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Key Component Capture and Safety Intelligent Analysis of Beam String Structure Based on Digital Twins

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Abstract: In the construction process of beam string structures, the environmental effect and corresponding mechanical properties of the structure are complex. The problem of the misjudgment of structural safety performance caused by the uncertainty of a structural mechanical parameter analysis under various factors needs to be solved. In this study, a method for capturing key components and an intelligent safety analysis of beam string structures based on digital twins (DTs) was proposed. Combined with the characteristics of DTs mapping feedback, a component capture and security analysis framework was formed. Driven by twin framework, multi-source data for structural safety analysis were obtained and the parameter association mechanism established. Considering the space-time evolution and the interaction between the virtual and real elements of the construction process, a multidimensional model was established. Driven by the Dempster–Shafer (D–S) evidence theory, the fusion of structural mechanics parameters was carried out. The safety of the structure was analyzed intelligently by capturing key structural components, thereby providing a basis for the safety maintenance of the structure. The integration of DTs modeling and multi-source data improves the accuracy and intelligence of structural construction safety analysis. In the analysis process, capturing the key components of the structure is the core step. Taking the construction process of a string supported beam roof (symmetrical structure) in a convention and exhibition center as an example, the outlined research method was applied. Based on DTs and D–S evidence theory, the variation degree of mechanical parameters of various components under temperature was determined. By comprehensively investigating the changes of various mechanical parameters, the key components of the structure were captured. Thus, the intelligent analysis of structural safety was realized. The comparison of data verified that the intelligent method can effectively analyze the safety performance of the structure.

Keywords: beam string structure; digital twin; data fusion; safety analysis; structural health monitoring

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1. Introduction

Prestressed steel structures have the advantages of a strong spanning capacity, pleasing shape, light weight and short construction period, which is widely used in practical engineering [1]. At present, prestressed steel structures mainly include cable dome structures [2], wheel spoke structures [3], cable truss structures without inner ring space [4], cable net structures [5], cable net shell structures [6] and so on. Among them, the chord-supported beam structure, as an important form of prestressed steel structures, has been widely used in public buildings such as large stadiums. The construction of a large-span spatial structure is also an important standard to measure the national construction technology and level. The mechanical properties of components directly determine the safety performance of the structure [7]. Because large-span spatial structures are mostly used in buildings of high importance and large quantities are involved in the construction process, the safety requirements of the construction process are precise [8–10].

Zhang et al. [11] proposed a joint-square double-brace structure to improve the force performance of the spatial structure and derived a formula for calculating the prestressing of the members of the structure under their self-weight, which provided theoretical support for the stability verification of the structure during the construction process. The theoretical formula proposed in this paper quickly and accurately obtains the actual prestress distribution of the combined double-strut cable dome by accounting for the structural weight, which can provide a reference for engineering designs. Guo et al. [12] investigated the effect of the initial cable length error in the prestressing state on the sensitivity of prestressed cables to length error. By controlling the length error, the prestress level during cable tensioning was effectively improved. In this study, the influence of the initial cable length error on the load state was compared when the prestress state was different and the position of the initial cable length error was different. The research results play a guiding role in the design and construction of cable dome structures. Wang et al. [13] focused on the analysis of the cable force, which is the most active parameter in the construction process of spatial structure prestressed cables. In the whole process of prestressed cable tension, the safety of the construction process of the structure can be ensured by analyzing the cable force. In this study, a hybrid method combining the incremental ratio method and degradation compensation method was proposed, and the corresponding iterative calculation process was provided to guide the construction process. Castillo et al. [14] derived the damage accumulation process of a cable system under normal loading and established a theoretical model for the fatigue life of a cable by considering load redistribution. The results provided a basis for the fatigue calculation of a cable. Arezki et al. [15] investigated the effect of temperature variation on the safety performance of cable truss structures and cables. The research results provided a theoretical basis for the analysis of the mechanical properties of space structures subjected to temperature loads. Basta et al. [16] conducted a quantitative evaluation of the deconstruction of cable net structures based on building information models. In the construction process of the structure, the intelligent evaluation method was provided to ensure the rationality of the structure.

Having analyzed the research of the above scholars, it is clear that there are still some deficiencies in the construction safety analysis. (i) For the whole construction process of the structure, there is no virtual mapping mechanism. Due to the lack of a dynamic model, the closed-loop control of structural safety assessments cannot be realized. (ii) There is no information fusion mechanism for multi-source data in the construction process. The mechanical properties of structural components are not comprehensively analyzed from multi-source data, resulting in inaccurate structural safety analyses. With the development of new generation information technology and the promotion of the industrial information system, the application of intelligent technology to engineering construction has become a research hotspot. In the process of structural analysis, it is necessary to accurately judge the key stress components by integrating various structural mechanical parameters. On the one hand, it provides the basis for the safety analysis and maintenance of the structure; on the other hand, it reduces the cost of data acquisition in the structural test process. The application of digital twins (DTs) and intelligent algorithms in engineering practices can significantly improve the accuracy and intelligence of structural performance analyses [17,18].

DTs simulates and depicts the state and behavior of physical entities with a dynamic virtual model with high fidelity. As a link between the real physical world and the virtual digital space, it is the key enabling technology used to realize intelligent construction [19,20]. Artificial intelligence has been applied in many disciplines and forms a variety of intelligent algorithms. The algorithm can extract high-level features from the original data for perceptual decision making, and improve the objectivity and accuracy of information evaluation [21]. Liu et al. [22] proposed a DTs-driven dynamic guidance method for fire evacuation. The method integrates the Dijkstra algorithm to achieve the real-time acquisition of environmental information, the three-dimensional visualization of indoor layouts and evacuation path planning. Based on the establishment of the structural twin model, Zeng et al. [23] used the Dempster–Shafer (D–S) evidence matrix theory for a damage identification analysis. The fusion of DTs and the D–S evidence theory improves the accuracy of the multi-position damage identification of truss string. Acharya et al. [24] proposed a visual positioning method to achieve the real-time and accurate positioning of indoor buildings. The 3D indoor model was used to eliminate the image-based indoor environment reconstruction requirements, and the deep convolution neural network was fused to fine-tune the image. Therefore, the integration of DTs and intelligent algorithms provides new ideas and methods for the intelligent transformation and upgrading of the construction industry.

The advantages of DTs and intelligent algorithms were analyzed according to the requirements of an intelligent safety analysis of structure construction. In this study, a method for capturing key components and an intelligent safety analysis of beam string structures based on DTs were proposed. Firstly, the difficulties of a safety analysis in the construction process of beam string structures were summarized. The intelligent analysis framework of key stress components capture and structural safety was built by integrating DTs and an intelligent algorithm. Driven by this framework, the correlation mechanism of multi-source data in capturing key structural components and a safety analysis in the construction process was clarified and a multi-dimensional model established, forming a safety analysis method. In the study method, capturing key stressed member elements was found to be the core step. Based on this, the D–S evidence theory was applied to the fusion of multiple mechanical parameters to find the key components and assist in the formulation of maintenance measures. The integration of DTs and the D–S evidence theory forms a safety analysis process and realizes the closed-loop control of structural safety. The theoretical method was applied to the construction safety analysis of a symmetrical structure (string beam). The key stress components were captured under the action of temperature, which provides the basis and reference for structural health monitoring. In short, this work makes the following contributions.

(1) Based on the stress characteristics of string supported beam structure, the concept of DTs is introduced. Additionally, the frame of capturing and a safety analysis of key components of string-supported beam structures based on digital twins are established.
(2) Driven by the theoretical framework, the structural mechanical performance analysis model is constructed. Based on the concept of DTs, the high fidelity twin model of the structure is established. In this model, the mechanical parameters of the structure can be accurately obtained to characterize the safety performance of the structure. At the same time, the twin model can effectively avoid the error of the actual structure data acquisition and reduce the miscellaneous cost caused by the sensing equipment.
(3) In order to accurately obtain the key stress components of the structure, the safety performance of the structure is analyzed by considering the multiple mechanical parameters extracted from the twin model driven by the D–S evidence theory. The application of the D–S evidence theory can help to effectively avoid the problem of the insufficient accuracy of key components evaluated by a single index.
(4) Through the case study, it is proved that the proposed method is superior to the intelligent method for improving the key stress components and analyzing the safety performance of structures.

On the one hand, this research method can accurately map the state of the structure and reduce the error and cost of the actual structure data acquisition. On the other hand, the key components of the structure can be accurately judged by integrating various mechanical parameters, which provides the basis for the safety analysis and maintenance of the structure. The article is structured as follows. Section 2 summarizes the difficulties of conducting a safety analysis in the process of structural construction, and builds a framework for capturing and performing a safety analysis of the key components of string-supported beam structures based on DTs. Section 3 describes the multi-source parameter association mechanism that affects structural safety performance, and a multi-dimensional model for structural safety analysis is proposed. Section 4 analyzes the data fusion mechanism based on the D–S evidence theory, and the key component capture and structural safety analysis process is formed. Section 5 illustrates the case study used to verify the proposed method. Finally, conclusions are drawn in Section 6 and further research avenues are outlined.

2. Key Component Capture and Construction Safety Analysis Framework of String Supported Beam Steel Structure Based on DTs

The formation of the structure requires a very complex construction process, and involves a number of time-varying problems. The material nonlinearity, geometric nonlinearity, state nonlinearity and path correlation should be fully considered in the construction process [25]. If the mechanical change mechanism and structural deformation of the construction process are not considered, it will not only cause a high level of investment in direct costs, but also lead to safety risks. In the construction process of beam string structures, the geometric shape, stiffness, load and boundary conditions of the whole structure are successively formed according to the construction order. During construction, the load on the installed structure has no effect on the uninstalled structure. The deformation and internal force of the installed structure are the initial deformation and initial internal force for the structure after the subsequent installation. Once the new components and new loads are formed, the stress state of the structure changes. The whole construction process requires a series of quasi-structural states to achieve the completion state. The stress state of the structure and the construction steps show a certain nonlinear relationship to some extent [26]. In the process of structural construction, there are four main difficulties in performing a safety analysis, as shown in Table 1.

Table 1. Difficulties in structural construction safety analysis.

Analysis Difficulties	Concrete Expression
Time-varying Simulation of Structure Solution Domain [27]	In the construction process, the geometry and mechanical properties of the structure are constantly changing. The solution area of the analysis object increases or decreases with time.
Spatial variation of structural solution domain [28]	The structural member undergoes considerable deformation or displacement within the construction step. The mechanical performance of the structural system also changes greatly.
Simulation of material time-varying [29]	During the tensioning process, the cable performance constantly changes, showing strong nonlinear characteristics.
Simulation of time varying boundary conditions [30]	For a large-span spatial structure, some degrees of freedom are released in the initial stage of construction. Constraints are imposed at the subsequent stage of construction or after molding, resulting in continuous changes in the boundary conditions of the structure.

Based on the construction characteristics of a beam string structure, the intelligent analysis of structural safety was carried out by considering the spatial-temporal evolution and the interaction between virtual and real. DTs integrates artificial intelligence, machine learning and data analysis functions to create real-time digital simulation models. Such models can not only learn and update from multiple sources, but also represent and predict the current and future status of physical counterparts [31,32]. In this study, DTs and intelligent algorithms were integrated to establish an intelligent analysis mechanism for the construction safety of beam string structures. Through the twin model of virtual space (geometric model, physical model, behavior model, rule model), the synchronous mapping from real construction processes in physical space to virtual space were realized. The mapping from physical space to virtual space realizes the visualization and digitization of the total elements of structural construction. Through the simulation of a virtual model, collaborative feedback allows the model to run synchronously with a physical tension system. The intelligent and collaborative analysis of multi-source data in the structural construction process was realized by integrating artificial intelligence, especially an intelligent algorithm. The mechanical parameters of the structure were obtained in the twin model. The safety performance of the structure was characterized by analyzing the mechanical parameters of the structure. In this study, the key stress components in the structure were accurately judged using the D–S evidence theory and considering various mechanical parameters. In the analysis process, the changes in various mechanical parameters of the structure were analyzed. The analysis process based on the D–S evidence theory is elaborated on in Section 4 and applied in Section 5. In order to accurately capture the key stress components in the structure, the change rate of each mechanical parameter was taken as the input of the mathematical model. The final output integrated the comprehensive rate of change of multiple types of parameters. The key components in the structure were then judged according to the comprehensive change rate, which provides the basis for the safety analysis and maintenance of the structure. The combination of DTs and an intelligent algorithm can intuitively map the state of the whole process of structural construction and analyze the safety information of the structure. As shown in Figure 1, an intelligent analysis framework of beam string structure construction safety was built. In this process, the visualization of the structure was first performed. The key components in the structure were captured by analyzing the mechanical parameters of the structure. Finally, the intelligent analysis of structural safety provides the basis for safety maintenance. On the one hand, digital twinning can achieve the dynamic perception and visual presentation of the structural construction process. On the other hand, the intelligent algorithm can realize the fusion of multi-source data in the process of structural construction and the intelligent analysis of safety performance. DTs and intelligent algorithms support each other. In the construction process of the structure, combined with the action factors, safe construction is finally guided.

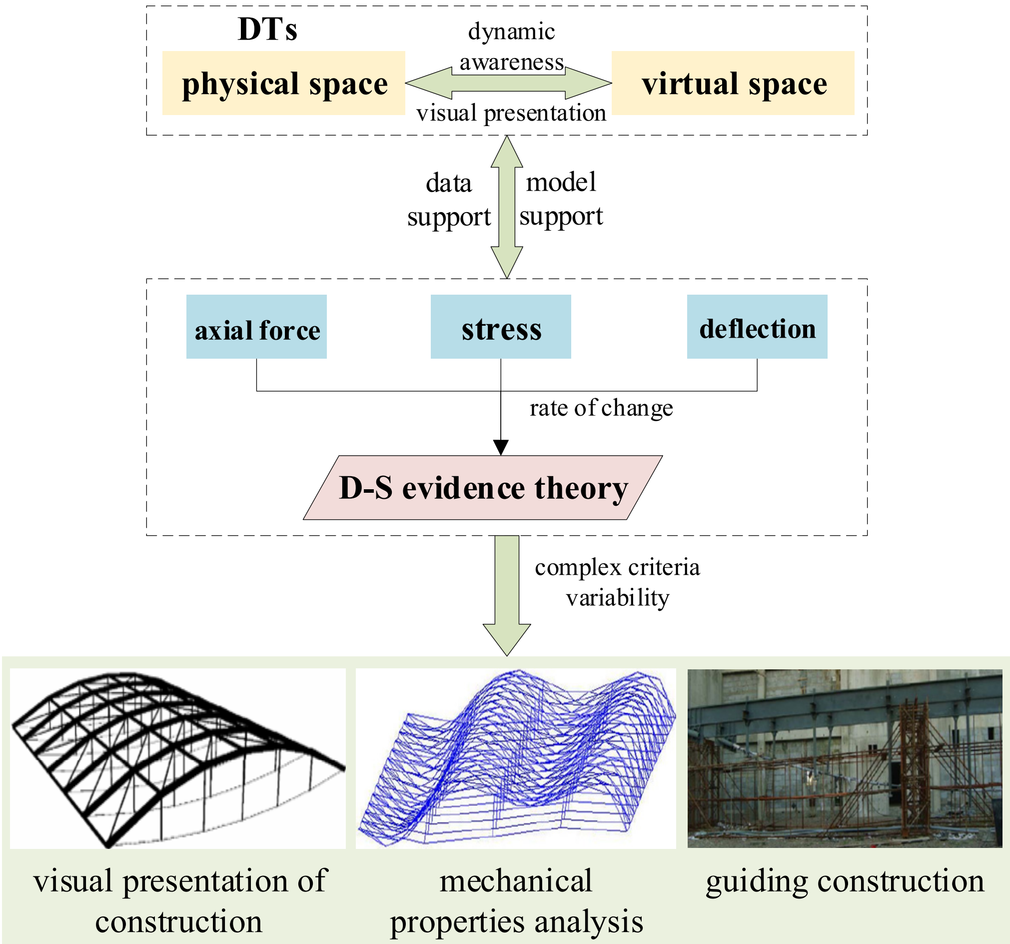


Figure 1. Construction Safety Intelligent Analysis Framework of Beam String Structure.

By means of the integration of DTs and artificial intelligence, the construction safety intelligent analysis mechanism of beam string structures has the following characteristics.

(1) Real-time perception is based on virtual reality. Through the information collection of the physical construction system and the establishment of the virtual model for the whole construction process, the visual monitoring of the construction process was achieved. The integration of all elements and multi-dimensional and multi-scale information of the construction system provides a synchronous operation model for the construction site.
(2) Data-driven intelligent diagnosis. The security analysis system can make full use of construction information such as historical data, real-time acquisition data and simulation data. By mining and analyzing all kinds of data, an intelligent diagnosis of construction process is realized, and construction risks can be avoided in time.
(3) Scientific analysis with virtual control. Using artificial intelligence technology, specifically an intelligent algorithm, helped to establish the data model. The simulation was carried out using the twin model and finally fed back to the real construction system.

3. Analysis Model of Structural Mechanical Performance under Multiple Influence Factors

Driven by the construction safety intelligent analysis framework of beam string structures, the visual presentation of the structure can be realized from the perspective of virtual–real interaction. Thus, the change in structural form can be directly judged. From the perspective of spatio-temporal evolution, the integration of DTs and intelligent algorithms can be used to intelligently analyze the mechanical changes of structures. Based on the analytical framework, specific implementation mechanisms should also be clarified. In this study, the multi-source data and their relationships with structural construction safety were first obtained. Driven by data, a multidimensional model for a structural safety analysis was established from five dimensions.

3.1. Classification and Correlation of Multi-Source Data

During the construction of a beam string structure, the safety analysis parameters are classified into two categories, namely influencing factors and mechanical parameters. The influencing factors represent the external environment that affects the safety in the process of structural construction. The mechanical parameters reflect the structure when subject to various influencing factors [33]. In this study, the change in mechanical parameters was used as an important basis for the analysis of structural safety performance. The source data of beam string structure construction safety analysis are expressed as Equations (1) and (2).

$$IF = \left(L_e,\quad W_l,\quad T_e\right) \tag{1}$$

$$MP = \left(C_f,\quad \omega,\quad \delta,\quad \varepsilon,\quad C\right) \tag{2}$$

In the Equation, IF represents the influencing factors of structural construction safety. By integrating the effect of various external influencing factors, the component length error (L_e), wind load effect (W_l) and temperature effect (T_e) are mainly captured and perceived. MP represents the mechanical parameters of the structural construction safety analysis. The key components of the safety analysis of a beam string structure in the construction process are the mechanical properties such as cable force (C_f), deflection (ω), stress (δ), strain (ε) and crack (C). In Section 5, the variation in axial force, stress and deflection of components are mainly captured for the structural safety performance analysis. Driven by the D–S evidence theory, the rate of change of three kinds of mechanical parameters is comprehensively analyzed to accurately judge the key stress members of the structure. Finally, it provides a basis for the safety analysis and maintenance of the structure.

The multi-source data for a safety analysis at the construction site can be directly obtained by the sensing equipment. On the basis of obtaining various parameters, the correlation mechanism can be established to provide data support for a structural safety analysis. The construction process of beam string structures has the characteristics of complexity, time-variations, and linkage. Considering the spatial and temporal evolution characteristics of structural construction safety information, the correlation model of multi-source data is established. The relational data model in this study analyzes the mechanical parameters driven by the influencing factors. All kinds of mechanical parameters in the real physical space are collectively called MP_p, and its mathematical language is expressed as Equation (3).

$$MP_p = \begin{pmatrix} C_{fp} \\ \omega_p \\ \delta_p \\ \varepsilon_p \\ C_p \end{pmatrix} = \begin{pmatrix} C_{fp1}, & C_{fp2}, & \cdots, & C_{fpn} \\ \omega_{p1}, & \omega_{p2}, & \cdots, & \omega_{pn} \\ \delta_{p1}, & \delta_{p2}, & \cdots, & \delta_{pn} \\ \varepsilon_{p1}, & \varepsilon_{p2}, & \cdots, & \varepsilon_{pn} \\ C_{p1}, & C_{p2}, & \cdots, & C_{pn} \end{pmatrix} \tag{3}$$

In Equation (3), $C_{fp}, \omega_p, \delta_p, \varepsilon_p$ and C_p represent the mechanical parameters of the structure in the physical space. $C_{fp1}, \quad C_{fp2}, \quad \cdots, \quad C_{fpn}$, indicate the cable force information of each component unit in the construction process of the structure in the physical space. $\omega_{p1}, \quad \omega_{p2}, \quad \cdots, \quad \omega_{pn}$ represent the deflection information of each component element in the construction process of the structure in the physical space. $\delta_{p1}, \quad \delta_{p2}, \quad \cdots, \quad \delta_{pn}$ represents the stress information of each component element in the construction process of the structure in the physical space. $\varepsilon_{p1}, \quad \varepsilon_{p2}, \quad \cdots, \quad \varepsilon_{pn}$ represents the strain information of each component element in the construction process of the structure in the physical space. $C_{p1}, \quad C_{p2}, \quad \ldots, \quad C_{pn}$ is the crack information of each component unit in the structural construction process in physical space. According to the information of physical space, the sensor simulation technology can be used to collect and transmit the information of various elements. At the same time, the simulation of various information can carried out in virtual space. Thus, the DTs information (MP_{DT}) is formed, and is expressed as Equation (4).

$$MP_{DT} = \begin{pmatrix} C_{fDT} \\ \omega_{DT} \\ \delta_{DT} \\ \varepsilon_{DT} \\ C_{DT} \end{pmatrix} = \begin{pmatrix} C_{fDT1}, & C_{fDT2}, & \cdots, & C_{fDTn} \\ \omega_{DT1}, & \omega_{DT2}, & \cdots, & \omega_{DTn} \\ \delta_{DT1}, & \delta_{DT2}, & \cdots, & \delta_{DTn} \\ \varepsilon_{DT1}, & \varepsilon_{DT2}, & \cdots, & \varepsilon_{DTn} \\ C_{DT1}, & C_{DT2}, & \ldots, & C_{DTn} \end{pmatrix} \tag{4}$$

In Equation (4), $C_{fDT}, \omega_{DT}, \delta_{DT}, \varepsilon_{DT}$ and C_{DT} represent the mechanical parameters of the structure in the twin space. $C_{fDT1}, \quad C_{fDT2}, \quad \cdots, \quad C_{fDTn}$ represents the cable force information of each component unit in the twin space during the structural construction. $\omega_{DT1}, \quad \omega_{DT2}, \quad \cdots, \quad \omega_{DTn}$ represents the deflection information of each component element in the twin space during the construction process. $\delta_{DT1}, \quad \delta_{DT2}, \quad \cdots, \quad \delta_{DTn}$ denotes the stress information of each component element in the twin space during construction. $\varepsilon_{DT1}, \quad \varepsilon_{DT2}, \quad \cdots, \quad \varepsilon_{DTn}$ equals the strain information of each component element in the construction process of the structure in the twin space. $C_{DT1}, \quad C_{DT2}, \quad \ldots, \quad C_{DTn}$ translated to the crack information of each component unit in the construction process of twin space. A simulation analysis of the construction process can be carried out in virtual space. The auxiliary formulation of correction decisions can be carried out for unsafe events. The feasibility analysis of decision making in the virtual model can eventually guide the maintenance of physical space. The resulting correlation mechanism of mechanical parameters is expressed as Equation (5).

$$IF \xrightarrow{\text{influence}} MP_p \overset{1:1}{\Leftrightarrow} MP_{DT} \underset{\text{maintenance measure}}{\overset{\text{data fusion}}{\rightleftarrows}} \text{Key component units} \tag{5}$$

In the correlation mechanism, the change in the structural mechanical parameters in physical space is caused by the influence of environment. At the same time, a mapping structure model is built in virtual space to form DTs information. Driven by the intelligent algorithm, the most critical component element affecting the safety of the structure is found through the fusion of multiple mechanical parameters. Through the maintenance of key units, the feasibility analysis in virtual space can be carried out to ensure the safe construction of the real structure.

3.2. Multidimensional Model Establishment for Structural Safety Analysis

In the intelligent analysis process for the construction safety of beam string structures, the core step is to find the most critical component unit of the structure. Driven by DTs, a multidimensional model is formed. The intelligent analysis of structural safety is realized by the interaction and cooperation of various levels of the multidimensional model. In the physical space, the function of the structure and the mechanical parameters of the structure are collected in real time. The acquisition process involves a variety of sensing devices. At the same time, the virtual model of solid structures is created in the process of a finite element analysis. By setting the corresponding conditions, the twin data of the structure's function and mechanical properties can be simulated. Thus, the twin simulation of the construction process is realized. Twin modeling provides a model basis, which further provides a basis for the analysis of structural construction safety performance. The multidimensional model for a structural safety analysis is established by Equation (6).

$$DT_M = \left(S_{pr}, \quad S_{vm}, \quad P_{td}, \quad L_{fa}, \quad C_n \right) \tag{6}$$

In the formula, DT_M is the multidimensional model for a structural safety analysis. S_{pr} represents the physical structure entity. S_{vm} represents the virtual structure model. P_{td} is the twin data processing layer. L_{fa} is the functional application layer. C_n represents a connection between components. The twin model can realize the simulation mapping of real structures and process the structural parameters of real monitoring and virtual simulation. Through a comprehensive analysis of various mechanical parameters, the key stress component elements can be determined. By evaluating the safety performance of the structure, accurate maintenance decisions can be made on the construction site. The multidimensional model for a structural safety analysis is shown in Figure 2. In the multi-dimensional model, the virtual–real interaction and spatio-temporal evolution of the structural construction process are fully considered. In the perspective of virtual–real interaction, the virtual structure model is established according to the physical structure entity to realize the real mapping of the construction process. The virtual model can provide feasibility verification for the final maintenance measures. From the perspective of spatio-temporal evolution, the twin data layer can be divided into two dimensions according to each step of the construction process. Finally, the safety performance is analyzed by fusing mechanical parameters. Driven by DTs and an intelligent algorithm, the real-time feedback of the structural construction process and closed-loop control of safety risk are achieved.

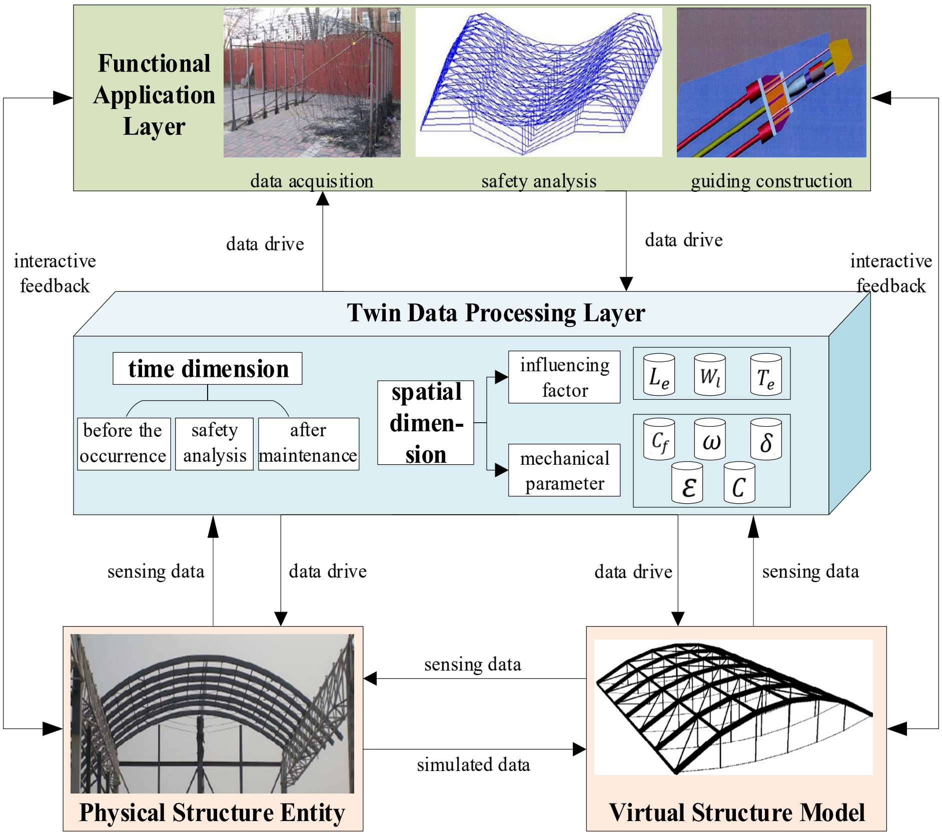


Figure 2. Multidimensional Model for Structural Safety Analysis.

4. Multi-Source Data Fusion and Structural Safety Analysis

In the process of a structural safety analysis, through the DTs model, the simulation data in the model can be directly extracted for analysis. The capturing of key stressed components is the core step in the analysis process. In this study, the D–S evidence theory was used to fuse the change degree of multiple mechanical parameters and finally determine the key components. The D–S theory, as a technique for the fusion of multi-source information, belongs to the category of artificial intelligence and has been applied in business administration and road transportation. In this study, it was applied to the fusion of structural mechanics parameters to judge the key components in the process of structural construction and improve the accuracy of a safety analysis [34].

4.1. Data Fusion Based on D–S Evidence Theory

The basic principle of the D–S evidence matrix is as follows: for a component, the degree of change of various mechanical parameters under the action of influencing factors is fused. The basic probability distribution function is determined by probability assignment for the judgment results of each mechanical parameter. According to the change degree of each parameter, the D–S combination rule was used for fusion to obtain the final discriminant results. The basic principles of the D–S evidence theory are shown in Table 2.

Table 2. Basic Principles of D–S Evidence Theory.

Component ID	Parameter 1	Parameter 2	Parameter 3	Comprehensive Evaluation
1	a_1	b_1	c_1	p_1
2	a_2	b_2	c_2	p_2
...	...	...	...	...
n	a_n	b_n	c_n	p_n

In the table, for a component, the change degree of mechanical properties is obtained by taking various mechanical parameters as the evaluation criteria. Among them are $\sum_{i=1}^{n} a_i = 1$, $\sum_{i=1}^{n} b_i = 1$, $\sum_{i=1}^{n} c_i = 1$. The variation degree of the obtained mechanical parameters is integrated by the D–S evidence theory. Finally, the mechanical properties of the components, after analyzing a comprehensive variety of mechanical parameters, are obtained. The integration of data effectively improves the accuracy of a safety analysis and assists in finding key stress component units.

Assume that φ is the beam string structure construction safety intelligent analysis framework, as expressed by Equation (7).

$$\varphi = (\alpha_1, \alpha_2, \cdots, \alpha_n) \tag{7}$$

In the Equation, α_i (i = 1, 2, . . . , n) is the mechanical property change of the ith member element in beam string structure.

All possible working conditions of the beam string structure member elements are represented by the power set "2^{φ}", which is specifically expressed as Equation (8).

$$2^{\varphi} = \left(\varnothing, \alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1 \cup \alpha_2, \alpha_i \cup \alpha_j \cup \cdots \cup \alpha_k \cdots\right) \tag{8}$$

In the formula, $\varnothing$ indicates that the mechanical properties of structural member elements do not change. α_i represents the change in mechanical properties of the ith member element in the structure. $\alpha_i \cup \alpha_j \cup \cdots \cup \alpha_k$ represents the change in mechanical properties of multiple component elements in the structure.

The basic probability distribution function (mass function) can be expressed as m: $2^{\varphi} \to [0, 1]$, satisfying Equation (9):

$$\begin{cases} m(\varnothing) = 0 \\ \sum_{A \subset \varphi} m(A) = 1 \end{cases} \tag{9}$$

In Equation (9), A is a certain working condition of the prestressed cable, and $m(A)$ is the basic probability distribution function of A.

Assume that $m_j(A_i)$ is the basic probability distribution function for the jth mechanical parameter sensitivity indicator. To determine the change in the mechanical properties of the ith component element, the synthesis rule of the D–S evidence matrix is Equation (10):

$$DF(A) = m_1 \oplus m_2 \oplus \cdots \oplus m_n(A) \begin{cases} \frac{\sum_{\cap A_i = A} \prod_{1 \le j \le n} m_j(A_i)}{K} & (A \neq \varnothing) \\ 0 & (A = \varnothing) \end{cases} \tag{10}$$

In Equation (10), $K = \sum_{\cap A_i \neq \varnothing} \prod_{1 \le j \le n} m_j(A_i) = \sum_{A_1 \cap A_2 \cap \cdots \cap A_n \neq \varnothing} m_1(A_1) m_2(A_2) \cdots m_n(A_n) = 1 - \sum_{A_1 \cap A_2 \cap \cdots \cap A_n = \varnothing} m_1(A_1) m_2(A_2) \cdots m_n(A_n)$.

4.2. Key Component Capture and Structure Safety Analysis Process

Driven by the DTs frame, the D–S evidence theory can be integrated to capture the most critical component units in the structure, and finally, the intelligent analysis of structural construction safety is realized. Firstly, twin modeling of the real construction structure is carried out to realize the real mapping of the real construction process. In the virtual model with high fidelity, the working conditions corresponding to the construction site are established to extract the mechanical performance index data of the model simulation. The relevant information of the index data is processed by probability, and the mechanical properties of each component element in the structural system are calculated using the D–S evidence theory algorithm. According to the degree of change analyzed by comprehensive mechanical parameters, the key stress components are captured, and then the maintenance measures of construction unsafe events are formed [35]. The maintenance measures are analyzed in the virtual model to guide the construction of physical space. In this study, the mechanical properties of each component element are characterized by a variation in the mechanical parameters. By sorting the change probability of mechanical parameters of each component element of the structure, the change degree of mechanical properties for each component element can be determined. The variation degree of multiple mechanical parameters can be used to visually determine the most critical component unit of the structure, so as to provide the basis for the safety performance evaluation and maintenance of the structure. Using the analysis of the change degree of mechanical parameters, the process of safety performance evaluation and the maintenance of the structure are represented in Figure 3.

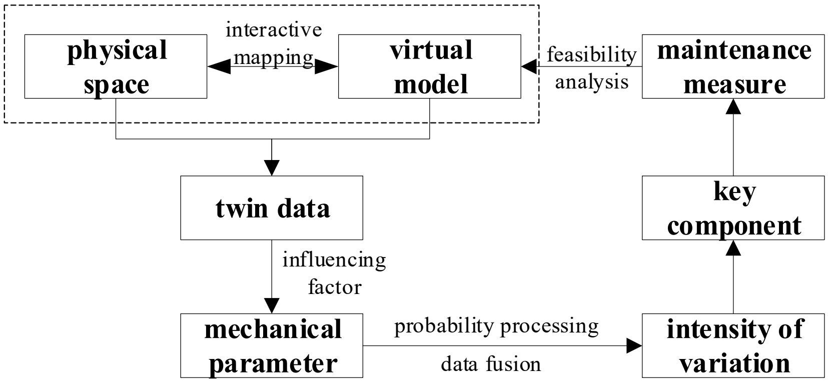


Figure 3. Security Performance Evaluation and Maintenance Process of Structure.

5. Key Component Capture and Safety Intelligent Analysis of Beam String Structure under Temperature

The integration of DTs and the D–S evidence theory forms the intelligent analysis method of structural construction safety. In this study, the theoretical method is applied to the construction process of the symmetrical structure (string beam). Under the action of temperature, the key stress components are captured. By capturing the key components, the safety analysis of the structure is realized. The effectiveness of this research method is verified via comparison with the field data collection.

5.1. Structural Model

Taking a string-supported beam roof as the research object, one of the plane string structures was selected for the calculation model. The model construction is shown in Figure 4. The span of the structure is L = 48 m, the rise height is 0 m, the sag is 3.5 m, and the number of struts is nine. The upper chord beam adopts H-shaped steel, Q345B. The pole adopts a round steel tube, Q235B. The lower chord cable adopts two types of 55 light circle stress-relief steel wires. The section size is shown in Table 3.

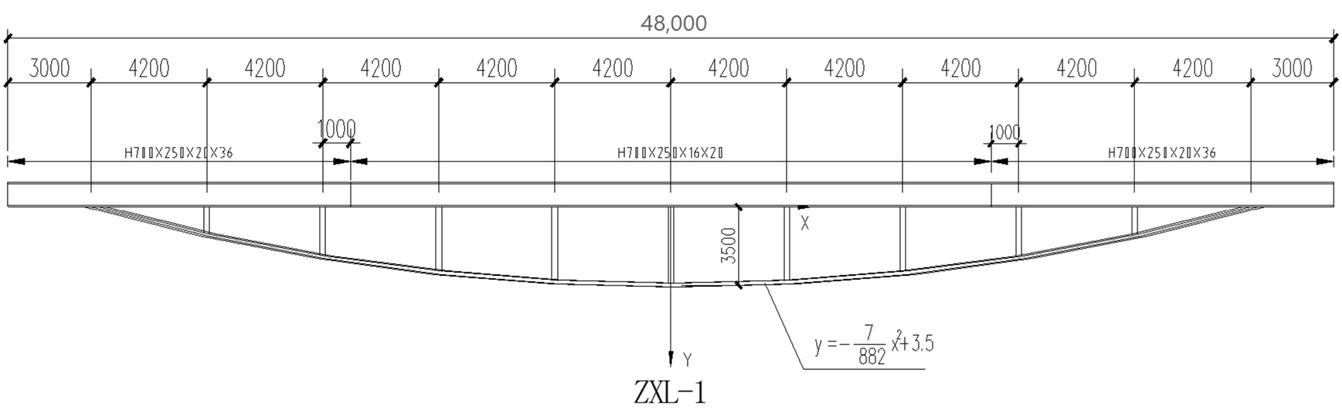


Figure 4. Structural model of string-supported beam during test.

Table 3. Sectional dimensions of components.

Section of Top Chord			Size of Strut		Cable Size	
Left Section (mm)	Midportion (mm)	Right Section (mm)	Model	Area (m^2)	Model	Area (m^2)
H700 × 250 × 20 × 36	H700 × 250 × 16 × 20	H700 × 250 × 20 × 36	$\varphi159 \times 8$	0.0038	$2SNS/S\varphi7 \times 55$	0.0021

The structure of this study is symmetrical. Therefore, in the process of conducting a construction safety analysis, half of the structure was utilized to judge the change in the mechanical parameters of each component. In the process of analysis, the components of the half structure were numbered as shown in Figure 5. The twin model of the structure was built in MIDAS. In this study, the cable and brace members were the main objects of analysis. In the finite element model, the strut and cable members were set as truss elements. Therefore, the cable unit is the tension only unit. The constraints of the cable and brace are articulated points. The finite element model can reflect the performance of the structure effectively. In the analysis process, the temperature effect is the influencing factor of structural safety performance. Driven by the integration of DTs and the D–S evidence theory, the changes in stress, deflection and axial force of each component were analyzed. Three kinds of mechanical parameters were integrated to judge the key components of the structure, and strain gauges were arranged on each component in the construction process to collect changes in real time. According to the collection results, the key stress components were found. Finally, the key components obtained by the two methods were compared to verify the effectiveness of the method. In this test structure, the change in the mechanical parameters of the upper chord beam was relatively small. In the analysis process, the change in the mechanical parameters of struts and lower chord cables (i.e., number 1–10) was analyzed carefully.

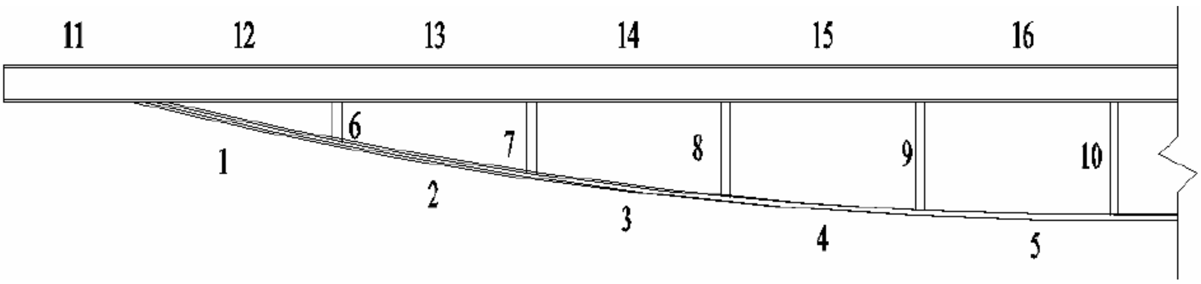


Figure 5. Numbers of components of half-span structures.

5.2. Analysis of Structural Mechanics Parameters

Combined with the temperature change of a practical project, it was concluded that the annual temperature change in the region is 50 °C. In order to simulate the state of the real structure, a test model was then built. In the process of a mechanical parameter fusion analysis, the middle point of each component can be used as the acquisition point of parameters. The virtual model of the structure was established using finite element modeling. In the construction process, the influence of a temperature rise of 50 °C on the structural safety performance was studied. At the same time, the temperature of the system was set to increase by 50 °C in the finite element model. The setting of temperature effect in the twin model is shown in Figure 6. In this study, stress, axial force and deflection in the component were analyzed. By analyzing the variation of three kinds of mechanical parameters comprehensively using the D–S evidence theory, the key stressed members in the structure were accurately judged.

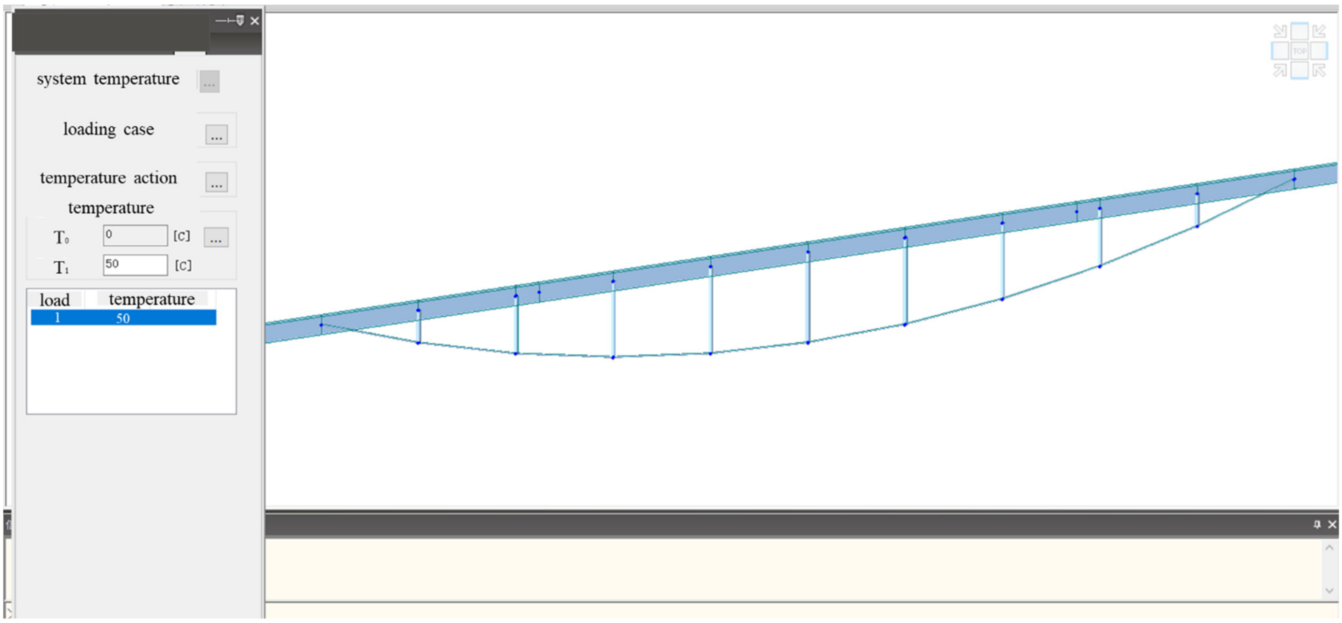


Figure 6. Setting of temperature action in twin model.

Under the action of temperature, the changes in various mechanical parameters of each component in the virtual model are shown in Table 4. In this study, the variation of various mechanical parameters in each component was obtained by comparing the results before and after the temperature increased by 50 °C under the condition of self-weight. The stress variation was then calculated using Equation (11).

$$P_i^* = \frac{\left|\delta_{fi} - \delta_{fi}^*\right|}{\delta_{fi}} \tag{11}$$

In the Equation, P_i^* represents the change of stress of the ith component. δ_{fi} means the stress of the ith member before the temperature rises by 50 °C. δ_{fi}^* represents the stress of the ith member after the temperature increases by 50 °C. The variation of other mechanical parameters was calculated by this method.

Table 4. Changes of various mechanical parameters of components.

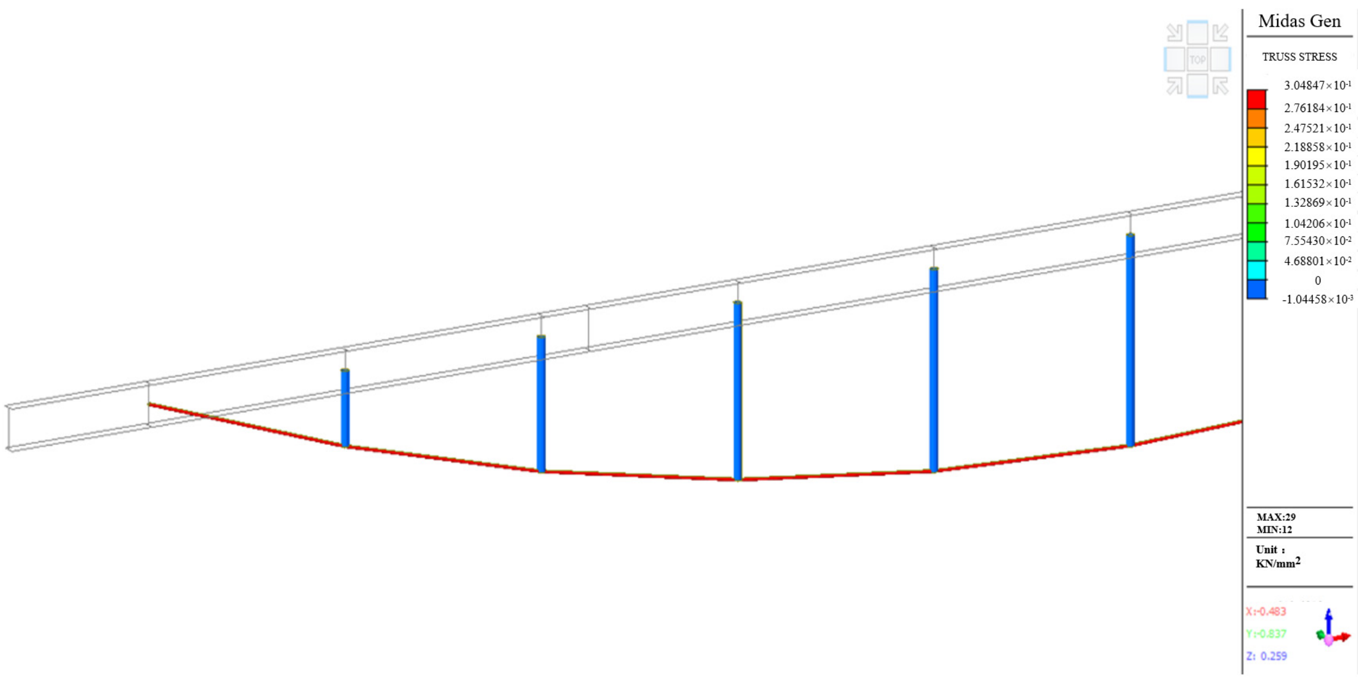
Component Number	Variation of Stress (%)	Variation of Axial Force (%)	Variation of Deflection (%)
1	34.13	34.13	33.27
2	34.13	34.12	30.89
3	34.10	34.13	28.03
4	34.13	34.14	26.13
5	34.16	34.15	25.12
6	35.93	35.92	31.17
7	35.64	35.64	28.19
8	35.56	35.56	25.97
9	35.73	35.73	24.53
10	34.18	34.13	24.05

5.3. Capture of Key Stressed Components

In Section 5.2, the changes in the mechanical parameters of the structure when subjected to a 50 °C temperature rise were identified using DTs modeling. However, the key stress components of the structure cannot be accurately determined according to a single mechanical parameter. According to the analysis of the D–S evidence theory in Section 4.1, three kinds of mechanical parameters are fused. Finally, the results of the change in the comprehensive parameters were used as the basis for capturing the key stress components. Firstly, the initial probability of Table 4 data was processed to ensure that the sum of the change probability of a certain mechanical parameter of each component was 1. Taking stress as an example, the stress change probability P_i of each component after correction can be calculated according to Equation (12).

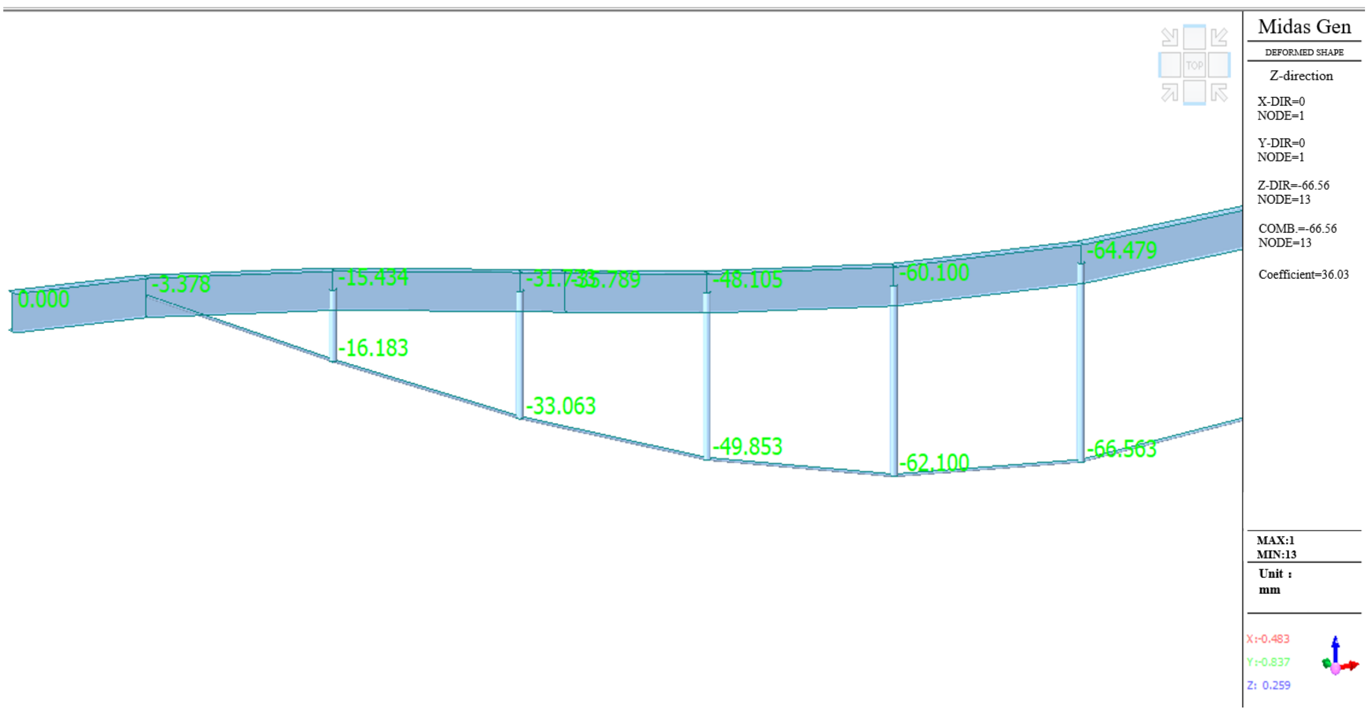
$$P_i = \frac{P_i^*}{\rho} \tag{12}$$

In the Equation, ρ denotes the probability correction coefficient, $\rho = \sum_{i=1}^{10} P_i^*$. The calculated probability can be used for the fusion analysis of D–S evidence theory, so as to determine the comprehensive changes in the mechanical parameters of each component. Similarly, the change probability of mechanical parameters of each component can be obtained using the axial force and deflection as the discriminant indexes. Combined with the D–S evidence theory, the response of the structure to temperature can be obtained by comprehensively judging the change in the mechanical parameters of each component. At the same time, the change rate of strain data measured by strain gauges in real structures can be used as the basis to verify the effectiveness of this research method. When the temperature rises by 50 °C, the structural mechanical parameters in the twin model are shown in Figure 7.



(**a**)

Figure 7. *Cont.*



(**b**)

Figure 7. Structural mechanical parameters in twin model. (**a**) stress. (**b**) perpendicular displacement.

According to the comparison between the analysis value of D–S evidence theory and the measured value of data this research method was found to improve the accuracy of safety performance analyses. It is inconsistent to judge the key components simply according to the change degree of a mechanical parameter in the virtual model simulation. On the basis of digital twinning modeling, the key components comprehensively judged using the D–S evidence theory and multiple mechanical parameters were consistent with the data analysis results of actual collection. In the physical space, the string supported beam roof was established for experimental verification. During the test, the working conditions consistent with the twin model were set. According to the literature [33], the rate of change of the cross-sectional area of each node on the member in real structure was taken as an important basis for the detection results of structural safety performance. During the test, the cross-section size of each node was collected according to the strain gauge. The change rate of the cross-sectional area of each node of the component on site was collected as shown in Figure 8. The comparison of data is shown in Figure 9. For ease of expression, the data represent the sum of probabilities for a processed rate of change of 1. That is, the sum of the change rates of each type of discriminant index for all components is 1. In this study, under the action of temperature, components 1 and 6 were analyzed as key stress components. Therefore, in the maintenance of structural safety performance, the stress performance changes of components 1 and 6 should be thoroughly analyzed. This study provides a new idea for the health monitoring of symmetrical structures based on the intelligent analysis method of structural construction safety formed by chord beams.

Figure 8. Condition setting and data acquisition in the test process.

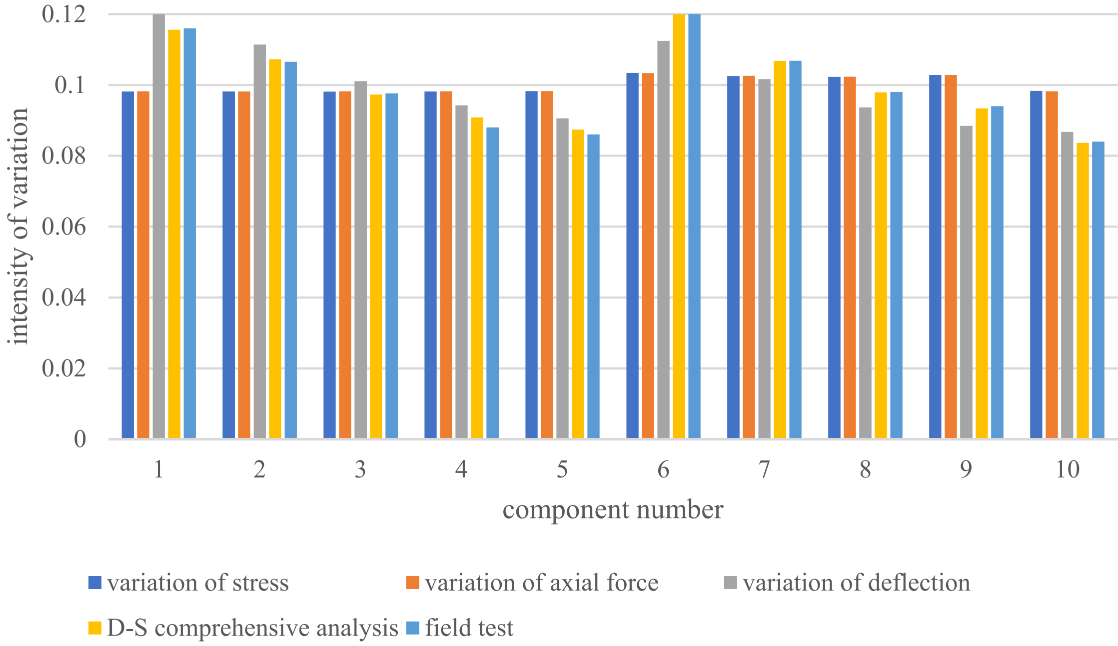


Figure 9. Comparison of field measured data and theoretical analysis data.

5.4. Analysis of the Research

In this study, an intelligent method for the capture and safety analysis of key components of string-supported beams based on DTs was established. This method can effectively avoid the problems of the insufficient accuracy of structural analysis and low efficiency of actual structural-data acquisition. Driven by the theoretical method, a high-fidelity twin model was established for the experimental structure. In this model, the mechanical properties of the structure can be truly reflected. The mechanical parameters of the structure were extracted by setting the analysis condition in the twin model. In order to accurately obtain the key force components of the structure, the changes in axial force, stress and deflection were comprehensively considered. Driven by the D–S evidence theory, the key components in the structure were identified. The analysis accuracy was effectively improved by integrating multiple mechanical parameters. This analysis method can help to avoid the problem of the inaccurate judgment of structural performance when using a single index [36]. During the test, the key components in the physical space were judged by the change in the section area of the components. Comparing the data collected in the test process with the data analyzed by the theoretical method, the results were found to be consistent. The twin model of the structure was established based on MIDAS, which can directly extract the mechanical parameters of the structure. Driven by the D–S evidence theory, the importance of each component can be obtained without an extensive parameter adjustment of the mathematical model. On the one hand, the method proposed in this study can accurately obtain the key stressed components, therefore providing the basis for the safety analysis and maintenance of the structure. On the other hand, this research method can effectively reflect the results of structural tests and reduce the cost of data collection of structural tests.

6. Conclusions and Prospects

6.1. Conclusions

Based on DTs modeling, this study integrated the D–S evidence theory into the data analysis process. By capturing the key stress components in the structure, the intelligent analysis of the construction safety of the structure was realized. In the process of analysis, the mechanical parameters of the structure can be directly extracted by establishing the twin model. The mechanical parameters were fused using the D–S evidence theory, and the mechanical properties of each component were comprehensively judged. Finally, the key components of the structure were found. In the process of finding key components, a variety of mechanical parameters were integrated to improve the accuracy of the analysis. In the research process of structural construction safety intelligent analysis method, the following main findings were obtained.

(1) Driven by the integration of DTs and the intelligent algorithm, a construction safety analysis framework for beam string structures was formed. Driven by this framework, the visualization of construction process, a mechanical property analysis and safety closed-loop control can be realized.
(2) Based on the analysis framework, the correlation mechanism of multi-source data in the construction process was first clarified. Considering space-time evolution and virtual–real interaction, a multidimensional model for structural safety analyses was formed. It is concluded that the core of structural safety analysis is the capturing of key components.
(3) In view of capturing the key components of the structure, the simulation data of the twin model were integrated with the D–S evidence theory. The structural safety analysis was achieved, thereby providing a reference for the health monitoring of practical engineering.

6.2. Prospects

The key component capture and safety intelligent analysis method of beam string structures formed in this study were applied to a convention and exhibition center. The validity of this research method was then verified by comparing the analytical value of the theoretical method with the measured value at the site. DTs visualization demonstrated the whole process of construction and provides model support for data extraction. The D–S evidence theory combines multiple mechanical parameters to judge key structural members. The integration of DTs and intelligent algorithms provides a basis and reference for the health monitoring of symmetric structures. For the safety analysis of structures, the aim of future research is to obtain the key influencing factors and quantify the influence degree of various factors on mechanical parameters [37]. Finally, the closed loop control of construction safety was carried out for all elements driven by DTs.

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References

1. Lu, J.; Xue, S.D.; Li, X.Y.; Liu, R.J. Key construction technology of annular crossed cable-truss structure. *J. Tianjin Univ. Sci. Technol.* **2021**, *54*, 101–110. (In Chinese)
2. Guo, J.M.; Zhou, D. Pretension simulation and experiment of a negative Gaussian curvature cable dome. *Eng. Struct.* **2016**, *127*, 737–747. [CrossRef]
3. Deng, H.; Zhang, M.; Liu, H.; Dong, S.; Zhang, Z.; Chen, L. Numerical analysis of the pretension deviations of novel crescent-shaped tensile canopy structural system. *Eng. Struct.* **2016**, *119*, 24–33. [CrossRef]
4. Lu, J.; Xue, S.; Li, X.; Liu, R. Study on membrane roof schemes of annular crossed cable-truss structure. *Int. J. Space Struct.* **2019**, *34*, 85–96. [CrossRef]
5. Xue, S.D.; Tian, X.S.; Liu, Y.; Li, X.Y.; Liu, R.J. Mechanical behavior of single-layer saddle-shape crossed cable net without inner-ring. *J. Build. Struct.* **2021**, *42*, 30–38. (In Chinese)
6. Liu, R.J.; Zou, Y.; Xue, S.D.; Li, X. Influence on static performance of loop-free suspen-dome after removal of cables. *J. Build. Struct.* **2020**, *41* (Suppl. S1), 1–9. (In Chinese)
7. Krishnan, S. Structural design and behavior of prestressed cable domes. *Eng. Struct.* **2020**, *209*, 110294. [CrossRef]
8. Chen, Z.H.; Ma, Q.; Yan, X.Y.; Lou, S.; Chen, R.; Si, B. Research on Influence of Construction Error and Controlling Techniques of Compound Cable Dome. *J. Hunan Univ. (Nat. Sci.)* **2018**, *45*, 47–56. (In Chinese)
9. Ge, J.Q.; Liu, B.N.; Wang, S.; Zhang, G.; Zhang, M.; Huang, J.; Liu, X. Study on design of prestressed tensegrity cable structures. *J. Build. Struct.* **2019**, *40*, 73–80. (In Chinese)
10. Thai, H.T.; Kim, S.E. Nonlinear static and dynamic analysis of cable structures. *Finite Elem. Anal. Des.* **2011**, *47*, 237–246. [CrossRef]
11. Zhang, A.L.; Sun, C.; Jiang, Z.Q. Calculation method of prestress distribution for levy cable dome with double struts considering self-weight. *Eng. Mech.* **2017**, *34*, 211–218. (In Chinese)
12. Guo, Y.L.; Zhang, X.Q. Experimental study on the influences of cable length errors in Geiger cable dome designed with unadjustable cable length. *China Civ. Eng. J.* **2018**, *51*, 52–68. (In Chinese)
13. Wang, Y.; Guo, Z.; Luo, B.; Shi, K. Study on the determination method for the equivalent pre-tension in cables of spatial prestressed steel structure. *China Civ. Eng. J.* **2013**, *46*, 53–61. (In Chinese)
14. Castillo, E.; Ramos, A.; Koller, R.; López-Aenlle, M.; Fernández-Canteli, A.F. A critical comparison of two models for assessment of atigue data. *Int. J. Fatigue* **2008**, *30*, 45–57. [CrossRef]
15. Arezki, S.; Kamel, L.; Amar, K. Effects of temperature changes on the behavior of a cable truss system. *J. Constr. Steel Res.* **2017**, *129*, 111–118.
16. Basta, A.; Serror, M.H.; Marzouk, M. A BIM-based framework for quantitative assessment of steel structure deconstructability. *Autom. Constr.* **2020**, *111*, 103064. [CrossRef]
17. Liu, Z.; Shi, G.; Zhang, A.; Huang, C. Intelligent Tensioning Method for Prestressed Cables Based on Digital Twins and Artificial Intelligence. *Sensors* **2020**, *20*, 7006. [CrossRef]
18. Yang, Y.; Zhang, Y.; Tan, X. Review on Vibration-Based Structural Health Monitoring Techniques and Technical Codes. *Symmetry* **2021**, *13*, 1998. [CrossRef]
19. Grieves, M. *Virtually Perfect: Driving Innovative and Lean Products through Product Lifecycle Management*; Space Coast Press: Merritt Island, FL, USA, 2011.
20. Shirowzhan, S.; Tan, W.; Sepasgozar, S.M.E. Digital Twin and Cyber GIS for Improving Connectivity and Measuring the Impact of Infrastructure Construction Planning in Smart Cities. *Int. J. Geo-Inf.* **2020**, *9*, 240.
21. LeCun, Y.; Bengio, Y.; Hinton, G. Deep learning. *Nature* **2015**, *521*, 436–444. [CrossRef]
22. Liu, Z.S.; Zhang, A.S.; Wang, W.S.; Wang, J.J. Dynamic Fire Evacuation Guidance Method for Winter Olympic Venues Based on Digital Twin-Driven Model. *J. Tongji Univ. (Nat. Sci.)* **2020**, *48*, 962–971. (In Chinese)
23. Zeng, B.; Zhou, Z.; Zhang, Q.F.; Xu, Q. Multi-position damage identification and anti-noise analysis of cable-stayed arch-truss based on data fusion. *J. Build. Struct.* **2020**, *41* (Suppl. S1), 36–43. (In Chinese)
24. Acharya, D.; Khoshelham, K.; Winter, S. BIM-PoseNet: Indoor camera localisation using a 3D indoor model and deep learning from synthetic images. *ISPRS J. Photogramm. Remote Sens.* **2019**, *150*, 245–258. [CrossRef]
25. Liu, Z.; Shi, G.; Jiao, Z.; Zhao, L. Intelligent Safety Assessment of Prestressed Steel Structures Based on Digital Twins. *Symmetry* **2021**, *13*, 1927. [CrossRef]
26. Zhao, L.; Cao, Z.; Wang, Z.; Fan, F. Initial prestress design and optimization of cable-stiffened latticed shells. *J. Constr. Steel Res.* **2021**, *184*, 106759. [CrossRef]

27. Zhu, H.; Wang, Y. Intelligent Analysis for Safety-Influencing Factors of Prestressed Steel Structures Based on Digital Twins and Random Forest. *Metals* **2022**, *12*, 646. [CrossRef]
28. Zhu, H.; Wang, Y. Intelligent Prediction of Prestressed Steel Structure Construction Safety Based on BP Neural Network. *Appl. Sci.* **2022**, *12*, 1442. [CrossRef]
29. Luo, Y.Z.; Fu, W.W.; Wan, H.P.; Shen, Y. Load-Effect Separation of a Large-Span Prestressed Structure Based on an Enhanced EEMD-ICA Methodology. *J. Struct. Eng.* **2022**, *148*, 04021288. [CrossRef]
30. Chen, H.P.; Yin, S.W.; Qiao, C.; He, J. Construction Effects on the Mechanical States of a Truss Structure. *J. Perform. Constr. Facil.* **2022**, *36*, 04021099. [CrossRef]
31. Lu, Q.; Parlikad, A.K.; Woodall, P.; Don, G. Developing a dynamic digital twin at building and city levels: A case study of the West Cambridge campus. *J. Manag. Eng.* **2019**, *36*, 1–19.
32. Turner, C.J.; Oyekan, J.; Stergioulas, L.; Griffin, D. Utilizing Industry 4.0 on the Construction Site: Challenges and Opportunities. *IEEE Trans. Ind. Inform.* **2021**, *17*, 746–756. [CrossRef]
33. Liu, Z.; Shi, G.; Jiang, A.; Li, W. Intelligent Discrimination Method Based on Digital Twins for Analyzing Sensitivity of Mechanical Parameters of Prestressed Cables. *Appl. Sci.* **2021**, *11*, 1485. [CrossRef]
34. Denoeux, T.; Shenoy, P. An Interval-Valued Utility Theory for Decision Making with Dempster-Shafer Belief Functions. International. *J. Approx. Reason.* **2020**, *124*, 194–216. [CrossRef]
35. Ma, C.; Zhao, T.; Li, G.; Zhang, A.; Cheng, L. Intelligent Anomaly Identification of Uplift Pressure Monitoring Data and Structural Diagnosis of Concrete Dam. *Appl. Sci.* **2022**, *12*, 612. [CrossRef]
36. Yang, Y.; Lu, H.; Tan, X.; Chai, H.K.; Wang, R.; Zhang, Y. Fundamental mode shape estimation and element stiffness evaluation of girder bridges by using passing tractor-trailers. *Mech. Syst. Signal Processing* **2022**, *169*, 108746. [CrossRef]
37. Yang, Y.; Ling, Y.; Tan, X.; Wang, S.; Wang, R.Q. Damage identification of frame structure based on approximate Metropolis–Hastings algorithm and probability density evolution method. *Int. J. Struct. Stab. Dyn.* **2022**, *22*, 2240014. [CrossRef]