

Topological Portals from Matter to Antimatter

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Abstract: We discuss possibilities of generating a Majorana mass for the neutron from topological quantum gravity effects which survive at mesoscopic scales from decoherence. We show how virtual micro-black hole (BH) pairs with skyrme/baryon hairs induce a neutron–antineutron transition which can be tested in next generation of experiments. Such effects do not destabilize the proton. We also discuss how BHs with mix ordinary and mirror baryon hairs can mediate neutron-mirror neutron mixings.

Keywords: neutron-antineutron physics; mirror matter; dark matter; quantum gravity, instantons; black hole hairs

1. Introduction and Main Ideas

Kuzmin’s idea of neutron–antineutron physics [1] is more actual and exciting than ever in light of next experimental possibilities [2]. Effective field theory would suggest to relate a neutron–antineutron transition, generated by a neutron Majorana mass $\epsilon nn + h.c.$, to $D = 9$ six quarks $|\Delta B| = 2$ violating effective operators $(udd)^2/\Lambda^5$ or $(qqd)^2/\Lambda^5$. Here, Λ is the new physics scale and, as it is well known, it is related to the effective Majorana mass as $\epsilon \simeq \Lambda_{QCD}^6/\Lambda^5$. The current best limits on $n - \bar{n}$ set an exclusion scale of $\Lambda > 300$ TeV [3] and there are future possibilities to push it towards 1000 TeV or so [2,4]. Thus, for generating a neutron Majorana mass testable in next future, we would need for new heavy fields having a combination of their masses on a multi-hundred TeV scale. For example, there are models with heavy vector-like colored scalar triplets and a sterile neutron see-saw partner [5–9] or GUT inspired scenarios involving colored scalar sextets [10–13]. Attempts to connect UV completion of the baryon violating six-quark operators with post-sphaleron baryogenesis were also proposed [11–13]. An exciting possibility is to consider a $n - \bar{n}$ oscillation from neutron-mirror neutron ($n - n'$) transitions ($|\Delta B| = 1$) [5,6,14–18]. In addition, in this case, $n - n'$ would be generated by effective $D = 9$ operators $(udd)(u'd'd')/\Lambda'^5$ or $(qqd)(q'q'd')/\Lambda'^5$ with a new physics scale considerably lower than the $n - \bar{n}$ scale [5,6,8,14–18].

Here, we will move on an alternatively radical path: we will assume that no any new heavy particle states are necessary for a neutron–antineutron transition. We propose that $n - \bar{n}$ can be obtained in Standard Model (SM) from non-perturbative quantum gravity effects surviving at *mesoscopic length scales* much larger than the Planck length L_{Pl} . It is a commonly accepted argument that quantum gravity effects mediate transitions violating any global symmetry such as Baryon/Lepton number conservations [19–22]. Nevertheless, these are expected to disappear at energies much below the Planck mass M_{Pl} . Apparently, in $L_{Pl} \rightarrow 0$ limit, there is no any way out from the fact that (minimal) SM cannot generate any $B - L$ violations, at both perturbative and non-perturbative level [23]. Indeed, while the perturbative lagrangian of the SM explicitly preserves B and L as accidental number conservations, electroweak non-perturbative effects such as sphalerons can only violate $B + L$, but not $B - L$, around the electroweak phase transition in the early Universe [24]. Nevertheless, sphalerons are exponentially suppressed in late Universe and practically untestable in low energy physics [24]. From strong QCD effects we would also not expect



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any surprising baryon violations, if the Vafa-Witten theorem [23] was not violated by any subtle dynamical. (An interesting conjecture proposed by Berezhiani is that an effective supersymmetrization of d.o.f in the Fermi world may contradict the VW-theorem [8]).

On the other hand, quantum gravity effects, although not well known around the Planck scale, are more theoretically controllable in low-energy regimes. Nevertheless, subtle and unexpected residual memory effects may survive at larger scales from quantum decoherence attacks if there was any *quantum amplification mechanism*.

Indeed, having defined \bar{L} as the length scale in renormalization group approach, there are several quantum gravity processes that do not scale as powers of $(L_{Pl}/\bar{L})^2$ and they can be interpreted as coherent semi-classical states. This is the case of gravitational instantons, that are classical saddle solutions of the euclidean path integral [25–28]. As it is known, instantons are generically interpreted as semi-classical and non-perturbative quantum tunneling processes in real space-time. Indeed, there is a correspondence among instantons and solitons (see for Example Ref. [29]). For example, in non-abelian Yang–Mills theories coupled to a scalar Higgs field, instantons can be interpreted as tunnelings of monopoles or antimonopoles out from domain walls or a Josephson junction [30]. In case of gravity, gravitational instanton dominating the euclidean path integral corresponds to a tunneling process of a Black Hole (BH) into a White Hole (WH), i.e., a virtual Wormhole [25–28]. As a quantum fluctuation of space-time, a gravitational instanton describes the appearance of a virtual BH pair. As every instantons, also gravitational instantons correspond to transition probabilities with a WKB exponential suppression as e^{-I_E} , where I_E is the Euclidean action of the instantons. Nevertheless, there is an important difference in gravitational case: the euclidean action is proportional to the BH entropy in correspondence to the instanton solution. Indeed, this is related to the fact that the euclidean quantum gravity path integral, in semiclassical regime, corresponds to the thermodynamical partition function of BH [25–28,31]. Thus, all thermodynamical quantities of the BH can be defined, including Bekeinstein–Hawking temperature, entropy, free-energy and so on. Now, as it is notoriously known, BH entropy scales as the BH Area rather than Volume and thus also the euclidean action of gravitational instantons is holographic:

$$S \sim I_E \sim (R_{BH}/L_{Pl})^2 + O(\log R_{BH}/L_{Pl}) \sim N + O(\log N), \quad (1)$$

where I_E is the Euclidean gravitational action, S is the BH entropy, R_{BH} is the BH radius, N is the number of *quantum hairs* or *planckian qubit* stored on the BH surface [32–35] while from the instanton side it may correspond to the topological winding number [36] (see also Ref. [37] for a fully controllable computation of horizon punctures from instantons in lower dimensional gravity). (As we recently discussed, the correspondence between the information storage and topological winding number may be related to a BH topological quantum memory protection from decoherence and noise; similarly to topological quantum computers [36] (see also Refs. [38,39] for its deep connection with *Holographic Naturalness* paradigm).) This also means that for a large BH, the tunneling probability to a WH is expected to be exponentially suppressed by its large entropy content, i.e., e^{-N} . Interestingly, gravitational instantons can generate exponentially suppressed new correlators for SM particles as an effect of fields' propagation on a non-trivial background [25–28,31].

Let us remark that GR admits BH solutions with topological baryon hairs as BH skyrme solutions [40–44]. Thus, there are also *B-hairy instantonic configurations* as well because of gravitational instanton/BH correspondence. (We never found such a remark on BH skyrme and gravitational instantons in any references in literature but it appears to us a direct consequence of the gravitational instanton/BH correspondence.) In BH Baryon/Skyrme solution, Baryon hair is preserved as a topological charge, associated to a topological conservation current, from quantum gravity effects [45]. This would suggest the emergence of topological selection rules which limit the possibility of B-violations from skyrme BHs: for example if the skyrme BH had $B = -1$ charge, a transition to a WH ($B = +1$) would violate the B-number of $|\Delta B| = 2$ and its topological current.

Nevertheless, following this logic, concerning neutrons, one may consider a transition process as $n + \widetilde{BH} \rightarrow \bar{n} + \widetilde{WH}$, where $\widetilde{BH}, \widetilde{WH}$ denote $B = \mp 1$ hairy BH(WH) skyrme/antiskyrmion (Figure 1). Such a process is completely Baryon-number preserving. We can think the neutron- \widetilde{BH} as a $B = 0$ bound state which can transform to an antineutron- \widetilde{WH} system, without any violations of the topological B -number conservation. Let us also remark that such instantons do not destabilize protons and neutrons as $p \rightarrow l^+ \pi^0, n \rightarrow l^+ \pi^-$ and in general into any lepton/meson decay channel. For example $n \widetilde{BH} \rightarrow \widetilde{WH} l^+ \pi^-$ does not preserve the baryon number, violating the B -topological protection while the $n \widetilde{BH} \rightarrow \widetilde{BH} l^+ \pi^-$ has the same transition probability of the SM-transition for energies well below the Planck scale. Indeed, no any skyrme BH can have a lepton number since originated from quarks, while lepto-quarks mediating proton decay would be problematic *di per se*, as well as beyond our no-new-particle approach.

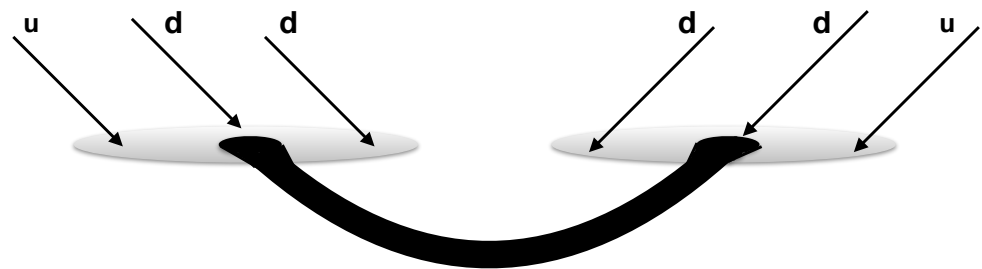


Figure 1. Gravitational skyrme instanton (black bridge) mediating a neutron–antineutron transition. (Anti)quarks composing the (anti)neutron enter in the (anti)skyrmion region (gray) collectively coupled with the (WH)BH (anti)skyrmion configuration.

Indeed, skyrme BHs can have topological B -hairs that are not localized in the BH radius but on a larger skyrmion radius. In other words, it is possible that micro-BH, even Planckian size ones, can have Baryon/skyrmion hairs spread on a much larger radius as $R_{\text{skyrmion}} \gg R_{\text{BH}}$. In this case, the skyrme BH is coupled to the neutron rather than the single quarks if $R_{\text{skyrmion}} \geq \Lambda_{\text{QCD}}^{-1}$ (see Figure 2).

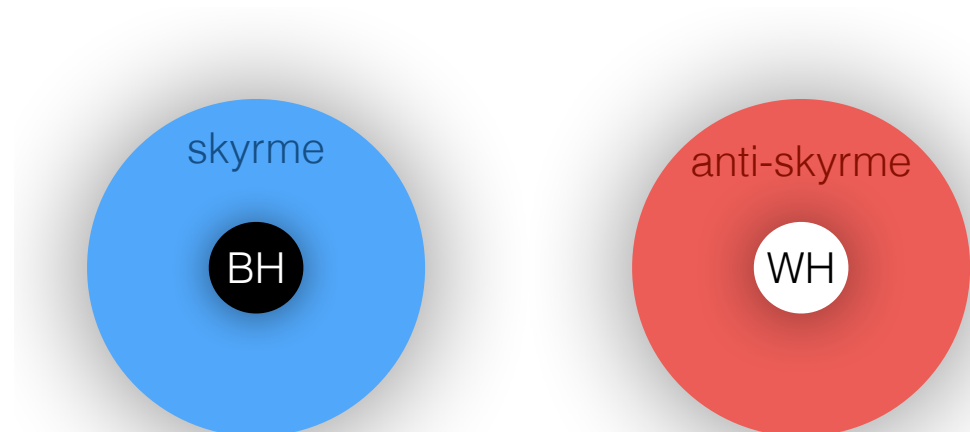


Figure 2. BH skyrmion is a bound state of a BH (Black circle) and a Baryon charged soliton localized on a higher radius than the BH horizon (Blue region). The anti-configuration is a WH anti-skyrmion bound state (WH as a White circle and anti-soliton in Red).

The neutron–antineutron transition rate induced by quantum gravity effect maximally scales as the Planck scale. Nevertheless an entropic factor suppression associated to the BH-WH transition has also to be included:

$$\langle n, BH | \bar{n}, WH \rangle \sim e^{-\bar{I}_E} \sim e^{-N} \rightarrow \Gamma(n \rightarrow \bar{n}) \simeq M_{Pl} e^{-N}. \quad (2)$$

where N is the number of qubit stored in the micro-BH. Another way to see the $n \rightarrow \bar{n}$ is as an oscillation on a non-trivial vacuum state of quantum gravity. The $n - \bar{n}$ transition can be induced by the gravitational skyrme instanton background as $\langle nn + h.c. \rangle_{\mathcal{M}} \rightarrow M_{Pl} e^{-I_E} nn + h.c$ where \mathcal{M} is the non-trivial euclidean topology of the gravitational instantons. In our case, the gravitational instanton is topologically equivalent to a $S_2 \times S_2$ double 2-sphere with two conical string defects, –since skyrme BH has conical angle defects in its metric as in the case of topological cosmic strings [40].

As it is well known, standard skyrme solutions, as well as string-like configurations eventually coupled to it, have fermionic zero modes. The appearance of Majorana fermion modes from skyrme-like or string-like topological defects is almost ubiquitous in condensed matter and topological materials. In our case, Majorana fermionic zero modes appear out as modes which get a (Majorana) mass gap outside the skyrme radius while inside they are (Majorana) massless. Thus we can consider a baryon preserving coupling of zero modes associated to the skyrme BH and neutrons as

$$\mathcal{L}_i = \mu_{\tau n} \tau n + h.c. \quad (3)$$

where τ has inverse baryon number than the neutron while $\mu_{\tau n}$ is a mass mixing. It is natural to assume as a missing singlet argument. (We mean that no any symmetry principle seems to protect such a mass scale to be arbitrarily large.) that such a mass is only cut off by quantum gravity energy scale, i.e., we will assume that it is of the order of the Planck scale. Let us consider a pair of identical τ zero modes localized on the BH skyrme configuration. Integrating on zero mode space, the instanton transition amplitude has an extra contribution from grassmanian integration

$$\int d^2 \tau e^{-\mu_{\tau n} \tau n} \propto nn \int d^2 \tau \tau \tau \rightarrow \frac{1}{M_{Pl}} \mu_{\tau n}^2 nn, \quad (4)$$

where $\mu_{\tau n}$ is naturally thought as $\simeq M_{Pl}, d^2 \tau$ because of two-zero modes and the M_{Pl} in denominator comes from the path integral normalization as quantum gravity scale. (In principle we can consider a larger zero mode space on the BH skyrme configuration but integral of higher species will only generate $M_n(nn)(nn)^N / \Lambda^{3N}$ ($N \leq 1$) operators.)

Incidentally, let us remark that such a mechanism exhibits an intriguing similarity with *exotic stringy instantons*, which also non-perturbatively generate new effective operators, integrating out supersymmetric moduli fields (see Refs. [46–48] for specific discussions of neutron–antineutron case).

Equation (4) is exponentially suppressed by the gravitational skyrme instanton euclidean action and thus we obtain a Majorana mass term for the neutron as

$$M_{Pl} e^{-\bar{I}_E} nn + h.c \rightarrow M_{Pl} e^{-N} nn + h.c. \quad (5)$$

In principle, we can also promote the B-number conservation to a $U_B(1)$ global symmetry. Equations (4) and (5) are subtly compatible with $U_B(1)$ symmetry: baryon transformation phase of nn part, $nn \rightarrow e^{2i\alpha} nn$ is exactly compensated by the imaginary part acquired by the $e^{-\bar{I}_E}$ part as an effect of euclidean action part of fermionic zero modes. On the other hand the $U_B(1)$ is spontaneously broken by the presence of the BH skyrme configuration.

In terms of quarks, the effective coupling among the skyrme zero modes and quarks are

$$\frac{\mu_{\tau n}}{\Lambda^3} \tau(udd) + h.c. \quad (6)$$

where the effective Λ cutoff will coincide with the QCD scale if the the gravitational instanton is paired with an effective skyrme with radius around the QCD length, i.e., $\Lambda = \Lambda_{\text{QCD}}$. $\mu_{\tau n}$ is the same mass scale defined in Equation (12). Integrating on the fermionic moduli space we obtain

$$\frac{\mu_{\tau n}^2}{M_{Pl}} e^{-\bar{I}_E} \frac{1}{\Lambda_{\text{QCD}}^6} (udd)^2 + h.c. \quad (7)$$

From this operator, we directly obtain Equation (5).

The emergent Majorana neutron mass term introduces an exponentially suppressed correlation between neutron and antineutron wave functions. In terms of the neutron–antineutron effective Hamiltonian matrix we have

$$\mathcal{H} = \begin{bmatrix} E_n & M_{Pl} e^{-\bar{I}_E} \\ M_{Pl} e^{-\bar{I}_E} & E_{\bar{n}} \end{bmatrix} \quad (8)$$

where in non-relativistic regime

$$E_n = m_n + \frac{p^2}{2m_n} + V_n, \quad E_{\bar{n}} = m_{\bar{n}} + \frac{p^2}{2m_{\bar{n}}} + V_{\bar{n}}. \quad (9)$$

CPT invariance guarantees that $m_n = m_{\bar{n}}$, which in principle may be questionable if quantum gravity induces measurable quantum decoherence violating unitarity. Here we will assume that both CPT and Lorentz invariance are preserved by quantum gravity effects. In general, it is also assumed that neutron and antineutron respect the strong or weak equivalence principle and that they feel the same gravitational field. In this case, if neutrons are not confined in a nucleus but just propagating on an external magnetic field, we will just have $V_n = -V_{\bar{n}} = \mu \cdot B$ where μ is the magnetic moment of the neutron.

Thus, for neutron–antineutron experiments the effective Hamiltonian would reduce to

$$\mathcal{H} = \begin{bmatrix} m_n + \mu \cdot B & M_{Pl} e^{-\bar{I}_E} \\ M_{Pl} e^{-\bar{I}_E} & m_n - \mu \cdot B \end{bmatrix}. \quad (10)$$

Now, in order to have a testable neutron Majorana mass from gravitational instantons we should have

$$M_{Pl} e^{-S} \sim 10^{-23} \text{ eV} \rightarrow e^{-S} \sim 10^{-50}. \quad (11)$$

Such a severe suppression corresponds to a reasonable entropy amount for a micro-BH: $S \sim (\bar{r}/L_{Pl})^2 \sim 115$ or so, related to the same amount of planckian qubits. As it is known, gravitational instantons are interpreted as the appearance of virtual BH(WH) pairs and quantum mechanically also virtual pairs with skyrme hairs have to appear out. In our considerations we are forced to assume that the skyrme BHs contributing to the neutron Majorana mass cannot have a smaller amount of qubits than around 115, otherwise the exponential suppression factor would not be competitive with the Planck mass scale and a too large Majorana mass would be generated out from our mechanism. Nevertheless it is also natural to assume that a virtual BH with baryon/skyrme hair necessarily has to contain a qubit number well larger than one since the very same skyrme solutions are generated out from a certain amount of degrees of freedom.

In this picture, the neutron–antineutron mixing seems to be related to a quantum entanglement of neutron and antineutron wave functions, which can be spatially separated by very large distances, rather than to a classical information exchange mediated by a new interaction. This is generated by the virtual BH pair, or a virtual BH-WH wormhole, as a new peculiar **EPR = ER** phenomenon [49]. In terms of kets, we can consider an entangled global state of neutron and antineutron for a certain position **I** and position **II** as $N(|n_I, \bar{n}_{II}\rangle + \epsilon e^{i\delta} |\bar{n}_I, n_{II}\rangle)$, where $N = 1/\sqrt{1 + \epsilon^2}$. If *Alice* hypothetically measures the particle in **I** and finds that it is a neutron, then *Bob* will detect an antineutron in **II**;

and viceversa. In this prospective, in a neutron–antineutron experiment, if we observed a final antineutron state this means that we would actually observe an antineutron *quantum tele-transported* to our experimental detector localized on the Earth, eventually from very long distances. The probability of it is very tiny and it is $\langle n_{\text{I}}, \bar{n}_{\text{II}} | \bar{n}_{\text{I}}, n_{\text{II}} \rangle = N(\epsilon)\epsilon \sim e^{-N}$. Such a quantum correlation is strongly suppressed if neutrons are bounded inside nuclei: nuclear binding energy would create an effective mass gap splitting among neutrons and antineutrons completely suppressing quantum entanglements as a decoherence effect. Large external magnetic fields as well.

Now, let us move on another interesting possibility concerning Mirror matter. Let us consider a Mirror standard model (MSM) with exactly the same gauge group of Ordinary SM but eventually opposite parity [50,51]. In this way parity violations in the ordinary weak sector is secretly restored in M- sector. As it is known, if Mirror symmetry is preserved (not explicitly or spontaneously broken), a fast O- and M- neutron transition will be possible without any matter destabilization, i.e., $n - \bar{n}'$ oscillations. Let us define a separate O- M- Baryon number for the O- and M- sectors and that $n(\bar{n})$ carries $B = 1(-1)$ while $n'(\bar{n}')$ carries a $B' = 1(-1)$. Then we can consider new BH skyrme solutions where the topological conserved charge is a combination of B, B' . For simplicity let us consider a $B + B'$. Indeed, BH skyrme to a WH mirror skyrme can be envisaged as $n\bar{B}\bar{H} \rightarrow \bar{n}'\bar{W}\bar{H}$ preserving the $B + B' = 0$ number, where $\bar{B}\bar{H} \equiv \bar{B}\bar{H}_{B+B'=B'=-1}$ and $\bar{W}\bar{H} \equiv \bar{W}\bar{H}_{B+B'=B'+1}$.

Similarly to the neutron–antineutron case, this generates a mixing mass as

$$M_{Pl}e^{-S'} n n' + h.c. \quad (12)$$

where $M_{Pl}e^{-S'}$ has exclusion limits which are at lower energy scale than the neutron Majorana mass of around 8th digits; but in terms of the exponential hierarchy with instanton entropy content is also around 100 qu-bit planck digits. Such a BH skyrme can be thought as coupled to a *mirror Alice string* which converts ordinary to mirror neutral particles, eventually interpretable as a topological string defect from $SO(20)$ unification between O- and M- standard model sector [52,53]. Similarly to the case of neutron Majorana mass, an operator in Equation (12) can be generated by the mixing of fermionic zero modes related to the $B + B'$ -hairy BH solution:

$$\mu_{\tau n} \tau n + \mu_{\tau' n'} \tau' n' + h.c., \quad (13)$$

$$\int d\tau d\tau' e^{\mu_{\tau n} \tau n + \mu_{\tau' n'} \tau' n'} \rightarrow \frac{1}{M_{Pl}} \mu_{\tau n} \mu_{\tau' n'} n n' + h.c. \quad (14)$$

where τ, τ' have O- and M- baryon numbers balancing O- and M- (anti)neutron and we assume one and one localized on the BH skyrme configuration.

As for neutron–antineutron, also $n - n'$ can be viewed as a quantum tele-transportation phenomenon. Such a mechanism avoids any early Universe bounds on $n - n'$ from re-thermalization proposed in Ref. [18] since not-related to any strong three O- M- quark collisions in early thermal bath.

2. Baryon Skyrme BHs and Gravitational Instantons

The skyrme is a solitonic solution derived from a set of N quark flavors (or in the easiest case from two flavors) having, in flat space-time, a baryon number as topological charge [54,55]:

$$B = \int d^3x J_0, \quad (15)$$

where

$$J_\mu = -\frac{1}{24\pi^2} \epsilon_{\mu\nu\rho\sigma} \text{Tr}(U \partial^\nu U U^{-1} \partial^\rho U U^{-1} \partial^\sigma U), \quad (16)$$

is the skyrme topological Chern-Simons current. Above we defined U as a $SU(N)$ flavor matrix that can be written as

$$U = e^{i\Pi_a(x)\tau_a/F} \quad (17)$$

where F is a characteristic constant, τ_a group matrices of $SU(N)$ and Π_a are meson fields related to $SU(N)$. In case of two quarks, $SU(N)$ is just $SU(2)$, τ_a are the Pauli matrices Π_a correspond to pion fields, F to the pion decay constant. In particular, the Skyrme effective lagrangian can be written in terms of U as in Ref. [54] and the skyrme current in Equation (16) can be easily derived from it. Typically, in the $SU(2)$ flavor case, skyrmion has a typical size and mass controlled by the pion decay constant and the electric charge as $L = e^{-1}F^{-1}$ and $M = Fe^{-1}$.

The action originating a BH skyrme solution reads as

$$I = I_m + \int_{\Sigma} \sqrt{-g} d^4x \left(\frac{1}{16\pi} (R - 2\Lambda) \right) + \frac{1}{8\pi} \int_{\partial\Sigma} \sqrt{h} d^3x (K - K^0). \quad (18)$$

where I_m includes

$$\int d^4x \sqrt{-g} \text{Tr} \left(\frac{F^2}{16} X^\mu X_\mu + \frac{1}{32e^2} Y^{\mu\nu} Y_{\mu\nu} \right), \quad (19)$$

$$X_\mu = U^{-1} \nabla_\mu U, \quad Y_{\mu\nu} = [X_\mu, X_\nu]. \quad (20)$$

In the case of a skyrme BH, we can consider a two sphere with radius r closing enclosing the BH horizon as $r \gg R_{BH}$ (R_{BH} the BH event horizon):

$$B = \int_{S_2} dx^\mu \wedge dx^\nu \mathcal{J}_{\mu\nu} \quad (21)$$

with $\mathcal{J}_{\mu\nu}$ the topological current coupled to the embedding coordinates x . Indeed J in Equation (16) represents the Hodge-dual of the exterior derivative of the two-form \mathcal{J} .

As remarked in Ref. [45], one can show with a simple argument that the baryon topological hairs can be detected through a memory phase shift effect *a la* Aharonov–Bohm. For instance, one can consider a path string loop surrounding a skyrmion BH solution considering an effective coupling g between the string and the skyrmion $g \int dX^\mu \wedge dX^\nu \mathcal{J}_{\mu\nu}$. Indeed, this can be the case of string flux-tubes of gauge fields, cosmic strings of fundamental strings. In this case, any physical processes in which the string world-volume encloses the skyrmion BH lead to the Aharonov–Bohm phase shift $\Delta\Phi = 2\pi g$.

Let us also remark that in a generic curved space-time, B has the integral form as

$$B = \int_{\Sigma} d^3x \sqrt{-g} J^0 = \int_{\partial\Sigma} dS J^a n_a \quad (22)$$

where dS is the surface element of $\partial\Sigma$ which delimits the volume Σ while n^a is the unit normal vector equipping the surface. Considerations about Aharonov–Bohm shifts induced by B-hair, introduced above, can be generalized in curved space-time.

As found by many authors, spherically symmetric skyrme BH solutions can be found from a skyrme lagrangian coupled to Einstein–Hilbert equation [40–43]. A large class of skyrme BHs are basically a bound state of a skyrme soliton and a ordinary spherically symmetric BH. In particular the ADM mass of the spherically symmetric skyrme BH is in many cases just the sum of the skyrme solitonic energy, the mass associated to the BH horizon and their binding energy:

$$M_{ADM} = M_s + M_h + E_B \quad (23)$$

where M_s is the mass associated to the skyrme configuration, M_h is the one contained within the BH horizon and E_B is a binding energy of the two. The three depends on the specific

metric profiles—see for example Ref. [44]. In our case, we consider $M_h \simeq 10M_{Pl} \gg M_s$, while $R_h \simeq 10L_{Pl} \ll R_s \simeq L_{QCD}$ with $R_{h,s}$ BH horizon and soliton radii respectively, $L_{QCD} = 1/\Lambda_{QCD}$. Such a case is naturally possible since the solitonic configuration has an energy density much smaller than the BH. Indeed this also implies that most of the entropy content of the bound state is (holographically) localized on the BH horizon, i.e., $S_{BH} \sim (R_h/L_{PL})^2 \gg S_s$ where S_s is the entropy content of the solitonic configuration.

Let us consider the partition function of the BH/skyrme solution:

$$Z = \int \mathcal{D}g \mathcal{D}\Psi e^{-I_E} \quad (24)$$

where I_E is the eucliden action of the system of Equation (18), g is the metric tensor and Ψ in general indicates all matter/gauge fields. As we know, gravitational instantons are saddle classical solutions of the Euclidean action. Gravitational instanton can be classified from Euler numbers and signature

$$\chi = \nu + B_2^+ + B_2^- = \frac{1}{128\pi^2} \int d^4x \sqrt{g} R_{abcd} R_{a'b'c'd'} \epsilon^{aba'b'} \epsilon^{cdc'd'} , \quad (25)$$

$$\tau = B_2^+ - B_2^- = \frac{1}{96\pi^2} \int d^4x \sqrt{g} R_{abcd} R_{a'b'c'd'} \epsilon^{aba'b'} , \quad (26)$$

where $\nu = 2$ for compact manifolds and $\nu = 1$ for non-compact ones and B_2^\pm are the second Betti numbers of the harmonic and anharmonic two forms, respectively. In case of gravity coupled to matter, without any topological defects in matter sector, one can individuate three main topological building blocks as $S_2 \times S_2$, CP^2 and K^3 ; but $S^2 \times S^2$ with $(\chi, \tau) = (4, 0)$ is the dominant class [25–28,31]. As mentioned, $S^2 \times S^2$ is interpreted as related to virtual BH pairs. In case of skyrme configuration, topology of $S^2 \times S^2$ is eventually punctured by opening string-like angles. Nevertheless this topological difference does not lead to any substantial difference or suppression/enancement compared to the $S^2 \times S^2$ configuration. As it is well known, the entropy related to the BH configuration is related to the partition function and the BH skyrme case follows the same thermodynamical rule:

$$S = \log Z + \beta E \simeq S_h \quad (27)$$

which, as mentioned before, is dominated by the entropy stored on the horizon surface. The $\widetilde{BH} - \widetilde{WH}$ transition probability is exponentially suppressed as the action evaluated on the gravitational skyrme instanton as

$$\Gamma = A e^{-I_{\mathcal{M}}} \sim e^{-c(R_h/L_{Pl})^2} \quad (28)$$

where A also includes quantum loop corrections, $I_{\mathcal{M}}$ is the action of the gravitational instanton with topology $\mathcal{M} \equiv S_2 \times S_2 - \{p\}$ with $\{p\}$ puncture set c a numerical coefficient not relevant for our discussions. (Similar considerations for the more standard $S_2 \times S_2$ case can be found in Refs. [56–59].) The rate can also be expressed in the terms of the imaginary part of the partition function as

$$\Gamma = -\frac{1}{\sqrt{3}\pi} \frac{1}{R_h} \frac{\text{Im}Z[\mathcal{M}]}{Z[S_4]} . \quad (29)$$

where S_4 is the Euclidean four-dimensional sphere.

3. Conclusions and Remarks

In this paper, we considered a novel mechanism generating a Majorana mass for the neutron from virtual skyrme/baryon black holes. In particular, quantum gravity effects may subtly survive at the Fermi scale generating a Majorana mass gap for the neutron rather than having a direct effect on single quark constituents. Indeed, GR admits BH/skyrme

bound systems which have the following properties: (i) they have a topological Baryon hair which generates a Aharanov–Bohm like effect on SM baryons; (ii) the BH and skyrme radii can be very different and the skyrme size can be much larger than the BH diameter; (iii) Baryon violating transitions generated by such configurations follow selection rules which allow for proton stability. We also showed that neutron mirror neutron mixing can be generated from virtual BHs with O- and M- skyrme/baryon hairs.

Contrary to typical mechanisms to generate a $n - \bar{n}$ or a $n - n'$ transition, here these mixings are effects of long-range correlations coherently surviving from Planck world to larger length scales. A neutron Majorana mass of about 10^{-23} eV is related to a long-range correlation between neutron and antineutron. This seems to suggest that such gravitational instantons, related to wormholes, can have a long-extension of warp throats, even if the corresponding BH(WH) sizes are small as few times the Planck length. Contrary to standard interactions, such a correlation is not suppressed by the distance but by the semiclassical probability of BH-WH skyrme/anti-skyrme tunneling, in turned exponentially suppressed as the BH(WH) entropy. This can be effectively viewed as a long-range gravitational entanglement between neutron and antineutron which may be interpreted as an intriguing manifestation of **ER = EPR** conjecture [49]. Indeed, this gravitational entanglement will be highly (but not fully) suppressed by decoherence effects if neutrons are confined in nuclei or in presence of strong external magnetic fields. Similarly, if mirror neutrons existed, a $n - n'$ long-range entanglement mediated by O/M skyrme BHs could be also envisaged. In this sense, quantum gravity has unexpected topological portals from matter to anti-matter (and viceversa) through *apparent* $|\Delta B| = 2$ and $|\Delta B| = 1$ violating transitions—another strong motivation towards neutron–antineutron searches.

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