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Comparative Analysis of Hybrid Fuzzy MCGDM Methodologies for Optimal Robot Selection Process

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Abstract: The generalized interval-valued trapezoidal fuzzy best-worst method (GITrF-BWM) provides more reliable and more consistent criteria weights for multiple criteria group decision making (MCGDM) problems. In this study, GITrF-BWM is integrated with the extended TOPSIS (technique for order preference by similarity to the ideal solution) and extended VIKOR (visekriterijumska optimizacija i kompromisno resenje) methods for the selection of the optimal industrial robot using fuzzy information. For a criteria-based selection process, assigning weights play a vital role and significantly affect the decision. Assigning weights based on direct opinions of decision makers can be biased, so weight deriving models, such as GITrF-BWM, overcome this discrepancy. In previous studies, generalized interval-valued trapezoidal fuzzy weights were not derived by using any MCGDM method for the robot selection process. For this study, both subjective and objective criteria are considered. The preferences of decision makers are provided with the help of linguistic terms that are then converted into fuzzy information. The stability and reliability of the methods were tested by performing sensitivity analysis, which showed that the ranking results of both the methodologies are not symmetrical, and the integration of GITrF-BWM with the extended TOPSIS method provides stable and reliable results as compared to the integration of GITrF-BWM with the extended VIKOR method. Hence, the proposed methodology provides robust optimal industrial robot selection.

Keywords: generalized interval-valued trapezoidal fuzzy best-worst method; extended VIKOR; extended TOPSIS; robot; hybrid MCGDM

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1. Introduction

Nowadays, the decision-making process includes subjective and uncertain criteria for the selection procedure. Industrial robots are used in all manufacturing industries with the aim of improving functionality and productivity. The market is full of industrial robots with various types of special features of industrial work, including many specifications and related skills. In such situations, industries require an expert team of decision makers in the selection process of robots that are suitable for a particular production and application. For multiple criteria group decision making (MCGDM) processes, decision makers/experts generally have different opinions about the same fact due to their knowledge structure, experience, and area of expertise.

In this paper, the generalized interval-valued trapezoidal fuzzy best-worst method (GITrF-BWM) is integrated with the generalized interval-valued trapezoidal fuzzy TOPSIS

(GITrF-TOPSIS) and generalized interval-valued trapezoidal fuzzy VIKOR (GITrF-VIKOR) methods. Both methods are distance-based but not symmetrical. For weight calculations, decision makers provided their preferences using linguistic terms. They felt more comfortable giving opinions using linguistic terms; then, the more reliable and consistent generalized interval-valued trapezoidal fuzzy weights (GITrFWs) were calculated using GITrF-BWM. Similarly, experts provided their preferences about the alternatives with respect to each subjective criteria using linguistic terms that were then converted into the GITrFNs. The provided fuzzy information was aggregated and normalized, and then using GITrFWs, fuzzy information was converted to a weighted normalized form. The ranking was obtained using two different integrated methods: (1) GITrF-BWM integrated with the GITrF-TOPSIS method and (2) GITrF-BWM integrated with the GITrF-VIKOR method. Rankings of each decision maker and aggregated decision ranking were tabulated and pictured. Sensitivity analysis was performed with respect to criteria; the results showed that GITrF-TOPSIS was more sensitive with respect to c_2 and c_3, and GITrF-VIKOR was more sensitive with respect to c_2, c_3, c_5 and c_6. This implies that the GITrF-VIKOR method provides better rank reversal with respect to criteria as compared to the GITrF-TOPSIS method, which is more stable.

The rest of the paper is organized as follows, Section 3 provides the basic definitions, and Section 4 describes the steps for GITrF-BWM. In Section 5, the steps for ranking using GITrF-TOPSIS are explained, and in Section 6, the steps for ranking using GITrF-VIKOR are explained. Section 7 consists of the robot selection process, Section 8 provides a sensitivity analysis of the methodology and Section 10 provides the conclusion of the research.

2. Literature Review

In 1965, Zadeh invented an extension of the classical set called the fuzzy set [1]. Bellman and Zadeh discovered in 1970 that the decision-making process involves fuzzy information as all humans have different ways of thinking [2]. There is extensive use of trapezoidal fuzzy numbers in multi-criteria decision making (MCDM) due to applied and reliable results [3–6]. Wei and Chen [7] discovered generalized interval-valued trapezoidal fuzzy numbers (GITrFNs) that are an extension of the generalized trapezoidal fuzzy numbers discovered by Chen in 1985 [8]. The success of these extensions in MCDM problems is obtaining more fame. Similarity measures of the GITrFNs based on geometric distance and the center of gravity were proposed by Wei and Chen [9]. A similarity measure based on the geometric distance of GITrFNs was applied by Wei and Chen for a risk analysis MCDM problem [7]. Liu discovered and applied aggregation operators of GITrFNs for the MCGDM problem [10]. Liu and Jin proposed the weighted geometric aggregation operators for MCGDM problems using GITrFNs [11]. Generalized trapezoidal numbers reduce the computational cost for a transportation problem using GITrFNs, as proposed by Ebrahimnejad [12]. Mathematical modeling and solutions to the models are important and their importance can be seen in management, social, and engineering applications [13–21]. Rashid, Beg, and Husnine [22] proposed an extended technique for order preference by similarity to the ideal solution (TOPSIS), which uses GITrFNs for the selection of robots considering both objective and subjective criteria but lacks a weights deriving procedure. Ali, Rashid and Chu [23,24] provided a best-worst methodology and a hybrid best-worst EDAS methodology for industrial robot selection but they did not have considered fuzzy information for the aggregation process. It is useful and significant to investigate MCGDM methods using generalized fuzzy information like GITrFNs, as its particular cases involve generalized trapezoidal, trapezoidal, interval-valued triangular, triangular and interval fuzzy numbers, etc.

Opricovic [25] developed the visekriterijumska optimizacija i kompromisno resenje (VIKOR) method to solve discrete MCDM problems that have non-commensurable and conflicting criteria. For the majority, "maximum group utility" and for the opponent, "minimum of an individual regret" are the key factors to provide a compromise solution using the VIKOR method as it is a serviceable tool for MCDM problems. A robot selection problem solved by Athawale, Chatterjee, and Chakraborty [26] using the VIKOR method determines the applicability and advantages of the technique. Chatterjee, Athawale, and Chakraborty [27] make a relative performance comparison of a robot selection problem solved using the elimination et choice translating reality (ELECTRE) and VIKOR methods. The interval-valued trapezoidal fuzzy numbers are integrated with the VIKOR method in a systematic and logical approach for the industrial robot selection problem. An extension of the VIKOR method for a triangular intuitionistic fuzzy environment was proposed by Devi [28], in which alternatives are evaluated using triangular intuitionistic fuzzy numbers. Both quantitative and qualitative criteria were considered for a robot selection novel MCDM method by Rao, Patel, and Parnichkun [29]. Tansel, Yurdakul, and Dengiz [30] presented a robot selection process based on a fuzzy analytic hierarchy process (FAHP) and the ROBSEL two-phase method that is less dependent on the experts' opinions for the selection process. Bairagi, Dey, Sarkar, and Sanyal [31] calculated weights using FAHP and utilize them to calculate ranking using the fuzzy VIKOR, fuzzy TOPSIS, and complex proportional assessment of alternatives with grey relations (COPRAS-G) methods. Liu et al. [32] solved a robot selection MCDM problem by using the TOPSIS method in which decision makers evaluated criteria using interval two-tuple linguistic fuzzy sets. Lanbaran et al. [33] evaluated investment opportunities proposing the fuzzy TOPSIS extension using an interval-valued fuzzy set. Parameshwaran, Kumar, and Saravanakumar [34] made criteria selection using the fuzzy delphi method for an educational robot selection; here, weights were calculated using the FAHP method, and alternative rankings were obtained using the fuzzy VIKOR and fuzzy TOPSIS methods. Bairagi, Dey, Sarkar, and Sanyal [35] solved a robot selection MCDM problem by presenting the technique of precise order preferences (TPOP) methodology. Samantra, Datta, and Mahapatra [36] proposed an extended VIKOR MCGDM method using GITrFNs for the selection of industrial robots but did not adopt a proper weight deriving procedure. Ghorabaee [37] utilized interval type-2 fuzzy numbers by extending the VIKOR method to a fuzzy environment for a robot selection problem for which he used the Spearman correlation coefficient to analyze the stability of the method. Jiang et al. [38] used the fuzzy DEMATEL MCDM method to identify the critical variables for sustainable manufacturing. Joshi and Kumar [39] extended the TOPSIS MCDM method by introducing a Choquet integral operator for an interval-valued intuitionistic hesitant fuzzy set for the alternative ranking purpose. BWM, TOPSIS, and VIKOR are very practical methods that have wide areas of application and success in decision processes [40–46].

3. Preliminaries

Zadeh invented the fuzzy set in 1965 by extending the classical set [1]. This theory is used to solve problems that have uncertain and vague environments. A fuzzy set in defined in the form of a pair (U, μ) where U represent a set of discourse and a function $\mu : U \rightarrow [0, 1]$ that maps each element $x \in U$ to the real number in the interval $[0, 1]$, called a membership function:

Definition 1 ([8]). *Equation (1) represents a generalized trapezoidal fuzzy number (GTrFN):*

$$G_T(g) = \begin{cases} h - \frac{m-g}{m-l}, & for \quad l < g \leqslant m; \\ h, & for \quad m \leqslant g \leqslant n; \\ h - \frac{g-n}{u-n}, & for \quad n \leqslant g < u; \\ 0, & for \quad g \leqslant l \ \ and \ \ g \geqslant u \end{cases} \tag{1}$$

that is denoted by $G = (l, m, n, u; h)$, where $h \in [0, 1]$ is the height of G, and $\forall\ l, m, n, u \in R$, $l \leqslant m \leqslant n \leqslant u$.

Definition 2 ([47]). *Let $G_T^L(g_i)$ and $G_T^U(g_i)$ be two GTrFNs, $h_{g_i}^L$ and $h_{g_i}^U$ denote the heights of $G_T^L(g_i)$ and $G_T^U(g_i)$, respectively, where $l_i^L, m_i^L, n_i^L, u_i^L, l_i^U, m_i^U, n_i^U, u_i^U \in R$. Let $G = \{g_1, g_2,$*

$g_3, \cdots, g_n\}$ be a universe of discourse, and a generalized interval-valued trapezoidal fuzzy set G_{Tr} defined on G is represented by Equation (2):

$$\begin{aligned} G_{Tr} &= \left\{ \left\langle g_i, \left[G_T^L(g_i), G_T^U(g_i) \right] \right\rangle | g_i \in G \right\} \\ &= \left\{ \left\langle g_i, \left[\left(l_i^L, m_i^L, n_i^L, u_i^L; h_{g_i}^L \right), \left(l_i^U, m_i^U, n_i^U, u_i^U; h_{g_i}^U \right) \right] \right\rangle | g_i \in G \right\} \end{aligned} \tag{2}$$

where $l_i^L \leqslant m_i^L \leqslant n_i^L \leqslant u_i^L, l_i^U \leqslant m_i^U \leqslant n_i^U \leqslant u_i^U, 0 \leqslant h_{g_i}^L \leqslant h_{g_i}^U \leqslant 1, l_i^U \leqslant l_i^L, u_i^L \leqslant u_i^U$. Moreover, $G_T^L(g_i) = \left(l_i^L, m_i^L, n_i^L, u_i^L; h_{g_i}^L \right), G_T^U(g_i) = \left(l_i^U, m_i^U, n_i^U, u_i^U; h_{g_i}^U \right)$.

A GITrFN $G_{Tr}(g) = \left[G_T^L(g), G_T^U(g) \right]$ consists of the two GTrFNs $G_T^L(g) = (l^L, m^L, n^L, u^L; h_g^L)$ and $G_T^U(g) = \left(l^U, m^U, n^U, u^U; h_g^U \right)$, where $G_T^L(g)$ is called the lower trapezoidal fuzzy number, and $G_T^U(g)$ is called the upper trapezoidal fuzzy number. The normal interval-valued trapezoidal fuzzy number is obtained by having $h_g^L = h_g^U = 1$ in the GITrFN $G_{Tr}(g)$. If $G_T^L(g) = G_T^U(g)$, then $G_{Tr}(g)$ becomes a GTrFN. The operational rules for the GITrFNs can be seen in Chen [47].

Definition 3. *The ranking of GITrFN $\tilde{G}_{Tr}$ is represented by the graded mean integration representation (GMIR) $R(G_{Tr})$ [48–50].*

The GMIR $R(G_{Tr})$ of GITrFN $G_{Tr}(g) = [(l^L, m^L, n^L, u^L; h^L), (l^U, m^U, n^U, u^U; h^U)]$ is calculated by Equation (3):

$$R(G_{Tr}) = \frac{l^L + l^U + 2(m^L + m^U) + 2(n^L + n^U) + u^L + u^U}{12} \tag{3}$$

Definition 4. *[51] Let $G_{1Tr} = \left[\left(l_{1i}^L, m_{1i}^L, n_{1i}^L, u_{1i}^L; h_{1i}^L \right), \left(l_{1i}^U, m_{1i}^U, n_{1i}^U, u_{1i}^U; h_{1i}^U \right) \right]$ and $G_{2Tr} = [(l_{2i}^L, m_{2i}^L, n_{2i}^L, u_{2i}^L; h_{2i}^L), (l_{2i}^U, m_{2i}^U, n_{2i}^U, u_{2i}^U; h_{2i}^U)]$ be two GITrFNs. Their distance d_T is defined by Equation (4):*

$$\begin{aligned} d_T(G_{1Tr}, G_{2Tr}) = \frac{1}{8} \Big(& \left| h_{1i}^L \times l_{1i}^L - l_{2i}^L \times h_{2i}^L \right| + \left| h_{1i}^L \times m_{1i}^L - m_{2i}^L \times h_{2i}^L \right| + \left| h_{1i}^L \times n_{1i}^L - n_{2i}^L \times h_{2i}^L \right| + \left| h_{1i}^L \times u_{1i}^L - u_{2i}^L \times h_{2i}^L \right| \\ & + \left| h_{1i}^U \times l_{1i}^U - l_{2i}^U \times h_{2i}^U \right| + \left| h_{1i}^U \times m_{1i}^U - m_{2i}^U \times h_{2i}^U \right| + \left| h_{1i}^U \times n_{1i}^U - n_{2i}^U \times h_{2i}^U \right| + \left| h_{1i}^U \times u_{1i}^U - u_{2i}^U \times h_{2i}^U \right| \Big) \end{aligned} \tag{4}$$

Definition 5 ([36])**.** *Let $G_{1Tr} = \left[\left(l_{1i}^L, m_{1i}^L, n_{1i}^L, u_{1i}^L; h_{1i}^L \right), \left(l_{1i}^U, m_{1i}^U, n_{1i}^U, u_{1i}^U; h_{1i}^U \right) \right] = \left[G_1^L, G_1^U \right]$ and $G_{2Tr} = \left[\left(l_{2i}^L, m_{2i}^L, n_{2i}^L, u_{2i}^L; h_{2i}^L \right), \left(l_{2i}^U, m_{2i}^U, n_{2i}^U, u_{2i}^U; h_{2i}^U \right) \right] = \left[G_2^L, G_2^U \right]$ be two GITrFNs. Their distance d_V is defined by Equation (5):*

$$d_V(G_{1Tr}, G_{2Tr}) = \frac{1}{2} \sqrt{ \left(y_{G_1^L} - y_{G_2^L} \right)^2 + \left(x_{G_1^L} - x_{G_2^L} \right)^2 + \left(y_{G_1^U} - y_{G_2^U} \right)^2 + \left(x_{G_1^U} - x_{G_2^U} \right)^2 } \tag{5}$$

where $\left(x_{G_1^L}, y_{G_1^L} \right)$, $\left(x_{G_1^U}, y_{G_1^U} \right)$, $\left(x_{G_2^L}, y_{G_2^L} \right)$, and $\left(x_{G_2^U}, y_{G_2^U} \right)$ are the coordinates of center of gravity (CoG) points belonging to the GTrFNs G_1^L, G_1^U, G_2^L, and G_2^U, respectively. Computational steps for evaluation of CoG points can be seen in Wei and Chen [7].

4. GITrF-BW Method

The GITrF-BWM method proposed by Rashid and Ali [52] is a fuzzy extension of the classical best-worst method (BWM) that is more consistent and more reliable compared to the classical BWM due to its lower consistency ratio and that it provides unique results. The generalized interval-valued trapezoidal fuzzy pairwise comparisons for a research object that have n criteria are performed using the following decision makers' linguistic terms: "Equally important (EI)", "Weakly important (WI)", "Fairly important (FI)", "Very important (VI)' and "Absolutely important (AI)". Table 1 and Figure 1 represent the transformation rules of decision makers' linguistic evaluation to the GITrFNs.

Table 1. Transformation of decision makers' linguistic terms to GITrFNs [52].

GITrFNs Transformation	Linguistic Terms
$[(1,1,1,1;1),(1,1,1,1;1)]$	Equally Important (EI)
$[(0.7857,1.0238,1.1429,1.381;0.8),(0.6667,0.9048,1.2619,1.5;1)]$	Weakly Important (WI)
$[(1.6429,1.9286,2.0714,2.3571;0.8),(1.5,1.7857,2.2143,2.5;1)]$	Fairly Important (FI)
$[(2.6429,2.9286,3.0714,3.3571;0.8),(2.5,2.7857,3.2143,3.5;1)]$	Very Important (VI)
$[(3.6429,3.9286,4.0714,4.3571;0.8),(3.5,3.7857,4.2143,4.5;1)]$	Absolutely Important (AI)

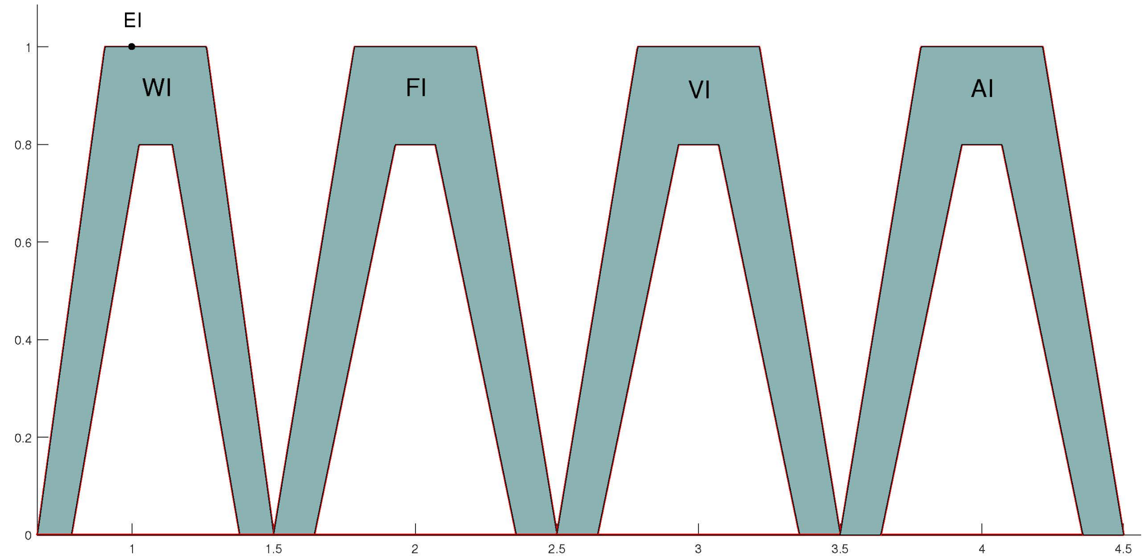


Figure 1. Graphical representation of fuzzy evaluation table for GITrF-BWM.

The pairwise comparison matrix using GITrFNs is represented by the Equation (6),

$$\tilde{P} = \begin{bmatrix} \tilde{p}_{11} & \tilde{p}_{12} & \cdots & \tilde{p}_{1n} \\ \tilde{p}_{21} & \tilde{p}_{22} & \cdots & \tilde{p}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{p}_{n1} & \tilde{p}_{n2} & \cdots & \tilde{p}_{nn} \end{bmatrix} \quad (6)$$

where $\tilde{p}_{ij}$ (a GITrFN) represents the relative generalized interval-valued trapezoidal fuzzy preference (GITrFP) of criterion i to criterion j; when $i = j$, $\tilde{p}_{ij} = \{(1,1,1,1;1),(1,1,1,1;1)\}$. The basic principle of BWM [53] is adopted to have the matrix $\tilde{P}$, the principle dictates that only fuzzy reference comparisons are sufficient instead of n pairwise comparisons.

Definition 6 ([52])**.** *A generalized interval-valued trapezoidal fuzzy reference comparison (GITrF-RC) is defined by $\tilde{p}_{ij}$, only if i is the best element and/or j is the worst element.*

By using GITrF-BWM, GITrFWs of criteria can be calculated. To determine GITrFWs of criteria/alternatives the following steps of GITrF-BWM are adopted.

4.1. Step 1. Selection of a Criteria Set

Decision makers determine a criteria set $K = \{k_1, k_2, k_3, \cdots, k_n\}$ for the evaluation of alternatives. For example, a set can be {safety, quality, price, style, comfort} for a car selection problem.

4.2. Step 2. Selection of Most Favorable Criterion and Least Favorable Criterion

Decision makers make a selection of a most favorable criterion and a least favorable criterion among all the criteria and call them the best criterion "denoted by k_B" and worst criterion "denoted by k_W", respectively. For a car selection, the price and style can be the best and worst criteria, respectively.

4.3. Step 3. GITrF-RC of the Best Criterion over All Other Criteria

The important part of the GITrF-BWM is the GITrF-RC. Definition 6 determines that the GITrF-RC has two parts; one is the pairwise comparison $\tilde{p}_{ij}$ where i is the best element $(k_i = s_B)$, and the other is the pairwise comparison $\tilde{p}_{ij}$ where j is the worst element $(k_j = s_W)$. Using the rules of transformation of linguistic terms to GITrFNs defined in Table 1 and Figure 1, the performed GITrFPs of the best criterion to all other criteria in the form of linguistic terms are converted to GITrFNs. In this way, the generalized interval-valued trapezoidal fuzzy vector for best to other criteria is obtained and is denoted by Equation (7):

$$\tilde{P}_B = (\tilde{p}_{B1}, \tilde{p}_{B2}, \tilde{p}_{B3}, \cdots, \tilde{p}_{Bn}) \tag{7}$$

where $\tilde{P}_B$ is the generalized interval-valued trapezoidal fuzzy best to another vector; $\tilde{p}_{Bj}$ represents the GITrFP of the best criterion k_B over criteria k_j, $j = 1, 2, 3, \cdots, n$.

4.4. Step 4. GITrF-RC of All of the Other Criteria over the Worst Criterion

Using rules of transformation of linguistic terms to GITrFNs defined in Table 1 and Figure 1, the performed GITrFP of all of the other criteria over the worst criteria in the form of linguistic terms are converted to GITrFNs. In this way, a generalized interval-valued trapezoidal fuzzy vector from all other criteria to the worst criterion is obtained and is denoted by Equation (8):

$$\tilde{P}_W = (\tilde{p}_{1W}, \tilde{p}_{2W}, \tilde{p}_{3W}, \cdots, \tilde{p}_{nW}) \tag{8}$$

where $\tilde{P}_W$ is the generalized interval-valued trapezoidal fuzzy other to worst vector; and $\tilde{p}_{iW}$ represents the GITrFP of all other criteria k_i over the worst criterion k_W, $i = 1, 2, 3, \cdots, n$.

4.5. Step 5. Determine the GITrFWs of Criteria

We can have optimal GITrFWs for each of the criteria for ${}^{t\tilde{w}_B}/_{t\tilde{w}_j} = \tilde{p}_{Bj}$ and ${}^{t\tilde{w}_j}/_{t\tilde{w}_W} = \tilde{p}_{jW}$, for each generalized interval-valued trapezoidal fuzzy pair ${}^{t\tilde{w}_B}/_{t\tilde{w}_j}$ and ${}^{t\tilde{w}_j}/_{t\tilde{w}_W}$, but it cannot be always true. Thus, the maximum absolute gaps $\left|{}^{t\tilde{w}_B}/_{t\tilde{w}_j} - \tilde{p}_{Bj}\right|$ and $\left|{}^{t\tilde{w}_j}/_{t\tilde{w}_W} - \tilde{p}_{jW}\right|$ are minimized, which satisfies these conditions for all j. Here $t\tilde{w}_B$, $t\tilde{w}_j$ and $t\tilde{w}_W$ are GITrFNs, which are very different from those in BWM. We used the GMIR method to transform the GITrFWs of criteria to crisp weights. Thus, to determine the GITrFWs $(t\tilde{w}_1^*, t\tilde{w}_2^*, t\tilde{w}_3^*, \cdots, t\tilde{w}_n^*)$, we can have the constrained optimization model (9):

$$\begin{aligned}
&\min \max_j \left\{ \left| \frac{t\tilde{w}_B}{t\tilde{w}_j} - \tilde{p}_{Bj} \right|, \left| \frac{t\tilde{w}_j}{t\tilde{w}_w} - \tilde{p}_{jW} \right| \right\} \\
&\text{Subject to:} \\
&\sum_{j=1}^{n} R(t\tilde{w}_j) = 1 \ ; \ l_j^{Lw} \leqslant m_j^{Lw} \leqslant n_j^{Lw} \leqslant u_j^{Lw} \ ; \\
&l_j^{Uw} \leqslant m_j^{Uw} \leqslant n_j^{Uw} \leqslant u_j^{Uw} \ ; \ 0 \leqslant h_j^{Lw} \leqslant h_j^{Uw} \leqslant 1 \ ; \\
&l_j^{Uw} \leqslant l_j^{Lw} \ ; \ u_j^{Lw} \leqslant u_j^{Uw} \ ; \ l_j^{Uw} \geqslant 0 \ ; \ j = 1, 2, 3, \cdots, n.
\end{aligned} \tag{9}$$

where:

$$\tilde{w}_B = \left[\left(l_B^{Lw}, m_B^{Lw}, n_B^{Lw}, u_B^{Lw}; h_B^{Lw}\right), \left(l_B^{Uw}, m_B^{Uw}, n_B^{Uw}, u_B^{Uw}; h_B^{Uw}\right)\right],$$
$$\tilde{w}_j = \left[\left(l_j^{Lw}, m_j^{Lw}, n_j^{Lw}, u_j^{Lw}; h_j^{Lw}\right), \left(l_j^{Uw}, m_j^{Uw}, n_j^{Uw}, u_j^{Uw}; h_j^{Uw}\right)\right],$$
$$\tilde{w}_W = \left[\left(l_W^{Lw}, m_W^{Lw}, n_W^{Lw}, u_W^{Lw}; h_W^{Lw}\right), \left(l_W^{Uw}, m_W^{Uw}, n_W^{Uw}, u_W^{Uw}; h_W^{Uw}\right)\right],$$
$$\tilde{p}_{Bi} = \left[\left(l_{Bj}^L, m_{Bj}^L, n_{Bj}^L, u_{Bj}^L; h_{Bj}^L\right), \left(l_{Bj}^U, m_{Bj}^U, n_{Bj}^U, u_{Bj}^U; h_{Bj}^U\right)\right],$$
$$\tilde{p}_{jW} = \left[\left(l_{jW}^L, m_{jW}^L, n_{jW}^L, u_{jW}^L; h_{jW}^L\right), \left(l_{jW}^U, m_{jW}^U, n_{jW}^U, u_{jW}^U; h_{jW}^U\right)\right].$$

The mathematical model (9) is converted into a non-linear optimization model that is represented by Equation (10).

$$\begin{aligned}
&\min \tilde{\xi} \\
&\text{subject to:} \\
&\left|\frac{t\tilde{w}_B}{t\tilde{w}_j} - \tilde{p}_{Bj}\right| \leq \tilde{\xi} \;;\; \left|\frac{t\tilde{w}_j}{t\tilde{w}_W} - \tilde{p}_{jW}\right| \leq \tilde{\xi} \\
&\sum_{j=1}^{n} R\left(t\tilde{w}_j\right) = 1 \;;\; l_j^{Lw} \leqslant m_j^{Lw} \leqslant n_j^{Lw} \leqslant u_j^{Lw}; \\
&l_j^{Uw} \leqslant m_j^{Uw} \leqslant n_j^{Uw} \leqslant u_j^{Uw} \;;\; 0 \leqslant h_j^{Lw} \leqslant h_j^{Uw} \leqslant 1; \\
&l_j^{Uw} \leqslant l_j^{Lw}; \quad u_j^{Lw} \leqslant u_j^{Uw}; \quad l_j^{Uw} \geqslant 0; \quad j = 1, 2, 3, \cdots, n.
\end{aligned} \tag{10}$$

where $\tilde{\xi} = \left[\left(l^{L\xi}, m^{L\xi}, n^{L\xi}, u^{L\xi}; h^{L\xi}\right), \left(l^{U\xi}, m^{U\xi}, n^{U\xi}, u^{U\xi}; h^{U\xi}\right)\right]$.
Consider $l^{L\xi} \leqslant m^{L\xi} \leqslant n^{L\xi} \leqslant u^{L\xi}$; $l^{U\xi} \leqslant m^{U\xi} \leqslant n^{U\xi} \leqslant u^{U\xi}$; by supposing $\tilde{\xi}^* = [(\varepsilon^*, \varepsilon^*, \varepsilon^*, \varepsilon^*; \varepsilon^*), (\varepsilon^*, \varepsilon^*, \varepsilon^*, \varepsilon^*; \varepsilon^*)]$, $\varepsilon^* \leqslant l^{U\xi}$, model (10) can be converted into model (11).

$$\begin{aligned}
&\min \tilde{\xi}^* \\
&\text{Subject to:} \\
&\left|\frac{\left[\left(l_B^{Lw}, m_B^{Lw}, n_B^{Lw}, u_B^{Lw}; h_B^{Lw}\right), \left(l_B^{Uw}, m_B^{Uw}, n_B^{Uw}, u_B^{Uw}; h_B^{Uw}\right)\right]}{\left[\left(l_j^{Lw}, m_j^{Lw}, n_j^{Lw}, u_j^{Lw}; h_j^{Lw}\right), \left(l_j^{Uw}, m_j^{Uw}, n_j^{Uw}, u_j^{Uw}; h_j^{Uw}\right)\right]} - \left[\left(l_{Bj}^L, m_{Bj}^L, n_{Bj}^L, u_{Bj}^L; h_{Bj}^L\right), \left(l_{Bj}^U, m_{Bj}^U, n_{Bj}^U, u_{Bj}^U; h_{Bj}^U\right)\right]\right| \\
&\qquad \leqslant [(\varepsilon^*, \varepsilon^*, \varepsilon^*, \varepsilon^*; \varepsilon^*), (\varepsilon^*, \varepsilon^*, \varepsilon^*, \varepsilon^*; \varepsilon^*)] \\
&\left|\frac{\left[\left(l_j^{Lw}, m_j^{Lw}, n_j^{Lw}, u_j^{Lw}; h_j^{Lw}\right), \left(l_j^{Uw}, m_j^{Uw}, n_j^{Uw}, u_j^{Uw}; h_j^{Uw}\right)\right]}{\left[\left(l_W^{Lw}, m_W^{Lw}, n_W^{Lw}, u_W^{Lw}; h_W^{Lw}\right), \left(l_W^{Uw}, m_W^{Uw}, n_W^{Uw}, u_W^{Uw}; h_W^{Uw}\right)\right]} - \left[\left(l_{jW}^L, m_{jW}^L, n_{jW}^L, u_{jW}^L; h_{jW}^L\right), \left(l_{jW}^U, m_{jW}^U, n_{jW}^U, u_{jW}^U; h_{jW}^U\right)\right]\right| \\
&\qquad \leqslant [(\varepsilon^*, \varepsilon^*, \varepsilon^*, \varepsilon^*; \varepsilon^*), (\varepsilon^*, \varepsilon^*, \varepsilon^*, \varepsilon^*; \varepsilon^*)] \\
&\sum_{j=1}^{n} R\left(t\tilde{w}_j\right) = 1 \;;\; l_j^{Lw} \leqslant m_j^{Lw} \leqslant n_j^{Lw} \leqslant u_j^{Lw} \;;\; l_j^{Uw} \leqslant m_j^{Uw} \leqslant n_j^{Uw} \leqslant u_j^{Uw} \;;\; 0 \leqslant h_j^{Lw} \leqslant h_j^{Uw} \leqslant 1 \;;\; l_j^{Uw} \leqslant l_j^{Lw}; \\
&u_j^{Lw} \leqslant u_j^{Uw}; \quad l_j^{Uw} \geqslant 0, \quad j = 1, 2, 3, \cdots, n.
\end{aligned} \tag{11}$$

The optimal GITrFWs $\left(t\tilde{w}_1^*, t\tilde{w}_2^*, t\tilde{w}_3^*, \cdots, t\tilde{w}_n^*\right)$ are obtained by the solution of Equations (11).

5. GITrF-TOPSIS Method

Generally, MCGDM problems involve fuzzy information. To handle fuzzy information, GITrFNs are very useful and provide good results. Decision makers provide their preferences using linguistic terms as they feel difficulty in providing their opinion using GITrFNs. The provided linguistic information is converted into fuzzy information. Preference conversion rules of linguistic terms to GITrFNs are provided in Table 2.

Table 2. The rules of preference conversion of linguistic terms to GITrFNs, according to Chen [47].

GITrFNs	Linguistic Terms
$[(0.0, 0.0, 0.0, 0.0; 1.0), (0.0, 0.0, 0.0, 0.0; 1.0)]$	Absolutely poor (AP)
$[(0.0075, 0.0075, 0.015, 0.0525; 0.8), (0.0, 0.0, 0.02, 0.07; 1.0)]$	Very poor (VP)
$[(0.0875, 0.12, 0.16, 0.1825; 0.8), (0.04, 0.10, 0.18, 0.23; 1.0)]$	Poor (P)
$[(0.2325, 0.255, 0.325, 0.3575; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]$	Medium poor (MP)
$[(0.4025, 0.4525, 0.5375, 0.5675; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]$	Medium (M)
$[(0.65, 0.6725, 0.7575, 0.79; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]$	Medium good (MG)
$[(0.7825, 0.815, 0.885, 0.9075; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]$	Good (G)
$[(0.9475, 0.985, 0.9925, 0.9925; 0.8), (0.93, 0.98, 1.0, 1.0; 1.0)]$	Very good (VG)
$[(1.0, 1.0, 1.0, 1.0; 1.0), (1.0, 1.0, 1.0, 1.0; 1.0)]$	Absolutely good (AG)

The ranking process with the help of the GITrF-TOPSIS method using GITrFNs has the following procedural steps:

5.1. Step 1. Experts' Alternatives and Criteria Sets

For an MCGDM problem $\tilde{P} = [\tilde{p}_{ij}]_{m\times n}$ to be a fuzzy decision matrix, $E = \{e_1, e_2, \cdots, e_k\}$ is the set of the experts involved in the decision process, $A = \{A_1, A_2, \cdots, A_m\}$ is the set of the considered alternatives and $K = \{k_1, k_2, \cdots, k_n\}$ is the set of the criteria used for evaluating the alternatives.

5.2. Step 2. Aggregation of Group Decision

The aggregated performance of alternative A_i with respect to criterion k_j for an MCGDM problem with k experts is calculated by Equation (12). Aggregated weights of criteria are calculated by Equation (13).

$$\tilde{p}_{ij} = \frac{1}{k}\left[\tilde{p}_{ij}^1 + \tilde{p}_{ij}^2 + \cdots + \tilde{p}_{ij}^k\right] \tag{12}$$

$$\tilde{tw}_j = \frac{1}{k}\left[\tilde{tw}_j^1 + \tilde{tw}_j^2 + \cdots + \tilde{tw}_j^k\right] \tag{13}$$

5.3. Step 3. Normalization Process

Given $\tilde{p}_{ij} = \left[\left(l_{ij}^L, m_{ij}^L, n_{ij}^L, u_{ij}^L; h_{x_{ij}}^L\right), \left(l_{ij}^U, m_{ij}^U, n_{ij}^U, u_{ij}^U; h_{x_{ij}}^U\right)\right]$ the normalized performance rating can be calculated by Equations (14) or (15) depending on the type of criteria.

$$\tilde{n}_{ij} = \left[\left(\frac{l_{ij}^L}{u_j^+}, \frac{m_{ij}^L}{u_j^+}, \frac{n_{ij}^L}{u_j^+}, \frac{u_{ij}^L}{u_j^+}; h_{x_{ij}}^L\right), \left(\frac{l_{ij}^U}{u_j^+}, \frac{m_{ij}^U}{u_j^+}, \frac{n_{ij}^U}{u_j^+}, \frac{u_{ij}^U}{u_j^+}; h_{x_{ij}}^U\right)\right], i = 1, 2, \cdots, n, j \in \mathrm{B} \tag{14}$$

where $u_j^+ = \max_i u_{ij}^U, j \in \mathrm{B}$.

$$\tilde{n}_{ij} = \left[\left(\frac{l_j^-}{u_{ij}^L}, \frac{l_j^-}{n_{ij}^L}, \frac{l_j^-}{m_{ij}^L}, \frac{l_j^-}{l_{ij}^L}; h_{x_{ij}}^L\right), \left(\frac{l_j^-}{u_{ij}^U}, \frac{l_j^-}{n_{ij}^U}, \frac{l_j^-}{m_{ij}^U}, \frac{l_j^-}{l_{uij}^U}; h_{x_{ij}}^U\right)\right], i = 1, 2, \cdots, n, j \in \mathrm{K} \tag{15}$$

where $l_j^- = \min_i l_{ij}^U, j \in \mathrm{K}$, where B and K are associated with benefit and cost criteria, respectively. This normalization method is used to preserve the property that the ranges of normalized interval numbers fall within the interval [0, 1]. The weighted normalized matrix $\tilde{V}$ can be constructed as follows: $\tilde{R} = [\tilde{r}_{ij}]_{n\times m}$; where $\tilde{r}_{ij} = \tilde{tw}_j \times \tilde{n}_{ij}$.

5.4. Step 4. GITrF-PIS and GITrF-NIS

Let B be a collection of benefit criteria (i.e., the larger k_j is, the greater the preference) and K be a collection of cost criteria (i.e., the smaller k_j is, the greater the preference).

The generalized interval-valued trapezoidal fuzzy positive-ideal solution (GITrF-PIS), denoted as $\tilde{A}^+ = (\tilde{V}_1^+, \tilde{V}_2^+, \cdots, \tilde{V}_n^+)$, and the generalized interval-valued trapezoidal fuzzy negative-ideal solution (GITrF-NIS), denoted as $\tilde{A}^- = (\tilde{V}_1^-, \tilde{V}_2^-, \cdots, \tilde{V}_n^-)$, are defined by Equations (16) and (17), respectively.

$$\begin{aligned} A^+ = \Big[&((\max_i v^L_{1ij}, \max_i v^L_{2ij}, \max_i v^L_{3ij}, \max_i v^L_{4ij}; \max_i h^L v_{ij}) | j \in \mathrm{B}, (\min_i v^L_{1ij}, \min_i v^L_{2ij}, \min_i v^L_{3ij}, \min_i v^L_{4ij}; \min_i h^L v_{ij}) | j \in \mathrm{K}), \\ &((\max_i v^U_{1ij}, \max_i v^U_{2ij}, \max_i v^U_{3ij}, \max_i v^U_{4ij}; \max_i h^U v_{ij}) | j \in \mathrm{B}, (\min_i v^U_{1ij}, \min_i v^U_{2ij}, \min_i v^U_{3ij}, \min_i v^U_{4ij}; \min_i h^U v^{ij}) | j \in \mathrm{K}) \Big], \\ & i = 1, 2, \cdots, m. \end{aligned} \tag{16}$$

$$\tilde{A}^+ = (\tilde{V}_1^+, \tilde{V}_2^+, \cdots, \tilde{V}_n^+).$$

where $\tilde{V}_j^+ = \left[(v_{1j}^{L+}, v_{2j}^{L+}, v_{3j}^{L+}, v_{4j}^{L+}; h_{vj}^{L+}), (v_{1j}^{U+}, v_{2j}^{U+}, v_{3j}^{U+}, v_{4j}^{U+}; h_{vj}^{U+}) \right]$, $j = 1, 2, \cdots, n$.

$$\begin{aligned} A^- = \Big[&((\min_i v^L_{1ij}, \min_i v^L_{2ij}, \min_i v^L_{3ij}, \min_i v^L_{4ij}; \min_i h^L v_{ij}) | j \in \mathrm{B}, (\max_i v^L_{1ij}, \max_i v^L_{2ij}, \max_i v^L_{3ij}, \max_i v^L_{4ij}; \max_i h^L v_{ij}) | j \in \mathrm{K}), \\ &((\min_i v^U_{1ij}, \min_i v^U_{2ij}, \min_i v^U_{3ij}, \min_i v^U_{4ij}; \min_i h^U v_{ij}) | j \in \mathrm{B}, (\max_i v^U_{1ij}, \max_i v^U_{2ij}, \max_i v^U_{3ij}, \max_i v^U_{4ij}; \max_i h^U v^{ij}) | j \in \mathrm{K}) \Big], \\ & i = 1, 2, \cdots, m. \end{aligned} \tag{17}$$

$$\tilde{A}^- = (\tilde{V}_1^-, \tilde{V}_2^-, \cdots, \tilde{V}_n^-).$$

where $\tilde{V}_j^- = \left[(v_{1j}^{L-}, v_{2j}^{L-}, v_{3j}^{L-}, v_{4j}^{L-}; h_{vj}^{L-}), (v_{1j}^{U-}, v_{2j}^{U-}, v_{3j}^{U-}, v_{4j}^{U-}; h_{vj}^{U-}) \right]$, $j = 1, 2, ..., n$.

5.5. Step 5. Ideal and Anti-Ideal Matrices

The ideal separation matrix is denoted by Equation (18) and the anti-ideal separation matrix is denoted by Equation (19).

$$D^+ = \begin{bmatrix} d_T(\tilde{r}_{11}, \tilde{V}_1^+) + d_T(\tilde{r}_{12}, \tilde{V}_2^+) + \cdots + d_T(\tilde{r}_{1n}, \tilde{V}_n^+) \\ d_T(\tilde{r}_{21}, \tilde{V}_1^+) + d_T(\tilde{r}_{22}, \tilde{V}_2^+) + \cdots + d_T(\tilde{r}_{2n}, \tilde{V}_n^+) \\ \vdots \quad\quad \vdots \quad\quad \ddots \quad\quad \vdots \\ d_T(\tilde{r}_{m1}, \tilde{V}_1^+) + d_T(\tilde{r}_{m2}, \tilde{V}_2^+) + \cdots + d_T(\tilde{r}_{mn}, \tilde{V}_n^+) \end{bmatrix} \tag{18}$$

$$D^- = \begin{bmatrix} d_T(\tilde{r}_{11}, \tilde{V}_1^-) + d_T(\tilde{r}_{12}, \tilde{V}_2^-) + \cdots + d_T(\tilde{r}_{1n}, \tilde{V}_n^-) \\ d_T(\tilde{r}_{21}, \tilde{V}_1^-) + d_T(\tilde{r}_{22}, \tilde{V}_2^-) + \cdots + d_T(\tilde{r}_{2n}, \tilde{V}_n^-) \\ \vdots \quad\quad \vdots \quad\quad \ddots \quad\quad \vdots \\ d_T(\tilde{r}_{m1}, \tilde{V}_1^-) + d_T(\tilde{r}_{m2}, \tilde{V}_2^-) + \cdots + d_T(\tilde{r}_{mn}, \tilde{V}_n^-) \end{bmatrix} \tag{19}$$

5.6. Step 6. Relative Closeness

The relative closeness of each alternative to the ideal solution is calculated by Equation (20).

$$RC(A_i) = \frac{D_i^-}{D_i^+ + D_i^-}, \quad i = 1, 2, \cdots, m. \tag{20}$$

where $D_i^- = \sum_{j=1}^{n} d_T\left(\tilde{r}_{ij}, \tilde{V}_j^-\right)$ and $D_i^+ = \sum_{j=1}^{n} d_T\left(\tilde{r}_{ij}, \tilde{V}_j^+\right)$.

5.7. Step 7. Ranking

The rank of alternatives $A_i (i = 1, 2, \cdots, m)$ is achieved by arranging $RC(A_i)$ in decreasing order, the greater the value $RC(A_i)$, the better the alternative A_i.

6. GITrF-VIKOR Method

For the fuzzy MCGDM problems, preferences provided by the decision makers are recorded in fuzzy information. GITrFNs are more applicable and widely used fuzzy numbers for MCGDM problems. The VIKOR method using GITrFNs consists of the following steps. Steps 1, 2, 3, and 4 are the same as in the GITrF-TOPSIS method.

6.1. Step 5. Calculation of S and R

In this step, the values of S_i and R_i $(i = 1, 2, \cdots, m)$ are calculated using Equations (21) and (22), respectively.

$$S_i = \sum_{j=1}^{n} w_j \frac{d_V\left(\tilde{V}_j^+, \tilde{r}_{ij}\right)}{d_V\left(\tilde{V}_j^+, \tilde{V}_j^-\right)} \tag{21}$$

$$R_i = \max_j \left[wj \frac{d_V\left(\tilde{V}_j^+, \tilde{r}_{ij}\right)}{d_V\left(\tilde{V}_j^+, \tilde{V}_j^-\right)} \right] \tag{22}$$

where $w_j = \frac{x_{t\tilde{w}_j} + y_{t\tilde{w}_j}}{\sum\limits_{j}^{n} x_{t\tilde{w}_j} + \sum\limits_{j}^{n} y_{t\tilde{w}_j}}$ and $(x_{t\tilde{w}_j}, y_{t\tilde{w}_j})$ are CoG points of $t\tilde{w}_j$.

6.2. Step 6. Compute Q Value

The values Q_i, $(i = 1, 2, \cdots, m)$ are calculated by Equation (23).

$$Q_i = v \frac{(S_i - S^*)}{(S^- - S^*)} + (1 - v) \frac{(R_i - R^*)}{(R^- - R^*)} \tag{23}$$

Here, $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$ and $R^- = \max_i R_i$, where v is introduced as the weight of the strategy of "the majority" of criteria (or "the maximum group utility"), here $v = 0.5$.

6.3. Step 7. Ranking

In this step, ranking of alternatives is performed by sorting the values S, R and Q, in ascending order, the lower value determines the better alternative.

7. Optimal Robot Selection Process

Assume that four decision makers (DMs) (DM1, DM2, DM3, DM4) of a company are given the task of best robot selection among five robots $(R_1, R_2, R_3, R_4, R_5)$. For this task, DMs consider six criteria, i.e., man–machine interface (C_1), programming flexibility (C_2), vendor's service contact (C_3), purchase cost (C_4), load capacity (C_5) and positioning accuracy (C_6). Among them, C_1, C_2 and C_3 are subjective criteria and C_4, C_5 and C_6 are objective criteria. Decision makers independently selected best and worst criteria by giving their preferences using linguistic variables from Table 1 that were recorded in Table 3 and their weights were calculated using GITrF-BWM (Model 11), and the calculated weights were aggregated using Equation (13) to have final aggregated weights of criteria recorded in Table 4.

The preferences of DMs for alternatives with respect to subjective criteria (C_1, C_2, C_3) were given using Table 2, recorded in Table 5 and aggregated using Equation (12). The ratings of objective criteria (C_4, C_5, C_6) were recorded in Table 6. Normalized ratings of robots with respect to criteria were calculated using Equations (14) and (15) and were recorded in Table 7. Weighted normalized ratings were calculated using aggregated weights of criteria from Table 4 and using normalized ratings of robots with respect to criteria in Table 7 and were recorded in Table 8. GITrF-PIS and GITrF-NIS were calculated using Equations (16) and (17) and were recorded in Table 9; here C_1, C_2, C_3 and C_5 are benefit criteria and C_4 and C_6 are non-benefit criteria.

Table 3. Decision makers' preferences for weight calculations.

DM1	$\tilde{p}_{21}$ WI	$\tilde{p}_{23}$ FI	$\tilde{p}_{24}$ AI	$\tilde{p}_{25}$ FI	$\tilde{p}_{26}$ WI	$\tilde{p}_{14}$ VI	$\tilde{p}_{34}$ FI	$\tilde{p}_{54}$ VI	$\tilde{p}_{64}$ VI
DM2	$\tilde{p}_{51}$ WI	$\tilde{p}_{52}$ FI	$\tilde{p}_{53}$ AI	$\tilde{p}_{54}$ WI	$\tilde{p}_{56}$ FI	$\tilde{p}_{13}$ VI	$\tilde{p}_{23}$ FI	$\tilde{p}_{43}$ VI	$\tilde{p}_{63}$ VI
DM3	$\tilde{p}_{12}$ WI	$\tilde{p}_{13}$ WI	$\tilde{p}_{14}$ FI	$\tilde{p}_{15}$ FI	$\tilde{p}_{16}$ AI	$\tilde{p}_{26}$ VI	$\tilde{p}_{36}$ FI	$\tilde{p}_{46}$ FI	$\tilde{p}_{56}$ VI
DM4	$\tilde{p}_{51}$ AI	$\tilde{p}_{52}$ FI	$\tilde{p}_{53}$ VI	$\tilde{p}_{54}$ FI	$\tilde{p}_{56}$ WI	$\tilde{p}_{21}$ VI	$\tilde{p}_{31}$ WI	$\tilde{p}_{41}$ FI	$\tilde{p}_{61}$ VI

Table 4. Weights of criteria and aggregated weights.

DMs	Weights	GITrFNs
DM1	w_1	$[(0.1997, 0.2109, 0.2204, 0.2417; 0.8), (0.1955, 0.2019, 0.2332, 0.2514; 1)]$
	w_2	$[(0.2434, 0.2578, 0.2702, 0.2950; 0.8), (0.2308, 0.2462, 0.2857, 0.3064; 1)]$
	w_3	$[(0.1410, 0.1472, 0.1569, 0.1769; 0.8), (0.1337, 0.1399, 0.1684, 0.1855; 1)]$
	w_4	$[(0.0673, 0.0689, 0.0664, 0.0779; 0.8), (0.0657, 0.0669, 0.0789, 0.0813; 1)]$
	w_5	$[(0.1709, 0.1915, 0.2064, 0.2394; 0.8), (0.1583, 0.1782, 0.2241, 0.2561; 1)]$
	w_6	$[(0.2008, 0.2107, 0.2205, 0.2419; 0.8), (0.1899, 0.2015, 0.2332, 0.2471; 1)]$
DM2	w_1	$[(0.1994, 0.2105, 0.2199, 0.2410; 0.8), (0.1895, 0.2014, 0.2327, 0.2508; 1)]$
	w_2	$[(0.1407, 0.1425, 0.1565, 0.1770; 0.8), (0.1344, 0.1118, 0.1679, 0.1848; 1)]$
	w_3	$[(0.0664, 0.0689, 0.0700, 0.0776; 0.8), (0.0649, 0.0670, 0.0742, 0.0809; 1)]$
	w_4	$[(0.1994, 0.2105, 0.2198, 0.2368; 0.8), (0.1903, 0.2012, 0.2343, 0.2508; 1)]$
	w_5	$[(0.2423, 0.2579, 0.2701, 0.2946; 0.8), (0.2307, 0.2462, 0.2858, 0.3060; 1)]$
	w_6	$[(0.1314, 0.1902, 0.2058, 0.2387; 0.8), (0.1577, 0.1778, 0.2241, 0.2491; 1)]$
DM3	w_1	$[(0.2595, 0.2745, 0.2910, 0.3306; 0.8), (0.2422, 0.2593, 0.3161, 0.3475; 1)]$
	w_2	$[(0.2496, 0.2646, 0.2833, 0.3282; 0.8), (0.2297, 0.2476, 0.3126, 0.3444; 1)]$
	w_3	$[(0.2024, 0.2101, 0.2222, 0.2553; 0.8), (0.1905, 0.1996, 0.2433, 0.2695; 1)]$
	w_4	$[(0.1913, 0.2021, 0.2135, 0.2500; 0.8), (0.1761, 0.1952, 0.2354, 0.2660; 1)]$
	w_5	$[(0.2326, 0.2626, 0.2808, 0.3260; 0.8), (0.2067, 0.2385, 0.3099, 0.3437; 1)]$
	w_6	$[(0.0830, 0.0877, 0.0909, 0.1046; 0.8), (0.0806, 0.0841, 0.0969, 0.1111; 1)]$
DM4	w_1	$[(0.0749, 0.0778, 0.0803, 0.0871; 0.8), (0.0738, 0.0758, 0.0850, 0.0973; 1)]$
	w_2	$[(0.1896, 0.2124, 0.2297, 0.2726; 0.8), (0.1746, 0.1970, 0.2511, 0.2857; 1)]$
	w_3	$[(0.1060, 0.1114, 0.1152, 0.1295; 0.8), (0.1021, 0.1069, 0.1225, 0.1369; 1)]$
	w_4	$[(0.1584, 0.1672, 0.1802, 0.2037; 0.8), (0.1541, 0.1590, 0.1932, 0.2129; 1)]$
	w_5	$[(0.2680, 0.2862, 0.3008, 0.3293; 0.8), (0.2556, 0.2722, 0.3192, 0.3492; 1)]$
	w_6	$[(0.2248, 0.2374, 0.2488, 0.2748; 0.8), (0.2124, 0.2253, 0.2691, 0.2864; 1)]$
Aggregate	w_1	$[(0.1834, 0.1934, 0.2029, 0.2251; 0.8), (0.1753, 0.1846, 0.2168, 0.2368; 1)]$
	w_2	$[(0.2058, 0.2193, 0.2349, 0.2682; 0.8), (0.1924, 0.2007, 0.2543, 0.2803; 1)]$
	w_3	$[(0.1290, 0.1344, 0.1411, 0.1598; 0.8), (0.1228, 0.1284, 0.1521, 0.1682; 1)]$
	w_4	$[(0.1541, 0.1622, 0.1700, 0.1921; 0.8), (0.1466, 0.1556, 0.1855, 0.2028; 1)]$
	w_5	$[(0.2285, 0.2496, 0.2645, 0.2973; 0.8), (0.2128, 0.2338, 0.2848, 0.3138; 1)]$
	w_6	$[(0.1600, 0.1815, 0.1915, 0.2150; 0.8), (0.1602, 0.1722, 0.2058, 0.2234; 1)]$

Table 5. Rating of robots under subjective criteria expressed in linguistic variables.

Criteria	Rotots	Decision Makers			
		D_1	D_2	D_3	D_4
C_1	R_1	M	M	G	VG
	R_2	M	G	M	G
	R_3	G	M	VG	MG
	R_4	VG	VG	MG	P
	R_5	MG	MG	G	G
C_2	R_1	G	P	G	MG
	R_2	VG	G	VG	G
	R_3	G	M	VG	VG
	R_4	P	MG	G	P
	R_5	MG	VG	MG	G
C_3	R_1	M	M	G	G
	R_2	G	M	VG	MG
	R_3	G	G	G	VG
	R_4	VG	VG	MG	G
	R_5	MP	MG	P	MP

Table 6. Ratings of robots with respect to objective criteria.

Criteria	Rotots	Rating of Criteria
C_4	R_1	$[(72.25, 72.50, 72.70, 73.00; 0.8), (72.00, 72.25, 73.50, 74.00; 1)]$
	R_2	$[(69.00, 69.50, 70.00, 71.00; 0.8), (68.50, 68.80, 71.50, 72.00; 1)]$
	R_3	$[(67.50, 68.00, 69.00, 69.50; 0.8), (67.00, 67.30, 70.00, 70.30; 1)]$
	R_4	$[(70.00, 70.25, 70.65, 71.00; 0.8), (69.75, 70.00, 71.50, 72.00; 1]$
	R_5	$[(68.50, 69.00, 70.00, 70.75; 0.8), (68.00, 68.50, 71.25, 72.25; 1)]$
C_5	R_1	$[(48.50, 49.00, 50.00, 50.50; 0.8), (48.00, 48.25, 51.00, 52.00; 1)]$
	R_2	$[(44.00, 44.50, 45.00, 45.50; 0.8), (43.50, 43.80, 46.00, 46.50; 1)]$
	R_3	$[(43.50, 44.00, 45.00, 45.50; 0.8), (43.00, 43.30, 47.00, 47.50; 1)]$
	R_4	$[(45.00, 45.50, 46.00, 47.00; 0.8), (44.50, 45.00, 47.50, 48.50; 1)]$
	R_5	$[(47.50, 48.00, 48.50, 49.00; 0.8), (46.50, 47.50, 49.25, 50.50; 1)]$
C_6	R_1	$[(0.115, 0.120, 0.130, 0.135; 0.8), (0.100, 0.110, 0.140, 0.142; 1)]$
	R_2	$[(0.150, 0.157, 0.165, 0.170; 0.8), (0.140, 0.145, 0.180, 0.185; 1)]$
	R_3	$[(0.162, 0.165, 0.170, 0.175; 0.8), (0.155, 0.160, 0.185, 0.190; 1)]$
	R_4	$[(0.130, 0.136, 0.145, 0.151; 0.8), (0.120, 0.125, 0.155, 0.166; 1)]$
	R_5	$[(0.173, 0.178, 0.182, 0.189; 0.8), (0.168, 0.172, 0.190, 0.199; 1)]$

Table 7. Normalized decision matrix.

Criteria	Robots	Normalized Rating Value of Criteria
C_1	R_1	$[(0.6926, 0.7391, 0.8067, 0.8292; 0.8), (0.6257, 0.7049, 0.8415, 0.8934; 1)]$
	R_2	$[(0.6475, 0.6926, 0.7773, 0.8060; 0.8), (0.5683, 0.6503, 0.8197, 0.8852; 1)]$
	R_3	$[(0.7602, 0.7992, 0.8668, 0.8900; 0.8), (0.6967, 0.7650, 0.9016, 0.9508; 1)]$
	R_4	$[(0.7193, 0.7548, 0.7930, 0.8081; 0.8), (0.6776, 0.7350, 0.8142, 0.8443; 1)]$
	R_5	$[(0.7828, 0.8128, 0.8975, 0.9276; 0.8), (0.7104, 0.7705, 0.9399, 1.0000; 1)]$
C_2	R_1	$[(0.5844, 0.6148, 0.6821, 0.7075; 0.8), (0.5228, 0.5812, 0.7157, 0.7690; 1)]$
	R_2	$[(0.8782, 0.9137, 0.9530, 0.9645; 0.8), (0.8376, 0.8934, 0.9746, 1.0000; 1)]$
	R_3	$[(0.7817, 0.8217, 0.8648, 0.8782; 0.8), (0.7360, 0.7995, 0.8883, 0.9188; 1)]$
	R_4	$[(0.4080, 0.4385, 0.4981, 0.5235; 0.8), (0.3503, 0.4086, 0.5279, 0.5812; 1)]$
	R_5	$[(0.7690, 0.7982, 0.8610, 0.8832; 0.8), (0.7132, 0.7665, 0.8934, 0.9365; 1)]$

Table 7. *Cont*.

Criteria	Robots	Normalized Rating Value of Criteria
C_3	R_1	$[(0.6061, 0.6483, 0.7276, 0.7545; 0.8), (0.5320, 0.6087, 0.7673, 0.8286; 1)]$
	R_2	$[(0.7116, 0.7481, 0.8114, 0.8331; 0.8), (0.6522, 0.7161, 0.8440, 0.8900; 1)]$
	R_3	$[(0.8427, 0.8772, 0.9329, 0.9501; 0.8), (0.7903, 0.8491, 0.9616, 1.0000; 1)]$
	R_4	$[(0.8510, 0.8843, 0.9277, 0.9418; 0.8), (0.8082, 0.8619, 0.9514, 0.9795; 1)]$
	R_5	$[(0.4143, 0.4399, 0.5115, 0.5422; 0.8), (0.3504, 0.4041, 0.5473, 0.6061; 1)]$
C_4	R_1	$[(0.9178, 0.9216, 0.9241, 0.9273; 0.8), (0.9054, 0.9116, 0.9273, 0.9306; 1)]$
	R_2	$[(0.9437, 0.9571, 0.9640, 0.9710; 0.8), (0.9306, 0.9371, 0.9738, 0.9781; 1)]$
	R_3	$[(0.9640, 0.9710, 0.9853, 0.9926; 0.8), (0.9531, 0.9571, 0.9955, 1.0000; 1)]$
	R_4	$[(0.9437, 0.9483, 0.9537, 0.9571; 0.8), (0.9306, 0.9371, 0.9571, 0.9606; 1)]$
	R_5	$[(0.9470, 0.9571, 0.9710, 0.9781; 0.8), (0.9273, 0.9404, 0.9781, 0.9853; 1)]$
C_5	R_1	$[(0.9327, 0.9423, 0.9615, 0.9712; 0.8), (0.9231, 0.9279, 0.9808, 1.0000; 1)]$
	R_2	$[(0.8462, 0.8558, 0.8654, 0.8750; 0.8), (0.8365, 0.8423, 0.8846, 0.8942; 1)]$
	R_3	$[(0.8365, 0.8462, 0.8654, 0.8750; 0.8), (0.8269, 0.8327, 0.9038, 0.9135; 1)]$
	R_4	$[(0.8654, 0.8750, 0.8846, 0.9038; 0.8), (0.8558, 0.8654, 0.9135, 0.9327; 1)]$
	R_5	$[(0.9135, 0.9231, 0.9327, 0.9423; 0.8), (0.8942, 0.9135, 0.9471, 0.9712; 1)]$
C_6	R_1	$[(0.7407, 0.7692, 0.8333, 0.8696; 0.8), (0.7042, 0.7143, 0.9091, 1.0000; 1)]$
	R_2	$[(0.5882, 0.6061, 0.6369, 0.6667; 0.8), (0.5405, 0.5556, 0.6897, 0.7143; 1)]$
	R_3	$[(0.5714, 0.5882, 0.6061, 0.6173; 0.8), (0.5263, 0.5405, 0.6250, 0.6452; 1)]$
	R_4	$[(0.6623, 0.6897, 0.7353, 0.7692; 0.8), (0.6024, 0.6452, 0.8000, 0.8333; 1)]$
	R_5	$[(0.5291, 0.5495, 0.5618, 0.5780; 0.8), (0.5025, 0.5263, 0.5814, 0.5952; 1)]$

Table 8. Weighted normalized decision matrix.

Criteria	Robots	Weighted Normalized Rating Value of Criteria
C_1	R_1	$[(0.1270, 0.1430, 0.1637, 0.1867; 0.8), (0.1097, 0.1301, 0.1824, 0.2115; 1)]$
	R_2	$[(0.1187, 0.1340, 0.1577, 0.1814; 0.8), (0.0996, 0.1200, 0.1777, 0.2096; 1)]$
	R_3	$[(0.1394, 0.1546, 0.1759, 0.2003; 0.8), (0.1221, 0.1412, 0.1954, 0.2251; 1)]$
	R_4	$[(0.1319, 0.1460, 0.1609, 0.1819; 0.8), (0.1187, 0.1357, 0.1765, 0.1999; 1)]$
	R_5	$[(0.1435, 0.1572, 0.1821, 0.2088; 0.8), (0.1245, 0.1422, 0.2037, 0.2368; 1)]$
C_2	R_1	$[(0.1203, 0.1349, 0.1602, 0.1897; 0.8), (0.1006, 0.1166, 0.1820, 0.2156; 1)]$
	R_2	$[(0.1807, 0.2004, 0.2239, 0.2587; 0.8), (0.1611, 0.1793, 0.2479, 0.2803; 1)]$
	R_3	$[(0.1609, 0.1802, 0.2032, 0.2355; 0.8), (0.1416, 0.1604, 0.2259, 0.2576; 1)]$
	R_4	$[(0.0840, 0.0962, 0.1170, 0.1404; 0.8), (0.0674, 0.0820, 0.1343, 0.1629; 1)]$
	R_5	$[(0.1583, 0.1751, 0.2023, 0.2369; 0.8), (0.1372, 0.1538, 0.2272, 0.2625; 1)]$
C_3	R_1	$[(0.0782, 0.0871, 0.1026, 0.1206; 0.8), (0.0653, 0.0781, 0.1167, 0.1394; 1)]$
	R_2	$[(0.0918, 0.1005, 0.1145, 0.1332; 0.8), (0.0801, 0.0919, 0.1284, 0.1497; 1)]$
	R_3	$[(0.1087, 0.1179, 0.1316, 0.1519; 0.8), (0.0970, 0.1090, 0.1463, 0.1682; 1)]$
	R_4	$[(0.1097, 0.1188, 0.1309, 0.1505; 0.8), (0.0992, 0.1106, 0.1447, 0.1648; 1)]$
	R_5	$[(0.0534, 0.0591, 0.0722, 0.0867; 0.8), (0.0430, 0.0519, 0.0832, 0.1020; 1)]$
C_4	R_1	$[(0.1414, 0.1495, 0.1571, 0.1781; 0.8), (0.1327, 0.1418, 0.1720, 0.1887; 1)]$
	R_2	$[(0.1454, 0.1552, 0.1639, 0.1865; 0.8), (0.1364, 0.1458, 0.1806, 0.1983; 1)]$
	R_3	$[(0.1486, 0.1575, 0.1675, 0.1907; 0.8), (0.1397, 0.1489, 0.1846, 0.2028; 1)]$
	R_4	$[(0.1454, 0.1538, 0.1621, 0.1839; 0.8), (0.1364, 0.1458, 0.1775, 0.1948; 1)]$
	R_5	$[(0.1459, 0.1552, 0.1650, 0.1879; 0.8), (0.1359, 0.1463, 0.1814, 0.1998; 1)]$
C_5	R_1	$[(0.2131, 0.2352, 0.2544, 0.2887; 0.8), (0.1965, 0.2169, 0.2793, 0.3138; 1)]$
	R_2	$[(0.1933, 0.2136, 0.2289, 0.2602; 0.8), (0.1780, 0.1969, 0.2519, 0.2806; 1)]$
	R_3	$[(0.1911, 0.2112, 0.2289, 0.2602; 0.8), (0.1760, 0.1947, 0.2574, 0.2866; 1)]$
	R_4	$[(0.1977, 0.2184, 0.2340, 0.2687; 0.8), (0.1821, 0.2023, 0.2601, 0.2926; 1)]$
	R_5	$[(0.2087, 0.2304, 0.2467, 0.2802; 0.8), (0.1903, 0.2135, 0.2697, 0.3047; 1)]$

Table 8. *Cont.*

Criteria	Robots	Weighted Normalized Rating Value of Criteria
C_6	R_1	$[(0.1185, 0.1396, 0.1596, 0.1870; 0.8), (0.1128, 0.1230, 0.1871, 0.2234; 1)]$
	R_2	$[(0.0941, 0.1100, 0.1220, 0.1433; 0.8), (0.0866, 0.0957, 0.1419, 0.1596; 1)]$
	R_3	$[(0.0914, 0.1068, 0.1161, 0.1327; 0.8), (0.0843, 0.0931, 0.1286, 0.1441; 1)]$
	R_4	$[(0.1060, 0.1252, 0.1408, 0.1654; 0.8), (0.0965, 0.1111, 0.1647, 0.1862; 1)]$
	R_5	$[(0.0847, 0.0997, 0.1076, 0.1243; 0.8), (0.0805, 0.0906, 0.1197, 0.1330; 1)]$

Table 9. GITrF-PIS and GITrF-NIS for criteria.

Criteria	GITrF-PIS/GITrF-NIS	Normalized Rating Value of Criteria
C_1	GITrF-PIS	$[(0.1435, 0.1572, 0.1821, 0.2088; 0.8), (0.1245, 0.1422, 0.2037, 0.2368; 1)]$
	GITrF-NIS	$[(0.1187, 0.1340, 0.1577, 0.1814; 0.8), (0.0996, 0.1200, 0.1765, 0.1999; 1)]$
C_2	GITrF-PIS	$[(0.1807, 0.2004, 0.2239, 0.2587; 0.8), (0.1611, 0.1793, 0.2479, 0.2803; 1)]$
	GITrF-NIS	$[(0.0840, 0.0962, 0.1170, 0.1404; 0.8), (0.0674, 0.0820, 0.1343, 0.1629; 1)]$
C_3	GITrF-PIS	$[(0.1097, 0.1188, 0.1316, 0.1519; 0.8), (0.0992, 0.1106, 0.1463, 0.1682; 1)]$
	GITrF-NIS	$[(0.0534, 0.0591, 0.0722, 0.0867; 0.8), (0.0430, 0.0519, 0.0832, 0.1020; 1)]$
C_4	GITrF-PIS	$[(0.1414, 0.1495, 0.1571, 0.1781; 0.8), (0.1327, 0.1418, 0.1720, 0.1887; 1)]$
	GITrF-NIS	$[(0.1486, 0.1575, 0.1675, 0.1907; 0.8), (0.1397, 0.1489, 0.1846, 0.2028; 1)]$
C_5	GITrF-PIS	$[(0.2131, 0.2352, 0.2544, 0.2887; 0.8), (0.1965, 0.2169, 0.2793, 0.3138; 1)]$
	GITrF-NIS	$[(0.1911, 0.2112, 0.2289, 0.2602; 0.8), (0.1760, 0.1947, 0.2519, 0.2806; 1)]$
C_6	GITrF-PIS	$[(0.0847, 0.0997, 0.1076, 0.1243; 0.8), (0.0805, 0.0906, 0.1197, 0.1330; 1)]$
	GITrF-NIS	$[(0.1185, 0.1396, 0.1596, 0.1870; 0.8), (0.1128, 0.1230, 0.1871, 0.2234; 1)]$

7.1. GITrF-TOPSIS Results

The ideal separation matrix (D^+), anti ideal separation matrix (D^-), relative closeness (RC) and robot ranking are presented in Table 10. Moreover, the ranking order of individual DMs and the aggregated ranking order of the GITrF-TOPSIS method are recorded in Table 11 and pictured in Figure 2.

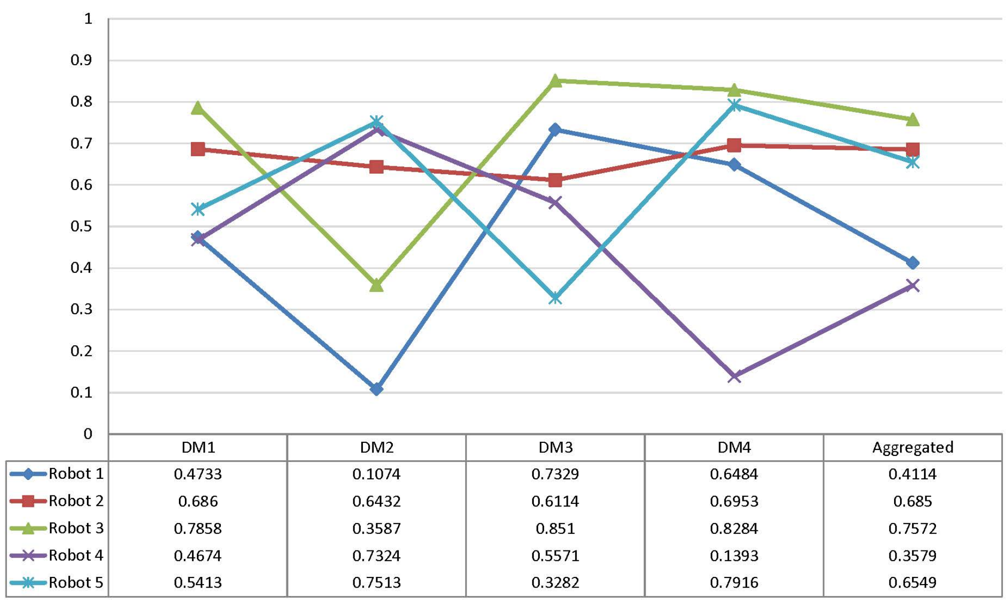


Figure 2. Decision makers' ranking and aggregated ranking of robots using the GITrF-TOPSIS method.

Table 10. D^+, D^-, RC and ranking of the GITrF-TOPSIS method.

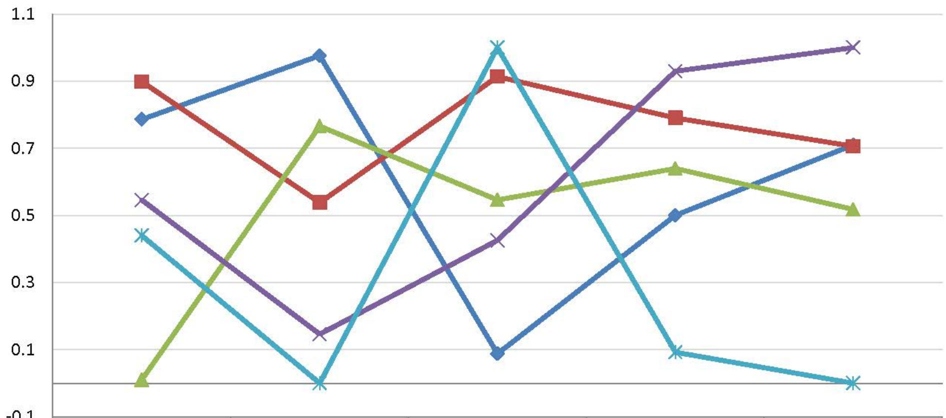
D^+	D^-	RC	Ranking
0.1485	0.1038	0.4114	4
0.0795	0.1728	0.6850	2
0.0613	0.1911	0.7572	1
0.1620	0.0903	0.3579	5
0.0871	0.1653	0.6549	3

Table 11. Ranking results of the GITrF-TOPSIS method.

	GITrF-TOPSIS Ranking
DM1	$R_3 \succ R_2 \succ R_5 \succ R_1 \succ R_4$
DM2	$R_5 \succ R_4 \succ R_2 \succ R_3 \succ R_1$
DM3	$R_3 \succ R_1 \succ R_2 \succ R_4 \succ R_5$
DM4	$R_3 \succ R_5 \succ R_2 \succ R_1 \succ R_4$
Aggregated	$R_3 \succ R_2 \succ R_5 \succ R_1 \succ R_4$

7.2. GITrF-VIKOR Results

S_i, R_i and Q_i were calculated using Equations (21)–(23) and were recorded in Table 12. Moreover, the ranking order of individual DMs and the aggregated ranking order of the GITrF-VIKOR method were recorded in Table 13 and pictured in Figure 3.



	DM1	DM2	DM3	DM4	Aggregate
Robot 1	0.7858	0.9758	0.0879	0.5	0.7099
Robot 2	0.8984	0.538	0.9135	0.79	0.7058
Robot 3	0.01	0.7655	0.5459	0.6395	0.5176
Robot 4	0.5456	0.146	0.4254	0.9297	1
Robot 5	0.44	0	0.9998	0.0922	0

Figure 3. Decision makers' ranking and aggregated ranking of robots using the GITrF-VIKOR method.

Table 12. S_i, R_i, Q_i and ranking of the GITrF-VIKOR method.

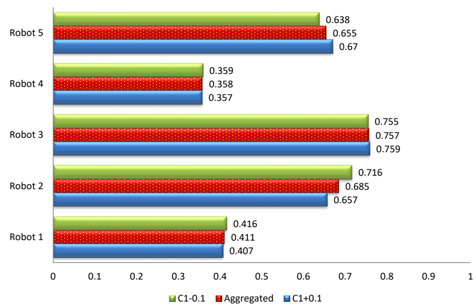
S_i	R_i	Q_i	Ranking
0.4760	0.1749	0.7099	4
0.5301	0.1693	0.7058	3
0.4381	0.1689	0.5176	2
0.6130	0.1761	1	5
0.3578	0.1505	0	1

Table 13. Ranking results of the GITrF-VIKOR method.

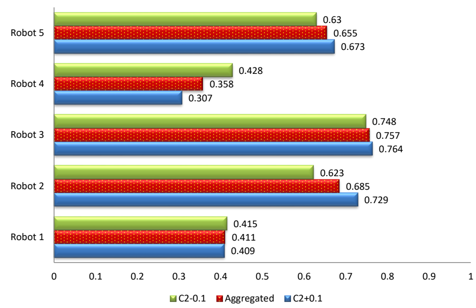
	GITrF-VIKOR Ranking
DM1	$R_3 \succ R_5 \succ R_4 \succ R_1 \succ R_2$
DM2	$R_5 \succ R_4 \succ R_2 \succ R_3 \succ R_1$
DM3	$R_1 \succ R_4 \succ R_3 \succ R_2 \succ R_5$
DM4	$R_5 \succ R_1 \succ R_3 \succ R_2 \succ R_4$
Aggregated	$R_5 \succ R_3 \succ R_2 \succ R_1 \succ R_4$

8. Sensitivity Analysis

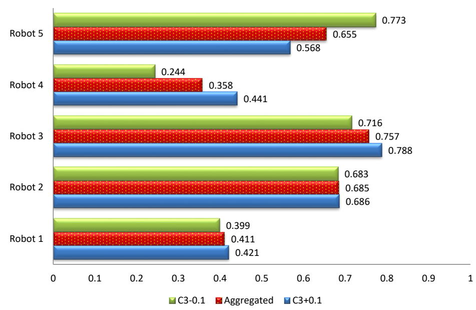
Sensitivity analysis was performed to test how the ranking order of an MCDM method is sensitive to the variation of the weights of criteria. Sensitivity analysis was performed first by adding 0.1 to each criterion and then subtracting 0.1 from each criterion separately and adjusting other criteria accordingly to test the results with little variation in the weights and see the rank reversal of the two methods, i.e., GITrF-TOPSIS and GITrF-VIKOR. The six cases of sensitivity analysis of the GITrF-TOPSIS method are shown in Figure 4a–f, where the higher value determines the better alternative. Here, the deviations of values from the aggregated values are noted. The results show that GITrF-TOPSIS was more sensitive with respect to criteria c_2 and c_3 as shown in Figures 5 and 6. The six cases of sensitivity analysis of the GITrF-VIKOR method are shown in Figure 7a–f, where the lower value determines the better alternative. Here, the deviations of values from the aggregated values are noted. The results show that GITrF-VIKOR was more sensitive with respect to criteria c_2, c_3, c_5, and c_6 as shown in Figures 8 and 9. It can be noted that the GITrF-VIKOR method provided more rank reversal behavior for small variations in criteria values, whereas the GITrF-TOPSIS method had less rank reversal in this scenario. The higher sensitivity and additional rank reversal behavior of the GITrF-VIKOR method is due to the distance formula (Equation (5)) used for the method. Thus, the overall sensitivity results show that the GITrF-TOPSIS method is more stable with respect to criteria as compared to the GITrF-VIKOR method.



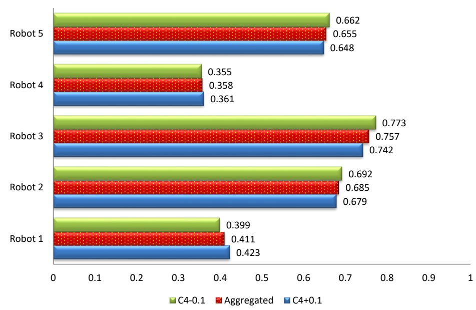
(**a**) Sensitivity with respect to C_1



(**b**) Sensitivity with respect to C_2

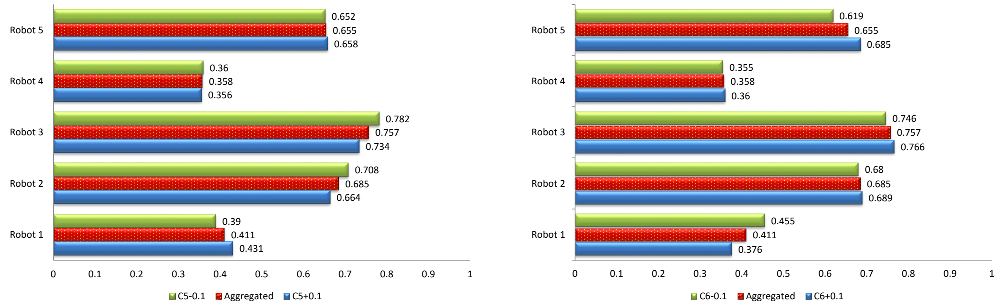


(**c**) Sensitivity with respect to C_3



(**d**) Sensitivity with respect to C_4

Figure 4. *Cont.*



(**e**) Sensitivity with respect to C_5 (**f**) Sensitivity with respect to C_6

Figure 4. Sensitivity analysis with respect to criteria for the GITrF-TOPSIS method.

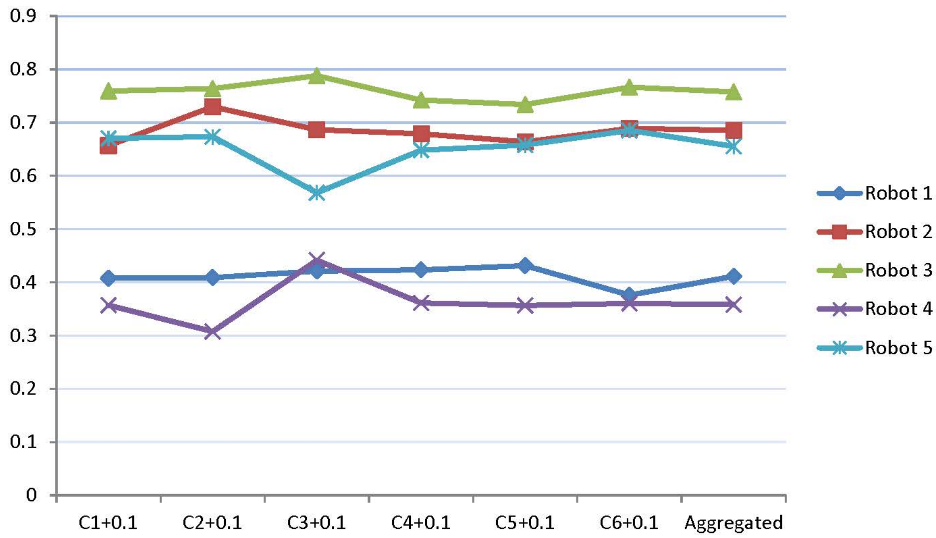


Figure 5. Sensitivity analysis of the GITrF-TOPSIS method by increased weights.

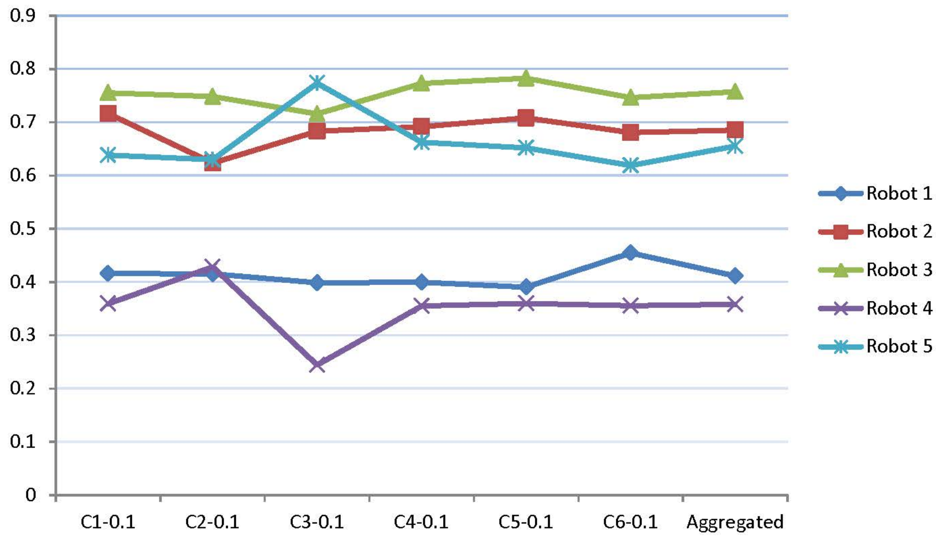
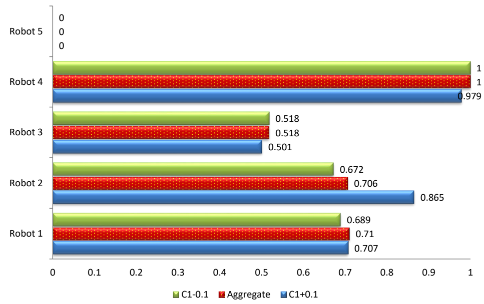
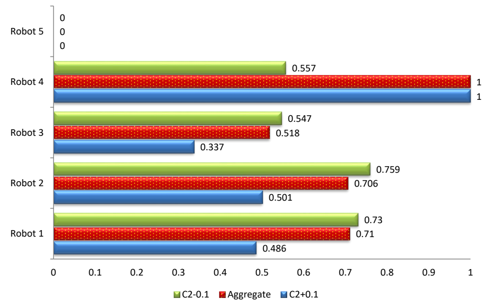


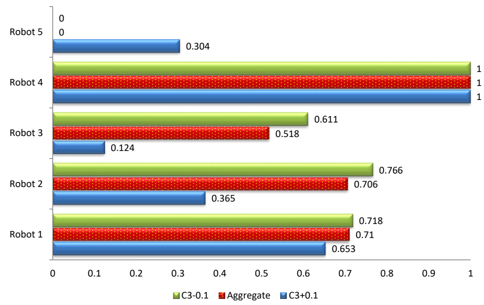
Figure 6. Sensitivity analysis of the GITrF-TOPSIS method by decreased weights.



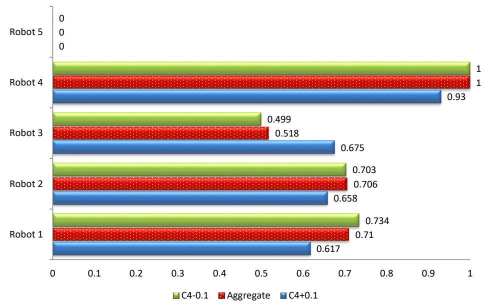
(**a**) Sensitivity with respect to C_1



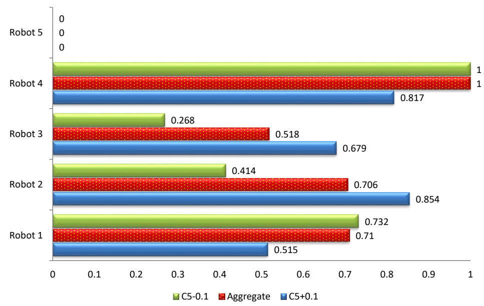
(**b**) Sensitivity with respect to C_2



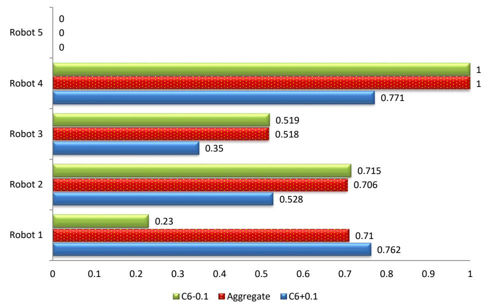
(**c**) Sensitivity with respect to C_3



(**d**) Sensitivity with respect to C_4



(**e**) Sensitivity with respect to C_5



(**f**) Sensitivity with respect to C_6

Figure 7. Sensitivity analysis with respect to criteria for the GITrF-VIKOR method.

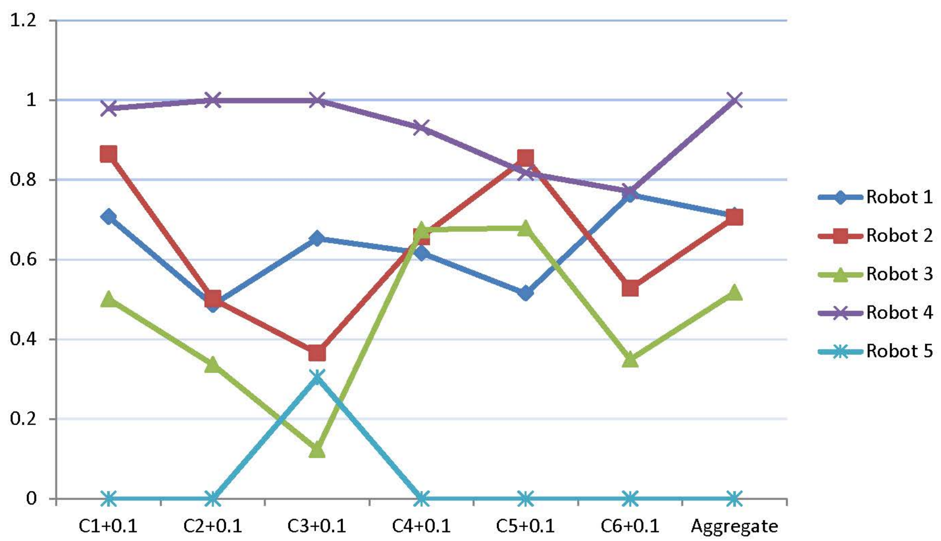


Figure 8. Sensitivity analysis of the GITrF-VIKOR method by increased weights.

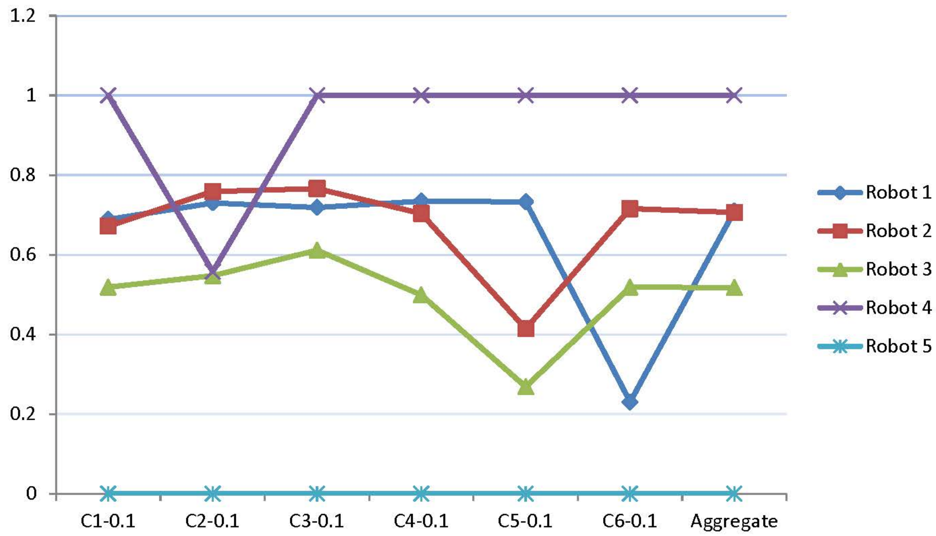


Figure 9. Sensitivity analysis of the GITrF-VIKOR method by decreased weights.

9. Discussion

Industrial robot selection is a very difficult task in this competitive marketplace. For this selection process, deciding weights for criteria is an important factor to consider. GITrFBWM is a robust and more consistent MCDM method that assigns weights to the criteria based on preferences provided by the decision makers by utilizing their knowledge structures, experience, and expertise. For ranking of robots, the GITrF-TOPSIS and GITrF-VIKOR methods were used, and their results were compared. This is a group decision-making process that provides slightly different results for both methods. The MCDM/MCGDM method that provides less rank reversal for a small variation in criteria weights is called a stable method. Sensitivity analysis showed that the GITrF-TOPSIS method is more stable and reliable for this selection process. The proposed hybrid methodologies GITrFBWM-GITrF-TOPSIS and GITrFBWM-GITrF-VIKOR are general methodologies that are not only useful for this proposed problem but can be applied to many similar types of selection processes.

10. Conclusions

The classical methods almost fail to convey the vagueness and imprecision of the linguistic assessment. Linguistic terms are used to assist experts in providing their opinions and then convert these terms to GITrFNs. Robot selection MCGDM problems were discussed with regards to two hybrid methodologies: (1) GITrF-BWM with GITrF-TOPSIS and (2) GITrF-BWM with GITrF-VIKOR. GITrFWs are derived using GITrF-BWM, and the ranking of the robots was performed using these weights along with the GITrF-TOPSIS and GITrF-VIKOR methods separately. There is uncertainty and vagueness in real problems as human thinking is fuzzy, and thus such situations are modeled and handled using fuzzy set theory. In this research, GITrFNs were used to obtain more convincing and reliable results, as GITrFNs cover a wide area of uncertainty and applications. GITrF-BWM gives more consistent and reliable weights of criteria due to a lower consistency ratio that is a vital part of any MCDM problem. The ranking results for each decision maker and aggregated ranking results were presented for each methodology. Sensitivity analysis was performed to see the most critical weights for this problem. More sensitive criteria need more attention as compared to less sensitive criteria; the results show that the GITrF-TOPSIS method is more stable with respect to the criteria as compared to the GITrF-VIKOR method. Direct opinion-based decisions can be biased, and thus hybrid MCDM/MCGDM methodologies are needed. The proposed hybrid methodologies remedy biasess in decision processes. This research also classifies the sensitivity behavior of the proposed methods. These methodologies are general and can be applied to any criteria-based selection problem, and especially to solve social, economic, waste management, and engineering problems.

The proposed hybrid MCGDM methodologies can be applied to different managerial and engineering applications, for example hospital site selection, recruitment selection, project selection, etc. In the future, We will integrate GITrFBWM with other fuzzy and classical MCDM/MCGDM methods such as fuzzy EDAS, fuzzy CODAS, etc., and will conduct a comparative study and analysis for different social, economic, waste management, and engineering applications.

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Abbreviations

The following abbreviations are used in this manuscript:

BWM	Best-worst method
COPRAS-G	COmplex PRoportional ASsessment of alternatives with Grey relations
CoG	Center of gravity
DM	Decision maker
ELECTRE	ELimination Et Choice Translating REality
FAHP	Analytic hierarchy process
GITrFNs	Generalized interval-valued trapezoidal fuzzy numbers
GITrFWs	Generalized interval-valued trapezoidal fuzzy weights
GITrF-BWM	Generalized interval-valued trapezoidal fuzzy best-worst method
GITrFP	Generalized interval-valued trapezoidal fuzzy preference
GITrF-RC	Generalized interval-valued trapezoidal fuzzy reference comparison
GITrF-PIS	Generalized interval-valued trapezoidal fuzzy positive-ideal solution
GITrF-NIS	Generalized interval-valued trapezoidal fuzzy negative-ideal solution
GMIR	Graded mean integration representation
MCDM	Multi-criteria decision making
MCGDM	Multiple criteria group decision-making
TOPSIS	Technique for Order Preference by Similarity to the Ideal Solution
TPOP	Technique of Precise Order Preferences
VIKOR	VIsekriterijumska optimizacija i KOmpromisno Resenje

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