b)},\ z \in \mathbb{D};$$

2. For the special case $a = 1$, $b = 1$, and $\mathcal{M}(n) = \frac{(a)_n (b)_n}{(c)_n}$, where $(k)_n$ represents the Pochhammer symbol, the function of the Definition 2 reduces to the Gaussian hypergeometric function

$$_2F_1(a, b; c; z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{\Gamma(n+1)},\ z \in \mathbb{D}.$$

Assuming that $\mathcal{M}(0) \neq 0$, define the function $_{\mathcal{M}}\mathbb{F}_{a,b}$ to be the normalized function obtained from $z\,\mathcal{F}_{a,b}(z)$ by

$$_{\mathcal{M}}\mathbb{F}_{a,b}(z) := \frac{\Gamma(b)}{\mathcal{M}(0)} \left(z\mathcal{F}_{a,b}(z)\right) = z + \sum_{n=2}^{\infty} \frac{\mathcal{M}(n-1)\Gamma(b)}{\mathcal{M}(0)\Gamma(a(n-1)+b)}\, z^n,\ z \in \mathbb{D}. \tag{3}$$

Note that if $\mathcal{M}(n) = (n+1)^{-r}$, $r \geq 0$, with $a = 0$ and $b = 1$, the operator (3) is the Sălăgean integral operator of order r (see [27]).

In the present article, first we will give a generalization of the normalized function $\mathbb{F}_{a,b}$ by using the q-gamma function, in order to be the q-Raina function, as follows:

$$_{q,\mathcal{M}}\mathbb{F}_{a,b}(z) := z + \sum_{n=2}^{\infty} \Phi_n(a,b,\mathcal{M},q)z^n,\ z \in \mathbb{D},$$

where

$$\Phi_n(a,b,\mathcal{M},q) := \frac{\mathcal{M}(n-1)\Gamma_q(b)}{\mathcal{M}(0)\Gamma_q(a(n-1)+b)}, \tag{4}$$

and $\mathrm{Re}\, a > 0$, $\mathrm{Re}\, b > 0$, $\mathcal{M}(0) \neq 0$.

In view of the quantum operator $\eth_q$, we introduce the following q-Raina differential operator $_{\mathcal{M}}\Delta_q^k : \Lambda \to \Lambda$ by

$$\begin{aligned}
&_{\mathcal{M}}\Delta_q^0(a,b)h(z) = h(z) * {}_{q,\mathcal{M}}\mathbb{F}_{a,b}(z),\\
&_{\mathcal{M}}\Delta_q^1(a,b)h(z) = z\eth_q\left({}_{\mathcal{M}}\Delta_q^0(a,b)h(z)\right),\\
&_{\mathcal{M}}\Delta_q^2(a,b)h(z) = {}_{\mathcal{M}}\Delta_q^1(a,b)\left({}_{\mathcal{M}}\Delta_q^1(a,b)h(z)\right),\\
&\ldots\\
&_{\mathcal{M}}\Delta_q^k(a,b)h(z) = {}_{\mathcal{M}}\Delta_q^1(a,b)\left({}_{\mathcal{M}}\Delta_q^{k-1}(a,b)h(z)\right),\ h \in \Lambda,\ k \in \mathbb{N},\ k \geq 2.
\end{aligned} \tag{5}$$

Using the above definition, it follows that if $h \in \Lambda$ has the form (1), then

$$\begin{aligned}
{}_{\mathcal{M}}\Delta_q^k(a,b)h(z) &= z + \sum_{n=2}^{\infty} [n]_q^k \frac{\mathcal{M}(n-1)\Gamma_q(b)}{\mathcal{M}(0)\Gamma_q(a(n-1)+b)} a_n z^n\\
&= z + \sum_{n=2}^{\infty} [n]_q^k \Phi_n(a,b,\mathcal{M},q) a_n z^n,\ z \in \mathbb{D},
\end{aligned}$$

where $\Phi_n(a,b,\mathcal{M},q)$ is given by (4) and

$$[n]_q^k := \left([n]_q\right)^k,\ k \in \mathbb{N}_0.$$

Note that, if $a = 0$ and $\mathcal{M}(n-1) = 1$ for all $n \geq 1$, we obtain the Sălăgean q-differential operator defined in [28]. Moreover, if $\mathcal{M}(n-1) = 1$ for all $n \geq 1$, we obtain the q-differential operator of [11]. With the aid of the q-differential operator defined by (5), we will define and study some new classes of analytic functions in the open unit disk.

Definition 3. *Let define by $\rho_{j,\wp}$ the convex analytic function in $\mathbb{D}$ as follows:*

$$\rho_{j,\wp}(z) := \begin{cases} \dfrac{1+z}{1-z}, & \text{if } j = 0,\\ \mho_1(j,\wp), & \text{if } j = 1,\\ \mho_2(j,\wp), & \text{if } 0 < j < 1,\\ \mho_3(j,\wp), & \text{if } j > 1, \end{cases}$$

where $\wp \in \mathbb{C} \setminus \{0\}$*, and the following functions are defined by (see [29])*

$$\mho_1(j,\wp)(z) = 1 + \frac{2\wp}{\pi^2}\left(\log\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\right)^2,$$

$$\mho_2(j,\wp)(z) = 1 + \frac{2\wp}{1-j^2}\sinh^2\left(\frac{2}{\pi}\arccos(j)\operatorname{arctanh}(\sqrt{z})\right),$$

$$\mho_3(j,\wp)(z) = 1 + \frac{\wp}{1-j^2} + \frac{\wp}{j^2-1}\sin\left(\frac{\pi}{2R(t)}\int_0^{\ell(z)/\sqrt{t}}\frac{d\zeta}{\sqrt{1-\zeta^2}\sqrt{1-(\zeta t)^2}}\right).$$

where $l(z) = \dfrac{z-\sqrt{t}}{1-\sqrt{tz}}$*,* $t \in (0,1)$*, is chosen such that* $t = \cosh\left(\dfrac{\pi R'(t)}{4R(t)}\right)$*, where* $R(t)$ *is the Legendre's complete elliptic integral of the first kind and* $R'(t)$ *is the complementary integral of* $R(t)$*, where* $(R'(t))^2 = 1 - (R(t))^2$*.*

Definition 4. *The function* $h \in \Lambda$ *is called to be in the class* $j - \mathcal{S}^k_{q,\wp}(a,b)$ *if and only if*

$$\frac{z\eth_q\left({}_{\mathcal{M}}\Delta^k_q(a,b)h(z)\right)}{{}_{\mathcal{M}}\Delta^k_q(a,b)h(z)} \prec \rho_{j,\wp}(z),$$

where (see also [29,30])

$$\rho_{j,\wp}(z) = 1 + \rho_1 z + \rho_2 z^2 + \ldots,\ z \in \mathbb{D}, \tag{6}$$

is defined in the Definition 3.

Definition 5. *The function* $h \in \Lambda$ *is called in the class* $j - \mathcal{S}^k_{\wp}(a,b)$ *if* $h \in j - \mathcal{S}^k_{q,\wp}(a,b)$ *and* $q \to 1^-$*.*

Lemma 1 ([31])**.** *Let* $G(z) = \sum\limits_{n=0}^{\infty} g_n z^n$ *be a univalent convex function in* $\mathbb{D}$ *satisfying the inequality*

$$H(z) = \sum_{n=0}^{\infty} h_n z^n \prec G(z).$$

Then, $|h_n| \leq |g_1|$ *for all* $n \geq 1$*.*

Lemma 2 ([32])**.** *Let* $P(z) = 1 + \sum\limits_{n=1}^{\infty} p_n z^n$ *be analytic in* $\mathbb{D}$ *satisfying the condition* $\operatorname{Re} P(z) > 0$*,* $z \in \mathbb{D}$*. Then,*

$$\left|p_2 - \Bbbk p_1^2\right| \leq 2\max\{1; |2\Bbbk - 1|\},\ \Bbbk \in \mathbb{C}.$$

2. Subordination and Coefficient Upper Bounds for the Class $j - \mathcal{S}^k_{q,\wp}(a,b)$

We start our first subordination result for the functions of the class $j - \mathcal{S}^k_{q,\wp}(a,b)$ when $q \to 1^-$, as follows:

Theorem 1. *If h is in the class* $j - \mathcal{S}^k_{\wp}(a,b)$*, then*

$$ {}_{\mathcal{M}}\Delta^k_q(a,b)h(z) \prec z\exp\left(\int_0^z \frac{\rho_{j,\wp}(\omega(\chi))-1}{\chi}d\chi\right),$$

where ω is a Schwarz function satisfying $\omega(0) = 0$ and $|\omega(z)| < 1$, $z \in \mathbb{D}$. Furthermore, for $|z| := \varrho < 1$ we have

$$\exp\left(\int_0^1 \frac{\rho_{j,\wp}(-\varrho) - 1}{\varrho} d\varrho\right) \leq \left|\frac{{}_{\mathcal{M}}\Delta_q^k(a,b)h(z)}{z}\right| \leq \exp\left(\int_0^1 \frac{\rho_{j,\wp}(\varrho) - 1}{\varrho} d\varrho\right).$$

Proof. If $z = 0$, the results in theorem are satisfied by defination of the subordination relation.

If $z \in \mathbb{D}$ and $z \neq 0$, since $h \in j - \mathcal{S}_\wp^k(a,b)$, then

$$\frac{\left({}_{\mathcal{M}}\Delta_q^k(a,b)h(z)\right)'}{{}_{\mathcal{M}}\Delta_q^k(a,b)h(z)} - \frac{1}{z} = \frac{\rho_{j,\wp}(\omega(z)) - 1}{z},$$

where ω is a Schwarz function satisfying $\omega(0) = 0$ and $|\omega(z)| < 1$, $z \in \mathbb{D}$.

Integrating both sides of the above relation it follows that

$${}_{\mathcal{M}}\Delta_q^k(a,b)h(z) \prec z\exp\left(\int_0^z \frac{\rho_{j,\wp}(\chi) - 1}{\chi} d\chi\right),$$

which is equivalent to

$$\frac{{}_{\mathcal{M}}\Delta_q^k(a,b)h(z)}{z} \prec \exp\left(\int_0^z \frac{\rho_{j,\wp}(\chi) - 1}{\chi} d\chi\right).$$

Since

$$\rho_{j,\wp}(-\varrho|z|) \leq \mathrm{Re}\left(\rho_{j,\wp}(\omega(z\varrho))\right) \leq \rho_{j,\wp}(\varrho|z|),$$

this yields

$$\int_0^1 \frac{\rho_{j,\wp}(-\varrho|z|) - 1}{\varrho} d\varrho \leq \int_0^1 \frac{\mathrm{Re}\left(\rho_{j,\wp}(\omega(z\varrho))\right) - 1}{\varrho} d\varrho \leq \int_0^1 \frac{\rho_{j,\wp}(\varrho|z|) - 1}{\varrho} d\varrho.$$

Combining the above inequalities we obtain

$$\int_0^1 \frac{\rho_{j,\wp}(-\varrho|z|) - 1}{\varrho} d\varrho \leq \log\left|\frac{{}_{\mathcal{M}}\Delta_q^k(a,b)h(z)}{z}\right| \leq \int_0^1 \frac{\rho_{j,\wp}(\varrho|z|) - 1}{\varrho} d\varrho,$$

and this leads to

$$\exp\left(\int_0^1 \frac{\rho_{j,\wp}(-\varrho) - 1}{\varrho} d\varrho\right) \leq \left|\frac{{}_{\mathcal{M}}\Delta_q^k(a,b)h(z)}{z}\right| \leq \exp\left(\int_0^1 \frac{\rho_{j,\wp}(\varrho) - 1}{\varrho} d\varrho\right).$$

□

The previous theorem represents a generalization of some earlier results, as we can see in the next two special cases:

Corollary 1 ([11], Theorem 6)**.** *If $\mathcal{M}(n) = 1$ for all $n \geq 1$, then*

$$\Delta_q^k(a,b)h(z) := {}_1\Delta_q^k(a,b)h(z) \prec z\exp\left(\int_0^z \frac{\rho_{j,\wp}(\chi) - 1}{\chi} d\chi\right),$$

where ω is a Schwarz function satisfying $\omega(0)=0$ and $|\omega(z)|<1$, $z\in\mathbb{D}$. Furthermore, for $|z|:=\varrho<1$ we have

$$\exp\left(\int_0^1 \frac{\rho_{j,\wp}(-\varrho)-1}{\varrho}d\varrho\right)\leq\left|\frac{\Delta_q^k(a,b)h(z)}{z}\right|\leq\exp\left(\int_0^1 \frac{\rho_{j,\wp}(\varrho)-1}{\varrho}d\varrho\right).$$

Corollary 2 ([33], Theorem 3.1)**.** *If $\mathcal{M}(n)=1$ for all $n\geq 1$ and $a=0$, $b=1$, then*

$$\Delta_q^k(0,1)h(z)\prec z\exp\left(\int_0^z \frac{\rho_{j,\wp}(\chi)-1}{\chi}d\chi\right),$$

where ω is a Schwarz function satisfying $\omega(0)=0$ and $|\omega(z)|<1$, $z\in\mathbb{D}$. Furthermore, for $|z|:=\varrho<1$ we have

$$\exp\left(\int_0^1 \frac{\rho_{j,\wp}(-\varrho)-1}{\varrho}d\varrho\right)\leq\left|\frac{\Delta_q^k(0,1)h(z)}{z}\right|\leq\exp\left(\int_0^1 \frac{\rho_{j,\wp}(\varrho)-1}{\varrho}d\varrho\right).$$

The following theorem gives us an upper bound for the Taylor coefficients of the functions from the class $j-\mathcal{S}_{q,\wp}^k(a,b)$:

Theorem 2. *If h belongs to the class $j-\mathcal{S}_{q,\wp}^k(a,b)$, then*

$$|a_2|\leq\frac{|\rho_1|}{[2]_q^k\Phi_2(a,b,\mathcal{M},q)\big([2]_q-1\big)},$$
$$|a_n|\leq\frac{|\rho_1|}{[n]_q^k\Phi_n(a,b,\mathcal{M},q)([n]_q-1)}\prod_{j=1}^{n-2}\left(1+\frac{|\rho_1|}{[j+1]_q-1}\right),\ n\geq 3,$$

where ρ_1 is defined by (6).

Proof. Letting

$$\frac{z\eth_q\left(_{\mathcal{M}}\Delta_q^k(a,b)h(z)\right)}{_{\mathcal{M}}\Delta_q^k(a,b)h(z)}=:P(z),\ z\in\mathbb{D},$$

where $P(z)=1+\sum\limits_{n=1}^{\infty}p_nz^n$, this brings that

$$z+\sum_{n=2}^{\infty}[n]_q^{k+1}\,\Phi_n(a,b,\mathcal{M},q)a_nz^n=\left(z+\sum_{n=2}^{\infty}[n]_q^k\,\Phi_n(a,b,\mathcal{M},q)a_nz^n\right)\left(1+\sum_{n=1}^{\infty}p_nz^n\right)$$
$$=\sum_{n=0}^{\infty}p_nz^{n+1}\cdot\sum_{n=0}^{\infty}p_nz^n\cdot\sum_{n=2}^{\infty}[n]_q^k\,\Phi_n(a,b,\mathcal{M},q)a_nz^n.$$

From the comparison of the coefficients of z^n of the above equality we obtain

$$[n]_q^{k+1}\,\Phi_n(a,b,\mathcal{M},q)a_n=[n]_q^k\,\Phi_n(a,b,\mathcal{M},q)a_n+\sum_{j=1}^{n-1}[j]_q^k\frac{\mathcal{M}(j-1)\Gamma_q(b)}{\mathcal{M}(0)\Gamma_q(a(j-1)+b)}a_j\,p_{n-j}.$$

Accordingly, we obtain

$$[n]_q^k([n]_q-1)\Phi_n(a,b,\mathcal{M},q)a_n=\sum_{j=1}^{n-1}[j]_q^k\frac{\mathcal{M}(j-1)\Gamma_q(b)}{\mathcal{M}(0)\Gamma_q(a(j-1)+b)}a_j\,p_{n-j},$$

and a calculation implies that

$$a_n = \frac{1}{[n]_q^k\big([n]_q - 1\big)\Phi_n(a,b,\mathcal{M},q)} \sum_{j=1}^{n-1} [j]_q^k \frac{\mathcal{M}(j-1)\Gamma_q(b)}{\mathcal{M}(0)\Gamma_q(a(j-1)+b)} a_j\, p_{n-j}.$$

In view of Lemma 1, since $|p_n| \leq |\rho_1|$, we obtain

$$|a_n| \leq \frac{|\rho_1|}{[n]_q^k\big([n]_q - 1\big)\Phi_n(a,b,\mathcal{M},q)} \sum_{j=1}^{n-1} [j]_q^k \frac{\mathcal{M}(j-1)\Gamma_q(b)}{\mathcal{M}(0)\Gamma_q(a(j-1)+b)} |a_j|.$$

For $n = 2$ we have

$$\begin{aligned} |a_2| &\leq \frac{|\rho_1|}{[2]_q^k\big([2]_q - 1\big)\Phi_2(a,b,\mathcal{M},q)} \sum_{j=1}^{1} [j]_q^k \frac{\mathcal{M}(j-1)\Gamma_q(b)}{\mathcal{M}(0)\Gamma_q(a(j-1)+b)} |a_j| \\ &= \frac{|\rho_1|}{[2]_q^k \Phi_2(a,b,\mathcal{M},q)\big([2]_q - 1\big)}, \end{aligned}$$

while if $n = 3$, then

$$|a_3| \leq \frac{|\rho_1|}{[3]_q^k \Phi_3(a,b,\mathcal{M},q)\big([3]_q - 1\big)} \Big(1 + [2]_q^k \Phi_2(a,b,\mathcal{M},q)|a_2|\Big).$$

Combining the last two inequalities we obtain

$$\begin{aligned} |a_3| &\leq \frac{|\rho_1|}{[3]_q^k \Phi_3(a,b,\mathcal{M},q)\big([3]_q - 1\big)} \left(1 + [2]_q^k \Phi_2(a,b,\mathcal{M},q)\left(\frac{|\rho_1|}{[2]_q^k \Phi_2(a,b,\mathcal{M},q)\big([2]_q - 1\big)}\right)\right) \\ &= \frac{|\rho_1|}{[3]_q^k \Phi_3(a,b,\mathcal{M},q)\big([3]_q - 1\big)} \left(1 + \frac{|\rho_1|}{[2]_q - 1}\right). \end{aligned}$$

Suppose that for a fixed $j \geq 2$ the next inequality is valid:

$$|a_j| \leq \frac{|\rho_1|}{[j]_q^k\, \Phi_j(a,b,\mathcal{M},q)([j]_q - 1)} \prod_{j=1}^{j-2} \left(1 + \frac{|\rho_1|}{[j+1]_q - 1}\right),\ n \geq 3.$$

Hence, we have

$$\begin{aligned} |a_{j+1}| \leq & \frac{|\rho_1|}{[j+1]_q^k\, \Phi_{j+1}(a,b,\mathcal{M},q)([j+1]_q - 1)} \cdot \left(1 + \frac{|\rho_1|}{[2]_q - 1} + \frac{|\rho_1|}{[3]_q - 1}\left(1 + \frac{|\rho_1|}{[2]_q - 1}\right) + \dots \right. \\ & \left. + \frac{|\rho_1|}{[j]_q^k\, \Phi_j(a,b,\mathcal{M},q)([j]_q - 1)} \prod_{j=1}^{j-2} \left(1 + \frac{|\rho_1|}{[j+1]_q - 1}\right)\right) \\ & \leq \frac{|\rho_1|}{[j]_q^k\, \Phi_j(a,b,\mathcal{M},q)([j]_q - 1)} \prod_{j=1}^{j-2} \left(1 + \frac{|\rho_1|}{[j+1]_q - 1}\right), \end{aligned}$$

which, according to the mathematical induction, completes our proof. □

Remark 2. *The results in Theorem 2, are sharp for the following functions which are belonging to the class* $j - \mathcal{S}_{q,\wp}^k(a,b)$*:*

$$f(z) = z + \frac{|\rho_1|}{[2]_q^k \Phi_2(a,b,\mathcal{M},q)\big([2]_q - 1\big)} z^2$$

and

$$f(z) = z + \left(\frac{|\rho_1|}{[n]_q^k \Phi_n(a,b,\mathcal{M},q)([n]_q - 1)} \prod_{j=1}^{n-2} \left(1 + \frac{|\rho_1|}{[j+1]_q - 1} \right) \right) z^n, \quad n \geq 3$$

where ρ_1 *is defined by* (6).

The next two special cases of this theorem were previously obtained by different authors:

Corollary 3 ([11], Theorem 8)**.** *If* $\mathcal{M}(n) = 1$ *for all* $n \geq 1$*, then*

$$|a_2| \leq \frac{|\rho_1|}{[2]_q^k \Phi_2(a,b,1,q)\big([2]_q - 1\big)},$$
$$|a_n| \leq \frac{|\rho_1|}{[n]_q^k \Phi_n(a,b,1,q)\big([n]_q - 1\big)} \prod_{j=1}^{n-2} \left(1 + \frac{|\rho_1|}{[j+1]_q - 1} \right), \ n \geq 3,$$

with ρ_1 *given by* (6).

Corollary 4 ([33], Theorem 3.2)**.** *If* $\mathcal{M}(n) = 1$ *for all* $n \geq 1$ *and* $a = 0$*,* $b = 1$*, then*

$$|a_2| \leq \frac{|\rho_1|}{[2]_q^k \Phi_2(0,1,1,q)\big([2]_q - 1\big)},$$
$$|a_n| \leq \frac{|\rho_1|}{[n]_q^k \Phi_n(0,1,1,q)\big([n]_q - 1\big)} \prod_{j=1}^{n-2} \left(1 + \frac{|\rho_1|}{[j+1]_q - 1} \right), \ n \geq 3,$$

where ρ_1 *was given by* (6).

3. Fekete-Szegő Problem for the Class $j - \mathcal{S}_{q,\wp}^k(a,b)$

First, we will give an estimate for the well-known Fekete–Szegő functional for the class $j - \mathcal{S}_{q,\wp}^k(a,b)$.

Theorem 3. *If* $h \in j - \mathcal{S}_{q,\wp}^k(a,b)$*, then*

$$|a_3 - \psi a_2^2| \leq \frac{|\rho_1|}{2[3]_q^k \Phi_3(a,b,\mathcal{M},q)\big([3]_q - 1\big)} \max\{1; |2\Psi - 1|\},$$

where $\psi \in \mathbb{C}$*, and*

$$\Psi := \Psi(a,b,\mathcal{M},q) = \frac{1}{2} \left(1 - \frac{\rho_2}{\rho_1} - \rho_1 \left(\frac{1}{[2]_q - 1} - \psi \frac{[3]_q^k([3]_q - 1)}{2\Phi_2(a,b,\mathcal{M},q)\Big([2]_q^k([2]_q - 1)\Big)^2} \right) \right), \tag{7}$$

with ρ_1 *and* ρ_2 *defined by* (6).

Proof. From the condition $h \in j - \mathcal{S}_{q,\wp}^k(a,b)$ we have

$$\frac{z\eth_q\Big({}_{\mathcal{M}}\Delta_q^k(a,b)h(z)\Big)}{{}_{\mathcal{M}}\Delta_q^k(a,b)h(z)} = \rho_{j,\wp}(\omega(z)),$$

where ω is a Schwarz functions satisfies $\omega(0) = 0$ and $|\omega(z)| < 1, z \in \mathbb{D}$.

Let $p \in \mathcal{P}$ be a function defined by

$$p(z) = \frac{1+\omega(z)}{1-\omega(z)} = 1 + p_1 z + p_2 z^2 + \ldots,\ z \in \mathbb{D},$$

which implies

$$\omega(z) = \frac{p_1}{2} z + \frac{1}{2}\left(p_2 - \frac{p_1^2}{2}\right) z^2 + \ldots,\ z \in \mathbb{D},$$

and

$$\rho_{j,\wp}(\omega(z)) = 1 + \frac{\rho_1 p_1}{2} z + \left(\frac{\rho_2 p_1^2}{4} + \frac{1}{2}\left(p_2 - \frac{p_1^2}{2}\right)\rho_1\right) z^2 + \ldots, z \in \mathbb{D}.$$

Therefore, we obtain

$$\frac{z\eth_q\left({}_{\mathcal{M}}\Delta_q^k(a,b)h(z)\right)}{{}_{\mathcal{M}}\Delta_q^k(a,b)h(z)} = 1 + [2]_q^k \Phi_2(a,b,\mathcal{M},q)\left([2]_q - 1\right) a_2 z$$
$$+\left([3]_q^k \Phi_3(a,b,\mathcal{M},q)\left([2]_q - 1\right) a_3 - \left([2]_q^k \Phi_2(a,b,\mathcal{M},q)\right)^2 \left([2]_q - 1\right) a_2^2\right) z^2 + \ldots,\ z \in \mathbb{D},$$

thus the initial coefficients can be determined as follows:

$$\begin{aligned}
a_2 &= \frac{\rho_1 p_1}{2[2]_q^k \Phi_2(a,b,\mathcal{M},q)\left([2]_q - 1\right)},\\
a_3 &= \frac{1}{[3]_q^k \Phi_3(a,b,\mathcal{M},q)\left([3]_q - 1\right)}\left(\frac{\rho_1 p_2}{2} + \frac{p_1^2}{4}\left(\rho_2 - \rho_1 + \frac{\rho_1^2}{[2]_q - 1}\right)\right)\\
a_3 - \psi a_2^2 &= \frac{1}{[3]_q^k \Phi_3(a,b,\mathcal{M},q)\left([3]_q - 1\right)}\left(\frac{\rho_1 p_2}{2} + \frac{p_1^2}{4}\left(\rho_2 - \rho_1 + \frac{\rho_1^2}{[2]_q - 1}\right)\right),\\
&\quad - \psi\left(\frac{\rho_1 p_1}{2[2]_q^k \Phi_2(a,b,\mathcal{M},q)\left([2]_q - 1\right)}\right)^2.
\end{aligned}$$

A simple computation yields

$$a_3 - \psi a_2^2 = \frac{\rho_1}{2[3]_q^k \Phi_3(a,b,\mathcal{M},q)\left([3]_q - 1\right)}\left(p_2 - \Psi p_1^2\right),$$

where Ψ is given by (7) and $\psi \in \mathbb{C}$. Hence, in view of Lemma 2 we obtain the desired result. □

The above theorem generalizes some previous results, as we can observe in the next two particular cases:

Corollary 5 ([11], Theorem 10). *If $h \in j - \mathcal{S}_{q,\wp}^k(a,b)$ with $\mathcal{M}(n) = 1$ for all $n \geq 1$, then*

$$|a_3 - \psi a_2^2| \leq \frac{|\rho_1|}{2[3]_q^k \Phi_3(a,b,1,q)\left([3]_q - 1\right)} \max\{1; |2\widehat{\Psi} - 1|\},$$

where $\psi \in \mathbb{C}$, and

$$\widehat{\Psi} := \Psi(a,b,1,q) = \frac{1}{2}\left(1 - \frac{\rho_2}{\rho_1} - \rho_1\left(\frac{1}{[2]_q - 1} - \psi\frac{[3]_q^k\left([3]_q - 1\right)}{2\Phi_2(a,b,1,q)\left([2]_q^k\left([2]_q - 1\right)\right)^2}\right)\right),$$

with ρ_1 and ρ_2 given by (6).

Corollary 6 ([33], Theorem 3.3)**.** *If $h \in j - \mathcal{S}^k_{q,\wp}(0,1)$ with $\mathcal{M}(n) = 1$ for all $n \geq 1$ and $a = 0$, $b = 1$, then*

$$|a_3 - \psi a_2^2| \leq \frac{|\rho_1|}{2[3]_q^k \Phi_3(0,1,1,q)\big([3]_q - 1\big)} \max\{1; |2\widetilde{\Psi} - 1|\},$$

where $\psi \in \mathbb{C}$, and

$$\widetilde{\Psi} := \Psi(0,1,1,q) = \frac{1}{2}\left(1 - \frac{\rho_2}{\rho_1} - \rho_1\left(\frac{1}{[2]_q - 1} - \psi\frac{[3]_q^k\big([3]_q - 1\big)}{2\Phi_2(0,1,1,q)\Big([2]_q^k\big([2]_q - 1\big)\Big)^2}\right)\right),$$

with ρ_1 and ρ_2 defined by (6)*.*

The last result deals with a sufficient condition for the coefficients of a function $h \in \Lambda$ to be in the class $j - \mathcal{S}^k_{q,\wp}(a,b)$.

Theorem 4. *Let $h \in \Lambda$ be of the form* (1)*. If*

$$\sum_{n=2}^{\infty} \big(\big([n]_q - 1\big)(j+1) + |\wp|\big)|\Phi_n(a,b,\mathcal{M},q)|[n]_q^k|a_n| \leq |\wp|,$$

then $h \in j - \mathcal{S}^k_{q,\wp}(a,b)$.

Proof. Obviously, we have

$$\left|\frac{z\eth_q\Big({}_{\mathcal{M}}\Delta_q^k(a,b)h(z)\Big)}{{}_m\Delta_q^k(a,b)h(z)} - 1\right| = \left|\frac{z\eth_q\Big({}_m\Delta_q^k(a,b)h(z)\Big) - {}_{\mathcal{M}}\Delta_q^k(a,b)h(z)}{{}_m\Delta_q^k(a,b)h(z)}\right|$$

$$= \left|\frac{\sum\limits_{n=2}^{\infty}\big([n]_q - 1\big)[n]_q^k\,\Phi_n(a,b,\mathcal{M},q)a_n z^n}{z + \sum\limits_{n=2}^{\infty}[n]_q^k\,\Phi_n(a,b,\mathcal{M},q)a_n z^n}\right| \leq \frac{\sum\limits_{n=2}^{\infty}\Big|\big([n]_q - 1\big)[n]_q^k\,\Phi_n(a,b,\mathcal{M},q)\Big||a_n|}{1 - \sum\limits_{n=2}^{\infty}|[n]_q^k\,\Phi_n(a,b,\mathcal{M},q)||a_n|},\ z \in \mathbb{D},$$

and from the assumption of the theorem

$$1 - \sum_{n=2}^{\infty}\Big|[n]_q^k\,\Phi_n(a,b,\mathcal{M},q)\Big||a_n| > 0.$$

Since

$$\left|\frac{j}{\wp}\left(\frac{z\eth_q\Big({}_{\mathcal{M}}\Delta_q^k(a,b)h(z)\Big)}{{}_{\mathcal{M}}\Delta_q^k(a,b)h(z)} - 1\right)\right| - \operatorname{Re}\left(\frac{1}{\wp}\left(\frac{z\eth_q\Big({}_{\mathcal{M}}\Delta_q^k(a,b)h(z)\Big)}{{}_{\mathcal{M}}\Delta_q^k(a,b)h(z)} - 1\right)\right)$$

$$\leq \frac{j}{|\wp|}\left|\left(\frac{z\eth_q\Big({}_m\Delta_q^k(a,b)h(z)\Big)}{{}_{\mathcal{M}}\Delta_q^k(a,b)h(z)} - 1\right)\right| + \left|\frac{1}{\wp}\right|\left|\frac{z\eth_q\Big({}_m\Delta_q^k(a,b)h(z)\Big)}{{}_{\mathcal{M}}\Delta_q^k(a,b)h(z)} - 1\right|$$

$$= \frac{j+1}{|\wp|}\left|\left(\frac{z\eth_q\Big({}_m\Delta_q^k(a,b)h(z)\Big)}{{}_{\mathcal{M}}\Delta_q^k(a,b)h(z)} - 1\right)\right| = \frac{j+1}{|\wp|}\left|\frac{z\eth_q\Big({}_{\mathcal{M}}\Delta_q^k(a,b)h(z)\Big) - {}_{\mathcal{M}}\Delta_q^k(a,b)h(z)}{{}_{\mathcal{M}}\Delta_q^k(a,b)h(z)}\right|$$

$$\leq \frac{j+1}{|\wp|}\left(\frac{\sum\limits_{n=2}^{\infty}|\big([n]_q - 1\big)[n]_q^k\,\Phi_n(a,b,\mathcal{M},q)||a_n|}{1 - \sum\limits_{n=2}^{\infty}|[n]_q^k\,\Phi_n(a,b,\mathcal{M},q)||a_n|}\right) \leq 1,\ z \in \mathbb{D},$$

we obtain $h \in j - \mathcal{S}^k_{q,\wp}(a,b)$. □

This theorem generalizes other previously obtained results, which we can observe in the next two special cases. Thus, taking in the above theorem $\mathcal{M}(n) = 1$ for all $n \geq 1$, and $\mathcal{M}(n) = 1$ for all $n \geq 1$ with $a = 0$, $b = 1$, we obtain the next special cases, respectively:

Corollary 7 ([11], Theorem 12)**.** *Let $h \in \Lambda$ be of the form* (1)*. If*

$$\sum_{n=2}^{\infty} \left(\left([n]_q - 1 \right)(j+1) + |\wp| \right) |\Phi_n(a,b,1,q)| [n]_q^k |a_n| \leq |\wp|,$$

then

$$\frac{z\eth_q \left(\Delta_q^k(a,b)h(z) \right)}{\Delta_q^k(a,b)h(z)} \prec \rho_{j,\wp}(z).$$

That is $h \in j - \mathcal{S}_{q,\wp}^k(a,b)$, when $\mathcal{M}(n) = 1$ for all $n \geq 1$.

Corollary 8 ([33], Theorem 3.4)**.** *Let $h \in \Lambda$ be of the form* (1)*. If*

$$\sum_{n=2}^{\infty} \left(\left([n]_q - 1 \right)(j+1) + |\wp| \right) |\Phi_n(0,1,1,q)| [n]_q^k |a_n| \leq |\wp|,$$

then

$$\frac{z\eth_q \left(\Delta_q^k(0,1)h(z) \right)}{\Delta_q^k(0,1)h(z)} \prec \rho_{j,\wp}(z).$$

That is $h \in j - \mathcal{S}_{q,\wp}^k(0,1)$, when $\mathcal{M}(n) = 1$ for all $n \geq 1$.

4. Conclusions

In light of Jackson's calculus, Raina's function in $\mathbb{D}$ is expanded. The proposed q-differential operator was applied to the normalized subclass, and the geometric behavior of the operator is investigated using differential inequalities. For more recent efforts (see [11,33]), some generalizations are provided, and finally, Theorem 4 gives the sufficient condition for a function to belong to this class. We also gave an estimate for the Fekete–Szegő functional for these newly defined classes of functions.

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References

1. Jackson, F.H. XI.–On q-functions and a certain difference operator. *Earth Environ. Sci. Trans. R. Soc. Edinburgh* **1909**, *46*, 253–281. [CrossRef]
2. Jackson, F.H. On Q-definite integrals. *Quart. J. Pure Appl. Math.* **1910**, 41, 193–203.
3. Ismail, M.E.H.; Merkes, E.; Styer, D. A generalization of starlike functions, A generalization of starlike functions, Complex Variables. *Theory Appl.* **1990**, *14*, 77–84. [CrossRef]
4. Seoudy, T.M.; Aouf, M.K. Coefficient estimates of new classes of q-starlike and q-convex functions of complex order. *J. Math. Inequal.* **2016**, *10*, 135–145. [CrossRef]
5. Zainab, S.; Shakeel, A.; Imran, M.; Muhammad, N.; Naz, H.; Malik, S.N.; Arif, M. Sufficiency criteria for starlike functions associated with cardioid. *J. Funct. Spaces* **2021**, *2021*, 9999213. [CrossRef]
6. Hadid, S.B.; Ibrahim, R.W.; Shaher, M. A new measure of quantum starlike functions connected with Julia functions. *J. Funct. Spaces* **2022**, *2022*, 4865785. [CrossRef]

7. Ibrahim, R.W. Geometric process solving a class of analytic functions using q-convolution differential operator. *J. Taibah Univ. Sci.* **2020**, *14*, 670–677. [CrossRef]
8. Ibrahim, R.W.; Băleanu, D. On quantum hybrid fractional conformable differential and integral operators in a complex domain. *Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. (RACSAM)* **2021**, *115*, 31. [CrossRef]
9. Ibrahim, R.W.; Elobaid, R.M.; Obaiys, S.J. Geometric inequalities via a symmetric differential operator defined by quantum calculus in the open unit disk. *J. Funct. Spaces* **2020**, *2020*, 6932739. [CrossRef]
10. Ibrahim, R.W.; Elobaid, R.M.; Obaiys, S.J. A class of quantum Briot-Bouquet differential equations with complex coefficients. *Mathematics* **2020**, *8*, 794. [CrossRef]
11. Noor, S.; Razzaque, A. New subclass of analytic function involving Mittag-Leffler function in conic domains. *J. Funct. Spaces* **2022**, *2022*, 8796837. [CrossRef]
12. Tang, H.; Khan, S.; Hussain, S.; Khan, N. Hankel and Toeplitz determinant for a subclass of multivalent q-starlike functions of order α. *AIMS Math.* **2021**, *6*, 5421–5439. [CrossRef]
13. Karthikeyan, K.R.; Lakshmi, S.; Varadharajan, S.; Umadevi, D.M.E. Starlike functions of complex order with respect to symmetric points defined using higher order derivative. *Fractal Fract.* **2022**, *6*, 116. [CrossRef]
14. Riaz, S.; Nisar, U.A.; Xin, Q.; Malik, S.N.; Raheem, A. On starlike functions of negative order defined by q-fractional derivative. *Fractal Fract.* **2022**, *6*, 30. [CrossRef]
15. Attiya, A.A. Some applications of Mittag-Leffler function in the unit disk. *Filomat* **2016**, *30*, 2075–2081. [CrossRef]
16. Haubold, H.J.; Mathai, A.M.; Saxena, R.K. Mittag-Leffler functions and their applications. *J. Appl. Math.* **2011**, *2011*, 298628. [CrossRef]
17. Ryapolov, P.A.; Postnikov, E.B. Mittag-Leffler function as an approximant to the concentrated ferrofluid's magnetization curve. *Fractal Fract.* **2021**, *5*, 147. [CrossRef]
18. Shukla, A.K.; Prajapati, J.C. On a generalization of Mittag-Leffler function and its properties. *J. Math. Anal. Appl.* **2007**, *336*, 797–811. [CrossRef]
19. Srivastava, H.M.; Kiliçman, A.; Abdulnaby, Z.E.; Ibrahim, R. Generalized convolution properties based on the modified Mittag-Leffler function. *J. Nonlinear Sci. Appl.* **2017**, *10*, 4284–4294. http://dx.doi.org/10.22436/jnsa.010.08.23. [CrossRef]
20. Duren, P.L. *Univalent Functions, Grundlehren Math*; Wissenschaften, Band 259; Springer: New York, NY, USA; Berlin/Heidelberg, Germany; Tokyo, Japan, 1983.
21. Bulboacă, T. *Differential Subordinations and Superordinations*; New Results; House of Scientific Book Publ.: Cluj-Napoca, Romania, 2005.
22. Ruscheweyh, S. *Convolutions in Geometric Function Theory*; Les Presses de L'Université de Montréal: Montréal, QC, Canada, 1982.
23. Carathéodory, C. Über den Variabilitätsbereich der Koeffizienten von Potenzreihen, die gegebene Werte nicht annehmen. *Math. Ann.* **1907**, *64*, 95–115. [CrossRef]
24. Carathéodory, C. Über den ariabilitätsbereich der fourier'schen konstanten von positiven harmonischen funktionen. *Rend. Circ. Mat. Palermo* **1911**, *32*, 193–217. [CrossRef]
25. Gil, A.; Javier, S.; Nico, M.T. *Numerical Methods for Special Functions*; Society for Industrial and Applied Mathematics: Philadelphia, PA, USA, 2007.
26. Raina, R.K. On generalized Wright's hypergeometric functions and fractional calculus operators. *East Asian Math. J.* **2005**, *21*, 191–203.
27. Sălxaxgean, G.S. *Subclasses of univalent functions, Complex Analysis-Fifth Romanian-Finnish Seminar, Part 1 (Bucharest, 1981)*; Lecture Notes in Mathematics; Springer: Berlin, Germany, 1983; Volume 1013, pp. 362–372.
28. Govindaraj, M.; Sivasubramanian, S. On a class of analytic functions related to conic domains involving q-calculus. *Anal. Math.* **2017**, *43*, 475–487. [CrossRef]
29. Kanas, S.; Altınkaya, Ş. Functions of bounded variation related to domains bounded by conic sections. *Math. Slovaca* **2019**, *69*, 833–842. [CrossRef]
30. Kanas, S. Techniques of the differential subordination for domains bounded by conic sections. *Int. J. Math. Math. Sci.* **2003**, *38*, 2389–2400. [CrossRef]
31. Rogosinski, W. On the coefficients of subordinate functions. *Proc. Lond. Math. Soc.* **1943**, *48*, 48–82. [CrossRef]
32. Ma, W.C.; Minda, D. A unified treatment of some special classes of univalent functions. In Proceedings of the Conference Complex Analysis, Tianjin, China, 19–23 June 1992; Conf. Proc. Lecture Notes Anal. I; International Press: Cambridge, MA, USA, 1994; pp. 157–169.
33. Hussain, S.; Khan, S.; Zaighum, M.A.; Darus, M. Certain subclass of analytic functions related with conic domains and associated with Sălăgean q-differential operator. *AIMS Math.* **2017**, *2*, 622–634. [CrossRef]