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On the Suppression of the Dark Matter-Nucleon Scattering Cross Section in the SE$_6$SSM

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Abstract: In the E_6 inspired $U(1)_N$ extension of the minimal supersymmetric (SUSY) standard model (MSSM), a single discrete $\tilde{Z}_2^H$ symmetry permits suppressing rapid proton decay and non-diagonal flavor transitions. If matter parity and $\tilde{Z}_2^H$ symmetry are preserved in this SUSY model (SE$_6$SSM), it may involve two dark matter candidates. In this article, we study a new modification of the SE$_6$SSM in which the cold dark matter is composed of gravitino and the lightest neutral exotic fermion. We argue that, in this case, the dark matter nucleon scattering cross-section can be considerably smaller than the present experimental limit.

Keywords: unified field theories and models; models beyond the standard model; supersymmetry; cold dark matter

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1. Introduction

The possible unification of all interactions remains one of the most appealing motivations for studying supersymmetric (SUSY) extensions of the Standard Model (SM). At very high energies, the minimal supersymmetric standard model (MSSM) and its extensions can be embedded into Grand Unified Theories (GUTs). In $N = 2$ SUSY GUTs based on the E_8 gauge group, all SM bosons and fermions can belong to a single 248 representation of E_8. This representation involves three $27 \oplus \overline{27}$, which are fundamental and anti-fundamental representations of E_6, adjoint 78 representation of E_6 and 8 components that do not participate in the E_6 gauge interactions. In SUSY GUTs, three fundamental representations of E_6 may include Higgs doublet and three families of the SM fermions. The SM gauge bosons are contained in the adjoint representation of E_6. It is expected that near the scale $M_0 \gtrsim M_X \sim 2 \cdot 10^{16}$ GeV the extended SUSY is broken down to $N = 1$ supersymmetry while the breakdown of E_6 (or E_8) gauge group can lead to

$$\begin{aligned} E_6 \to SO(10) \times U(1)_\psi \to SU(5) \times U(1)_\chi \times U(1)_\psi \to \\ \to SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_\psi \times U(1)_\chi \,, \end{aligned} \tag{1}$$

where $SU(3)_C \times SU(2)_W \times U(1)_Y$ is a SM gauge group.

The exceptional supersymmetric standard model (E$_6$SSM) [1,2] (for recent review, see Ref. [3]) implies that around the GUT scale M_X rank-6 model with two extra symmetries $U(1)_\psi$ and $U(1)_\chi$ is reduced further to an effective rank-5 model based on the SM gauge group together with an additional $U(1)_N$ factor, i.e.,

$$U(1)_\psi \times U(1)_\chi \to U(1)_N \times Z_2^M \,, \tag{2}$$

where $Z_2^M = (-1)^{3(B-L)}$ is the so-called matter parity, which is a discrete subgroup of $U(1)_\psi$ and $U(1)_\chi$. In Equation (2) the $U(1)_N$ gauge symmetry is defined as

$$U(1)_N = \frac{1}{4} U(1)_\chi + \frac{\sqrt{15}}{4} U(1)_\psi \,. \tag{3}$$

In these E_6 inspired $U(1)_N$ extensions of the MSSM the right-handed neutrinos do not participate in the gauge interactions and may be superheavy [1,2]. Therefore, a see-saw mechanism can be used to shed light on the origin of the mass hierarchy in the lepton sector, providing a comprehensive understanding of the neutrino oscillations data. The successful leptogenesis is a distinctive feature of the $U(1)_N$ extensions of the MSSM because the heavy Majorana right-handed neutrinos may decay into final states with lepton number $L = \pm 1$, creating a lepton asymmetry in the early Universe [4,5].

In the E_6 inspired $U(1)_N$ extensions of the MSSM gauge anomalies become cancelled if the particle spectrum at low-energies contains complete representations of E_6. To ensure that the E$_6$SSM is anomaly-free one is forced to extend the minimal matter content by extra matter beyond the MSSM which, together with ordinary SM fermions, form three complete 27-plets of E_6 (27_i with $i = 1, 2, 3$). Each 27-dimensional representations of E_6 involves one generation of ordinary matter, a SM singlet field S_i, that carries non-zero $U(1)_N$ charge, up- and down-type Higgs doublets H_i^u and H_i^d, as well as charged $\pm 1/3$ exotic quarks D_i and $\bar{D}_i$. The presence of extra exotic matter may give rise to non-diagonal flavor transitions and rapid proton decay. In the E$_6$SSM a set of discrete symmetries can be used to suppress the corresponding operators [1,2].

In this article, we examine the dark matter-nucleon scattering cross section within the modification of the E$_6$SSM (SE$_6$SSM) [6,7] in which a single discrete $\tilde{Z}_2^H$ symmetry forbids tree-level flavor-changing transitions, as well as the most dangerous baryon and lepton number violating operators. The conservation of $\tilde{Z}_2^H$ symmetry and matter parity/R-parity implies the existence of at least two stable states that may account for all or some of the observed cold dark matter density. At the same time null results from direct detection experiments [8–10] placed stringent limits on the dark matter nucleon scattering cross-section. Here, we explore a new variant of the SE$_6$SSM in which two stable states are gravitino and the lightest neutral exotic fermion. In general, the spin-independent part of the dark matter-nucleon scattering cross section can be much larger in this case than the corresponding experimental limit. Nevertheless, the results of our analysis indicate that there is a part of the SE$_6$SSM parameter space, in which this cross section is sufficiently strongly suppressed.

The paper is organised as follows. In the next section, we specify a new variant of the SE$_6$SSM which is considered in this article. In Section 3, we investigate the dependence of the dark matter-nucleon scattering cross section on the parameters of the SE$_6$SSM. Section 4 concludes the paper.

2. The SE$_6$SSM

Since 2006, several modifications of the E$_6$SSM have been explored [1,2,6,7,11–19]. The implications of the $U(1)_N$ extensions of the MSSM were considered for Z–Z' mixing [20], neutralino sector [20–22], electroweak (EW) symmetry breaking (EWSB) [21,23,24], the renormalization group (RG) flow of couplings [21,25], the renormalization of vacuum expectation values (VEVs) [26,27], non-standard neutrino models [28], and dark matter [19,29,30]. Within the E$_6$SSM, the upper bound on the lightest Higgs mass near the quasi-fixed point was examined in [31]. The corresponding quasi-fixed point is an intersection of the invariant and quasi-fixed lines [32,33]. The particle spectrum in the constrained E$_6$SSM (cE$_6$SSM) and its modifications was analyzed in [34–37]. The degree of fine tuning and threshold corrections were explored in Refs. [38,39] and [40], respectively. Extra exotic matter in the E$_6$SSM may result in distinctive LHC signatures [1,2,12,15,41–44] and can give rise to non-standard Higgs decays [7,45,46].

The SE$_6$SSM implies that below the scale M_X three complete 27–plets are accompanied by SM singlet superfield ϕ, which does not participate in the E_6 interactions, and a set of pairs of supermultiplets M_l and $\overline{M}_l$, which belong to additional $27'_l$ and $\overline{27'}_l$ representations, respectively. Because M_l and $\overline{M}_l$ carry opposite $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_N$ quantum numbers gauge anomalies still cancel. In the simplest case, the set of M_l and $\overline{M}_l$ involves three pairs of $SU(2)_W$ doublets, i.e., L_4 and $\overline{L}_4$, H_u and $\overline{H}_u$, H_d and $\overline{H}_d$, as well as a pair of superfields, S and $\overline{S}$. The field content of the SE$_6$SSM may originate from the E_6 orbifold GUT model in six dimensions in which the appropriate splitting of the bulk $27'$ supermultiplets can be achieved [6].

The supermultiplets ϕ, S, $\overline{S}$, H_u, H_d, L_4, and $\overline{L}_4$ are required to be even under the $\tilde{Z}_2^H$ custodial symmetry whereas all other supermultiplets are odd [7]. In the SE$_6$SSM superpotential, the $\tilde{Z}_2^H$ symmetry allows the interactions that comes from $27'_l \times 27'_m \times 27'_n$ and $27'_l \times 27_i \times 27_k$ but it forbids all terms which originate from $27_i \times 27_j \times 27_k$, where indexes l, m, n are associated with the supermultiplets M_l while family indexes $i, j, k = 1, 2, 3$ run over three generations. Such structure of the superpotential ensures that it does not involve any operators that give rise to rapid proton decay. Because the set of supermultiplets M_l includes only two Higgs doublets H_d and H_u, the up-type quarks couple to just H_u, whereas the down-type quarks and charged leptons couple to H_d only. Therefore, the flavor-changing processes are suppressed in the SE$_6$SSM at tree-level.

In the simplest scenario $\overline{H}_u$ and $\overline{H}_d$ become combined with the superposition of the corresponding components from the 27_i, forming vector-like states with masses of order M_X. The components of the supermultiplets L_4 and $\overline{L}_4$, as well as S and $\overline{S}$ are expected to gain the TeV scale masses. The presence of L_4 and $\overline{L}_4$ at low energies permits the lightest exotic quarks to decay within a reasonable time, as well as facilitates the gauge coupling unification [25] and the generation of the baryon asymmetry of the Universe [5].

Using the method discussed in [47], it was revealed that in the E$_6$SSM and its simplest modifications the lightest R-parity odd states (lightest SUSY particles) have to be lighter than 60–65 GeV [46]. These states are predominantly linear superpositions of the fermion components of the superfields S_i. Although the couplings of these lightest exotic fermions to the SM particles are very small the lightest SUSY particle (LSP) could account for some of the cold dark matter relic density if the LSP had a mass close to half the Z boson mass $M_Z/2$ [46]. However, in this case the SM-like Higgs boson decays mostly into the lightest exotic fermions whereas its other branching ratios are suppressed. LHC experiments have already excluded such a scenario. When the lightest exotic fermions are substantially lighter than M_Z, the LSP annihilation cross section tends to be too small resulting in too large cold dark matter density. In the simplest phenomenologically viable scenario, the sparticle spectrum of the SE$_6$SSM includes the lightest SUSY particles with masses which are much smaller than 1 eV. These lightest exotic fermions form hot dark matter in our Universe. Nevertheless, they give a minor contribution to the total density of dark matter. The existence of neutral fermions with tiny masses may lead to very interesting implications for the neutrino physics [48].

The variant of the SE$_6$SSM explored here implies that in addition to all states mentioned above the low-energy matter content of the model involves at least three E_6 singlet superfields ϕ_i which are odd under the $\tilde{Z}_2^H$ symmetry. This allows to avoid the appearance of the lightest exotic fermions with tiny masses in the particle spectrum. It is expected that the components of the supermultiplets

$$\begin{gathered}(Q_i,\, u_i^c,\, d_i^c,\, L_i,\, e_i^c) + (D_i,\, \bar{D}_i) + S_i + \phi_i + (H_\alpha^u,\, H_\alpha^d) \\ + L_4 + \overline{L}_4 + S + \overline{S} + H_u + H_d + \phi\,, \end{gathered} \tag{4}$$

where $\alpha = 1, 2$ and $i = 1, 2, 3$, have masses either of the order of 10 TeV or considerably lower. The right-handed neutrino superfields N_i^c are assumed to be much heavier then 10 TeV. The $U(1)_Y$ and $U(1)_N$ charges of all matter supermultiplets in the SE$_6$SSM are summarised in Table 1.

Table 1. The $U(1)_Y$ and $U(1)_N$ charges of matter supermultiplets in the SE$_6$SSM. The superfields N_i^c, ϕ_i, and ϕ have zero $U(1)_Y$ and $U(1)_N$ charges.

	Q_i	u_i^c	d_i^c	L_i, L_4	e_i^c	S_i, S	H_α^u, H_u	H_α^d, H_d	D_i	$\overline{D}_i$	$\overline{L}_4$	$\overline{S}$
$\sqrt{\frac{5}{3}}Q_i^Y$	$\frac{1}{6}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	0
$\sqrt{40}Q_i^N$	1	1	2	2	1	5	-2	-3	-2	-3	-2	-5

Integrating out the right-handed neutrino superfields N_i^c and neglecting all suppressed non-renormalisable interactions, the low-energy superpotential of the new variant of the SE_6SSM can be written as

$$\begin{aligned} W_{\mathrm{SE_6SSM}} &= \lambda S(H_u H_d) - \sigma\phi S\overline{S} + \frac{\kappa}{3}\phi^3 + \frac{\mu}{2}\phi^2 + \Lambda\phi + \mu_L L_4\overline{L}_4 + \tilde{\sigma}\phi L_4\overline{L}_4 + W_{IH} \\ &+ \kappa_{ij}S(D_i\overline{D}_j) + g^D_{ij}(Q_i L_4)\overline{D}_j + h^E_{i\alpha}e^c_i(H^d_\alpha L_4) + g_{ij}\phi_i\overline{L}_4 L_j + W_{\mathrm{MSSM}}(\mu=0)\,. \end{aligned} \tag{5}$$

It is worth noting that the extra $U(1)_N$ gauge symmetry forbids the term $\mu_0 H_d H_u$ in the superpotential (5), but allows all other terms in $W_{\mathrm{MSSM}}(\mu = 0)$. In Equation (5) the part of the superpotential W_{IH} describes the interactions of ϕ_i, S_i, H^u_α and H^d_α with the $\tilde{Z}_2^H$ even supermultiplets ϕ, S, $\overline{S}$, H_u, and H_d.

$$\begin{aligned} W_{IH} &= \tilde{M}_{ij}\phi_i\phi_j + \tilde{\kappa}_{ij}\phi\phi_i\phi_j + \tilde{\lambda}_{ij}\overline{S}\phi_i S_j + \lambda_{\alpha\beta}S(H^d_\alpha H^u_\beta) \\ &+ \tilde{f}_{i\alpha}S_i(H^d_\alpha H_u) + f_{i\alpha}S_i(H_d H^u_\alpha)\,. \end{aligned} \tag{6}$$

The sector responsible for the breakdown of the gauge symmetry in the SE_6SSM is formed by the scalar components of ϕ, S, $\overline{S}$, H_u, and H_d. In the limit $\sigma \to 0$ the $U(1)_N$ D-term contribution to the effective scalar potential can force the minimum of such potential to be along the D-flat direction [49]. Indeed, if the coupling σ vanishes the part of the scalar potential that depends on S and $\overline{S}$ takes the form

$$V_S(S,\overline{S}) = m_S^2|S|^2 + m_{\overline{S}}^2|\overline{S}|^2 + \frac{Q_S^2 g_1'^2}{2}\left(|S|^2 - |\overline{S}|^2\right)^2, \tag{7}$$

where Q_S is the $U(1)_N$ charge of S and $\overline{S}$, g_1' is the $U(1)_N$ gauge coupling whereas m_S^2 and $m_{\overline{S}}^2$ are the soft SUSY breaking mass parameters. The last term in Equation (7) is associated with the $U(1)_N$ D-term contribution. For $\langle S\rangle = \langle\overline{S}\rangle$ the quartic term vanishes. If, in this case $(m_S^2 + m_{\overline{S}}^2) < 0$, then the scalar potential (7) has a run-away direction $\langle S\rangle = \langle\overline{S}\rangle \to \infty$. The F-term contribution to the scalar potential (7) induced by the small coupling σ stabilizes the run-away direction so that the SM singlet superfields acquire VEVs which are much larger than the sparticle mass scale M_S, i.e.,

$$\langle\phi\rangle \sim \langle S\rangle \simeq \langle\overline{S}\rangle \sim \frac{M_S}{\sigma}\,, \tag{8}$$

resulting in an extremely heavy Z' boson. All extra exotic states can be also rather heavy in this limit.

The conservation of Z_2^M and $\tilde{Z}_2^H$ symmetries implies that R-parity and Z_2^E symmetry are also conserved where $\tilde{Z}_2^H = Z_2^M \times Z_2^E$ [6]. The transformation properties of different supermultiplets under the $\tilde{Z}_2^H$, Z_2^M and Z_2^E symmetries are summarized in Table 2. Here, we assume that gravitino is the lightest R-parity odd state. Therefore, it has to be stable and potentially contributes to the density of dark matter. There is a large class of models in which gravitino can be much lighter than the superpartners of other particles [50–53]. Recently, the cosmological implications of the gravitino with mass $m_{3/2} \sim$ KeV were discussed in Ref. [54]. Since gravitino is the lightest SUSY particle with $Z_2^E = +1$ the lightest exotic state with $Z_2^E = -1$ must be absolutely stable as well [6].

Table 2. Transformation properties of different supermultiplets under the discrete symmetries $\tilde{Z}_2^H$, Z_2^M and Z_2^E. The signs $+$ and $-$ correspond to the states which are even and odd under different Z_2 symmetries.

	$Q_i, u^c_i, d^c_i, L_i, e^c_i, N^c_i$	$\overline{D}_i, D_i, H^d_\alpha, H^u_\alpha, S_i, \phi_i$	$H_d, H_u, S, \overline{S}, \phi$	$L_4, \overline{L}_4$
$\tilde{Z}_2^H$	$-$	$-$	$+$	$+$
Z_2^M	$-$	$+$	$+$	$-$
Z_2^E	$+$	$-$	$+$	$-$

The scalar components of the supermultiplets ϕ_i, S_i, H^u_α, and H^d_α do not develop VEVs. Their fermion components compose the exotic chargino and neutralino states. In the limit, when all components of ϕ_i are considerably heavier than the bosons and fermions from the supermultiplets S_i, H^u_α, and H^d_α, the superfields ϕ_i can be integrated out so that the part of the SE$_6$SSM superpotential W_{IH} reduces to:

$$W_{IH} \to \widetilde{W}_{IH} \simeq -\widetilde{\mu}_{ij} S_i S_j + \lambda_{\alpha\beta} S(H^d_\alpha H^u_\beta) + \tilde{f}_{i\alpha} S_i (H^d_\alpha H_u) + f_{i\alpha} S_i (H_d H^u_\alpha) + \dots \tag{9}$$

Hereafter, we use the field basis in which $\widetilde{\mu}_{ij} = \widetilde{\mu}_i\, \delta_{ij}$ and $\lambda_{\alpha\beta} = \lambda_{\alpha\alpha}\, \delta_{\alpha\beta}$.

In this article, we focus on the scenarios in which the lightest exotic state with $Z^E_2 = -1$ is predominantly formed by the fermion components of the supermultiplets H^u_1 and H^d_1 while all sparticles except gravitino and all other exotic states have masses which are much larger than 1 TeV. If H^u_1 and H^d_1 mostly interact with H_u, H_d and S_1, whereas all other couplings of H^u_1 and H^d_1 are negligibly small, the part of the mass matrix, that determines the masses of the lightest exotic neutralino states, can be written in the following form:

$$M^{ab} = -\begin{pmatrix} 0 & \mu_{11} & \dfrac{\tilde{f}_{11}}{\sqrt{2}} v_2 \\ \mu_{11} & 0 & \dfrac{f_{11}}{\sqrt{2}} v_1 \\ \dfrac{\tilde{f}_{11}}{\sqrt{2}} v_2 & \dfrac{f_{11}}{\sqrt{2}} v_1 & \widetilde{\mu}_1 \end{pmatrix}, \tag{10}$$

where $\mu_{11} \simeq \lambda_{11} \langle S \rangle$, v_1 and v_2 are the VEVs of the Higgs doublets H_d and H_u, i.e., $\langle H_d \rangle = v_1/\sqrt{2}$ and $\langle H_u \rangle = v_2/\sqrt{2}$. Instead of v_1 and v_2 it is more convenient to use $\tan\beta = v_2/v_1$ and $v = \sqrt{v_1^2 + v_2^2} \approx 246\,\text{GeV}$. Hereafter, we neglect the contribution of loop corrections to the mass matrix (10). In this case, the mass of the charged fermion components of H^u_1 and H^d_1 is determined by μ_{11}, i.e., $m_{\chi^\pm_1} = \mu_{11}$.

When $|\widetilde{\mu}_1|$ is much larger than $|\mu_{11}|$ and v, the perturbation theory method can be used to diagonalise the mass matrix (10) (see, for example, [55–58]). This method yields

$$\begin{gathered} m_{\chi_1} \simeq m_{\chi^\pm_1} - \Delta_1\,, \qquad m_{\chi_2} \simeq m_{\chi^\pm_1} + \Delta_2\,, \qquad m_{\chi_2} \simeq \widetilde{\mu}_1 + \Delta_1 + \Delta_2\,, \\ \Delta_1 \simeq \frac{(\tilde{f}_{11} v \sin\beta + f_{11} v \cos\beta)^2}{4(\widetilde{\mu}_1 - m_{\chi^\pm_1})}\,, \qquad \Delta_2 \simeq \frac{(\tilde{f}_{11} v \sin\beta - f_{11} v \cos\beta)^2}{4(\widetilde{\mu}_1 + m_{\chi^\pm_1})}\,. \end{gathered} \tag{11}$$

From Equation (11), it follows that the masses of the lightest exotic chargino and neutralino states in the leading approximation are set by μ_{11}. If the mass difference $m_{\chi_2} - m_{\chi_1}$ is as small as $O(100\,\text{KeV})$, the inelastic scattering processes $\chi_1 N \to \chi_2 N$, where N is a nucleon, may occur. In this article, we restrict our consideration to the part of the SE$_6$SSM parameter space where $m_{\chi_2} - m_{\chi_1} > 200\,\text{MeV}$. Because of this the inelastic scattering processes do not take place. Moreover, the lifetime of χ_2 is much smaller than 1 s that allows to preserve the success of the Big Bang Nucleosynthesis (BBN).

In the scenarios under consideration, the contribution of the lightest neutral exotic fermion to the cold dark matter relic density can be estimated using the approximate formula:

$$\Omega_{\tilde{H}} h^2 \simeq 0.1 \left(\frac{\mu_{11}}{1\,\text{TeV}}\right)^2. \tag{12}$$

which was derived in the case of the Higgsino dark matter within the MSSM (see, for example, [59,60]). Because the Planck observations lead to [61]:

$$(\Omega h^2)_{\text{exp}} = 0.1188 \pm 0.0010\,, \tag{13}$$

in the phenomenologically viable scenarios μ_{11} is expected to be lower than 1.1 TeV. If $\mu_{11} \lesssim 1.1$ TeV then gravitino may account for some or major part of the observed cold dark matter density.

In general, to find a viable cosmological scenario with stable gravitino one has to ensure that the decay products of the lightest unstable R-parity odd (or exotic) particle Y do not alter the abundances of light elements induced by BBN. The decays of state Y change the abundances of light elements the more the longer its lifetime τ_Y is. This problem can be evaded if sparticle Y decays before BBN, i.e., $\tau_Y \lesssim 1$ s. The lifetime of state Y decaying into its SM partner (or lightest exotic fermion) and gravitino can be estimated as [62]:

$$\tau_Y \sim 48\pi \frac{m_{3/2}^2 M_P^2}{m_Y^5}\,, \tag{14}$$

where m_Y is its mass and $M_P = (8\pi G_N)^{-1/2} \simeq 2.4 \cdot 10^{18}$ GeV is the reduced Planck mass. In order to obtain $\tau_Y \lesssim 1$ sec for $m_Y \simeq 1$ TeV we should restrict our consideration to $m_{3/2} \lesssim 1$ GeV.

If gravitinos mostly originate from scattering processes of particles in the thermal bath then their abundance is approximately proportional to the reheating temperature T_R after inflation. In the leading approximation one finds [63,64]:

$$\Omega_{3/2}h^2 \sim 0.27 \left(\frac{T_R}{10^8 GeV}\right)\left(\frac{1\,\text{GeV}}{m_{3/2}}\right)\left(\frac{M_{\tilde{g}}}{1\,\text{TeV}}\right)^2, \tag{15}$$

where $M_{\tilde{g}}$ is a gluino mass. Taking into account that $\Omega_{3/2}h^2 \le 0.12$, for $M_{\tilde{g}} \gtrsim 2-3$ TeV and $m_{3/2} \simeq 1$ GeV one obtains an upper bound on the reheating temperature, i.e., $T_R \lesssim 10^{6-7}$ GeV [65]. However even for so low reheating temperatures the appropriate amount of the baryon asymmetry can be induced within the SE$_6$SSM via the decays of the lightest right-handed neutrino/sneutrino into exotic states [5].

3. Dark Matter Nucleon Scattering Cross-section

Since the couplings of gravitino to the SM particles are negligibly small, in the scenarios under consideration the interactions of the cold dark matter with the baryons are determined by the couplings of the lightest neutral exotic fermion χ_1. The low energy χ_1–quark effective Lagrangian is:

$$\mathcal{L}_{\chi_1 q} = \sum_q \Big(a_q \bar{\chi}_1 \chi_1 \bar{q} q + d_q \bar{\chi}_1 \gamma^\mu \gamma_5 \chi_1 \bar{q} \gamma_\mu \gamma_5 q\Big)\,. \tag{16}$$

The first term in the brackets gives rise to a spin-independent interaction, while the second one is associated with a spin-dependent interaction.

For spin-independent interactions experiments quote the cross-section to scatter off a nucleon, which is given by [66,67]:

$$\sigma_{SI} = \frac{4m_r^2}{\pi}\frac{(Zf^p + (A-Z)f^n)^2}{A^2}\,, \qquad\qquad m_r = \frac{m_{\chi_1} m_N}{m_{\chi_1} + m_N}\,, \tag{17}$$

where A and Z are the nucleon number and charge of the target nucleus, whereas f^p and f^n are related to the quantities entering $\mathcal{L}_{\chi_1 q}$. To simplify our analysis, we assume that $f^p \approx f^n \approx f^N$ while:

$$\frac{f^N}{m_N} = \sum_{q=u,d,s} \frac{a_q}{m_q} f^N_{Tq} + \frac{2}{27} \sum_{Q=c,b,t} \frac{a_Q}{m_Q} f^N_{TQ}\,, \tag{18}$$

$$m_N f^N_{Tq} = \langle N | m_q \bar{q} q | N \rangle\,, \qquad\qquad f^N_{TQ} = 1 - \sum_{q=u,d,s} f^N_{Tq}\,.$$

From Equations (17) and (18), one can see that σ_{SI} depends rather strongly on the hadronic matrix elements, i.e., the coefficients f^N_{Tq}, which are related to the π-nucleon σ

term and the spin content of the nucleon. Here, we set $f^N_{Tu} \simeq 0.0153$, $f^N_{Td} \simeq 0.0191$, and $f^N_{Ts} \simeq 0.0447$. These values of the hadronic matrix elements are the default values used in micrOMEGAs, as determined in Ref. [68] from lattice results (see also Refs. [69–72]).

Because in the SE$_6$SSM the exotic neutralino states do not couple to quarks and squarks, the coefficients a_q receives only contributions from the t-channel exchange of CP-even Higgs bosons. In the scenarios under consideration, the mass of the lightest Higgs particle m_{h_1} is much smaller than the masses of all other Higgs bosons. Therefore, in our analysis we ignore all contributions induced by the heavy Higgs exchange. Moreover, in this case, the lightest Higgs scalar manifests itself in the interactions with the SM fermions and gauge bosons as a SM-like Higgs so that:

$$\frac{a_q}{m_q} \simeq \frac{a_Q}{m_Q} \simeq \frac{g_{h\chi\chi}}{v m^2_{h_1}}\,. \tag{19}$$

When $\Delta_1 \ll \mu_{11} \ll \tilde{\mu}_1$ the coupling $g_{h\chi\chi}$ of the SM-like Higgs to the lightest exotic neutralino is given by:

$$|g_{h\chi\chi}| \simeq \frac{\Delta_1}{v}\,. \tag{20}$$

The dominant contribution to the coefficients d_q in Equation (16) comes from t-channel Z boson exchange. In the field basis $(\tilde{H}^{d0}_1,\, \tilde{H}^{u0}_1,\, \tilde{S}_1)$, the two lightest exotic neutralinos are made up of the following superposition of interaction states:

$$\chi_\alpha = N^1_\alpha \tilde{H}^{d0}_1 + N^2_\alpha \tilde{H}^{u0}_1 + N^3_\alpha \tilde{S}_1\,, \tag{21}$$

where $\alpha = 1, 2$. In Equation (21), N^a_i is the exotic neutralino mixing matrix defined by:

$$N^a_i M^{ab} N^b_j = m_i \delta_{ij}, \qquad \text{no sum on } i. \tag{22}$$

where M^{ab} is 3×3 exotic neutralino mass matrix given by Equation (10). Using the above compositions of the lightest exotic neutralino states (21), it is straightforward to derive the couplings of these states to the Z-boson. The part of the Lagrangian that describes the interactions of Z with χ_1 and χ_2 can be written as:

$$\mathcal{L}_{Z\chi\chi} = \sum_{\alpha,\beta} \frac{M_Z}{2v} Z_\mu \left(\chi^T_\alpha \gamma_\mu \gamma_5 \chi_\beta\right) R_{Z\alpha\beta}\,, \qquad R_{Z\alpha\beta} = N^1_\alpha N^1_\beta - N^2_\alpha N^2_\beta\,. \tag{23}$$

Then, the coefficients d_q and the corresponding cross-section can be presented in the following form:

$$\sigma^{p,n} = \frac{12 m^2_r}{\pi} \left(\sum_{q=u,d,s} d_q \Delta^{p,n}_q \right)^2, \qquad d_q = \frac{T_{3q}}{2v^2} R_{Z11}\,, \tag{24}$$

where T_{3q} is the third component of isospin. For fractions of the nucleon spin carried by a given quark q we use (see also [60])

$$\Delta^p_u = \Delta^n_d = 0.842\,, \qquad \Delta^p_d = \Delta^n_u = -0.427\,, \qquad \Delta^p_s = \Delta^n_s = -0.085\,. \tag{25}$$

Using the compositions of χ_1 and χ_2 one can obtain the analytical expressions for the couplings of these states to the lightest Higgs boson, i.e.,

$$g_{h\alpha\beta} = -\frac{1}{\sqrt{2}} \left(f_{11} N^3_\alpha N^2_\beta \cos\beta + \tilde{f}_{11} N^3_\alpha N^1_\beta \sin\beta \right). \tag{26}$$

The perturbation theory method allows to compute the entries of the exotic neutralino mixing matrix N^a_i. Substituting N^a_i into Equation (26) one can reproduce the analytical

Formula (20) for $g_{h11} = g_{h\chi\chi}$ and find the approximate expression for R_{Z11}. If $\widetilde{\mu}_1 \gg \mu_{11} > 0$ and μ_{11} is larger than $\tilde{f}_{11} v \sin\beta$ and $f_{11} v \cos\beta$, we get:

$$R_{Z11} \simeq \frac{v^2(f_{11}^2 \cos^2\beta - \tilde{f}_{11}^2 \sin^2\beta)}{4\mu_{11}(\widetilde{\mu}_1 - \mu_{11})} \,. \tag{27}$$

Here, we restrict our considerations to low values of $\tan\beta \simeq 2$. In this case, the particle spectrum of the SE$_6$SSM may include the SM-like Higgs state with mass around 125 GeV if $\lambda \gtrsim g'_1$. Indeed, in this SUSY model the upper bound on the lightest Higgs boson mass m_{h_1} is given by:

$$m_{h_1}^2 \lesssim \frac{\lambda^2}{2} v^2 \sin^2 2\beta + M_Z^2 \cos^2 2\beta + g_1'^2 v^2 (Q_{H_d} \cos^2\beta + Q_{H_u} \sin^2\beta)^2 + \tilde{\Delta} \,, \tag{28}$$

where Q_{H_d} and Q_{H_u} are the $U(1)_N$ charges of H_d and H_u whereas $\tilde{\Delta}$ is the contribution of loop corrections. When $\lambda \gtrsim \sqrt{2}(M_Z/v) \simeq 0.52 \gtrsim g'_1$ the sum of the first two terms can be larger than M_Z^2 for moderate values of $\tan\beta$. As a consequence, for $\tan\beta \simeq 2$ the 125 GeV Higgs state can be obtained. In this part of the SE$_6$SSM parameter space, all Higgs states except the lightest Higgs boson tend to have masses beyond the multi-TeV range and cannot be observed at the LHC experiments [1,2,41,42].

Within the SE$_6$SSM the masses of new exotic states including the Z' boson are set by the VEVs of S and $\overline{S}$. LHC constraints require the extra $U(1)_N$ gauge boson to be heavier than 4.5 TeV [73,74]. When $\langle S \rangle \simeq \langle \overline{S} \rangle$ for the mass of the extra vector boson $M_{Z'}$ one finds:

$$M_{Z'} \approx 2 g'_1 Q_S \langle S \rangle \,. \tag{29}$$

In order to obtain $M_{Z'} \gtrsim 4.5$ TeV we choose $\langle S \rangle \simeq \langle \overline{S} \rangle \gtrsim 6$ TeV. The experimental constraints on $Z - Z'$ mixing are also satisfied in this case. To avoid the experimental lower limit on the mass of the lightest exotic chargino and to ensure that the lightest exotic neutralino state gives rise to the phenomenologically acceptable dark matter density the interval of variation of μ_{11} is chosen so that $\mu_{11} \gtrsim 200$ GeV and $\mu_{11} \lesssim 1$ TeV. This corresponds to relatively small values of $\lambda_{11} \lesssim 0.17$. To simplify our analysis we set $\widetilde{\mu}_1 \simeq 2$ TeV. In addition, we also require the validity of perturbation theory up to the scale M_X that, in particular, constrains the allowed range of f_{11} and $\tilde{f}_{11}$.

The results of our analysis are summarised in Figures 1 and 2. From the approximate expressions (20) and (27), it follows that the couplings $g_{h\chi\chi}$ and R_{Z11} vanish if $f_{11} \approx -\tilde{f}_{11} \tan\beta$. In other words, the interactions of the lightest exotic neutralino with the baryons can be extremely weak in some part of the SE$_6$SSM parameter space. Our numerical analysis indicates that the perturbation theory method describes the dependence of $g_{h\chi\chi}$ and R_{Z11} on the SE$_6$SSM parameter quite well when $\widetilde{\mu}_1$ is sufficiently large (see Figures 1 and 2). In particular, $g_{h\chi\chi}$ and σ_{SI} grow whereas R_{Z11} and $\sigma^{p,n}$ diminish with increasing μ_{11} from 200 GeV to 1 TeV. Since in the scenario under consideration $\tilde{f}_{11}$ is chosen to be negative while f_{11} is positive, $g_{h\chi\chi}$ and R_{Z11}, as well as the spin-dependent and spin-independent cross sections decreases when f_{11} grows.

The calculated values of σ_{SI} and $\sigma^{p,n}$ should be compared with the corresponding experimental limits. For $m_{\chi_1} \approx \mu_{11} < 1.1$ TeV the value of $\Omega_{\tilde{H}} h^2$ can be substantially smaller than $(\Omega h^2)_{\rm exp}$ that leads to some reduction in the number density of the lightest neutral exotic fermions. Thereby, the direct detection limits become weakened, i.e.,

$$\sigma_{SI} < \frac{(\Omega h^2)_{\rm exp}}{\Omega_{\tilde{H}} h^2} (\sigma_{SI})_{\rm exp} \,, \qquad \sigma^{p,n} < \frac{(\Omega h^2)_{\rm exp}}{\Omega_{\tilde{H}} h^2} (\sigma^{p,n})_{\rm exp} \,, \tag{30}$$

where σ_{SI}, $\sigma^{p,n}$ and $\Omega_{\tilde{H}} h^2$ are computed values of these quantities for each set of the parameters of the SE$_6$SSM, while $(\sigma_{SI})_{\rm exp}$ and $(\sigma^{p,n})_{\rm exp}$ are the experimental limits on the spin-independent and spin-dependent cross-sections at the given mass m_{χ_1}. Any set of the

SE$_6$SSM parameters that does not satisfy the conditions (30) is basically ruled out. Hereafter, we just compare σ_{SI} and $\sigma^{p,n}$ with $(\sigma_{SI})_{\text{exp}}$ and $(\sigma^{p,n})_{\text{exp}}$.

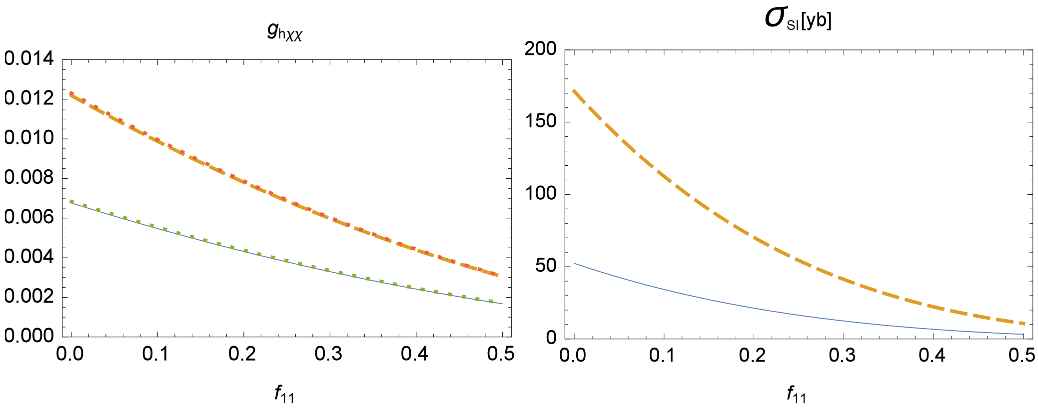

Figure 1. (**Left**) The coupling $g_{h\chi\chi}$ and (**Right**) the cross-section σ_{SI} as a function of f_{11} for $\tilde{f}_{11} = -0.5$, $\tan\beta = 2$, $\tilde{\mu}_1 = 2\,\text{TeV}$, $\mu_{11} = 200\,\text{GeV}$ (solid lines), and $\mu_{11} = 1\,\text{TeV}$ (dashed lines). The dotted lines correspond to the approximate expression for $g_{h\chi\chi}$ (20).

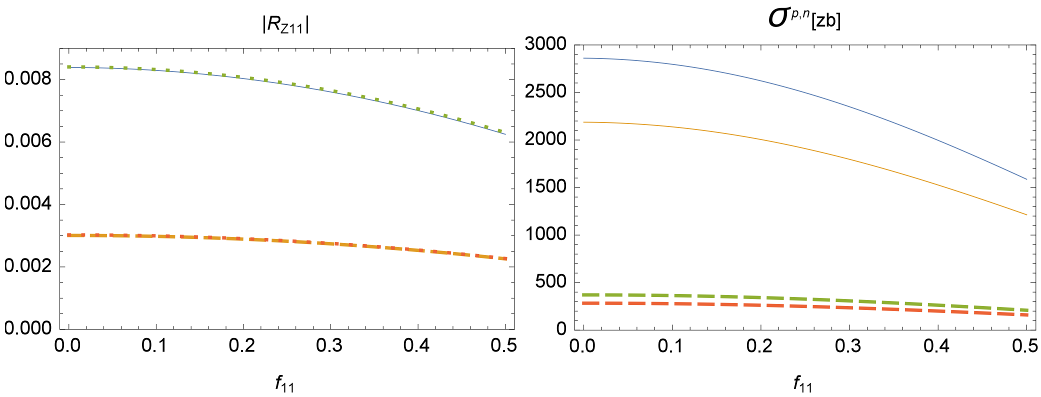

Figure 2. (**Left**) The coupling R_{Z11} and (**Right**) the cross-section $\sigma^{p,n}$ as a function of f_{11} for $\tilde{f}_{11} = -0.5$, $\tan\beta = 2$, $\tilde{\mu}_1 = 2\,\text{TeV}$, $\mu_{11} = 200\,\text{GeV}$ (solid lines), and $\mu_{11} = 1\,\text{TeV}$ (dashed lines). The upper solid and upper dashed lines represent σ^p while lower solid and lower dashed lines are associated with σ^n. The dotted lines correspond to the approximate expression for R_{Z11} (27).

The spin-independent cross-sections shown in Figure 1 remain always smaller than 60 yb for $m_{\chi_1} \approx \mu_{11} \approx 200\,\text{GeV}$ and 300 yb for $m_{\chi_1} \approx \mu_{11} \approx 1\,\text{TeV}$ ($1\,\text{yb} = 10^{-48}\,\text{cm}^2$) which are the values of $(\sigma_{SI})_{\text{exp}}$ obtained by the LUX–ZEPLIN (LZ) experiment [10]. The most stringent experimental upper bound on the WIMP-proton spin-dependent cross-section was set by the PICO-60 experiment [75]. For $m_{\chi_1} \approx \mu_{11}$ equals to 200 GeV (1 TeV) the value of $(\sigma^p)_{\text{exp}} \approx 10^5\,\text{zb}\,(4 \cdot 10^5\,\text{zb})$, where $1\,\text{zb} = 10^{-45}\,\text{cm}^2$. The WIMP-neutron spin-dependent cross-section is more tightly constrained. If $m_{\chi_1} \approx \mu_{11} \approx 200\,\text{GeV}$ (1 TeV) the LZ experiment set the experimental limit $(\sigma^n)_{\text{exp}} \approx 0.9 \cdot 10^4\,\text{zb}\,(5 \cdot 10^4\,\text{zb})$ [10]. All experimental bounds on the spin-dependent cross sections mentioned here are substantially larger than the values of $\sigma^{p,n}$ presented in Figure 2.

In the part of the SE$_6$SSM parameter space, where $|g_{h\chi\chi}|$ and $|R_{Z11}|$ become smaller than 10^{-3}, one cannot ignore quantum corrections to the effective Lagrangian (16) which are induced by the one-loop diagrams involving the electroweak gauge bosons [76,77]. The inclusion of such corrections results in $\sigma_{SI} \sim 0.1$ yb even when $|g_{h\chi\chi}| \ll 10^{-3}$ at the tree level [78].

The values of $\sigma^{p,n}$ and σ_{SI} presented in Figures 1 and 2 are substantially smaller than the maximal possible values of the spin-dependent and spin-independent χ_1-nucleon scattering cross-sections in the SE$_6$SSM. In the part of the parameter space examined in

Figures 1 and 2, the suppression of $\sigma^{p,n}$ and σ_{SI} is caused by the large value of $\widetilde{\mu}_1$, which is associated with the sparticle mass scale, as well as by the partial cancellations of different contributions to $g_{h\chi\chi}$ and R_{Z11}. The maximal possible values of $\sigma^{p,n}$ and σ_{SI} are attained when $\mu_{11} \simeq \widetilde{\mu}_1$ and $\tilde{f}_{11} \sim f_{11} \sim 1$. In this case, the spin-independent χ_1-nucleon scattering cross section can reach 20–30 zb which is two orders of magnitude larger than the present experimental limit on σ_{SI} [10]. However, even when $\mu_{11} \simeq \widetilde{\mu}_1$, the desirable suppression of σ_{SI} can be achieved if $\tilde{f}_{11} \sim f_{11} \ll 0.1$.

4. Conclusions

To ensure anomaly cancellation the $U(1)_N$ extensions of the MSSM (E_6SSM) are expected to include at least three fundamental representations of E_6 below the GUT scale M_X. These fundamental representations of E_6, in particular, contain three families of Higgs-like doublets H_i^d and H_i^u. One pair of such Higgs-like doublet supermultiplets ($H_u \equiv H_3^u$ and $H_d \equiv H_3^d$) gains VEVs resulting in the breakdown of the EW symmetr,y while two other families (H_α^d and H_α^u, where $\alpha = 1, 2$) are normally assumed to be inert.

In addition to three 27-plets the low-energy matter content of the SE$_6$SSM also involves an E_6 singlet superfield ϕ, a pair of superfields S and $\overline{S}$ with opposite $U(1)_N$ charges, as well as a pair of $SU(2)_W$ doublets L_4 and $\overline{L}_4$ with opposite $SU(2)_W \times U(1)_Y \times U(1)_N$ quantum numbers. The supermultiplets S, $\overline{S}$, L_4 and $\overline{L}_4$ can come from additional $27'_l$ and $\overline{27}'_l$. Within the SE$_6$SSM a single $\tilde{Z}_2^H$ symmetry prevents rapid proton decay and tree-level non-diagonal flavor transitions. In this paper, we have considered new variant of this SUSY model in which the low-energy particle spectrum is supplemented by three E_6 singlet superfields ϕ_i which are odd under the $\tilde{Z}_2^H$ symmetry. This permits to avoid the presence of neutral exotic fermions with tiny masses ($\lesssim 1\,\text{eV}$) in the spectrum of the SE$_6$SSM. The scalar components of the $\tilde{Z}_2^H$ even superfields ϕ, S and $\overline{S}$ may acquire very large VEVs, i.e., $\langle\phi\rangle \sim \langle S\rangle \sim \langle\bar{S}\rangle \gg 10\,\text{TeV}$, breaking the $U(1)_N$ gauge symmetry and generating masses of all extra exotic fermions which can be much heavier than all SM particles.

The conservation of R–parity and the $\tilde{Z}_2^H$ symmetry within the SE$_6$SSM ensures that at least two neutral states can be stable, giving rise to dark matter density. Since no firm indication of the presence of dark matter has been observed at the direct detection experiments we assume that one of the stable states is gravitino with mass $m_{3/2} \lesssim 1\,\text{GeV}$. Another stable state can be the lightest exotic fermion which is mostly composed of the neutral fermion components of the supermultiplets H_1^d and H_1^u. In this case, the lightest and second lightest exotic neutralino states (χ_1 and χ_2), as well as the lightest exotic chargino $\chi_1^\pm$ are almost degenerate. If these exotic fermions have masses below 1.1 TeV they can lead to the phenomenologically acceptable density of the dark matter.

The interactions of dark matter with the baryons are defined by the couplings of χ_1 because the interaction of gravitino with the SM fermions and bosons is extremely weak. In this article, we have explored the scenario in which all scalars and exotic fermions except the lightest Higgs boson, gravitino, χ_1, χ_2, and $\chi_1^\pm$ have masses of the order of a few TeV or even higher. Therefore, the spin-dependent and spin-independent χ_1-nucleon cross-sections ($\sigma^{p,n}$ and σ_{SI}) are dominated by the t-channel exchanges of the Z boson and the lightest Higgs scalar, respectively. Our analysis revealed that for a fixed mass of the lightest exotic fermion m_{χ_1} the values of $\sigma^{p,n}$ and σ_{SI} vanish in the leading approximation if $f_{11} = -\tilde{f}_{11}\tan\beta$. As a result for a given m_{χ_1} the spin-independent χ_1-nucleon cross-section varies from its maximal value, which is about 20–30 zb, to zero. When $f_{11} \approx -\tilde{f}_{11}\tan\beta$ so that $|g_{h\chi\chi}| \ll 10^{-3}$ and $|R_{Z11}| \ll 10^{-3}$, one cannot ignore the contributions to $\sigma^{p,n}$ and σ_{SI} induced by the t-channel exchanges of the heavy Higgs states. Moreover, in this case, one has to take into account the quantum corrections to the effective Lagrangian (16), which is beyond the scope of the present paper. Nevertheless, the results presented in Figures 1 and 2 demonstrate that there is a part of the SE$_6$SSM parameter space in which $\sigma^{p,n}$ and σ_{SI} can be considerably smaller than the corresponding experimental limits. In particular, the spin-independent dark matter nucleon scattering cross section may be less than 10 yb. In the near future, the experiments XENONnT [79], LZ [80], DarkSide-20k

[81], and DARWIN [82] may set even more stringent bounds on the spin-dependent and spin-independent WIMP-nucleon scattering cross-sections constraining further the SE$_6$SSM parameter space.

As mentioned before, the scenario under consideration implies that two lightest exotic neutralino states and the lightest exotic chargino must be lighter than 1.1 TeV. Several collider experiments have searched for such particles. However, if the mass splitting between these states is small the decay products of χ_2 and $\chi_1^{\pm}$ are very soft and may escape detection. This happens, for example, with the lightest ordinary neutralino and chargino states within natural SUSY, where the splitting between the heavier neutralino or the chargino and the LSP is at least a few GeV [83–85]. In this case, the results of the search for the lightest exotic fermions depend on $\Delta = m_{\chi_1^{\pm}} - m_{\chi_1}$.

At the LHC the pair production of such states can occur through off-shell W and Z bosons. ATLAS excluded the exotic chargino with masses below 193 GeV and 140 GeV for $\Delta \simeq 4.7$ GeV and $\Delta \simeq 2$ GeV, respectively [86], while CMS ruled out such chargino with masses below 112 GeV for $\Delta = 1$ GeV [87]. On the other hand, if $\Delta \lesssim 150$ MeV the exotic charginos may be long-lived. When their lifetime is longer than the time needed to pass through the detector, these states appear as charged stable massive particles. LHC experiments excluded such charginos with masses below 1090 GeV [88].

The last experimental limit is not applicable in the case of the SE$_6$SSM scenario discussed here because Δ tends to be larger than 300 MeV [78,89]. The part of the SE$_6$SSM parameter space explored in Figures 1 and 2 is associated with relatively small mass splitting between χ_1, χ_2, and $\chi_1^{\pm}$. For $m_{\chi_1^{\pm}} \simeq \mu_{11} = 200$ GeV the value of $\Delta \approx \Delta_1$ estimated using Equation (11) changes from 1.67 GeV to 0.42 GeV when f_{11} increases from 0 to 0.5. As a consequence the lightest exotic chargino decays mainly into hadrons and the lightest exotic neutralino so that it seems to be rather problematic to discover the set of these states at hadron colliders. The discovery prospects for such fermions at future International Linear Collider look more promising (for a review see [90]).

Thus, in this article we argued that within the new variant of the SE$_6$SSM, in which the cold dark matter density is formed by two stable states, there are some regions of the parameter space that are safe from all current constraints. The SUSY model under consideration predicts the existence of exotic fermions and exotic scalars which are colored and carry baryon and lepton numbers simultaneously [6]. In collider experiments, such exotic quarks/squarks can only be created in pairs. The stringent LHC lower limits on the masses of scalar leptoquarks [91–93] are not directly applicable in the case of the SE$_6$SSM. Indeed, in the E_6 inspired SUSY models ordinary scalar leptoquark can decay either to an up-type quark u_i and charged lepton ℓ_j or to down-type quark d_i and left-handed neutrino ν_j. On the other hand, in the SE$_6$SSM the lightest exotic colored particle D_1 (scalar or fermion) is odd under $\tilde{Z}_2^E$ symmetry. Therefore, its decay always gives rise to the missing energy and transverse momentum in the final state associated with the lightest exotic fermion, i.e.,

$$D_1 \to u_i(d_i) + \ell_j(\nu_j) + E_T^{\rm miss} + X\,.$$

Here, X may include extra charged leptons and/or jets that can come from the decays of intermediate states. Although the masses of the exotic colored particles are expected to be in the multi TeV range D_1 can be lighter than 2 TeV. If D_1 is exotic colored fermion the LHC pair production cross section for D_1 decreases from 10 fb to 1 fb when its mass changes from 1.3 TeV to 1.7 TeV [94]. The LHC pair production cross-section for the lightest exotic squarks tends to be an order of magnitude smaller. The pair production of D_1 may lead to some enhancement of the cross sections of $pp \to jj\ell^+\ell^- + E_T^{\rm miss} + X$ and $pp \to jj + E_T^{\rm miss} + X$ if such states are relatively light. The discovery of these exotic colored states will provide a smoking gun signal of the model allowing to distinguish it from other extensions of the SM.

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