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# Gas Cooled Graphite Moderated and Pressurized Water Reactor the Optimal Choice for Nuclear Power Plants Based on a New Group Decision-Making Technique

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**Abstract:** The aim of this work is to introduce the novel concept of an m-polar fuzzy soft set, including various types of algorithms and their fundamental operations. We created mathematical modeling to analyze operational rules and discuss the advantages, disadvantages, and natural aspects of algorithms for six types of nuclear power plants. It has been determined that emerging trends and the benefits of algorithms are increasing step by step. The suggested modeling with an m-polar fuzzy soft set is integrated into the fuzzy mean environment to analyze the effect of the correlation between decision factors and decision results without an excessive duty cycle, thus minimizing energy use and other adverse effects. Based on a new group decision-making technique considering an asymmetric weight vector, we proved that Gas Cooled, Graphite-Moderated, and Pressurized Water Reactors are the optimal choices for nuclear power plants. In the end, a numerical illustration is provided for selecting the best photo to demonstrate the use of the generated technique and to exhibit its adequacy.

**Keywords:** nuclear power plants; Gas Cooled Graphite Moderated; Pressurized Water Reactor; Boiling Water Reactors; Heavy Water Cooled and Moderated; Reactor Boiling Light Water; *m*-polar fuzzy soft set



**Citation:** Khalaf, M.M.; Ismail, R.; Al-Shamiri, M.M.A.; Abdelwahab, A.M. Gas Cooled Graphite Moderated and Pressurized Water Reactor the Optimal Choice for Nuclear Power Plants Based on a New Group Decision-Making Technique. *Symmetry* **2022**, *14*, 2621. https://doi.org/10.3390/sym14122621

Academic Editor: Dragan Pamučar

Received: 24 October 2022
Accepted: 2 December 2022
Published: 11 December 2022



## 1. Introduction

The fundamental idea of fuzzy set theory (FS), which is an extension of classical set theory for managing ambiguous and unclear information, was first introduced by Zadeh [1] in 1965. The worth of a membership degree in a fuzzy set was found in [0, 1]. A new multi-criteria group decision-making (MCGDM) analysis using m-polar fuzzy soft expert sets was introduced by Akram et al. [2]. For the fuzzy decision-making problem, we propose two techniques using the inf product or sup product operations of possible m-polar fuzzy soft sets. We devise an algorithm utilizing a possible m-polar fuzzy soft set to address the decision-making problems and provide a numerical example to show how it can be used. We conclude from the study that the presented approach can effectively manage uncertainty when dealing with decision-making challenges. It has been difficult for researchers and decision makers to deal with the inability of traditional FS models when tackling ambiguity and imprecision in every field of life, including the social sciences, information technology, economics, and business management. To address this issue, numerous attempts have already been made. In 1999, Molodtsov [3] proposed the soft set. The application of soft fuzzy sets (SfS) to choose the finest housing was demonstrated by Akram and Maji [4,5]. The fuzzy soft sets were defined, and their underlying properties were examined by Maji et al. [6]. Group decision making based on the TOPSIS method of m-polar fuzzy linguistics was introduced by Arooj et al. [7]. A novel multi-attribute decision-making technique based on m-polar fuzzy soft rough sets was studied by Akram et al. in [8]. Later, Karaaslan et al. [9] reformulated bipolar fuzzy soft sets and focused on both the application and the decision-making processes. We were given characteristics reduction algorithms for m-polar fuzzy relational decision systems by Akram et al. [10], and their basic operations were outlined. Neha et al. [11] used a variety of multi-criteria decision-making strategies in their study of multi-attribute decision making based on m-polar fuzzy Hamacher aggregation operators. Then, Akram et al. [12] employed the TOPSIS and ELECTRE I methods to diagnose medical conditions utilizing bipolar fuzzy data. The idea of fuzzy soft sets is applied to evaluate binary evaluation-based data. For non-binary discrete evaluation systems, it is unsuccessful. Most systems today undergo evaluation through the use of a rating system. These systems use a variety of symbols to score the options, including stars, checkmarks, dots, numbers, and so on. Fatimah et al. [13] developed set theoretic operations and decision-making algorithms that are helpful for capturing ordered graded information to address these issues by giving the stimulating idea of an N-soft set. An effective method for dealing with many decision-making circumstances is known as multi-criteria group decision making (MCGDM), in which various experts weigh in on the various criteria that influence the decision. Due to the complexity of the problem and the presence of complicated, unclear information, MCGDM research is difficult. The primary objective of this study was to develop a new MAGDM model that includes expert evaluation of the criteria. By combining m-polar fuzzy sets and soft expert sets, a new hybrid model known as the m-polar fuzzy soft expert set was created that could study soft expert sets in the m-polar fuzzy environment. Numerical examples have been used to examine the properties of this hybrid model. The operations of subset hood, complement, intersection, and union, as well as the OR and AND operators, have also been introduced, along with an investigation of their fundamental features. As an alternative, a wide range of reactor designs have been put forth, some of which have reached the prototype and commercial levels. The energy found within atoms can be independently captured and transformed into electricity at a nuclear power plant. This electricity is used by all of us. The energy held within atoms is transformed into electricity at nuclear power plants. To find the best nuclear power plants, this research aims to create models and algorithms. Since both alternatives and qualities are subject to change over time, linear programming is usually used to assess objects that are sometimes related to the attributes. Therefore, it is beneficial to investigate nuclear plants with m-bipolar fuzzy soft sets as attributes, and the significance of the links between them is both theoretical and practical.

### 1.1. Presented Manuscript's Contribution

The following statements sum up the main contributions of this manuscript:

1. This study introduces a novel skillful hybrid model, named the m-polar fuzzy soft set, and extends it to include pursuing the periodicity seen in real-world situations.
2. We have shown how the novel model works effectively as a tool for grading-based parameterized two-dimensional bipolar fuzzy information.
3. We also provided some fundamental procedures and outcomes for an m-polar fuzzy soft set environment. In addition, we developed three nimble algorithms for selecting the optimal answer to multi-attribute decision-making scenarios. The rigorous evaluation of a real-world application also supports the methods.
4. This innovative model has the parametric properties of a flexible soft set as well as the distinctive properties of an m-polar fuzzy soft set to handle the double-sided periodic ambiguous data. Table 1 lists the technical specifications as well as the key financial and safety features of each type of thermal reactor (adapted from [14]).

**Table 1.** The main economic and safety characteristics of each of the thermal reactor types. The primary economic and safety features of each type of thermal reactor.

| Comparison Approach | Fuel | Moderator | Heat Extraction | Outlet Temp. | Pressure |
|---|---|---|---|---|---|
| Gas Cooled Graphite-Moderated (Magnox) | Natural uranium metal (0.7% $U^{235}$) Magnesium alloy cladding | Graphite | Fuel heated carbon dioxide gas produces steam in a steam generator | 360 °C | 300 psia |
| Gas Cooled Graphite-Moderated (AGR) | Uranium dioxide enriched to 2.3% $U^{235}$ Stainless steel cladding | Graphite | Fuel heated carbon dioxide gas creates steam in a steam generator | 650 °C | 600 psia |
| Pressurized Water Reactor (PWR) | Uranium dioxide enriched to 3.2% $U^{235}$ Zirconium alloy cladding | Light Water | Pumping pressurized light water to a steam generator that generates steam in a different circuit | 317 °C | 2235 psia |
| Boiling Water Reactors (BWR), | Uranium dioxide enriched to 2.4% $U^{235}$ Zirconium alloy cladding | Light Water | Steam produced when pressurized light water boils in the pressure vessel directly runs a turbine | 286 °C | 1050 psia |
| Heavy Water Cooled and Moderated (CANDU)), | Unenriched uranium dioxide (0.7% $U^{235}$) Zirconium alloy cladding | Heavy water | A steam generator in a separate circuit generates steam from heavy water that is pumped under pressure over the fuel | 305 °C | 1285 psia |
| Reactor Boiling Light Water (RBMK)) | Uranium dioxide enriched to 1.8% $U^{235}$ | Graphite | Light water boiled with pressure, steam employed to power a turbine | 284 °C | 1000 psia |
| Comparison approach | Spent Fuel Reprocessing | Steam Cycle Efficiency | Main Economic and Safety Characteristics | | |
| Gas-Cooled Graphite-Moderated (Magnox) | Usually within a year, for practical purposes | 31% | Coolant's inability to change phases has a safety benefit. Additional potential for high availability comes from the ability to refuel while operating | | |
| Gas-Cooled Graphite-Moderated (AGR) | Can be kept underwater for tens of years, although storage in a dry environment may last longer | 42% | Higher operating temperatures and pressures provide the same operational and safety benefits as Magnox while lowering capital costs and increasing steam cycle efficiencies | | |
| Pressurized Water Reactor (PWR) | Long-term storage underwater allows for flexibility in waste management | 32% | Low manufacturing costs as a result of the design's suitability for production in factory-built subassemblies. Worldwide, a wealth of operational expertise has been accumulated. Refueling required after offloading | | |
| Boiling Water Reactors (BWR), | Regarding PWR | 32% | Similar PWR construction cost benefits improved by the lack of a heat exchanger are compensated by the need for some steam circuit and turbine shielding. Offload refueling is required | | |
| Heavy Water Cooled and Moderated (CANDU), | Regarding PWR | 30% | Good operational history, but infrastructure is needed to produce large volumes of heavy water at affordable prices | | |
| Reactor Boiling Light Water(RBMK) | Information is unavailable | 31% | Information unavailable, although they were present throughout the old USSR in large numbers. believed to be inherently less safe in the West | | |

### *1.2. Overview of the Manuscript Presented*

The rest of the article is organized as follows: Section 2 introduces some basis for developing the model of soft sets, and describes and generates the mathematical framework of the suggested m-bipolar fuzzy soft set models. Section 3 and a summary of the main thermal reactor types provide technicalities and the primary safety and economic properties of each of the thermal reactor types by providing the first algorithm for a decision-making problem. Based on these findings, we can reorganize the power plant according to equality as (1) Gas-Cooled Graphite-Moderated, (2) Boiling Water Reactors, (3) Reactor Boiling Light Water, (4) Gas-cooled graphite-moderated, (5) Pressurized Water Reactor (shown in Figure 1), (6) Heavy Water Cooled and Moderated (shown in Figure 2). The three algorithms are presented in Section 4 for use in making decisions. The ideal alternative for nuclear power plant appropriateness is centered on a 2-polar fuzzy soft set, and is the best option among nuclear power plants under the influence of key alternative factors. Section 5 introduces the findings, future scope, constraints, future goals, and key contributions.

**Figure 1.** Pressurized Water Reactor (PWR).

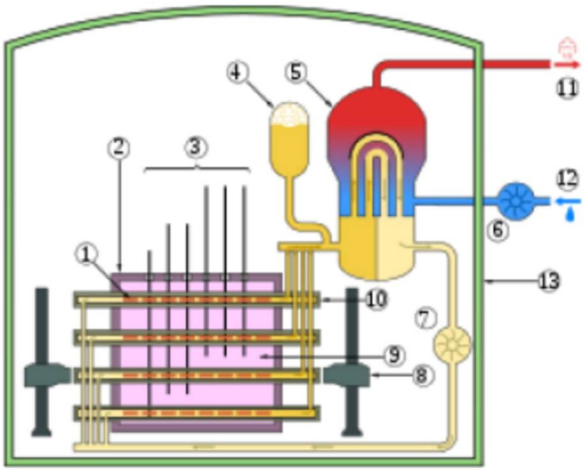


**Figure 2.** Heavy Water Cooled and Moderated (CANDU).

Some of the distinctive features of this research article include the following:

1. To identify the best nuclear power facilities, we used models and algorithms.
2. To deal with circumstances involving collective decision making where the qualities are interrelated, we presented a family of MAGDM and linear programming to assess objects where the linkages between the attributes are occasionally present.
3. A method for multi-attribute group decision making (MAGDM) was devised that is based on an m-bipolar fuzzy set.
4. The m-bipolar fuzzy set is given some formal definitions, examples, and qualities that are deduced.
5. A new MAGDM method for estimating nuclear power reactors is provided that is based on an m-bipolar fuzzy set.

## 2. Materials and Methods

Note that every single valued neutrosophic set [15] on a set X is actually a 3-polar fuzzy set on X and that every bipolar fuzzy set on a set X (which has been studied by several papers, e.g., [16–18]) can be looked at as a 2-polar fuzzy set on X. In this section, we present the core notion (i.e., m-polar fuzzy set [19,20], where m is an arbitrary cardinality.

**Definition 1.** *(Molodtsov [3]) Let $E$ be a non-empty finite set of attributes (parameters, characteristics, or properties) that the objects in $U$ possess and let $P(U)$ denote the family of all subsets of $U$. A pair $(F, A)$ is called a soft set over $U$, where $A \subseteq E$ and $F : A \to P(U)$ is a set-valued* *mapping. In other words, a soft set $(F, A)$ over $U$ is a parameterized family of subsets of $U$ where each parameter $e \in A$ is associated with a subset $F(e)$ of $U$. The set $F(i)$ contains the objects of $U$ having the property $i$ and is called the set of $i$-approximate elements in $(F, A)$.*

**Definition 2.** *([19,20]) Elements $\left([0,1]^m\right)^X$ (the set of all mappings from $X$ to $[0,1]^m$ with the pointwise order) are called m-polar fuzzy sets or $[0,1]^m$-sets (roughly, many-polar fuzzy sets or M-polar fuzzy sets) on X, such that m is an arbitrary cardinality. A subset $A = \{A_k\}_{k\in K} \subseteq \left([0,1]^m\right)^X$ (which can be looked at as a mapping $A : K \to \left([0,1]^m\right)^X$ satisfying $A(k) = A_k \ \forall\ k \in K$ is called an m-polar fuzzy soft set (roughly, many-polar fuzzy soft set or M-polar fuzzy soft set) on X.*

Here we make no difference between symbols $\left([0,1]^m\right)^X$ and $\left([0,1]^X\right)^m$; similarly, $\left[\left([0,1]^m\right)^X\right]^I$ and $\left[\left([0,1]^m\right)^I\right]^X$. Notice that $[0,1]^m = \{\{a_i\}_{i<m} | a_i \in [0,1] (\forall i \in m)$ is a Hutton algebra (i.e., a completely distributive complete lattice equipped with an order-reversing involution). Additionally, keep in mind that 2-polar fuzzy sets on X are also known as bipolar fuzzy sets on X and that 2-m-polar fuzzy sets on X will be referred to as possibility 2-m-polar fuzzy sets on X (because of their associations with the word ‘possibility’ in real-world issues).

When solving specific real-world issues, m-polar fuzzy sets and m-polar fuzzy soft sets are more helpful than fuzzy sets since there is less information lost when using the first two than when using the third [4–21].

**Example 1.** *Let us say a company needs to fill a position. The set of alternatives $X = \{a_1, a_2\}$ consists of two choices. The hiring committee, which consists of three expert groups, takes into consideration a set of parameters $I = \{i_1, i_2, i_3\}$ of parameters, where $i_1$ denotes the parameter ‘experience’, $i_2$ denotes the parameter ‘computer knowledge’, and $i_3$ denotes the parameter ‘age’. The data provided by the committee for decision-making use is the following 3-polar fuzzy soft set $A \in \left[\left([0,1]^3\right)^X\right]^I = \left[\left([0,1]^3\right)^X\right]^I = \left([0,1]^3\right)^{X\times I} = \left([0,1]^3\right)^{I\times X}$:*

$$A(a_1) = \left\{\frac{(0.6733, 0.4325)}{i_1}, \frac{(0.2455, 0.1985)}{i_2}, \frac{(0.8771, 0.4765)}{i_3}\right\}$$
$$A(a_2) = \left\{\frac{(0.9325, 0.6325)}{i_1}, \frac{(0.7342, 0.5675)}{i_2}, \frac{(0.0815, 0.0421)}{i_3}\right\}$$

where $A(a_1)(i_1) = (0.6733, 0.4325)$ means that the experience score $a_1$ given by group 1 (resp., by group 2) is 0.6733 (resp., 0.4325), meanings of $A(a_s)(i_t)$ can be explained similarly (s = 1,2, t = 1,2,3). To find the best choice from X, let us first compute the 3-polar fuzzy set $\underline{A} = \left([0,\ 1]^3\right)^X$ defined by

$$P_k{}^\circ \underline{A}(a) = 1 \wedge \sum_{i=1} P_k \circ\ A(a)(i)\ (\forall a \in X)$$

where $P_k : [0,1]^3 \to [0,1]$ is the kth projection $(k = 1, 2)$. $P_1{}^\circ \underline{A}(a_1) = 1 \wedge (0.6733 + 0.2455 + 0.8771) = 1$. Similarly, $P_2{}^\circ \underline{A}(a_1) = P_1{}^\circ \underline{A}(a_2) = P_2{}^\circ \underline{A}(a_2) = 1$. Therefore

$$\underline{A} = \left\{\frac{(1,1)}{a_1}, \frac{(1,1)}{a_2}\right\}$$

Next, we compute the score $S(a) = A(x)e^{\to}$ of $a \in X$ based on the given asymmetric weight vector $e^{\to} = (0.4,\ 0.6\ )^T$. $S(a_1) = (0.6733, 0.4325)\ e^{\to} = 1 \times 0.4 + 1 \times 0.6 = 1 = S(a_2)$

As $a_1 = a_2$ have the highest value, the best choice by experts should be $a_1$ *or* $a_2$.

**Definition 3.** *Let* $\{A_k\}_{k\in K} \in [([0,1]^m)^X]^{I_k}$ $(k \in K,\ k$ *is a set*$)$. *Define two m-polar fuzzy soft sets*

$$\wedge_{k\in K}\{A_k\},\quad \vee_{k\in K}\{A_k\} \in [([0,1]^m)^X]^I (I = \prod_{k\in K} I_k) \text{ by putting}$$

$$(\wedge_{k\in K}\{A_k\})(\{i_k\}_{k\in K}) = \wedge_{k\in K}\{A_k\}(i_k)(\forall\{i_k\}_{k\in K} \in I).$$

$$(\vee_{k\in K}\{A_k\})(\{i_k\}_{k\in K}) = \vee_{k\in K}\{A_k\}(i_k)(\forall\{i_k\}_{k\in K} \in I).$$

## 3. A Summary of Basic Thermal Reactor Types

The technical specifications as well as the key financial and safety features of each type of thermal reactor are outlined in the Table 2 and Figure 3.

**Table 2.** The specific technical information as well as the key financial and safety features of each type of thermal reactor.

| Comparison Approach | Fuel | Moderator | Heat Extension | Outlet Temp. | Pressure | Spent Fuel Reprocessing | Steam Cycle Efficiency | Degree of Economic and Safety Levels |
|---|---|---|---|---|---|---|---|---|
| Magnox | 0.70% | 0.8 | 0.9 | 360 | 300 | 0.5 | 31% | 0.9 |
| AGR | 2.30% | 0.8 | 0.9 | 650 | 600 | 0.8 | 42% | 0.95 |
| PWR | 3.20% | 0.8 | 0.8 | 317 | 2235 | 0.7 | 32% | 0.6 |
| BWR | 2.40% | 0.8 | 0.7 | 286 | 1050 | 0.7 | 32% | 0.65 |
| CANDU | 0.70% | 0.85 | 0.7 | 305 | 1285 | 0.7 | 30% | 0.5 |
| RBMK | 1.80% | 0.8 | 0.6 | 284 | 1000 | N/A | 31% | 0.51 |

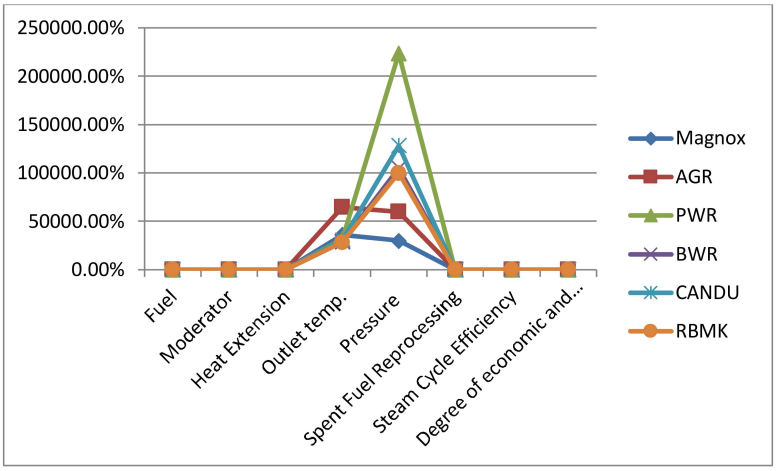


**Figure 3.** Demonstrates technical specifics as well as the key safety and financial features of each type of thermal reactor.

### *Algorithm for Decision Making for the Best Possible Option for Nuclear Power Plants by m-Polar Fuzzy Soft Set*

We decided to use nuclear power plants, including the Heavy Water Cooled and Moderated (CANDU), Gas Cooled Graphite-Moderated (AGR), Pressurized Water Reactor (PWR), Boiling Water Reactors (BWR), and Reactor Boiling Light Water (RBMK) models. Examining the best option for nuclear power plant suitability is our goal including the fuel, modulator, heat extension, outlet temperature, pressure, reprocessing of spent fuel, steam cycle efficiency, and amount of economic and safety levels. Using an m-polar fuzzy soft set, the primary role and best alternative for a nuclear power plant is stated. Therefore, let us say that we examine nuclear power plants, including the Gas Cooled, Graphite-Moderated (Magnox), Gas Cooled, Graphite-Moderated (AGR), Pressurized Water Reactor

(PWR), Boiling Water Reactors (BWR), Heavy Water Cooled and Moderated (CANDU), and Reactor Boiling Light Water (RBMK) models, with the following characteristics: steam cycle efficiency, spent fuel reprocessing, fuel, moderator, heat extension, outlet temperature, pressure, degree of economics, and safety. Our goal is to determine the best station under these circumstances. Therefore, let the set $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ denote, respectively, the nuclear power plants (Gas Cooled Graphite-Moderated (Magnox), Gas Cooled Graphite-Moderated (AGR), Pressurized Water Reactor (PWR), Boiling Water Reactors (BWR), Heavy Water Cooled and Moderated (CANDU), and Reactor Boiling Light Water (RBMK)) with the parameters $I = \{i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8\}$ where $i_1$ denotes fuel, $i_2$ denotes moderator, $i_3$ denotes heat extension, $i_4$ denotes outlet temp, $i_5$ denotes pressure, $i_6$ denotes spent fuel reprocessing, $i_7$ denotes steam cycle efficiency, $i_8$ denotes a degree of economic and safety levels. These parameters have an impact on the degree.

As demonstrated below, we now create an algorithm (see Algorithm 1) for a decision-making problem.

**Algorithm 1:** Using a 3-polar Fuzzy soft set.

**Step 1**. Provide $A \in \left[\left([0,2250]^3\right)^I\right]^X$.
**Step 2.** Calculate $p_k{}^\circ \underline{A} = 2250 \wedge \sum_{i \in I} p_k{}^\circ A(x)$ $(\forall\, x \in X)$, where $p_k : [0,2250]^3 \to [0,2250]$ is the $k$-the projection ($k$ =1,2,3).
**Step 3**. Calculate $\underline{A} \in \left([0,2250]^3\right)^X$.
**Step 4**. Put a reasonable weight vector $e^{\to} = (1.00, -10.00, -100.00)^T$ and calculate the score $S(x) = \underline{A}(x)e^{\to}$ for each $x \in X$.
**Step 5**. The optimum option for the suitability of nuclear power plants based on a 3-polar fuzzy soft set is stated by $S(\tilde{x})$ at its maximum value.

The important values of Fuel, Moderator, Heat Extension, Outlet temp, Pressure, Spent Fuel Reprocessing, Steam Cycle Efficiency, Degree of economic and safety are given in Table 3 and depicted in Figure 4.

**Table 3.** The important values of Fuel, Moderator, Heat Extension, Outlet temp, Pressure, Spent Fuel Reprocessing, Steam Cycle Efficiency, Degree of economic and safety.

| Fuel | Moderator | Heat Extension | Outlet temp. | Pressure | Reprocessing of Spent Fuel | Efficiency of the Steam Cycle | Economic and Safety Levels |
|---|---|---|---|---|---|---|---|
| 3.20% | 0.85 | 0.6 | 650 | 1285 | 0.7 | 32% | 0.65 |

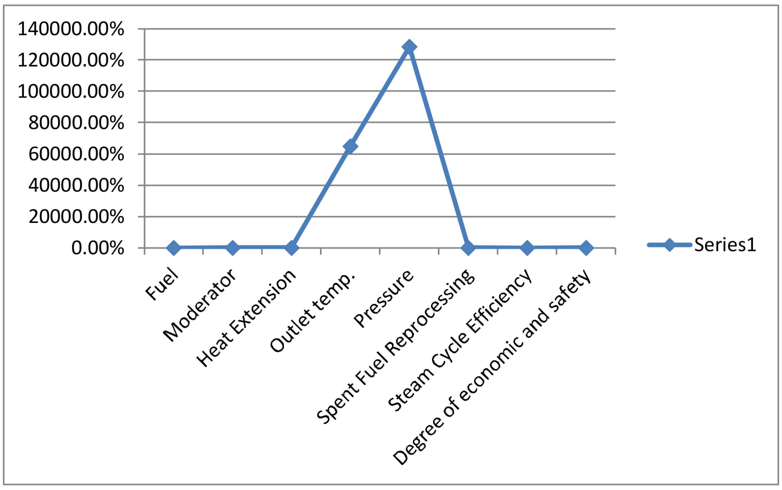


**Figure 4.** Important values of Fuel, Moderator, Heat Extension, Outlet temp, Pressure, Reprocessing of Spent Fuel, Efficiency of the Steam Cycle, Economic and safety levels.

Three experts in nuclear power plants gave the degree of equality of the alternative $I = \{i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8\}$ for nuclear power plants.

The data provided by the committee for decision-making use are the following 3-polar fuzzy soft sets $\mathcal{A} \in [([0,2250]^3)^I]^X = [([0,2250]^3)^X]^I = ([0,2250]^3)^{I\times X} = ([0,2250]^3)^{X\times I}$ defined by:

$$\mathcal{A}(x_1) = \left\{ \frac{(0.004,0.006,0.003)}{i_1}, \frac{(0.80,0.7,0.6)}{i_2}, \frac{(0.8,0.7,0.8)}{i_3}, \frac{(317,386,286)}{i_4}, \frac{(234,272,298)}{i_5}, \frac{(0.4733,0.4325,0.3325)}{i_6}, \frac{(0.30,0.29,0.27)}{i_7}, \frac{(0.7,0.8,0.6)}{i_8} \right\}$$

$$\mathcal{A}(x_2) = \left\{ \frac{(0.015,0.021,0.012)}{i_1}, \frac{(0.7,0.7,0.6)}{i_2}, \frac{(0.8,0.8,0.6)}{i_3}, \frac{(517,486,586)}{i_4}, \frac{(450,372,498)}{i_5}, \frac{(0.6733,0.43,0.7325)}{i_6}, \frac{(0.41,0.40,0.39)}{i_7}, \frac{(0.91,0.88,0.90)}{i_8} \right\}$$

$$\mathcal{A}(x_3) = \left\{ \frac{(0.031,0.032,0.030)}{i_1}, \frac{(0.6,0.7,0.77)}{i_2}, \frac{(0.8771,0.765,0.8654)}{i_3}, \frac{(317,286,286)}{i_4}, \frac{(1534,221,2000)}{i_5}, \frac{(0.7,0.5325,0.7)}{i_6}, \frac{(0.22,0.30,0.25)}{i_7}, \frac{(0.6,0.7,0.6)}{i_8} \right\}$$

$$\mathcal{A}(x_4) = \left\{ \frac{(0.022,0.021,0.023)}{i_1}, \frac{(0.6,0.5,0.6)}{i_2}, \frac{(0.771,0.765,0.654)}{i_3}, \frac{(282,277,222)}{i_4}, \frac{(1000,999,890)}{i_5}, \frac{(0.6,0.7,0.66)}{i_6}, \frac{(0.3,0.31,0.30)}{i_7}, \frac{(0.6,0.64,0.55)}{i_8} \right\}$$

$$\mathcal{A}(x_5) = \left\{ \frac{(0.002,0.005,0.005)}{i_1}, \frac{(0.8,0.7,0.4)}{i_2}, \frac{(0.771,0.765,0.654)}{i_3}, \frac{(300,226,0.300)}{i_4}, \frac{(1100,1200,1198)}{i_5}, \frac{(0.55,0.7,0.65)}{i_6}, \frac{(0.3,20,0.3)}{i_7}, \frac{(0.46,0.5,0.44)}{i_8} \right\}$$

$$\mathcal{A}(x_6) = \left\{ \frac{(0.007,0.005,0.006)}{i_1}, \frac{(0.74,0.66,0.616)}{i_2}, \frac{(0.6,0.5,0.6)}{i_3}, \frac{(277,235,264)}{i_4}, \frac{(876,976,890)}{i_5}, \frac{(0.87,0.50,0.97)}{i_6}, \frac{(0.3,28.0,0.3)}{i_7}, \frac{(0.46,0.5,0.44)}{i_8} \right\}$$

where $\mathcal{A}(x_1)(i_1) = (0.004, 0.006, 0.003)$ means that the fuel of nuclear power plants $x_1$ (Gas Cooled Graphite-Moderated (Magnox)), given by expert 1 (resp., by expert 2, by expert 3) is 0.004 (resp.,$0.006, 0.003$);

The decisions of experts in nuclear power plants $\mathcal{A}(x_s)(i_t)$ can be explained in the above same fashion $(s = 1,2,3,4,5,6;\ t = 1,2,3,4,5,6,7,8)$.

To find the best choice from $X$, let us first compute the 3-polar fuzzy set $\overline{\mathcal{A}} \in ([0,2250]^3)^X$, defined by $p_k \circ \overline{\mathcal{A}} = 2250 \wedge \sum_{i\in I} p_k \circ \mathcal{A}(x)$ $(\forall\, x \in X)$, where $p_k : [0,2250]^3 \to [0,2250]$ is the $k$-th projection ($k$ =1, 2, 3).

$$p_1(x_1) = 2250 \wedge (0.004 + 0.80 + 0.8 + 317 + 234 + 0.4733 + 0.30 + 0.7) = 2250 \wedge\ 553.2773$$

= 553.2773. Similarly,

Table 4 explains the final values of computing the 3-polar fuzzy set $\overline{\mathcal{A}} \in ([0,1050]^3)^X$ and are also shown in Figure 5.

**Table 4.** The 3-polar fuzzy set $\overline{\mathcal{A}} \in ([0,1050]^3)^X$.

| | $\mathbf{x}_1$ | $\mathbf{x}_2$ | $\mathbf{x}_3$ | $\mathbf{x}_4$ | $\mathbf{x}_5$ | $\mathbf{x}_6$ |
|---|---|---|---|---|---|---|
| $p_1$ | 553.2773 | 969.4683 | 1854.028 | 1285.091 | 1402.883 | 1105.907 |
| $p_2$ | 660.2693 | 861.231 | 510.3175 | 1278.936 | 1448.67 | 1241.165 |
| $\mathbf{p}_3$ | 586.6055 | 1389.235 | 2250 | 1114.787 | 1200.749 | 1156.932 |

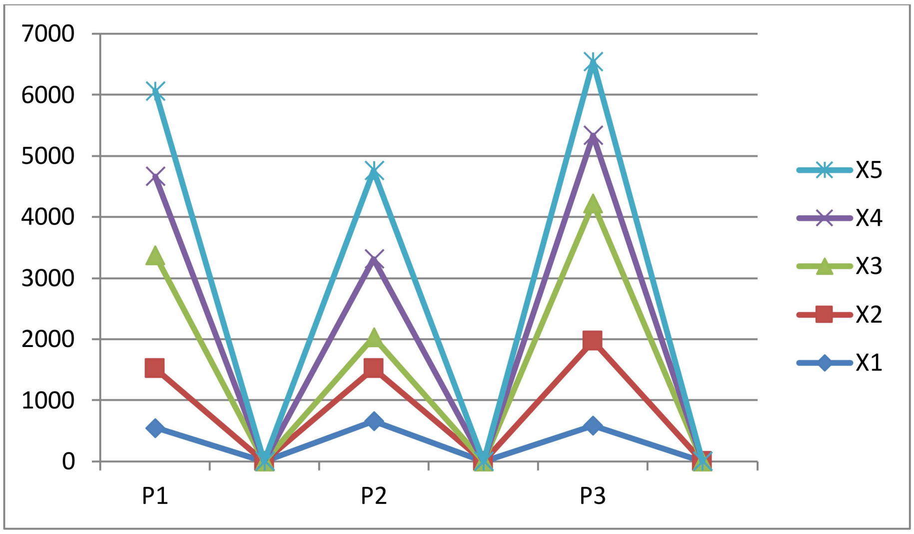


**Figure 5.** The 3-polar fuzzy set $\overline{\mathcal{A}} \in \left([0,1050]^3\right)^X$.

Therefore,

$$\overline{\mathcal{A}} = \left\{ \begin{array}{l} \frac{(553.2773,660.2693,586.6055)}{x_1}, \frac{(969.4683,861.231,1389.235)}{x_2}, \frac{(1854.028,510.3175,2250)}{x_3} \\ , \frac{(1285.091,1278.936,1114.787)}{x_4}, \frac{(1402.883,1448.67,1200.749)}{x_5}, \frac{(1105.907,1241.165,1156.932)}{x_6} \end{array} \right\}$$

Based on the weight vector $e^{\rightarrow} = (1.00, -10.00, -100)^T$ we compute the score $S(x) = \mathcal{A}(x)e^{\rightarrow}$ for each $x \in X$. Then:

$$S(x_1) = (553.2773, 660.2693, 586.6055)\ e^{\rightarrow} = 553.2773 \times 1.00 + 660.2693 \times (-10.0) + \\ 586.6055 \times (-100.0) = -64709.9657$$

By the model, we complete and obtain $S(x_2)$ = −146,566.3417, $S(x_3)$ = −228,249.147, $S(x_4)$ = −122,982.969, $S(x_5)$ = −133,158.717, and $S(x_6)$ = −126,998.943.

As $S(x_1)$ = −64,709.9657, the Gas Cooled Graphite-Moderated (Magnox) nuclear power plant under the values of fuel, moderator, heat extension, outlet temp, pressure, spent fuel reprocessing, steam cycle efficiency, and degree of economic and safety levels has the highest value.

The best choice by experts should be the Gas Cooled Graphite-Moderated (Magnox) model as the most suitable for nuclear power plants based on an m3-polar fuzzy soft set. Depending on these results, we can opt for rearrangement of the power plant according to equality as shown in Table 5 and Figure 6.

**Table 5.** The rearrangement of the nuclear power plants according to the equality.

| Gas Cooled Graphite-Moderated (Magnox), | Boiling Water Reactors (BWR), | Reactor Boiling Light Water (RBMK)) | Heavy Water Cooled and Moderated (CANDU) | Gas Cooled Graphite-Moderated (AGR), | Pressurized Water Reactor (PWR), |
|---|---|---|---|---|---|
| −64,709.9657 | −146,566.3417 | −228,249.147 | −122,982.969 | −133,158.717 | −126,998.943 |

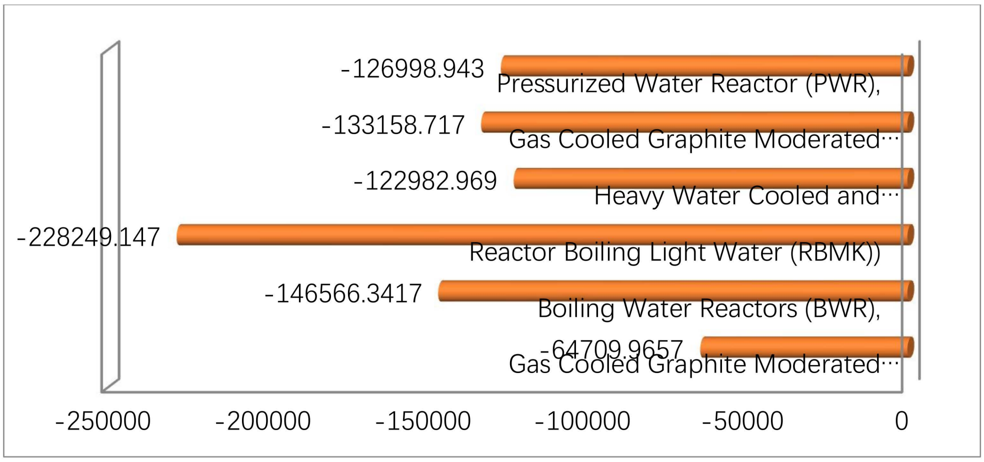


**Figure 6.** The rearrangement of the nuclear power plants according to the equality.

## 4. The Finest Nuclear Power Plant Selection Takes into Account the Key Alternative Factors and Has a Greater Impact Thanks to a 2-Polar Fuzzy Soft Set

Here, we select the relevant real-world possibilities for the nuclear power plants' fuel, outlet temperature, pressure, and steam cycle efficiency [14]. Table 6 describes significant alternative parameters and their greater impact (fuel, outlet temp, pressure, steam cycle efficiency).

**Table 6.** The crucial substitute criteria that have a greater impact (Fuel, Outlet temp, Pressure, Steam Cycle Efficiency).

| Comparison Approach | Fuel | Outlet Temp. | Pressure | Steam Cycle Efficiency |
|---|---|---|---|---|
| Magnox | Natural uranium metal (0.7% $U^{235}$) Magnesium alloy cladding | 360 °C | 300 psia | 31% |
| AGR | Uranium dioxide enriched to 2.3% $U^{235}$ Stainless steel cladding | 650 °C | 600 psia | 42% |
| PWR | Uranium dioxide enriched to 3.2% $U^{235}$ Zirconium alloy cladding | 317 °C | 2235 psia | 32% |
| BWR | Uranium dioxide enriched to 2.4% $U^{235}$ Zirconium alloy cladding | 286 °C | 1050 psia | 32% |
| CANDU | Unenriched uranium dioxide (0.7% $U^{235}$) Zirconium alloy cladding | 305 °C | 1285 psia | 30% |

**Table 6.** *Cont.*

| Comparison Approach | Fuel | Outlet Temp. | Pressure | Steam Cycle Efficiency |
|---|---|---|---|---|
| RBMK | Uranium dioxide enriched to 1.8% $U^{235}$ | 284 °C | 1000 psia | 31% |
| Magnox | (0.7% $U^{235}$) | 360 °C | 300 psia | 31% |
| AGR | 2.3% $U^{235}$ | 650 °C | 600 psia | 42% |
| PWR | 3.2% $U^{235}$ | 317 °C | 2235 psia | 32% |
| BWR | 2.4% $U^{235}$ | 286 °C | 1050 psia | 32% |
| CANDU | (0.7% $U^{235}$) | 305 °C | 1285 psia | 30% |
| RBMK | 1.8% $U^{235}$ | 284 °C | 1000 psia | 31% |

Assume $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ denotes, respectively, the nuclear power plants, the Gas Cooled Graphite-Moderated (Magnox), Gas Cooled Graphite-Moderated (AGR), Pressurized Water Reactor (PWR), Boiling Water Reactors (BWR), Heavy Water Cooled and Moderated (CANDU), and Reactor Boiling Light Water (RBMK) models having the parameters $I = \{i_1, i_2, i_3, i_4\}$ where $i_1$ denotes fuel, $i_2$ denotes outlet temp., $i_3$ denotes pressure, and $i_4$ denotes steam cycle efficiency. Then the factors have a steam cycle efficiency at 40%, outlet temperature pf 300, pressure of 2250, and uranium dioxide enriched to 2.5%) is the best nuclear power plant appropriateness based on a 2-polar fuzzy soft set.

We provide the following algorithm (Algorithm 2) for decision-making problems (and similar ones) because of the aforementioned problem:

**Algorithm 2:** Using 2-polar Fuzzy soft set.

**Step 1**. Input the possibility m-polar fuzzy soft set
$A \in [([0,2250]^2)^X \times ([0,2250]^2)^X]^I = [([0,2250]^4)^X]^I = [([0,2250]^4)^I]^X = ([0,2250]^4)^{X\times I}$
defined by two experts

**Step 2.** Compute the 2-polar fuzzy set $\underline{A} \in ([0,2250]^2)^{X\times I}$ defined by

$$\underline{A}(x,i) = 2250 \wedge \sum_{k=1}^{2} (p_k{\circ}p_1{\circ}A(x,i) \times p_k{\circ}p_2{\circ}A(x,i)) \quad \forall i \in I,\ \forall x \in X)$$

where $p_k : [0,2250]^2 \to [0,2250]$ is the $k$-the projection ($k$ = 1,2);

**Step 3**. Calculate the choice value of $A(x)(i)$ $(\forall(i,x) \in I \times X)$ by constructing the table
$m_i = \sum_{k=1}^{6}(x_k)(i), \quad x \in X, \quad (i = 1,2,3,4)$ and compute $r_i = \sum_i^4 m_i - m_j \quad (j = 1,2,3,4,5,6)$,

**Step 4**. The maximal value of the score $S(x) = r_i$. The nuclear power plant's maximum score and condition are based on a 2-polar fuzzy soft set.

Now calculate,

$$A \in [([0,2250]^2)^X \times ([0,2250]^2)^X]^I = [([0,2250]^4)^X]^I = [([0,2250]^4)^I]^X = ([0,2250]^4)^{X\times I}.$$

The following information is provided by the parameters as measured by two experts:

$$\mathcal{A}(x_1) = \left\{ \begin{array}{c} \frac{\langle(0.021,0.020),(0.021,0.019)\rangle}{i_1}, \frac{\langle(299,290),(260,250)\rangle}{i_2}, \\ \frac{\langle(2250,2240),(2240,2250)\rangle}{i_3}, \frac{\langle(0.4,0.3),(0.3,0.25)\rangle}{i_4} \end{array} \right\}$$

$$\mathcal{A}(x_2) = \left\{ \begin{array}{c} \frac{\langle(0.019,0.018),(0.021,0.020)\rangle}{i_1}, \frac{\langle(290,280),(270,250)\rangle}{i_2}, \\ \frac{\langle(2020,2210),(2240,2200)\rangle}{i_3}, \frac{\langle(0.4,0.3),(0.3,0.28)\rangle}{i_4} \end{array} \right\}$$

$$\mathcal{A}(x_3) = \left\{ \begin{array}{c} \frac{\langle(0.021,0.018),(0.021,0.020)\rangle}{i_1}, \frac{\langle(289,240),(200,199)\rangle}{i_2}, \\ \frac{\langle(2240,2230),(2240,2210)\rangle}{i_3}, \frac{\langle(0.38,0.3),(0.3,0.25)\rangle}{i_4} \end{array} \right\}$$

$$\mathcal{A}(x_4) = \left\{ \frac{\langle (0.020,0.017),(0.021,0.011) \rangle}{i_1}, \frac{\langle (269,220),(230,210) \rangle}{i_2}, \frac{\langle (2150,2140),(2240,2210) \rangle}{i_3}, \frac{\langle (0.4,0.3),(0.3,0.25) \rangle}{i_4} \right\}$$

$$\mathcal{A}(x_5) = \left\{ \frac{\langle (0.021,0.018),(0.021,0.017) \rangle}{i_1}, \frac{\langle (249,230),(260,250) \rangle}{i_2}, \frac{\langle (2130,2120),(2210,2200) \rangle}{i_3}, \frac{\langle (0.36,0.3),(0.3,0.25) \rangle}{i_4} \right\}$$

$$\mathcal{A}(x_6) = \left\{ \frac{\langle (2050,2240),(2240,2210) \rangle}{i_1}, \frac{\langle (286,220),(200,210) \rangle}{i_2}, \frac{\langle (2180,2140),(2140,2050) \rangle}{i_3}, \frac{\langle (0.36,0.3),(0.3,0.15) \rangle}{i_4} \right\}$$

$$\mathcal{A}(x_1) = \left\{ \frac{\langle (0.021,0.020),(0.021,0.019) \rangle}{i_1}, \frac{\langle (299,290),(260,250) \rangle}{i_2}, \frac{\langle (2250,2240),(2240,2250) \rangle}{i_3}, \frac{\langle (0.4,0.3),(0.3,0.25) \rangle}{i_4} \right\}$$

$$\mathcal{A}(x_2) = \left\{ \frac{\langle (0.019,0.018),(0.021,0.020) \rangle}{i_1}, \frac{\langle (290,280),(270,250) \rangle}{i_2}, \frac{\langle (2020,2210),(2240,2200) \rangle}{i_3}, \frac{\langle (0.4,0.3),(0.3,0.28) \rangle}{i_4} \right\}$$

$$\mathcal{A}(x_3) = \left\{ \frac{\langle (0.021,0.018),(0.021,0.020) \rangle}{i_1}, \frac{\langle (289,240),(200,199) \rangle}{i_2}, \frac{\langle (2240,2230),(2240,2210) \rangle}{i_3}, \frac{\langle (0.38,0.3),(0.3,0.25) \rangle}{i_4} \right\}$$


$$\mathcal{A}(x_4) = \left\{ \frac{\langle (0.020,0.017),(0.021,0.011) \rangle}{i_1}, \frac{\langle (269,220),(230,210) \rangle}{i_2}, \frac{\langle (2150,2140),(2240,2210) \rangle}{i_3}, \frac{\langle (0.4,0.3),(0.3,0.25) \rangle}{i_4} \right\}$$

$$\mathcal{A}(x_5) = \left\{ \frac{\langle (0.021,0.018),(0.021,0.017) \rangle}{i_1}, \frac{\langle (249,230),(260,250) \rangle}{i_2}, \frac{\langle (2130,2120),(2210,2200) \rangle}{i_3}, \frac{\langle (0.36,0.3),(0.3,0.25) \rangle}{i_4} \right\}$$

$$\mathcal{A}(x_6) = \left\{ \frac{\langle (2050,2240),(2240,2210) \rangle}{i_1}, \frac{\langle (286,220),(200,210) \rangle}{i_2}, \frac{\langle (2180,2140),(2140,2050) \rangle}{i_3}, \frac{\langle (0.36,0.3),(0.3,0.15) \rangle}{i_4} \right\}$$


where, $A(x_1)(i_1) = \langle(((0.021, 0.020), (0.021, 0.021))\rangle$ signifies that the energy $x_1$ of the parameter $i_1$ (fuel) in the Gas Cooled Graphite-Moderated (Magnox) nuclear reactor increases or decreases by 0.021 or 0.020 in aspects, and by the second measure, increases or decreases by 0.021 or 0.019, respectively;

The meaning of $A(x_s)(i_t)$ can be explained similarly ($s = 1, 2, 3.4, 5, 6, 7, 8, 9$; $t = 1, 2, 3, 4$). To find the best option from $X$, let us first calculate the 2-polar fuzzy set $\underline{A} \in ([-1050, 1050]^2)^{X \times I}$ given by

$$\underline{A}(x,i) = 2250 \wedge \sum_{k=1}^{2} (p_k{}^\circ p_1{}^\circ A(x,i) \times p_k{}^\circ p_2{}^\circ A(x,i)) \quad \forall i \in I,\ \forall x \in X)$$

where $p_k : [0, 2250]^2 \to [0, 2250]$ is the $k$-the projection ($k$ =1,2);

$$\underline{A}(x_1)(i_1) = [(0.021 \times 0.021) + (0.020 \times 0.019)] = 0.000819.$$

Similarly

$$\underline{A}(x_1) = \left\{ \frac{0.000821}{i_1}, \frac{2250}{i_2}, \frac{2250}{i_3}, \frac{0.195}{i_4} \right\} \underline{A}(x_2) = \left\{ \frac{0.000759}{i_1}, \frac{2250}{i_2}, \frac{2250}{i_3}, \frac{0.204}{i_4} \right\}$$

$$\underline{A}(x_3) = \left\{ \frac{0.000801}{i_1}, \frac{2250}{i_2}, \frac{2250}{i_3}, \frac{0.189}{i_4} \right\} \underline{A}(x_4) = \left\{ \frac{0.000607}{i_1}, \frac{2250}{i_2}, \frac{2250}{i_3}, \frac{0.195}{i_4} \right\}$$

$$\underline{A}(x_5) = \left\{ \frac{0.000747}{i_1}, \frac{2250}{i_2}, \frac{2250}{i_3}, \frac{0.183}{i_4} \right\} \underline{A}(x_6) = \left\{ \frac{2250}{i_1}, \frac{2250}{i_2}, \frac{2250}{i_3}, \frac{0.153}{i_4} \right\}$$

Now, we compute $m_i = \sum_{k=1}^{6} (x_k)(i),\ x \in X,\ (i = 1,2,3,4)$ as shown in Table 7 and in Figure 7.

**Table 7.** Computation of $m_i = \sum_{k=1}^{9} (x_k)(i),\ x \in X,\ (i = 1,2,3,4)$.

| $X$ | $i_1$ | $i_2$ | $i_3$ | $i_4$ | $m_i$ |
|---|---|---|---|---|---|
| $x_1$ | 0.000821 | 2250 | 2250 | 0.195 | 4500.195821 |
| $x_2$ | 0.000759 | 2250 | 2250 | 0.204 | 4500.204 |
| $x_3$ | 0.000801 | 2250 | 2250 | 0.189 | 4500.189801 |
| $x_4$ | 0.000607 | 2250 | 2250 | 0.195 | 4500.195607 |
| $x_5$ | 0.000747 | 2250 | 2250 | 0.183 | 4500.183747 |
| $x_6$ | 2250 | 2250 | 2250 | 0.153 | 6750.153 |

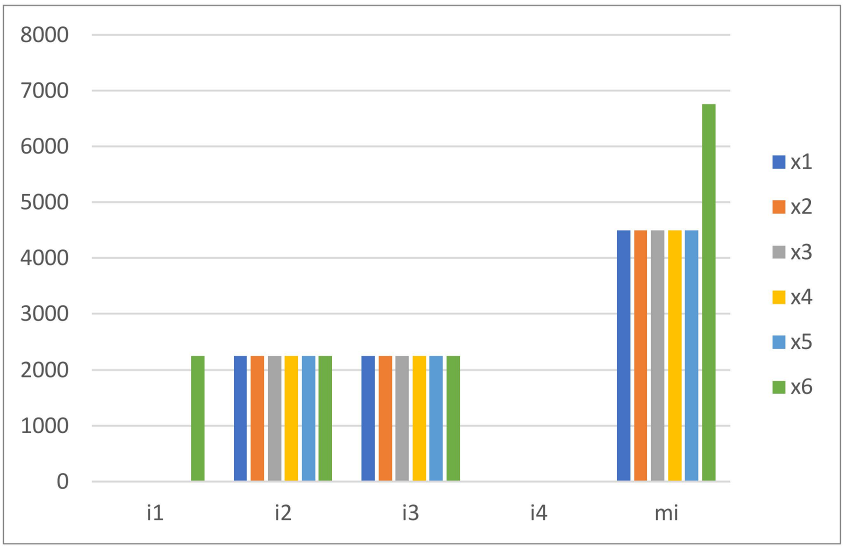


**Figure 7.** Shows computation of $m_i = \sum_{k=1}^{9} (x_k)(i),\ x \in X,\ (i = 1,2,3,4)$.

Now, compute $r_i = \sum_i^6 m_i - m_j$ $(j = 1,2,3,4,5,6)$, then

$$r_1 = (m_1 - m_1) + (m_1 - m_2) + (m_1 - m_3) + (m_1 - m_4) + (m_1 - m_5) + (m_1 - m_6) = (4500.195821 - 4500.195821) + (4500.195821 - 4500.204) + (4500.195821 - 4500.189801) + (4500.195821 - 4500.195607) + (4500.195821 - 4500.183747) + (4500.195821 - 6750.153) = -2249.94705,$$

Similarly, $r_2 = -2249.9$, $r_3 = -2249.98$, $r_4 = -2249.95$, $r_5 = -2250.02$, and $r_6 = 11{,}249.8$.

Since the score $S(x) = r_i$, then the maximum score is $r_6 = 11{,}249.8$ and the nuclear power plant selected based on a 2-polar fuzzy soft set is $r_6$ (Reactor Boiling Light Water (RBMK)).

*Suitability of Nuclear Power Plants Based on Two Operations ($\wedge$ and $\vee$) of 2-Polar Fuzzy Soft Sets*

In this section, we explore the problem by using two operations ( $\wedge$ *and* $\vee$) of 2-polar fuzzy soft sets. we give *nuclear power plants* (Gas Cooled Graphite-Moderated (Magnox), Gas Cooled Graphite Moderated (AGR), Pressurized Water Reactor (PWR), Boiling Water Reactors (BWR)). The set $X = \{x_1, x_2, x_3, x_4\}$ has the parameters $I = \{i_1, i_2, i_3, i_4\}$ where $i_1$ stands for fuel, $i_2$ stands for outlet temp., $i_3$ stands for pressure, and $i_4$ stands for steam cycle; these parameters are important for the degree. (Uranium dioxide enriched to 2.5%, outlet temp. 300, pressure 2250 and ($Steam\ Cycle\ Efficiency$ 40%)) The data for the best alternative for the suitability of nuclear power plants is based on a 2-polar fuzzy soft set, taking into account each individual's needs.

In response to the aforementioned issue, we provide the following algorithm (Algorithm 3) for problem solving:

**Algorithm 3:** Using 3-polar Fuzzy soft set.

**Step 1**. State
$A, B \in [(([0,2250]^2)^X \times ([0,2250]^2)^X]^I = [([0,2250]^4)^X]^I = [([0,2250]^4)^I]^X = ([0,2250]^4)^{X\times I}$
**Step 2**. Compute $C = A \wedge B \in [(([0,2250]^2)^X \times ([0,2250]^2)^X]^I$
**Step 3**. Compute the 3-polar Fuzzy soft set $\hat{C} \in ([0,2250]^{2\times 3})^X$, defined by

$$\hat{C}(x)(i,j) = 2250 \wedge \sum_{k=1}^{3}(p_k{}^\circ p_1{}^\circ C(x)(i,j) \times p_k{}^\circ p_2{}^\circ C(x)(i,j)\ ) \quad \forall (i,j) \in I^2,\ \forall x \in X)$$

where $p_k : [0,2250]^2 \to [0,2250]$ is the $k$-the projection $(k = 1,2,3)$;
**Step 4**. $C_M : X \to R$, by $C_M(x) = \sum_{(i,j)\in I^2} \beta(x)(i,j)$, where

$$\beta(x)(i,j) = \{C(x)(i,j),\ C(x)(i,j)\} = maxC(x)(s,t)\ : (s,t) \in I^2\ \},\ 0\ Otherwise.$$

**Step 5**. The maximal value of $C_M$ to state nuclear power plants based on a 2-polar fuzzy soft set X Based on a 3-polar Fuzzy soft set.

Now calculate,

$$A \in [(([0,2250]^2)^X \times ([0,2250]^2)^X]^I = [([0,2250]^4)^X]^I = [([0,2250]^4)^I]^X = ([0,2250]^4)^{X\times I}.$$

We take the parameter's negative sign when it is defined by three measurements to state the parameter's measure.

$$\mathcal{A}(\mathrm{i}_1) = \left\{ \begin{array}{l} \frac{\langle(0.021,0.020),(0.021,0.019),(0.018,0.017)\rangle}{x_1}, \frac{\langle(299,290),(260,250),(270,280)\rangle}{x_2}, \\ \frac{\langle(2250,2240),(2240,2250),(2240,2230)\rangle}{x_3}, \frac{\langle(0.4,0.3),(0.3,0.25),(0.4,0.3)\rangle}{x_4} \end{array} \right\}$$

$$\mathcal{A}(\mathrm{i}_2) = \left\{ \begin{array}{l} \frac{\langle(0.019,0.018),(0.021,0.020),(0.016,0.016)\rangle}{x_1}, \frac{\langle(290,280),(270,250),(250,240)\rangle}{x_2}, \\ \frac{\langle(2020,2210),(2240,2200),(2220,2210)\rangle}{x_3}, \frac{\langle(0.4,0.3),(0.3,0.28),(0.5,0.4)\rangle}{x_4} \end{array} \right\}$$

$$\mathcal{A}(\mathrm{i}_3) = \left\{ \begin{array}{l} \frac{\langle(0.021,0.018),(0.021,0.020),(0.018,0.016)\rangle}{x_1}, \frac{\langle(289,240),(200,199),(279,230)\rangle}{x_2}, \\ \frac{\langle(2240,2230),(2240,2210).(2230,2240)\rangle}{x_3}, \frac{\langle(0.38,0.3),(0.3,0.25),(0.36,0.16)\rangle}{x_4} \end{array} \right\}$$

$$\mathcal{A}(\mathrm{i}_4) = \left\{ \begin{array}{l} \frac{\langle(0.020,0.017),(0.021,0.011),(0.019,0.012)\rangle}{x_1}, \frac{\langle(269,220),(230,210),(249,240)\rangle}{x_2}, \\ \frac{\langle(2150,2140),(2240,2210),(2220,2240)\rangle}{x_3}, \frac{\langle(0.4,0.3),(0.3,0.28),(0.32,0.33)\rangle}{x_4} \end{array} \right\}$$

where $A(i_1)(x_1) = \langle(0.021, 0.020), (0.021, 0.019), (0.018, 0.017)\rangle$ means that parameter $i_1$ (fuel) of the (Gas Cooled Graphite-Moderated (Magnox)) is $x_1$. According to the first measure, growth has increased by 0.021 percent and decreased by 0.020 percent.

Using the second measure, the rise is equal to 0.021 and the reduction is equal to 0.019.

The third measurement uses the values 0.018 for the rise and 0.017 for reduction. The meaning of $A(i_s)(x_t)$ can be explained similarly ($s = 1,2,3.4\ ;\ t = 1,2,3,4$). Similarly, A subset $B = \{B_i\}_i : I \to ([0,2250]^{2\times 3})^X$ is also called a 3-polar fuzzy soft set on $X$, defined by $B(i) = B_i\ \forall i \in I$, and the data of nuclear power plants based on a 3-polar fuzzy soft set are given by another three measures $B \in [([0,2250]^{2\times 3})^X]^I = ([0,\ 2250]^{2\times 3})^{X\times I}$ defined by

$$\mathcal{B}(\mathrm{i}_1) = \left\{ \begin{array}{l} \frac{\langle(0.019,0.020),(0.017,0.019),(0.016,0.019)\rangle}{x_1}, \frac{\langle(289,291),(250,270),(260,270)\rangle}{x_2}, \\ \frac{\langle(2230,2250),(2220,2240),(2250,2240)\rangle}{x_3}, \frac{\langle(0.4,0.4),(0.3,0.4),(0.41,0.29)\rangle}{x_4} \end{array} \right\}$$

$$\mathcal{B}(\mathrm{i}_2) = \left\{ \begin{array}{l} \frac{\langle(0.016,0.015),(0.016,0.020),(0.018,0.017)\rangle}{x_1}, \frac{\langle(299,288),(260,255),(254,260)\rangle}{x_2}, \\ \frac{\langle(2019,2210),(2250,2210),(2210,2230)\rangle}{x_3}, \frac{\langle(0.3,0.5),(0.4,0.27),(0.7,0.31)\rangle}{x_4} \end{array} \right\}$$

$$\mathcal{B}(\mathrm{i}_3) = \left\{ \begin{array}{c} \frac{\langle(0.018,0.016),(0.020,0.021),(0.017,0.015)\rangle}{x_1}, \frac{\langle(279,250),(201,198),(289,235)\rangle}{x_2}, \\ \frac{\langle(2250,2240),(2230,2220).(2235,2241)\rangle}{x_3}, \frac{\langle(0.36,0.4),(0.33,0.24),(0.35,0.15)\rangle}{x_4} \end{array} \right\}$$

$$\mathcal{B}(\mathrm{i}_4) = \left\{ \begin{array}{c} \frac{\langle(0.021,0.016),(0.018,0.013),(0.018,0.013)\rangle}{x_1}, \frac{\langle(259,210),(220,220),(239,230)\rangle}{x_2}, \\ \frac{\langle(2152,2130),(2230,2220),(2230,2220)\rangle}{x_3}, \frac{\langle(0.3,0.31),(0.2,0.20),(0.3,0.36)\rangle}{x_4} \end{array} \right\}$$

Now we need to find the best choice from $X$ based on $\mathbb{C} = \mathcal{A} \wedge \mathcal{B}$. Thus compute $\mathbb{C}$.

$$\mathbb{C}(\mathrm{i}_1, \mathrm{i}_1) = \left\{ \begin{array}{c} \frac{\langle(0.019,0.020),(0.017,0.019),(0.016,0.017)\rangle}{x_1}, \frac{\langle(289,290),(250,250),(260,270)\rangle}{x_2}, \\ \frac{\langle(2230,2240),(2220,2240),(2240,2230)\rangle}{x_3}, \frac{\langle(0.4,0.3),(0.3,0.25),(0.4,0.29)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_1, \mathrm{i}_2) = \left\{ \begin{array}{c} \frac{\langle(0.016,0.015),(0.016,0.019),(0.018,0.017)\rangle}{x_1}, \frac{\langle(299,288),(260,250),(254,260)\rangle}{x_2}, \\ \frac{\langle(2019,2210),(2240,2210),(2210,2230)\rangle}{x_3}, \frac{\langle(0.3,0.3),(0.3,0.25),(0.4,0.3)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_1, \mathrm{i}_3) = \left\{ \begin{array}{c} \frac{\langle(0.018,0.016),(0.020,0.019),(0.017,0.015)\rangle}{x_1}, \frac{\langle(279,250),(201,198),(270,235)\rangle}{x_2}, \\ \frac{\langle(2250,2240),(2230,2220).(2235,2230)\rangle}{x_3}, \frac{\langle(0.36,0.3),(0.3,0.24),(0.35,0.15)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_1, \mathrm{i}_4) = \left\{ \begin{array}{c} \frac{\langle(0.021,0.016),(0.018,0.013),(0.018,0.013)\rangle}{x_1}, \frac{\langle(259,210),(220,220),(239,230)\rangle}{x_2}, \\ \frac{\langle(2152,2130),(2230,2220),(2230,2220)\rangle}{x_3}, \frac{\langle(0.3,0.3),(0.29,0.20),(0.3,0.3)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_2, \mathrm{i}_1) = \left\{ \begin{array}{c} \frac{\langle(0.019,0.018),(0.017,0.019),(0.016,0.016)\rangle}{x_1}, \frac{\langle(289,280),(250,250),(250,240)\rangle}{x_2}, \\ \frac{\langle(2020,2210),(2220,2200),(2220,2210)\rangle}{x_3}, \frac{\langle(0.4,0.3),(0.3,0.28),(0.41,0.29)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_2, \mathrm{i}_2) = \left\{ \begin{array}{c} \frac{\langle(0.016,0.015),(0.061,0.020),(0.016,0.016)\rangle}{x_1}, \frac{\langle(290,280),(260,250),(250,240)\rangle}{x_2}, \\ \frac{\langle(2019,2210),(2240,2200),(2210,2210)\rangle}{x_3}, \frac{\langle(0.3,0.3),(0.3,0.27),(0.5,0.31)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_2, \mathrm{i}_3) = \left\{ \begin{array}{c} \frac{\langle(0.018,0.016),(0.020,0.020),(0.016,0.015)\rangle}{x_1}, \frac{\langle(279,250),(201,198),(250,240)\rangle}{x_2}, \\ \frac{\langle(2220,2210),(2230,2220).(2220,2210)\rangle}{x_3}, \frac{\langle(0.36,0.3),(0.3,0.24),(0.35,0.15)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_2, \mathrm{i}_4) = \left\{ \begin{array}{c} \frac{\langle(0.019,0.016),(0.018,0.013),(0.016,0.013)\rangle}{x_1}, \frac{\langle(259,210),(220,220),(239,230)\rangle}{x_2}, \\ \frac{\langle(2120,2110),(2230,2200),(2220,2210)\rangle}{x_3}, \frac{\langle(0.3,0.3),(0.29,0.20),(0.3,0.36)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_3, \mathrm{i}_1) = \left\{ \begin{array}{c} \frac{\langle(0.019,0.018),(0.017,0.019),(0.061,0.016)\rangle}{x_1}, \frac{\langle(289,240),(200,199),(260,230)\rangle}{x_2}, \\ \frac{\langle(2230,2230),(2220,2210).(2230,2240)\rangle}{x_3}, \frac{\langle(0.38,0.3),(0.3,0.25),(0.36,0.16)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_3, \mathrm{i}_2) = \left\{ \begin{array}{c} \frac{\langle(0.016,0.015),(0.016,0.020),(0.018,0.016)\rangle}{x_1}, \frac{\langle(289,240),(200,199),(254,230)\rangle}{x_2}, \\ \frac{\langle(2019,2210),(2240,2210),(2210,2230)\rangle}{x_3}, \frac{\langle(0.3,0.3),(0.3,0.25),(0.3,0.16)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_3, \mathrm{i}_3) = \left\{ \begin{array}{c} \frac{\langle(0.018,0.016),(0.020,0.020),(0.017,0.015)\rangle}{x_1}, \frac{\langle(279,240),(200,198),(279,230)\rangle}{x_2}, \\ \frac{\langle(2240,2230),(2230,2210).(2230,2240)\rangle}{x_3}, \frac{\langle(0.36,0.4),(0.3,0.24),(0.35,0.15)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_3, \mathrm{i}_4) = \left\{ \begin{array}{c} \frac{\langle(0.021,0.016),(0.018,0.013),(0.018,0.013)\rangle}{x_1}, \frac{\langle(259,210),(200,199),(239,230)\rangle}{x_2}, \\ \frac{\langle(2240,2130),(2230,2210).(2230,2220)\rangle}{x_3}, \frac{\langle(0.3,0.3),(0.29,0.20),(0.3,0.16)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_4, \mathrm{i}_1) = \left\{ \begin{array}{c} \frac{\langle(0.019,0.017),(0.017,0.011),(0.016,0.012)\rangle}{x_1}, \frac{\langle(269,220),(230,210),(249,240)\rangle}{x_2}, \\ \frac{\langle(2150,2140),(2220,2210),(2220,2240)\rangle}{x_3}, \frac{\langle(0.4,0.3),(0.3,0.25),(0.32,0.29)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_4, \mathrm{i}_2) = \left\{ \begin{array}{c} \frac{\langle(0.016,0.015),(0.016,0.011),(0.018,0.012)\rangle}{x_1}, \frac{\langle(269,220),(230,210),(249,240)\rangle}{x_2}, \\ \frac{\langle(2019,2140),(2240,2210),(2210,2230)\rangle}{x_3}, \frac{\langle(0.3,0.3),(0.3,0.25),(0.32,0.31)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_4, \mathrm{i}_3) = \left\{ \begin{array}{c} \frac{\langle(0.018,0.016),(0.020,0.011),(0.017,0.015)\rangle}{x_1}, \frac{\langle(269,220),(201,198),(249,235)\rangle}{x_2}, \\ \frac{\langle(2250,2240),(2230,2210).(2220,2240)\rangle}{x_3}, \frac{\langle(0.36,0.3),(0.3,0.24),(0.32,0.15)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_4, \mathrm{i}_4) = \left\{ \frac{\langle(0.020,0.016),(0.018,0.011),(0.018,0.012)\rangle}{x_1}, \frac{\langle(259,210),(220,210),(239,230)\rangle}{x_2}, \frac{\langle(2150,2130),(2230,2210),(2220,2220)\rangle}{x_3}, \frac{\langle(0.3,0.3),(0.29,0.20),(0.3,0.33)\rangle}{x_4} \right\}$$

Secondly, compute the 3-polar fuzzy soft set $\hat{\mathbb{C}} \in \left([[0,2250]]^{2\times3}\right)^X$, defined by

$$\hat{\mathbb{C}}(x)(i,j) = 2250 \wedge \sum_{k=1}^{3} (p_k{}^\circ p_1{}^\circ \mathbb{C}(x)(i,j) \times p_k{}^\circ p_2{}^\circ \mathbb{C}(x)(i,j)) \ \ \forall (i,j) \in I^2, \ \forall x \in X)$$

where $p_k : [[0,2250]]^2 \to [[0,2250]]$ is the $k$-th projection ($k$ =1, 2, 3);

$$\hat{\mathbb{C}}(x_1)(i_1,j_1) = (2250) \wedge [(0.019 \times 0.017 \times 0.016) + (0.020 \times 0.019 \times 0.017)] = (2250) \wedge (0.000011628)$$
$$= 0.000011628;$$

Similarly, in Table 8, and Figure 8, compute $\hat{\mathbb{C}}(x)(i,j) \in ([0,2250]^{2\times3})^X$.

**Table 8.** Compute $\hat{\mathbb{C}}(x)(i,j) \in ([0,2250]^{2\times3})^X$.

| $\hat{\mathbb{C}}$. | $\hat{\mathbb{C}}(x_1)$ | $\hat{\mathbb{C}}(x_2)$ | $\hat{\mathbb{C}}(x_3)$ | $\hat{\mathbb{C}}(x_4)$ |
|---|---|---|---|---|
| $(i_1,j_1)$ | 0.000011628 | 2250 | 2250 | 0.06975 |
| $(i_1,j_2)$ | 0.000009453 | 2250 | 2250 | 00.0585 |
| $(i_1,j_3)$ | 0.000010680 | 2250 | 2250 | 00.0486 |
| $(i_1,j_4)$ | 0.000009508 | 2250 | 2250 | 00.0441 |
| $(i_2,j_1)$ | 0.000010640 | 2250 | 2250 | 0.07356 |
| $(i_2,j_2)$ | 0.000008896 | 2250 | 2250 | 000.7011 |
| $(i_2,j_3)$ | 0.000010560 | 2250 | 2250 | 0.037908 |
| $(i_2,j_4)$ | 0.000010880 | 2250 | 2250 | 0.0477 |
| $(i_3,j_1)$ | 00.00001064 | 2250 | 2250 | 0.05304 |
| $(i_3,j_2)$ | 0.000009408 | 2250 | 2250 | 0.039 |
| $(i_3,j_3)$ | 00.00001096 | 2250 | 2250 | 0.0522 |
| $(i_3,j_4)$ | 0.000009508 | 2250 | 2250 | 0.0357 |
| $(i_4,j_1)$ | 0.000007412 | 2250 | 2250 | 0.06015 |
| $(i_4,j_2)$ | 0.000004796 | 2250 | 2250 | 0.05205 |
| $(i_4,j_3)$ | 0.00000876 | 2250 | 2250 | 0.08316 |
| $(i_4,j_4)$ | 0.000088592 | 2250 | 2250 | 0.0459 |

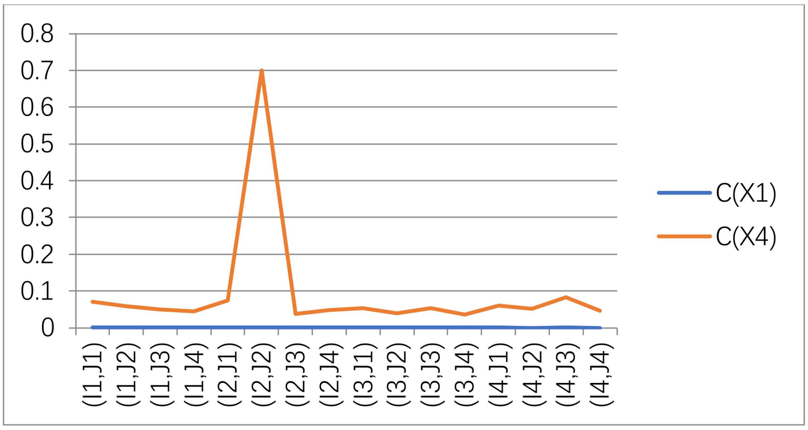


**Figure 8.** Shows computation of $\hat{\mathbb{C}}(x)(i,j) \in ([0,60]^{2\times3})^X$.

Therefore,

$$\hat{\mathbb{C}}(x_1) = \left\{ \begin{array}{c} \frac{0.000011628}{(i_1,j_1)}, \frac{0.000009453}{(i_1,j_2)}, \frac{00.00001068}{(i_1,j_3)}, \frac{0.000009508}{(i_1,j_4)}, \\ \frac{0.000010640}{(i_2,j_1)}, \frac{0.000008896}{(i_2,j_2)}, \frac{0.000010560}{(i_2,j_3)}, \frac{0.000010880}{(i_2,j_4)}, \\ \frac{00.00001064}{(i_3,j_1)}, \frac{0.000009408}{(i_3,j_2)}, \frac{00.00001096}{(i_3,j_3)}, \frac{0.000009508}{(i_3,j_4)}, \\ \frac{0.000007412}{(i_4,j_1)}, \frac{0.000004796}{(i_4,j_2)}, \frac{0.00000876}{(i_4,j_3)}, \frac{0.000088592}{(i_4,j_4)} \end{array} \right\}$$

$$\hat{\mathbb{C}}(x_2) = \left\{ \begin{array}{c} \frac{2250}{(i_1,j_1)}, \frac{2250}{(i_1,j_2)}, \frac{2250}{(i_1,j_3)}, \frac{2250}{(i_1,j_4)}, \\ \frac{2250}{(i_2,j_1)}, \frac{2250}{(i_2,j_2)}, \frac{2250}{(i_2,j_3)}, \frac{2250}{(i_2,j_4)}, \\ \frac{2250}{(i_3,j_1)}, \frac{2250}{(i_3,j_2)}, \frac{2250}{(i_3,j_3)}, \frac{2250}{(i_3,j_4)}, \\ \frac{2250}{(i_4,j_1)}, \frac{2250}{(i_4,j_2)}, \frac{2250}{(i_4,j_3)}, \frac{2250}{(i_4,j_4)} \end{array} \right\}$$

$$\hat{\mathbb{C}}(x_3) = \left\{ \begin{array}{c} \frac{2250}{(i_1,j_1)}, \frac{2250}{(i_1,j_2)}, \frac{2250}{(i_1,j_3)}, \frac{2250}{(i_1,j_4)}, \\ \frac{2250}{(i_2,j_1)}, \frac{2250}{(i_2,j_2)}, \frac{2250}{(i_2,j_3)}, \frac{2250}{(i_2,j_4)}, \\ \frac{2250}{(i_3,j_1)}, \frac{2250}{(i_3,j_2)}, \frac{2250}{(i_3,j_3)}, \frac{2250}{(i_3,j_4)}, \\ \frac{2250}{(i_4,j_1)}, \frac{2250}{(i_4,j_2)}, \frac{2250}{(i_4,j_3)}, \frac{2250}{(i_4,j_4)} \end{array} \right\}$$

$$\hat{\mathbb{C}}(x_4) = \left\{ \begin{array}{c} \frac{0.06975}{(i_1,j_1)}, \frac{00.0585}{(i_1,j_2)}, \frac{00.0486}{(i_1,j_3)}, \frac{00.0441}{(i_1,j_4)}, \\ \frac{0.07356}{(i_2,j_1)}, \frac{000.7011}{(i_2,j_2)}, \frac{0.037908}{(i_2,j_3)}, \frac{0.0477}{(i_2,j_4)}, \\ \frac{0.05304}{(i_3,j_1)}, \frac{0.039}{(i_3,j_2)}, \frac{0.0522}{(i_3,j_3)}, \frac{0.0357}{(i_3,j_4)}, \\ \frac{0.06015}{(i_4,j_1)}, \frac{0.05205}{(i_4,j_2)}, \frac{0.08316}{(i_4,j_3)}, \frac{0.0459}{(i_4,j_4)} \end{array} \right\}$$

Now we make a decision in two ways:

(1) First way:

Define a mapping $\mathbb{C}_M : X \to R,$ by $\mathbb{C}_M(x) = \sum_{(i,j)\in I^2} \beta(x)(i,j)$, where

$$\beta(x)(i,j) = \begin{cases} \mathbb{C}(x)(i,j), & \mathbb{C}(x)(i,j) = max\{\mathbb{C}(x)(s,t) \ : (s,t) \in I^2 \}, \\ 0 & Otherwise. \end{cases}$$

In Table 9 and Figure 9, compute a mapping

$$\mathbb{C}_M : X \to R, \text{ by } \mathbb{C}_M(x) = \sum_{(i,j)\in I^2} \beta(x)(i,j)$$

**Table 9.** Computes a mapping $\mathbb{C}_M : X \to R,$ by $\mathbb{C}_M(x) = \sum_{(i,j)\in I^2} \beta(x)(i,j))$.

| $\hat{\mathbb{C}}$. | $\hat{\mathbb{C}}(\boldsymbol{x}_1)$ | $\hat{\mathbb{C}}(\boldsymbol{x}_2)$ | $\hat{\mathbb{C}}(\boldsymbol{x}_3)$ | $\hat{\mathbb{C}}(\boldsymbol{x}_4)$ |
|---|---|---|---|---|
| $(i_1,j_1)$ | 0.000011628 | 2250 | 2250 | 0.06975 |
| $(i_1,j_2)$ | 0.000009453 | 2250 | 2250 | 00.0585 |
| $(i_1,j_3)$ | 00.00001068 | 2250 | 2250 | 00.0486 |
| $(i_1,j_4)$ | 0.000009508 | 2250 | 2250 | 00.0441 |
| $(i_2,j_1)$ | 0.000010640 | 2250 | 2250 | 0.07356 |
| $(i_2,j_2)$ | 0.000008896 | 2250 | 2250 | 000.7011 |
| $(i_2,j_3)$ | 0.000010560 | 2250 | 2250 | 0.037908 |
| $(i_2,j_4)$ | 0.000010880 | 2250 | 2250 | 0.0477 |
| $(i_3,j_1)$ | 00.00001064 | 2250 | 2250 | 0.05304 |
| $(i_3,j_2)$ | 0.000009408 | 2250 | 2250 | 0.039 |
| $(i_3,j_3)$ | 00.00001096 | 2250 | 2250 | 0.0522 |
| $(i_3,j_4)$ | 0.000009508 | 2250 | 2250 | 0.0357 |
| $(i_4,j_1)$ | 0.000007412 | 2250 | 2250 | 0.06015 |
| $(i_4,j_2)$ | 0.000004796 | 2250 | 2250 | 0.05205 |
| $(i_4,j_3)$ | 0.00000876 | 2250 | 2250 | 0.08316 |
| $(i_4,j_4)$ | 0.000088592 | 2250 | 2250 | 0.0459 |
| $\mathbb{C}_M$ | 0.000232321 | 36,000 | 36,000 | 1.502418 |

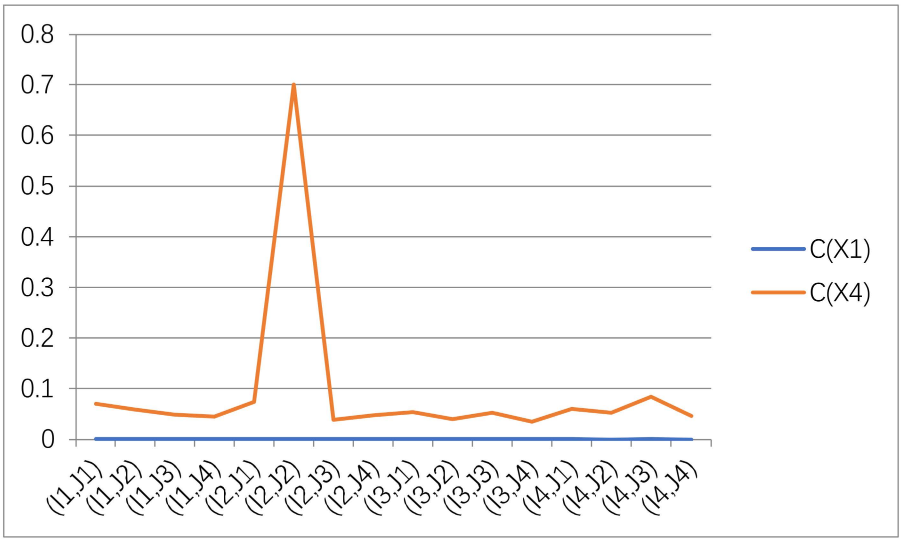


**Figure 9.** Shows mapping $\mathbb{C}_M : X \to R$, by $\mathbb{C}_M(x) = \sum_{(i,j)\in I^2} \beta(x)(i,j)$.

Since $\mathbb{C}_{\boldsymbol{M}}((x_2, (x_3) = 36000 = \max\mathbb{C}_{\boldsymbol{M}}$ , then the optimal alternative for the suitability of the nuclear power plants based on a 2-polar fuzzy soft set X based on a 3-polar fuzzy soft set is $x_2,\ x_3$ (the Gas Cooled Graphite-Moderated (AGR) and Pressurized Water Reactor (PWR) models).

(2) The second way: First give the algorithm (Algorithm 4) to explain the method

**Algorithm 4:** Using 3-polar fuzzy soft set.

**Step 1**. Compute
$\mathcal{A}, \mathcal{B} \in \left[\left([0,2250]^2\right)^X \times \left([0,2250]^2\right)^X\right]^I = \left[\left([0,2250]^4\right)^X\right]^I = \left[\left([0,2250]^4\right)^I\right]^X = \left([0,2250]^4\right)^{X\times I}$

**Step 2**. Compute $\mathbb{C} = (\mathcal{A} \wedge \mathcal{B}) \in \left[\left([0,2250]^2\right)^X \times \left([0,2250]^2\right)^X\right]^I$
**Step 3**. Compute the 3-polar fuzzy soft set $\hat{\mathbb{C}} \in \left([0,2250]^{2\times 3}\right)^X$, defined by

$$\hat{\mathbb{C}}(x)(i,j) = 2250 \wedge \sum_{k=1}^{3} \left(p_k{}^\circ p_1{}^\circ\mathbb{C}(x)(i,j) \times p_k{}^\circ p_2{}^\circ\mathbb{C}(x)(i,j)\right) \quad \forall (i,j) \in I^2,\ \forall x \in X)$$

where $p_k : [0,2250]^2 \to [0,2250]$ is the $k$-the projection $(k = 1,\ 2,\ 3)$;
**Step 4**. $\mathbb{C}_M : X \to R$, by where

$$\beta(x)(i,j) = \begin{cases} \mathbb{C}(x)(i,j), & \mathbb{C}(x)(i,j) = max\{\mathbb{C}(x)(s,t)\ : (s,t) \in I^2\ \}, \\ 0 & Otherwise. \end{cases}$$

**Step 5**. Compute $m_i = \sum_{k=1}^{4} (x_k)(i,j),\quad x \in X,\quad (i,j) \in (I \times I)$ and compute
$r_i = \sum_{j=1}^{4} \left(m_i - m_j\right) \quad (i = 1,2,3,4)$,
**Step 6**. The maximal value of $r_i = \sum_{j=1}^{4} \left(m_i - m_j\right) \quad (i,j = 1,2,3,4)$ to state the optimal alternative for the suitability of nuclear power plants based on a 2-polar fuzzy soft set X based on a 3-polar fuzzy soft set. Of nuclear power plants based on 2-polar fuzzy soft set X based on 3-polar fuzzy soft set.
The second way, compute $m_i = \sum_{k=1}^{4} (x_k)(i,j),\quad x \in X,\quad (i,j) \in (I \times I)$ and compute
$r_i = \sum_{j=1}^{4} \left(m_i - m_j\right) \quad (i = 1,2,3,4)$, then, use Table 10 and follow Table 10 to compute
$m_i = \sum_{k=1}^{4} (x_k)(i,j),\quad x \in X,\quad (i,j) \in (I \times I))$. Table 10: Compute
$m_i = \sum_{k=1}^{4} (x_k)(i,j),\quad x \in X,\quad (i,j) \in (I \times I)$.

**Table 10.** Compute $m_i = \sum_{k=1}^{4}(x_k)(i,j),\ x \in X,\ (i,j) \in (I \times I)$.

| | $\mathbf{x}_1$ | $\mathbf{x}_2$ | $\mathbf{x}_3$ | $\mathbf{x}_4$ |
|---|---|---|---|---|
| $(i_1, j_1)$ | 0.000011628 | 2250 | 2250 | 0.06975 |
| $(i_1, j_2)$ | 0.000009453 | 2250 | 2250 | 00.0585 |
| $(i_1, j_3)$ | 00.00001068 | 2250 | 2250 | 00.0486 |
| $(i_1, j_4)$ | 0.000009508 | 2250 | 2250 | 00.0441 |
| $(i_2, j_1)$ | 0.000010640 | 2250 | 2250 | 0.07356 |
| $(i_2, j_2)$ | 0.000008896 | 2250 | 2250 | 000.7011 |
| $(i_2, j_3)$ | 0.000010560 | 2250 | 2250 | 0.037908 |
| $(i_2, j_4)$ | 0.000010880 | 2250 | 2250 | 0.0477 |
| $(i_3, j_1)$ | 00.00001064 | 2250 | 2250 | 0.05304 |
| $(i_3, j_2)$ | 0.000009408 | 2250 | 2250 | 0.039 |
| $(i_3, j_3)$ | 00.00001096 | 2250 | 2250 | 0.0522 |
| $(i_3, j_4)$ | 0.000009508 | 2250 | 2250 | 0.0357 |
| $(i_4, j_1)$ | 0.000007412 | 2250 | 2250 | 0.06015 |
| $(i_4, j_2)$ | 0.000004796 | 2250 | 2250 | 0.05205 |
| $(i_4, j_3)$ | 0.00000876 | 2250 | 2250 | 0.08316 |
| $(i_4, j_4)$ | 0.000088592 | 2250 | 2250 | 0.0459 |
| $\mathrm{m}_i$ | 0.000232321 | 36,000 | 36,000 | 1.502418 |

$$m_1 = 0.000232321, m_2 = 36000, \ \ m_3 = 36000, m_4 = 1.502418$$

Now, calculate

$$r_1 = (m_1 - m_1) + (\mathrm{m}_1 - m_2) + (m_1 - m_3) + (m_1 - m_4) = (0.000232321 - 0.000232321) + (0.000232321 - 36000) + (0.000232321 - 36000) + (0.000232321 - 1.502418) \ = -7201.501721;$$

Similarly, $r_2 = 7198.49735$, $r_3 = 7198.49735$, $r_4 = -7195.492978$.

Since $r_3, r_2 = 7198.49735 = \max r_i$, then the optimal alternative for the suitability of nuclear power plants based on a 2-polar fuzzy soft set X based on a 3-polar fuzzy soft set is $x_2,\ x_3$ (the Gas Cooled Graphite-Moderated (AGR) and Pressurized Water Reactor (PWR) models).

Now, find the optimal alternative for the suitability of nuclear power plants based on 2-polar fuzzy soft set X based on 3-polar fuzzy soft set by using the operator $\vee$,

First compute $\mathbb{C} = \mathcal{A} \vee \ \mathcal{B}$. Thus compute $\mathbb{C}$.

$$\mathbb{C}(\mathrm{i}_1,\ \mathrm{i}_1) = \left\{ \begin{array}{c} \frac{\langle(0.021,0.020),(0.021,0.019),(0.018,0.019)\rangle}{x_1}, \frac{\langle(299,291),(260,270),(270,280)\rangle}{x_2}, \\ \frac{\langle(2250,2250),(2240,2250),(2250,2240)\rangle}{x_3}, \frac{\langle(0.4,0.4),(0.3,0.4),(0.41,0.3)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_1, \mathrm{i}_2) = \left\{ \begin{array}{c} \frac{\langle(0.021,0.020),(0.021,0.020),(0.018,0.017)\rangle}{x_1}, \frac{\langle(299,290),(260,255),(270,280)\rangle}{x_2}, \\ \frac{\langle(2250,2240),(2250,2250),(2240,2230)\rangle}{x_3}, \frac{\langle(0.4,0.3),(0.4,0.27),(0.7,0.31)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_1, \mathrm{i}_3) = \left\{ \begin{array}{c} \frac{\langle(0.021,0.020),(0.021,0.021),(0.018,0.017)\rangle}{x_1}, \frac{\langle(299,290),(260,250),(289,280)\rangle}{x_2}, \\ \frac{\langle(2250,2240),(2240,2250),(2240,2241)\rangle}{x_3}, \frac{\langle(0.4,0.4),(0.33,0.25),(0.4,0.3)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_1, \mathrm{i}_4) = \left\{ \begin{array}{c} \frac{\langle(0.021,0.020),(0.021,0.019),(0.018,0.017)\rangle}{x_1}, \frac{\langle(299,290),(260,250),(270,280)\rangle}{x_2}, \\ \frac{\langle(2250,2240),(2240,2250),(2240,2230)\rangle}{x_3}, \frac{\langle(0.4,0.31),(0.3,0.25),(0.4,0.36)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_2, \mathrm{i}_1) = \left\{ \begin{array}{c} \frac{\langle(0.019,0.020),(0.021,0.020),(0.016,0.019)\rangle}{x_1}, \frac{\langle(290,291),(270,270),(260,270)\rangle}{x_2}, \\ \frac{\langle(2230,2250),(2240,2240),(2250,2240)\rangle}{x_3}, \frac{\langle(0.4,0.4),(0.3,0.4),(0.5,0.4)\rangle}{x_4} \end{array} \right\}$$

$$\mathbb{C}(\mathrm{i}_2,\mathrm{i}_2)=\left\{\begin{array}{c}\frac{\langle(0.019,0.018),(0.021,0.020),(0.018,0.017)\rangle}{x_1},\frac{\langle(299,288),(270,255),(254,260)\rangle}{x_2},\\ \frac{\langle(2020,2210),(2250,2200),(2220,2230)\rangle}{x_3},\frac{\langle(0.4,0.5),(0.4,0.28),(0.7,0.4)\rangle}{x_4}\end{array}\right\}$$

$$\mathbb{C}(\mathrm{i}_2,\mathrm{i}_3)=\left\{\begin{array}{c}\frac{\langle(0.019,0.018),(0.021,0.021),(0.017,0.016)\rangle}{x_1},\frac{\langle(290,280),(270,250),(289,240)\rangle}{x_2},\\ \frac{\langle(2250,2240),(2240,2220),(2235,2241)\rangle}{x_3},\frac{\langle(0.4,0.4),(0.33,0.28),(0.5,0.4)\rangle}{x_4}\end{array}\right\}$$

$$\mathbb{C}(\mathrm{i}_2,\mathrm{i}_4)=\left\{\begin{array}{c}\frac{\langle(0.021,0.018),(0.021,0.020),(0.018,0.016)\rangle}{x_1},\frac{\langle(290,280),(270,250),(250,240)\rangle}{x_2},\\ \frac{\langle(2252,2230),(2240,2220),(2230,2220)\rangle}{x_3},\frac{\langle(0.4,0.31),(0.3,0.28),(0.5,0.4)\rangle}{x_4}\end{array}\right\}$$

$$\mathbb{C}(\mathrm{i}_3,\mathrm{i}_1)=\left\{\begin{array}{c}\frac{\langle(0.021,0.020),(0.021,0.020),(0.018,0.019)\rangle}{x_1},\frac{\langle(289,291),(250,270),(279,270)\rangle}{x_2},\\ \frac{\langle(2240,2250),(2240,2240).(2250,2240)\rangle}{x_3},\frac{\langle(0.4,0.4),(0.3,0.4),(0.41,0.29)\rangle}{x_4}\end{array}\right\}$$

$$\mathbb{C}(\mathrm{i}_3,\ \mathrm{i}_2)=\left\{\begin{array}{c}\frac{\langle(0.021,0.018),(0.021,0.020),(0.018,0.017)\rangle}{x_1},\frac{\langle(299,288),(260,255),(279,260)\rangle}{x_2},\\ \frac{\langle(2240,2230),(2250,2210).(2230,2240)\rangle}{x_3},\frac{\langle(0.38,0.5),(0.4,0.27),(0.7,0.31)\rangle}{x_4}\end{array}\right\}$$

$$\mathbb{C}(\mathrm{i}_3,\mathrm{i}_3)=\left\{\begin{array}{c}\frac{\langle(0.021,0.018),(0.021,0.021),(0.018,0.016)\rangle}{x_1},\frac{\langle(289,250),(201,199),(289,235)\rangle}{x_2},\\ \frac{\langle(2250,2240),(2240,2220).(2235,2241)\rangle}{x_3},\frac{\langle(0.38,0.4),(0.33,0.25),(0.36,0.16)\rangle}{x_4}\end{array}\right\}$$

$$\mathbb{C}(\mathrm{i}_3,\mathrm{i}_4)=\left\{\begin{array}{c}\frac{\langle(0.021,0.018),(0.021,0.020),(0.018,0.016)\rangle}{x_1},\frac{\langle(289,240),(220,220),(279,230)\rangle}{x_2},\\ \frac{\langle(2252,2230),(2240,2210).(2230,2240)\rangle}{x_3},\frac{\langle(0.38,0.31),(0.3,0.25),(0.36,0.36)\rangle}{x_4}\end{array}\right\}$$

$$\mathbb{C}(\mathrm{i}_4,\mathrm{i}_1)=\left\{\begin{array}{c}\frac{\langle(0.020,0.020),(0.021,0.019),(0.019,0.019)\rangle}{x_1},\frac{\langle(289,291),(250,270),(260,270)\rangle}{x_2},\\ \frac{\langle(2230,2250),(2240,2240),(2250,2240)\rangle}{x_3},\frac{\langle(0.4,0.4),(0.3,0.4),(0.41,0.33)\rangle}{x_4}\end{array}\right\}$$

$$\mathbb{C}(\mathrm{i}_4,\mathrm{i}_2)=\left\{\begin{array}{c}\frac{\langle(0.020,0.017),(0.021,0.020),(0.019,0.017)\rangle}{x_1},\frac{\langle(299,288),(260,255),(254,260)\rangle}{x_2},\\ \frac{\langle(2150,2210),(2250,2210),(2220,2240)\rangle}{x_3},\frac{\langle(0.4,0.5),(0.4,0.27),(0.7,0.33)\rangle}{x_4}\end{array}\right\}$$

$$\mathbb{C}(\mathrm{i}_4,\mathrm{i}_3)=\left\{\begin{array}{c}\frac{\langle(0.020,0.017),(0.021,0.021),(0.019,0.015)\rangle}{x_1},\frac{\langle(279,250),(230,210),(289,240)\rangle}{x_2},\\ \frac{\langle(2150,2240),(2240,2220),(2235,2241)\rangle}{x_3},\frac{\langle(0.4,0.4),(0.33,0.25),(0.35,0.33)\rangle}{x_4}\end{array}\right\}$$

$$\mathbb{C}(\mathrm{i}_4,\mathrm{i}_4)=\left\{\begin{array}{c}\frac{\langle(0.021,0.017),(0.021,0.013),(0.019,0.013)\rangle}{x_1},\frac{\langle(269,220),(230,220),(249,240)\rangle}{x_2},\\ \frac{\langle(2152,2140),(2240,2220),(2230,2240)\rangle}{x_3},\frac{\langle(0.4,0.31),(0.3,0.25),(0.32,0.36)\rangle}{x_4}\end{array}\right\}$$

Secondly, compute the 3-polar fuzzy soft set $\hat{\mathbb{C}} \in \left([0,2250]^{2\times 3}\right)^X$, defined by

$$\hat{\mathbb{C}}(x)(i,j) = 2250 \wedge \sum_{k=1}^{3}\left(p_k{}^\circ p_1{}^\circ \mathbb{C}(x)(i,j) \times p_k{}^\circ p_2{}^\circ \mathbb{C}(x)(i,j)\ \right) \quad \forall (i,j) \in I^2,\ \forall x \in \mathrm{X})$$

where $p_k : [0,2250]^2 \to [0,2250]$ is the $k$-th projection ($k$ =1, 2, 3);

$$\hat{\mathbb{C}}(x_1)(i_1,j_1) = (2250) \wedge [(0.5 \times 0.4 \times 0.4) + (0.3 \times 0.2 \times 0.3)] = (70) \wedge (\ 0.098) = 0.098;$$

Similarly, as in Table 11 and Figure 10, compute $\hat{\mathbb{C}}(x)(i,j) \in \left([0,2250]^{2\times 3}\right)^X$.

**Table 11.** Compute $\hat{\mathbb{C}}(x)(i,j) \in \left([0, 2250]^{2\times 3}\right)^X$.

| $\hat{\mathbb{C}}$ | $\hat{\mathbb{C}}(x_1)$ | $\hat{\mathbb{C}}(x_2)$ | $\hat{\mathbb{C}}(x_3)$ | $\hat{\mathbb{C}}(x_4)$ |
|---|---|---|---|---|
| $(i_1, j_1)$ | 0.000015158 | 2250 | 2250 | 0.1812 |
| $(i_1, j_2)$ | 0.000021538 | 2250 | 2250 | 0.15383 |
| $(i_1, j_3)$ | 0.000015078 | 2250 | 2250 | 0.07125 |
| $(i_1, j_4)$ | 0.000351168 | 2250 | 2250 | 0.0759 |
| $(i_2, j_1)$ | 0.000013984 | 2250 | 2250 | 1.260 |
| $(i_2, j_2)$ | 0.000013302 | 2250 | 2250 | 0.168 |
| $(i_2, j_3)$ | 0.000313140 | 2250 | 2250 | 0.1108 |
| $(i_2, j_4)$ | 0.000013698 | 2250 | 2250 | 0.12944 |
| $(i_3, j_1)$ | 0.000015538 | 2250 | 2250 | 0.0956 |
| $(i_3, j_2)$ | 0.000014058 | 2250 | 2250 | 0.14825 |
| $(i_3, j_3)$ | 0.000013988 | 2250 | 2250 | 0.06114 |
| $(i_3, j_4)$ | 0.000013698 | 2250 | 2250 | 0.06894 |
| $(i_4, j_1)$ | 0.000015200 | 2250 | 2250 | 0.102 |
| $(i_4, j_2)$ | 0.000013760 | 2250 | 2250 | 0.15655 |
| $(i_4, j_3)$ | 0.000000119 | 2250 | 2250 | 0.0792 |
| $(i_4, j_4)$ | 0.000011250 | 2250 | 2250 | 0.0663 |

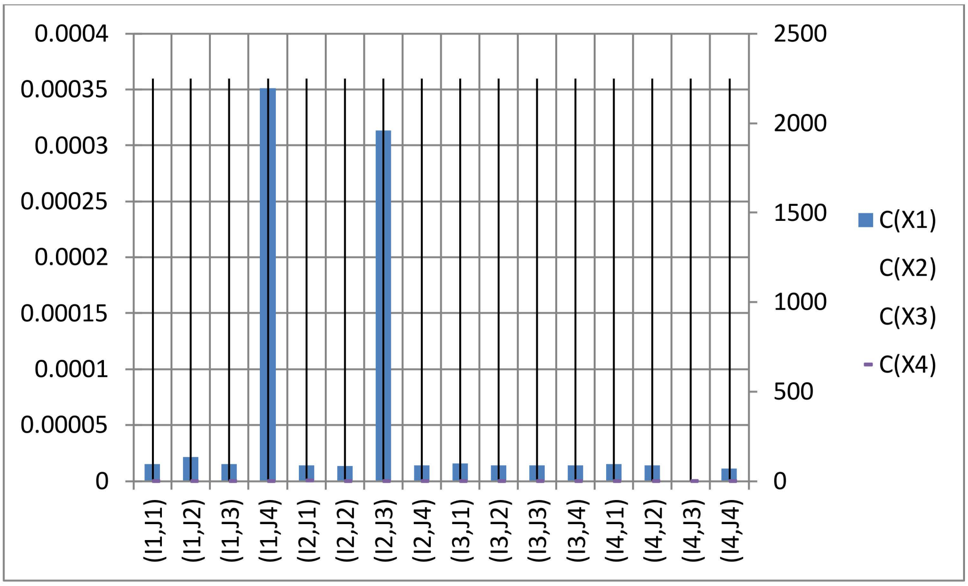


**Figure 10.** Compute $\hat{\mathbb{C}}(x)(i,j) \in \left([0, 2250]^{2\times 3}\right)^X$.

Therefore,

$$\hat{\mathbb{C}}(x_1) = \begin{Bmatrix} \frac{0.000015158}{(i_1,j_1)}, & \frac{0.000015158}{(i_1,j_2)}, & \frac{0.000015078}{(i_1,j_3)}, & \frac{0.000351168}{(i_1,j_4)}, \\ \frac{0.000013984}{(i_2,j_1)}, & \frac{0.000013302}{(i_2,j_2)}, & \frac{0.000313140}{(i_2,j_3)}, & \frac{0.000013698}{(i_2,j_4)}, \\ \frac{0.000015538}{(i_3,j_1)}, & \frac{0.000014058}{(i_3,j_2)}, & \frac{0.000013988}{(i_3,j_3)}, & \frac{0.000013698}{(i_3,j_4)}, \\ \frac{0.000015200}{(i_4,j_1)}, & \frac{0.000013760}{(i_4,j_2)}, & \frac{0.000000119}{(i_4,j_3)}, & \frac{0.000011250}{(i_4,j_4)} \end{Bmatrix}$$

$$\hat{\mathbb{C}}(x_2) = \begin{Bmatrix} \frac{2250}{(i_1,j_1)}, & \frac{2250}{(i_1,j_2)}, & \frac{2250}{(i_1,j_3)}, & \frac{2250}{(i_1,j_4)}, \\ \frac{2250}{(i_2,j_1)}, & \frac{2250}{(i_2,j_2)}, & \frac{2250}{(i_2,j_3)}, & \frac{2250}{(i_2,j_4)}, \\ \frac{2250}{(i_3,j_1)}, & \frac{2250}{(i_3,j_2)}, & \frac{2250}{(i_3,j_3)}, & \frac{2250}{(i_3,j_4)}, \\ \frac{2250}{(i_4,j_1)}, & \frac{2250}{(i_4,j_2)}, & \frac{2250}{(i_4,j_3)}, & \frac{2250}{(i_4,j_4)} \end{Bmatrix}$$

$$\hat{\mathbb{C}}(x_3) = \left\{ \begin{array}{l} \frac{2250}{(i_1,j_1)}, \frac{2250}{(i_1,j_2)}, \frac{2250}{(i_1,j_3)}, \frac{2250}{(i_1,j_4)}, \\ \frac{2250}{(i_2,j_1)}, \frac{2250}{(i_2,j_2)}, \frac{2250}{(i_2,j_3)}, \frac{2250}{(i_2,j_4)}, \\ \frac{2250}{(i_3,j_1)}, \frac{2250}{(i_3,j_2)}, \frac{2250}{(i_3,j_3)}, \frac{2250}{(i_3,j_4)}, \\ \frac{2250}{(i_4,j_1)}, \frac{2250}{(i_4,j_2)}, \frac{2250}{(i_4,j_3)}, \frac{2250}{(i_4,j_4)} \end{array} \right\} \hat{\mathbb{C}}(x_4) = \left\{ \begin{array}{l} \frac{0.1812}{(i_1,j_1)}, \frac{0.15383}{(i_1,j_2)}, \frac{0.07125}{(i_1,j_3)}, \frac{0.0759}{(i_1,j_4)}, \\ \frac{1.260}{(i_2,j_1)}, \frac{0.168}{(i_2,j_2)}, \frac{0.1108}{(i_2,j_3)}, \frac{0.12944}{(i_2,j_4)}, \\ \frac{0.0956}{(i_3,j_1)}, \frac{0.14825}{(i_3,j_2)}, \frac{0.06114}{(i_3,j_3)}, \frac{0.06894}{(i_3,j_4)}, \\ \frac{0.102}{(i_4,j_1)}, \frac{0.15655}{(i_4,j_2)}, \frac{0.0792}{(i_4,j_3)}, \frac{0.0663}{(i_4,j_4)} \end{array} \right\}$$

Now we make a decision in two ways:

(1) First way:

Define a mapping $\mathbb{C}_M : X \to R$, by $\mathbb{C}_M(x) = \sum_{(i,j)\in I^2} \beta(x)(i,j)$, where

$$\beta(x)(i,j) = \begin{cases} \mathbb{C}(x)(i,j), & \mathbb{C}(x)(i,j) = max\{\mathbb{C}(x)(s,t) \ : (s,t) \in I^2 \}, \\ 0 & Otherwise. \end{cases}$$

In Table 12, we compute the mapping $\mathbb{C}_M : X \to R$, by $\mathbb{C}_M(x) = \sum_{(i,j)\in I^2} \beta(x)(i,j))$.

**Table 12.** Compute the mapping $\mathbb{C}_M : X \to R$, by $\mathbb{C}_M(x) = \sum_{(i,j)\in I^2} \beta(x)(i,j)$.

| $\hat{\mathbb{C}}$ | $\hat{\mathbb{C}}(x_1)$ | $\hat{\mathbb{C}}(x_2)$ | $\hat{\mathbb{C}}(x_3)$ | $\hat{\mathbb{C}}(x_4)$ |
|---|---|---|---|---|
| $(i_1, j_1)$ | 0.000015158 | 2250 | 2250 | 0.1812 |
| $(i_1, j_2)$ | 0.000021538 | 2250 | 2250 | 0.15383 |
| $(i_1, j_3)$ | 0.000015078 | 2250 | 2250 | 0.07125 |
| $(i_1, j_4)$ | 0.000351168 | 2250 | 2250 | 0.0759 |
| $(i_2, j_1)$ | 0.000013984 | 2250 | 2250 | 1.260 |
| $(i_2, j_2)$ | 0.000013302 | 2250 | 2250 | 0.168 |
| $(i_2, j_3)$ | 0.000313140 | 2250 | 2250 | 0.1108 |
| $(i_2, j_4)$ | 0.000013698 | 2250 | 2250 | 0.12944 |
| $(i_3, j_1)$ | 0.000015538 | 2250 | 2250 | 0.0956 |
| $(i_3, j_2)$ | 0.000014058 | 2250 | 2250 | 0.14825 |
| $(i_3, j_3)$ | 0.000013988 | 2250 | 2250 | 0.06114 |
| $(i_3, j_4)$ | 0.000013698 | 2250 | 2250 | 0.06894 |
| $(i_4, j_1)$ | 0.000015200 | 2250 | 2250 | 0.102 |
| $(i_4, j_2)$ | 0.000013760 | 2250 | 2250 | 0.15655 |
| $(i_4, j_3)$ | 0.000000119 | 2250 | 2250 | 0.0792 |
| $(i_4, j_4)$ | 0.000011250 | 2250 | 2250 | 0.0663 |
| $\mathbb{C}_M$ | 0.000854677 | 36,000 | 36,000 | 2.9284 |

Since $\mathbb{C}_M((x_2,\ x_3) = 36000 = \ \max\mathbb{C}_M$,

Then the optimal alternative for the suitability of nuclear power plants based on a 2-polar fuzzy soft set X based on a 3-polar fuzzy soft set is $x_2,\ x_3$ (the Gas Cooled Graphite-Moderated (AGR) and Pressurized Water Reactor (PWR) models). Motivated by the above problem, we give the following algorithm (Algorithm 5) for a decision-making problem:

**Algorithm 5:** Using 3-polar fuzzy soft set.

**Step 1**. State $\mathcal{A}, \mathcal{B} \in \left[\left([0,2250]^2\right)^X \times \left([0,2250]^2\right)^X\right]^I = \left[\left([0,2250]^4\right)^X\right]^I = \left[\left([0,2250]^4\right)^I\right]^X = \left([0,2250]^4\right)^{X\times I}$

**Step 2**. Compute $\mathbb{C} = \mathcal{A} \vee \mathcal{B} \in \left[\left([0,2250]^2\right)^X \times \left([0,2250]^2\right)^X\right]^I$

**Step 3**. Compute the 3-polar fuzzy soft set $\hat{\mathbb{C}} \in \left([0,2250]^{2\times 3}\right)^X$, defined by

$$\hat{\mathbb{C}}(x)(i,j) = 2250 \wedge \sum_{k=1}^{3} (p_k{}^\circ p_1{}^\circ \mathbb{C}(x)(i,j) \times p_k{}^\circ p_2{}^\circ \mathbb{C}(x)(i,j)\ ) \quad \forall (i,j) \in I^2,\ \forall x \in X)$$

where $p_k : [0,2250]^2 \to [0,2250]$ is the $k$-the projection $(k = 1,2,3)$;

**Step 4**. $\mathbb{C}_M : X \to R$, by $\mathbb{C}_M(x) = \sum_{(i,j)\in I^2} \beta(x)(i,j)$, where

$$\beta(x)(i,j) = \begin{cases} \mathbb{C}(x)(i,j), & \mathbb{C}(x)(i,j) = max\{\mathbb{C}(x)(s,t) \ : (s,t) \in I^2 \}, \\ 0 & Otherwise. \end{cases}$$

**Step 5**. The maximal value of $\mathbb{C}_M$ to state optimal alternative for suitability of nuclear power plants based on 2-polar fuzzy soft set X based on 3-polar fuzzy soft set.

(2) The second way:

Compute $m_i = \sum_{k=1}^{4}(x_k)(i,j),\ x \in X,\ (i,j) \in (I \times I)$ as shown in Table 13 and compute $r_i = \sum_{j=1}^{4} \left(m_i - m_j\right)$ $(i = 1,2,3,4)$, then

**Table 13.** Compute $m_i = \sum_{k=1}^{4} (x_k)(i,j),\ x \in X,\ (i,j) \in (I \times I)$.

| $\hat{\mathbb{C}}$ | $\mathbf{x}_1$ | $\mathbf{x}_2$ | $\mathbf{x}_3$ | $\mathbf{x}_4$ |
|---|---|---|---|---|
| $(i_1, j_1)$ | 0.000015158 | 2250 | 2250 | 0.1812 |
| $(i_1, j_2)$ | 0.000021538 | 2250 | 2250 | 0.15383 |
| $(i_1, j_3)$ | 0.000015078 | 2250 | 2250 | 0.07125 |
| $(i_1, j_4)$ | 0.000351168 | 2250 | 2250 | 0.0759 |
| $(i_2, j_1)$ | 0.000013984 | 2250 | 2250 | 1.260 |
| $(i_2, j_2)$ | 0.000013302 | 2250 | 2250 | 0.168 |
| $(i_2, j_3)$ | 0.000313140 | 2250 | 2250 | 0.1108 |
| $(i_2, j_4)$ | 0.000013698 | 2250 | 2250 | 0.12944 |
| $(i_3, j_1)$ | 0.000015538 | 2250 | 2250 | 0.0956 |
| $(i_3, j_2)$ | 0.000014058 | 2250 | 2250 | 0.14825 |
| $(i_3, j_3)$ | 0.000013988 | 2250 | 2250 | 0.06114 |
| $(i_3, j_4)$ | 0.000013698 | 2250 | 2250 | 0.06894 |
| $(i_4, j_1)$ | 0.000015200 | 2250 | 2250 | 0.102 |
| $(i_4, j_2)$ | 0.000013760 | 2250 | 2250 | 0.15655 |
| $(i_4, j_3)$ | 0.000000119 | 2250 | 2250 | 0.0792 |
| $(i_4, j_4)$ | 0.000011250 | 2250 | 2250 | 0.0663 |
| $\mathbb{C}_M$ | 0.000854677 | 36,000 | 36,000 | 2.9284 |

From Table 13 and following 13 we obtain

$$m_1 = \ 0.000854677, m_2 = 36000,\ \ m_3 = 36000, m_4 = 2.9284$$

Now, calculate

$$r_1 = (m_1 - m_1) + (\mathrm{m}_1 - m_2) + (m_1 - m_3) + (m_1 - m_4) = 0 + (0.000854677 - 36000) + (0.000854677 - 36000) + (0.000854677 - 2.9284) = -72002.9258$$

Similarly, $r_2 = 71997.07075 = r_3 = 71997.07075$, $r_4 = -71991.21565$.

Since $r_2 = 71997.07075 = r_3 = \max r_i$,

Then the optimal alternative for the suitability of nuclear power plants based on a 2-polar fuzzy soft set X based on a 3-polar fuzzy soft set is $x_2,\ x_3$ (the Gas Cooled Graphite-Moderated (AGR) and Pressurized Water Reactor (PWR) models).

Motivated by the above problem, we give the following algorithm (Algorithm 6) for a decision- making problem:

**Algorithm 6:** Using 3-polar fuzzy soft set.

**Step 1**. State $\mathcal{A}, \mathcal{B} \in [(([0,2250]^2)^X \times ([0,2250]^2)^X]^I = [(([0,2250]^4)^X]^I = [(([0,2250]^4)^I]^X = ([0,2250]^4)^{X \times I}$

**Step 2**. Compute $\mathbb{C} = \mathcal{A} \vee \mathcal{B} \in [(([0,2250]^2)^X \times ([0,2250]^2)^X]^I$

**Step 3**. Compute the 3-polar fuzzy soft set $\hat{\mathbb{C}} \in ([0,2250]^{2\times 3})^X$, defined by

$$\hat{\mathbb{C}}(x)(i,j) = 2250 \wedge \sum_{k=1}^{3} (p_k {}^\circ p_1 {}^\circ \mathbb{C}(x)(i,j) \times p_k {}^\circ p_2 {}^\circ \mathbb{C}(x)(i,j)\ ) \quad \forall (i,j) \in I^2,\ \forall x \in X)$$

where $p_k : [0,2250]^2 \to [0,2250]$ is the $k$-the projection $(k = 1,2,3)$;

**Step 4**. Compute $m_i = \sum_{k=1}^{4} (x_k)(i,j),\quad x \in X,\quad (i,j) \in (I \times I)$ and compute $r_i = \sum_{j=1}^{4} \left(m_i - m_j\right) \quad (i = 1,2,3,4,5)$,

**Step 5**. The maximal value of $r_i = \sum_{j=1}^{4} \left(m_i - m_j\right) \quad (i,j = 1,2,3,4,5)$ to state the optimal alternative for suitability of nuclear power plants based on 2-polar fuzzy soft set X based on 3-polar fuzzy soft set.

## 5. Conclusions and Future Directions

A useful extension of soft sets, the notion of m-polar fuzzy sets (introduced by Chen et al. [19]), is emerging as a way to address real-world decision-making challenges. In the current study, we strengthened the theoretical underpinnings of m-polar fuzzy sets in ways supported by their effects on certain contexts and their capacity in terms of the number of opinions. Considering this, we developed an inventive technique called the algorithmic technique. In the end, this paper enhances outranking techniques based on m-polar fuzzy soft sets to explore coordinated data from six different nuclear power plant kinds. The proposed algorithm technique is developed by considering an asymmetric two-component weight vector, which can be generalized to a symmetric $n$-component weight vector. We calculate the results by the program Maple 7.

### *5.1. Limitations*

This paper's main goal is to emphasize the idea of a potential possibility m-polar fuzzy soft set (because it is useful in decision-making and other similar problems). Several operations (such as subset, equal, complement, union, intersection, inf product, and sup product) over the possibility of m-polar fuzzy soft sets are introduced for the simplicity of practical applications. For the fuzzy decision-making problem, we offer two techniques using the inf product or sup product operations of possibility m-polar fuzzy soft sets. To tackle the decision-making difficulties, we design an algorithm employing a possibility m-polar fuzzy soft set, and we demonstrate its viability using a numerical example. We draw the conclusion from the study that the suggested approach can effectively manage uncertainty when dealing with decision-making challenges.

### *5.2. Future Targets*

We will focus on the idea of the possibility m-polar fuzzy soft set, which can be thought of as a new possibility for m-polar fuzzy soft models, in light of these advancements. Theoretical operations of possible m-polar fuzzy soft sets are discussed, along with some of their structural aspects, including subset, equal, complement, inf product, sup product, union, and intersection. An example is given to illustrate the created methodology before a novel method for solving decision-making problems based on a possibility m-polar fuzzy soft set is shown.

- We can use the suggested approach to solve significant MAGDM issues that arise in real-world settings, such as those related to water waste management, forest management, medical sciences, and other issues.
- Additionally, the work may be expanded to include the most comprehensive complex T-spherical fuzzy N-soft environment. Additionally, because of the adaptability of the innovative m-polar fuzzy soft set model concept, we can also introduce the group decision-supporting scheme.

The following list summarizes the paper's main contributions:

1. Based on the m-polar fuzzy soft set, we have arrived at a criterion for the best choice for the suitability of nuclear power plants.
2. In the literature already in existence, a novel design and model of real-life applications have been developed and presented, pointing the way to the best alternative for the applicability of nuclear power plants.
3. After analyzing the data, we decided on the following nuclear power plants: Heavy Water Cooled and Moderated, Gas Cooled Graphite-Moderated, Pressurized Water Reactors, Boiling Water Reactors, and Boiling Light Water.
4. The algorithms for the analyses' results have also been noted, and the best option for applications in nuclear power plants is chosen using an m-polar fuzzy soft set decision-making criterion. In the future, we shall apply more advanced theories to Pythagorean fuzzy set decision making based on a Pythagorean fuzzy set.

**Author Contributions:** Methodology, M.M.K. and A.M.A.; Data curation, R.I.; Funding acquisition, M.M.A.A.-S. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research received no external funding.

**Acknowledgments:** The authors extend their appreciation to Deanship of Scientific Research at King Khalid University for funding this work through General Research project under grant number (R.G.P.1/277/43).

**Conflicts of Interest:** The authors declare no conflict of interest.

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