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Review

Turbulence in Two-Phase Flows with Macro-, Micro- and Nanoparticles: A Review

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Abstract: Turbulent flows are nonstationary in nature. Since the turbulent fluctuations of most flow parameters satisfy a symmetric Gaussian distribution, the turbulent characteristics have the property of symmetry in the statistical meaning. A widespread simplest model of turbulent flows is the model of "symmetric" turbulence, namely, homogeneous isotropic turbulence (HIT). The presence of particles with non-uniform distribution of their concentration in the turbulent flow, even under HIT conditions, can lead to redistribution of different components of fluctuation velocities of the carrier gas, i.e., to the appearance of asymmetry. The subject of the review is turbulent flows of gas with solid particles. Particular attention is paid to the problem of the back influence of particles on carrier gas characteristics (first of all, on the turbulent kinetic energy). A review of the results of experimental and computational-theoretical studies of the effect of the presence of the dispersed phase in the form of particles on the parameters of the turbulent flow of the carrier gas phase has been carried out. The main physical mechanisms and dimensionless criteria determining the direction and magnitude of the impact of particles of different inertia on the carrier gas phase turbulence energy are described and analyzed. The peculiarities of the influence of particles on the turbulence energy of the gas for different classes of flows: homogeneous isotropic turbulence, homogeneous shear flow, and wall turbulence in a pipe (channel) have been considered. For the near-wall flow in the pipe, it is shown that the turbulizing effect of extremely low-inertia particles of sub-micrometer size (nanoparticles) is replaced by the laminarizing effect of low-inertia particles of micrometer size (microparticles), and then again it is replaced by turbulizing due to additional generation of turbulence in the wakes of large particles of millimeter size (macroparticles). The review is intended to some extent to fill in the currently existing gap associated with the absence of dimensionless criteria (or complexes of physical parameters) responsible for the direction (attenuation or enhancement) of turbulence modification, and the value of this change. Possible directions for further researches are given in the conclusion of the review.

Keywords: two-phase flow; turbulent kinetic energy; particle back influence; microparticles; nanoparticles

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1. Introduction

In recent decades, one can observe a steady growth in the interest of numerous groups of researchers around the world in the study of multiphase (two-phase, heterogeneous) flows.

Gas flows carrying dispersed impurity in the form of solid particles (droplets) occur in a number of natural phenomena (air tornadoes, sandstorms, volcanic eruptions, forest fires, rains, etc.). Examples of technical devices in which two-phase flows are used are combustion chambers of gas turbines, paths of solid and liquid jet engines, fire extinguishing systems, devices for sand and blast treatment of various surfaces, pneumatic conveyors of bulk materials, dust collectors of various types, etc.

Studying the behavior of solid particles in turbulent flow and their inverse effect on the characteristics of the carrier gas is one of the fundamental problems of mechanics of two-phase (heterogeneous) media. Features of particle motion and intensity of occurring interphase processes largely depend on inertia of the dispersed phase and its concentration in the flow.

It should be noted that inertia of particles is determined not only by physical size, physical density of the dispersed phase and viscosity of the carrier phase but also by the characteristic spatial and temporal scales of the flow in which their movement occurs. In some conditions, particles of a given size and density will have low inertia, while in other conditions they will have high inertia.

There are many experimental, theoretical and computational works devoted to various issues related to turbulent flows with solid particles. A general overview of these issues is provided by a number of review papers [1–5]. The above reviews do not analyze the features of the influence of particles on different classes of two-phase flows (homogeneous isotropic turbulence, homogeneous shear flow, near-wall turbulence) and do not attempt to generalize the results to the case when the inertia of particles varies in an extremely wide range (by several orders of magnitude). It is this gap that the present review aims to fill.

The vast majority of the works analyzed in this review were performed under such conditions, when extremely low-inertia particles have submicrometric sizes (nanoparticles), relatively low-inertia particles have micrometer sizes (microparticles), and large particles have millimeter sizes (macroparticles). This circumstance explains the use of these terms in the title of the review. In recent years, researchers have been attracted by a variety of aspects related to nanofluids [6–10].

The subject of the review is turbulent gas flows which carry solid particles. The purpose of this work is to review the results of experimental and computational-theoretical studies of the peculiarities of particle motion and their inverse effect on the turbulence characteristics of the carrier gas flow and, above all, on the turbulence energy.

The review is structured as follows. The second section describes the main characteristics of turbulent two-phase flows. The third section presents and analyses the results of early experimental and computational-theoretical studies devoted to the study of particle motion in turbulent flows and their inverse effect on the characteristics of the carrier phase. Further, the main physical mechanisms of particle impact on the turbulence energy of gas and the corresponding dimensionless criteria are described. In the final section, the peculiarities of the influence of particles on the turbulence energy of the carrier phase for different classes of flow: homogeneous isotropic turbulence, homogeneous shear flow, wall turbulence in a pipe (channel) are discussed in detail. Conclusions are given in the Conclusions section, following from the performed review.

2. Motion of Particles in Gas Flows (Basic Characteristics)

The motion of particles in turbulent flows, as well as their inverse effect on the parameters of the carrier gas, is determined by their inertia and concentration [11,12]. To analyze and generalize the results of single-phase flows studies, criteria that are well known are used. In this section, the characteristics and criteria specific to two-phase turbulent flows are presented. In the future, these characteristics will be used in the description and analysis of research results.

2.1. Inertia of Particles: Dynamic Relaxation Time

The behavior of particles in turbulent flows of the carrier medium and their inverse effect on the characteristics of the gas phase is largely determined by their inertia. The inertia of solid particles moving in the flow depends primarily on their size (diameter) d_p and physical density ρ_p.

The complex characteristic of the inertia of particles is the dynamic relaxation time τ_p, which is represented as:

$$\tau_p = \tau_{p0}/C(\mathrm{Re}_p) = \frac{\rho_p\, d_p^2}{18\mu\, C(\mathrm{Re}_p)} \quad (1)$$

Here, τ_{p0}—dynamic relaxation time of a Stokesian particle; μ—dynamic viscosity. Note that the relaxation time of a particle also depends on the dynamic viscosity of the medium in which it moves. The correction function $C(\mathrm{Re}_p)$ takes into account the effect of inertial forces on the relaxation time of a non-Stokesian particle. Thus, in the case of a non-Stokesian particle motion, its inertia also depends on the particle's Reynolds number Re_p, which is calculated from the relative velocity between phases and the particle diameter (see Section 2.3 below).

2.2. Concentration of Particles: Interphase Interaction Regimes

An extensive physical characteristic of two-phase flows is the concentration of the dispersed impurity. There are three concentrations of particles—mass M, volumetric Φ and numerical N. The flows of gases with solid particles considered in this review are characterized by a relatively small volumetric particle content ($\Phi << 1$). In this case, there are obvious relations linking the mass, volume and counting particle concentrations in the form:

$$\mathrm{M} = \Phi\, \rho_p/\rho, \tag{2}$$

$$\Phi\ = N\, (\pi\, d_p^3/6) \tag{3}$$

In [13] a classification of two-phase flows depending on the volume concentration of the dispersed phase Φ is proposed.

When modeling the motion of particles in low dusty flows, when $\Phi \leq O(10^{-6})$, i.e., at low volume concentration of the dispersed phase, the main attention is usually paid to establishing the characteristics (behavior) of particles in their interaction with the turbulent vortices of the carrier flow. Such calculations are called "one-way coupling" [3], which means taking into account only one-directional influence of a carrying flow on particles suspended in it, which fully determines features of their behavior.

As the concentration of particles increases, i.e., at $O(10^{-6}) < \Phi \leq O(10^{-3})$, they, in turn, begin to have an inverse effect on the characteristics (all without exception) of the carrier medium. Taking into account the mutual influence of the dispersed and carrier phases significantly complicates mathematical modelling of two-phase flow (such calculations are called "two-way coupling") [13].

A further increase in the particle concentration, i.e., at $\Phi > O(10^{-3})$, leads to the necessity of taking into account the contribution of interparticle interactions to the momentum and energy transfer of the dispersed phase. In strongly dusted flows the determining role in formation of statistical properties of the disperse phase is played by interparticle collisions. Consideration of paired (binary) collisions of particles complicates mathematical modelling even more (in the literature such calculations are called "four-way coupling").

2.3. Reynolds Numbers

The Reynolds numbers in single-phase flows are well known, for the construction of which the radius (diameter) of the channel or jet, the longitudinal coordinate in the boundary layer, etc. are used as a characteristic geometric dimension.

In two-phase flows, the main criteria determining the flow regime is the particle Reynolds number, calculated from the relative velocity between the phases $\mathbf{w}$ and the particle diameter d_p

$$\mathrm{Re}_p = \frac{|\mathbf{w}|\, d_p}{\nu} = \frac{|\mathbf{u} - \mathbf{v}|\, d_p}{\nu} \tag{4}$$

where $\mathbf{u}$—velocity vector of carrier gas, $\mathbf{v}$—velocity vector of particles, ν—kinematic viscosity of the carrier gas.

Considering that velocity of carrying gas is a function of spatial coordinates and time, it is obvious that the value of reduced Reynolds number can undergo significant changes along the trajectory of particle motion. This circumstance should be taken into account when carrying out calculations of particle motion.

When analyzing turbulent flows, the Taylor Reynolds number of the carrier gas is often used, which has the following form:

$$\mathrm{Re}_\lambda = \frac{\left(\overline{u'^2}\right)^{1/2}\lambda}{\nu} = (2k/3)^{1/2}\lambda/\nu, \tag{5}$$

where $\left(\overline{u'^2}\right)^{1/2}$—rms value of gas fluctuation velocity, λ—Taylor spatial scale of turbulence, k—kinetic energy of the carrier gas turbulence (TKE).

To analyze turbulent flows, researchers also use the "viscous" Reynolds number, defined by the friction velocity, namely:

$$\mathrm{Re}_\tau = \frac{u_\tau \delta}{\nu}, \tag{6}$$

where u_τ—dynamic velocity (friction velocity) of the gas, $\delta = R = H/2$—characteristic geometric dimension, which can be the radius of the pipe R or the half-width of the channel $H/2$.

2.4. Stokes Numbers: Classification by Inertia of Particles

Single-phase turbulent flows are characterized by a number of spatial and corresponding temporal scales. Therefore, it is possible to construct a number of dimensionless criteria—Stokes numbers, determining the inertia of particles with respect to certain scales of the flow, in the following form:

$$\mathrm{Stk}_i = \frac{\tau_p}{T_i}, \tag{7}$$

where T_i—some characteristic time of the carrier phase.

In works [11,12,14], three basic dimensionless criteria are distinguished: Stk_f, Stk_L, and Stk_K—Stokes numbers in the time-averaged, large-scale fluctuation, and small-scale fluctuation motions, respectively. When constructing these criteria as characteristic times one choose T_f, T_L and τ_K—the characteristic times of the carrier phase in the time-averaged motion, large-scale fluctuation motion (time Lagrangian integral scale of turbulence) and small-scale fluctuation motion (temporal Kolmogorov scale of turbulence), respectively.

Depending on the range of changes in Stokes numbers, several basic classes of two-phase flows are distinguished in [14]. Let us briefly describe them.

1. Equilibrium flow. A distinctive feature of this flow is the equality of the time-averaged ($U_i = V_i$) and fluctuation ($u'_i = v'_i$) velocities of gas and particles and the following range of variation of the basic Stokes numbers: $\mathrm{Stk}_f \to 0$, $\mathrm{Stk}_L \to 0$ и $\mathrm{Stk}_K \approx O(1)$;

2. Quasi-equilibrium flow. The peculiarity of such flow is equality of time-averaged ($U_i = V_i$) and difference of fluctuation ($u'_i \neq v'_i$) velocities of gas and particles, as well as the following range of variation of basic Stokes numbers: $\mathrm{Stk}_f \to 0$, $\mathrm{Stk}_L \approx O(1)$, $\mathrm{Stk}_K \approx O(1)$;

3. Nonequilibrium flow. When this class of flow is realized, there is a difference of both time-averaged ($U_i \neq V_i$), and fluctuation ($u'_i \neq v'_i$) velocities of gas and particles and the following range of variation of the basic Stokes numbers: $\mathrm{Stk}_f \approx O(1)$, $\mathrm{Stk}_L \approx O(1)$, $\mathrm{Stk}_K \to \infty$;

4. Flow with large particles. The feature of such two-phase flow is the difference in time-averaged ($U_i \neq V_i$) velocities of phases, complete inertness of particles in relation to turbulent fluctuations of gas velocity ($v'_i = 0$) and the following range of the basic Stokes numbers: $\mathrm{Stk}_f \approx O(1)$, $\mathrm{Stk}_L \to \infty$, $\mathrm{Stk}_K \to \infty$.

It should be noted that the sign «=», used above is conventional. It is clear that the particles possessing inertia cannot completely trace neither the time-averaged, nor moreover the fluctuation motion of gas. Therefore for correctness it is possible to put that a particle completely involved in time-averaged (fluctuation) motion of gas is a particle, time-averaged (fluctuation) velocity of which differs no more than by 1% from corresponding velocities of the carrier phase. Note that the case of inertialess (in relation to small-scale gas fluctuations) particles is realized at $\mathrm{Stk}_K \to 0$.

It should also be noted that the above classification has a local scale and for two-phase flows, characterized by high spatiotemporal gradients of changes in the velocity of the carrier phase (which leads to changes in the characteristic time scales of gas T_i), one class of flows will transform (transition) into another.

3. Early Studies on the Effect of Particles on the Turbulence Characteristics of the Carrier Gas

This section describes and analyzes the results of early experimental and computational theoretical studies devoted to the study of particle motion in turbulent flows and their inverse effect on the characteristics of the carrier phase.

3.1. Experimental Studies

Historically, more development has taken place in the theory of turbulent two-phase jets. This is due to the obviousness of practical application of this class of two-phase flows and relative simplicity in setting up experimental studies. In early works on two-phase jets [15,16], it was shown that the presence of particles leads to a decrease in the intensity of turbulence of the carrier gas. The particles also changed the energy spectrum of turbulence by suppressing the high-frequency components. Later studies (e.g., [17–20], etc.) conducted for a wide range of concentrations and particle sizes, as well as phase density ratios, confirmed the results obtained.

Early studies of two-phase flows in channels include works [21–26]. They are mainly devoted to flows with spherical particles in vertical tubes with mass flow concentration of $\mathrm{M} \leq 5$. A detailed review of the works published before 1969 is contained in [25]. Typical tools in these experiments were Pitot tubes for measuring the velocity of the carrier gas and various photographic techniques for measuring the velocity of particles. These studies revealed the effect of particles on the profile of the averaged velocity of the carrier phase in the case of mass concentration of $\mathrm{M} > 1$. Direct measurements of gas turbulence intensity were not possible. However, in [26] the behavior of gas turbulence in the presence of particles was investigated by means of gas indicator diffusion. These measurements, valid near the pipe axis, did not establish any influence of particles on the turbulence intensity. Nevertheless, it was shown that the Lagrangian integral scale of two-phase flow turbulence decreases.

Later studies of two-phase currents in channels were carried out mainly using various modifications of laser Doppler anemometry (LDA) and particle image anemometry (PIV—Particle Image Velocimetry, the internationally accepted name for this method), which allow to measure velocities of both phases.

When investigating single-phase flows using the LDA and PIV methods [27–31], extremely low-inertia particles (tracer particles) are used. The instantaneous velocity of tracer particles obtained as a result of measurements is associated with the instantaneous velocity of gas. However, it should be kept in mind that the volume (mass) concentration of low-inertia tracer particles introduced into single-phase flow is negligibly small. As a consequence, the inverse effect of particles on the carrier phase turbulence characteristics (in particular, on the distribution of averaged velocity) is also negligible. With increasing concentration of such particles, their influence on gas should be increasing.

One of the first works where detailed measurements of averaged air velocities in the presence of particles and the particles themselves for a nonequilibrium two-phase flow were carried out were studies [32–34].

In [32,33], the authors studied the upward developed turbulent flow of air in tubes of diameter of $D = 38$ mm and $D = 56$ mm in the presence of spherical particles of glass ($d_p = 45$ μm и $d_p = 136$ μm) and copper ($d_p = 93$ μm). The mass flow concentration of the dispersed phase varied in the range M = 0.1–0.54. The velocity of the carrier air on the axis of the tube was $U_{xc} = 4.1–5.7$ m/s. The experiments demonstrated that the averaged particle velocity profile was more flat in comparison with the corresponding profile for the carrier phase. The flatness of the velocity profile increased with increasing inertia of the particles. It was also found that at concentration values, the presence of particles led to a significant flattening of the profile of the averaged velocity of the gas phase. This effect also increased with increasing inertia of the dispersed phase.

In article [34], an upward fully developed turbulent air flow in a tube of diameter $D = 42$ mm with glass particles ($d_p = 100–800$ µm) was studied. The mass flow concentration of the dispersed phase ranged from M = 1.2 for fine particles to M = 2.5 for heavy particles. The speed of the carrier air was $U_{xc} = 5.7$ m/s. Studies have shown that large particles flatten the profile of the average gas velocity, reducing its value near the axis and increasing it near the wall.

The most comprehensive study of nonequilibrium two-phase flows in pipes is presented by works [35–39]. In [35,36] measurements were also carried out for flows with large millimeter-sized particles (macroparticles).

In Ref. [36], detailed data on distributions of averaged velocities of "clean" air and air in the presence of particles ($d_p = 200$ µm, $\rho_p = 1000$ kg/m^3) for upward turbulent flow in a pipe with diameter of $D = 30.5$ mm were obtained. In general, the results obtained are consistent with the conclusions of [22–24]. In particular, it is shown that at values of concentration of M $\geq$ 1.3 the presence of particles leads to a significant filling of the profile of the averaged velocity of carrier air.

In work [36] the velocity fluctuation intensity distributions of "pure" air and air in the presence of plastic particles were obtained ($d_p = 200$ µm, $\rho_p = 1000$ kg/m^3). The results of measurements allowed one to make an explicit conclusion that the presence of relatively low-inertia particles leads to suppression of the intensity of turbulent fluctuations of carrier air throughout the cross section of the pipe. With increasing mass concentration of the dispersed phase, this effect increases up to M = 1.3, and then at M = 1.9 and M = 3.2, it weakens a little. At high values of mass concentration of particles, the profile of air fluctuation velocity in a considerable area of the tube ($r/R = 0–0.8$) becomes practically homogeneous. This is probably due to the fact that the influence of particles on the distribution of averaged air velocity at these concentrations of particles becomes significant. The flatness of the averaged velocity profile of the carrier phase is the reason for reduction of velocity fluctuations and their alignment along the pipe cross section. Thus, the influence of particles on gas turbulence intensity is mediated through their impact on the profile of averaged velocity of the carrier phase.

In paper [37], an upward turbulent air flow in a tube with a diameter of $D = 64$ mm with nonspherical aluminum oxide particles ($d_p = 50$ µm, $\rho_p = 3950$ kg/m^3) and spherical glass particles ($d_p = 50$ µm and $d_p = 100$ µm). The mass flow rate concentration of particles varied in the range M = 0.12–0.39. The average velocity of the carrier air on the tube axis was $U_{xc} = 6.4$ m/s.

The measurements [37] clearly showed that the influence of particles on the distribution of the time-averaged velocity of the carrier phase is negligible. Analysis of the measured profiles of fluctuation velocities of the carrier phase of the two-phase flow allowed the following conclusions to be made: (1) all particles used in the experiments reduced the intensity of longitudinal and transverse fluctuations of carrier air practically along the tube cross section; (2) maximum suppression of longitudinal and transverse fluctuations was observed near the tube axis; and (3) the degree of suppression of longitudinal and transverse velocity fluctuations increases with increasing mass concentration of particles and decreasing their inertia.

The main reason for suppression of turbulent fluctuations by particles is their involvement in large-scale fluctuation motion due to interaction with energy-carrying turbulent vortices of the supporting medium. The greater the degree of particle involvement in fluctuation motion, the greater their influence on the air fluctuation velocity. The parameter of dynamic inertia of particles in large-scale fluctuation motion is the corresponding Stokes number Stk_L.

As the inertia of the particles increases, they acquire less and less fluctuation velocity in their interaction with the energy-carrying turbulent vortices and, therefore, take less turbulence energy from the gas carrying them. Thus, in the near-wall region the particle inertia parameter (Stokes number) increases sharply compared to its value near the pipe axis due to a decrease in the characteristic lifetime of energy-carrying turbulent vortices (there are vortices with small characteristic sizes and high values of velocity fluctuations near the wall). Therefore, the involvement of particles in the fluctuation motion, and, consequently, the additional dissipation of turbulence caused by the presence of particles is not significant as near the pipe axis.

In Ref. [38] the downward turbulent flow of air in a tube with a diameter of $D = 46$ mm with spherical glass particles ($d_p = 50$ µm) was studied. The mass flow rate concentration of particles varied in the range of M = 0.05–1.2. The average velocity of the carrier air on the tube axis was $U_{xc} = 5.2$ m/s. This work reveals the effect of the excess of longitudinal fluctuations of particle velocities over the corresponding fluctuations of the carrier gas. The main physical mechanisms causing particle velocity fluctuations were analyzed: (1) turbulent fluctuations of particle velocity associated with their involvement in the fluctuation motion by turbulent vortices of gas; (2) fluctuations of particle velocity due to their polydispersity, i.e., the presence in the flow of particles of different sizes with different values of time-averaged velocities; (3) particle velocity fluctuations due to changes in their velocity in the interaction between themselves and with the channel wall; and (4) fluctuation velocity due to particle migration in the area with a shift of the time-averaged velocity of dispersed phase.

The effect of the excess of particle velocity fluctuations over the carrier phase fluctuations revealed in the experiments was first predicted theoretically in [39]. This effect was also revealed in works [40,41] devoted to simulation of particle dynamics by the large vortex method during flow in a channel and in a homogeneous shear layer. An excess of particle velocity fluctuations over carrier gas fluctuations was obtained in [42] when analyzing particle motion in inhomogeneous turbulent flow using the kinetic equation for the probability density function of particle velocities. Growth of intensity of particle velocity fluctuations as they approach the wall was recorded experimentally in [43,44]. In [45] an excess of longitudinal fluctuations of glass particles with diameter of 100 µm over fluctuations of carrying air was also revealed practically throughout the whole pipe cross section at low concentration of disperse phase. In this study a strong dependence of longitudinal fluctuations of particles on local concentration of the dispersed phase under conditions of essentially nonuniform distribution of the latter over the tube cross section was found.

Now a few words about flows with large millimeter-sized particles (macroparticles). This type of two-phase flows is characterized by the fact that the relaxation time of particles significantly exceeds the characteristic time of large-scale turbulent vortices, i.e., $\mathrm{Stk}_L \to \infty$ (see Section 2.4). Such particles will not react to turbulent fluctuations of the carrier phase velocity, and their average velocity distributions will be practically homogeneous over the channel (pipe) cross section. This is exactly what was confirmed by data [36] on distributions of averaged velocities of "pure" air, air in the presence of particles and plastic particles themselves ($d_p = 3000$ µm, $\rho_p = 1000$ kg/m^3) over pipe cross section. The measurement results also allowed us to conclude that there is significant dynamic slip between the phases in the time-averaged motion. The presence of phase slip leads to intense momentum exchange between gas and particles, which will be the cause of filling the profile of the time-averaged velocity of the carrier phase.

Let us consider the influence of large particles (macro-particles) on the intensity of carrier air velocity fluctuations using data from [36]. This paper presents the results of measurements of fluctuation (rms) velocity distributions of "pure" air and air in the presence of plastic particles ($d_p = 3000$ µm, $\rho_p = 1000$ kg/m^3) over pipe cross section [36].

It is concluded that the presence of large particles in the flow leads to a significant increase in the intensity of turbulent gas pulsations. The main reason for the observed phenomenon is the formation of turbulent trails behind the moving particles, which leads to additional generation of turbulence. The effect of generation of gas velocity fluctuations increases with increasing concentration of particles and distance from the pipe wall.

3.2. Theoretical Studies

This section will describe the results of early theoretical and computational-analytical studies of the effect of particles on the turbulence of the carrier gas flow.

The effect of relatively low-inertia particles on the laminar flow stability was studied in research [46]. Both considered types of particles (large and small) were so low-inertia that they completely tracked the averaged motion of the carrier gas (the time-averaged velocities of the particles and the carrier phase are equal). It was found that in the case of flows with extremely low-inertia particles ($\tau_p < \tau_e$, where τ_e—characteristic time of flow perturbation) their presence destabilizes the flow, while more inertia particles ($\tau_p > \tau_e$) have a stabilizing effect. These effects were interpreted as follows. The ultimate low-inertia particles move together with perturbation so that the effect of their presence is as if equivalent to an increase in the density of the carrying medium. If we assume that the particles have no effect on the dynamic viscosity, then they will decrease the kinematic viscosity by increasing the density. Thus, the presence of extremely low-inertia particles in a laminar flow causes the Reynolds number to increase, destabilizing the flow. More inertial particles cannot follow the flow when it is perturbed and are similar to sources of additional friction that take energy away from the perturbation and increase the critical Reynolds number, stabilizing the flow.

In Refs. [47,48], the mechanism of interaction between turbulent fluctuations and particles moving in a locally homogeneous, isotropic and stationary turbulent flow is considered for the case of vanishingly small Reynolds numbers of relative averaged motion of particles and gas. Interaction of a particle with turbulent fluctuations of different scales is determined by the ratio of the fluctuation frequency of the given scale ω_i to which depends on the properties of the particle and the characteristics of the medium but does not depend on the structure of turbulence. If $\omega_p < \omega_L$ (ω_L—frequency of the most large-scale fluctuations), then the particle has a large inertia, and even the largest-scale fluctuations during the period practically do not have time to set it in motion. In the other boundary case, when $\omega_p > \omega_l$ (ω_l—the frequency of the smallest scale fluctuations), the particle tends to follow fluctuations of all scales. If $\omega_L < \omega_p < \omega_l$, then the particle will be entrained by those fluctuations whose frequencies are small compared to ω_p, and will be flown by fluctuations whose frequencies are large compared to ω_p.

It was also shown in [47,48] that the additional energy dissipation in the two-phase flow, which is due to the presence of particles, becomes comparable to the dissipation in a pure gas if the phase density ratio of $\rho_p/\rho = O(10^3)$ (i.e., for the case with solid particles) is already at volume concentration $\Phi = O(10^{-3})$. For solid particles located in a liquid ($\rho_p/\rho = O(1)$), the presence of a small volume content of the dispersed phase practically does not change the value of dissipated energy. The following explanation was given. Energy dissipation on the particles is due to relative motion of particles and fluid. If particles are suspended in gas, the relative velocity will be large due to significant difference in densities and energy dissipation on particles will be comparable with dissipation in pure gas. At the same time, when particles move in liquid and densities of liquid and particles are comparable, entrainment by liquid and particles will be practically full and energy dissipation will be small.

The effect of the presence of Stokesian particles on the fluctuation velocity of the carrier phase is considered in [49]. Two assumptions were made for the analysis: (1) particles do not affect the time-averaged velocity profile of the carrier medium; (2) the presence of particles does not change the integral scale of turbulence. As a result, it was obtained that low-inertia particles dissipate turbulence in accordance with the following dependences [49]:

$$\frac{\left(\overline{u_i'^2}\right)^{1/2}}{\left(\overline{u_{i0}'^2}\right)^{1/2}} \approx (1+\mathrm{M})^{-1/2}, \text{ when } \tau_p << \tau_e; \tag{8}$$

$$\frac{\left(\overline{u_i'^2}\right)^{1/2}}{\left(\overline{u_{i0}'^2}\right)^{1/2}} \approx \left(1+\mathrm{M}\frac{\tau_e}{\tau_p}\right)^{-1/2}, \text{ when } \tau_p \geq \tau_e. \tag{9}$$

where $\left(\overline{u_i'^2}\right)^{1/2}$, $\left(\overline{u_{i0}'^2}\right)^{1/2}$—the root-mean-square (rms) value of the fluctuation velocities of two-phase and single-phase flows, respectively, τ_e—characteristic time of energy-containing vortices.

The simplest theoretical model describing turbulence dissipation by fine dispersed impurity as applied to two-phase jets was proposed by G.N. Abramovich [50]. In this pioneering work, within the framework of semi-empirical Prandtl turbulence theory, the fluctuation velocities of the carrier and dispersed phases were determined. The developed model is based on the equation of conservation of momentum of turbulent vortex and particles moving in it, as well as the equation of fluctuation motion of the dispersed phase within the vortex. It is assumed that low-inertia particles are involved in the fluctuation motion by turbulent vortices of the carrier phase, due to which the fluctuation velocity of the gas decreases. The fluctuation velocities obtained for the carrier and dispersed phases are used to find correlations by multiplying the corresponding fluctuation quantities, which is a very approximate method. As a result, the following expression for the relationship between the Reynolds stresses of two-phase and single-phase flows was obtained [50].

$$\frac{\overline{u_i' u_j'}}{\overline{u'_{i0} u'_{j0}}} = \frac{1}{(1+\mathrm{M})^2} \tag{10}$$

It should be noted that G.N. Abramovich's model was further developed in the works [51–55] and others.

3.3. Data Generalization: Search for Dimensionless Parameters

The first attempts to generalize the available experimental data to find dimensionless parameters (criteria) determining the effect of solid particles on the turbulent gas flow were undertaken in [56–60].

Thus, in [56] it was proposed to use as the main dimensionless parameter determining the interaction of particles with turbulence, the ratio of particle diameter to the characteristic length scale of the turbulent flow—d_p/l. It was shown that there is a critical value of this parameter, below which the particles suppress turbulence and above which they generate it. This critical value is equal to $d_p/l \approx 0.1$. The flow length scale (the size of the energy-containing vortices) l_e was determined from the data [61]. In [61], it is shown that for flows in tubes, $l \approx 0.2R$ near the axis and decreases to zero at the wall, starting from $r/R = 0.7$. Then, in [57] it was found that the critical value d_p/l increases linearly with distance from the pipe axis and reaches a value $d_p/l \approx 0.3$ near the wall. It was noted that this parameter only answers the question about the direction of turbulence modification (generation or dissipation) but not about the magnitude of this change.

The effect of another dimensionless parameter, the particle Reynolds number Re_p, determined by the averaged relative velocity between phases and particle diameter, on the interaction between the disperse impurity and the turbulence of the carrier gas was studied in [58]. Using data from [56], it was assumed that large particles ($\mathrm{Re}_p > 400$) cause vortices behind them that destabilize the flow and transform the energy of averaged motion into high-frequency components of the energy spectrum of turbulence. Fine particles ($\mathrm{Re}_p < 110$) mainly suppress turbulence energy, spending it on their own acceleration (involvement in fluctuation motion). As for medium-sized particles ($110 < \mathrm{Re}_p < 400$), they will have a mixed effect on turbulence.

In paper [59], attempts were continued to find criteria that determine the change in turbulence intensity caused by the presence of the dispersed phase. Nine physical quantities describing the nature of turbulent two-phase flows were identified. Application of the π-theorem led to the following six dimensionless criteria [59]:

$$\frac{\left(\overline{u'^2_x}\right)^{1/2}}{\left(\overline{u'^2_{x0}}\right)^{1/2}} = f\left(\frac{\rho_p}{\rho}, \frac{\rho\, D\, U_x}{\mu}, \frac{\rho\, W_x\, d_p}{\mu}, \frac{l}{d_p}, \frac{\left(\overline{u'^2_{x0}}\right)^{1/2}}{W_x}, \mathrm{M}\right), \tag{11}$$

where $\left(\overline{u'^2_{x0}}\right)^{1/2}$—fluctuation root-mean-square (rms) velocity of single-phase flow, D—tube diameter (duct width), U_x—time-averaged gas velocity, $W_x = |U_x - V_x|$—averaged relative velocity between phases, l—characteristic size of energy-containing vortices, M—mass concentration of particles.

In [60] it was noted that only four of these criteria (where there is no W) can be determined with sufficient accuracy without conducting experiments. An attempt to combine these criteria into some modified Stokes number—$\mathrm{Stk}_{Lm} = \rho_p d_p^2\, U_x/18\mu l$ and use it to describe the available experimental data did not yield a positive result.

There are several other papers (in addition to those described above), where the authors have also tried to find the parameters determining the interaction of suspended particles and turbulence in the carrier gas by analytical means [61–65].

Analysis of additional dissipation of turbulence in a two-phase flow containing low-inertia particles with neglect of the volume they occupy was carried out in [61]. For this purpose, the equation for turbulent energy of the carrier phase was used [61].

$$\frac{Dk}{D\tau} = P - \varepsilon - \frac{1}{\tau_p}\sum_i \left[\mathrm{M}\left(\overline{u'_i u'_i} - \overline{u'_i v'_i}\right) + (U_i - V_i)\overline{m' u'_i} + \left(\overline{m' u'_i u'_i} - \overline{m' u'_i v'_i}\right)\right], \tag{12}$$

where P and ε—generation and dissipation of turbulence (similar to the corresponding terms in the equation for single-phase flow).

Further in [61] some simplifications of Equation (12) are made with regard to the boundary layer conditions on a flat plate. First, for flows with particles with $Stk_L > 1$, the fluctuation component of the mass concentration m' s negligibly small, so that the triplet correlations of the right-hand sides of equation $\overline{m' u'_i u'_i}$ and $\overline{m' u'_i v'_i}$ are small compared to the other terms. Secondly, based on experimental data, it was shown that the time-averaged relative velocity between particles and carrying gas in longitudinal direction is of the same order as the gas fluctuation velocity. The relative velocity in the normal direction is small and the averaged relative velocity in the azimuthal direction is practically zero. Thus, the term $(U_i - V_i)\,\overline{m' u'_i}$ is of the order of triple correlation and can also be neglected.

After these simplifications, the equation for turbulence energy (12) takes the form [61].

$$\frac{Dk}{D\tau} = P - \varepsilon - \sum_i \frac{\mathrm{M}\left(\overline{u'_i u'_i} - \overline{u'_i v'_i}\right)}{\tau_p}. \tag{13}$$

In [43] it was found that $\overline{u'_y v'_y} << \overline{u'_y u'_y}$ and $\overline{u'_z v'_z} << \overline{u'_z u'_z}$. It has also been noted that $\overline{u'_x v'_x}$ may constitute a significant portion of the $\overline{u'_x u'_x}$. As a result, in [62] it was concluded that the additional dissipation of turbulence in a stream with low-inertia particles will depend on the value of: (1) the time-averaged value of the mass concentration; (2) the dynamic relaxation time of the particle τ_p; and (3) the correlation $\overline{u'_x v'_x}$.

In [62], a simple theoretical model was developed that takes into account the processes of additional dissipation and additional generation of turbulence energy in two-phase flows with low-inertia (microparticles) and large (macroparticles) particles, respectively. We will not give the final expression for determining the magnitude of the turbulence energy of the carrier gas in the presence of particles, obtained in this work, since mathematical inaccuracies were admitted in its derivation. However, in paper [63], an important conclusion was made that turbulence dissipation in case of low-inertia particles is proportional to the cube of particle diameter and turbulence generation is proportional to the square of particle diameter.

A mathematical model describing the generation and dissipation of turbulence in flows with solid particles is proposed in [63]. The model is based on the provisions of the pioneering work of G.N. Abramovich [50] on the effect of solid particles on the fluctuation velocity of the carrier gas. The proposed model is based on a modified Prandtl mixing path theory and takes into account two main sources of turbulence generation in two-phase flows: the time-averaged velocity gradient of the carrying gas and turbulent wakes behind the moving particles. The initial system of equations includes: (1) the momentum conservation equation for the individual turbulent vortex and the particles moving in it; (2) the equation of particle motion within the turbulent vortex; and (3) some relations for the flow in the particle wake.

As a result of analytical solution of the obtained system of equations, four dimensionless criteria [63], responsible for modification of turbulence in two-phase flows, were found: (1) dimensionless particle diameter $\overline{d}_p = d_p/l$, where l—characteristic size of energy-containing vortices; (2) modified Reynolds number of the particle, $\mathrm{Re}_{pm} = \left(\overline{u'^2_{x0}}\right)^{1/2} d_p/\nu$; (3) mass concentration of particles M; and (4) dimensionless density (ratio of densities of dispersed and carrier phases) $\overline{\rho} = \rho_p/\rho$.

It was also shown in [63] that in the case of flows with relatively low-inertia particles ($\mathrm{Re}_p < 400$) the modification of turbulence is determined only by the value of the mass concentration of particles M. For a flow with large particles ($\mathrm{Re}_p > 1000$) the change of turbulent fluctuation will depend on the ratio of mass concentration and dimensionless density $\mathrm{M}/\overline{\rho}$, i.e., determined by the volume concentration of disperse impurity Φ.

Comparison of turbulence intensity of polydisperse and monodisperse two-phase flows was carried out in [64]. The polydisperse flow was understood as a two-phase flow, in which the dispersed phase is represented by particles of two different sizes. Both considered types of flows contained spherical particles of relatively small sizes, such that the effect of turbulent wakes behind them had negligible influence, i.e., for the case when $\mathrm{Re}_p < 110$.

As a result of obtained relations from solution of system of equations similar to [63] it was found: (1) intensity of polydisperse flow turbulence is determined by the following parameters—total mass concentration of the disperse phase, mass concentrations of small and large particles, the ratio of diameters of large and small particles, the ratio of phase densities, Reynolds number of small and large particles, and the ratio of mixing path length to diameter of large (fine) particles; and (2) if all other parameters are constant, increasing mass concentration of small particles leads to an increase in additional dissipation of turbulence, and increasing mass concentration of large particles, on the contrary, leads to a decrease in dissipation of turbulence of polydisperse flow.

Based on the analogy of momentum and heat transfer processes, in [65] the process of suppression of turbulent temperature fluctuations in two-phase flows is analyzed. The intensity of temperature fluctuations of polydisperse and monodisperse two-phase flows was compared.

Further development of mathematical models that take into account the inverse effect of particles on the turbulence energy of the carrier gas phase was obtained in Refs. [66–68]. Separately, the laminarizing (dissipative) effect of fine impurities was analyzed in article [66], the turbulizing effect of large particles due to wake formation—in [67] and the combined effect of both mechanisms on turbulent gas flow in a pipe—in Ref. [68].

The differential energy transfer equation for turbulent gas k in the presence of particles can be written in compressed form:

$$\frac{Dk}{D\tau} = D + P - \varepsilon + P_p - \varepsilon_p, \tag{14}$$

where D—diffusion of turbulence energy; P—generation of turbulence energy; ε—dissipation of turbulence energy; P_p—the term responsible for the additional generation of turbulence energy due to the presence of particles; ε_p—additional dissipation caused by particles.

Thus, taking into account the modification of turbulence in two-phase flows implies a correct description of the terms of Equation (14) that are responsible for additional generation P_p and additional dissipation ε_p.

One important observation should be made. In [62,69,70], based on semiempirical considerations, additional terms are introduced into the turbulent energy balance equation of the supporting continuous phase due to generation of turbulent fluctuations of velocity at large Reynolds numbers of particles flowing. In [63], the turbulization of the flow with large particles was estimated on the basis of direct use of the automodel solution for the long-range axisymmetric turbulent wake. Naturally, this approach is valid only at very low volume concentration of the dispersed phase, when there is no interference of wakes behind individual particles. In [67,68], the solution for the automodel turbulence wake is involved not to calculate the turbulence characteristics of the carrier flow directly, but to determine the additional generation of turbulence in the fluctuation energy balance equation. This interpretation of the automodel solution for the long-range wake (i.e., using the solution in the local rather than in the integral sense) makes the proposed model applicable to various two-phase turbulent flows and allows one to hope for its validity not only at low, but also at moderate volume concentrations of particles.

In References [66–78], Equation (14) is analyzed for the case of stationary hydrodynamically developed flow when its left part is zero. In addition, in order to obtain a simple analytical relation for the influence of particles on turbulence, the analysis was carried out in the so-called diffusion-free (algebraic) approximation, i.e., without taking into account the contribution of the diffusion term in (14).

In these works the assumption was made that the influence of the dispersed phase on the time-averaged gas velocity profile, as well as on the mixing length distribution, when analyzing the effect on the turbulent energy intensity (in the first approximation) can be neglected.

As a result, in paper [68] a transcendental expression for turbulence energy in two-phase flow was obtained, which contains two sets of physical parameters responsible for the values of additional generation (parameter complex Y) and additional dissipation (parameter complex X) of turbulence energy of the carrier gas:

$$\frac{k}{k_0} = \frac{1 + (k_0/k)^{1/2} b \mathrm{Y}}{1 + 2\mathrm{X}/C_\mu^{1/2}}, \tag{15}$$

where $b = \frac{a}{C_\mu^{3/4}\beta^{4/3}}$, $\mathrm{Y} = \frac{C_D^{4/3}\Phi\, \overline{W}^3 \bar{l}}{\bar{d}_p\, \bar{k}_0^{3/2}}$, $\mathrm{X} = \frac{\mathrm{M}}{1+C_\mu^{1/4}(\bar{\tau}_p\, \bar{k}_0^{1/2}/\bar{l})(k/k_0)^{1/2}}$, $C_\mu = 0.09$, $a = 0.027$, $\beta = 0.2$, $\overline{W} = W/u_{\tau 0}$—dimensionless relative velocity, $\bar{d}_p = d_p/R$—dimensionless diameter of particles, $\bar{l} = l/R$—dimensionless length of the Prandtl–Nikuradze mixing path, $\bar{\tau}_p = \tau_p\, u_{\tau 0}/R$—dimensionless dynamic relaxation time, $\bar{k}_0 = k_0/u_{\tau 0}^2$—dimensionless turbulence energy of single-phase flow. Thus, the dynamic velocity (friction velocity) of the single-phase flow acts as the velocity scale $u_{\tau 0}$, and as a geometric scale—the radius of the tube R.

According to (15), the effect of relatively small particles on turbulence is determined primarily by mass concentration and dimensionless relaxation time $\bar{\tau}_p$. As the inertia of the particles decreases (to a certain degree), the laminarizing effect of the dispersed phase on the flow increases. The parameter $\bar{l}$ n (15) takes into account the increasing influence of the dispersed phase with increasing distance from the wall, which is explained by a decrease in the relative inertia of the particles due to an increase in the time scale of turbulence.

In (15), it can be seen that the effect of large particles on turbulence is mainly determined by the volume concentration Φ (rather than mass concentration M, as in the case of fine impurities), dimensionless speed of interfacial slip $\overline{W}$ and dimensionless diameter $\overline{d}_p$. As the particle size increases, the turbulizing effect of the dispersed phase on the flow increases (since the value $C_D^{4/3}\overline{W}^3/\overline{d}_p$ is increasing). The parameter $\bar{l}$ in (15) shows that the turbulizing effect of large particles, as well as the laminarizing effect of small particles accordingly, increases with increasing distance from the wall.

The experimental data [37], summarized in coordinates k/k_0 from X (for low-inertia particles Y $\equiv$ 0). We can conclude that the experimental points are sufficiently tightly grouped as a dependence of k/k_0 on X and are adequately described by the Formula (15). It should be emphasized that Formula (15) does not satisfy the obvious limit transition for the case of inertialess particles: $k \to k_0$ when $\tau_p \to 0$. Therefore, Formula (15) is valid for the analysis of influence on turbulence of particles, relaxation time of which is longer than time microscale. In order to provide a correct limit transition at calculating the additional dissipation coefficient, the relation given in [57,58] can be used.

Paper [36] presents experimental data, generalized in coordinates k/k_0 from Y (for large particles X $\equiv$ 0). It can be seen that the experimental points generalize quite well in the form of dependence of k/k_0 on Y and are well described by Formula (15).

Good correlation between the calculation data and the available experimental results gives hope for the efficiency of the described mathematical model in case of non-equilibrium flow (see Section 2.4), when both mechanisms (laminarizing and turbulizing) of particles influence on turbulence may act together.

4. Studies on the Effect of Particles on Turbulence

The works of recent years will be described below. Additionally, in this section, the peculiarities of the influence of particles on the turbulence energy of the carrier phase for different classes of flows are considered: homogeneous isotropic turbulence, homogeneous shear flow, and near-wall turbulence in a pipe (channel).

In [71], an original two-parameter turbulence model for gas flow containing solid particles is developed. The main assumptions of the model are: (1) the motion of incompressible viscous gas carrying small heavy particles is considered, i.e., $\rho_p/\rho = O(10^3)$; (2) particle size does not exceed the Kolmogorov spatial microscale ($d_p \le \eta$); (3) the volume concentration of particles is assumed to be small $\Phi << 1$, so that the collisions of particles with each other can be neglected; (4) the mass concentration M can be quite significant.

Under these assumptions, the equations for the carrying gas phase were written in the point forces approximation. In the work [71], the main force determining the behavior of particles in a turbulent flow and their inverse effect on its characteristics was the aerodynamic drag force.

In Ref. [71], a very important conclusion was made that the presence of particles affects the vast majority of the terms of the turbulence energy transfer equation. The gas turbulent energy balance equation $k = \sum_i \frac{\overline{u_i' u_i'}}{2}$ was obtained in the following form [71]:

$$K = D + P - \varepsilon + A_k, \tag{16}$$

where $K = \frac{\partial k}{\partial \tau} + \sum_j U_j \frac{\partial k}{\partial x_j}$, $D = \sum_j \frac{\partial}{\partial x_j}\left(\nu \frac{\partial k}{\partial x_j} - \frac{\overline{u_i' u_i' u_j'}}{2} - \frac{\overline{u_j' p'}}{\rho}\right)$, $P = -\sum_j \sum_i \overline{u_i' u_j'} \frac{\partial U_i}{\partial x_j}$, $\varepsilon = \nu \sum_j \sum_i \overline{\frac{\partial u_i'}{\partial x_j} \frac{\partial u_i'}{\partial x_j}}$, $A_k = -K_p + D_p + P_p - \varepsilon_p + G_p$.

The terms of the right side of the expression for A_k can be interpreted [71] as additional convection K_p caused by the presence of particles in the flow, additional diffusion

D_p, additional generation P_p, additional dissipation ε_p and the effect of inhomogeneous distribution of particles in space G_p.

These terms have the form [71]:

$$K_p = \mathrm{M}\, f_{u1}\left(\frac{\partial\, k}{\partial \tau} + \sum_j U_j \frac{\partial\, k}{\partial x_j}\right), \tag{17}$$

$$D_p = -\mathrm{M}\, f_{u1} \sum_j \frac{\partial \overline{u_i'\, u_i' u_j'}_p}{\partial x_j}\ , \tag{18}$$

$$P_p = -\mathrm{M} \sum_j \sum_i \overline{u_i' u_j'} \left(g_u \frac{\partial V_i}{\partial x_j} - l_u \frac{\partial U_i}{\partial x_j}\right), \tag{19}$$

$$\varepsilon_p = \frac{2\mathrm{M}}{\tau_p}(1 - f_u)k, \tag{20}$$

$$G_p = \mathrm{M}\, g_u \sum_j (U_{pi} - V_i)\, \overline{u_i' u_j'} \frac{\partial \ln \Phi}{\partial x_j}. \tag{21}$$

Here, U_{pi}—gas velocity "visible" by particles. The lower index «p» of the triple correlation in (18) explains its origin. Triple correlation occurs due to the presence of fluctuation drift velocity between gas and particles due to inhomogeneous distribution of the dispersed phase.

The values f_u, f_{u1}, g_u and l_u are coefficients of particle involvement in the turbulent gas flow and are functions of the dynamic relaxation time, the integral and differential timescales characterizing the interaction of particles with turbulent vortices. For simplicity, they are assumed to be scalars and are described by relations corresponding to the bi-exponential autocorrelation function [72].

The second basic equation of the mathematical model [71] was the turbulent energy dissipation equation of the form:

$$K_\varepsilon = D_\varepsilon + C_{\varepsilon 1}\frac{\varepsilon}{k}P - C_{\varepsilon 2}\frac{\varepsilon^2}{k} + C_{\varepsilon 3}\frac{\varepsilon}{k}A_k, \tag{22}$$

where $K_\varepsilon = \frac{\partial\, \varepsilon}{\partial \tau} + \sum\limits_j U_j \frac{\partial\, \varepsilon}{\partial x_j}$, a $C_{\varepsilon 1}$, $C_{\varepsilon 2}$, $C_{\varepsilon 3}$—constants. It should be noted that, compared to the case of single-phase flow, Equation (22) contains an additional constant $C_{\varepsilon 3}$. Various values of the constant $C_{\varepsilon 3}$, for example, have been used in the literature: 1.2 [73], 2.0 [74], 1.9 [75]. In [66], the value $C_{\varepsilon 3}$ was assumed to depend on the mass concentration of particles, with $C_{\varepsilon 3} = 1.2$ when $\mathrm{M} \to 0$.

In the study [71], $C_{\varepsilon 3}$ is determined from the requirement that inertia-free particles contribute to convection, diffusion and generation of turbulent energy by increasing the effective flow density, but do not affect the destruction of turbulence dissipation. For this purpose it is necessary to take $C_{\varepsilon 3} = C_{\varepsilon 2}$. In this case, Equation (22) takes the form [71]:

$$K_\varepsilon = D_\varepsilon + C_{\varepsilon 1}\frac{\varepsilon}{k}P - C_{\varepsilon 2}\frac{\varepsilon(\varepsilon - A_k)}{k}. \tag{23}$$

Based on the developed model, it was possible to analyze various cases of two-phase flows [71].

4.1. Inertialess Particles

In the case of gas flow with inertialess particles ($\tau_p \to 0$, $f_u = f_{u1} = 1$, $\partial V_i/\partial x_j = \partial U_i/\partial x_j$, $g_u - l_u = 1$) asymptotic relations follow from (16)–(21) [71]:

$$(\lim K_p)_{\tau_p \to 0} = -\mathrm{M}K, \tag{24}$$

$$(\lim D_p)_{\tau_p \to 0} = -\mathrm{M}D, \tag{25}$$

$$(\lim P_p)_{\tau_p \to 0} = -\mathrm{M}P, \tag{26}$$

$$(\lim \varepsilon_p)_{\tau_p \to 0} = (\lim G_p)_{\tau_p \to 0} = 0. \tag{27}$$

Thus, for the term A_k, characterizing the impact of particles on the turbulence energy, in the inertialess limit from (16) we have [61]:

$$(\lim A_k)_{\tau_p \to 0} = -\mathrm{M}(K - D - P). \tag{28}$$

Substituting (28) into (16) allows us to obtain the turbulence energy balance equation in the following form [71]:

$$K = D + P - \frac{\varepsilon}{1 + \mathrm{M}}, \tag{29}$$

Expression (29) has a deep physical meaning and allows us to draw an important conclusion about the influence of particles on convection, diffusion and turbulence generation by increasing the flow density by a factor of $(1 + \mathrm{M})$ and no influence on dissipation due to viscosity. This conclusion is consistent with the conclusions of [77–80].

4.2. Homogeneous Isotropic Turbulence (HIT)

Homogeneous isotropic turbulence is the simplest class of turbulent flows. The first studies devoted to direct numerical simulation (DNS) of the behavior of small particles in homogeneous isotropic turbulence (HIT) were [81–83]. An important characteristic is the ratio of the particle size d_p to the shortest turbulence length, the Kolmogorov spatial scale η, i.e., d_p/η. If the particle size is smaller than the Kolmogorov spatial scale ($d_p < \eta$), then their effect on turbulence will be determined by their response, defined by the value of τ_p, on the Kolmogorov time scale τ_K. At sufficiently large Stokes number ($\mathrm{Stk_K} >> 1$) the influence of particles will be determined by the process of their interaction with large vortices, characterized by other time scales, it is clear that the Stokes number $\mathrm{Stk_K}$ in this case ceases to be a defining parameter.

In [71] the case of unsteady degenerate turbulence behind the lattice in the absence of gravity is considered. In this case diffusion and generation of turbulence are absent ($D = 0$, $D_\varepsilon = 0$, $P = 0$), and Equations (16) and (22) and take a form [71]:

$$(1 + \mathrm{M}f_{u1})\frac{dk}{d\tau} = -\left[\varepsilon + \frac{2\mathrm{M}}{\tau_p}(1 - f_u)k\right], \tag{30}$$

$$(1 + \mathrm{M}f_{u1})\frac{d\varepsilon}{d\tau} = -C_{\varepsilon 2}\frac{\varepsilon}{k}\left[\varepsilon + \frac{2\mathrm{M}}{\tau_p}(1 - f_u)k\right], \tag{31}$$

where $C_{\varepsilon 2} = 1.8$.

The initial conditions for (30) and (31) are as follows:

$$k = k_0,\ \varepsilon = \varepsilon_0 \text{ when } \tau = 0. \tag{32}$$

The system of Equations (30) and (31), taking into account (32), has an integral $\varepsilon = \varepsilon_0 (k/k_0)^{C_{\varepsilon 2}}$.

The coefficients of particle involvement in turbulent gas motion f_u and f_{u1}, as before, are assumed to be scalars and are described by relations corresponding to the bi-exponential autocorrelation function [72].

The results of solving problem (30)–(32) on turbulent energy attenuation are shown by Ref. [71]. The initial Reynolds number Re_λ was equal to 75. Particles were introduced into the flow at a certain time moment $\overline{\tau} = 1$ ($\overline{\tau}$—dimensionless time [75]) with mass concentration $\mathrm{M} = 1$.

It can be seen [71] that the introduction of extremely low-inertia particles into the flow (Stokes number in small-scale pulsation at the moment of particle introduction into the flow $\mathrm{Stk}_K = 0.1$) leads to deceleration of turbulence degeneration rate. This effect is associated with the contribution of microparticles to the turbulent energy transfer due to an increase in the effective density of the two-phase flow. In the limit of inertialess particles ($\mathrm{Stk}_K \to 0$, $f_u = f_{u1} = 1$) the task (30)–(32) is reduced to the corresponding one-phase flow problem by introducing the effective time $\widetilde{\tau} = \tau/(1 + \mathrm{M})$.

The influence of more inertial particles ($\mathrm{Stk}_K = 0.25$) on the turbulent energy of the carrier gas is small. A further increase in the inertia of the particles leads to a more rapid decay of the turbulence energy due to an increase in the additional dissipation of turbulence ε_p and a reduced role of additional convection K_p due to a decrease in the coefficient of involvement f_{u1}.

Note that the above calculation results [71] agree well with the data [79] obtained by direct numerical simulation (DNS).

Among the recent studies, we should mention papers [84–86] devoted mainly to direct numerical simulation (DNS) of the forced (stationary) homogeneous isotropic flow [84], damping homogeneous isotropic turbulence [85], as well as the formation of heavy particle clusters in the homogeneous isotropic turbulence (HIT) [86].

In [84], a direct numerical simulation investigates the interaction between a stationary homogeneous isotropic turbulent flow and inertial particles, taking into account inter-particle collisions (four-way coupling). The calculations were performed for a 128^3 periodic cubic cell for two values of the Taylor Reynolds number ($\mathrm{Re}_\lambda = 35.4$ and $\mathrm{Re}_\lambda = 58$) while varying the volume concentration of particles (from $\Phi = 1.37 \cdot 10^{-5}$ to $\Phi = 8.22 \cdot 10^{-5}$) and the Stokes numbers ($\mathrm{Stk_K} = \tau_p/\tau_\mathrm{K} = 0.19$–$12.7$). The disperse phase was elastic spherical particles with a diameter of $d_p = 67.6$ µm, which corresponds to $d_p/\eta = 0.1$. The Stokes number was changed by varying the density of the particles over a very wide range: $\rho_p = 150$–$150{,}000\ \mathrm{kg/m^3}$.

The results of [84] demonstrated that the dissipation decreases up to 32% with increasing Stokes number and volume concentration of particles. It is shown that the indicated value of the maximum dissipation decrease is overestimated by 7% if interparticle collisions are taken into account. Spectral analysis revealed energy transfer by particles from large to small scales, which explains the difference in dissipation.

In [85], by direct numerical simulation, the effect of monodisperse sub Kolmogorov ($d_p < \eta$) Stokesian ($\mathrm{Re_p} << 1$) particles on the damping of homogeneous isotropic turbulence. The paper emphasized the influence of the numerical concentration of particles. The calculations varied independently the Stokes number ($\mathrm{Stk_K} = 0.3$–4.8), the mass concentration of particles ($\mathrm{M} = 0.001$–0.3) and the number of particles in the Kolmogorov vortex ($\mathrm{N}_\eta = 0.07$–17). The numerical particle concentration N_0 and the number of particles in the Kolmogorov vortex N_η are related as $\mathrm{N}_\eta = \mathrm{N}_0\eta^3$.

The calculations made allowed us to clearly distinguish two modes. At $\mathrm{Stk_K} < 1$ the presence of particles leads to a decrease in turbulence energy attenuation (first mode). At $\mathrm{Stk_K} > 1$ particles accelerate turbulence attenuation (second mode).

In [86], the dynamics of formation and decay of clusters of small heavy particles under homogeneous isotropic turbulence was studied by numerical simulation. The knowledge of cluster dynamics is important for understanding the processes of interparticle interaction and the inverse effect of the dispersed phase on the turbulence of the carrier flow. Lagrangian simulations of particle dynamics are performed to track them before, during, and after they become part of a coherent cluster. A certain criteria, which answers the question of cluster survival on consecutive time steps and is used to characterize its life cycle, is proposed. The calculations performed have found that cluster life has a typical duration at several Kolmogorov time scales with a positive correlation between cluster size and lifetime. Increasing of inertia and gravitational settling lead to longer lifetimes. Small clusters arise as a result of coagulation of unclustered particles, followed quickly by decay into mostly unclustered particles. In contrast, large clusters arise from recombination of other large clusters. The birth of a cluster is preceded by an exponential contraction of the particle cloud, and its demise coincides with the beginning of a slower exponential expansion.

4.3. Homogeneous Shear Flow

The homogeneous flow with a constant shear rate is of great fundamental importance since in this case the triple correlations of gas and particle velocities are equal to zero, due to what the chain of equations breaks at the level of double correlations.

Due to the homogeneity of the flow, the concentration of particles does not change in space, and the gradients of the carrier and dispersed phases are equal and are given in the form [71]:

$$\frac{dU_i}{dx_j} = \frac{dV_i}{dx_j} = S\,\delta_{ix}\delta_{jy}, \tag{33}$$

where S—shear rate, and x with y—coordinates in axial and transverse direction.

In this case, Equations (16) and (22), taking into account (33) and the expression for turbulent viscosity [71], take the form:

$$(1 + \mathrm{M} f_{u1})\frac{dk}{d\tau} = (1 + \mathrm{M} f_{u1})P - (\varepsilon + \varepsilon_p), \tag{34}$$

$$(1 + \mathrm{M} f_{u1})\frac{d\varepsilon}{d\tau} = \frac{C_{\varepsilon 1}\varepsilon(1 + \mathrm{M} f_{u1})P}{k} - C_{\varepsilon 2}\frac{\varepsilon}{k}(\varepsilon + \varepsilon_p), \tag{35}$$

$$P = \nu_t S^2,\ \nu_t = \frac{C_\mu(1 + \mathrm{M}\, f_{u1})k^2}{\varepsilon + \varepsilon_p + \left[(1 + \mathrm{M}\, f_{u1})P - (\varepsilon + \varepsilon_p)\right]/C_1},\ \varepsilon_p = \frac{2\mathrm{M}}{\tau_p}(1 - f_u)k. \tag{36}$$

The constants in (34)–(36) are usually taken to be $C_{\varepsilon 1} = 1.5$, $C_{\varepsilon 2} = 1.8$, $C_\mu = 0.09$, $C_1 = 1.1$.

Equations (34) and (35) in the absence of turbulence generation due to velocity shift ($S = 0$, $P = 0$) transform to Equations (30) and (32). Initial conditions for Equations (34) and (35) are set as (32).

The results of solving problem (34)–(36) on the time evolution of the turbulent energy of a homogeneous shear flow is shown in [71]. The initial Reynolds number Re_λ was assumed to be equal to 20. Particles were introduced into the flow at some point of time $\overline{\tau} = 1$ ($\overline{\tau}$—dimensionless time [80]) at the moment of particle introduction into the flow and the mass concentration of particles were varied M.

The most low-inertia of the considered particles ($\mathrm{Stk}_K = 0.233$) cause the growth of turbulent energy due to their contribution to the generation from the averaged motion. Note that in the limit of inertialess particles ($\mathrm{Stk}_K \to 0$, $f_u = f_{u1} = 1$) the problem (34)–(36) s reduced to the corresponding problem for a single-phase flow by introducing the effective time $\widetilde{\tau} = \tau/(1 + \mathrm{M})$ and shear $\widetilde{S} = (1 + \mathrm{M})S$. Thus, the turbulization of the flow due to the presence of microparticles is determined by two physical mechanisms: (1) a decrease in the effective damping time of the initial turbulence and (2) an increase in the effective shear rate. The introduction of more inertial particles ($\mathrm{Stk}_K = 0.583$ and $\mathrm{Stk}_K = 2.33$) into the flow leads to a decrease in turbulence energy due to an increase in additional dissipation of turbulent energy ε_p. Increasing of mass concentration leads to an increase in the effect of particles on turbulence. Note that the calculation results given above are in a good agreement with the data [80] obtained by direct numerical simulation.

Among the recent studies of this class of two-phase flows, we note the work [87]. In this work, a numerical simulation with interface-resolved modification of statistically steady-state homogeneous shear turbulence (HST) by finite-sized particles with neutral buoyancy was performed. Two forms of particles—spherical and flattened at different volume concentrations of the latter (up to $\Phi = 0.2$). Calculations have demonstrated that equilibrium turbulence, when there is a balance between turbulence energy generation and dissipation rate, is inherent not only to single-phase flows, but also to two-phase flows. The turbulent kinetic energy (TKE) was found to exhibit non-monotonic behavior with increasing volume concentration of particles: a decrease in TKE up to a certain particle concentration and then an increase in TKE at higher concentrations. This behavior is observed at lower volume concentrations for flattened particles than for spherical particles. The suppression of TKE at low concentrations is explained by an increase in the turbulence dissipation rate near the particle surface. At higher concentrations, the interaction of neighboring particles forms regions of high Reynolds stresses, causing the TKE to increase.

4.4. Near-Wall Turbulence in the Pipe (Channel)

The study of particle motion in the flow field of carrying gas, when there are gradients of time-averaged and fluctuation velocities and temperatures (in case of non-isothermal flow) in radial direction, is by itself a difficult problem. Gradient profiles of carrying gas leads to inhomogeneity of force factors acting on a particle in longitudinal and radial directions. This is the reason for the formation of significantly heterogeneous profiles of time-averaged and fluctuation velocities, temperatures and particle concentrations. The presence of shear profiles of particle characteristics significantly complicates the study of their inverse effect on the parameters of the carrying medium. Thus, the complexity of two-phase flows in pipes (channels) has led to the fact that they remain poorly studied, despite the significant number of available studies.

In [71] a near-wall flow in a pipe is considered, where there is an approximate equality of generation and dissipation of turbulent energy. By neglecting convective transport K_ε and using a simple algebraic relation for D_ε, and assuming that the time-averaged longitudinal velocity profile U_x and the spatial scale of turbulence l in the change only slightly, we analyze the effect of particles on turbulence energy. As a result of the analysis, the following expression was obtained [71]:

$$\frac{k}{k_0} = \frac{1 + \mathrm{M}\, f_{u1}}{1 + \varepsilon_p/\varepsilon}, \tag{37}$$

where $k_0 = (l^2/C_\mu^{1/2})(\partial U_x/\partial y)^2$ and $\varepsilon_p/\varepsilon = (2\mathrm{M}(1 - f_u))/(C_\mu^{1/2}\Omega)$—single-phase flow turbulence energy and the ratio of additional dissipation of turbulence energy to viscous dissipation, respectively.

The resulting expression (37) has a perfectly clear physical meaning [71]: the numerator characterizes the increase in turbulence energy due to the contribution of particles to turbulence generation, and the denominator describes its decrease due to additional dissipation.

In addition, Ref. [71] emphasizes the effect of particle inertia (Stokes number in large-scale fluctuation motion Stk_L) on turbulence intensity (ratio of turbulence energies in two-phase and single-phase flows). The calculation was performed using relation (37) at $\mathrm{M} = 1$ ratio of differential and integral time scales of turbulence). It confirms that with increasing Stokes number, turbulizing effect of particles due to their contribution to turbulence generation is replaced by laminarizing effect due to additional dissipation. As follows from (37), at high values of Stokes number the role of additional dissipation ε_p becomes insignificant and influence of particles on turbulence disappears. As the parameter decreases, the area of small values of Stokes number, where the turbulizing effect of particles takes place, narrows and disappears at $z \to 0$.

Among recent studies, let us highlight the works [88–94] devoted to measurements of statistical characteristics of particle motion in a vertical channel [88], numerical modeling of the effect of several stationary ones on the turbulence of the carrier gas [90], the effect of small particles on turbulence and drag [91], the effect of particles of different inertia in resolving the interphase boundary [92]. In [93,94] an attempt has been made to take into account all three main mechanisms of particle influence on the turbulence energy of the carrier phase when the inertia of the latter varies in a very wide range—macro-, micro- and nanoparticles.

In [89] the results of an experimental study of the motion of glass particles in an ascending flow of air in a vertical channel at Reynolds numbers $\mathrm{Re}_\tau = 235$ and $\mathrm{Re}_\tau = 335$. The particle size was chosen so that the Stokes numbers, constructed using the Kolmogorov and viscous scales, were $\mathrm{Stk_K} = O(10)$ and $\mathrm{Stk}_\tau = O(100)$ respectively. Two values of the volume concentration of particles were considered—$\Phi = 3 \cdot 10^{-6}$ и $\Phi = 5 \cdot 10^{-5}$. At low concentrations, the distribution of particles has maximums in the near-wall region and near the channel axis. The time-averaged particle velocity was equal to the velocity of the carrier air except for the near-wall region. Particle velocity fluctuations are close to single-phase flow fluctuations in the whole flow core, exceeding the latter in the wall region. At high concentration of particles all statistical characteristics of disperse phase motion undergo changes. The near-wall concentration peak became more distinct, and the maximum in the center of the channel disappeared. The time-averaged particle velocity decreased in the logarithmic and buffer regions. The ripple velocities of particles and their deposition rate on the wall increased. The difference between the behavior of particles in the central and wall regions was revealed. In the channel core, dense clusters of particles are formed, elongated in longitudinal direction and having higher velocities. In the near-wall region the particles are arranged in the form of strongly elongated strips having length of several channel heights and located in the region $y^+ = O(100)$. This suggests that the particles are concentrated in the formation of low-velocity streaks in the carrier air, which are inherent to near-wall turbulence.

In [90] the results of direct numerical simulation (DNS) of turbulent two-phase flow in a channel are presented. Several stationary particles located in the near-wall region were introduced into the flow. The aim of the work was to study the interaction of wall-generated turbulence with particles and the influence of particles on the global turbulence statistics. A complete analysis of all terms of the equation for TKE. It is revealed that particles modify turbulence through two competing mechanisms. The first mechanism is a decrease in TKE due to a decrease in the time-averaged velocity shift due to aerodynamic drag caused by the presence of particles. The second mechanism is an increase in TKE due to displacing the flowing fluid due to the presence of particles. An analysis of the component-by-component equation for turbulent kinetic energy revealed that the presence of particles leads to a more isotropic distribution of kinetic energy between the three velocity components.

In [91], the effect of the presence of small inertial particles on the modification of the wall turbulence was studied. Three values of the mass concentration of particles were considered—M = 0.0034, M = 0.0337 and M = 0.337. It is noted that only at two large values of concentration the effect of particles on turbulence was revealed. In the case of extremely low concentrations, the particles had almost no effect on the statistical characteristics of turbulence, but their accumulation in the near-wall region caused an increase in drag, similar to what happens in single-phase flows with a slight increase in the Reynolds number. At higher concentrations, the particles change the turbulence dynamics throughout the flow (not just in the wall region), and the interaction between the phases becomes more complicated. Although the Reynolds stresses decreased at moderate concentrations, the particles caused an increase in drag in the wall region due to correlated velocity fluctuations, which ultimately resulted in an increase in overall drag as well.

The paper [92] presents the results of direct numerical simulation (DNS) with interface-resolved turbulent two-phase upward flow in a vertical channel. The influence of the Reynolds number of the particle ($\mathrm{Re}_p < 227$), particle size, bulk Reynolds number ($\mathrm{Re}_b = 5746$ and $\mathrm{Re}_b = 12{,}000$), phase density ratios ($\rho_p/\rho = 2$–100), the ratio of the particle radius to the half-width of the channel ($2r_p/H = 0.05$–0.15), volume concentration of particles ($\Phi = 3 \cdot 10^{-3}$–$2.36 \cdot 10^{-2}$) on the intensity of the carrier phase fluctuation velocity was studied. The calculation results show that at small values Re_p the turbulence intensity decreases throughout the channel cross section. At medium values of Re_p the turbulence intensity increases in the central region of the channel and decreases in the near-wall region. At high values of Re_p the turbulence intensity increases across the entire channel cross section. The critical value Re_p increases with increasing volume Reynolds number, particle size and phase density ratio, and also increases with decreasing volume concentration of particles.

Recently published research results [93–100] are a continuation of works [57–59]. Using a two-parameter $k-\varepsilon$ turbulence model for two-phase flow, an expression was obtained to analyze the effect of particles of different inertia on the turbulence energy of the carrier phase:

$$\frac{k}{k_0} = \frac{1 + (P_{pS} + P_{pL})/P}{1 + \varepsilon_p/\varepsilon}. \tag{38}$$

where $k_0 = (l^2/C_\mu^{1/2})(\partial U_x/\partial y)^2$—turbulent energy of a single-phase flow, $P_{pS} = \mathrm{M}\, f_\varepsilon\, P$—additional generation of turbulence energy by extremely low-inertia particles [62], $P_{pL} = a(C_D/\beta)^{4/3}\Phi\left(W^3/d_p\right)$—additional generation of turbulence energy in the wakes behind large particles [58], P—the term responsible for the generation of turbulence energy in the single-phase flow, $\varepsilon_p/\varepsilon = 2\mathrm{M}(1-f_u)/(C_\mu^{1/2}\Omega)$—the ratio of the additional dissipation caused by the presence of particles to the viscous dissipation [72].

The formula (38) expresses, in equilibrium approximation, the influence of particles on the intensity of near-wall turbulence and has a clear physical meaning: the numerator describes an increase in turbulence energy due to the particle contribution to turbulence generation by increasing the effective viscosity and turbulization in the wake of large particles, while the denominator describes its decrease due to additional dissipation. Thus, expression (38) takes into account all three main mechanisms of the influence of particles on the turbulence energy of the carrier gas.

Let us illustrate this by the example of calculations performed in [84]. The turbulence energy of the two-phase flow was calculated using relation (38), taking into account the expressions for P_{pS}, P_{pL}, and $\varepsilon_p/\varepsilon$ when $z = 0.2$ for three values of the mass concentration of particles. The calculations were performed with the following single-phase flow parameters: $k_0 = 0.75\ \mathrm{m}^2/\mathrm{s}^2$, $T_L = 0.015$ sec and $l = 0.007$ m.

Article [94] consists the effect of particle inertia (Stokes number in large-scale fluctuation motion Stk_L) on turbulence intensity (the ratio of turbulence energies in two-phase and single-phase flows. The data above indicates that as Stokes number increases, the turbulizing effect ($k/k_0 > 1$) of particles due to their contribution to turbulence generation is replaced by a laminarizing effect ($k/k_0 < 1$) due to additional dissipation and, then again, is replaced by a turbulizing effect ($k/k_0 > 1$) due to generation of turbulence in the wakes behind large particles.

5. Conclusions

An analysis of works devoted to the study of the motion of solid particles in gas flows, and their inverse effect on the parameters of the carrier phase is performed. The main physical mechanisms and dimensionless criteria determining the direction and magnitude of the impact of particles of different inertia on the turbulence energy of the carrier phase are described. The analysis of studies carried out in the review clearly showed that, in the general case, the presence of particles in a gas flow affects all terms without exception in the turbulent kinetic energy balance equation. The peculiarities of the influence of particles on the turbulence energy of gas for different classes of flows: homogeneous isotropic turbulence, homogeneous shear flow, wall turbulence in a pipe (channel) are analyzed. The review has shown that in various specific cases of great theoretical or practical importance, some terms of the turbulent kinetic energy balance can be neglected and, consequently, the influence of particles on these terms can also be neglected.

This review allowed us to formulate priority areas for future research in turbulent two-phase flows:

(i) Detailed experimental studies of particle motion and its inverse effect on the characteristics of the carrier gas when the inertia of the dispersed phase changes in a very wide range (by several orders of magnitude);

(ii) Detailed modeling of the motion features of particles of different inertia in turbulent flows with resolution of the interphase boundary and taking into account interparticle collisions;

(iii) Physical and numerical modeling of turbulence induced by large particles;
(iv) Studies that would focus on the features of turbulence modification for two cases: the presence in the flow of particles that do not differ greatly in size (ordinary polydisperse particles) and the presence in the flow of particles that differ greatly in size (having a hybrid mixed effect on turbulence).

In conclusion, it is important to note the importance of studying high-speed compressible two-phase flows.

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Nomenclature

d_p	particle diameter, m
ρ_p	particle radius, m
D	pipe diameter, m
R	pipe radius, m
H	channel width, m
δ	characteristic geometric size, m
l	characteristic time of turbulence, Prandtl mixing length, m
τ	time, s
τ_p	dynamic relaxation time of particle, s
τ_{p0}	dynamic relaxation time of Stokesian particle, s
T_f	characteristic time of gas in time-averaged motion, s
T_L	characteristic time of gas in large-scale fluctuation motion, s
τ_K	Kolmogorov time microscale, s
τ_e	characteristic time of disturbance, s
ω_p	particle characteristic frequency, s^{-1}
ω	characteristic frequency of disturbance, s^{-1}
λ	Taylors spatial scale of turbulence, m
η_K	Kolmogorov spatial microscale, m
ρ	carrier gas density, kg/m^3
ρ_p	particle density, kg/m^3
$\mathbf{u}$	vector of carrier gas actual velocity, m/s
$\mathbf{v}$	vector of particle actual velocity, m/s
$\mathbf{w}$	vector of relative actual velocity, m/s
U_x, U_y, U_z	projections of carrier gas time-averaged velocity, m/s
u'_x, u'_y, u'_z	projections of carrier gas fluctuation velocity, m/s
V_x, V_y, V_z	projections of particle time-averaged velocity, m/s
v'_x, v'_y, v'_z	projections of particle fluctuation velocity, m/s
W_x	relative velocity in axial direction, m/s
u_τ	dynamic (friction) velocity, m/s
μ	dynamic viscosity of carrier gas, kg/(ms)
ν	kinematic viscosity of carrier gas, m^2/s
k	turbulent kinetic energy, m^2/s^2
K	convection term in turbulent kinetic energy equation, m^2/s^2
D	diffusion term in turbulent kinetic energy equation, m^2/s^3
P	production term in turbulent kinetic energy equation, m^2/s^3
ε	turbulent kinetic energy dissipation rate, m^2/s^3
K_p	additional convection term due to particles, m^2/s^2

D_p	additional diffusion term due to particles, m^2/s^2
P_p	additional production term due to particles, m^2/s^3
ε_p	additional energy dissipation rate due to particles, m^2/s^3
N	numerical concentration of particles, m^{-3}
Dimensionless parameters	
f_u, f_{u1}, g_u, l_u	coefficient of particle involvement in turbulent motion
C_D	aerodynamic drag coefficient
M	mass concentration of particles
Φ	volume concentration of particles
Re_p	Reynolds number of particle
Re_λ	Taylor Reynolds number
Re_τ	viscous Reynolds number
Stk_f	Stokes number in time-averaged motion
Stk_L	Stokes number in large-scale fluctuation motion
Stk_K	Stokes number in small-scale fluctuation motion
Indexes	
$\overline{(\ldots)}$	nondimensional value, time-averaged value
$(\ldots)'$	fluctuation value
$\sqrt{(\ldots)'^2}$	root mean square (rms) value
Subscripts	
0	one-phase flow
c	pipe axis, channel center
L	largest scale fluctuation
l	smallest scale fluctuation
i	i-th component of velocity
i	i-th value
m	modified value

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