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$(I, J^P) = (1, 1/2^+)$ ΣNN Quasibound State

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Abstract: JLab has recently found indications of the possible existence of a ΣNN resonance at $(3.14 \pm 0.84) - i(2.28 \pm 1.2)$ MeV. In the past, using models that exploit symmetries between the two-baryon sector with and without strangeness, hyperon–nucleon interactions that reproduce the experimental data of the strangeness -1 sector have been derived. We make use of these interactions to review the existing Faddeev studies of the $\Lambda NN - \Sigma NN$ system that show theoretical evidence of a $(I, J^P) = (1, 1/2^+)$ ΣNN quasibound state near the threshold. The calculated position of the pole is at 2.92 $-i$ 2.17 MeV, which is in reasonable agreement with the experimental findings.

Keywords: multiquarks; quark models; few-body systems

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The Hall A Collaboration at Jefferson Lab made use of the $(e, e'K^+)$ reaction to study the possible existence of neutral three-body Λ and Σ hypernuclei [1]. They reported an excess of events around the Σ thresholds. The most significant enhancement appeared 3.14 ± 0.84 MeV below the $\Sigma^0 nn$ threshold and had a width of $\sigma \approx 2.28 \pm 1.2$ MeV. This possibly hints at a bound $\Sigma^0 nn$ $(I = 1)$ state.

The existing experimental data and the expected forthcoming optimized data call for theoretical studies that could help with their interpretation. In this letter, it is our purpose to emphasize the relevant findings of the existing Faddeev studies of the $\Lambda NN - \Sigma NN$ system. The theoretical results obtained are a valuable tool for analyzing the Hall A Collaboration data.

We carried out a detailed study of the $\Lambda NN - \Sigma NN$ three-body system at the threshold to look for bound states or resonances [2]. The strangeness -1 two-body interactions were derived from the chiral quark cluster model (CQCM) [3] by exploiting the symmetries with the two-nucleon sector. In the CQCM, hadrons are clusters of massive (constituent) quarks. As color carriers, massive quarks are confined through a confining potential. They interact through a one-gluon exchange potential arising from the perturbative effects of quantum chromodynamics (QCD). The non-perturbative effects generate one-boson exchange potentials between quarks [3,4].

The nucleon–nucleon (NN) and hyperon–nucleon (YN) interactions describe the NN and YN two-body observables reasonably well [4]. In particular, the low-energy parameters, NN $S-$wave phase shifts, and triton binding energy in a two-nucleon system were described correctly [3]. In addition, there was a reasonable agreement with the hyperon–nucleon elastic and inelastic scattering cross-sections and the hypertriton binding energy. Finally, the isospin one-Λnn system was found to be unbound [2,4,5].

At the two-body level, the $N\Lambda - N\Sigma$ coupling and the tensor force, which is responsible for the coupling between S and D waves, have been considered. The $\Lambda \leftrightarrow \Sigma$ conversion is crucial in order to have a correct description of the ΛNN system [2]. The NN and YN interactions contain sizable non-central terms that are responsible for, among other things, the deuteron binding energy. The relevance of the YN tensor force becomes apparent when studying the $\Sigma^- p \to \Lambda n$ reaction. This process is controlled by the $\Sigma N(\ell = 0) \to$

$\Lambda N(\ell = 2)$ transition, so if only the central interaction $\Sigma N(\ell = 0) \to \Lambda N(\ell = 0)$ is considered, the cross-section cannot be correctly described [6]. The non-central $N\Lambda - N\Sigma$ interaction induces a three-body force through the coupling between the YNN channels with $(\ell, \lambda) = (0, 0)$ and $(\ell, \lambda) = (2, 2)$, where the relative orbital angular momentum of the YN is denoted by ℓ, and λ stands for that of the spectator nucleon in the YN system.

For this study, different models were designed by choosing sets of spin-singlet and spin-triplet ΛN scattering lengths that correctly described the available experimental data. In particular, in addition to its reasonable description of the YN cross-sections, the hypertriton binding energy corresponded to its experimental value within the error bars of $B_{0,1/2} = 0.130 \pm 0.050$ MeV [7]. The upper limit of the ΛN spin-triplet scattering length, $a^{\Lambda N}_{1/2,1}$, was established by requiring that the $(I, J^P) = (0, 3/2^+)$ ΛNN state does not become bound [2]. The lower limit was set by requiring a correct description of the YN cross-sections, which deteriorated markedly as the ΛN spin-triplet scattering length decreased. Thus, it was found that $1.41 \le a^{\Lambda N}_{1/2,1} \le 1.58$ fm. Once the ΛN spin-triplet scattering length was defined, the ΛN spin-singlet scattering length, $a^{\Lambda N}_{1/2,0}$, was constrained by demanding for the hypertriton binding energy to be in the experimental interval of $B = 0.130 \pm 0.050$ MeV, leading to $2.33 \le a^{\Lambda N}_{1/2,0} \le 2.48$ fm. Without loss of generality, we take the model with $a^{\Lambda N}_{1/2,1} = 1.41$ and $a^{\Lambda N}_{1/2,0} = 2.48$ as the reference model. All calculations were performed for several models within the scattering length intervals, and the conclusions remained unchanged. For the reference model, the hypertriton binding energy obtained was 129 keV. To illustrate the relevant role played by the D waves of the three-body system, it is worth noting that when considering only S-wave three-body channels, the hypertriton binding energy is 89 keV, which is out of the experimental range.

The solutions of the three-body problem have been described elsewhere [2] and are out of the scope of this letter. We focus on the results concerning the possible existence of a ΣNN $(I, J^P) = (1, 1/2^+)$ resonance [1].

Let us first discuss the attractive or repulsive character of the different $J^P = 1/2^+$ ΣNN channels. Figure 1 shows the Fredholm determinant of the $J^P = 1/2^+$ ΣNN channels below the Σd threshold, where the continuum starts. The Fredholm determinant of the $I = 0$ and 1 channels is complex because the ΛNN channels are open. The imaginary part is small and uninteresting. It can be seen that the channel showing the most attractive character is $(I, J^P) = (1, 1/2^+)$. For an attractive channel, the Fredholm determinant, D_F, is smaller than 1, and it becomes negative if a bound state exists [5]. Thus, the fact that the Fredholm determinant is very close to zero at the Σd threshold is a clear indication of a quasibound state. The $(I, J^P) = (0, 1/2^+)$ channel is also attractive, but far less so than the $I = 1$ one. This can be easily understood as follows. We show in Table 1 the two-body channels that contribute to a given $J^P = 1/2^+$ $\Lambda NN - \Sigma NN$ state with isospin I. The most attractive two-body channels, in particular, the ΣN $^3S_1(I = 1/2)$ and $^1S_0(I = 3/2)$ and the NN $^3S_1(I = 0)$ channels, contribute to the $(I, J^P) = (1, 1/2^+)$ ΣNN state. However, the last two are forbidden for the $(I, J^P) = (0, 1/2^+)$ ΣNN state, with one of them being the deuteron channel.

The most interesting result in connection with the results reported in [1] is the prediction of a ΣNN $(I, J^P) = (1, 1/2^+)$ quasibound state in the region near the threshold. We show in Figure 2 the real, $\mathrm{Re}(A_{1,1/2})$, and imaginary, $\mathrm{Im}(A_{1,1/2})$, parts of the Σd scattering length as a function of the attraction in the three-body channel. The real part becomes negative, while the imaginary part has a maximum, which are the typical signals of a quasibound state [8]. The position of the pole almost does not change for the different models, and it is at $2.92 - i\,2.17$ MeV for the reference model. The width of this state comes mainly from the coupling to a D-wave ΛNN channel. It is worth emphasizing that the enhancement suggested as a possible ΣNN resonance by the Hall A Collaboration at Jefferson Lab appeared at about $(3.14 \pm 0.84) - i(2.28 \pm 1.2)$ MeV [1], which is in very good agreement with the theoretical results reported in this study.

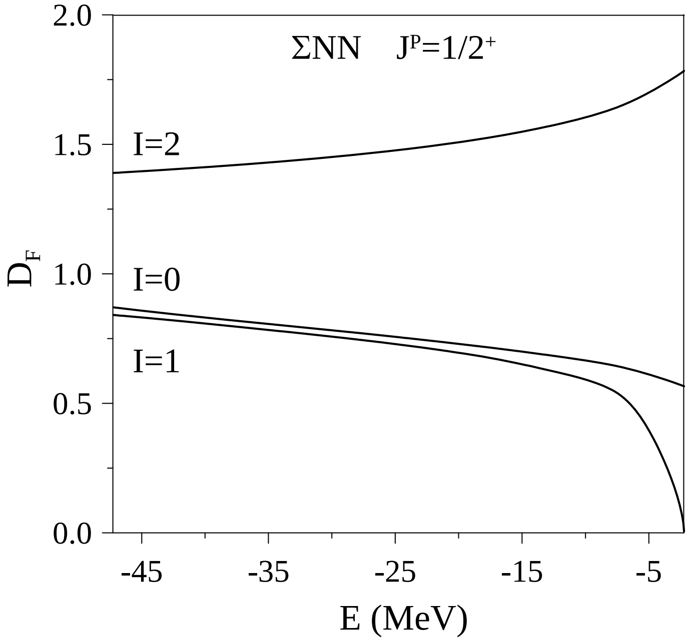

Figure 1. The Fredholm determinant, D_F, for the $J^P = 1/2^+$ ΣNN channels when utilizing the reference model, in which the deuteron binding energy is $E = -2.225$ MeV [2].

Table 1. Two-body ΣN channels (i_Σ, s_Σ), ΛN channels (i_Λ, s_Λ), NN channels with a Σ spectator $(i_{N(\Sigma)}, s_{N(\Sigma)})$, and NN channels with a Λ spectator $(i_{N(\Lambda)}, s_{N(\Lambda)})$ that contribute to a given $J^P = 1/2^+$ $\Lambda NN - \Sigma NN$ state with total isospin I.

I	(i_Σ, s_Σ)	(i_Λ, s_Λ)	$(i_{N(\Sigma)}, s_{N(\Sigma)})$	$(i_{N(\Lambda)}, s_{N(\Lambda)})$
0	(1/2,0),(1/2,1)	(1/2,0),(1/2,1)	(1,0)	(0,1)
1	(1/2,0),(3/2,0),(1/2,1),(3/2,1)	(1/2,0),(1/2,1)	(0,1),(1,0)	(1,0)
2	(3/2,0),(3/2,1)		(1,0)	

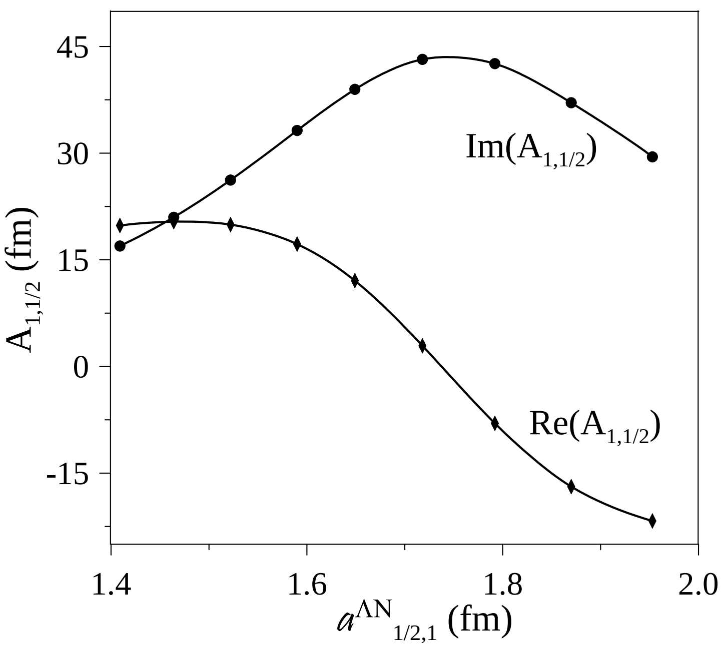

Figure 2. Real and imaginary parts of the Σd scattering length, $A_{1,1/2}$.

The existence of a $(I, J^P) = (1, 1/2^+)$ ΣNN quasibound state was suggested in a variational calculation for the investigation of the structure of $A = 3$ Σ-hypernuclei [9]. Similar results were obtained by Harada and Hirabayashi [10] by using a distorted-wave impulse approximation within a coupled $(2N - \Lambda) + (2N - \Sigma)$ model with a spreading potential. Afnan and Gibson found a near-threshold $I = 0$ resonance while exploring Λd elastic scattering with a continuum Faddeev calculation [11]. Recent preliminary calculations [12] suggested that the pole for the $I = 1$ resonance is also located near the ΣNN threshold, but the two resonances are unlikely to be distinguished experimentally.

To conclude, the Hall A Collaboration at Jefferson Lab [1] has found indications of the possible existence of a ΣNN resonance at $(3.14 \pm 0.84) - i(2.28 \pm 1.2)$ MeV. The state is likely a $\Sigma^0 nn$ state, although this has to be confirmed in future experiments. We have presented a detailed study of the $\Lambda NN - \Sigma NN$ system by using the hyperon–nucleon and nucleon–nucleon interactions derived from a chiral constituent quark model with full inclusion of the $\Lambda \leftrightarrow \Sigma$ conversion and by taking all three-body configurations with S- and D-wave components into account. In the case of the ΣNN system, there exists a narrow quasibound state near the threshold in the $(I, J^P) = (1, 1/2^+)$ channel. The position of the pole is at 2.92 $-i$ 2.17 MeV. There is a reasonable agreement with the enhancement suggested as a possible ΣNN resonance by the Hall A Collaboration at Jefferson Lab, appearing at about $(3.14 \pm 0.84) - i(2.28 \pm 1.2)$ MeV, and our result is inside the error bar of the experimental data.

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