

## Article

# Analytical Solutions of Nonlocal Thermoelastic Interaction on Semi-Infinite Mediums Induced by Ramp-Type Heating

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**Abstract:** A novel nonlocal model with one thermal relaxation time is presented to investigate the propagation of waves in a thermoelastic semi-infinite medium. We used Eringen's theory of the nonlocal continuum to develop these models. Analytical solutions in all physical quantities are provided by using Laplace transforms and eigenvalue techniques. All physical quantities are presented as symmetric and asymmetric tensors. The temperature, the displacement, and the stress variations in semi-infinite materials have been calculated. The effects of nonlocal parameters, ramp type heating, and the thermal relaxation times on the wave propagation distribution of physical fields for mediums are graphically displayed and analyzed.

**Keywords:** eigenvalue approach; Laplace transforms; nonlocal thermoelastic model; thermal relaxation time



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## 1. Introduction

The nonlocal elastic theory was first advocated by Eringen [1]. After a period of 2 years, the theory of nonlocal thermoelasticity was explored by Eringen [2]. In continuum mechanics, he addressed constitutive relations, governing equations, laws of equilibrium, and displacement equations/temperature under a nonlocal elastic model. According to the nonlocal elastic model, strain depends on the applied stress of continuous bodies at a place  $x$  that is impacted not only by the strain point but also by the strains of the bodies in every other region near this point  $x$ . In the uniqueness reference, Altan [3] studied the nonlocal linear elastic theory in depth. Wang and Dhaliwal [4] explained the uniqueness of the theory of nonlocal thermoelasticity. Eringen [5] studied nonlocal electromagnetic solids and superconductivity under the theory of elasticity. Povstenko [6] recommended the nonlocal elastic model to take into account the forces of actions between atoms. Nonlocal theories of field elasticity have been explained in detail by Eringen [7] concerning continuum mechanics. Narendar and Gopalakrishnan [8] studied the effects of nonlocal scale on the ultrasonic wave properties of nanorods. Yu, Tian, and Liu [9] studied Eringen's nonlocal theory of thermoelasticity with a size-dependent model. Zenkour and Abouelregal [10] studied the vibrations of thermal conductivity under the nonlocal thermoelastic theory due to harmonically variable heat sources.

When motivated by the rule of Fourier heat conduction, Biot [11] established the coupled thermoelasticity theory (CD theory), which became acceptable for modern engineering applications, particularly in high-temperature cases. On the other hand, the thermoelastic models are physically undesirable at low temperatures and cannot achieve equilibrium. Lord and Shulman [12] (LS) added one relaxation period to the heat conduction equation to resolve the conflict.

Eringen [13] researched micromorphic materials' mechanics. The theory of micromorphic materials with memory was established by Eringen [14]. In micromorphic thermoelasticity, He et al. [15] established expanded variational ideas. Abbas [16] looked at analytical solutions for thermo-elastic hollow spherical free vibrations. Variable thermal conductivity and the magnetic field under the LS model on an indefinitely long annular cylinder were examined by Abo-Dahab and Abbas [17]. The magneto-thermo-elastic reactions of FG cylinders were investigated using a finite element approach by Zenkour and Abbas [18]. Liu et al. [19] investigated the thermal oscillation that occurs when a porous hierarchy is subjected to a heat shock and its application. He et al. [20] investigated the insights into partial slips and temperature jumps of a nanofluid flow over a stretched or shrinking surface. Hobiny and Abbas [21] used an eigenvalue technique to offer analytical solutions for photothermal and elastic waves in a non-homogeneous semiconductor material. Sharma et al. [22] studied the effects of DPL theory on the free vibration of an isotropic homogenous nonlocal thermo-elastic hollow sphere containing voids. Zhang and He [23] discussed the generalized thermo-elastic problem with memory-dependent derivative and nonlocal effects under moving heat sources. Yu et al. [24] studied the nonlocal thermoelastic model based on nonlocal elasticity and nonlocal thermal conduction. The nonlocal model of thermo-elastic materials with voids and fractional derivatives heat transmission was investigated by Bachher and Sarkar [25]. Abouelregal [26] investigated the temperature-dependent physical properties of rotating nonlocal nanobeams subjected to a variable heat source and a dynamic load. Recently, the authors of [27–44] investigated several problems for thermo-elastic waves employing the eigenvalue approach and numerical method.

This study aims to see how the LS model affects wave propagation in a nonlocal thermoelastic medium. The Laplace transform and eigenvalue technique was used to solve the field functions. The eigenvalue method was used to depict the fluctuations in displacement, temperature, and stress.

## 2. Mathematical Model

Following Eringen [45,46] and Lord and Shulman [12] in the absence of a heat source and body force, the basic equations for a nonlocal thermoelastic medium are as follows:

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ji} - \gamma_t T_{,i} = \rho \left(1 - n_e \epsilon^2 \nabla^2\right) \frac{\partial^2 u_i}{\partial t^2} \quad (1)$$

$$K \nabla^2 T = \left(1 - n_t \epsilon^2 \nabla^2\right) \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \left(\rho c_e \frac{\partial T}{\partial t} + \gamma_t T_0 \frac{\partial u_{i,i}}{\partial t}\right) \quad (2)$$

$$\left(1 - n_e \epsilon^2 \nabla^2\right) \sigma_{ij} = \mu (u_{i,j} + u_{j,i}) + (\lambda u_{k,k} - \gamma_t T) \delta_{ij} \quad (3)$$

This model can be summarized in the following points:

- (i) NTE refers to the nonlocal thermoelastic model

$$n_t = n_e = 1, \epsilon \neq 0$$

- (ii) NT refers to the nonlocal thermal model

$$n_t = 1, n_e = 0, \epsilon \neq 0$$

- (iii) NE refers to the nonlocal elastic model

$$n_e = 1, n_t = 0, \epsilon \neq 0$$

- (iv) LTE refers to the local thermoelastic model

$$n_t = n_e = 1, \epsilon = 0$$

The medium state only depends on  $x$  and  $t$  when considering a semi-infinite half-space  $x \geq 0$ . As a result, the formulas can be written as follows:

$$(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma_t \frac{\partial T}{\partial x} = \rho \left( 1 - n_e \epsilon^2 \frac{\partial^2}{\partial x^2} \right) \frac{\partial^2 u}{\partial t^2} \quad (4)$$

$$K \frac{\partial^2 T}{\partial x^2} = \left( 1 - n_t \epsilon^2 \frac{\partial^2}{\partial x^2} \right) \left( 1 + \tau_o \frac{\partial}{\partial t} \right) \left( \rho c_e \frac{\partial T}{\partial t} + \gamma_t T_o \frac{\partial^2 u}{\partial t \partial x} \right) \quad (5)$$

$$\left( 1 - n_e \epsilon^2 \frac{\partial^2}{\partial x^2} \right) \sigma_{xx} = \sigma = (\lambda + 2\mu) \frac{\partial u}{\partial x} - \gamma_t T \quad (6)$$

### 3. Application

The initial and corresponding boundary conditions are presumptively assumed as follows:

$$u(x, 0) = 0, T(x, 0) = 0, \frac{\partial u(x, 0)}{\partial t} = 0, \frac{\partial T(x, 0)}{\partial t} = 0 \quad (7)$$

$$u(0, t) = 0 \quad (8)$$

$$T(0, t) = T_1 \begin{cases} 0, & t \leq 0, \\ \frac{t}{t_o}, & 0 < t \leq t_o \\ 1, & t > t_o, \end{cases} \quad (9)$$

The non-dimensional variables are used to obtain main fields in a dimensionless form as follows:

$$(t', t'_o, \tau'_o) = \eta c^2 (t, t_o, \tau_o), (x', u', \epsilon') = \eta c (x, u, \epsilon), T' = \frac{\gamma_t T}{\rho c^2}, \sigma' = \frac{\sigma}{\rho c^2} \quad (10)$$

where  $c^2 = \frac{\lambda + 2\mu}{\rho}$ ,  $\eta = \frac{\rho c_e}{K}$ .

Using Equation (10) for the basic equations (without the superscript '), the following results are obtained:

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial T}{\partial x} = \left( 1 - n_e \epsilon^2 \frac{\partial^2}{\partial x^2} \right) \frac{\partial^2 u}{\partial t^2} \quad (11)$$

$$\frac{\partial^2 T}{\partial x^2} = \left( 1 - n_t \epsilon^2 \frac{\partial^2}{\partial x^2} \right) \left( 1 + \tau_o \frac{\partial}{\partial t} \right) \left( \frac{\partial T}{\partial t} + \zeta \frac{\partial^2 u}{\partial t \partial x} \right) \quad (12)$$

$$\sigma = \frac{\partial u}{\partial x} - T \quad (13)$$

$$u = 0, T = T_1 \begin{cases} 0, & t \leq 0, \\ \frac{t}{t_o}, & 0 < t \leq t_o \\ 1, & t > t_o, \end{cases} \text{ on } x = 0 \quad (14)$$

where  $\zeta = \frac{\gamma_t^2 T_o}{\rho c_e (\lambda + 2\mu)}$ .

For the  $G(x, t)$  function, the Laplace transforms are given as [47]

$$\bar{G}(x, s) = L[G(x, t)] = \int_0^\infty G(x, t) e^{-st} dt \quad (15)$$

Therefore, the basic equations can be given by

$$\frac{d^2 \bar{u}}{dx^2} - \frac{d\bar{T}}{dx} = \left( 1 - n_e \epsilon^2 \frac{d^2}{dx^2} \right) s^2 \bar{u} \quad (16)$$

$$\frac{d^2 \bar{T}}{dx^2} = \left( 1 - n_t \epsilon^2 \frac{d^2}{dx^2} \right) s(1 + s\tau_o) \left( \bar{T} + \zeta \frac{d\bar{u}}{dx} \right) \quad (17)$$

$$\bar{\sigma} = \frac{d\bar{u}}{dx} - \bar{T} \quad (18)$$

$$\bar{u} = 0, \bar{T} = T_1 \frac{1 - e^{-t_0 s}}{t_0 s^2} \quad \text{on } x = 0 \quad (19)$$

Using the eigenvalue techniques proposed in [48–55], we can obtain solutions to the coupled differential Equations (16) and (17). By substituting from Equation (16) into Equation (17), (16) and (17) can be given by

$$\frac{d^2 \bar{u}}{dx^2} = c_{31} \bar{u} + c_{34} \frac{d\bar{T}}{dx} \quad (20)$$

$$\frac{d^2 \bar{T}}{dx^2} = c_{41} \bar{u} + c_{42} \bar{T} + c_{43} \frac{d\bar{u}}{dx} \quad (21)$$

$$\text{where } c_{31} = \frac{s^2}{(1+n_t \epsilon^2 s^2)}, c_{34} = \frac{1}{(1+n_t \epsilon^2 s^2)}, c_{41} = \frac{-n_t \epsilon^2 (s+s^2 \tau_0) v c_{31}}{(1+n_t \epsilon^2 (s+s^2 \tau_0) + n_t \epsilon^2 (s+s^2 \tau_0) \zeta c_{34})},$$

$$c_{42} = \frac{(s+s^2 \tau_0)}{(1+n_t \epsilon^2 (s+s^2 \tau_0) + n_t \epsilon^2 (s+s^2 \tau_0) \zeta c_{34})}, \text{ and } c_{43} = \frac{(s+s^2 \tau_0) \epsilon}{(1+n_t \epsilon^2 (s+s^2 \tau_0) + n_t \epsilon^2 (s+s^2 \tau_0) \zeta c_{34})}.$$

Equations (20) and (21)'s matrix-vectors can thus be represented as

$$\frac{dV}{dx} = CV \quad (22)$$

$$\text{where } C = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ c_{31} & 0 & 0 & c_{34} \\ c_{41} & c_{42} & c_{43} & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} \bar{u} & \bar{T} & \frac{d\bar{u}}{dx} & \frac{d\bar{T}}{dx} \end{pmatrix}^T.$$

Thus, the characteristic relation of the matrix  $C$  is taken as

$$\omega^4 - \omega^2(c_{31} + c_{42} + c_{34}c_{43}) + \omega c_{34}c_{41} + c_{31}c_{42} = 0 \quad (23)$$

The eigenvalues of the matrix described by  $\omega_1, \omega_2, \omega_3$ , and  $\omega_4$  are the solutions of Equation (23). The eigenvectors  $X$  corresponding to eigenvalues  $\omega$  are then calculated as follows:

$$X_1 = \omega c_{34}, X_2 = \omega^2 - c_{31}, X_3 = \omega^2 c_{34}, X_4 = \omega(\omega^2 - c_{31}) \quad (24)$$

The eigenvectors  $X$  corresponding to the eigenvalue  $\omega_i, i = 1 \dots 4$  can be easily calculated using Equation (24). As a result, the analytical solutions to Equation (22) are as follows:

$$V(x, s) = \sum_{i=1}^4 B_i X_i e^{\omega_i x} \quad (25)$$

where  $B_1, B_2, B_3$ , and  $B_4$  are constants that are computed using the problem's boundary conditions. The general solutions of all variables concerning  $x$  and  $s$  can be provided in the following forms using (25) and (22):

$$\bar{T}(x, s) = \sum_{i=1}^4 B_i T_i e^{\omega_i x} \quad (26)$$

$$\bar{u}(x, s) = \sum_{i=1}^4 B_i U_i e^{\omega_i x} \quad (27)$$

$$\bar{\sigma}(x, s) = \sum_{i=1}^4 B_i (\omega_i U_i - T_i) e^{\omega_i x} \quad (28)$$

Finally, to obtain numerical inversions of physical quantities, the Stehfest [56] numerical inversions approach has been used as in [57].

#### 4. Results and Discussion

This section illustrates the numerical results of the analytical expressions of previously provided physical quantities. To discuss the numerical calculations, the authors consider

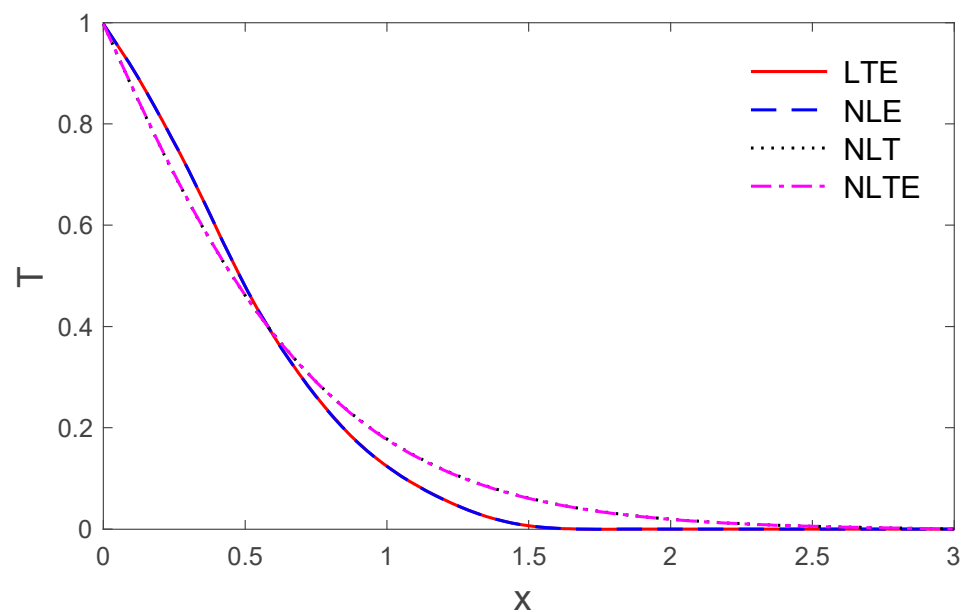
the material characteristics of the copper material, whose physical data are provided below [58].

$$\alpha_t = 1.78 \times 10^{-5} (k^{-1}), \lambda = 7.76 \times 10^{10} (N) (m^{-2}), \rho = 8954 (kg) (m^{-3})$$

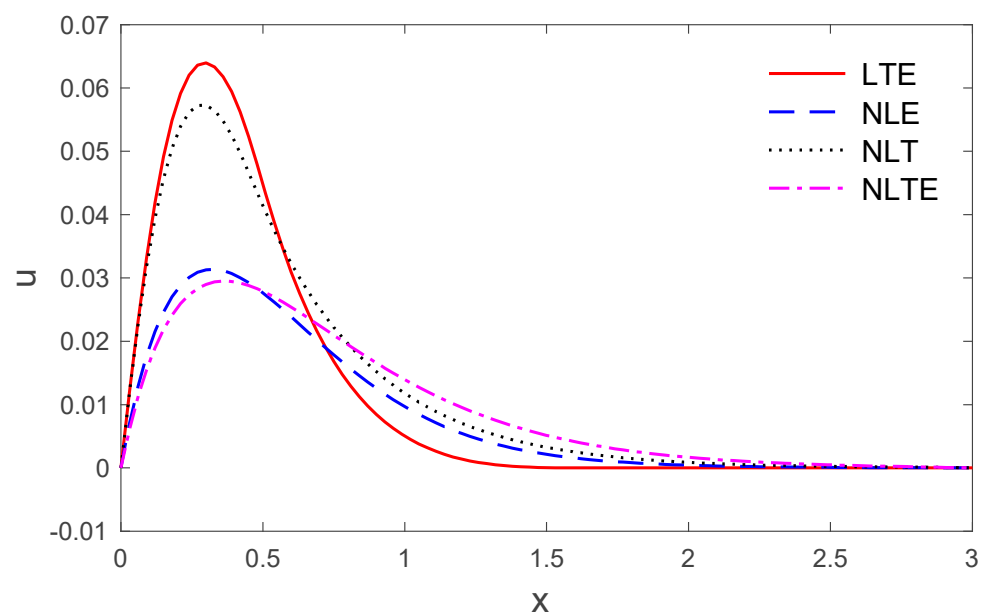
$$\mu = 3.86 \times 10^{10} (N) (m^{-2}), c_e = 383.1 (m^2) (k^{-1}), T_o = 293 (k)$$

$$K = 386 (N) (k^{-1}) (s^{-1}), \tau_o = 0.1, t_o = 0.4, t = 0.5, \epsilon = 0.3$$

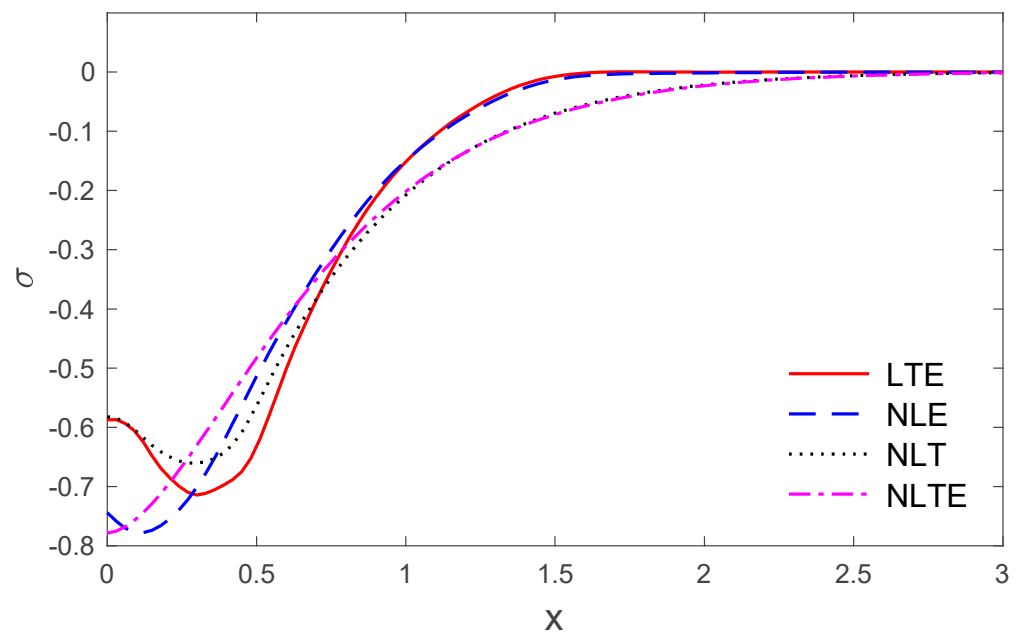
The temperatures, the displacement, and the variations of stress along the distance  $x$  are numerically calculated using the nonlocal thermo-elastic (NLTE) model. Figures 1–9 show the computations of physical values (numerically) via the distance  $x$ , the effects of nonlocal parameters, the heating ramp type, and the relaxation time based on the physical data set.



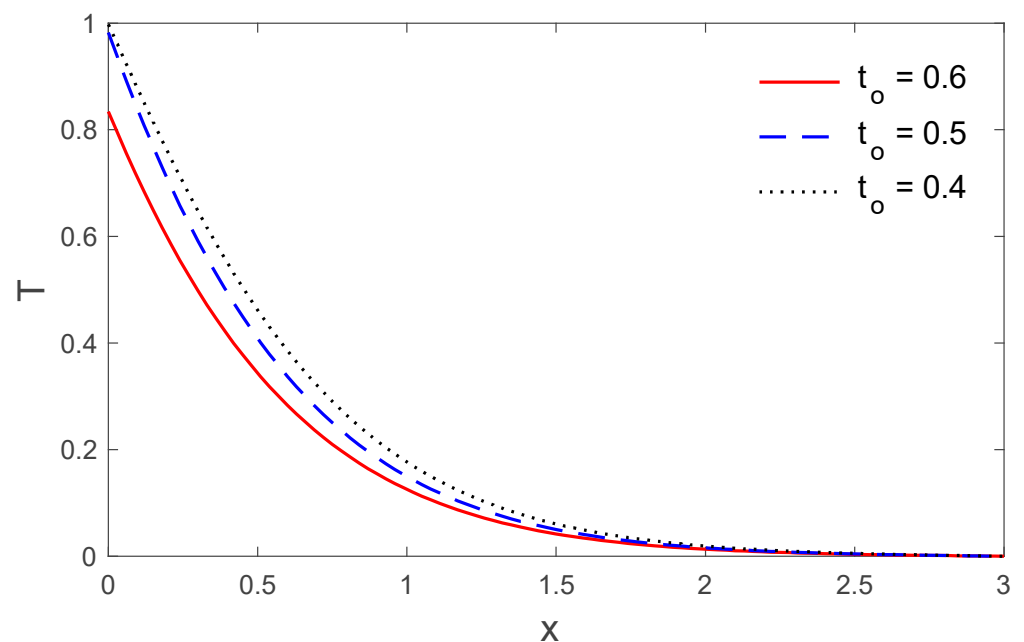
**Figure 1.** The temperature variation via  $x$  for four models.



**Figure 2.** The displacement variation via  $x$  for four models.



**Figure 3.** The stress variations via  $x$  for four models.



**Figure 4.** The temperature variations via  $x$  for different values of  $t_o$ .

Among these, Figures 1, 4 and 7 display the variations of temperature with respect to the distances. It is clear that the non-dimensional temperature starts from one, satisfies the boundary conditions of the problem, then decreases with the increase of distance to reach zero values. Figures 2, 5 and 7 display the displacement variations along the distance. It is noticed that the displacement starts from zero to obey the problem boundary conditions, then increases to attain maximum values, then decreases again to reach zero values. Figures 3, 6 and 9 show the variation of stress along the distances. It is clear that the magnitudes of stress start from maximum values then decrease with the increasing of  $x$  until finished. From the figures, the comparisons between the local generalized thermoelastic theory (LTE), the generalized thermoelastic theory with nonlocal elasticity only (NLE), the generalized thermoelastic theory with nonlocal thermal conduction only (NLT), and the generalized thermoelastic model with nonlocal thermal conduction and elasticity

(NLTE) are presented as in Figures 1–3. Figures 4–6 show the effects of the ramp type heating parameter when  $(t_o > t, t_o = 0.6)$ ,  $(t_o = t, t_o = 0.5)$ , and  $(t_o < t, t_o = 0.4)$  under the NLTE theory. In Figures 7–9, the generalized thermoelastic theory with nonlocal thermal conduction and elasticity (NLTE) is used to show the effects of thermal relaxation time in all studying fields. Finally, from the comparisons among the results, one can conclude that the nonlocal thermo-elasticity theory (nonlocal thermal conduction and elasticity) is a great phenomenon and has a considerable effect on the distribution of the physical quantities.

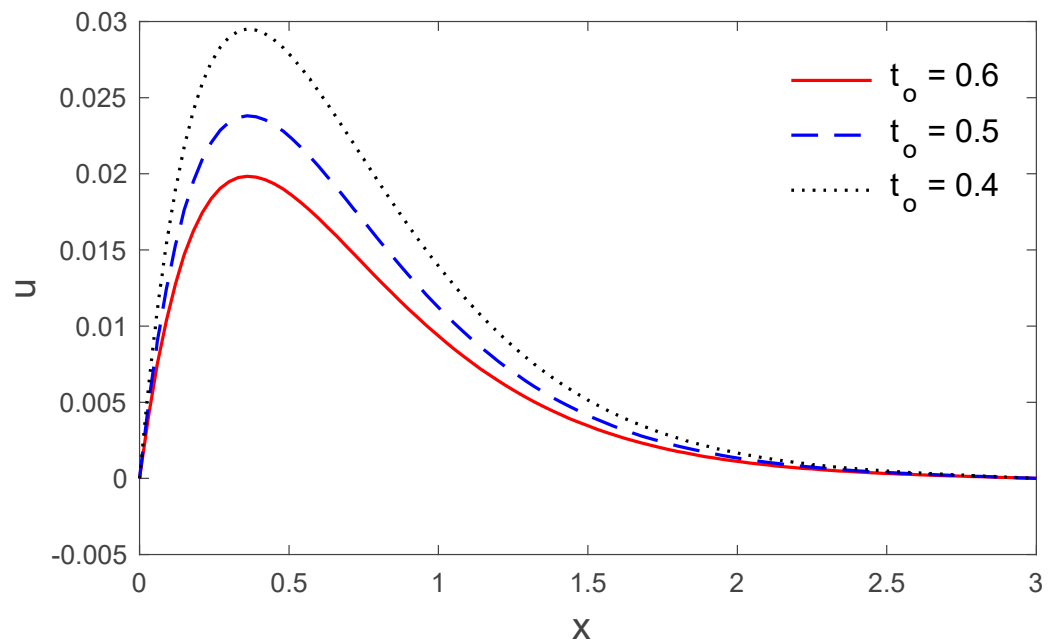


Figure 5. The displacement variation via  $x$  for different values of  $t_o$ .

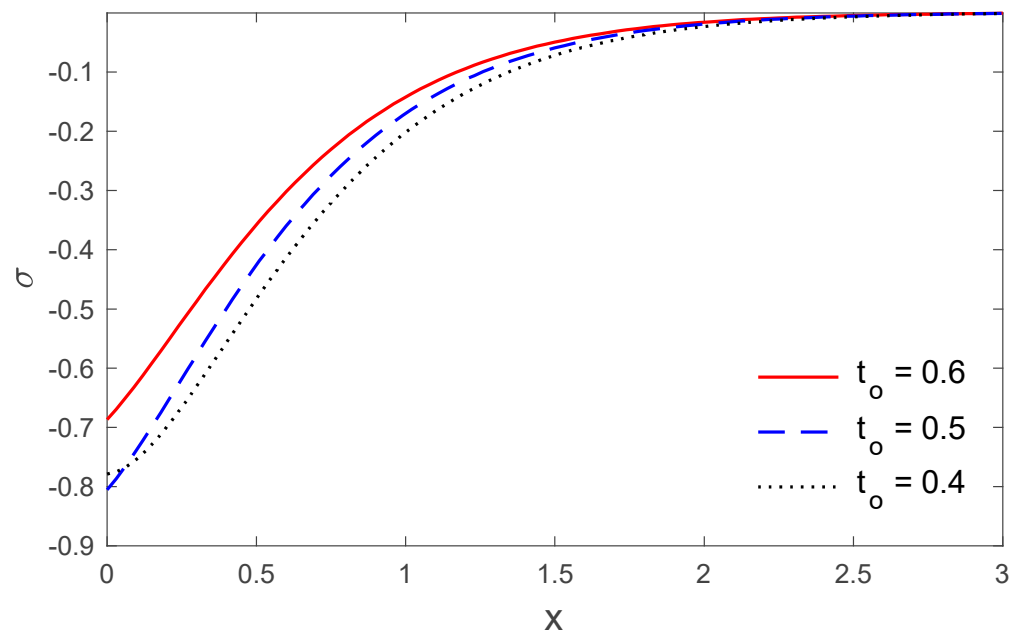
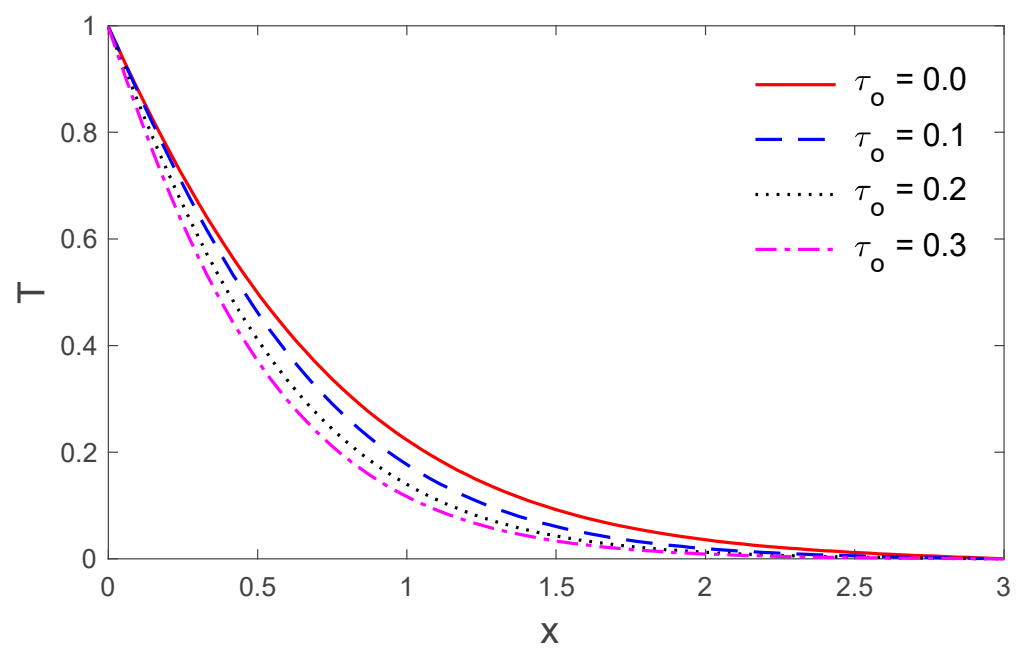
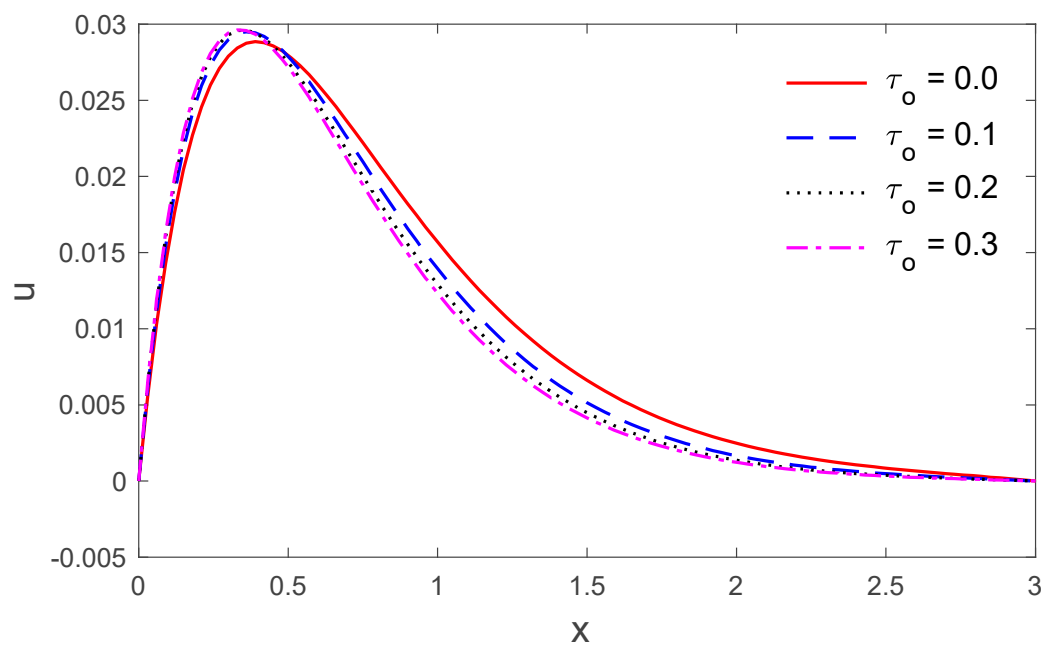


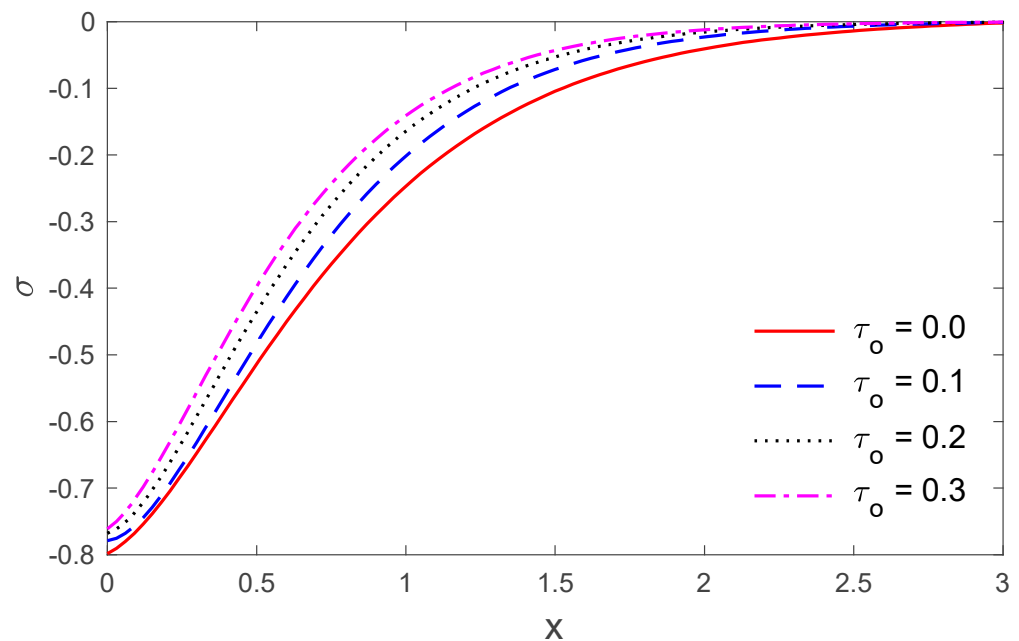
Figure 6. The stress variations via  $x$  for different values of  $t_o$ .



**Figure 7.** The variation of temperature via  $x$  with and without relaxation time.



**Figure 8.** The displacement variation with respect to  $x$  with and without relaxation time.



**Figure 9.** The stress variations along  $x$  with and without relaxation time.

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## Nomenclature

|                                                  |                                             |
|--------------------------------------------------|---------------------------------------------|
| $t$                                              | the time                                    |
| $u_i$                                            | the displacements                           |
| $T = T^* - T_0, T^*$                             | the temperature variations                  |
| $T_0$                                            | the reference temperature                   |
| $t_0$                                            | the ramp type heating parameter             |
| $\tau_0$                                         | the thermal relaxation time                 |
| $c_e$                                            | the specific heat at constant strain        |
| $\rho$                                           | the density of material                     |
| $\gamma_t = (3\lambda + 2\mu)\alpha_t, \alpha_t$ | the coefficient of linear thermal expansion |
| $\sigma_{ij}$                                    | the stresses components                     |
| $K$                                              | the heat conductivity                       |
| $\lambda, \mu$                                   | the Lamé's constants                        |

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