Article

Control of a Symmetric Chaotic Supply Chain System Using a New Fixed-Time Super-Twisting Sliding Mode Technique Subject to Control Input Limitations

Bo Wang [1,2], **Hadi Jahanshahi** [3] , **Christos Volos** [4,*] , **Stelios Bekiros** [5,6] , **Abdullahi Yusuf** [7,8] , **Praveen Agarwal** [9,10,11] **and Ayman A. Aly** [12]

1 School of Electronic Information and Automation, Aba Teachers University, Wenchuan 623002, China; coolbie@163.com or bwang@mail.xhu.edu.cn
2 School of Applied Mathematics, University Electronic Science and Technology of China, Chengdu 610054, China
3 Department of Mechanical Engineering, University of Manitoba, Winnipeg, MB R3T 5V6, Canada; Jahanshahi.hadi90@gmail.com or hadi_jahanshahi@ut.ac.ir
4 Laboratory of Nonlinear Systems-Circuits and Complexity, Physics Department, Aristotle University of Thessaloniki, GR-54124 Thessaloniki, Greece
5 Department of Banking and Finance, FEMA, University of Malta, MSD 2080 Msida, Malta; stelios.bekiros@eui.eu
6 Department of Economics, European University Institute, Via delle Fontanelle, 18, I-50014 Florence, Italy
7 Department of Computer Engineering, Biruni University, Istanbul 34010, Turkey; yusufabdullahi@fud.edu.ng
8 Department of Mathematics, Federal University Dutse, Jigawa 7156, Nigeria
9 Department of Mathematics, Anand International College of Engineering, Jaipur 303012, India; goyal.praveen2011@gmail.com or praveen.agarwal@anandice.ac.in
10 Nonlinear Dynamics Research Center (NDRC), Ajman University, Ajman AE 346, United Arab Emirates
11 International Center for Basic and Applied Sciences, Jaipur 302029, India
12 Department of Mechanical Engineering, College of Engineering, Taif University, P.O. Box 11099, Taif 21944, Saudi Arabia; aymanaly@tu.edu.sa
* Correspondence: volos@physics.auth.gr

Abstract: Control of supply chains with chaotic dynamics is an important, yet daunting challenge because of the limitations and constraints there are in the amplitude of control efforts. In real-world systems, applying control techniques that need a large amplitude signal is impractical. In the literature, there is no study that considers the control of supply chain systems subject to control input limitations. To this end, in the current study, a new control scheme is proposed to tackle this issue. In the designed control input, limitations in control inputs, as well as robustness against uncertainties, are taken into account. The proposed scheme is equipped with a fixed time disturbance observer to eliminate the destructive effects of uncertainties and disturbances. Additionally, the super-twisting sliding mode technique guarantees the fixed-time convergence of the closed-loop system. After that, a symmetric supply chain system is presented, and its chaotic attractors are demonstrated. Finally, the proposed controller is applied to the symmetric supply chain system. Numerical simulations exhibit the proposed scheme's excellent performance even though the system is subjected to control input limitations and time-varying uncertainties.

Keywords: symmetric supply chain system; constrained control input super-twisting sliding mode; fixed-time disturbance observer; chaos control

1. Introduction

During recent decades, academics and decision makers have been striving for a reliable approach to handle advanced supply chain systems. In the past, decisions were often made based on intuition and experience [1,2]. However, over time, supply chains have become more and more complicated, resulting in the failure of traditional management

techniques [3]. Business entities in the supply chain are tightly intertwined with uncertainties. Additionally, most supply chain systems possess nonlinear dynamics that sometimes provide chaotic responses. Hence, applying up-to-date techniques for their manganates is of crucial importance [4–6].

Nonlinear dynamical systems with an infinite number of different periodic responses that are susceptible to initial conditions are known as chaotic systems [7–10]. Although chaos can be useful in certain situations, it is often an unwelcome occurrence. The chaotic oscillations must be suppressed; thus, regulation mechanisms are needed [11,12]. Furthermore, due to minor faults, chaotic processes are prone to take entirely separate paths. As a result, control and stabilization are needed in chaotic systems [13,14]. The regulation of chaotic processes has been one of the most significant issues in the control engineering field after Ott, Grebogi, and Yorke (1990) successfully controlled a chaotic system [15]. To date, different approaches for dealing with chaotic processes have been suggested and tested [16–18].

Among all control schemes that have been used to control the actions of complex systems over the last few decades, sliding mode control (SMC), as the most common robust controller, has received a lot of attention because of its features, including ease of execution, guaranteed stability, and resistance to parameter variations [19–21]. In addition, numerous experiments have recently been conducted on the use of sliding mode control in the control of chaotic systems. However, SMC has some drawbacks [22–25]. For instance, SMC does not guarantee convergence in finite time. To this end, terminal sliding mode control (TSMC) was developed to overcome the issue of finite-time convergence [26,27].

As mentioned, a significant challenge in this field is designing control and management approaches that ensure the systems' appropriate performance. Control theory has established a firm foundation for handling nonlinear dynamics [28–34]. Hence, the application of control theory can provide luminating results in managing supply chains with nonlinear dynamics [35]. Numerous methods developed in the control literature have been applied to supply chains over the last half-century [36]. Model predictive control (MPC) and neuro-dynamic programming are examples of increasingly advanced control methodologies applied to supply chain systems [37,38]. Despite a lot of efforts made by researchers, some issues still require more attention. For instance, no study in the literature considers finite-time convergence and limitations in control inputs in the control of supply chains.

Most financial processes possess nonlinear dynamics subjected to disturbances and control input limitations [39,40]. Hence, many studies have proposed robust techniques to control economic and financial systems [41]. Since the external disturbance cannot be directly evaluated in a nonlinear environment, nonlinear observers should be utilized to detect their dynamics [21,42]. On the other hand, input saturation, a nonsmooth nonlinearity, should be considered in real-world applications [43,44].

The abovementioned issues have motivated the current study. The present research proposes a new control scheme. The suggested scheme includes a fixed time disturbance observer to minimize the negative consequences of uncertainties and disruptions. Then, an uncertain, symmetric chaotic supply chain system has been considered with limited control inputs, and the performance of the designed control scheme has been assessed by presenting some numerical simulation results.

It is noteworthy that the proposed control technique improves some issues which occur in its counterparts. Compared to its state-of-the-art counterparts, the suggested controller has important advantages. For example, Chen et al. [45] have proposed a TSMC which is equipped with a finite time observer. Although their study was a breakthrough in this field of study, it suffers from several critical disadvantages. For instance, chattering was not investigated in their controller, and in some situations, that control scheme can result in severe vibration in systems. Huang has offered a fast TSMC with a novel fuzzy disturbance observer [46], in which the error of disturbance observation in that method converges asymptotically, and in real applications, it may take a long time to estimate

unknown functions. In 2018, Liu et al. [47] proposed a disturbance observer-based fuzzy TSMC for a hypersonic vehicle. Although the finite-time convergence of the disturbance observer and the closed-loop system was proven in their study, the existence of control input saturation and chattering was ignored, which lowers the performance of that control scheme considerably. On the contrary, the super-twisting sliding surface we propose in the current research is chatter-free and takes the effects of control input saturation into account. More recently, an observer-based fuzzy TSMC has been offered by Vahidi-Moghaddam et al. [48] for MIMO systems. Convergence time is, however, a considerable drawback in their controller. They have only demonstrated the disturbance observer's finite-time convergence; nonetheless, the closed-loop system is exponentially stable, and the convergence of the closed-loop system in fixed time is not guaranteed. As a result, convergence time may be extremely long in some circumstances. On the other hand, in the current study, we prove the fixed-time convergence of the disturbance observer and the closed-loop system simultaneously.

The rest of the current paper is organized as follows: In Section 2, the design procedure of the proposed scheme, including effects of uncertainties and limitation in control input, fixed-time disturbance observer, and super-twisting SMC, are delineated. Additionally, the stability of the closed-loop system is proven through the Lyapunov stability theorem. Then, in Section 3, the chaotic behavior symmetric supply chain system is described. In Section 4, the proposed control scheme is applied to control the chaotic behavior of the symmetric supply chain system, and various numerical simulations are presented. Finally, in Section 5, concluding remarks are given.

2. Controller Design

In this paper, we investigate the control of MIMO nonlinear supply chains. Let the state space of the MIMO system be as described below:

$$\dot{x}(t) = f(x) + \Delta f(x) + (g(x) + \Delta g(x))u + d_0(t) \tag{1}$$

where $x = [\, x_1, x_2, \ldots, x_n \,]^T$ is the state vector, f and g are nonlinear functions. The structural variation and uncertainty that there are in the dynamics of the system are represented by Δf and Δg. Additionally, $d_0 = [\, d_1, d_2, \ldots, d_n \,]^T$ stands for the external disturbance. In addition, $u = [\, u_1, u_2, \ldots, u_n \,]^T$ denotes the control input.

2.1. Effects of Uncertainties and Limitation in Control Input

The whole unknown terms produced by structural variations, uncertainties, and disturbances, i.e., Δf, Δg, and the external disturbance (d_0), are considered as a single disturbance term $d(t)$ as follows:

$$d(t) = \Delta f(x) + \Delta g(x)\, u + d_0(t) \tag{2}$$

Furthermore, control input limitations that may occur in many real systems act like nonsmooth nonlinearities. Since control input limitations are common to most systems, ignoring them in the design of control schemes for real-world applications could dramatically ruin the performance of the systems. To tackle this problem in the current study, we consider the effects of control limitations and saturation in the designed control. Under the control input constraints, the limited control input u is given by

$$u = \begin{cases} u_{max} & if \ u_c > u_{max} \\ u_c & \\ u_{min} & if \ u_c < u_{min} \end{cases} \tag{3}$$

where u_c is designed control command, and u denotes the limited control input applied to the system. u_{min} and u_{max} are the bounds of the control input. Defining $u = u - u_c$ and substituting Equations (2) and (3) into Equation (1) result in

$$\dot{x}(t) = f(x) + g(x)u_c + D$$
$$D(t) = g(x)u + d(t) = g(x)u + \Delta f(q) + \Delta g(q)u + d_0(t) \tag{4}$$

where D is the compound disturbance imposed on the system.

2.2. Fixed-Time Disturbance Observer

Lemmas 1 and 2 are used in the design process of the fixed-time disturbance observer.

Lemma 1. *Let $V(t)$ be a continuous positive definite function satisfying the following inequality* [49]

$$\dot{V}(t) + \vartheta V(t) + \xi V^\chi \leq 0, \qquad \forall t > t_0 \tag{5}$$

As a result, $V(t)$ converges to its equilibrium point in the finite time t_s which is given by

$$t_s \leq t_0 + \frac{1}{\vartheta(1+\chi)} \ln \frac{\vartheta V^{1-\chi}(t_0) + \xi}{\xi} \tag{6}$$

where $\vartheta > \xi > 0$, and $0 < \chi < 1$.

Lemma 2. *Based on the triangle inequality, if $0 < n < 1$ and $a_\Delta > 0$, $\Delta = 1, 2, \ldots, m$, then*

$$\left(\sum_{\Delta=1}^m a_\Delta \right)^n \leq \sum_{\Delta=1}^m a_\Delta^n \tag{7}$$

The following auxiliary variables are defined for the disturbance observer

$$s = z - x \tag{8}$$

where z is given by

$$z = -ks - \beta sign(s) - \varepsilon s^{\frac{p_0}{q_0}} - |f(x)| sign(s) + g(x)u_c \tag{9}$$

In which $\beta > D_1$. Additionally, $p_0 < q_0$ and both p_0 and q_0 are odd positive integers. Furthermore, k and ε are positive constants. The fixed-time disturbance observer $\hat{D}$ is designed as [45,48]

$$\hat{D} = -ks - \beta sign(s) - \varepsilon s^{p_0/q_0} - |f(x)| sign(s) - f(x) \tag{10}$$

Taking the derivative of (10) and considering (8), (9), and (4) give

$$\dot{s} = \dot{z} - \dot{x} = -ks - \beta \text{sign}(s) - \varepsilon s^{p_0/q_0} - |f(x)| \text{sign}(s) - f(x) - D \tag{11}$$

Based on Equations (3), (10), and (11), the following equation is reached

$$\begin{aligned} D &= \hat{D} - D = -ks - \beta \text{sign}(s) - \varepsilon s^{\frac{p_0}{q_0}} - |f(x)| \text{sign}(s) - f(x) - D \\ &= -ks - \beta sign(s) - \varepsilon s^{\frac{p_0}{q_0}} - |f(x)| sign(s) - f(x) - \dot{x} + f(x) + g(x)u_C \\ &= -ks - \beta sign(s) - \varepsilon s^{\frac{p_0}{q_0}} - |f(x)| sign(s) + g(x)u_c - \dot{x} = \dot{z} - \dot{x} = \dot{s}. \end{aligned} \tag{12}$$

Theorem 1. *Using the proposed fixed-time disturbance observer (8)–(10), the disturbance estimation error (12) converges to zero.*

Proof. Let a Lyapunov function be

$$V_0 = \frac{1}{2}s^T s \tag{13}$$

By taking the time derivative of V_0, we reach

$$
\begin{aligned}
\dot{V}_0 = s^T \dot{s} \quad &= s^T \left(-ks - \beta sign(s) - \varepsilon s^{\frac{p_0}{q_0}} - f(x)_1 sign(s) - f(x) - d \right) \\
&\leq -ks^T s - \beta s^T \text{sign}(s) - \varepsilon s^T s^{\frac{p_0}{q_0}} - f(x)_1 \ s^T sgn(s) \\
&\quad -s^T f(x) - s^T d \\
&\leq -ks^T s - \beta s^T{}_1 - \varepsilon s^T s^{\frac{p_0}{q_0}} - f(x)_1 s^T{}_1 - s^T f(x) \\
&\quad + s^T{}_1 d_1 \leq -ks^T s - \varepsilon s^T s^{\frac{p_0}{q_0}} \\
&\leq -2kV_0 - 2^{(p_0+q_0)/2q_0} \varepsilon V_0^{(p_0+q_0)/2q_0}
\end{aligned}
\tag{14}
$$

The last line of the equation above has been calculated through Lemma 3. $\square$

Lemma 3.

$$
\begin{aligned}
V_0 &= \tfrac{1}{2}s^T s = \tfrac{1}{2}\left(s_1^2 + s_2^2 + \ldots + s_n^2\right) \\
V_0^{(p_0+q_0)/2q_0} &= \left(\tfrac{1}{2}\left(s_1^2 + s_2^2 + \ldots + s_n^2\right) \right)^{(p_0+q_0)/2q_0} \\
&\leq \tfrac{1}{2^{(p_0+q_0)/2q_0}} \left(s_1^{(p_0+q_0)/q_0} + s_2^{(p_0+q_0)/q_0} + \ldots + s_n^{(p_0+q_0)/q_0} \right)
\end{aligned}
\tag{15}
$$

Thus, we have

$$2^{(p_0+q_0)/2q_0} V_0^{(p_0+q_0)/2q_0} \leq s_1^{(p_0+q_0)/q_0} + s_2^{(p_0+q_0)/q_0} + \ldots + s_n^{(p_0+q_0)/q_0} \tag{16}$$

As is evident, $s_1^{(p_0+q_0)/q_0} + s_2^{(p_0+q_0)/q_0} + \ldots + s_n^{(p_0+q_0)/q_0} = s^T s^{p_0/q_0}$ *, consequently, we have*

$$
\begin{aligned}
2^{(p_0+q_0)/2q_0} V_0^{(p_0+q_0)/2q_0} &\leq s^T s^{\frac{p_0}{q_0}} \\
\xrightarrow{\text{yields}} -\varepsilon s^T s^{p_0/q_0} &\leq -\varepsilon 2^{(p_0+q_0)/2q_0} V_0^{(p_0+q_0)/2q_0}
\end{aligned}
\tag{17}
$$

Therefore, following Lemmas 1, 2, and 3 as well as Equations (17) and (12), the auxiliary vector s and the disturbance approximation error D converge to zero in the fixed time.

Remark 1. *Based on Lemma 1, the convergence time of disturbance approximation is as follows:*

$$t_{s\Delta} < t_{0\Delta} + \frac{q_0}{k(p_0 + 3q_0)} \ln\left(\frac{k s_{0\Delta}^{(q_0-p_0)/\,q_0} t_{0\Delta}}{\varepsilon} + 1 \right) \qquad \Delta = 1,2,3\ldots,m \tag{18}$$

where $t_{0\Delta}$ denotes the initial time; hence, by choosing parameters of the disturbance and considering Equation (18), we could reach a fixed-time disturbance observer.

2.3. Super-Twisting SMC

The super-twisting algorithm is a well-known second-order sliding mode algorithm introduced in [50], and it has been widely used for control and observation. In the literature, the effectiveness of super-twisting sliding mode controllers in terms of the accuracy of the control scheme and reducing chattering has been proven. Using the disturbance observer developed in Section 2.1, the fixed-time super-twisting TSMC is designed for system (1). The tracking error of the system is given by

$$e(t) = [e_1(t), e_2(t), \ldots, e_n(t)]^T = x(t) - x_d(t) \tag{19}$$

where $x_d(t)$ is the required value for the state $x(t)$. We define the sliding surface as follows.

$$s_t(t) = \tau e(t) + s(t), \tag{20}$$

In which $s_d(t)$ is the auxiliary variable, given in Equation (8), and τ is an appositive use defined constant. Finally, we propose the disturbance observer-based fixed-time super-twisting for system (1) as

$$
\begin{aligned}
u_c &= -g^{-1}(x)\left(f(x) - \dot{x}_d + u_{st1} + \hat{D}\right), \\
u_{st1} &= -k_1|s_t|^{\frac{1}{2}}sign(s_t) + u_{st2} \\
\dot{u}_{st2} &= -k_2 sign(s_t)
\end{aligned}
\tag{21}
$$

where parameters k_1 and k_2 are positive.

Theorem 2. *Using the control law (21), the closed-loop system states (1) converge to the desired value in fixed time.*

Proof. The time-derivative of the sliding surface (Equation (20)) is given by

$$
\begin{aligned}
\dot{s}_t &= (f(x) + g(x)\,u_c - \dot{x}_d + \dot{s}_d) \\
&= \left(f(x) - \left(f(x) - \dot{x}_d + u_{st1} + \hat{d}\right) - \dot{x}_d + \dot{s}_d\right) \\
&= \left(-, u_{st1}, +, d, -, \hat{d}, +, \dot{s}_d\right)
\end{aligned}
\tag{22}
$$

Based on Equation (12), $d - \hat{d} = -d = -\dot{s}_d$, which results in

$$\dot{s}_t = -u_{st1} \tag{23}$$

Hence, we have

$$
\begin{aligned}
\dot{s}_t &= -k_1|s_t|^{\frac{1}{2}}sign(s_t) + u_{st2} \\
\dot{u}_{st2} &= -k_2 sign(s_t)
\end{aligned}
\tag{24}
$$

Defining new variables as $w_1 = s_t$ and $w_2 = u_{st2}$ and rewriting the above equation, we obtain

$$
\begin{aligned}
\dot{w}_1 &= -k_1|w_1|^{\frac{1}{2}}sign(w_1) + w_2 \\
\dot{w}_2 &= -k_2 sign(w_1)
\end{aligned}
\tag{25}
$$

Equation (25) denotes a second-order super-twisting algorithm. Based on Theorem 2, in [51] and its proof, we consider the following Lyapunov function:

$$V_0 = \varsigma^{\mathrm{T}} P \varsigma \tag{26}$$

where $\varsigma = [\varsigma_1, \varsigma_2]^T = \left[|w_1|^{\frac{1}{2}}sign(w_1), w_2\right]^T$. V_0 is quadratic, robust, and strict and with symmetric and positive definite matrix P will fulfill

$$\dot{V}_0 = -|w_1|^{\frac{1}{2}}\varsigma^{\mathrm{T}} Q \varsigma \tag{27}$$

for symmetric and positive definite matrix Q. In addition, the trajectory originating at w(0) will reach the origin in a fixed time t_f which is given by

$$t_f = t_{s\Delta} + \frac{2\lambda_{max}\{P\}}{\lambda_{min}^{\frac{1}{2}}\{P\}\lambda_{min}\{Q\}} V_0^{\frac{1}{2}}(t_0) \tag{28}$$

The matrices P and Q of the Lyapunov function can be selected in accordance with the procedure given in [51], which guarantees that the sliding variables w_1 and w_2 go to zero in the fixed time. $\square$

3. Symmetric Supply Chain System

Thus far, many researchers have tried to model supply chain systems. Among all models proposed for supply chain systems, the nonlinear model introduced by Anne et al. [52] has attracted a lot of attention. They introduced a nonlinear supply chain model that covers safety stock, information distortion, and retailer order satisfaction. That model is given by

$$\begin{aligned}
\dot{x}_1 &= m\, x_2 - (n+1)x_1 \\
\dot{x}_2 &= r\, x_1 - x_2 - x_1 x_3 \\
\dot{x}_3 &= x_1\, x_2 + (k-1)x_3
\end{aligned} \qquad (29)$$

where x_1 is the quantity demanded by the retailer in the current period, x_2 denotes the quantity distributors can supply in the current period, and x_3 indicates the quantity produced in the current period depending on the order. In addition, m is the rate of customer demand satisfaction with the retailer. n stands for the inventory level of distributors. k is the safety stock coefficient of the manufacturer, and r is the rate of information distortion of products demanded by the retailer.

This system is rotation symmetric around the x_3-axis, since it is invariant under the change of coordinates $(x_1,\, x_2,\, x_3)\ (-x_1, -x_2,\, x_3)$. The chaotic attractor of the supply chain system (29) when the system parameter values are selected as follows: $(m,\, n,\, r,\, k) = \left(12,\, 7,\, 45,\, -\frac{7}{3}\right)$ as shown in Figures 1 and 2. As illustrated in these figures, the nonlinear supply chain system shows chaotic behavior in this condition.

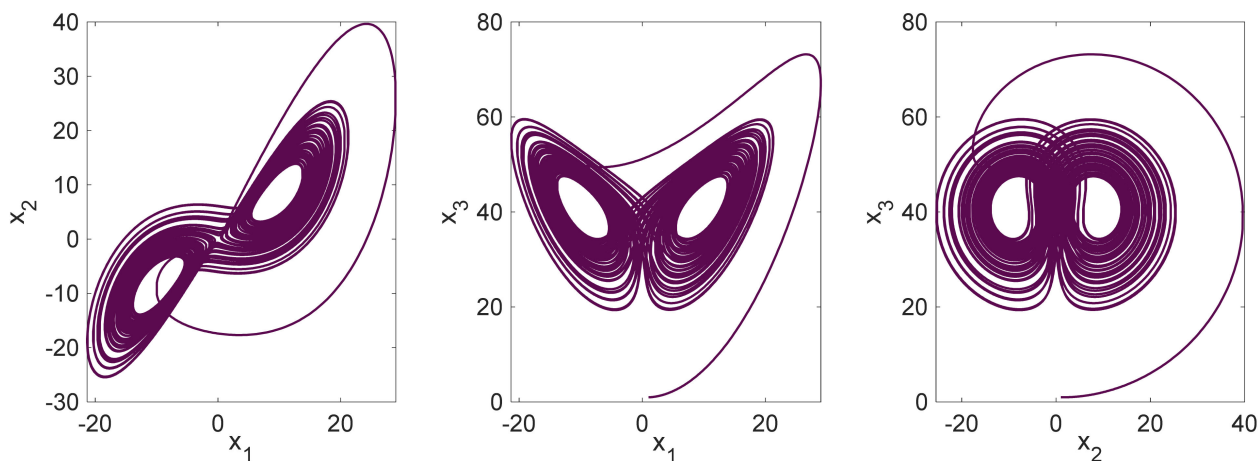**Figure 1.** The 2D phase portraits of the symmetric three-echelon supply chain system.
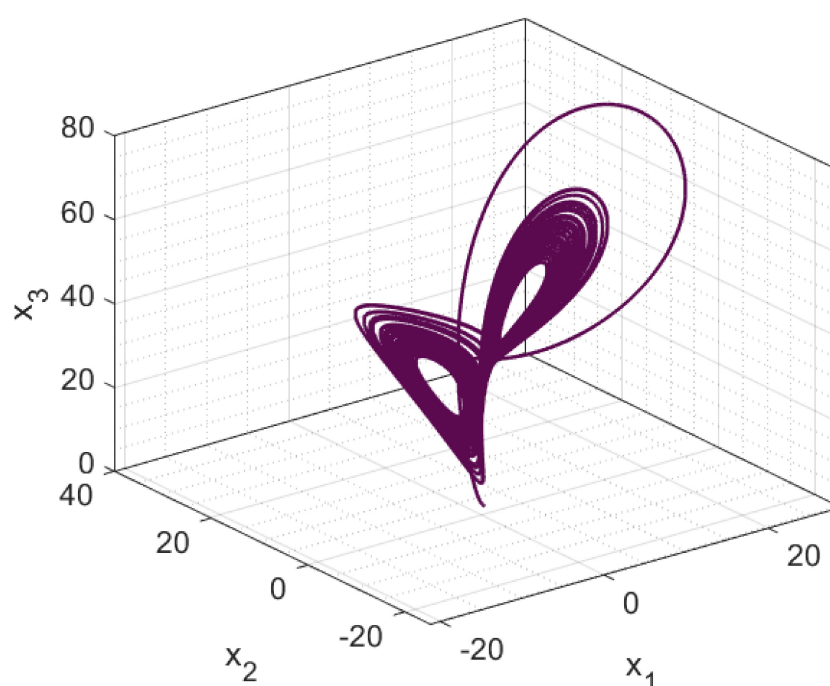
Figure 2. The 3D phase portrait of the symmetric three-echelon supply chain system.

4. Simulation Study by Applying Control Input

In this section, the proposed scheme is applied to the symmetric chaotic supply chain system. In this section, it is assumed that all parameters of the system are varying by time through unknown functions. This way, in all simulations, the effects of uncertainties on the system are taken to account. The parameters of the system are considered as

$$m = 15 + \Delta m, \ n = 7 + \Delta n, \ r = 45 + \Delta r, \ k = -\frac{7}{3} + \Delta k. \tag{30}$$

where for the simulations, we consider the unknown functions as

$$\Delta m = 5\sin(2t), \ \Delta n = 4\cos(2t), \Delta r = 8\cos(t), \ \Delta k = 2\sin(t) \tag{31}$$

Additionally, the user-defined parameters of the controller are designed as

$$\tau = [10, 10, 10]^T, \ k_1 = [20, 20, 20]^T, \ k_2 = [10, 10, 10]^T,$$
$$k = [10, 10, 10]^T, \ \beta = [10, 10, 10]^T, \ \varepsilon = [1, 1, 1]^T \tag{32}$$

4.1. Stabilization without Control Input Limitations

In this part, we aim to stabilize the symmetric supply chain system. At first, we consider no control limitation. Figure 3 shows the states of the system. As is shown in this figure, after a very short period of time (less than 0.1 time units), the system is completely stabilized. Additionally, the response of the systems is chatter-free. Figure 4 demonstrates the control input that is applied to the system. The control signals are chatter-free as well. However, now the most important issue regarding the control inputs is their large amplitudes. For instance, as is shown in Figure 4, the maximum value of u_2 is near 200, and it could be impossible to apply such a large control input in economic systems. This shows the importance of the proposed control input limitations. In the next subsection, we consider the control input limitations.
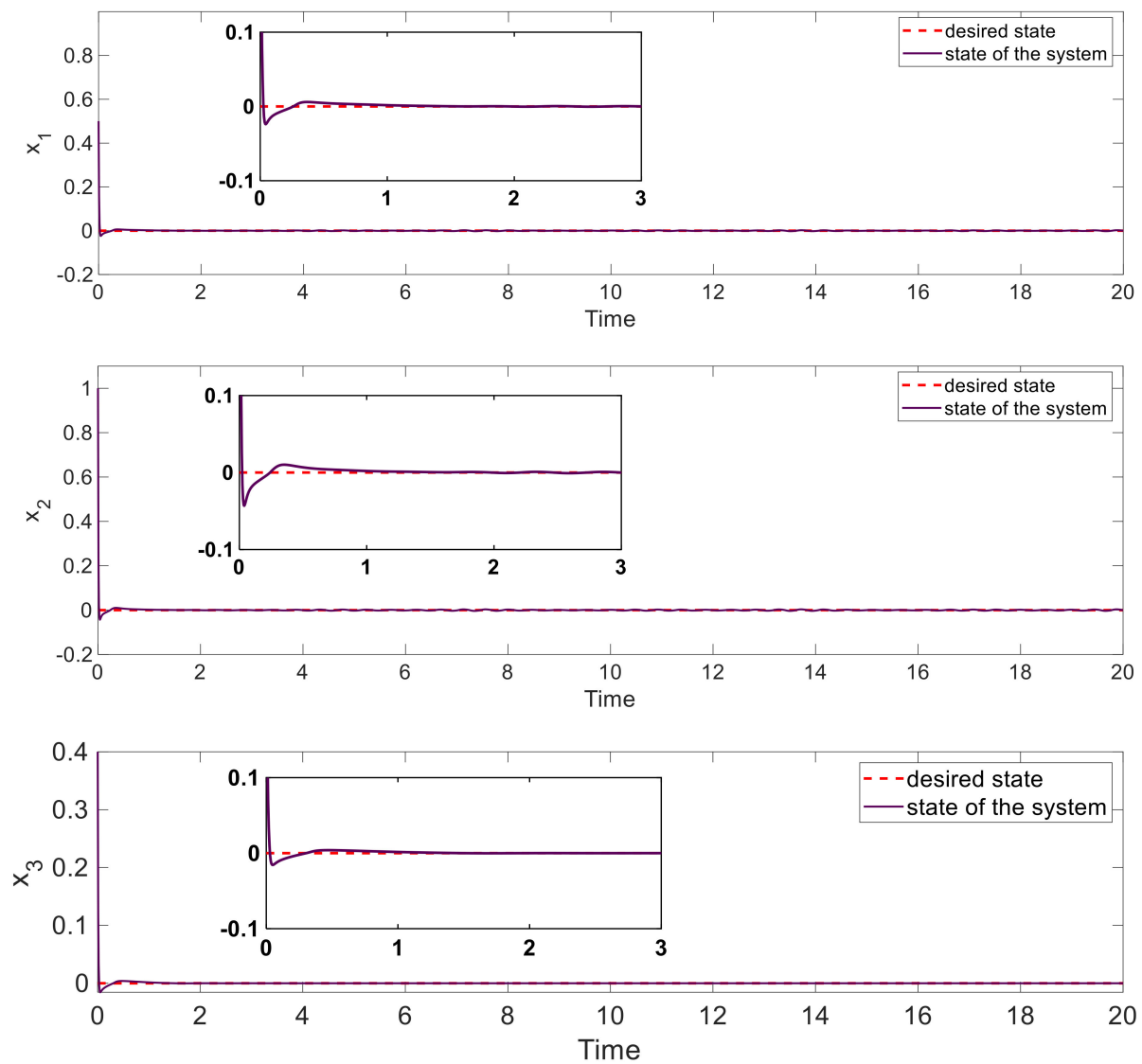
Figure 3. The states of the symmetric supply chain system under the proposed fixed-time disturbance observer-based super-twisting sliding mode technique.
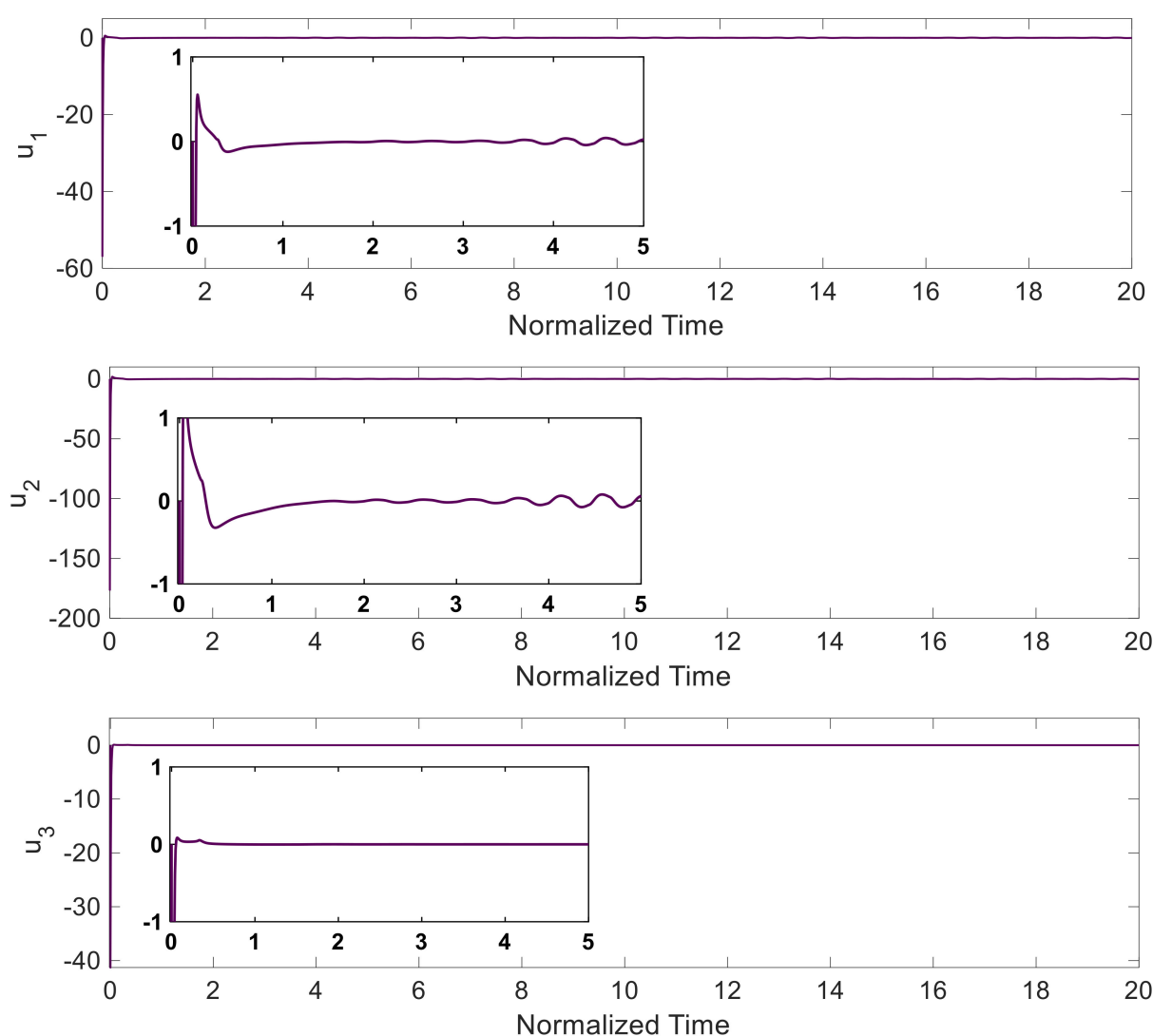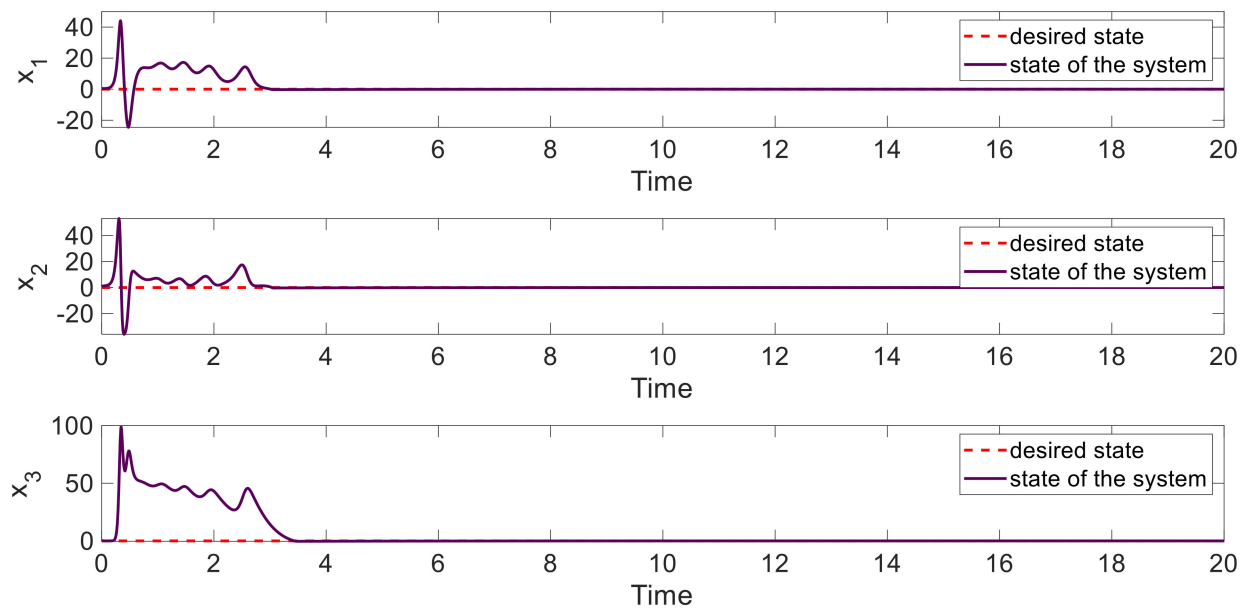
Figure 4. The control input based on the proposed fixed-time disturbance observer-based super-twisting sliding mode technique.

4.2. Stabilization with Limited Control Input

The bounds of the control signals are considered as

$$u_{1_{max}} = u_{2_{max}} = u_{3_{max}} = 15 u_{x1_{min}} = u_{x2_{min}} = u_{x3_{min}} = -15 \qquad (33)$$

The stabilized states of the system are illustrated in Figure 5. Although the control signals are limited, in the fixed time, the states of the system converge to zero. Figure 6 shows the limited control signals obtained through the proposed super-twisting sliding mode technique. It is noteworthy that when we limit the control input, the convergence time will be longer, but in this condition, the controller can be applied to practical systems.

Figure 5. The states of the symmetric supply chain system subject to control input limitations under the proposed fixed-time disturbance observer-based super-twisting sliding mode technique.
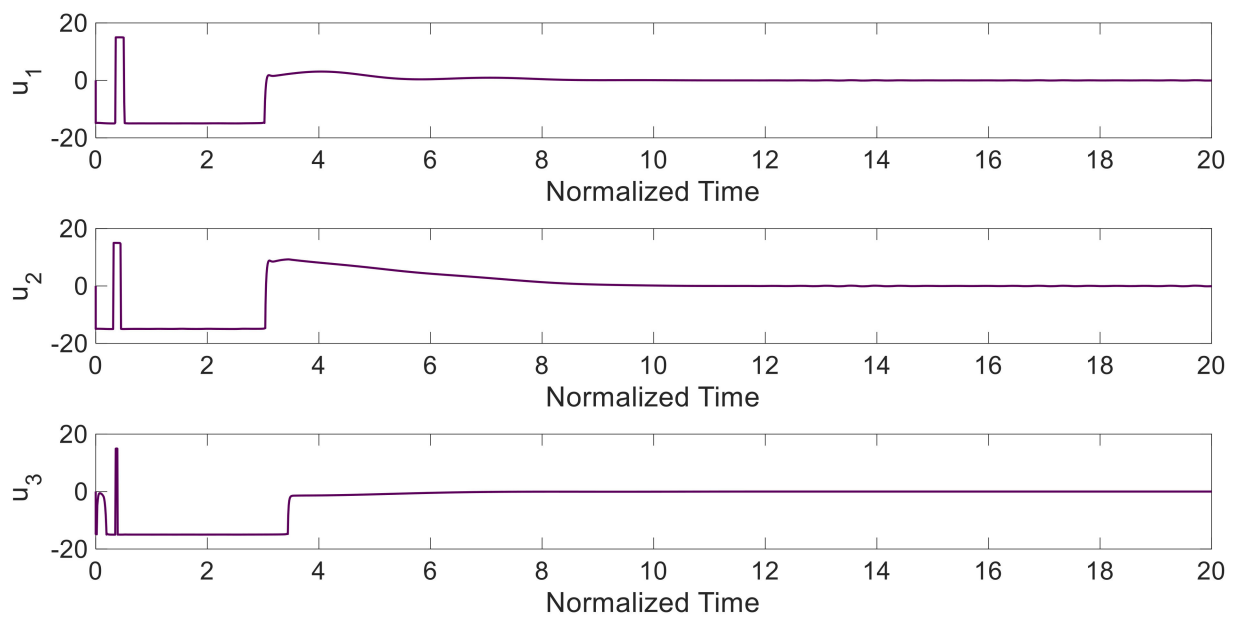
Figure 6. The limited control input based on the proposed fixed-time disturbance observer-based super-twisting sliding mode technique.

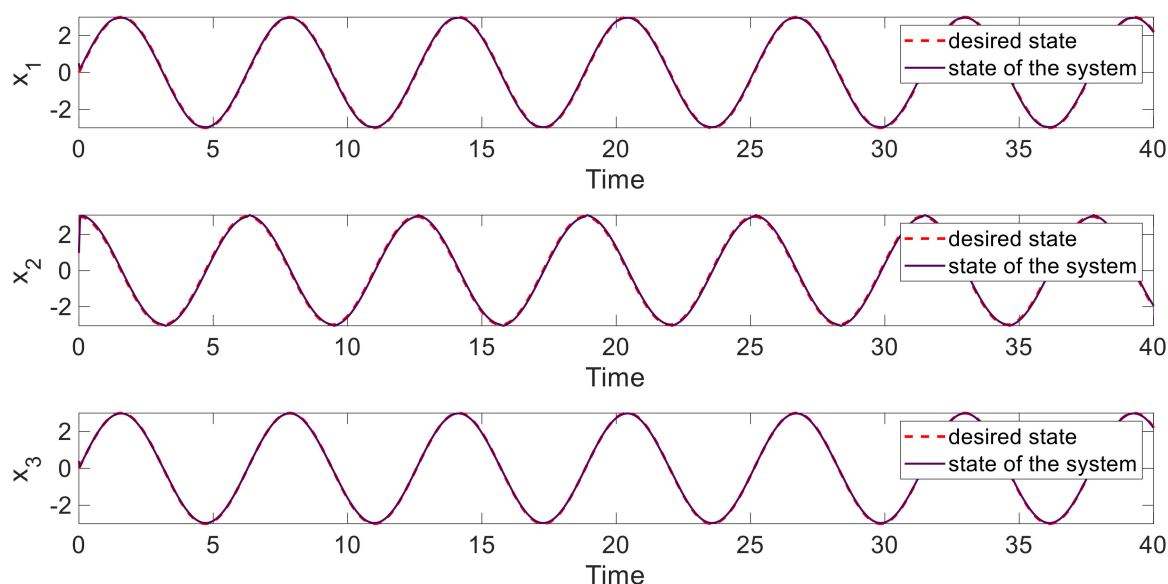### 4.3. Tracking Control with Limited Control Input

To investigate the performance of the proposed scheme, we consider the desired states as

$$x_d = 3\sin(10t), \ y_d = 3\cos(10t), \ z_d = 3\cos(10t), \tag{34}$$

Herein, the bounds of the control signals are considered as

$$u_{1_{max}} = 50, \ u_{2_{max}} = 150, \ u_{3_{max}} = 40 \tag{35}$$

The states of the system and the limited control input are displayed in Figures 7 and 8. As shown in the tracking control results, the proposed control scheme satisfies the expected performance.

Figure 7. The states of the symmetric supply chain system subject to control input limitations under the proposed technique in the tracking control.
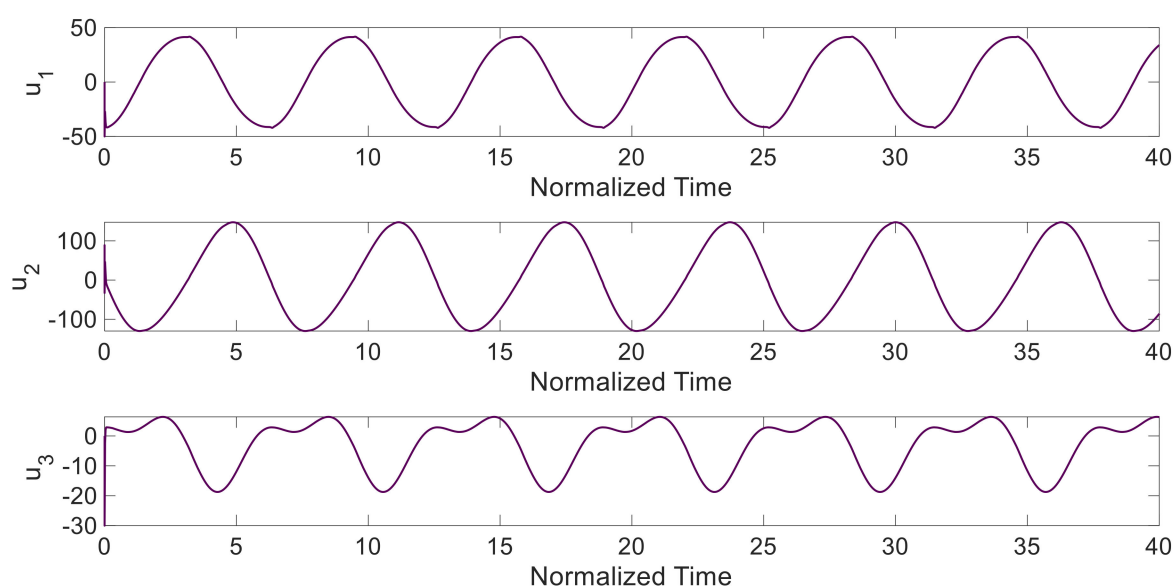
Figure 8. The limited control input based on the proposed technique in the tracking control.

5. Conclusions

In this paper, we studied stabilization and tracking control of a symmetric chaotic supply chain system subject to control input limitations. To this end, we introduced a new control scheme which is an extension of sliding mode control that can provide chatter-free and fixed-time responses. In the proposed control scheme, the effects of the control input limitation, as well as uncertainties, are taken into account. The fixed-time convergence and stability of the system were proven through the Lyapunov stability theorem. Then, the chaotic response of the symmetric supply chain system was demonstrated. Lastly, through various numerical simulations, the proposed scheme's effectiveness in different scenarios,

including stabilization without input limitations, stabilization with input limitations, and tracking control with input limitations, was assessed. Numerical simulations clearly confirm all theoretical claims about the proposed control technique. The proposed scheme's excellent performance even when the system is in the presence of control input limitation and time-varying uncertainties was displayed. As a feature suggestion, the proposed controller can be promoted through self-tuning algorithms. Additionally, the proposed method has a great potential to be customized or upgraded for various systems, including fractional-order ones.

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