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Editorial

Special Issue Editorial "Solvability of Nonlinear Equations with Parameters: Branching, Regularization, Group Symmetry and Solutions Blow-Up"

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1. Introduction

Nonlinear dynamical models with parameters are at the heart of natural science, and they serve as essential instrument to analyze and solve various appealing problems in engineering areas. Indeed, parameter-dependent models are widely employed in mathematical modeling in many scientific research fields formulated in direct and inverse problems. Many critical processes in fluid dynamics, thermodynamics, and space plasma physics are modeled using the cutting-edge theory of nonlinear differential-operator equations. The Vlasov–Maxwell systems and other kinetic models play an important role in contemporary mathematical physics. The problems discussed in this volume have an immediate mathematical appeal and are of increasing importance in thermal and plasma physics, computer tomography, mechanics, and material science but are not yet as widely known as they should be to researchers interested in these contemporary research fields.

The Lyapunov–Schmidt methods are the basement of the bifurcation theory for the functional equations initiated by A. M. Lyapunov and E. Schmidt in the beginning of the 20th century and developed by other famous mathematicians. For excellent reviews, readers may refer to the seminal publications of A. I. Nekrasov, L. A. Lusternik, M. M. Vainberg and V. A. Trenogin, M. A. Krasnoselsky, B. Buoni, E. N. Dancer and J. Toland, S.-N. Chow, J. K Hale, and many other authors. These pivotal studies were initiated the construction of modern nonlinear analysis with sound applications in fluid dynamics and mathematical modeling.

The main objective of this Special Issue is to survey the relations between the various kinds of differential-operators, kinetic equations, and bifurcation theory from a constructive point of view. We collected new results derived to further expand the differential-operator and kinetic equations theory.

The investigation of the branching (bifurcating) solutions of such equations is one of the most important aspects in the analysis of such models. The foundations of the theory of bifurcations for the functional equations were laid in the well-known publications by A. M. Lyapunov on equilibrium forms of rotating liquids. The approach proposed by Lyapunov has been thoroughly developed and is presently known as the Lyapunov–Schmidt method. Here, readers may refer to the landmark results of M. M. Vainberg, V. A. Trenogin, and N. A. Sidorov. A valuable part in the foundations of the bifurcation theory belongs to Jules Henri Poincaré's ideas. Later, to the end of proving the theorems on the existence of bifurcation points, infinite-dimensional generalizations of topological and variational methods were proposed by M. A. Krasnoselsky, M. M. Vainberg, and others. Here, readers may refer to recent book [1] and references therein. A great contribution to the development and applications of the bifurcation theory has been made by a number of famous 20th century pure and applied mathematicians.

Well-known are the applications of the bifurcation theory in mechanics (convection, wave theory, oscillations, aero-hydro-elasticity, bending of bars, membranes, and shells) described in profound investigations of A. I. Nekrasov, T. Levi-Chivit, N. Kochin, D. Stroyk, M. A. Lavrentiev, K. O. Fridrichs, J. Stoker, D. Joseph, J. Keller, J. Toland, A. M. Ter-Krikorov, V. A. Trenogin, I. I. Vorovitch, A. S. Wolmir, M. Berger, S. Ya. Sekerzh-Zen'kovich, Y. I. Yudovich, B. Y. Loginov and N. A. Sidorov, L. S. Srubschik, V. V. Pukhnachov, V. Y. Bolotin, and many others. The sphere of applications of both the Lyapunov–Schmidt method and the theory of bifurcations has been extending since the time of their advent. Currently, it has embraced many new areas of natural science, economics, and engineering.

Starting with the seminal works of A. M. Lyapunov, A. Poincaré, and E. Schmidt, the branching theory of nonlinear parameter-dependent equations enabled various essential applications in natural sciences and engineering over the course of the last hundred years. V. I. Yudovich pioneered the application of symmetry methods in branching theory. A series of applications of the Lyapunov–Schmidt method, the Conley index theory, and the central manifold methods in the conditions of group symmetry were reported in many seminal works during the last decades. Various critical processes in plasma physics, fluid dynamics, and thermo-dynamics are modeled using the branching theory of nonlinear differential-operator parameter-dependent equations.

The blow-up theory for the differential-operator equations depending on parameters in cases when the solutions are extended in the vicinity of special points (bifurcation points, in particular) is also critical for nonlinear systems analysis. In the analytical case here, an effective method may be to construct asymptotics of solutions in the form of Laurent series and logarithm-step polynomials. The phenomenon of the destruction of a solution can also be connected with the irregularity of the problem when we look for a solution of the system with an irreversible operator in the main part (see the works by G. A. Sviridyuk, M. O. Korpusov, and others).

Methods for studying the solution blow-up phenomenon for the nonlinear partial derivative equations were developed by M. O. Korpusov, A. G. Sveshnikov, V. A. Galaktionov, S. I. Pohožaev, and others.

It is to be noted that blow-up theory is a relatively young field of modern nonlinear analysis, which is still under active studies by many research groups. The phenomenon of solution blow-up for various classes of nonlinear equations is not only of the pure mathematical but also of physical interest, since it has relation to the catastrophe theory and many processes in thermal and plasma physics, heat and power engineering, fluid mechanics, and other applications.

The objective of this Special Issue is to report on the cutting edge development of the advanced branching theory of nonlinear equations and their applications. The Special Issue brings together experts in the qualitative theory of differential-operator equations, numerical analysts, and practitioners in the various applied fields of contemporary natural sciences. The results on the solvability of non-standard nonlinear equations with parameters will be reported, focusing on the analysis of the problems associated with branching, regularization, group symmetry, and solution blow-up phenomena.

This volume provides the comprehensive introduction to the modern theory of differential-operator and kinetic models including Fredholm, Lyapunov–Schmidt branching equations, and Vlasov–Maxwell equations to name a few.

The following manuscripts were selected for publications. Articles were prepared by scientists working in leading universities and research centers in China, Colombia, India, Russia, and Saudi Arabia. Some results were announced at the International Conference "Dynamical Systems and Computer Science: Theory and Application" (DYSC 2020 and DYSC 2021) in Irkutsk, Russia.

S. S. Santra at al., in the paper "Impulsive Differential Equations under Canonical and Non-Canonical Conditions" [2], studied the nonoscillatory and oscillatory behavior of the solutions of nonlinear neutral impulsive systems with forcing terms. The various ranges of the neutral coefficient were studied. The sufficient conditions were derived for the existence of positive bounded solutions of the impulsive system.

Y. Shen et al., in the paper "CT Image Reconstruction via Nonlocal Low-Rank Regularization and Data-Driven Tight Frame" [3], propose a novel computer comography image reconstruction model using the nonlocal low-rank regularity and data-driven tight frame model. The model employed an asymmetric treatment for image reconstruction and Radon domain inpainting. This makes it possible to effectively combine the nonlocal low-rank approximation method for spatial domain CT image reconstruction and data-driven tight frame-based regularization for Radon domain image inpainting.

S. V. Meleshko and V. V. Pukhnachev, in the paper "Group Analysis of the Boundary Layer Equations in the Models of Polymer Solutions" [4], constructed the boundary layer equations of two mathematical models of the flow of aqueous polymer solutions and applied the group analysis method to study them.

A. Kazakov et al., in the paper "Analytical Solutions to the Singular Problem for a System of Nonlinear Parabolic Equations of the Reaction-Diffusion Type" [5], studied the analytic solvability of a boundary value problem with the degeneration for the reaction–diffusion system of nonlinear second-order parabolic equations. A. Kazakov, L. Spevak et al., in the paper "On the Analytical and Numerical Study of a Two-Dimensional Nonlinear Heat Equation with a Source Term" [6], considered the heat wave propagation problem with a specified zero front in the case of two spatial variables. The two-dimensional boundary-value problem for the degenerate nonlinear parabolic equation with a source term is studied both analytically and numerically.

N. A. Sidorov et al. derived the necessary and sufficient conditions of existence of the nonlinear operator equations' branches of solutions in the neighborhood of branching points in the paper "Solvability and Bifurcation of Solutions of Nonlinear Equations with Fredholm Operator" [7]. The proposed methods are illustrated in the solution of nonlinear integral equations.

E. M. Rojas et al. proved the existence of non-negative solutions for nonlinear singular system of second-order ordinary differential equations for modeling the magnetic (non) insulated regime on a plane diode in the paper "A Boundary Value Problem for Noninsulated Magnetic Regime in a Vacuum Diode" [8].

This Special Issue aims to fill a gap in the considerable body of existing academic literature on the analytical methods of studies of the complex behavior of differential-operator equations and kinetic models.

This volume will be of interest to mathematicians, physicists, and engineers interested in the theory of such non-standard systems.

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