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# Damage Effectiveness Calculation of Hitting Targets with Ammunition Based on Bayesian Multinomial Distribution

**Haobang Liu and Xianming Shi ***

Shijiazhuang Campus, Army Engineering University, Shijiazhuang 050003, China; liuwuliu_2020@tju.edu.cn
* Correspondence: sxm@nudt.edu.cn

**Abstract:** Owning to the fact that ammunition can cause varying degrees of damage to its target, this article presents a damage effectiveness calculation method of hitting targets with ammunition based on Bayesian multinomial distribution to solve the problems of complex processes, few trial times and difficult calculations of damage probability in target-hitting tests with high-tech ammunition, according to a calculation index of damage effectiveness about the occurrence probability of different damage. Based on the concept of symmetry, the idea of "divide damage level—determine distribution—integrate information—solve distribution" is adopted. Firstly, this paper describes the damage effectiveness test of ammunition attacking targets with multiple distributions; secondly, this paper integrates the damage effectiveness information of ammunition strike targets with Dempster–Shafer evidence theory (D–S evidence theory) and symmetry advantage; finally, this paper attempts to solve the symmetric posterior distribution of damage effectiveness parameters with Bayesian theory and the Markov chain Monte Carlo (MCMC) method. The result demonstrates that this method is very significant in improving the calculation accuracy of ammunition damage effectiveness, which could describe the damage effectiveness of ammunition in detail by integrating the prior information with multiple types of damage effectiveness.

**Keywords:** damage effectiveness; ammunition; Bayesian; multinomial distribution; Markov chain Monte Carlo; Dempster–Shafer evidence theory

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## 1. Introduction

With the application and development of high-tech ammunition, the impacts of ammunition damage effectiveness on ammunition ratio, ammunition type selection and ammunition demand calculations are also increasing [1,2]. There are more accurate requirements for the damage effectiveness calculation of hitting targets with ammunition because of the precision operation and striking in modern war at present [3].

Through the research on the damage effectiveness of ammunition hitting targets, it will be more efficient to use ammunition and make a fire strike plan. In recent years, the US military has made some progress in the study of ammunition damage effectiveness, who take the battle damage assessment (BDA) system as a priority development project [4]. The US military has also compiled the "Joint Ammunition Effectiveness Manual", which covers the damage effectiveness data of many ammunitions and establishes an evaluation system about multiple damage effectiveness [5,6]. The research has greatly helped the US military to make decisions in ammunition development and use planning. The present study takes the "single shot damage probability" and "ammunition consumption with expected damage probability" as the quantitative indicators of ammunition damage effectiveness in China [7]. Wu et al. [8] analyzed the law of damage effectiveness changing with actual combat factors such as direction deviation angle and falling angle according to the index of damage probability. Wang et al. [9] used the amount of ammunition to damage ships to measure the damage effectiveness of semi armor piercing shells to small ship targets. Chen et al. [10] analyzed the functional vulnerability of ship targets with the method of

Monte Carlo simulation and established a calculation model of damage effectiveness based on standards of moderate and severe damage.

The present research in damage effectiveness is mainly based on the binomial distribution of the damage standard after hitting targets with ammunition. However, in fact, the damage level caused by each attacked target with ammunition are different [11]. It could better reveal the damage effectiveness characteristics of ammunition to describe damage effectiveness tests in terms of multiple distributions according to the quantitative characterization index of damage effectiveness with the probability of different damage levels of strike targets. Limited by the test environment and test cost, the damage effectiveness data of attack targets are insufficient due to the small number of attack tests with high-tech ammunition under outfield conditions. Therefore, it is also necessary to consider the calculation of damage effectiveness under the condition of small samples. Bayesian theory is widely used in the field of small sample engineering at present. Liu et al. [12] estimated the average residual strength of multi-state component systems based on non-parametric Bayesian method. Zhao et al. [13] studied the ammunition consumption associated with different damage levels under small sample data using the Bayesian method to integrate multi-source prior information. Zhao et al. [14] used the Bayesian method which integrates various types of reliability information to estimate the remaining life of on orbit satellite components. Aimed at the problem of small sample testing in the development of aerospace valves, Wang et al. [15] made full use of various existing test data to complete a reliability evaluation of aerospace valves based on the Bayes principle. Ming et al. [16] used a binomial distribution algorithm to design an identification test scheme for a success-or-failure test using the Bayes method. Gu et al. [17] studied parameter estimation for a type of fractional diffusion equation based on a compact difference scheme by using the Bayesian method. However, the Bayesian method, which has been used in many fields, is less applied in the damage effectiveness evaluation of ammunition, so the problem of insufficient sample data of damage effectiveness still exists.

Based on the above analysis, this article provides a description of the damage effectiveness test in terms of multiple distributions according to the quantitative characterization index of damage effectiveness with the probability of different damage ranks of strike targets. This article uses the Bayesian method and incorporates the idea of symmetry to solve the small sample problem [18], and it uses Dempster–Shafer evidence theory (D–S evidence theory) to integrate the prior information. The posterior distribution is solved by Gibbs sampling using the Markov chain Monte Carlo (MCMC) method. More comprehensive results of damage effectiveness calculation are obtained at last.

## 2. Hypothesis of Damage Effectiveness Distribution for Ammunition Hitting Targets

"Single shot damage probability" is used as a quantitative indicator of ammunition damage effectiveness. Single shot damage probability refers to the probability that the target hit by a single shot of ammunition reaches the damage standard, which means the greater the probability, the higher the damage effectiveness. In order to calculate the damage effectiveness of ammunition hitting the target, it is necessary to divide the damage level of the target and assume the probability distribution of the damage effectiveness test.

### *2.1. Classification of Target Damage Level*

According to the degree of impact of ammunition hitting on target combat effectiveness, the damage grades of targets are divided [19] as shown in Table 1.

**Table 1.** Target damage classification.

| *K* | Target Damage Grades | Damage |
|---|---|---|
| 1 | Zero damage | The target is slightly damaged or undamaged, and the combat effectiveness loss is within 5%. |
| 2 | Mild damage | Target performance suffers a slight loss. If repairs are not carried out in time, combat performance will be affected and combat effectiveness will be lost by 5% to 20%. |
| 3 | Moderate damage | The target is seriously damaged and requires replacement parts for repair, resulting in a 20% to 50% loss of combat effectiveness. |
| 4 | Severely damage | The target is seriously damaged and needs to be returned to the factory for overhaul, the repair takes a long time and the combat effectiveness is lost by 50% to 80%. |
| 5 | Destroy the annihilation | The target is completely destroyed and cannot be repaired, and the combat effectiveness loss is more than 80%. |

### *2.2. Multinomial Distribution Hypothesis of Damage Effectiveness Test*

It can be seen from the above that the damage grades caused by ammunition hitting the target are divided into 5, and a total of $n$ hit tests are carried out, and the conditions of each test are the same. Then, five basic events $S_1, S_2, S_3, S_4, S_5$ can be generated, which represent the results of different damage grades of ammunition hitting targets. Note that $p_i = P\{S_i\}, i = 1, 2, 3, 4, 5$ and $p_i$, represents the probability of different damage grades, and the sum of the probabilities is 1. Let $x_i$ represent the number of times event $S_i$ occurs in $n$ strike trials, then $\boldsymbol{x} = (x_1, x_2, x_3, x_4, x_5)$ obeys the multinomial distribution with parameter $\boldsymbol{p} = (p_1, p_2, p_3, p_4, p_5)$:

$$f(\boldsymbol{x}|n, \boldsymbol{p}) = \frac{n! \prod_{i=1}^{5} {p_i}^{x_i}}{\prod_{i=1}^{5} x_i!} \tag{1}$$

where $\sum_{i=1}^{5} x_i = n$, and $\sum_{i=1}^{5} p_i = 1$.

Based on the multinomial distribution hypothesis, Bayesian inference of damage effectiveness can be performed.

## 3. Determination of Prior Distribution of Damage Effectiveness Parameters

Before using the Bayesian method, the prior distribution of the damage effectiveness parameter $\boldsymbol{p}$ needs to be determined. The prior distribution contains sufficient prior information of damage effectiveness, such as simulation test information, expert knowledge information, etc. The importance of making full use of prior information has become a consensus in the field of small sample research.

### *3.1. Bayesian Theory Parameter Estimation*

Different from classical statistics, which only utilize sample information, the Bayesian method can not only utilizes sample information but also integrate prior information and overall information [20]. Therefore, the Bayesian method can be selected to solve the problem of insufficient sample size in damage effectiveness tests. The solution of the Bayesian method is as follows:

$$\pi(\boldsymbol{p}|\boldsymbol{x}) = \frac{L(\boldsymbol{x}|\boldsymbol{p})\pi(\boldsymbol{p})}{m(\boldsymbol{x})} = \frac{L(\boldsymbol{x}|\boldsymbol{p})\pi(\boldsymbol{p})}{\int_{\Theta} L(\boldsymbol{x}|\boldsymbol{p})\pi(\boldsymbol{p})\mathrm{d}\boldsymbol{p}} \tag{2}$$

In the formula, $L(\boldsymbol{x}|\boldsymbol{p})$ is the likelihood function of the field sample of the damage effectiveness test, $\pi(\boldsymbol{p})$ is the prior distribution of the damage effectiveness parameter $\boldsymbol{p}$ and $m(\boldsymbol{x})$ is the marginal distribution of the sample and has nothing to do with the parameter $\boldsymbol{p}$; the above formula can be simplified as:

$$\pi(\boldsymbol{p}|\boldsymbol{x}) \propto L(\boldsymbol{x}|\boldsymbol{p})\pi(\boldsymbol{p}) \tag{3}$$

It can be seen from the above formula that the posterior distribution of the damage effectiveness parameter $\boldsymbol{p}$ is the combination of prior information and sample information. Therefore, scientific and reasonable prior information is needed to determine the prior distribution of parameters.

### *3.2. Selection of Prior Distribution of Damage Effectiveness Parameters*

Common prior distribution selection methods in Bayesian theory include the maximum entropy method [21], conjugate prior distribution method [22], uninformative prior distribution [23], etc. Considering that there is sufficient prior information and in order to facilitate the Bayesian solution, the conjugate prior distribution method was selected to determine the prior distribution. By consulting the Bayesian conjugate prior distribution family, it can be seen that the conjugate prior distribution of the multinomial distribution is the Dirichlet distribution [24], so the probability density function of the damage effectiveness parameter $\boldsymbol{p}$ can be expressed as:

$$f(\boldsymbol{p};\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5) = \frac{1}{B(\boldsymbol{\alpha})}\prod_{i=1}^{5}(p_i)^{\alpha_i-1} \tag{4}$$

where $\boldsymbol{\alpha} = (\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5)$ is the hyperparameter in the prior distribution and $B(\boldsymbol{\alpha})$ is the multivariate Beta function, namely

$$B(\boldsymbol{\alpha}) = \frac{\prod_{i=1}^{5}\Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^{5}\alpha_i)} \tag{5}$$

In the formula, $\Gamma(\alpha_i)$ is the Gamma function.

### *3.3. D–S Evidence Theory Information Fusion Method*

Before the field strike test, a large number of simulation tests or scientific research firing tests will be carried out. These test data can be used as prior information after passing the consistency test with the strike test data under the outfield conditions. In addition, some prior information can also be obtained by processing the strike test data of similar types of ammunition and collecting the knowledge and opinions of experts in relevant fields. This article mainly collected the ammunition damage effectiveness probability assignment of simulation information data and expert knowledge information data, as shown in Table 2.

**Table 2.** Probability assignment of prior information.

| Damage Level Events | Simulation Test Information $m_1$ | Expert Knowledge Information $m_2$ |
|---|---|---|
| $S_1$ | 0.16 | 0.17 |
| $S_2$ | 0.24 | 0.23 |
| $S_3$ | 0.26 | 0.25 |
| $S_4$ | 0.17 | 0.19 |
| $S_5$ | 0.17 | 0.16 |

In order to further refine the prior distribution of parameters, Dempster–Shafer evidence theory (D–S evidence theory) was introduced to fuse prior information [25]. The evidence fusion formula is as follows:

$$m_{1\oplus 2}(S) = \begin{cases} \frac{\sum\limits_{A\cap B=S} m_1(A)m_2(B)}{1-K} & S \neq \varnothing \\ 0 & S = \varnothing \end{cases} \tag{6}$$

where, $K = \sum\limits_{A\cap B\neq\varnothing} m_1(A)m_2(B) = 1 - \sum\limits_{A\cap B=\varnothing} m_1(A)m_2(B)$ is the normalization coefficient; $S$ represents the set of events of different damage levels caused by ammunition hitting the target and $m_1(A)$ and $m_2(B)$ represent the mass functions, respectively, representing the views of simulation information and expert knowledge information on the occurrence probability of events of different damage levels, which meet the following requirements:

$$\begin{aligned} &m(\varnothing) = 0 \\ &\sum_{S\subseteq\Theta} m(S) = 1 \end{aligned} \tag{7}$$

D–S evidence theory is used to integrate simulation test data and expert knowledge data, and the identification framework of D–S evidence theory is represented by $\Theta = \{S_1, S_2, S_3, S_4, S_5\}$, which represents the collection of events with different damage levels. The prior information fusion processing is completed according to the probability assignment of the prior information in Table 2.

1. Calculate the normalization coefficient:

$$1 - K = \sum_{i=1}^{5} m_1(A_i)m_2(B_i) \tag{8}$$

where $m_1(A_i)$ is the probability assignment of damage level event $S_i$ of the simulation test information, such as $m_1(A_1) = 0.16$ and $m_2(B_i)$ is the probability assignment of damage level event $S_i$ of the expert knowledge information, such as $m_2(B_1) = 0.17$.

2. Perform fusion solution for each damage level event $S_i$; calculate the corresponding combined mass function: The combined mass function of zero damage $S_1$ is:

$$\begin{aligned} m_{1\oplus 2}(S_1) &= m_1(A_1) \oplus m_2(B_1) \\ &= \tfrac{m_1(A_1)m_2(B_1)}{1-K} \end{aligned} \tag{9}$$

The combined mass function of the mild damage $S_2$ is:

$$\begin{aligned} m_{1\oplus 2}(S_2) &= m_1(A_2) \oplus m_2(B_2) \\ &= \tfrac{m_1(A_2)m_2(B_2)}{1-K} \end{aligned} \tag{10}$$

Similarly, the combined mass function, $m_{1\oplus 2}(S_3)$, of moderate damage $S_3$, $m_{1\oplus 2}(S_4)$ of severely damage $S_4$ and $m_{1\oplus 2}(S_5)$ of destroy the annihilation $S_5$ can be obtained. The value of $m_{1\oplus 2}(S_i)$ is the prior information probability assignment of the damage efficacy parameters after fusion. After fusing the simulation test data and expert knowledge data, a more accurate understanding of the prior information of the ammunition damage effectiveness can be obtained.

## 4. Bayesian Derivation of Ammunition Damage Effectiveness

### *4.1. Prior Information Consistency Check*

The scientificity and credibility of prior information have an important influence on the Bayesian derivation results of damage effectiveness. In order to use prior information in Bayesian derivation, it is necessary to check for consistency with the information of strike experiments under field conditions to prove the rationality of the prior infor-

mation. It is assumed that $X = (x_1, x_2, x_3, x_4, x_5)$ is the priori information data, and $X' = (x_1', x_2', x_3', x_4', x_5')$ is the field strike test data. Through the prior information of the ammunition damage effectiveness and the multinomial distribution hypothesis, the distribution function of ammunition damage effectiveness of prior information can be obtained:

$$F(\boldsymbol{x}|n,\boldsymbol{p}) = \int_{\Omega} f(\boldsymbol{x}|n,\boldsymbol{p})d\boldsymbol{p} \tag{11}$$

For the experimental data of field strikes, the parameter distribution of damage effectiveness is difficult calculate due to the small quantity and large error of parameter estimation. Therefore, the goodness-of-fit test of the distribution is carried out by selecting the field strike test data $X' = (x_1', x_2', x_3', x_4', x_5')$. The model was constructed using the EDF-type goodness-of-fit test [26]:

$$\begin{array}{l} H_0 : F(\boldsymbol{x}'|n,\boldsymbol{p}) = F(\boldsymbol{x}|n,\boldsymbol{p}) \\ H_1 : F(\boldsymbol{x}'|n,\boldsymbol{p}) \neq F(\boldsymbol{x}|n,\boldsymbol{p}) \end{array} \tag{12}$$

$H_0$ indicates that the prior information passes the consistency test, and $H_1$ indicates that it fails the consistency test.

Define the empirical distribution of the outfield hitting samples as:

$$F(\boldsymbol{x}'|n,\boldsymbol{p}) = \frac{1}{n}\sum_{i=1}^{n} I_{[x_n' \leq x']} = \frac{\#\{\boldsymbol{x}_i' \leq \boldsymbol{x}', i = 1, \cdots, n\}}{n} \tag{13}$$

where $I_{[x_n' \leq x']}$ represents the representative function and $\#\{\cdot\}$ represents the number of elements in the set $\{\cdot\}$. The empirical distribution function of the order statistic $x_i'$ is expressed as:

$$\widehat{F}(\boldsymbol{x}'|n,\boldsymbol{p}) = \begin{cases} 0, \boldsymbol{x}' < \boldsymbol{x}_1' \\ \frac{i}{n}, \boldsymbol{x}_i' \leq \boldsymbol{x}' < \boldsymbol{x}_{i+1}', i = 1, \cdots, n-1 \\ 1, \boldsymbol{x}' \geq \boldsymbol{x}_n' \end{cases} \tag{14}$$

The Kolmogorov distance is introduced to measure the degree of consistency between $F(\boldsymbol{x}'|n,\boldsymbol{p})$ and $F(\boldsymbol{x}|n,\boldsymbol{p})$ [27]:

$$K = \sup_{\boldsymbol{x}' \in \Theta} \left| F(\boldsymbol{x}'|n,\boldsymbol{p}) - F(\boldsymbol{x}|n,\boldsymbol{p}) \right| \tag{15}$$

According to the actual empirical distribution function calculation, the following equation can be obtained:

$$\widehat{K} = \max_{\boldsymbol{x}' \in \Theta} \left| \widehat{F}(\boldsymbol{x}'|n,\boldsymbol{p}) - F(\boldsymbol{x}|n,\boldsymbol{p}) \right| \tag{16}$$

Test consistency at a given significance level of $\alpha$:

$$P(K \leq \widehat{K}) = 1 - \alpha \tag{17}$$

When $\alpha$ is greater than the given significance level, it passes the consistency test, otherwise it fails. The significance level of 0.5 is selected for testing, and the result accepted was $H_0$, indicating that the prior information is more reliable.

### *4.2. Posterior Distribution Derivation of Ammunition Damage Effectiveness*

According to the derivation in the previous section, the Dirichlet distribution of conjugate prior distribution of multinomial distribution is selected as the prior distribution of the ammunition damage effectiveness parameters. Then, the posterior distribution of parameters can be solved by using the Bayesian formula to fuse the prior information and the experimental data in the field. The derivation process of posterior distribution is as follows:

Firstly, the prior distribution of the parameter $\boldsymbol{p}$ is obtained according to the prior information, that is, it obeys the Dirichlet distribution:

$$\pi(\boldsymbol{p}) \sim \text{Dirichlet}(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) \tag{18}$$

Secondly, construct the likelihood function for the sample condition from the field shooting test data:

$$\begin{aligned} L(\boldsymbol{x}|\boldsymbol{p}) &= f(x_1, x_2, x_3, x_4, x_5|n, p_1, p_2, p_3, p_4, p_5) \\ &= \frac{n!}{x_1!x_2!x_3!x_4!x_5!} {p_1}^{x_1} {p_2}^{x_2} {p_3}^{x_3} {p_4}^{x_4} {p_5}^{x_5} \end{aligned} \tag{19}$$

Finally, the posterior distribution of the damage effectiveness parameter $\boldsymbol{p}$ can be obtained by using the Bayesian formula:

$$\begin{aligned} \pi(\boldsymbol{p}|\boldsymbol{x}) &= \frac{L(\boldsymbol{x}|\boldsymbol{p})\pi(\boldsymbol{p})}{\int L(\boldsymbol{x}|\boldsymbol{p})\pi(\boldsymbol{p})d\boldsymbol{p}} \\ &= \frac{\frac{n!}{\prod_{i=1}^{5} x_i!} \prod_{i=1}^{5} {p_i}^{x_i} \frac{\Gamma(\sum_{i=1}^{5} \alpha_i)}{\prod_{i=1}^{5} \Gamma(\alpha_i)} \prod_{i=1}^{5} (p_i)^{\alpha_i - 1}}{\int \frac{n!}{\prod_{i=1}^{5} x_i!} \prod_{i=1}^{5} {p_i}^{x_i} \frac{\Gamma(\sum_{i=1}^{5} \alpha_i)}{\prod_{i=1}^{5} \Gamma(\alpha_i)} \prod_{i=1}^{5} (p_i)^{\alpha_i - 1} d\boldsymbol{p}} \\ &= \frac{\frac{\Gamma(\sum_{i=1}^{5} \alpha_i)}{\prod_{i=1}^{5} \Gamma(\alpha_i)} \prod_{i=1}^{5} (p_i)^{\alpha_i + x_i - 1}}{\int \frac{n!}{\prod_{i=1}^{5} x_i!} \frac{\Gamma(\sum_{i=1}^{5} \alpha_i)}{\prod_{i=1}^{5} \Gamma(\alpha_i)} \prod_{i=1}^{5} (p_i)^{\alpha_i + x_i - 1} d\boldsymbol{p}} \\ &= \frac{\Gamma(\sum_{i=1}^{5} \alpha_i + x_i)}{\prod_{i=1}^{5} \Gamma(\alpha_i + x_i)} \prod_{i=1}^{5} (p_i)^{\alpha_i + x_i - 1} \end{aligned} \tag{20}$$

It can be seen that the posterior distribution and the prior distribution obey the same distribution family and are both Dirichlet distributions, which also conforms to the properties of conjugate prior distribution.

The prior information is fused according to the D–S evidence theory, and the posterior distribution is solved by combining the Bayesian formula, and the posterior density function of the damage effectiveness parameter $\boldsymbol{p}$ can be obtained:

$$\pi(\boldsymbol{p}|\boldsymbol{x}) \sim \text{Dirichlet}(\alpha_1 + x_1, \alpha_2 + x_2, \alpha_3 + x_3, \alpha_4 + x_4, \alpha_5 + x_5) \tag{21}$$

According to the posterior probability density function, the posterior expected value of each element in the parameter $\boldsymbol{p} = (p_1, p_2, p_3, p_4, p_5)$ can be obtained:

$$E(p_i\,|\boldsymbol{x}) = \int_0^1 p_i \pi(\boldsymbol{p}|\boldsymbol{x}) dp_i \tag{22}$$

According to the obtained posterior expectation, it is possible to have a more comprehensive understanding of the probability $p_i$ of the ammunition hitting the target causing different damage levels.

## 5. Model Solving Method Based on MCMC Method

Due to the large number of parameters in Bayesian estimation, it is necessary to solve the high-dimensional integration problem of the posterior distribution to obtain the posterior distribution or the posterior expected value of the parameter, $\boldsymbol{p}$. To overcome this problem, the Markov Chain Monte Carlo (MCMC) method is adopted to solve the posterior distribution through sampling simulation of the parameters [28]. The basic process is as follows:

(1) Determine the initial state of parameter $\boldsymbol{p}$, and select $\boldsymbol{p}_0 = (p_1^0, p_2^0, p_3^0, p_4^0, p_5^0)$ from the original sub-sample as the initial value of parameter estimation.
(2) The Markov chain is constructed for parameter state transfer. When the traversal mean $\frac{1}{k}\sum_{i=1}^{k} p^{(i)}$ reaches the convergent state, the Markov chain is judged to reach a stationary state.
(3) Continue to perform iterative transfer to generate a new point sequence $\left[\boldsymbol{p}^{(k+1)}, \boldsymbol{p}^{(k+2)}, \cdots \boldsymbol{p}^{(k+M)}\right]$.
(4) Remove the $k$ point sequence before the Markov chain reaches the convergent state and select the new point sequence as the sampling value of the parameter $\boldsymbol{p}$. The posterior expected value of parameter $\boldsymbol{p}$ can be obtained according to the mean value of sampling:

$$E(\boldsymbol{p}|\boldsymbol{o}_2) = \frac{1}{M-k}\sum_{t=k+1}^{M} \boldsymbol{p}^{(t)} \tag{23}$$

Due to the high dimension of the parameter, $\boldsymbol{p}$, it is not easy to directly extract samples. In this case, it is necessary to use the Gibbs sampling method to transform the complex problem of high-dimensional samples into the one-dimensional sampling problem in which only one parameter is extracted at a time so that sampling is easy to achieve [29]. The steps based on the Gibbs sampling method are as follows:

To determine the univariate conditional distribution of the posterior distribution of parameters, it is much easier to extract samples from the univariate conditional distribution than directly from the multivariate posterior distribution. The posterior univariate conditional distribution can be obtained according to the posterior distribution formula:

$$\begin{cases} \pi_1(p_1|X, p_2, p_3, p_4, p_5) \\ \pi_2(p_2|X, p_1, p_3, p_4, p_5) \\ \vdots \\ \pi_5(p_5|X, p_1, p_2, p_3, p_4) \end{cases} \tag{24}$$

Iterative sampling starts from the original sub-sample $\boldsymbol{p}_0 = (p_1^0, p_2^0, p_3^0, p_4^0, p_5^0)$. If the value of the parameter, $\boldsymbol{p}$, is $\boldsymbol{p}^{(k-1)} = (p_1^{(k-1)}, p_2^{(k-1)}, p_3^{(k-1)}, p_4^{(k-1)}, p_5^{(k-1)})$ at the beginning of the $k$ iteration, the process of the $k$ iteration is as follows:

Extract $p_1^{(k)}$ from the posterior univariate conditional distribution $\pi_1(p_1 \left| X, p_2^{(k-1)}, p_3^{(k-1)}, p_4^{(k-1)}, p_5^{(k-1)}\right.)$;

Extract $p_2^{(k)}$ from the posterior univariate conditional distribution $\pi_2(p_2 \left| X, p_1^{(k-1)}, p_3^{(k-1)}, p_4^{(k-1)}, p_5^{(k-1)}\right.)$;

...

Extract $p_5^{(k)}$ from the posterior univariate conditional distribution $\pi_5(p_5 \left| X, p_1^{(k-1)}, p_2^{(k-1)}, p_3^{(k-1)}, p_4^{(k-1)}\right.)$;

From this, the new sample $\boldsymbol{p}^{(k)} = (p_1^{(k)}, p_2^{(k)}, p_3^{(k)}, p_4^{(k)}, p_5^{(k)})$ generated by the $k$ iteration can be obtained.

Select the mean of the point series during the period when the Markov chain reaches a stationary state as the parameter point estimate:

$$\begin{aligned} \hat{p}_1 &= \frac{p_1^{(k+1)} + p_1^{(k+2)} + \cdots + p_1^{(M)}}{M-k} \\ \hat{p}_2 &= \frac{p_2^{(k+1)} + p_2^{(k+2)} + \cdots + p_2^{(M)}}{M-k} \\ &\vdots \\ \hat{p}_5 &= \frac{p_5^{(k+1)} + p_5^{(k+2)} + \cdots + p_5^{(M)}}{M-k} \end{aligned} \tag{25}$$

Then, the Bayesian estimates of the probability of ammunition hitting the target with different damage levels can be obtained.

## 6. Example Analysis

In this study, the target damage level is divided into 5 levels, namely zero damage, mild damage, moderate damage, severe damage, and destruction and annihilation. The test of a new type of ammunition hitting armored targets is selected as the research object, and the damage effectiveness of the ammunition is calculated. Due to the limitation of test cost and test environment, the number of strike tests under field conditions is small, which makes the estimation of damage effectiveness inaccurate. In this regard, the Bayesian method was chosen to overcome the problem of insufficient sample size to calculate the damage effectiveness.

After the D–S evidence theory fused the simulation information and expert knowledge information, the probability distribution of the damage effectiveness parameters under the prior information can be obtained, as shown in Table 3.

**Table 3.** Probability distribution of damage effectiveness under prior information.

| | Probability of Zero Damage | Probability of Mild Damage | Probability of Moderate Damage | Probability of Severely Damage | Probability of Destroy the Annihilation |
|---|---|---|---|---|---|
| single-shot ammunition | 0.131 | 0.267 | 0.314 | 0.157 | 0.131 |

In order to further obtain more accurate information on the damage effectiveness of the ammunition, a strike test was carried out under field conditions, and a total of 3 groups of strike tests were carried out. The first group fired 10 rounds, the second group fired 20 rounds, and the third group fired 30 rounds. The target frequencies formed are shown in Table 4.

**Table 4.** Frequency distribution of hit targets with different damage levels.

| Number of Ammo Fired | Zero Damage | Mild Damage | Moderate Damage | Severely Damage | Destroy the Annihilation |
|---|---|---|---|---|---|
| 10 | 1 | 3 | 3 | 2 | 1 |
| 20 | 3 | 5 | 6 | 4 | 2 |
| 30 | 4 | 8 | 8 | 5 | 5 |

Due to length reasons, this article only counts the test results of the 30 rounds of ammunition in the third group, and the calculation process is the same for the number of ammunition 10 and 20. It can be obtained that in 30 strike tests, the probability of zero damage is 0.133, the probability of mild damage is 0.267, the probability of moderate damage is 0.267, the probability of severely damage is 0.167, and the probability of destroy the annihilation is 0.167.

Based on the WinBUGS software environment, this experiment solved the Bayesian estimation of ammunition damage effectiveness [30]. The posterior distribution is inferred by the Markov chain Monte Carlo method, and the damage effectiveness parameter, $\boldsymbol{p}$, is iteratively sampled from the conditional probability distribution by the Gibbs sampling method to generate the Markov chain. Set the number of iterations to 1500 and generate the iterative trace diagram of parameters $p_1$, $p_2$, $p_3$, $p_4$, $p_5$, as shown in Figure 1. Observe the autocorrelation diagram to determine whether the Markov chain has reached convergence, as shown in Figure 2. It is judged that Markov chains converge after observation, indicating that the sampling results can truly describe the posterior distribution of the parameters. Drawing the kernel density diagram of the parameters, as shown in Figure 3, can give a more intuitive understanding of the parameter distribution.

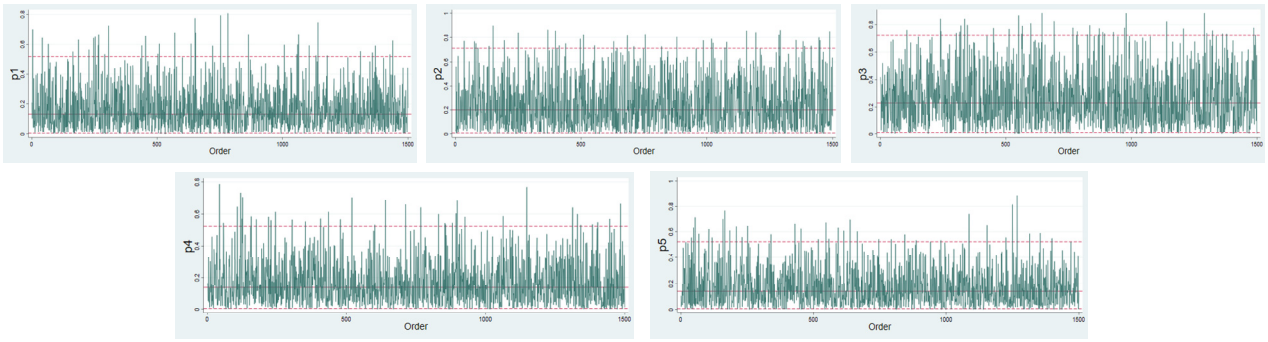



**Figure 1.** Iteration trace of parameters $p_1, p_2, p_3, p_4, p_5$.

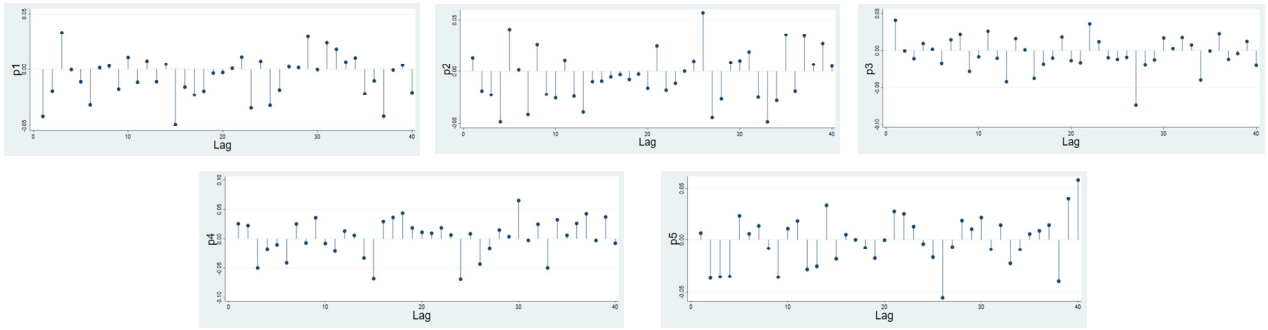



**Figure 2.** Autocorrelation diagram of parameters $p_1, p_2, p_3, p_4, p_5$.

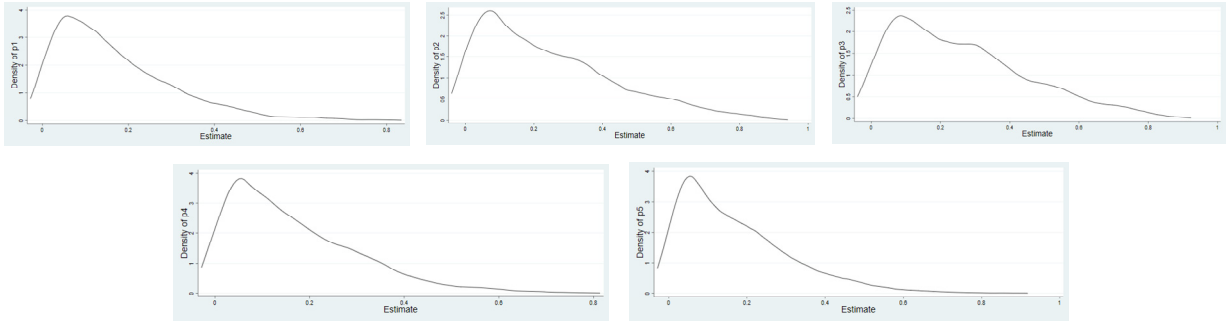



**Figure 3.** Kernel density of parameters $p_1, p_2, p_3, p_4, p_5$.

At the same time, the binomial distribution Bayesian model of damage efficiency was constructed with severe damage as standard. Selecting the conjugate prior distribution Beta distribution of the binomial distribution for calculation, the iterative trace diagram, autocorrelation diagram, and kernel density diagram of the damage probability can be obtained, as shown in Figure 4.

Statistical analysis of the sampling results of each Markov chain is shown in Table 5.

**Table 5.** Posterior statistics of parameters $p_1, p_2, p_3, p_4, p_5, p$.

| Parameter | *n* | Mean | sd | MC Error | Val2.5pc | Median | Val97.5pc |
|---|---|---|---|---|---|---|---|
| $p_1$ | 1500 | 0.132 | 0.134 | 0.001949 | 0.003379 | 0.134 | 0.4252 |
| $p_2$ | 1500 | 0.267 | 0.192 | 0.004337 | 0.008285 | 0.263 | 0.6903 |
| $p_3$ | 1500 | 0.280 | 0.193 | 0.003794 | 0.007537 | 0.284 | 0.6985 |
| $p_4$ | 1500 | 0.163 | 0.137 | 0.0023 | 0.003861 | 0.166 | 0.4328 |
| $p_5$ | 1500 | 0.158 | 0.141 | 0.002043 | 0.003657 | 0.170 | 0.4626 |
| $p$ | 1500 | 0.321 | 0.241 | 0.006453 | 0.012861 | 0.324 | 0.8854 |

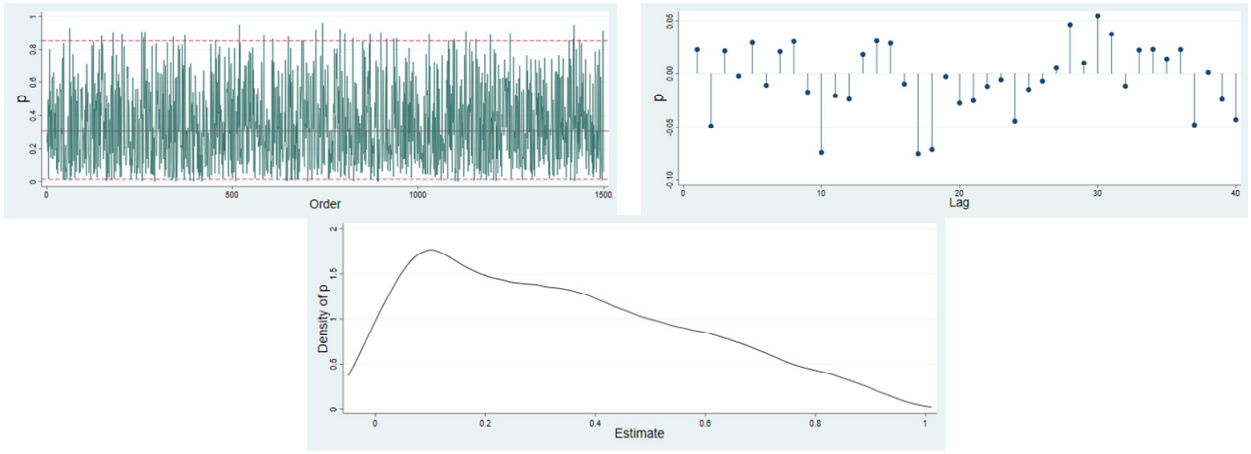



**Figure 4.** Damage probability, $p$, of binomial distribution method.

According to the results of the Bayesian posterior distribution in Table 5, the mean value of the zero damage probability of the new ammunition to the armored target under a single-shot strike is 0.132 and the standard deviation is 0.134; the mean value of the mild damage probability is 0.267 and the standard deviation is 0.192; the mean probability of moderate damage is 0.280 and the standard deviation is 0.193; the mean probability of severely damage is 0.163, while the standard deviation is 0.137; the mean probability of destroy the annihilation is 0.158 and the standard deviation is 0.141. The results of the Bayesian method regard the damage probability parameters as random variables obeying a certain distribution rather than fixed values, which has better applicability. Using the binomial distribution Bayesian estimation method with the damage standard of seriously damage, the mean value of damage probability of the ammunition is 0.321 and the standard deviation is 0.241. The results of the binomial distribution method cannot obtain the probability of occurrence of each damage level, which is not conducive to the scientific use and planning of ammunition. Moreover, compared with only relying on the test results of strike tests under field conditions, the Bayesian method can incorporate prior information, effectively increasing the amount of information about the damage effectiveness of ammunition.

In addition, compared with the Bayesian inference results of 10, 20, and 30 ammunition fired under field conditions, it can be found that with the increase in the number of sample tests under field conditions, the Bayesian results are more inclined to the sample information. The results show that when the number of field strike samples increases, it is more dominant in the calculation of damage effectiveness, and the results are more in line with the actual situation.

## 7. Conclusions

Compared with the binomial distribution method based on damage or not, this article provides a calculation method of ammunition attack target damage effectiveness based on Bayesian multinomial distribution to further divide the results of damage level and describe the ammunition damage effectiveness more accurately. It also provides a reference for the calculation of ammunition damage effectiveness under the condition of small samples by integrating various prior information with Bayesian inference method. The Bayesian inference process is as follows:

(1) The probability of different damage levels caused by a single ammunition hitting the target is used as the basis for damage effectiveness calculations. Multiple distribution assumptions are made for the ammunition damage effectiveness test, which provides a basis for the Bayesian method.

(2) In order to simplify the solution process of Bayesian inference, Dirichlet distribution of conjugate distribution of multinomial distribution is selected as the prior distribution of this study.
(3) In order to obtain accurate prior distribution of parameters, D–S evidence theory is introduced to fuse prior information. At the same time, the prior information is checked for consistency to prove the rationality of the prior information.
(4) The posterior distribution of ammunition damage effectiveness is deduced by using Bayesian formula, and the Markov chain Monte Carlo method is used to overcome the high-dimensional integration problem of the posterior distribution, and the parameters are sampled by the Gibbs sampling method. The Bayesian estimation of the damage effectiveness parameter was obtained according to the sampling results.

Through the above inference process, we can calculate the probability of different damage levels caused by ammunition hitting targets and comprehensively understand the damage effectiveness.

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