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Editorial

Symmetry in Functional Equations and Analytic Inequalities II

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1. Introduction

The field of functional equations is an ever-growing branch of mathematics with far-reaching applications; it is increasingly used to investigate problems in mathematical analysis, combinatorics, biology, information theory, statistics, physics, the behavioral sciences, and engineering.

Inequalities play a significant role in all fields of mathematics and present a highly active and attractive field of research.

The symmetry properties of functions used to define an equation or inequality can be studied in order to determine solutions with particular properties. As far as inequalities are concerned, the study of special functions such as hypergeometric functions and special polynomials considering their symmetry properties may provide some interesting outcomes. Studies on symmetry properties for different types of operators associated with the concept of quantum calculus may also be investigated.

This Special Issue promotes an exchange of ideas between eminent mathematicians from around the world, dedicated to functional equations and analytic inequalities. The aim of the issue is to boost cooperation among mathematicians working on a broad variety of pure and applied mathematical areas.

This volume of ideas and mathematical methods will include a wide area of applications in which equations, inequalities, and computational techniques relevant to their solutions, play an important role. These ideas and methods have a significant effect on everyday life, as new tools are constantly being developed to achieve revolutionary research results, bringing scientists even closer to exact sciences, and encouraging the emergence of new approaches, techniques, and perspectives in functional equations, analytical inequalities, etc. Please note that all submitted papers should be within the scope of the journal.

2. Brief Overview of the Contributions

In [1], Ala Amourah, Basem Aref Frasin, Morad Ahmad and Feras Yousef construct a new subclass of analytic bi-univalent functions defined on symmetric domain by means of the Pascal distribution series and Gegenbauer polynomials. They provide estimates of Taylor–Maclaurin coefficients $|a2|$ and $|a3|$ for functions in the aforementioned class, and next, they solve the Fekete–Szegö functional problem. Moreover, some interesting findings for new subclasses of analytic bi-univalent functions emerge by reducing the parameters in the main results.

The authors of the paper [2], Amal Mohammed Darweesh, Waggas Galib Atshan, Ali Hussein Battor and Alina Alb Lupaş, study suitable classes of admissible functions, and establish the properties of third-order differential subordination by making use of certain differential operators of analytic functions in U and the normalized Taylor–Maclaurin series. Some new results on differential subordination with some corollaries are obtained. These properties and results are symmetrical to the properties of the differential superordination that form the sandwich theorems.

In the paper [3], Bingren Chen, Qi Liu and Yongjin Li firstly investigate the constant H(X) proposed by Gao by discussing several properties of it that have not yet been discovered. Secondly, they focus on a new constant GL(X) closely related to H(X), along with

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a variety of geometric properties. In addition, they show several relations among it, and several basic geometric constants via a few inequalities. Finally, the authors explicitly characterize the geometric properties of its generalized forms GL(X,p) and CL(X).

Adriana Cătaş, Emilia Borşa and Loredana Iambor in [4] present a survey on subordination and superordination theorems related to the class of analytic functions defined in a symmetric domain, which is the open unit disc. The results were deduced by making use of a new differential operator. They present two properties of this operator from which they construct the final results. Moreover, based on the obtained outcomes, two sandwich-type theorems are given. Some interesting further consequences are also taken into consideration.

The goal of the investigation made by Loriana Andrei and Vasile-Aurel Caus in [5] is to introduce a new class of analytic functions (Kt,q), defined in the open unit disk, by means of the q-difference operator, which may have symmetric or assymetric properties, and to establish the relationship between the new defined class and appropriate subordination. The derived relationships of this class obtain sufficient conditions for an analytic function to be Kt,q. Finally, in the concluding section, they take the decision to restate the clearly proven fact that any attempt to create the rather simple (p,q)-variations of the results, which they have provided in the paper, will be a rather inconsequential and trivial work, simply because the added parameter p is obviously redundant.

It is a well-known fact that convex and non-convex fuzzy mappings play a critical role in the study of fuzzy optimization. Due to the behavior of its definition, the idea of convexity also plays a significant role in the subject of inequalities. The concepts of convexity and symmetry have a tight connection. In [6], the authors Muhammad Bilal Khan, Pshtiwan Othman Mohammed, José António Tenreiro Machado and Juan L.G. Guirao use information learned from both concepts, owing to the significant correlation that has developed between both in recent years. They introduce a new class of harmonically convex fuzzy-interval-valued functions, which are known as harmonically h-convex fuzzy-interval-valued functions (abbreviated as harmonically h-convex F-I-V-Fs) by means of fuzzy order relation. This fuzzy order relation is defined level-wise through the Kulisch–Miranker order relation defined on interval space. Some properties of this class are investigated. By using fuzzy order relation and h-convex F-I-V-Fs, Hermite–Hadamard-type inequalities for harmonically are developed via fuzzy Riemann integral. They have also obtained some new inequalities for the product of harmonically h-convex F-I-V-Fs. Moreover, they establish Hermite–Hadamard–Fejer inequality for harmonically h-convex F-I-V-Fs via fuzzy Riemann integral. These outcomes are a generalization of a number of previously known results, and many new outcomes can be deduced as a result of appropriate parameters and real-valued function selections. For the validation of the main results, they have added some nontrivial examples.

The paper [7] is devoted to studying approximations of symmetric continuous functions by symmetric analytic functions on a Banach space X with a symmetric basis. The authors, Mariia Martsinkiv and Andriy Zagorodnyuk, obtain some positive results for the case when X admits a separating polynomial using a symmetrization operator. However, even in this case, there is a counter-example, because the symmetrization operator is well defined only on a narrow, proper subspace of the space of analytic functions on X. For $X = c_0$, they introduce ε-slice G-analytic functions that have a behavior similar to G-analytic functions at points $x \in c_0$ such that all coordinates of x are greater than ε, and they prove a theorem on approximations of uniformly continuous functions on c_0 by ε-slice G-analytic functions.

Owning to the importance and great interest of differential operators, two generalized differential operators, which may be symmetric or assymetric, are newly introduced in paper [8]. Motivated by the familiar Jackson's second and third Bessel functions, Loriana Andrei and Vasile-Aurel Caus derive necessary and sufficient conditions for which the new generalized operators belong to the class of q-starlike functions of order alpha. Several corollaries and consequences of the main results are also pointed out.

The theory of fractional analysis has been a focal point of fascination for scientists in mathematical science, given its essential definitions, properties, and applications in handling real-life problems. In the last few decades, many mathematicians have shown their considerable interest in the theory of fractional calculus and convexity due to their wide range of applications in almost all branches of applied sciences, especially in numerical analysis, physics, and engineering. The objective of the article [9] is to establish Hermite–Hadamard-type integral inequalities by employing the *k*-Riemann–Liouville fractional operator and its refinements, whose absolute values are twice-differentiable h-convex functions. Moreover, Soubhagya Kumar Sahoo, Muhammad Tariq, Hijaz Ahmad, Ayman A. Aly, Bassem F. Felemban and Phatiphat Thounthong also present some special cases of presented results for different types of convexities. Moreover, they also study how q-digamma functions can be applied to address the newly investigated results. Mathematical integral inequalities of this class and the associated arrangements have applications in diverse domains, in which symmetry presents a salient role.

The motivation of the article [10] is to define the ($p-$q)-Wanas operator in geometric function theory by the symmetric nature of quantum calculus. Abbas Kareem Wanas and Luminiţa-Ioana Cotîrlă initiate and explore certain new families of holormorphic and bi-univalent functions AE(λ,σ,δ,s,t,*p*,q;ϑ) and SE(μ,γ,σ,δ,s,t,*p*,q;ϑ), which are defined in the unit disk U associated with the (p$-$q)-Wanas operator. The upper bounds for the initial Taylor–Maclaurin coefficients and Fekete–Szegö-type inequalities for the functions in these families are obtained. Furthermore, several consequences of the results are pointed out based on the various special choices of the involved parameters.

The operator defined as the fractional integral of confluent hypergeometric function was introduced and studied by Alina Alb Lupaş in previously written papers, in view of the classical theory of differential subordination. In the paper [11], the same operator is studied using concepts from the theory of fuzzy differential subordination and superordination. The original theorems contain fuzzy differential subordinations and superordinations for which the fuzzy best dominant and fuzzy best subordinant are given, respectively. Interesting corollaries are obtained for particular choices of the functions acting as fuzzy best dominant and fuzzy best subordinant. A sandwich-type theorem is stated combining the results given in two theorems proven in this paper, using the two dual theories of fuzzy differential subordination and fuzzy differential superordination.

Paper [12] continues the study made by Alina Alb Lupaş and Adriana Cătaş on the relatively new concept of fuzzy differential subordination conducted in some recently published cited papers. Certain fuzzy subordination results for analytical functions involving the Atangana–Baleanu fractional integral of Bessel functions are presented. Theorems giving the best dominants for some fuzzy differential subordinations are proven, and interesting corollaries are provided with the use of particular functions as fuzzy best dominants.

In the paper [13], a new operator denoted by D$-$λzLnα is defined by using the fractional integral of Sălăgean and Ruscheweyh operators. By means of the newly obtained operator, the subclass Sn(δ,α,λ) of analytic functions in the unit disc is introduced, and various properties and characteristics of this class are derived by applying techniques specific to the differential subordination concept. By studying the operator D$-$λzLnα, some interesting differential subordinations are also given by Alina Alb Lupaş and Georgia Irina Oros.

In the paper [14] Alina Alb Lupaş studies strong differential subordinations for the extended new operator IRmλ,l defined by the Hadamard product of the extended multiplier transformation I(m,λ,l) and the extended Ruscheweyh derivative Rm, on the class of normalized analytic functions.

Marek T. Malinowski consider functional set-valued differential equations in their integral representations that possess integral symmetrically on both sides of the equations in the paper [15]. The solutions have values that are the non-empty compact and convex subsets. The main results contain a Peano-type theorem on the existence of the solution and a Picard type theorem on the existence and uniqueness of the solution to such equations.

The proofs are based on sequences of approximations that are constructed with appropriate Hukuhara differences of sets. An estimate of the magnitude of the solution's values is provided as well. The authors show the closeness of the unique solutions when the equations differ slightly.

In the paper [16], Hari Mohan Srivastava and Sheza M. El-Deeb investigate several fuzzy differential subordinations that are connected with the Borel distribution series $B(\lambda,\alpha,\beta)(z)$ of the Mittag-Leffler type, which involves the two-parameter Mittag-Leffler function $E\alpha,\beta(z)$. Using the above-mentioned operator $B(\lambda,\alpha,\beta)$, they also introduce and study a class $MF\lambda,\alpha,\beta(\eta)$ of holomorphic and univalent functions in the open unit disk Δ. The Mittag-Leffler-type functions, which they have used in the investigation, belong to the significantly wider family of the Fox–Wright function $p\Psi q(z)$, whose p numerator parameters and q denominator parameters possess a kind of symmetry behavior, in the sense that they remain invariant (or unchanged) when the order of the p numerator parameters or the q denominator parameters are arbitrarily changed. They have used such special functions in the study of a general Borel-type probability distribution, which may be symmetric or asymmetric. As symmetry is generally present in most works involving fuzzy sets and fuzzy systems, the usages here of fuzzy subordinations and fuzzy membership functions potentially possess local or non-local symmetry features.

The Special Issue Book "Symmetry in Functional Equations and Analytic Inequalities II" presents a collection of articles dealing with relevant topics in the field of functional equations and analytic inequalities. Various mathematical and computational techniques and approaches were presented to solve a range of problems. The success of the Special Issue, "Functional Equations and Analytic Inequalities", has motivated the editors to propose a new Special Issue, "Symmetry in Functional Equations and Analytic Inequalities II", that has complemented the first one with a focus on modern applications of functional equations and analytic inequalities. We invite the research community to submit novel contributions covering functional equations and analytic inequalities.

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