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Iterative Learning Control for Actuator Fault Uncertain Systems

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Abstract: An iterative learning fault-tolerant control method is designed for an actuator fault intermittent process with simultaneous uncertainties for the system parameters. First, an intermittent fault tolerance controller is designed using 2D system theory, and the iterative learning control (ILC) intermittent process is transformed into a 2D Roesser model. Secondly, sufficient conditions for the controller's existence are analyzed using the linear matrix inequality (LMI) technique, and the control gain matrices are obtained by convex optimization with LMI constraints. Under these conditions for all additive uncertainties for the system parameters and admissible failures, the controller can ensure closed-loop fault-tolerant performance in both the time and batch directions, and it can also meet the H∞ robust performance level against outside disturbances. Eventually, the algorithm's computational complexity is analyzed, and the effectiveness of the algorithm is verified by simulation with respect to an injection molding machine model. Compared with traditional ILC laws, which do not consider actuator faults, the proposed algorithm has a better convergence speed and stability when the time-invariant and time-variant actuator faults occur during implementation.

Keywords: actuator fault; uncertainties; intermittent process; iterative learning fault-tolerant control; LMI

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1. Introduction

The intermittent process is an important production method in modern industry which is widely applied in the manufacturing process of products with high accessional value such as food, medicine, agrochemicals, coatings, sensors, and polymers [1–4]. Correspondingly, the systems involved often need to be controlled under complex conditions, which will highlight the influence of uncertainty in the establishment of mathematical models, increase the system failure probability, and make system control more difficult. To ensure system reliability and security, it is particularly necessary to develop a fault-tolerant controller and take effective corrective measures for the failure of the uncertain intermittent process.

The fault-tolerant control topic of a continuous process has attracted the attention of many scholars in recent years. The authors of in [5] studied the robustness of the gyroscope system when the sensor fails. In [6], a fault-tolerant controller was designed to achieve attitude tracking control under the condition of flywheel failure. In [7,8], different fault tolerant control strategies were introduced to control the pitch angle of a wind turbine with actuator and sensor faults, uncertainties, and external disturbances. Recently, some artificial intelligence technologies were also used to control non-repetitive and time-varying parameter systems [9–11]. The design of the above controllers must be carried out on the premise that the specific fault information is known. Reliable control is a kind of fault-tolerant control method [12–15], and it is widely used. In [5], the sensor failure of the linear system was considered, and the Riccati equation was used to design a reliable quadratic Gaussian controller. In [13], the aim was to create a model with actuator failure that analyzed the H∞ robust performance of the system in this situation. The above research all aimed at the fault-tolerant control of a continuous process, but in recent years, the fault-tolerant control

of the intermittent process attracted scholars' attention. Although an intermittent process can be treated as continuous in each batch, a fault-tolerant method of reliable control cannot guarantee the performance of a batch process over a limited period, nor can it improve control performance on a batch basis [16]. Therefore, to improve the control performance of the current batch, the ILC algorithm can be combined with the research based on the repeatability and periodicity of the intermittent process. In ILC, the intermittent process can be transformed into corresponding two-dimensional models [17–19], and 2D system theory can be applied to break through various of constraints in one-dimensional systems [19]. Therefore, when designing the controller, the control effectiveness can be optimized by iteratively learning the information of the previous batch, and LMI can be applied to solve the parameters of the controller effectively. The authors of [17] proposed an ILC algorithm using 2D robust control theory and discussed the asymptotic stability of uncertain intermittent systems in both the iterative and time directions. For a time-varying and time delay intermittent process with parameter uncertainty, a feedback ILC algorithm was discussed in [20], and a learning-based identification technique for uncertain parameters in a system was obtained based on 2D system theory.

In addition, the ILC controller design scheme has been used to resolve the fault-tolerant control problem of the intermittent process with actuator failure in recent years. The corresponding 2D F-M model was then proposed for a new ILC strategy. However, the above control approaches do not fully consider the system uncertainty and controllers under the constraints of multiple performance indicators which are not balanced or optimized to achieve satisfactory performance and fault tolerance. Therefore, the main contribution of this paper is that an iterative learning fault-tolerant control algorithm is defined for an intermittent process affected by actuator faults and uncertainties. First, the intermittent process is transformed to a corresponding 2D Roesser system model. Secondly, an ILC controller is designed to give sufficient conditions in the form of LMI to ensure system stability in both the time and batch dimensions, and based on this condition, existence of the H∞ robust fault-tolerant control matrix can be simultaneously guaranteed under the influence of unknown disturbances, and a balance and optimization method is used to resolve the LMI to obtain a satisfactory controller. Finally, the results of the injection molding machine simulation experiment verify the algorithm's effectiveness. Therefore, the algorithm can be used for some specific industry area with repetitive operation, such as injection molding machines, robot manipulators, or agricultural irrigation. It has better performance and stability when an actuator fault occurs. However, there are also some limitations of the designed algorithm in this paper, such as the uncertain parameters being difficult to estimate, the actuator fault extent being unknown, and the computational resource being high. Because of this, overcoming these limitations will be the area of our deep cultivation in the future.

In this paper, matrices X, $X^{\top}$, and $X^{\perp}$ represent its transposed and orthogonal complements. For any matrix $M \in R^{n \times n}$, $M > 0$ represents M being a positive definite symmetric matrix, sym(X) represents $(X + X^{T})$, I and 0 represent the identity and 0 matrices, respectively, the character "*" represents the transpose at the corresponding position of the matrix, and the symbol "∀" means any. For a two-dimensional signal $w(t,k)$ and any integer N_1,$N_2 > 0$ that satisfies $\|w\|_{2e} = \sqrt{\sum_{t\to 0}^{N_1}\sum_{k\to 0}^{N_2}\|w(t,k)\|^2} < \infty$, we then say $w \in \iota_{2e}$.

2. Problem Description

2.1. System Description

Consider a batch process of the form given below:

$$\begin{cases} x(t+1,k) = (A + \Delta A(t))x(t,k) + ((B + \Delta B(t))u(t,k)) + Dw(t,k) \\ y(t,k) = (C + \Delta C(t))x(t,k)) \end{cases} \tag{1}$$

in the above formula , $x(0,k) = x_{0,k}, t = 0,1,\ldots,$ and $k = 1,2,\ldots,$ which is the number of iterations. $x(t,k) \in R^n$, $u(t,k) \in R^{m,}$ and $y(t,k) \in R^l$ are the system state, input and

output at time t, and batch k, while $x_{0,k}$ represents the system's initial condition at batch k, $w(t,k)$ is the system's outside disturbance, A, B, C, and D are dimensionality matrices, and $\Delta A(t,k)$,$\Delta B(t,k)$, and $\Delta C(t,k)$ are uncertain disturbances of the system state and output corresponding to A, B, and C, respectively, at time t. We define

$$\begin{cases} \Delta A(t) = E_1 \Delta_1(t) F_1 \\ \Delta B(t) = E_1 \Delta_2(t) F_2 \\ \Delta C(t) = E_2 \Delta_1(t) F_3 \end{cases} \tag{2}$$

where $\{E_1, F_1, F_2, E_2, F_3\}$ is a real constant matrix with known suitable dimensions and $\{\Delta_i(t)\}_{i=1,2,3}$ is an unknown, bounded uncertainty, non-repetitive system perturbation which at the same time satisfies

$$\Delta_i^T(t)\Delta_i(t) \le I \qquad (\forall t \ge 0; i = 1,2,3) \tag{3}$$

for the input $u_i(t,k)(i = 1,2,\ldots,n)$, let $u_i^F(t,k)$ represent the actuator fault output, and define the actuator fault model as follows [21]:

$$u_i^F(t,k) = \Gamma_i u_i(t,k)), \quad i = 1,2,\ldots,n \tag{4}$$

among which

$$0 \le \underline{\Gamma}_i \le \Gamma_i \le \overline{\Gamma}_i, \quad i = 1,2,\ldots,n \tag{5}$$

where Γ_i is an unknown quantity, but its range of variation is known. If $\Gamma_i = 1$, then the actuator is normal $u_i^F = u_i$. If $\Gamma_i = 0$, then it is a complete failure. If $\Gamma_i > 0$, then it is a partial failure. We define

$$\begin{array}{l} u^F = diag\{u_1^F, u_2^F, \ldots, u_n^F\} \\ \overline{\Gamma}_i = diag\{\overline{\Gamma}_1, \overline{\Gamma}_2, \ldots, \overline{\Gamma}_n\} \\ \underline{\Gamma}_i = diag\{\underline{\Gamma}_1, \underline{\Gamma}_2, \ldots, \underline{\Gamma}_n\} \\ \Gamma_i = diag\{\Gamma_1, \Gamma_2, \ldots, \Gamma_n\} \end{array} \tag{6}$$

meanwhile, the following definitions are introduced:

$$\begin{array}{l} q = diag\{q_1, q_2, \ldots, q_n\} \\ q_0 = diag\{q_1 0, q_2 0, \ldots, q_n 0\} \\ q_i = (\overline{\Gamma}_i + \underline{\Gamma}_i)/2, \quad i = 1,2,\ldots,n \\ q_{i0} = (\overline{\Gamma}_i - \underline{\Gamma}_i)/(\overline{\Gamma}_i + \underline{\Gamma}_i), \quad i = 1,2,\ldots,n \end{array} \tag{7}$$

from Equations (6) and (7), for the unknown matrix Γ_0, Γ can be expressed as

$$\Gamma = (I + \Gamma_0)q \tag{8}$$

and

$$\|\Gamma_0\| \le q_0 \le I \tag{9}$$

among which $\Gamma_0 = diag\{\Gamma_{01}, \Gamma_{02}, \ldots, \Gamma_{0n}\}$ and $|\Gamma_0| = diag\{|\Gamma_{01}|, |\Gamma_{02}|, \ldots, |\Gamma_{0n}|\}$.

Therefore, the intermittent process model for the condition of actuator failure can be expressed as

$$\begin{cases} x(t+1,k) = (A + \Delta A(t))x(t,k) + (B + \Delta B(t))\Gamma u(t,k) + Dw(t,k) \\ y(t,k) = (C + \Delta C(t))x(t,k) \\ x(0,k) = x_{0,k}; \quad t = 0,1,\ldots,t_p; \quad k = 1,2,\ldots \end{cases} \tag{10}$$

2.2. Equivalent Two-Dimensional Model

For the uncertain batch process in Equation (1), we design the design control law as shown below:

$$\begin{aligned} u(t,k) &= u(t,k-1) + r(t,k) \\ u(t,0) &= 0, \quad t = 0,1,\ldots,t_p \end{aligned} \tag{11}$$

where $u(t,0)$ is the system's initial input and $r(t,k)$ is the update law.

The variables of the iterative fault-tolerant control system refer to the independent functions of time t and batch k, which conform to two-dimensional dynamic characteristics. The fault-tolerant control of ILC in this paper learns from the faulty system in Equation (10), so the actual output $y(t,k)$ can track the expected trajectory $y_d(t)$ after multiple learning batches. The control tracking error definition of the intermittent process is

$$e(t,k) = y_d(t) - y(t,k). \tag{12}$$

To be convenient, for arbitrary variables $f(x,w\ldots)$, the following notation is introduced:

$$\delta f(t,k) = f(t,k) - f(t,k-1). \tag{13}$$

According to Equation (10), the control law in Equation (11), and Equation (12), we can find

$$\delta x(t+1,k) = \bar{A}\delta x(t,k) + \bar{B}\Gamma r(t,k) + D\delta w(t,k) \tag{14}$$

$$e(t+1,k) = e(t+1,k-1) - \bar{C}\delta x(t+1,k) = e(t+1,k-1) - \bar{C}\bar{A}\delta x(t1,k) - \bar{C}\bar{B}\Gamma r(t,k) - \bar{C}D\delta w(t,k) \tag{15}$$

where $\bar{A} = (A + \Delta A(t))$ and $\bar{B} = (B + \Delta B(t))$, $\bar{C} = (C + \Delta C(t))$. A 2D Roesser model can be acquired by combining Equations (14) and (15):

$$\begin{cases} \begin{bmatrix} \delta x(t+1,k) \\ e(t+1,k) \end{bmatrix} = \hat{A}\begin{bmatrix} \delta x(t,k) \\ e(t+1,k)-1 \end{bmatrix} + \hat{B}\Gamma r(t,k) + \hat{D}\delta w(t,k) \\ z(t,k) = G\begin{bmatrix} \delta x(t,k) \\ e(t+1,k-1) \end{bmatrix} \end{cases} \tag{16}$$

where $\hat{A} = \begin{bmatrix} \bar{A} & 0 \\ -\bar{C}\bar{A} & I \end{bmatrix}$, $\hat{B} = \begin{bmatrix} \bar{B} \\ -\bar{C}\bar{B} \end{bmatrix}$, $\hat{D} = \begin{bmatrix} \bar{D} \\ -\bar{C}\bar{D} \end{bmatrix}$, $G = \begin{bmatrix} 0 & I \end{bmatrix}$.

For the 2D Roesser model in Equation (16), we design a fault-tolerant control law:

$$r(t,k) = K\begin{bmatrix} \delta x(t,k) \\ e(t+1,k-1) \end{bmatrix} = K_1\delta x(t,k) + K_2 e(t+1,k-1) \tag{17}$$

where $K_1 \in R^n$ and $K_2 \in R^l$ are the unknown gain matrices. Then, the 2D Roesser system close-loop form from Equation (16) is

$$\begin{cases} \begin{bmatrix} \delta x(t+1,k) \\ e(t+1,k) \end{bmatrix} = (\hat{A} + \hat{B}\Gamma K)\begin{bmatrix} \delta x(t,k) \\ e(t+1,k-1) \end{bmatrix} + \hat{D}\delta w(t,k) \\ z(t,k) = G\begin{bmatrix} \delta x(t,k) \\ e(t+1,k-1) \end{bmatrix} \end{cases} \tag{18}$$

3. 2D Roesser System Control Theory

To analyze the system in Equation (18), robustness stability, and monotonic convergence in the iterative and time directions, we consider the following F-M mode [22]:

$$\begin{cases} X(t+1,k) = A_1X(t,k) + A_2X(t+1,k-1) + B\Gamma r(t,k) + C\delta w(t,k) \\ z(t,k) = GX(t,k) \\ r(t,k) = K_1X(t,k) + K_2X(t+1,k-1) \end{cases} \tag{19}$$

where $X(t,0) = X^{t,0}$, $X(0,k) = X^{0,k}$, $t = 0,1,\ldots,T$, k=1,2,..., $X(t,k) \in R^n$ is the state variable, $r(t,k) \in R^m$ is the control law, $z(t,k) \in R^l$ is the output variable, A_1, A_2, B, C, G, K_1, and K_2 are the dimensional real matrices, Γ is the fault information, $X^{t,0}$ is initial state of time t, and $X^{0,k}$ is the initial state of the batch k.

The time axis of the system is marked as the T-axis, the batch axis is marked as the K-axis, and the boundaries $X_2^{t,0}$ and $X_2^{0,k}$ are defined as the T-boundary, respectively. We let $X(t,k) = \begin{bmatrix} \delta x(t+1,k) \\ e(t+1,k) \end{bmatrix}$, and the 2D Roesser system in Equation (16) is transformed into an F-M model:

$$\begin{cases} X(t+1,k) = \begin{bmatrix} \bar{A} & 0 \\ 0 & 0 \end{bmatrix} X(t,k) + \begin{bmatrix} 0 & 0 \\ -\bar{C}\bar{A} & I \end{bmatrix} X(t+1,k-1) + \begin{bmatrix} \bar{B} \\ -\bar{C}\bar{B} \end{bmatrix} \Gamma r(t,k) + \hat{D}\delta w(t,k) \\ z(t,k) = \begin{bmatrix} 0 & I \end{bmatrix} X(t+1,k-1) \end{cases}$$

therefore, the conclusions of the system in Equation (19) also apply to the system in Equation (18). The purpose is to design suitable ILC laws to ensure the rejection of actuator faults and external disturbance. The ILC laws in this paper are $K1$ and $K2$. The controller design framework has four steps. The first step is to discretize the plant model, the second step is to estimate the uncertain parameters, the third step is to develop LMI in reference to the inequalities combining all constraints, and the fourth step is to solve the LMI to find the best ILC laws. Then, the ILC laws are used to iterate the controller input. Figure 1 below was drawn to illustrate the framework.

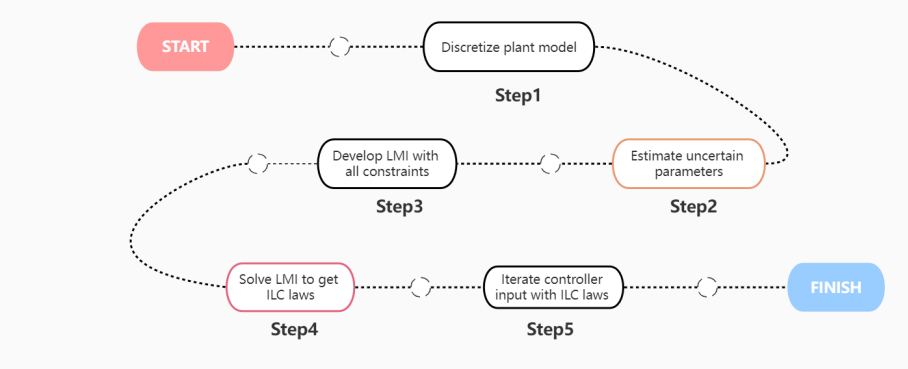


Figure 1. Controller design framework.

Definition 1 ([23]). *For a scalar $\gamma > 0$, if the F-M model (Equation (19)) is asymptotically stable, and for the zero-boundary condition and any disturbance w(t,k),the system output satisfies Equation*

$$\|z\|_2 < \gamma \|\delta w\|_2 \tag{20}$$

then the system is said to have robustness H∞ in its performance, where γ represents the maximum output sensitivity to disturbances. The smaller the value of γ, the more robust the system becomes.

Definition 2. *The 2D system in Equation (19) is two-dimensional fault-tolerant if there exists a function $V(\cdot)$ and a scalar $0 < \rho < 1$ that satisfy the following:*

1. *$V(x) \geq 0$ for $\forall x \in R^n$, and moreover, $V(x) = 0 \leftrightarrow x = 0$;*
2. *When $\|x\| \to \infty$, $V(x) \to \infty$;*
3. *For arbitrary boundary conditions and actuator faults satisfying Equation (5), there exists*

$$\sum_{\substack{t+k=T_0+K_0+i+1 \\ T_0 \leq t \leq T_0+i \\ K_0 \leq k \leq K_0+i}} V(x(t,k)) < \rho \sum_{\substack{t+k=T_0+K_0+i \\ T_0 \leq t \leq T_0+i \\ K_0 \leq k \leq K_0+i}} V(x(t,k)) \quad (\forall T_0, i, K_0 > 0) \tag{21}$$

among this, the smallest ρ satisfying Equation (19) is defined as a two-dimensional convergence index (2D-CI) of the system in Equation (19).

The smaller the value of the 2D-CI, the faster the convergence of system. Therefore, an appropriate 2D-CI value is selected to optimize the fault tolerance and convergence performance of the system in Equation (19):

Definition 3. *Assume that the 2D system in Equation (19) has functions $V(\cdot)$ and scalars $0 < \alpha < 1$ that satisfy the following:*

1. *$V(x) \geq 0$ for $\forall x \in R^n$, and moreover, $V(x) = 0 \leftrightarrow x = 0$;*
2. *When $\|x\| \to \infty$, $V(x) \to \infty$;*
3. *Let the K-bound condition $X_2^{t,0}$ be 0, and for any T-bound $X_2^{0,k}$, any integer $N > 0$ and any tolerable actuator fault have*

$$\sum_{k=1}^{N} V(x(t+1,k)) < \alpha \sum_{k=1}^{N} V(x(t,k))) \quad (\forall t > 0) \tag{22}$$

among this, the minimum value α which meets Equation (22) is called the T-fault tolerance index (T-CI) of the system in Equation (19).

Definition 4. *Assume that the 2D system in Equation (19) has functions $V(\cdot)$ and scalars $0 < \beta < 1$ that satisfy:*

1. *$V(x) \geq 0$ for $\forall x \in R^n$, and moreover, $V(x) = 0 \leftrightarrow x = 0$;*
2. *When $\|x\| \to \infty$, $V(x) \to \infty$;*
3. *Let the T-bound condition $X_2^{0,k}$ be 0, and for any T-bound $X_2^{t,0}$, any integer $N > 0$ and any tolerable actuator fault have*

$$\sum_{t=1}^{N} V(x(t+1,k)) < \beta \sum_{t=1}^{N} V(x(t,k))) \quad (\forall k > 0) \tag{23}$$

among this, the minimum value β which meets Equation (23) is called the K-fault tolerance index (K-CI) of the system in Equation (16).

The T-CI is relevant to tracking the error in the time direction, and the K-CI is related to tracking the error in the batch direction. The smaller their values, the faster the error convergence, and the better the fault tolerance performance. Appropriate practical parameters should be chosen to balance and optimize the two-dimensional system performance in both directions. Therefore, in Section 4, the fault-tolerant performance index and H∞ performance index are discussed individually to obtain the appropriate practical parameters:

Lemma 1 ([22]). *For the 2D system in Equation (19), when $\delta w(t,k) \equiv 0$, the system is two-dimensional fault-tolerant, and when T-CI < α, K-CI < β, and 2D-CI < ρ, then there must be a positive definite matrix P, Q_1, $Q_2 > 0$ such that*

$$\begin{cases} \begin{bmatrix} (A_1 + B\Gamma K_1)^T \\ (A_2 + B\Gamma K_2)^T \end{bmatrix} P [(A_1 + B\Gamma K_1)^T (A_2 + B\Gamma K_2)^T] - \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} < 0 \\ \dfrac{1}{\alpha} Q_1 + \dfrac{1}{\beta} Q_2 < P \end{cases} \tag{24}$$

4. Iterative Learning Controller Design

The system in Equation (18) is a 2D Roesser system with uncertainties and outside disturbances. Although the duration of the system in Equation (18) is limited, the above conclusions can still be used for iterative learning updates to the law $r(t,k)$ in the design of

a 2D system (Equation (18)). The design goal is to make the system in Equation (15) two-dimensional fault-tolerant and have a satisfactory 2D-CI, T-CI and K-CI, when $w(t,k) \neq 0$, the system robustness H∞ performance is the most optimal.

In the analysis process of this section, the following lemma is used:

Lemma 2 ([24]). *Assuming that X and Y are two matrices of suitable dimensions, for any positive definite matrix R > 0, the below inequalities hold:*

$$XY + Y^T X^T \leq XRX^T + Y^T R^{-1} Y. \tag{25}$$

Let $R = \varepsilon^{-1} I(\varepsilon > 0)$, *Then, we have*

$$XY + Y^T X^T \leq \varepsilon XX^T + \varepsilon Y^T Y. \tag{26}$$

Lemma 3 ([24]). *Assuming that W, L, and V are given matrices of appropriate dimensions, W and V are positive definite matrices, and then* $L^T V L - W < 0$ *is equivalent to*

$$\begin{bmatrix} -W & L^T \\ L & -V^{-1} \end{bmatrix} < 0 \tag{27}$$

or

$$\begin{bmatrix} -V^{-1} & L \\ L^T & -W \end{bmatrix} < 0. \tag{28}$$

Lemma 4 (The Projection Theorem [25]). *Let* Ψ, Λ, *and* Σ *be a real matrix of appropriate dimensions. Where* $\Psi = \Psi^T$, *there is a matrix W that holds with the below inequality*

$$\Psi + \Lambda^T W \sum + \sum^T W^T \Lambda < 0 \tag{29}$$

if and only if the below inequality holds for Ψ*:*

$$\begin{cases} \Lambda^{\perp T} \Psi \Lambda^{\perp} < 0 \\ \sum^{\perp T} \Psi \sum^{\perp} < 0 \end{cases} \tag{30}$$

Theorem 1. *Suppose* $\delta w(t,k) \equiv 0$. *For any given scalar* $0 < \alpha < 1$ *and* $0 < \beta < 1$, *if there are positive definite matrices* $Q_T \in R^{n \times n}$ *and* $Q_K \in R^{l \times l}$, *matrices* $R_1 \in R^{m \times n}$ *and* $R_2 \in R^{m \times l}$, *dimensional matrices* W_1, W_2, W_3, *and* W_4, *and scalars* $\varepsilon_i > 0 (i = 1,2,3,4)$, *the below LMI holds:*

$$\begin{bmatrix} -\alpha Q_T & * & * & * \\ 0 & -\beta Q_K & * & * \\ AQ_T + BW_{31}^T & BW_{32}^T & -Q_T + BW_{33}^T + W_{33}B^T + \varepsilon_1 E_1 E_1^T + \varepsilon_2 E_2 E_2^T & -Q_K - CW_{14}^T - W_{14}C^T + \varepsilon_3 E_3 E_3^T \\ -CW_{11}^T & Q_K - CW_{12}^T & -CW_{13}^T + W_{34}B^T & -W_{14}^T - W_2 C^T + BW_{34}^T \\ -W_{11}^T + AQ_T + BW_{31}^T & -W_{12}^T + BW_{32}^T & -W_{13}^T + BW_{33}^T + W_{35}B^T + \varepsilon_1 E_1 E_1^T + \varepsilon_2 E_2 E_2^T & -W_{34}^T \\ -W_{31}^T + qR_1 & -W_{32}^T + qR_2 & -W_{33}^T + W_4 B^T & 0 \\ F_1 Q_T & 0 & 0 & 0 \\ F_3 W_{11}^T & F_3 W_{12}^T & F_3 W_{13}^T & F_3 W_{14}^T \\ F_2 W_{31}^T & F_2 W_{32}^T & F_2 W_{33}^T & F_2 W_{34}^T \\ qR_1 & qR_2 & 0 & 0 \end{bmatrix} \rightarrow$$

$$\leftarrow \begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ sym(BW_{35}^T) + \varepsilon_1 E_1 E_1^T + \varepsilon_2 E_2 E_2^T & * & * & * & * & * \\ -W_{35}^T + W_4 B^T & -W_4^T + W_4 + \varepsilon_4 q_0 q_0^T & * & * & * & * \\ 0 & 0 & -\varepsilon_1 I & * & * & * \\ F_3 W_7^T & 0 & * & -\varepsilon_3 I & * & * \\ F_2 W_{35}^T & F_2 W_4^T & 0 & 0 & -\varepsilon_2 I & * \\ 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I \end{bmatrix} < 0 \tag{31}$$

Among this, $Q = diag\{Q_T \quad Q_K\}$, $R = [R_1 \quad R_2] = KQ = [K_1 Q_T \quad K_2 Q_K]$, *and* $Q(\alpha, \beta) = diag\{\alpha Q_T \quad \beta Q_K\}$. *Then, for the fault-tolerant performance index of the system in Equation (18) T-CI <* α, *K-CI <* β, *and 2D-CI <* ρ.

Proof. See Appendix A. □

To ensure the control performance in the time and batch directions. α and β should be minimized at the same time. Equation (31) becomes a nonlinear optimization problem, and a more applied and flexible optimization method needs to be designed at this time. Therefore, the following two weighting factors are proposed: $\lambda(0 \le \lambda \le 1)$ and ρ, which satisfy $\alpha = 1 - \lambda + \lambda\rho$ and $\beta = \lambda + (1-\lambda)\rho$. In addition, the larger λ is, the more emphasis is placed on the T-fault tolerance, and the smaller λ is, the more emphasis there is on the K-fault tolerance. The specific satisfactory fault-tolerant performance index depends on the specific engineering requirements. At this time, the optimization problem in Equation (31) can be transformed into

$$Minimize_{Q,M,\varepsilon_{1,2,3}}\rho \quad (Subject \quad to \quad (31)) \tag{32}$$

Theorem 2. *For any given scalar $\gamma > 0$, $0 < \alpha < 1$, and $0 < \beta < 1$ (Appendix A), if there are positive definite matrices $Q_T \in R^{n*n}$ and $Q_K \in R^{l*l}$, matrices $R_1 \in R^{m*n}$ and $R_2 \in R^{m*l}$, dimensional matrices W_1, W_2, W_3, and W_4, and positive definite symmetric matrices $\varepsilon = diag\{\varepsilon_2, \varepsilon_1, \varepsilon_3\} > 0$, then this makes the following linear matrices unequal:*

$$\begin{bmatrix} -\alpha Q_T & * & * & * & * \\ 0 & -\beta Q_K & * & * & * \\ 0 & 0 & -\gamma I & * & * \\ AQ_T + BW_{31}^T & BW_{32}^T & D + BW_{33}^T & -Q_T + sym(BW_{34}^T) + \varepsilon_1 E_1 E_1^T + \varepsilon_2 E_2 E_2^T & * \\ -CW_{11}^T & Q_K - CW_{12}^T & -CD - CW_{13}^T & -CW_{14}^T + W_{35}B^T & -Q_K - sym(CW_{15}^T) + \varepsilon_3 E_3 E_3^T \\ \mathbf{0} & Q_K & 0 & W_{36}B^T & -W_{16}C^T \\ -W_{11}^T + AQ_T + BW_{31}^T & -W_{12}^T + BW_{32}^T & -W_{13}^T + BW_{33}^T & -W_{14}^T + BW_{34}^T + W_{37}B^T + \varepsilon_1 E_1 E_1^T + \varepsilon_2 E_2 E_2^T & -W_{15}^T - W_2 C^T + BW_{35}^T \\ -W_{31}^T + qR_1 & -W_{32}^T + qR_2 & -W_{33}^T & -W_{34}^T + W_4 B^T & -W_{35}^T \\ F_1 Q_T & 0 & 0 & 0 & 0 \\ F_3 W_{11}^T & F_3 W_{12}^T & F_3 D + F_3 W_{13}^T & F_3 W_{14}^T & F_3 W_{15}^T \\ F_2 W_{31}^T & F_2 W_{32}^T & F_3 W_{33}^T & F_2 W_{34}^T & F_2 W_{35}^T \\ qR_1 & qR_2 & 0 & 0 & 0 \end{bmatrix} \rightarrow$$

$$\leftarrow \begin{bmatrix} * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ -\gamma I & * & * & * & * & * & * \\ -W_{16}^T + BW_{36}^T & -sym(W_2 - BW_{37}^T) + \varepsilon_1 E_1 E_1^T + \varepsilon_2 E_2 E_2^T & * & * & * & * & * \\ -W_{36}^T & -W_{37}^T + W_4 B^T & -sym(W_4) + \varepsilon_4 q_0 q_0^T & * & * & * & * \\ 0 & 0 & 0 & -\varepsilon_1 I & * & * & * \\ F_3 W_{16}^T & F_3 W_2^T & 0 & 0 & -\varepsilon_3 I & * & * \\ F_2 W_{36}^T & F_2 W_{37}^T & F_2 W_4^T & 0 & 0 & -\varepsilon_2 I & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I \end{bmatrix} < 0 \tag{33}$$

where $Q = diag\{Q_T, Q_K\}$, $R = \begin{bmatrix} R_1 & R_2 \end{bmatrix} = KQ = \begin{bmatrix} K_1 Q_T & K_2 Q_K \end{bmatrix}$, and $Q(\alpha, \beta) = diag\{\alpha Q_T, \beta Q_K\}$. Then, the system in Equation (18) has two-dimensional fault tolerance.

Proof. See Appendix B. □

For a given α and β, the LMI (Equations (32) and (33)) can be solved with the LMI toolbox [26]. If both α and β are regarded as decisive variables, Equation (32) and the inequality in Equation (33) are nonlinear matrix inequalities, α and β are regarded as scalars between 0 and 1, and the optimization solution is carried out.

For the LMI (Equation (33)), $\gamma > 0$ is upper limit of the robustness H∞ performance index, which is further considered as an optimization variable, and the minimum upper limit of the robust H∞ controller for the system in Equation (18) can be acquired by resolving the below obtained eigenvalue problem (EVP) [24]:

$$Minimize_{Q,M,\varepsilon_{1,2,3}}\gamma \quad (Subject \quad to \quad (33)). \tag{34}$$

The method in this paper does not need to introduce a robust tracking controller comparing the existing literature, which greatly reduces the influence of uncertainty and external disturbances on the system. This is simpler and more flexible than the cost function method for optimizing and solving the eigenvalue problem (EVP) of a certain index. The main steps of the controller design are in Algorithm 1.

Algorithm 1 Actuator Fault Uncertainty System ILC Algorithm

Input: System array $A, B, C, E1, F1, F2, F3$, actuator fault Γ, initial input signal u_0, target trajectory y^*, initial ILC law $K1^*$ and $K2^*$.
Output: Optimized controller input signal u^*, actual trajectory y.
1: **initialization:** Trial number $k = 0$.
2: Send the initial input signal u_0 to the controller, measure the initial output trajectory y_0, and obtain the initial error $\hat{e}_0$.
3: Record the initial input signal u_0 and motion trajectory y_0.
4: **while not** $\|\hat{e}_k\| < \epsilon$ **do**
5: **if** $(\Gamma = 1)$: then $K1 = K1^*$, $K2 = K2^*$.
6: **else**: Develop and Solve LMI to get optimal $K1$ and $K2$ according to Theorem 1 and 2.
7: Update the input signal u_{k+1} with $K1$ and $K2$ according to Equation (17).
8: Set $k = k + 1$ to perform the next ILC trial.
9: Send the input signal u_k to the controller and measure the output trajectory y_k.
10: Record the input signal u_k and actual profile y_k.
11: **end while**
12: **return** $u^* = u_k$ and $y^* = y_k$.

There is only one loop inside the algorithm, and in each loop, there is an LMI loop when actuator faults occur, so the calculation complexity is $o(n^2)$, where n is the number of iterations.

5. Simulation Results

To prove the feasibility of the algorithm in this paper, a typical batch system injection molding machine is taken as the simulation object. Each batch of the injection molding process includes three key processes: injection, pressure holding, and cooling [17]. In the pressure-holding process, the nozzle pressure is the key controlled factor, and it needs be controlled near the set value to ensure the product's quality. The research in [17] identified the relationship between the nozzle pressure and valve opening as the following ARX model:

$$\begin{aligned}&\{1 - 1.607(\pm 5\%)z^{-1} + 0.6086(\pm 5\%)z^{-2}\}y(t,k) = \\ &\{1.239(\pm 5\%)z^{-1} - 0.9282(\pm 5\%)z^{-2}\}u(t,k) + w(t,k)\end{aligned} \tag{35}$$

where $\pm 5\%$ represents the relative uncertainty of the parameter and$w(t,k)$ is the unknown disturbance.

The output also introduces the same uncertainty parameter, and Equation (35) can be transformed into the following state space format:

$$\begin{cases} x(t+1,k) = \left(\begin{bmatrix} 1.607 & 1 \\ -0.6086 & 0 \end{bmatrix} + \Delta A(t) \right) x(t,k) + \left(\begin{bmatrix} 1.239 \\ -0.9282 \end{bmatrix} + \Delta B(t) \right) u(t,k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(t,k) \\ y(t,k) = \left(\begin{bmatrix} 1 & 0 \end{bmatrix} + \Delta C(t) \right) x(t,k) \end{cases} \tag{36}$$

The parameters of each uncertainty are

$$\Delta A(t) = \begin{bmatrix} 0.0804\delta_1(t) & 0 \\ -0.0304\delta_2(t) & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Delta_1(t) \begin{bmatrix} 0.0804 & 0 \\ -0.0304 & 0 \end{bmatrix}$$

$$\Delta B(t) = \begin{bmatrix} 0.062\delta_1(t) \\ -0.0464\delta_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Delta_1(t) \begin{bmatrix} 0.062 \\ -0.0464 \end{bmatrix}$$

$$\Delta C(t) = \begin{bmatrix} 0.05\delta_3(t) & 0 \end{bmatrix} = \begin{bmatrix} 0.05 & 0 \end{bmatrix} \Delta_2(t) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where the scalar $\delta(t)(i = 1, 2, 3)$ varies between –1 and 1 arbitrarily and $|\delta_i(t)| \leq 1$.

Suppose the initial state is $x(0,k) \equiv \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. If there is an unknown actuator fault Γ which satisfies $0.6 = \underline{\Gamma} \leq \Gamma \leq \overline{\Gamma} = 1$, then q = 0.8, and $q_0 = 0.25$. The output target value is selected as follows:

$$y_d(t) = \begin{cases} 200 & 0 \leq t \leq 100 \\ 200 + 5(t-100) & 100 \leq t \leq 120 \\ 300 & 120 \leq t \leq 200 \end{cases}. \tag{37}$$

To evaluate the tracking performance, the introduced performance indicators are the following:

$$H(k) = \sqrt{\sum_{i=1}^{200} e^2(t,k)}. \tag{38}$$

If $H(k)$ is smaller, then the tracking effect in the batch direction is better. In this experiment, suppose that a fault occurs at the 21st time on the batch axis.

5.1. Scenario 1: Steady Faults and Repetitive Disturbances

In this case, suppose $\Gamma = 0.85$, $w(t,k) = sint$. Then, $\delta w(t,k) = 0$. In this paper, we assume $\lambda = 0.5$. Using the LMI optimization toolbox to solve Equation (32), we can obtain $\rho^* = 0.6293$, $K_1 = \begin{bmatrix} -1.703 & -1.1858 \end{bmatrix}$, $K_2 = 0.3516$, and $\alpha = \beta = 0.8146$. The simulation results are shown in Figures 2 and 3.

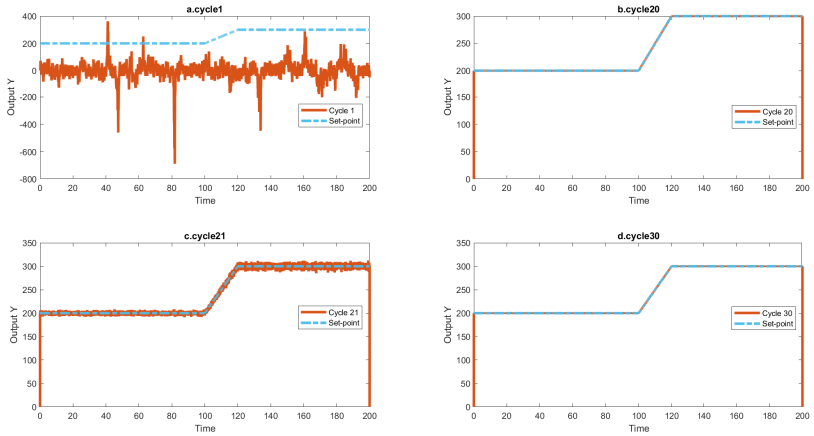


Figure 2. The output of the injection molding system under a steady fault.

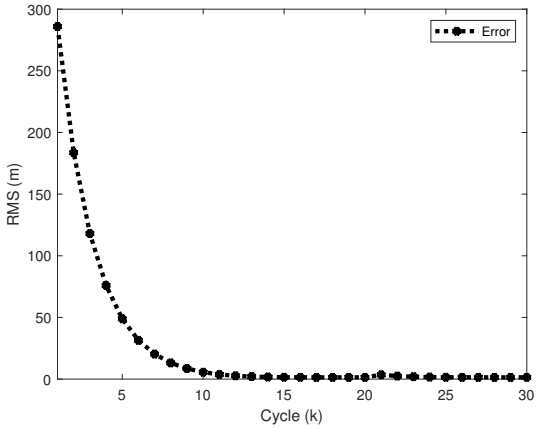


Figure 3. Means square error trajectory.

Before the system fails, the tracking error is gradually stabilized in the time direction, and the tracking performance is also optimized gradually in the intermittent direction,

as shown in Figure 2a,b. After the failure in cycle 21, as shown in Figure 2c,d, the tracking performance decreased compared with before, but after several batches, the tracking performance returned to a more ideal level. This conclusion can also be drawn from Figure 3, which shows H values for different batches, showing that the proposed algorithm was effective in both the normal and fault scenarios.

5.2. Scenario 2:Time-Varying Faults and Non-Repeating Disturbances

On this condition, suppose that in $\Gamma = 0.85 + 0.15sin(t)$, the outside disturbance $w(t,k)$ is a random variable evenly distributed on the interval $\begin{bmatrix} -1 & 1 \end{bmatrix}$. Because $\delta w(t,k) \neq 0$, we use $\alpha = \beta = 0.8146$ to solve Equation (34) with the LMI optimization toolbox, and we can find $\gamma^* = 1.5288$, $K_1 = \begin{bmatrix} -1.6991 & -1.1762 \end{bmatrix}$, and $K_2 = 0.3377$. Figures 4 and 5 show that when the system has time-varying faults and non-repetitive disturbances, the algorithm in this paper can also optimize the control performance in both the batch and time directions in the normal and fault scenarios. For non-repetitive external disturbances, it has a certain inhibitory effect. In the meantime, according to the convergece speed and final H value, we can see that the system stability was also verified in this simulation.

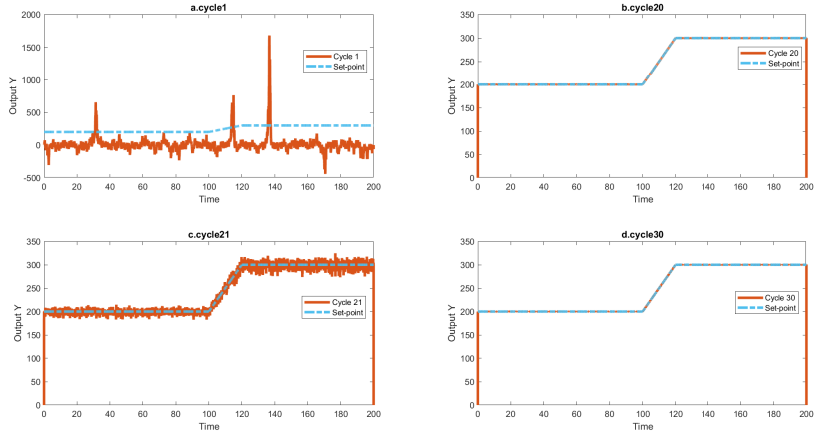


Figure 4. Injection system output under time-varying conditions.

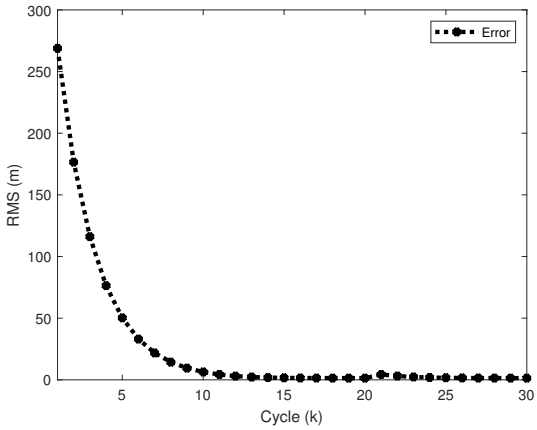


Figure 5. Means square error trajectory.

6. Conclusions and Future Work

In this paper, an intermittent process with uncertain parameters, actuator faults, and external disturbance was taken as the controlled object, and the system's passive fault-tolerant control problem was studied. Based on 2D system theory, an iterative learning fault-tolerant control algorithm was designed. First, the intermittent process was transformed into a 2D Roesser system model. Secondly, a satisfactory fault-tolerant controller was

designed. The LMI technique was used to analyze the system stability and the sufficient conditions for robustness H∞ in performance under unknown disturbances, and the convex optimization method was used to obtain a controller gain matrix. Finally, the better performance in the simulation experiment of the injection molding process of the injection machine proved the proposed algorithm's effectiveness.

Research in the future should include a study on H∞ robust performance with external non-repetitive disturbances in the states and output variables. Another potential direction concerns dyanmic ILC which would be a more complex controller structure to be designed, and the extension to states and input delays shall be investigated too. Aside from that, in this paper, the actuator fault was a suspicion which was not practical, so to estimate the actuator fault is also our following study direction, and experiments or simulations shall be performed to validate the theory's results.

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Appendix A

Proof of Theorem 1. For the system in Equation (18), according to Lemma 1, when $\delta w(t,k) \equiv 0$, there is a matrix P = diag$\{P_T, P_K\}$ to yield

$$(\hat{A} + \hat{B}\Gamma K)^T P(\hat{A} + \hat{B}\Gamma K) - P(\alpha, \beta) < 0. \tag{A1}$$

According to Lemma 3, we obtain

$$\begin{bmatrix} -P(\alpha,\beta) & * \\ \hat{A}+\hat{B}\Gamma K & -P^{-1} \end{bmatrix} = \begin{bmatrix} -\alpha P_T & * & * & * \\ 0 & -\beta P_K & * & * \\ \bar{A}+\bar{B}\Gamma K_1 & \bar{B}\Gamma K_2 & -P_T^{-1} & * \\ -\bar{C}(\bar{A}+\bar{B}\Gamma K_1) & I-\bar{C}\bar{B}\Gamma K_2 & 0 & -P_K^{-1} \end{bmatrix} < 0. \tag{A2}$$

We define $Q_T = P_T^{-1}, Q_K = P_K^{-1}, R_1 = K_1 Q_T, R_2 = K_2 Q_K$. We can multiply the left matrix in Equation (A1) by the left and right $diag\{Q_T, Q_K, I, I\}$ to obtain

$$\begin{aligned} &\begin{bmatrix} -\alpha Q_T & * & * & * \\ 0 & -\beta Q_K & * & * \\ \bar{A}Q_T+\bar{B}\Gamma R_1 & \bar{B}\Gamma R_2 & -Q_T & * \\ -\bar{C}(\bar{A}Q_T+\bar{B}\Gamma R_1) & Q_K-\bar{C}\bar{B}\Gamma R_2 & 0 & -Q_K \end{bmatrix} = \begin{bmatrix} -\alpha Q_T & * & * & * \\ 0 & -\beta Q_K & * & * \\ \bar{A}Q_T+\bar{B}\Gamma R_1 & \bar{B}\Gamma R_2 & -Q_T & * \\ 0 & Q_K & 0 & -Q_K \end{bmatrix} \\ &+ \begin{bmatrix} 0 & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \\ -\bar{C}\bar{A}Q_T-\bar{C}\bar{B}\Gamma R_1 & -\bar{C}\bar{B}\Gamma R_2 & 0 & 0 \end{bmatrix} = \Omega_1 + \Sigma_1 < 0 \end{aligned} \tag{A3}$$

among which

$$\Omega_1 = \begin{bmatrix} -\alpha Q_T & * & * & * \\ 0 & -\beta Q_K & * & * \\ \bar{A}Q_T + \bar{B}\Gamma R_1 & \bar{B}\Gamma R_2 & -Q_T & * \\ 0 & Q_K & 0 & -Q_K \end{bmatrix}, \Sigma_1 = \begin{bmatrix} 0 & * & ** & \\ 0 & 0 & ** & \\ 0 & 0 & 0 & * \\ -\bar{C}\bar{A}Q_T - \bar{C}\bar{B}\Gamma R_1 & -\bar{C}\bar{B}\Gamma R_2 & 0 & 0 \end{bmatrix}.$$

The matrix Σ_1 can be further decomposed into

$$\Sigma_1 = M_1 N_1^T + N_1 M_1^T \tag{A4}$$

where $M_1^T = \begin{bmatrix} 0 & 0 & 0 & -\bar{C}^T \end{bmatrix}$, $N_1^T = \begin{bmatrix} \bar{A}Q_T + \bar{B}\Gamma R_1 & \bar{B}\Gamma R_2 & 0 & 0 \end{bmatrix}$. Then, the establishment condition in Equation (A2) is transformed into $\Omega_1 + M_1 N_1^T + N_1 M_1^T < 0$, which is

$$\begin{bmatrix} I & M_1 \end{bmatrix} \begin{bmatrix} \Omega_1 & N_1 \\ N_1^T & 0 \end{bmatrix} \begin{bmatrix} I \\ M_1^T \end{bmatrix} < 0. \tag{A5}$$

Let $\Pi_1 = \begin{bmatrix} \Omega_1 & N_1 \\ N_1^T & 0 \end{bmatrix}$, $\Lambda = I$, $\Sigma_1^\perp = \begin{bmatrix} I \\ M_1^T \end{bmatrix}$, $\Sigma_1 = \begin{bmatrix} M_1^T & -I \end{bmatrix}$, $W_A = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}$. From Lemma 4, we know that Equation (A5) holds if and only if the matrix W_1 satisfies

$$\Pi_1 + \Lambda^T W_A \Sigma_1 + \Sigma_1^T W_A^T \Lambda < 0 \tag{A6}$$

which is

$$\Pi_1 + \begin{bmatrix} W_1 M_1^T + M_1 W_1^T & -W_1 + M_1 W_2^T \\ W_2 M_1^T - W_1^T & -W_2 - W_2^T \end{bmatrix} < 0. \tag{A7}$$

Let $W_1 = \begin{bmatrix} W_{11} \\ W_{12} \\ W_{13} \\ W_{14} \end{bmatrix}$. Then, the formula in Equation (A7) is

$$\Omega_2 + M_2 N_2^T + N_2 M_2^T < 0 \tag{A8}$$

where $M_2^T = \begin{bmatrix} 0 & 0 & \bar{B}^T & 0 & \bar{B}^T \end{bmatrix}$, $N_2^T = \begin{bmatrix} \Gamma R_1 & \Gamma R_2 & 0 & 0 & 0 \end{bmatrix}$, and

$$\Omega_2 = \begin{bmatrix} -\alpha Q_1 & * & * & * & * \\ 0 & -\beta Q_K & * & * & * \\ \bar{A}Q_T & 0 & -Q_T & * & * \\ -\bar{C}W_{11}^T & Q_K - \bar{C}W_{12}^T & -\bar{C}W_{13}^T & -Q_K - \bar{C}W_{14}^T - W_{14}\bar{C}^T & * \\ -W_{11}^T + \bar{A}Q_T & -W_{12}^T & -W_{13}^T & -W_{14}^T - W_2^T\bar{C}^T & -W_2 - W_2^T \end{bmatrix}.$$

Using Lemma 4 again, we obtain

$$\Pi_2 + \Lambda^T W_B \Sigma_2 + \Sigma_2^T W_B^T \Lambda < 0 \tag{A9}$$

where $\Pi_2 = \begin{bmatrix} \Omega_2 & N_2 \\ N_2^T & 0 \end{bmatrix} =$

$$\begin{bmatrix} -\alpha Q_T & * & * & * & * & * \\ 0 & -\beta Q_K & * & * & * & * \\ \bar{A}Q_T & 0 & -Q_T & * & * & * \\ -\bar{C}W_{11}^T & Q_K - \bar{C}W_{12}^T & -\bar{C}W_{13}^T & -Q_K - \bar{C}W_{14}^T - W_{14}\bar{C}^T & * & * \\ -W_{11}^T + \bar{A}Q_t & -W_{12}^T & -W_{13}^T & -W_{14}^T - W_2^T\bar{C}^T & -W_2 - W_2^T & * \\ \Gamma R_1 & \Gamma R_2 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$W_B = \begin{bmatrix} W_3 \\ W_4 \end{bmatrix}$, $W_3 = \begin{bmatrix} W_{31}^T & W_{32} & W_{33} & W_{34} & W_{35} \end{bmatrix}^T$, $\Sigma_2^\perp = \begin{bmatrix} I \\ M_2^T \end{bmatrix}$, $\Sigma_2 = \begin{bmatrix} M_2^T & -I \end{bmatrix}$.

That is, we have

$$\Phi_1 + X_1 \bar{\Delta}_1 Y_1 + (X_1 \bar{\Delta}_1 Y_1)^T < 0 \tag{A10}$$

where

$$X_1 = \begin{bmatrix} 0 & 0 & E_1^T & 0 & E_1^T & 0 \\ 0 & 0 & 0 & -E_3^T & 0 & 0 \end{bmatrix}^T, \bar{\Delta}_1 = \begin{bmatrix} \Delta_1(t) & 0 \\ 0 & \Delta_3(t) \end{bmatrix}, Y_1 = \begin{bmatrix} F_1 Q_T & 0 & 0 & 0 & 0 & 0 \\ F_3 W_{11}^T & F_3 W_{12}^T & F_3 W_{13}^T & F_3 W_{14}^T & F_3 W_2^T & 0 \end{bmatrix},$$

$$\Phi_1 = \begin{bmatrix} -\alpha Q_T & * & * & * & * & * \\ 0 & -\beta Q_K & * & * & * & * \\ AQ_T + \bar{B}W_{31}^T & \bar{B}W_{32}^T & -Q_T + W_{33}\bar{B}^T + \bar{B}W_{33}^T & * & * & * \\ -CW_{11}^T & Q_K - CW_{12}^T & -CW_{13}^T + W_{34}\bar{B}^T & -Q_K - CW_{14}^T - W_{14}C^T & * & * \\ -W_{11}^T + AQ_T + \bar{B}W_{11}^T & -W_{12}^T + \bar{B}W_{32}^T & -W_{13}^T + W_{35}\bar{B}^T + \bar{B}W_{33}^T & -W_{14}^T - W_2C^T + \bar{B}W_{34}^T & sym(\bar{B}W_{35}^T - W_2) & * \\ -W_{31}^T + \Gamma R_1 & -W_{32}^T + \Gamma_2 R_2 & -W_{33}^T + W_4\bar{B}^T & -W_{34}^T & -W_{35}^T + W_4^T\bar{B}^T & -sym(W_4) \end{bmatrix}.$$

Furthermore, it can be known from Lemma 2 that if Equation (A10) holds, then there exists $\varepsilon = diag\{\varepsilon_1, \varepsilon_3\} > 0$ to make the following inequality hold:

$$\Phi_1 + X_1 \varepsilon X_1^T + Y_1^T \varepsilon^{-1} Y_1 < 0. \tag{A11}$$

Using Lemma 3 again, we obtain

$$\begin{bmatrix} -\alpha Q_T & * & * & * \\ 0 & -\beta Q_K & * & * \\ AQ_T + \bar{B}W_{31}^T & \bar{B}W_{32}^T & \varepsilon_1 E_1 E_1^T - Q_T + W_{33}\bar{B}^T + \bar{B}W_{33}^T & * \\ -CW_{11}^T & Q_K - CW_{12}^T & -CW_{13}^T + W_{34}\bar{B}^T & \varepsilon_3 E_3 E_3 - Q_K - sym(CW_{14}^T)^T \\ -W_{11}^T + AQ_T + \bar{B}W_{11}^T & -W_{12}^T + \bar{B}W_{32}^T & \varepsilon_1 E_1 E_1^T - W_{13}^T + W_{35}\bar{B}^T + \bar{B}W_{33}^T & -W_{14}^T - W_2C^T + \bar{B}W_{34}^T \\ -W_{31}^T + \Gamma R_1 & -W_{32}^T + \Gamma_2 R_2 & -W_{33}^T + W_4\bar{B}^T & -W_{34}^T \\ F_1 Q_T & 0 & 0 & 0 \\ F_3 W_{11}^T & F_3 W_{12}^T & F_3 W_{13}^T & F_3 W_{14}^T \end{bmatrix} \rightarrow$$

$$\leftarrow \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ sym(\bar{B}W_{35}^T - W_2) + \varepsilon_1 E_1 E_1^T & * & * & * \\ -W_{35}^T + W_4^T\bar{B}^T & -sym(W_4) & * & * \\ 0 & 0 & -\varepsilon_1 I & * \\ F_3 W_2^T & 0 & 0 & -\varepsilon_1 I \end{bmatrix} < 0. \tag{A12}$$

In this sense, Equation (A12) can be transformed into

$$\Phi_2 + X_2 \bar{\Delta}_2 Y_2 + (X_2 \bar{\Delta}_2 Y_2)^T < 0 \tag{A13}$$

where

Φ_2=

$$\begin{bmatrix} -\alpha Q_T & * & * & * \\ 0 & -\beta Q_K & * & * \\ AQ_T + \bar{B}W_{31}^T & BW_{32}^T & \varepsilon_1 E_1 E_1^T - Q_T + sym(W_{33}B^T) & * \\ -CW_{11}^T & Q_K - CW_{12}^T & -CW_{13}^T + W_{34}B^T & \varepsilon_3 E_3 E_3^T - Q_K - sym(CW_{14}^T) \\ -W_{11}^T + AQ_T + BW_{11}^T & -W_{12}^T + BW_{32}^T & \varepsilon_1 E_1 E_1^T - W_{13}^T + W_{35}B^T + BW_{33}^T & -W_{14}^T - W_2C^T + BW_{34}^T \\ -W_{31}^T + qR_1 & -W_{32}^T + qR_2 & -W_{33}^T + W_4B^T & -W_{34}^T \\ F_1 Q_T & 0 & 0 & 0 \\ F_3 W_{11}^T & F_3 W_{12}^T & F_3 W_{13}^T & F_3 W_{14}^T \end{bmatrix} \rightarrow$$

$$\leftarrow \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ sym(BW_{35}^T - W_2) + \varepsilon_1 E_1 E_1^T & * & * & * \\ -W_{35}^T + W_4\bar{B}^T & -sym(W_4) & * & * \\ 0 & 0 & -\varepsilon_1 I & * \\ F_3 W_2^T & 0 & 0 & -\varepsilon_1 I \end{bmatrix} < 0,$$

$$X_2 = \begin{bmatrix} 0 & 0 & E_2^T & 0 & E_2^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & q_0^T & 0 & 0 \end{bmatrix}, \bar{\Delta}_2 = \begin{bmatrix} \Delta_2(t) & 0 \\ 0 & I \end{bmatrix},$$
$$Y_2 = \begin{bmatrix} F_2 W_{31}^T & F_2 W_{32}^T & F_2 W_{33}^T & F_2 W_{34}^T & F_2 W_{35}^T & F_2 W_4^T & 0 & 0 \\ qR_1 & qR_2 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Then, according to Lemma 2, it can be known that Equation (A13) holds if and only if $\bar{\varepsilon} = diag\{\varepsilon_2, \varepsilon_4\} > 0$, and the following inequality holds:

$$\Phi_2 + X_2 \varepsilon X_2^T + Y_2^T \varepsilon^{-1} Y_2 < 0 \tag{A14}$$

Using Lemma 3 again, we find the form of Equation (31). □

Appendix B

Proof of Theorem 2. For the system in Equation (18), when $\delta w(t,k) \neq 0$, according to the conditions of Lemma 1 and Lemma 3, there is the following inequality:

$$\begin{bmatrix} (\hat{A} + \hat{B}\Gamma K)^T \\ \hat{D}^T \end{bmatrix} P \begin{bmatrix} \hat{A} + \hat{B}\Gamma K & \hat{D} \end{bmatrix} - \begin{bmatrix} P(\alpha,\beta) - \gamma^{-1} G^T G & 0 \\ 0 & \gamma I \end{bmatrix} < 0. \tag{A15}$$

According to Lemma 4, Equation (A15) can be transformed into

$$\begin{bmatrix} -P(\alpha,\beta) & * & * & * \\ 0 & -\gamma I & * & * \\ \hat{A} + \hat{B}\Gamma K & \hat{D} & -P^{-1} & * \\ \hat{G} & 0 & 0 & -\gamma I \end{bmatrix} < 0. \tag{A16}$$

We define $P_T^{-1} = Q_T$, $P_K^{-1} = Q_K$, $Q = diag\{Q_T, Q_K\}$, and $P = diag\{P_T, P_K\}$. By multiplying the left matrix in Equation (A16) by $diag\{Q, I, I, I\}$ before and after, we can obtain

$$\begin{bmatrix} -\alpha Q_T & * & * & * & * & * \\ 0 & -\beta Q_K & * & * & * & * \\ 0 & 0 & -\gamma I & * & * & * \\ \bar{A} Q_T + \bar{B}\Gamma R_1 & \bar{B}\Gamma R_2 & D & -Q_T & * & * \\ -\bar{C}(\bar{A} Q_T + \bar{B}\Gamma R_1) & Q_K - \bar{C}\bar{B}\Gamma R_2 & -\bar{C} D & 0 & -Q_K & * \\ 0 & Q_K & 0 & 0 & 0 & -\gamma I \end{bmatrix} < 0. \tag{A17}$$

The remaining proof is similar to Theorem 1, so it was omitted. Next, it has been proven that the robustness H∞ performance metric of the system in Equation (18) is not greater than γ.

We define the following:

$$J(t,k) = \Delta V(t,k) + \gamma^{-1} \|\delta x(t,k)\|_{GG^T} - \gamma \|\delta w(t,k)\|_I \tag{A18}$$

where

$$\Delta V(t,k) = \left\| \begin{bmatrix} \delta x(t+1,k) \\ e(t+1,k) \end{bmatrix} \right\|_P - \| \begin{bmatrix} \delta x(t+1,k) \\ e(t+1,k) \end{bmatrix} \|_{P(\alpha,\beta)}. \tag{A19}$$

According to Equation (18), we have

$$J(t,k) = \| \begin{bmatrix} \delta x(t,k) \\ e(t+1,k-1) \\ \delta w(t,k) \end{bmatrix} \|_\Theta < 0 \tag{A20}$$

where $\Theta = \left(\begin{bmatrix} (\hat{A} + \hat{B}\Gamma K)^T \\ \hat{D}^T \end{bmatrix} P \begin{bmatrix} \hat{A} + \hat{B}\Gamma K & \hat{D} \end{bmatrix} - \begin{bmatrix} P(\alpha,\beta) - \gamma^{-1} G^T G & 0 \\ 0 & \gamma I \end{bmatrix} \right)$.

For any integer N_1, N_2, when all boundary conditions of the system in Equation (18) are zero, we have

$$\begin{array}{l}\sum_{t=0}^{N_1}\sum_{k=1}^{N_2}\Delta V(t,k)=\sum_{t=0}^{N_1}\sum_{k=1}^{N_2}\left(\left\|\begin{array}{c}\delta x(t+1,k)\\ e(t+1,k)\end{array}\right\|_P-\left\|\left[\begin{array}{c}\delta x(t,k)\\ e(t+1,k-1)\end{array}\right]\right\|_{P_{(\alpha,\beta)}}\right)=\\ \sum_{t=0}^{N_1}\sum_{k=1}^{N_2}((1-\alpha)\|\delta x(t+1,k)\|_P)+\sum_{t=0}^{N_1}(\|e(t+1,N_2)\|_P)\sum_{t=0}^{N_1}\sum_{k=1}^{N_2-1}((1-\beta)\|e(t+1,k)\|_P)+\\ \sum_{k=1}^{N_2}(\|\delta x(N_1+1,k)\|_P)\geq 0.\end{array} \tag{A21}$$

Therefore, we obtain

$$\begin{array}{l}\sum_{t=1}^{N_1}\sum_{k=1}^{N_2}\left(\gamma^{-1}\|\delta x(t,k)\|_{GG^T}-\gamma\|\delta w(t,k)\|_I\right)\\ \leq\sum_{t=0}^{N_1}\sum_{k=1}^{N_2}\left(\Delta V(t,k)+\gamma^{-1}\|\delta x(t,k)\|_{GG^T}-\gamma\|\delta w(t,k)\|_I\right)=\sum_{t=0}^{N_1}\sum_{k=1}^{N_2}J(t,k)<0\end{array} \tag{A22}$$

which is $\|z\|_{2\theta}<\gamma\|w\|_{2\theta}$, showing that the robustness H∞ performance under the fault-tolerant control of the 2D Roesser system is fewer than the index γ. □

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