symmetry

Correction

Correction: Cabrera Martínez et al. On the Secure Total Domination Number of Graphs. *Symmetry* 2019, *11*, 1165

Abel Cabrera Martínez, **Luis P. Montejano** * and **Juan A. Rodríguez-Velázquez**

Departament d'Enginyeria Informàtica i Matemàtiques, Universitat Rovira i Virgili, Av. Països Catalans 26, 43007 Tarragona, Spain; abel.cabrera@urv.cat (A.C.M.); juanalberto.rodriguez@urv.cat (J.A.R.-V.)
* Correspondence: luispedro.montejano@urv.cat

Citation: Cabrera Martínez, A.; Montejano, L.P.; Rodríguez-Velázquez, J.A. Correction: Cabrera Martínez et al. On the Secure Total Domination Number of Graphs. *Symmetry* 2019, *11*, 1165. *Symmetry* **2021**, *13*, 1668. https://doi.org/10.3390/sym13091668

Received: 30 August 2021
Accepted: 31 August 2021
Published: 10 September 2021

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.

The authors wish to make the following corrections on paper [1]:

(1) Eliminate Lemma 1 because we have found that this lemma is not correct.

(2) Theorem 3 states that for any graph G with no isolated vertex,

$$\gamma_{st}(G) \leq \alpha(G) + \gamma(G).$$

The result is correct, but the proof uses Lemma 1. For this reason, we propose the following alternative proof for Theorem 3.

Proof. Let D be a $\gamma(G)$-set. Let I be an $\alpha(G)$-set such that $|D \cap I|$ is at its maximum among all $\alpha(G)$-sets. Notice that for any $x \in D \cap I$,

$$epn(x, D \cup I) \cup ipn(x, D \cup I) \subseteq epn(x, I). \tag{1}$$

We next define a set $S \subseteq V(G)$ of minimum cardinality among the sets satisfying the following properties.

(a) $D \cup I \subseteq S$.

(b) For every vertex $x \in D \cap I$,

- (b1) if $epn(x, D \cup I) \neq \varnothing$, then $S \cap epn(x, D \cup I) \neq \varnothing$;
- (b2) if $epn(x, D \cup I) = \varnothing$, $ipn(x, D \cup I) \neq \varnothing$ and $epn(x, I) \setminus ipn(x, D \cup I) \neq \varnothing$, then either $epn(x, I) \setminus D = \varnothing$ or $S \cap epn(x, I) \setminus D \neq \varnothing$;
- (b3) if $epn(x, D \cup I) = \varnothing$ and $epn(x, I) = ipn(x, D \cup I) \neq \varnothing$, then $S \cap N(epn(x, I)) \setminus \{x\} \neq \varnothing$;
- (b4) if $epn(x, D \cup I) = ipn(x, D \cup I) = \varnothing$, then $N(x) \setminus (D \cup I) = \varnothing$ or $S \cap N(x) \setminus (D \cup I) \neq \varnothing$.

Since D and I are dominating sets, from (a) and (b) we conclude that S is a TDS. From now on, let $v \in V(G) \setminus S$. Observe that there exists a vertex $u \in N(v) \cap I \subseteq N(v) \cap S$, as $I \subseteq S$ is an $\alpha(G)$-set. To conclude that S is a STDS, we only need to prove that $S' = (S \setminus \{u\}) \cup \{v\}$ is a TDS of G.

First, notice that every vertex in $V(G) \setminus N(u)$ is dominated by some vertex in S', because S is a TDS of G. Let $w \in N(u)$. Now, we differentiate two cases with respect to vertex u.

Case 1. $u \in I \setminus D$. If $w \notin D$, then there exists some vertex in $D \subseteq S'$ which dominates w, as D is a dominating set. Suppose that $w \in D$. If $w \in ipn(u, D \cup I)$, then $I' = (I \cup \{w\}) \setminus \{u\}$ is an $\alpha(G)$-set such that $|D \cap I'| > |D \cap I|$, which is a contradiction. Hence, $w \notin ipn(u, D \cup I)$, which implies that there exists some vertex in $(D \cup I) \setminus \{u\} \subseteq S'$ which dominates w.

Case 2. $u \in I \cap D$. We first suppose that $w \notin D$. If $w \notin epn(u, D \cup I)$, then w is dominated by some vertex in $(D \cup I) \setminus \{u\} \subseteq S'$. If $w \in epn(u, D \cup I)$, then by (b1) and the fact that in this case all vertices in $epn(u, D \cup I)$ form a clique, w is dominated by some vertex in

$S \setminus \{u\} \subseteq S'$. From now on, suppose that $w \in D$. If $w \notin ipn(u, D \cup I)$, then there exists some vertex in $(D \cup I) \setminus \{u\} \subseteq S'$ which dominates w. Finally, we consider the case in that $w \in ipn(u, D \cup I)$.

We claim that $ipn(u, D \cup I) = \{w\}$. In order to prove this claim, suppose that there exists $w' \in ipn(u, D \cup I) \setminus \{w\}$. Notice that $w' \in D$. By (1) and the fact that all vertices in $epn(u, I)$ form a clique, we prove that $ww' \in E(G)$, and so $w \notin ipn(u, D \cup I)$, which is a contradiction. Therefore, $ipn(u, D \cup I) = \{w\}$ and, as a result,

$$epn(u, D \cup I) \cup \{w\} \subseteq epn(u, I). \tag{2}$$

In order to conclude the proof, we consider the following subcases.

Subcase 2.1. $epn(u, D \cup I) \neq \varnothing$. By (2), (b1), and the fact that all vertices in $epn(u, I)$ form a clique, we conclude that w is adjacent to some vertex in $S \setminus \{u\} \subseteq S'$, as desired.

Subcase 2.2. $epn(u, D \cup I) = \varnothing$ and $epn(u, I) \setminus \{w\} \neq \varnothing$. By (2), (b2), and the fact that all vertices in $epn(u, I)$ form a clique, we show that w is dominated by some vertex in $S \setminus \{u\} \subseteq S'$, as desired.

Subcase 2.3. $epn(u, D \cup I) = \varnothing$ and $epn(u, I) = \{w\}$. In this case, by (b3) we deduce that w is dominated by some vertex in $S \setminus \{u\} \subseteq S'$, as desired.

According to the two cases above, we can conclude that S' is a TDS of G, and so S is a STDS of G. Now, by the the minimality of $|S|$, we show that $|S| \leq |D \cup I| + |D \cap I| = |D| + |I|$. Therefore, $\gamma_{st}(G) \leq |S| \leq |I| + |D| = \alpha(G) + \gamma(G)$, which completes the proof. □

The authors would like to apologize for any inconvenience caused to the readers by these changes. The changes do not affect the scientific results.

Reference

1. Cabrera Martínez, A.; Montejano, L.P.; Rodríguez-Velázquez, J.A. On the secure total domination number of graphs. *Symmetry* **2019**, *11*, 1165. [CrossRef]