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Editorial

Symmetry, Special Functions and Number Theory

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Editorial of Papers Published in 2020–2022 in the Journal *Symmetry*

This editorial is a short review of papers accepted in *Symmetry* in 2020–2022 about the topic of Number Theory. They belong to four Special Issues *Diophantine Number Theory*, edited by Guest Ákos Pintér, Hungary (2 papers); *Number Theory and Discrete Mathematics*, edited by Guest Elena Deza, Russia (2 papers); *Recent Advances in Number Theory and Their Applications*, edited by Guests Abdelmejid Bayad, France, and Yilmaz Simsek, Turkey (9 papers); and *Fibonacci and Lucas Numbers and the Golden Ratio in Physics and Biology*, edited by Guest Editor Tidjani Negadi (2 papers).

The first topic covered in [1] is built around the Gauss hypergeometric function ${}_2F_1$, the Kummer hypergeometric function ${}_1F_1$, and their generalization ${}_rF_s$. Summation formulae contiguous to the q-Kummer summation theorems are established. They are relevant to the theory of partitions. May be it is a good place to quote other works where hypergeometric function ${}_2F_1$ plays a role in number theory with the concepts of 'dessins d'enfants' and the Painlevé VI equation [2,3]. These references enable the placing of the subject in a broad perspective that links algebraic number theory (through the Belyi function of dessins), symmetry (through the monodromy group around the singularities), and Painlevé transcendents. Hypergeometric Bernoulli numbers and polynomials connect to the hypergeometric function ${}_1F_1$ which is the subject of [4]. The later paper explores hypergeometric polynomials with Lagrange polynomials in several variables and other polynomials. In a similar vein, ref. [5] shows the role of generating functions for parametrically generalized polynomials. They are related to the combinatorial numbers, the Bernoulli polynomials and numbers, the Euler polynomials and numbers, etc.

The second topic is about analytic number theory, the Riemann zeta function $\zeta(s)$ and other functions found in number theory. In [6], the authors prove an asymptotic formula for the sum of the values of the periodic zeta-function at the non-trivial zeros of $\zeta(s)$. Their proof holds unconditionally (irrespective of the Riemann hypothesis). Ref. [7] focuses upon the Riemann hypothesis and random walks. The method allows the calculation of critical zeros of $\zeta(s)$ at very large order, such as the 10^{100}-th zero. Again in the context of analytic number theory, the main aim of article [8] is to investigate some interesting symmetric identities for the Dirichlet-type multiple (p,q)-L function in relation to Euler polynomials and generalized Euler polynomials. Ref. [9] defines p-adic and q-Dedekind type sums. Dedekind reciprocity laws and their symmetries are a cornerstone of this study. Then, ref. [10] defines a new form of Carlitz's type degenerate twisted (p,q)-Euler numbers and polynomials and study some theories of the Carlitz's type degenerate twisted (p,q)-Euler numbers and polynomials. Finally, paper [11] investigates certain identities associated with (p,q)-binomial coefficients and (p,q)-Stirling polynomials of the second kind.

The note on a triple integral [12], still about the second topic, deserves a special mention due to its originality and the perspectives it offers in mathematical physics. A general integral theorem is developed in terms of the Lerch function, Hurwitz zeta function, polylogarithm function, and the Riemann zeta function. Ref. [13] is a survey of some recent developments on higher transcendental functions of analytic number theory and applied mathematics. The later two papers can be studied in parallel to contemplate the richness of the connection between analytical number theory and special functions.

A third topic of recurrent interest is about Fibonacci and Lucas numbers, and the Golden ratio. Ref. [14–18] illustrate their applications to chemistry, biology, physics, social sciences, anthropology, etc.

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References

1. Vyas, Y.; Srivastava, H.M.; Pathak, S.; Fatawat, K. General Summation Formulas Contiguous to the q-Kummer Summation Theorems and Their Applications. *Symmetry* **2021**, *13*, 1102. [CrossRef]
2. Kitaev, A.V. Grothendieck's dessins d'enfants, their deformations, and algebraic solutions of the sixth Painlevé and Gauss hypergeometric equations. *St. Petersburg Math. J.* **2006** , *17*, 169–206. [CrossRef]
3. Boalch, P. Towards a nonlinear Schwarz's list. *arxiv* **2008**, arXiv:0707.3375.
4. Muhiuddin, G.; Khan, W.A.; Duran U.; Al-Kadi, D. A New Class of Higher-Order Hypergeometric Bernoulli Polynomials Associated with Lagrange–Hermite Polynomials. *Symmetry* **2021**, *13*, 648. [CrossRef]
5. Bayad, A.; Simsek, Y. On Generating Functions for Parametrically Generalized Polynomials Involving Combinatorial, Bernoulli and Euler Polynomials and Numbers. *Symmetry* **2022**, *14*, 654. [CrossRef]
6. Tongsomporn, J.; Wananiyakui, S.; Steuding, J. The Values of the Periodic Zeta-Function at the Nontrivial Zeros of Riemann's Zeta-Function. *Symmetry* **2021**, *13*, 2410 [CrossRef]
7. LeClair, A. Riemann Hypothesis and RandomWalks: The Zeta Case. *Symmetry* **2021**, *13*, 2014. [CrossRef]
8. Hwang K.W.; Agarwal, R.P.; Ryoo, C.S. Symmetric Properties for Dirichlet-Type Multiple (p,q)-L-Function. *Symmetry* **2021**, *13*, 95. [CrossRef]
9. Bayad, A.; Simsek, Y. p-Adic q-Twisted Dedekind-Type Sums. *Symmetry* **2021**, *13*, 1756. [CrossRef]
10. Ryoo, C.S. Some Symmetry Identities for Carlitz's Type Degenerate Twisted (p,q)-Euler Polynomials Related to Alternating Twisted (p,q)-Sums. *Symmetry* **2021**, *13*, 1371. [CrossRef]
11. Usman, T.; Saif. M.; Choi, J. Certain Identities Associated with (p,q)-Binomial Coefficients and (p,q)-Stirling Polynomials of the Second Kind. *Symmetry* **2020**, *12*, 1436. [CrossRef]
12. Reynolds, R.; Stauffer, A. A Note on a Triple Integral. *Symmetry* **2021**, *13*, 2056. [CrossRef]
13. Srivastava, H.M. A Survey of Some Recent Developments on Higher Transcendental Functions of Analytic Number Theory and Applied Mathematics. *Symmetry* **2021**, *13*, 2294. [CrossRef]
14. Battaloglu, R.; Simsek, Y. On New Formulas of Fibonacci and Lucas Numbers Involving Golden Ratio Associated with Atomic Structure in Chemistry. *Symmetry* **2021**, *13*, 1334. [CrossRef]
15. Park, H.; Cho, B.; Cho, D.; Cho, Y.D.; Park, J. Representation of Integers as Sums of Fibonacci and Lucas Numbers. *Symmetry* **2020**, *12*, 1625. [CrossRef]
16. Gunes, A.Y.; Delen, S.; Demirci, M.; Cevik, A.S.; Cangui, I.N. Fibonacci Graphs. *Symmetry* **2020**, *12*, 1383. [CrossRef]
17. Laipaporn, K.; Phibul, K.; Khachorncharoenkul, P. The metallic ratio of pulsating Fibonacci sequences. *Symmetry* **2022**, *14*, 1204. [CrossRef]
18. Larsen, S.H. DNA structure and the Golden ratio revisited. *Symmetry* **2021**, *13*, 1949. [CrossRef]