symmetry

MDPI

Article

M-Parameterized N-Soft Topology-Based TOPSIS Approach for Multi-Attribute Decision Making

Muhammad Riaz [1], Ayesha Razzaq [1], Muhammad Aslam [2] and Dragan Pamucar [3,*]

[1] Department of Mathematics, University of the Punjab, Lahore 54590, Pakistan; mriaz.math@pu.edu.pk (M.R.); ayesharazzaq061@gmail.com (A.R.)
[2] Department of Mathematics, College of Sciences, King Khalid University, Abha 61413, Saudi Arabia; muamin@kku.edu.sa
[3] Department of Logistics, Military Academy, University of Defence in Belgarde, 11000 Belgarde, Serbia
* Correspondence: dragan.pamucar@va.mod.gov.rs

Abstract: In this article, we presented the notion of M-parameterized N-soft set (MPNSS) to assign independent non-binary evaluations to both attributes and alternatives. The MPNSS is useful for making explicit the imprecise data which appears in ranking, rating, and grading positions. The proposed model is superior to existing concepts of soft set (SS), fuzzy soft sets (FSS), and N-soft sets (NSS). The concept of M-parameterized N-soft topology (MPNS topology) is defined on MPNSS by using extended union and restricted intersection of MPNS-power whole subsets. For these objectives, we define basic operations on MPNSSs and discuss various properties of MPNS topology. Additionally, some methods for multi-attribute decision making (MADM) techniques based on MPNSSs and MPNS topology are provided. Furthermore, the TOPSIS (technique for order preference by similarity to an ideal solution) approach under MPNSSs and MPNS topology is established. The symmetry of the optimal decision is illustrated by interesting applications of proposed models and new MADM techniques are demonstrated by certain numerical illustrations and well justified by comparison analysis with some existing techniques.

Keywords: M-parameterized N-soft set; M-parameterized N-soft topology; algorithms; TOPSIS; MADM

Citation: Riaz, M.; Razzaq, A.; Aslam, M.; Pamucar, D. M-Parameterized N-Soft Topology-Based TOPSIS Approach for Multi-Attribute Decision Making. *Symmetry* **2021**, *13*, 748. https://doi.org/10.3390/sym13050748

Academic Editor: José Carlos R. Alcantud

Received: 12 April 2021
Accepted: 23 April 2021
Published: 25 April 2021

1. Introduction

The information in various complex real life problems is generally imprecise, ambiguous, and imperfect. Fuzzy modeling and fuzzy decision making are very helpful to capture these uncertainties. Conventionally, the information about an alternative is considered by the crisp numbers or linguistic numbers. The researchers have introduced various mathematical models to handle such realistic issues. Zadeh [1] innovated fuzzy set theory, rough set theory introduced by Pawlak in (1982) [2] and soft set theory established by Molodtsov [3] are powerful tools towards uncertainties. These theories are independent generalizations of classical sets or crisp sets. The notion of intuitionistic fuzzy set (IFS) innovated by Atanassov [4] is the extension of fuzzy set(FS) and a Pythagorean fuzzy set (PFS) established by Yager [5,6] is the expansion of IFS.

Soft set is the parametric representation of objects of universe that provides the binary evaluation to the objects. Numerous researchers have studied soft set to handle uncertainties. Fatimah et al. [7] invented the idea of N-soft set (NSS) to handle situations when non-binary assessments are expected to demonstrate the objects real importance. Recently, Riaz et al. [8] innovated the concept of N-soft topology (NS-topology) and its applications to MAGDM. Akram et al. [9,10] extended this concept to fuzzy N-soft sets (FNSSs) and hesitant N-soft sets (HNSSs) for MAGDM applications. Akram and Adeel [11] established the TOPSIS method for MAGDM with interval-valued hesitant fuzzy N-soft sets.

Some researchers established various hybrid mathematical structures of soft sets (see [12–18]). Soft topology on soft sets was proposed by Cagman et al. [19], and Shabir and Naz [20]. Riaz and Tehrim [21] introduced bipolar fuzzy soft topology on bipolar fuzzy soft sets and developed an important application in medical diagnosis. Soft set theory and fuzzy set theory have been studied for decision-making and modeling uncertainties in recent decades (see [22–31]). Garg and Arora [32,33] introduced Dual hesitant fuzzy soft aggregation operators and Generalized intuitionistic fuzzy soft power aggregation operator. Pamucar and Jankovic [34] presented an application of the hybrid interval rough weighted Power-Heronian operators. Riaz et al. [35] introduced hesitant fuzzy soft topology and its applications to MADM. Riaz et al. [36,37] introduced soft rough topology and soft multi rough topology with new properties and applications to MADM. The concept of linear Diophantine fuzzy Set (LDFS) introduced by Riaz and Hashmi [38]. Kamaci [39] introduced new algebraic structures of LDFSs.

In N-soft set environments, the ranking, rating. or grading is assigned to alternatives/objects only. Meanwhile, there is a lack of independent non-binary evaluations to the attributes which may effect the decision analysis phenomena. To enhance the significance of attributes there is a need for non-binary grading positions given to the attributes. The main objective of this study is to handle these difficulties with M-parameterized N-soft set (MPNSS) and MPNS topology. The proposed model of MPNSS is very helpful to assign independent non-binary evaluations to both attributes and alternatives. Additionally, this model is useful for developing strong MADM techniques to select most convincible alternative and make a robust optimal decision.

To facilitate our discussion, the classification of the paper is presented as follows: In Section 2, a few basic concepts of soft set, NSS, and FPSS are given. In Section 3, the notion of M-parameterized N-soft set (MPNSS) is introduced. The concepts of empty, universal, bottom weak complements, top weak complements, weak complements, restricted intersection, and extended union of MPNSSs are presented. In Section 4, the construction of MPNS topology is defined on MPNSS by using MPNS-power whole subsets, extended union and restricted intersection of MPNSSs. Several key properties of MPNS topology, as well as their implications, are well identified. In Section 5, MPNS topology- based MADM methods and their corresponding Algorithms 1 and 2 are developed to estimate the losses, formed extensive damage, displaced and affecting several people in the most affected districts in Sindh province, south-east Pakistan, during historical flooding of August 2011. Section 6 develops and illustrates a robust MADM method of TOPSIS with MPNSSs and MPNS topology using a numerical illustration. Finally, in Section 7, we summarize the findings of this research study.

2. Preliminaries

In the section presented, we discuss some rudiments of soft set (SS), N-soft sets (NSS), fuzzy soft set (FSS) and FP soft sets (FPSS) that are helpful in understanding the contributions in rest of the paper.

Definition 1 ([3])**.** *Suppose Λ be the universal set, $\Upsilon \neq \emptyset$ be the class of decision variables or parameters, and $\lambda \widetilde{\sqsubseteq} \Upsilon$. A soft set (SS) defined on Λ is a set of order pairs, denoted by $(\mathfrak{C}, \lambda)$ and can be represented as,*

$$(\mathfrak{C}, \lambda) = \left\{ \langle \rho, \mathfrak{C}(\rho) \rangle : \rho \in \lambda, \mathfrak{C}(\rho) \in 2^{\Lambda} \right\}$$

where $\mathfrak{C} : \lambda \to 2^{\Lambda}$ is a set valued mapping. In short, $(\mathfrak{C}, \lambda)$ can also be denoted as $\mathfrak{C}_{\lambda}$.

Definition 2 ([7])**.** *Let Λ be the universe of discourse, $\Upsilon \neq \hat{\emptyset}$ be the collection of decision variables or parameters. Suppose $\mathfrak{H} = \{0, 1, 2, \cdots, N-1\}$ is the grading set, where $N \in \{2, 3, \cdots\}$. The N-soft set (NSS) over Λ is formalized by $\mathfrak{C}_N = (\mathfrak{C}, \Upsilon, N)$ where $\mathfrak{C} : \Upsilon \to 2^{\Lambda \times \mathfrak{H}}$ in such a manner that for every $\rho \in \Upsilon$ there exist a specific $(\xi, \mathfrak{I}_{\rho}(\xi)) \in \Lambda \times \mathfrak{H}$ for all $\xi \in \Lambda$, $\rho \in \Upsilon$.*

Definition 3 ([7]). *Let $\mathfrak{C}_N$ be NSS defined over Λ. The weak complement of NSS $(\mathfrak{C}_N)$, specified as $\mathfrak{C}_N^{\hat{c}} = (\mathfrak{C}^c, \Upsilon, N)$, where $\mathfrak{C}_N^{\hat{c}}(\rho) \widetilde{\sqcap} \mathfrak{C}_N(\rho) = \hat{\emptyset}$, for each $\rho \in \Upsilon$.*

Definition 4 ([7]). *Let $\mathfrak{C}_N$ be NSS defined over Λ. The top weak complement of NSS $(\mathfrak{C}_N)$ is a NSS, defined by $\mathfrak{C}_N^{\hat{t}}$ where*

$$\mathfrak{C}_N^{\hat{t}}(\rho) = \begin{cases} N-1, & if\ \mathfrak{C}_N(\rho) < N-1 \\ 0, & if\ \mathfrak{C}_N(\rho) = N-1 \end{cases},$$

Definition 5 ([7]). *Let $\mathfrak{C}_N$ be NSS defined over Λ. The bottom weak complement of NSS $(\mathfrak{C}_N)$ is defined by $\mathfrak{C}_N^{\hat{b}}$, where*

$$\mathfrak{C}_N^{\hat{b}}(\rho) = \begin{cases} 0, & if\ \mathfrak{C}_N(\rho) > 0 \\ N-1, & if\ \mathfrak{C}_N(\rho) = 0 \end{cases},$$

Definition 6 ([40]). *Let Λ be the collection of universal elements, 2^Λ is the aggregation of subsets of Λ, Υ is the collection of decision variables and $\mathfrak{K}$ be fuzzy set over Υ. A Fuzzy Parameterized soft set (FPSS), denoted by $\mathfrak{C}_{\mathfrak{K}}$ on the universe Λ is defined as,*

$$\mathfrak{C}_{\mathfrak{K}} = \left\{ \left\langle \frac{\mu_{\mathfrak{K}}(\rho)}{\rho}, \mathfrak{C}_{\mathfrak{K}}(\rho) \right\rangle : \rho \in \Upsilon, \mathfrak{C}_{\mathfrak{K}}(\rho) \in 2^\Lambda, \mu_{\mathfrak{K}}(\rho) \in [0,1] \right\},$$

where $\mathfrak{C}_{\mathfrak{K}} : \Upsilon \to 2^\Lambda$ is a set valued mapping and $\mu_{\mathfrak{K}} : \Upsilon \to [0,1]$ is called membership function.

3. M-Parameterized N-Soft Set (MPNSS)

Fatima et al. [7] presented the idea of NSS as an extension of SS to cope up with situations in which non-binary assessment is required. This section is devoted to the establishment of M-parameterized N-soft set (MPNSS) which is superior than NSS and SS. For $M = 2$ MPNSS becomes NSS and for $M = N = 2$ it reduces to SS.

Definition 7. *Let Λ be the universe, $\lambda \widetilde{\sqsubseteq} \Upsilon$ is a collection of attributes. Consider two different sets for grading or rating $\mathcal{H} = \{0, 1, 2, \cdots N-1\}$ and $\Re = \{0, 1, 2, \cdots M-1\}$, where $M, N \in \{2, 3, \cdots\}$. Then the M-Parameterized N-soft set (MPNSS) over Λ, designated as Λ_N^M or $\Lambda(M, N)$ and defined by*

$$\Lambda_N^M = \left\{ \left\langle \frac{\mathcal{I}_\lambda(\delta_\jmath)}{\delta_\jmath}, \{(\xi_\imath, \mathcal{I}_{\delta_\jmath}(\xi_\imath))\} \right\rangle : \mathcal{I}_{\delta_\jmath}(\xi_\imath) \in \mathcal{H}, \mathcal{I}_\lambda(\delta_\jmath) \in \Re, \delta_\jmath \in \lambda, \xi_\imath \in \Lambda, \imath, \jmath \in \aleph \right\}.$$

Table 1 gives the matrix representation of MPNSS as follows.

Table 1. Matrix representation of Λ_N^M.

Λ_N^M	$\frac{\mathcal{I}_\lambda(\delta_1)}{\delta_1}$	$\frac{\mathcal{I}_\lambda(\delta_2)}{\delta_2}$	$\cdots$	$\frac{\mathcal{I}_\lambda(\delta_\jmath)}{\delta_\jmath}$
ξ_1	$\mathcal{I}_{\delta_1}(\xi_1)$	$\mathcal{I}_{\delta_2}(\xi_1)$	$\cdots$	$\mathcal{I}_{\delta_\jmath}(\xi_1)$
ξ_2	$\mathcal{I}_{\delta_1}(\xi_2)$	$\mathcal{I}_{\delta_2}(\xi_2)$	$\cdots$	$\mathcal{I}_{\delta_\jmath}(\xi_2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\xi_\imath$	$\mathcal{I}_{\delta_1}(\xi_\imath)$	$\mathcal{I}_{\delta_2}(\xi_\imath)$	$\cdots$	$\mathcal{I}_{\delta_\jmath}(\xi_\imath)$

Then MPNSS(Λ) represents the collection of all MPNSSs.

Example 1. *Let $\Lambda = \{\xi_1, \xi_2, \xi_3, \xi_4\}$ be the collection of different cities of Pakistan and $\Upsilon = \{\delta_1, \delta_2, \cdots, \delta_8\}$ is a collection of attributes, where*

$$\begin{aligned}
\delta_1 &= \textit{historical places, public parks, play grounds} \\
\delta_2 &= \textit{safe residential housing schemes} \\
\delta_3 &= \textit{beautiful shopping centers} \\
\delta_4 &= \textit{wonderful weather,} \\
\delta_5 &= \textit{cleanliness, wide roads, foot paths} \\
\delta_6 &= \textit{friendly and cooperative people} \\
\delta_7 &= \textit{high standard educational institutes} \\
\delta_8 &= \textit{rivers, canals, hills}
\end{aligned}$$

The attributes can be evaluated with the scales as follows.

$$\begin{aligned}
\textit{Extremly important} &= \diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit \textit{ means } 5 \\
\textit{Very important} &= \diamondsuit\diamondsuit\diamondsuit\diamondsuit \textit{ means } 4 \\
\textit{Important} &= \diamondsuit\diamondsuit\diamondsuit \textit{ means } 3 \\
\textit{Moderately important} &= \diamondsuit\diamondsuit \textit{ means } 2 \\
\textit{Slightly important} &= \diamondsuit \textit{ means } 1 \\
\textit{Not important} &= \bullet \textit{ means } 0
\end{aligned}$$

For alternatives, the evaluation scales are,

$$\begin{aligned}
\textit{Higly recommended} &= \dagger\dagger\dagger\dagger \textit{ means } 4 \\
\textit{Recommended} &= \dagger\dagger\dagger \textit{ means } 3 \\
\textit{Moderately recommended} &= \dagger\dagger \textit{ means } 2 \\
\textit{Slightly recommended} &= \dagger \textit{ means } 1 \\
\textit{Not recommended} &= \bullet \textit{ means } 0
\end{aligned}$$

According to comprehensive properties of the cities, the public give assessment scores to the evaluation attributes and cities, presented in Table 2 and matrix form of 6P6S-set is presented in Table 3.

Table 2. Evaluation of data provided by public.

$\Upsilon \rightarrow$ $\Lambda \downarrow$	δ_1 ◇◇◇◇	δ_2 ◇◇◇	δ_3 ◇◇	δ_4 ◇◇◇◇	δ_5 ◇◇◇◇◇	δ_6 ◇	δ_7 ◇◇◇◇	δ_8 ◇◇
ξ_1	†††	•	†††	†††††	†††	††††	††	•
ξ_2	††	†††††	††††	•	††	†††††	•	†††
ξ_3	†††††	††††	•	††	†	††	†††	††
ξ_4	†	††	†††††	†	•	†††	††††	†††

Table 3. Tabular representation of corresponding 6P6S (Λ_6^6).

Λ_6^6	$\frac{4}{\delta_1}$	$\frac{3}{\delta_2}$	$\frac{2}{\delta_3}$	$\frac{4}{\delta_4}$	$\frac{5}{\delta_5}$	$\frac{1}{\delta_6}$	$\frac{4}{\delta_7}$	$\frac{2}{\delta_8}$
ξ_1	3	0	3	5	3	4	2	0
ξ_2	2	5	4	0	2	5	0	3
ξ_3	5	4	0	2	1	2	3	2
ξ_4	1	2	5	1	0	3	4	3

$$\begin{aligned}\Lambda_6^6 \ = \ &\Big\{\Big\langle\frac{4}{\delta_1},\{(\xi_1,3),(\xi_2,2),(\xi_3,5),(\xi_4,1)\}\Big\rangle,\Big\langle\frac{3}{\delta_2},\{(\xi_1,0),(\xi_2,5),(\xi_3,4),(\xi_4,2)\}\Big\rangle,\\ &\Big\langle\frac{2}{\delta_3},\{(\xi_1,3),(\xi_2,4),(\xi_3,0),(\xi_4,5)\}\Big\rangle,\Big\langle\frac{4}{\delta_4},\{(\xi_1,5),(\xi_2,0),(\xi_3,2),(\xi_4,1)\}\Big\rangle,\\ &\Big\langle\frac{5}{\delta_5},\{(\xi_1,3),(\xi_2,2),(\xi_3,1),(\xi_4,0)\}\Big\rangle,\Big\langle\frac{1}{\delta_6},\{(\xi_1,4),(\xi_2,5),(\xi_3,2),(\xi_4,3)\}\Big\rangle,\\ &\Big\langle\frac{4}{\delta_7},\{(\xi_1,2),(\xi_2,0),(\xi_3,3),(\xi_4,4)\}\Big\rangle,\Big\langle\frac{1}{\delta_8},\{(\xi_1,0),(\xi_2,3),(\xi_3,2),(\xi_4,3)\}\Big\rangle,\end{aligned}$$

Definition 8. *Let Λ be universe, Υ is the set of attributes and $\lambda\widetilde{\sqsubseteq}\Upsilon$. Then the empty MPNSS, denoted by $\mathfrak{P}_0^0$ or $\mathfrak{P}(0,0)$, is defined as*

$$\mathfrak{P}_0^0 = \Big\{\Big\langle\frac{0}{\delta_\jmath},\ \{(\xi_\imath,0)\}\Big\rangle : \forall\delta_\jmath \in \Upsilon,\ \xi_\imath \in \Lambda,\ \imath,\jmath \in \aleph\Big\},$$

that is, $\mathcal{I}_\lambda(\delta_\jmath) = 0\quad \forall\delta_\jmath \in \Upsilon$ and $\mathcal{I}_{\delta_\jmath}(\xi_\imath) = 0\quad \forall\xi_\imath \in \Lambda$ and $\delta_\jmath \in \Upsilon$.

Definition 9. *Let Λ be universe, Υ is the set of attributes and $\lambda\widetilde{\sqsubseteq}\Upsilon$. Then the universal MPNSS, denoted by $\mathfrak{S}_{N-1}^{M-1}$ or $\mathfrak{S}(M-1,\ N-1)$ and defined as*

$$\mathfrak{S}_{N-1}^{M-1} = \Big\{\Big\langle\frac{M-1}{\delta_\jmath},\ \{(\xi_\imath,\ N-1)\}\Big\rangle :\ \forall\delta_\jmath \in \Upsilon,\ \xi_\imath \in \Lambda,\ \imath,\jmath \in \aleph\Big\},$$

that is, $\mathcal{I}_\lambda(\delta_\jmath) = M-1\quad \forall\delta_\jmath \in \lambda$ and $\mathcal{I}_{\delta_\jmath}(\xi_\imath) = N-1\quad \forall\xi_\imath \in \Lambda,\ \delta_\jmath \in \lambda$.

Definition 10. *Let Λ be a set of universal elements and $\lambda\widetilde{\sqsubseteq}\Upsilon$ is the set of attributes. The weak compliment of MPNSS (Λ_N^M) over Λ, indicated by $(\Lambda_N^M)^{\hat{c}}$ and described as*

$$(\Lambda_N^M)^{\hat{c}} = \Big\{\Big\langle\frac{\mathcal{I}_\lambda^{\hat{c}}(\delta_\jmath)}{\delta_\jmath},\ \{(\xi_\imath,\ \mathcal{I}_{\delta_\jmath}^{\hat{c}}(\xi_\imath))\}\Big\rangle :\ \mathcal{I}_{\delta_\jmath}^{\hat{c}}(\xi_\imath) \in \mathcal{H},\ \mathcal{I}_\lambda^{\hat{c}}(\delta_\jmath) \in \Re\ ,\ \forall\delta_\jmath \in \lambda,\ \xi_\imath \in \Lambda,\ \imath,\jmath \in \aleph\Big\},$$

where

$$\begin{aligned}\mathcal{I}_\lambda^{\hat{c}}(\delta_\jmath)\widetilde{\sqcap}\mathcal{I}_\lambda(\delta_\jmath) &= \hat{\emptyset}\quad \forall\delta_\jmath \in \lambda\\ \mathcal{I}_{\delta_\jmath}^{\hat{c}}(\xi_\imath)\widetilde{\sqcap}\mathcal{I}_{\delta_\jmath}(\xi_\imath) &= \hat{\emptyset}\quad \forall\xi_\imath \in \Lambda\end{aligned}$$

Example 2. *Consider a 6P6S-set(Λ_6^6) as given in Example 1. The weak compliment of Λ_6^6 is given in Table 4.*

Table 4. Tabular representation of $(\Lambda_6^6)^{\hat{c}}$.

$(\Lambda_6^6)^{\hat{c}}$	$\frac{3}{\delta_1}$	$\frac{1}{\delta_2}$	$\frac{5}{\delta_3}$	$\frac{3}{\delta_4}$	$\frac{2}{\delta_5}$	$\frac{4}{\delta_6}$	$\frac{2}{\delta_7}$	$\frac{1}{\delta_8}$
ξ_1	2	1	4	4	5	3	5	4
ξ_2	1	0	3	2	3	4	1	5
ξ_3	3	2	1	3	0	0	2	3
ξ_4	0	3	0	4	2	2	3	1

Definition 11. *Let Λ be universe and $\lambda\widetilde{\sqsubseteq}\Upsilon$ is the collection of attributes. A top weak compliment of MPNSS (Λ_N^M) over Λ, identified by $(\Lambda_N^M)^{\hat{t}}$ and demonstrated as*

$$(\Lambda_N^M)^{\hat{t}} = \Big\{\Big\langle\frac{\mathcal{I}_\lambda^{\hat{t}}(\delta_\jmath)}{\delta_\jmath},\ \{(\xi_\imath,\ \mathcal{I}_{\delta_\jmath}^{\hat{t}}(\xi_\imath))\}\Big\rangle : \mathcal{I}_{\delta_\jmath}^{\hat{t}}(\xi_\imath) \in \mathcal{H},\ \mathcal{I}_\lambda^{\hat{t}}(\delta_\jmath) \in \Re,\ \forall\delta_\jmath \in \lambda,\ \xi_\imath \in \Lambda,\ \imath,\jmath \in \aleph\Big\},$$

where

$$\mathcal{I}_{\lambda}^{\hat{t}}(\delta_{\jmath}) = \begin{cases} M-1, & if\ \mathcal{I}_{\lambda}(\delta_{\jmath}) < M-1, \\ 0, & if\ \mathcal{I}_{\lambda}(\delta_{\jmath}) = M-1. \end{cases}$$

and

$$\mathcal{I}_{\delta_{\jmath}}^{\hat{t}}(\xi_{\mathfrak{i}}) = \begin{cases} N-1, & if\ \mathcal{I}_{\delta_{\jmath}}(\xi_{\mathfrak{i}}) < N-1, \\ 0, & if\ \mathcal{I}_{\delta_{\jmath}}(\xi_{\mathfrak{i}}) = N-1. \end{cases}$$

Example 3. *Consider a 6P6S-set(Λ_6^6) as given in Example 1. A top weak compliment of Λ_6^6 is given in Table 5.*

Table 5. Tabular representation of $(\Lambda_6^6)^{\hat{t}}$.

$(\Lambda_6^6)^{\hat{t}}$	$\frac{5}{\delta_1}$	$\frac{5}{\delta_2}$	$\frac{5}{\delta_3}$	$\frac{5}{\delta_4}$	$\frac{0}{\delta_5}$	$\frac{5}{\delta_6}$	$\frac{5}{\delta_7}$	$\frac{5}{\delta_8}$
ξ_1	5	5	5	0	5	5	5	5
ξ_2	5	0	5	5	5	0	5	5
ξ_3	0	5	5	5	5	5	5	5
ξ_4	5	5	0	5	5	5	5	5

Definition 12. *Let Λ be universe and $\lambda\widetilde{\sqsubseteq}\mathrm{Y}$ is the collection of attributes. A bottom weak compliment of MPNSS (Λ_N^M) over Λ, denoted by $(\Lambda_N^M)^{\hat{b}}$ and defined as,*

$$(\Lambda_N^M)^{\hat{b}} = \left\{\left\langle\frac{\mathcal{I}_{\lambda}^{\hat{b}}(\delta_{\jmath})}{\delta_{\jmath}}, \left\{\left(\xi_{\mathfrak{i}}, \mathcal{I}_{\delta_{\jmath}}^{\hat{b}}(\xi_{\mathfrak{i}})\right)\right\}\right\rangle : \mathcal{I}_{\delta_{\jmath}}^{\hat{b}}(\xi_{\mathfrak{i}}) \in \mathcal{H},\ \mathcal{I}_{\lambda}^{\hat{b}}(\delta_{\jmath}) \in \Re\ \forall\delta_{\jmath} \in \lambda,\ \xi_{\mathfrak{i}} \in \Lambda,\ \mathfrak{i},\jmath \in \aleph\right\},$$

where

$$\mathcal{I}_{\lambda}^{\hat{b}}(\delta_{\jmath}) = \begin{cases} 0, & if\ \mathcal{I}_{\lambda}(\delta_{\jmath}) > 0, \\ M-1, & if\ \mathcal{I}_{\lambda}(\delta_{\jmath}) = 0. \end{cases}$$

and

$$\mathcal{I}_{\delta_{\jmath}}^{\hat{b}}(\xi_{\mathfrak{i}}) = \begin{cases} 0, & if\ \mathcal{I}_{\delta_{\jmath}}(\xi_{\mathfrak{i}}) > 0, \\ N-1, & if\ \mathcal{I}_{\delta_{\jmath}}(\xi_{\mathfrak{i}}) = 0. \end{cases}$$

Example 4. *Consider a 6P6S-set(Λ_6^6) as given in Example 1. The bottom weak compliment of Λ_6^6 is given in Table 6.*

Table 6. Tabular representation of $(\Lambda_6^6)^{\hat{b}}$.

$(\Lambda_6^6)^{\hat{b}}$	$\frac{0}{\delta_1}$	$\frac{0}{\delta_2}$	$\frac{0}{\delta_3}$	$\frac{0}{\delta_4}$	$\frac{0}{\delta_5}$	$\frac{0}{\delta_6}$	$\frac{0}{\delta_7}$	$\frac{0}{\delta_8}$
ξ_1	0	5	0	0	0	0	0	5
ξ_2	0	0	0	5	0	0	5	0
ξ_3	0	0	5	0	0	0	0	0
ξ_4	0	0	0	0	5	0	0	0

Definition 13. *Let $\Lambda_{N_1}^{M_1}, \Lambda_{N_2}^{M_2}$ be two MPNSSs defined on set of attributes $\lambda\widetilde{\sqsubseteq}\mathrm{Y}$ and $\eth\widetilde{\sqsubseteq}\mathrm{Y}$ respectively. Their extended union is symbolized as $\Lambda_{N_3}^{M_3} = \Lambda_{N_1}^{M_1}\widetilde{\sqcup}_{\mathcal{E}}\Lambda_{N_2}^{M_2}$ and defined as:*

$$\Lambda_{N_3}^{M_3} = \left\{\left\langle\frac{\mathcal{I}_{\mathfrak{X}}(\delta_{\jmath})}{\delta_{\jmath}}, \left\{\left(\xi_{\mathfrak{i}}, \mathcal{I}_{\delta_{\jmath}}(\xi_{\mathfrak{i}})\right)\right\}\right\rangle : \forall\delta_{\jmath} \in \mathfrak{X} = \lambda\widetilde{\sqcup}\eth,\ \xi_{\mathfrak{i}} \in \Lambda,\ \mathcal{I}_{\delta_{\jmath}}(\xi_{\mathfrak{i}}) \in \mathcal{H},\ \mathcal{I}_{\mathfrak{X}}(\delta_{\jmath}) \in \Re,\ \mathfrak{i},\jmath \in \aleph\right\},$$

where

$$\begin{aligned}
\mathcal{I}_{\mathfrak{X}}(\text{ffi}_{\jmath}) &= \max\{\mathcal{I}_{\lambda}(\delta_{\jmath}), \mathcal{I}_{\eth}(\delta_{\jmath})\}, \\
\mathcal{I}_{\delta_{\jmath}}(\xi_{\mathrm{i}}) &= \max\{\mathcal{I}^{1}_{\delta_{\jmath}}(\xi_{\mathrm{i}}), \mathcal{I}^{2}_{\delta_{\jmath}}(\xi_{\mathrm{i}})\}, \\
N_3 &= \max\{N_1, N_2\}, \\
M_3 &= \max\{M_1, M_2\}.
\end{aligned}$$

Example 5. *Consider a 6P6S-set(Λ^{6}_{6}) as given in Example 1, also consider another 5P4S-set(Λ^{5}_{4}) defined on Λ, as given in Table 7.*

Table 7. Tabular representation of Λ^{5}_{4}.

Λ^{5}_{4}	$\frac{4}{\delta_1}$	$\frac{3}{\delta_2}$	$\frac{2}{\delta_3}$	$\frac{4}{\delta_4}$	$\frac{1}{\delta_5}$	$\frac{4}{\delta_6}$	$\frac{2}{\delta_7}$	$\frac{4}{\delta_8}$
ξ_1	2	1	0	1	2	3	1	2
ξ_2	0	3	2	0	3	2	1	0
ξ_3	0	2	3	2	3	1	0	2
ξ_4	3	1	2	3	0	1	2	1

The extended union of Λ^{6}_{6} and Λ^{5}_{4} is defined as $\Lambda^{6}_{6}\widetilde{\sqcup}_{\mathcal{E}}\Lambda^{5}_{4} = \Lambda^{6}_{6}$, given in Table 8.

Table 8. Tabular representation of Λ^{6}_{6}.

Λ^{6}_{6}	$\frac{4}{\delta_1}$	$\frac{3}{\delta_2}$	$\frac{2}{\delta_3}$	$\frac{4}{\delta_4}$	$\frac{5}{\delta_5}$	$\frac{4}{\delta_6}$	$\frac{4}{\delta_7}$	$\frac{4}{\delta_8}$
ξ_1	3	1	3	5	3	4	2	2
ξ_2	2	5	4	0	3	5	1	3
ξ_3	5	4	3	2	3	2	3	2
ξ_4	3	2	5	3	0	3	4	3

Definition 14. *Let $\Lambda^{M_1}_{N_1}, \Lambda^{M_2}_{N_2} \in MPNSS(\Lambda)$. Their restricted intersection is symbolized by $\Lambda^{M_3}_{N_3} = \Lambda^{M_1}_{N_1}\widetilde{\sqcap}_{\mathcal{R}}\Lambda^{M_2}_{N_2}$ and defined as:*

$$\Lambda^{M_3}_{N_3} = \left\{\left\langle \frac{\mathcal{I}_{\Im}(\delta_{\jmath})}{\delta_{\jmath}},\ \left\{(\xi_{\mathrm{i}},\ \mathcal{I}_{\delta_{\jmath}}(\xi_{\mathrm{i}}))\right\}\right\rangle : \forall \delta_{\jmath} \in \Im = \lambda\widetilde{\sqcap}\eth,\ \xi_{\mathrm{i}} \in \Lambda,\ \mathcal{I}_{\delta_{\jmath}}(\xi_{\mathrm{i}}) \in \mathcal{H},\ \mathcal{I}_{\Im}(\delta_{\jmath}) \in \Re,\ \mathrm{i}, \jmath \in \aleph\right\},$$

where

$$\begin{aligned}
\mathcal{I}_{\Im}(\text{ffi}_{\jmath}) &= \min\{\mathcal{I}_{\lambda}(\delta_{\jmath}), \mathcal{I}_{\Im}(\delta_{\jmath})\}, \\
\mathcal{I}_{\delta_{\jmath}}(\xi_{\mathrm{i}}) &= \min\{\mathcal{I}^{1}_{\delta_{\jmath}}(\xi_{\mathrm{i}}), \mathcal{I}^{2}_{\delta_{\jmath}}(\xi_{\mathrm{i}})\}, \\
N_3 &= \min\{N_1, N_2\}, \\
M_3 &= \min\{M_1, M_2\}.
\end{aligned}$$

Example 6. *Consider Λ^{6}_{6} and Λ^{4}_{5} as given in Example 5. The restricted intersection is defined by $\Lambda^{6}_{6}\widetilde{\sqcap}_{\mathcal{R}}\Lambda^{5}_{4} = \Lambda^{5}_{4}$, given in Table 9.*

Table 9. Tabular representation of Λ^{5}_{4}.

Λ^{5}_{4}	$\frac{4}{\delta_1}$	$\frac{3}{\delta_2}$	$\frac{2}{\delta_3}$	$\frac{4}{\delta_4}$	$\frac{1}{\delta_5}$	$\frac{1}{\delta_6}$	$\frac{2}{\delta_7}$	$\frac{2}{\delta_8}$
ξ_1	2	0	0	1	2	3	1	0
ξ_2	0	3	2	0	2	2	0	0
ξ_3	0	2	0	2	1	1	0	2
ξ_4	1	1	2	1	0	1	2	1

4. M-Parameterized N-Soft Topology

The concept of M-parameterized N-soft topology (MPNS topology) based on MPNSS is introduced in this section. Certain properties of MPNS topology are expressed and their corresponding results are established.

Definition 15. *Let Λ_N^M be a MPNSS over Λ, Υ is the collection of attributes, $\mathcal{H} = \{0, 1, 2, \cdots, N - 1\}$, $\Re = \{0, 1, 2, \cdots, M - 1\}$ be two grading sets. The M-parameterized N-soft power whole set (MPNSPW-set) of the Λ_N^M indicated as, $\mathbb{P}(\Lambda_N^M)$ and defined as,*

$$\mathbb{P}(\Lambda_N^M) = \big\{\Lambda_{(\mathrm{i})} : \Lambda_{(\mathrm{i})} \widetilde{\sqsubseteq} \Lambda_N^M,\ \mathrm{i} \in \widehat{\mathcal{I}} \widetilde{\sqsubseteq} \aleph\big\}.$$

The cardinality of MPNSPW-set is defined by

$$|\mathbb{P}(\Lambda_N^M)| = 2^{\sum\limits_{\delta_j \in F} |(\xi_{\mathrm{i}}, \mathcal{I}_{\delta_j}(\xi_{\mathrm{i}}))|}$$

Example 7. *Let $\Lambda = \{\xi_1, \xi_2, \xi_3\}$ is a collection of different restaurants under consideration and $\Upsilon = \{\delta_1, \delta_2\}$ is a set of attributes, where*
δ_1 *= good food quality,*
δ_2 *= economical.*
Consider the 8P8S-set as given below

$$\Lambda_8^8 = \Big\{\Big\langle \frac{7}{\delta_1}, \big\{(\xi_1, 6), (\xi_2, 4), (\xi_3, 5)\big\}\Big\rangle, \Big\langle \frac{6}{\delta_2}, \big\{(\xi_1, 3), (\xi_2, 2)\big\}\Big\rangle\Big\}.$$

The cardinality of 8P8SPW-set is

$$|\mathbb{P}(\Lambda_8^8)| = 2^{3+2} = 2^5 = 32.$$

The list of all possible MPNS-power whole subsets of 8P8S-set (Λ_8^8) is as follows:

$$\begin{aligned}
\Lambda_{(1)} &= \mathfrak{P}_0^0 \\
\Lambda_{(2)} &= \Big\{\Big\langle \frac{7}{\delta_1}, \big\{(\xi_1, 6)\big\}\Big\rangle\Big\} \\
\Lambda_{(3)} &= \Big\{\Big\langle \frac{7}{\delta_1}, \big\{(\xi_2, 4)\big\}\Big\rangle\Big\} \\
\Lambda_{(4)} &= \Big\{\Big\langle \frac{7}{\delta_1}, \big\{(\xi_3, 5)\big\}\Big\rangle\Big\} \\
\Lambda_{(5)} &= \Big\{\Big\langle \frac{6}{\delta_2}, \big\{(\xi_1, 3)\big\}\Big\rangle\Big\} \\
\Lambda_{(6)} &= \Big\{\Big\langle \frac{6}{\delta_2}, \big\{(\xi_2, 2)\big\}\Big\rangle\Big\} \\
\Lambda_{(7)} &= \Big\{\Big\langle \frac{7}{\delta_1}, \big\{(\xi_1, 6), (\xi_2, 4)\big\}\Big\rangle\Big\} \\
\Lambda_{(8)} &= \Big\{\Big\langle \frac{7}{\delta_1}, \big\{(\xi_1, 6), (\xi_3, 5)\big\}\Big\rangle\Big\} \\
\Lambda_{(9)} &= \Big\{\Big\langle \frac{7}{\delta_1}, \big\{(\xi_2, 4), (\xi_3, 5)\big\}\Big\rangle\Big\} \\
\Lambda_{(10)} &= \Big\{\Big\langle \frac{7}{\delta_1}, \big\{(\xi_1, 6), (\xi_2, 4), (\xi_3, 5)\big\}\Big\rangle\Big\} \\
\Lambda_{(11)} &= \Big\{\Big\langle \frac{6}{\delta_2}, \big\{(\xi_1, 3), (\xi_2, 2)\big\}\Big\rangle\Big\}
\end{aligned}$$

$$
\begin{aligned}
\Lambda_{(12)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_1,6)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_1,3)\}\Big\rangle\Big\}\\
\Lambda_{(13)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_1,6)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_2,2)\}\Big\rangle\Big\}\\
\Lambda_{(14)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_2,4)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_1,3)\}\Big\rangle\Big\}\\
\Lambda_{(15)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_2,4)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_2,2)\}\Big\rangle\Big\}\\
\Lambda_{(16)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_3,5)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_1,3)\}\Big\rangle\Big\}\\
\Lambda_{(17)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_3,5)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_2,2)\}\Big\rangle\Big\}\\
\Lambda_{(18)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_1,6),(\xi_2,4)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_1,3)\}\Big\rangle\Big\}\\
\Lambda_{(19)} &= \Big\{\Big\langle\frac{7}{s_1},\{(\xi_1,6),(\xi_3,5)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_1,3)\}\Big\rangle\Big\}\\
\Lambda_{(20)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_2,4),(\xi_3,5)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_1,3)\}\Big\rangle\Big\}\\
\Lambda_{(21)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_1,6),(\xi_2,4)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_2,2)\}\Big\rangle\Big\}\\
\Lambda_{(22)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_1,6),(\xi_3,5)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_2,2)\}\Big\rangle\Big\}\\
\Lambda_{(23)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_2,4),(\xi_3,5)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_2,2)\}\Big\rangle\Big\}\\
\Lambda_{(24)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_1,6),(\xi_2,4),(\xi_3,5)\}\Big\rangle,\langle\frac{6}{\delta_2},\{(\xi_1,3)\}\Big\rangle\Big\}\\
\Lambda_{(25)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_1,6),(\xi_2,4),(\xi_3,5)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_2,2)\}\Big\rangle\Big\}\\
\Lambda_{(26)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_1,6)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_1,3),(\xi_2,2)\}\Big\rangle\Big\}\\
\Lambda_{(27)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_2,4)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_1,3),(\xi_2,2)\}\Big\rangle\Big\}\\
\Lambda_{(28)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_3,5)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_1,3),(\xi_2,2)\}\Big\rangle\Big\}\\
\Lambda_{(29)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_1,6),(\xi_2,4)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_1,3),(\xi_2,2)\}\Big\rangle\Big\}\\
\Lambda_{(30)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_1,6),(\xi_3,5)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_1,3),(\xi_2,2)\}\Big\rangle\Big\}\\
\Lambda_{(31)} &= \Big\{\Big\langle\frac{7}{\delta_1},\{(\xi_2,4),(\xi_3,5)\}\Big\rangle,\Big\langle\frac{6}{\delta_2},\{(\xi_1,3),(\xi_2,2)\}\Big\rangle\Big\}\\
\Lambda_{(32)} &= \Lambda_8^8
\end{aligned}
$$

Definition 16. *Let Λ is the collection of universal elements and Λ_N^M is a MPNSS on Λ. A collection $\breve{\mathfrak{T}}$ of power whole MPNS-subsets of Λ_N^M is called MPNS topology defined on a MPNSS Λ_N^M, if the following conditions hold,*

(1) $\mathfrak{P}_0^0, \Lambda_N^M \in \breve{\mathfrak{T}}$.

(2) *Arbitrary union of elements of $\breve{\mathfrak{T}}$ is a member of $\breve{\mathfrak{T}}$,*
i.e., $\big\{\Lambda_{(\mathfrak{i})} \widetilde{\sqsubseteq} \Lambda_N^M : \mathfrak{i} \in \widehat{\mathcal{I}} \widetilde{\sqsubseteq} \aleph\big\} \widetilde{\sqsubseteq} \breve{\mathfrak{T}} \Rightarrow \widetilde{\sqcup}_{\mathfrak{i}\in\widehat{\mathcal{I}}} \Lambda_{(\mathfrak{i})} \in \breve{\mathfrak{T}}$.

(3) *Finite intersection of elements of $\breve{\mathfrak{T}}$ is a member of $\breve{\mathfrak{T}}$*
i.e., $\big\{\Lambda_{(\mathfrak{i})} \widetilde{\sqsubseteq} \Lambda_N^M : 1 \le \mathfrak{i} \le \mathfrak{n}, \mathfrak{n} \in \aleph\big\} \widetilde{\sqsubseteq} \breve{\mathfrak{T}} \Rightarrow \widetilde{\sqcup}_{1\le\mathfrak{i}\le\mathfrak{n}} \Lambda_{(\mathfrak{i})} \in \breve{\mathfrak{T}}$.

The MPNS-topological space is indicated as, $\left(\Lambda_N^M, \breve{\mathfrak{T}}\right)$. The MPNS-open sets are members of a MPNS topology $\breve{\mathfrak{T}}$ and MPNS-closed sets are their bottom weak complements.

Example 8. *Consider the 8P8S-subsets of Λ_8^8, as given in Example 7. Then,*

$$\breve{\mathfrak{T}}_1 = \left\{\Lambda_{(1)}, \Lambda_{(2)}, \Lambda_{(7)}, \Lambda_{(10)}, \Lambda_{(32)}\right\}$$
$$\breve{\mathfrak{T}}_1 = \left\{\mathfrak{P}_0^0, \left\{\langle\frac{7}{\delta_1}, \{(\xi_1, 6)\}\right\rangle\right\}, \left\{\left\langle\frac{7}{\delta_1}, \{(\xi_1, 6), (\xi_2, 4)\}\right\rangle\right\}, \left\{\left\langle\frac{7}{\delta_1}, \{(\xi_1, 6), (\xi_2, 4), (\xi_3, 5)\}\right\rangle\right\}, \Lambda_8^8\right\}$$

is the 8P8S-topology on Λ_8^8. But

$$\breve{\mathfrak{T}}_2 = \left\{\Lambda_{(1)}, \Lambda_{(5)}, \Lambda_{(8)}, \Lambda_{(9)}, \Lambda_{(32)}\right\}$$
$$\breve{\mathfrak{T}}_2 = \left\{\mathfrak{P}_0^0, \left\{\left\langle\frac{7}{\delta_1}, \{(\xi_1, 3)\}\right\rangle\right\}, \left\{\left\langle\frac{7}{\delta_1}, \{(\xi_1, 6), (\xi_3, 5)\}\right\rangle\right\}, \left\{\left\langle\frac{7}{\delta_1}, \{(\xi_1, 6), (\xi_3, 5)\}\right\rangle, \left\langle\frac{6}{\delta_2}, \{(\xi_1, 3)\}\right\rangle\right\}, \Lambda_8^8\right\}$$

is not a 8P8S-topology on Λ_8^8.

Example 9. $\breve{\mathfrak{T}}_3 = \{\mathfrak{P}_0^0, \Lambda_8^8\}$ *is 8P8S-discrete topology and* $\breve{\mathfrak{T}}_4 = \mathbb{P}(\Lambda_N^M)$ *is 8P8S-indiscrete topology.*

Theorem 1. *Suppose $\left(\Lambda_N^M, \breve{\mathfrak{T}}\right)$ be a MPNS-topological space, the following conditions hold,*

(1) *The universal MPNSS $(\mathfrak{S}_{N-1}^{M-1})$ and $(\Lambda_N^M)^{\hat{b}}$ are MPNS-closed sets.*
(2) *Finite MPNS-union of the MPNS-closed sets are MPNS-closed sets.*
(3) *Arbitrary MPNS-intersection of the MPNS-closed sets are MPNS-closed sets.*

Proof. (1) $(\mathfrak{S}_{N-1}^{M-1})^{\hat{b}} = \mathfrak{P}_0^0$ and $((\mathfrak{S}_{N-1}^{M-1})^{\hat{b}})^{\hat{b}} = \mathfrak{S}_{N-1}^{M-1}$ are MPNS-closed sets.

(2) If $\left\{\Lambda_{(\mathfrak{i})} : \Lambda_{(\mathfrak{i})}^{\hat{b}} \in \breve{\mathfrak{T}}, \mathfrak{i} \in \widehat{\mathcal{I}} \widetilde{\sqsubseteq} \aleph\right\}$ is a given collection of MPNS-closed sets, then

$$\widetilde{\sqcup}_{\mathfrak{i}\in\widehat{\mathcal{I}}} \Lambda_{(\mathfrak{i})}^{\hat{b}} = \left(\widetilde{\sqcap}_{\mathfrak{i}\in\widehat{\mathcal{I}}} \Lambda_{(\mathfrak{i})}\right)^{\hat{b}}$$

is MPNS-open set. So that $\widetilde{\sqcap}_{\mathfrak{i}\in\widehat{\mathcal{I}}} \Lambda_{(\mathfrak{i})}$ is a MPNS-closed set.

(3) In the same way, if $\Lambda_{(\mathfrak{i})}$ is MPNS-closed set for $\mathfrak{i} = 1, 2, \cdots, \mathfrak{n}$, then

$$\widetilde{\sqcap}_{\mathfrak{i}=1}^{\mathfrak{n}} \Lambda_{(\mathfrak{i})}^{\hat{b}} = \left(\widetilde{\sqcup}_{\mathfrak{i}=1}^{\mathfrak{n}} \Lambda_{(\mathfrak{i})}\right)^{\hat{b}}$$

is MPNS-open set. Hence, $\widetilde{\sqcup}_{\mathfrak{i}=1}^{\mathfrak{n}} \Lambda_{(\mathfrak{i})}$ is a MPNS-closed set.
□

Definition 17. *Let $\left(\Lambda_N^M, \breve{\mathfrak{T}}_1\right)$ and $\left(\Lambda_N^M, \breve{\mathfrak{T}}_2\right)$ are two MPNS-topologies.*

(1) *$\breve{\mathfrak{T}}_1$ and $\breve{\mathfrak{T}}_2$ are said to be equivalent MPNS-topologies, if either $\breve{\mathfrak{T}}_1 \widetilde{\sqsubseteq} \breve{\mathfrak{T}}_2$ or $\breve{\mathfrak{T}}_2 \widetilde{\sqsubseteq} \breve{\mathfrak{T}}_1$.*
(2) *If $\breve{\mathfrak{T}}_1 \widetilde{\sqsubseteq} \breve{\mathfrak{T}}_2$ then $\breve{\mathfrak{T}}_2$ is MPNS-finer than $\breve{\mathfrak{T}}_1$ or $\breve{\mathfrak{T}}_1$ is MPNS-coarser than $\breve{\mathfrak{T}}_2$.*

Example 10. *Consider 8P8S-topologies on Λ_8^8 as given in Example 9. $\breve{\mathfrak{T}}_3$ is 8P8S-coarser than $\breve{\mathfrak{T}}_4$ or $\breve{\mathfrak{T}}_4$ is 8P8S-finer than $\breve{\mathfrak{T}}_3$.*

Proposition 1. *Let $\left(\Lambda_N^M, \breve{\mathfrak{T}}_1\right)$ and $\left(\Lambda_N^M, \breve{\mathfrak{T}}_2\right)$ be two MPNS-topological spaces over the same MPNSS$(\breve{\Lambda}_N^M)$, then $\left(\Lambda_N^M, \breve{\mathfrak{T}}_1 \widetilde{\sqcap} \breve{\mathfrak{T}}_2\right)$ is a MPNS-topological space defined on MPNSS (Λ_N^M).*

Proof. (1) $\mathfrak{P}_0^0$, $\mathfrak{S}_{N-1}^{M-1} \in \breve{\mathfrak{T}}_1 \widetilde{\sqcap} \breve{\mathfrak{T}}_2$.

(2) Let $\{\Lambda_{(\mathfrak{i})} : \mathfrak{i} \in \widehat{\mathcal{I}} \widetilde{\sqsubseteq} \aleph\}$ be a collection of MPNSSs in $\breve{\mathfrak{T}}_1 \widetilde{\sqcap} \breve{\mathfrak{T}}_2$. Then $\Lambda_{(\mathfrak{i})} \in \breve{\mathfrak{T}}_1$ and $\Lambda_{(\mathfrak{i})} \in \breve{\mathfrak{T}}_2$, $\forall \mathfrak{i} \in \widehat{\mathcal{I}}$, thus $\widetilde{\sqcup}_{\mathfrak{i}\in\widehat{\mathcal{I}}} \Lambda_{(\mathfrak{i})} \in \breve{\mathfrak{T}}_1$ and $\widetilde{\sqcup}_{\mathfrak{i}\in\widehat{\mathcal{I}}} \Lambda_{(\mathfrak{i})} \in \breve{\mathfrak{T}}_2$. Thus, $\widetilde{\sqcup}_{\mathfrak{i}\in\widehat{\mathcal{I}}} \Lambda_{(\mathfrak{i})} \in \breve{\mathfrak{T}}_1 \widetilde{\sqcap} \breve{\mathfrak{T}}_2$.

(3) Let $\Lambda_{(1)}, \Lambda_{(2)} \in \breve{\mathfrak{T}}_1 \widetilde{\sqcap} \breve{\mathfrak{T}}_2$. Then $\Lambda_{(1)}, \Lambda_{(2)} \in \breve{\mathfrak{T}}_1$ and $\Lambda_{(1)}, \Lambda_{(2)} \in \breve{\mathfrak{T}}_2$. Since $\Lambda_{(1)} \widetilde{\sqcap} \Lambda_{(2)} \in \breve{\mathfrak{T}}_1$ and $\Lambda_{(1)} \widetilde{\sqcap} \Lambda_{(2)} \in \breve{\mathfrak{T}}_2$, therefore $\Lambda_{(1)} \widetilde{\sqcap} \Lambda_{(2)} \in \breve{\mathfrak{T}}_1 \widetilde{\sqcap} \breve{\mathfrak{T}}_2$.
Consequently, $\breve{\mathfrak{T}}_1 \widetilde{\sqcap} \breve{\mathfrak{T}}_2$ establishes MPNS topology on Λ_N^M and $\left(\Lambda_N^M, \breve{\mathfrak{T}}_1 \widetilde{\sqcap} \breve{\mathfrak{T}}_2\right)$ is a MPNS-topological space on universal MPNSS (Λ_N^M).
□

Definition 18. *Suppose $(\Lambda_{N_1}^{M_1}, \breve{\mathfrak{T}})$ be a MPNS-topological space and $\Lambda_{N_2}^{M_2} \widetilde{\sqsubseteq} \Lambda_{N_1}^{M_1}$. The MPNS-subspace topology, denoted by $\widehat{\mathcal{T}}$ is the collection*

$$\widehat{\mathcal{T}} = \left\{\Lambda_{(\mathrm{i})} \widetilde{\sqcap} \Lambda_{N_2}^{M_2} : \Lambda_{(\mathrm{i})} \in \breve{\mathfrak{T}}, \mathrm{i} \in \widehat{\mathcal{I}} \widetilde{\sqsubseteq} \aleph\right\}$$

$(\Lambda_{N_2}^{M_2}, \widehat{\mathcal{T}})$ is called subspace of $(\Lambda_{N_1}^{M_1}, \breve{\mathfrak{T}})$.

Example 11. *Consider a $(\Lambda_8^8, \breve{\mathfrak{T}}_1)$ is 8P8S-topological space as given in Example 8. Let*

$$\Lambda_{(18)} = \left\{\left\langle \frac{7}{\delta_1}, \{(\xi_1, 6), (\xi_2, 4)\}\right\rangle, \left\langle \frac{6}{\delta_2}, \{(\xi_1, 3)\}\right\rangle\right\} \widetilde{\sqsubseteq} \Lambda_8^8$$

8P8S-subspace can be obtained as

$$\begin{aligned}
\Lambda_{(18)} \widetilde{\sqcap} \Lambda_{(1)} &= \mathfrak{P}_0^0 \\
\Lambda_{(18)} \widetilde{\sqcap} \Lambda_{(2)} &= \Lambda_{(2)} \\
\Lambda_{(18)} \widetilde{\sqcap} \Lambda_{(7)} &= \Lambda_{(7)} \\
\Lambda_{(18)} \widetilde{\sqcap} \Lambda_{(10)} &= \Lambda_{(7)} \\
\Lambda_{(18)} \widetilde{\sqcap} \Lambda_{(32)} &= \Lambda_{(18)}
\end{aligned}$$

Hence $\widehat{\mathcal{T}} = \left\{\mathfrak{P}_0^0, \Lambda_{(2)}, \Lambda_{(7)}, \Lambda_{(18)}\right\}$ is 8P8S-subspace topology.

Theorem 2. *Suppose $\left(\Lambda_{N_1}^{M_1}, \breve{\mathfrak{T}}\right)$ be a MPNS-topological space and $\Lambda_{N_2}^{M_2} \widetilde{\sqsubseteq} \Lambda_{N_1}^{M_1}$. Then a MPNS-subspace topology on $\Lambda_{N_2}^{M_2}$ is a MPNS topology.*

Proof. Indeed, $\widehat{\mathcal{T}}$ contains $\mathfrak{P}_0^0$ and $\Lambda_{N_2}^{M_2}$ because $\mathfrak{P}_0^0 \widetilde{\sqcap} \Lambda_{N_2}^{M_2} = \mathfrak{P}_0^0$ and $\Lambda_{N_1}^{M_1} \widetilde{\sqcap} \Lambda_{N_2}^{M_2} = \Lambda_{N_2}^{M_2}$, where $\mathfrak{P}_0^0, \Lambda_{N_1}^{M_1} \in \breve{\mathfrak{T}}_1$. Since $\breve{\mathfrak{T}} = \{\Lambda_{(\mathrm{i})} : \Lambda_{(\mathrm{i})} \widetilde{\sqsubseteq} \Lambda_{N_1}^{M_1}, \mathrm{i} \in \widehat{\mathcal{I}} \widetilde{\sqsubseteq} \aleph\}$, it is closed under finite MPNS-intersections and MPNS-unions,

$$\begin{aligned}
\widetilde{\sqcap}_{\mathrm{i}=1}^{n}\left(\Lambda_{(\mathrm{i})} \widetilde{\sqcap} \Lambda_{N_2}^{M_2}\right) &= \left(\widetilde{\sqcap}_{\mathrm{i}=1}^{n} \Lambda_{(\mathrm{i})}\right) \widetilde{\sqcap} \Lambda_{N_2}^{M_2} \\
\widetilde{\sqcup}_{\mathrm{i} \in \widehat{\mathcal{I}}}\left(\Lambda_{(\mathrm{i})} \widetilde{\sqcup} \Lambda_{N_2}^{M_2}\right) &= \left(\widetilde{\sqcap}_{\mathrm{i} \in \widehat{\mathcal{I}}} \Lambda_{(\mathrm{i})}\right) \widetilde{\sqcap} \Lambda_{N_2}^{M_2}
\end{aligned}$$

□

Definition 19. *Let Λ_N^M is a MPNSS. A basis is an assemblage of subsets of Λ_N^M, for a topology on Λ_N^M, which holds the following conditions,*

(1) *There exists one or multiple elements β containing $\Lambda_{(\mathrm{i})}$, for each $\Lambda_{(\mathrm{i})} \in \Lambda_N^M$*
(2) *If intersection of β_1 and β_2 contains $\Lambda_{(\mathrm{i})}$ then there must exist a β_3 containing $\Lambda_{(\mathrm{i})}$ in such a way that $\beta_3 \widetilde{\sqsubseteq} \beta_1 \widetilde{\sqcap} \beta_2$.*

Example 12. *Consider a 8P8S-topology $\breve{\mathfrak{T}}_1$ defined on 8P8S-set as given in Example 8. Then*

$$\begin{aligned}
\beta &= \left\{\Lambda_{(1)}, \Lambda_{(2)}, \Lambda_{(3)}, \Lambda_{(4)}\right\} \\
\text{or} \quad \beta &= \left\{\mathfrak{P}_0^0, \{\langle \frac{7}{\delta_1}, \{(\xi_1, 6)\}\rangle\}, \{\langle \frac{7}{\delta_1}\{(\xi_2, 4)\}\rangle\}, \{\langle \frac{7}{\delta_1}, \{(\xi_3, 5)\}\rangle\}\right\}
\end{aligned}$$

is a 8P8S-basis for the 8P8S-topology $\breve{\mathfrak{T}}_1$.

Definition 20. *Let $(\Lambda_{N_1}^{M_1}, \breve{\mathfrak{T}})$ be a MPNS-topological space and $\Lambda_{N_2}^{M_2}$ be a subset of $\Lambda_{N_1}^{M_1}$. The MPNS-interior of $\Lambda_{N_2}^{M_2}$ is the MPNS-union of all open subsets of $\Lambda_{N_2}^{M_2}$ and it is indicated by $(\Lambda_{N_2}^{M_2})^\circ$.*

Remark 1. *The interior of Λ_N^M is the union of all subsets of Λ_N^M which are open in $\breve{\mathfrak{T}}$.*

Example 13. *Consider a 8P8S-topology $\breve{\mathfrak{T}}_1$ defined on 8P8SS(Λ_8^8) as given in Example 8. Let $\Lambda_{(18)} = \Big\{\Big\langle \delta_1, \{(\xi_1,2),(\xi_2,4),(\xi_3,6)\}\Big\rangle, \Big\langle \delta_2, \{(\xi_1,1),(\xi_3,5)\}\Big\rangle\Big\} \tilde{\sqsubseteq} \Lambda_8^8$. The open subsets of $\Lambda_{(18)}$ are $\Lambda_{(1)}, \Lambda_{(2)}, \Lambda_{(7)}$. Hence 8P8S-interior is*

$$(\Lambda_{(18)})^\circ = \Lambda_{(1)} \tilde{\sqcup} \Lambda_{(2)} \tilde{\sqcup} \Lambda_{(7)} = \Lambda_{(7)} = \Big\{\Big\langle \frac{7}{\delta_1}, \{(\xi_1,6),(\xi_2,4)\}\Big\rangle\Big\}.$$

Theorem 3. *Let $(\Lambda_{N_1}^{M_1}, \breve{\mathfrak{T}})$ be a MPNS-topological space and $\Lambda_{N_2}^{M_2} \tilde{\sqsubseteq} \Lambda_{N_1}^{M_1}$. $\Lambda_{N_2}^{M_2}$ is a MPNS-open set iff $\Lambda_{N_2}^{M_2} = (\Lambda_{N_2}^{M_2})^\circ$.*

Proof. If $\Lambda_{N_2}^{M_2}$ is a MPNS-open set, then the largest open set, that $\Lambda_{N_2}^{M_2}$ is containing is equal to $\Lambda_{N_2}^{M_2}$. Consequently, $\Lambda_{N_2}^{M_2} = (\Lambda_{N_2}^{M_2})^\circ$.
Conversely, As we know, $(\Lambda_{N_2}^{M_2})^\circ$ is a MPNS-open set and if $\Lambda_{N_2}^{M_2} = (\Lambda_{N_2}^{M_2})^\circ$, then $\Lambda_{N_2}^{M_2}$ is MPNS-open set. □

Theorem 4. *Let $(\Lambda_{N_1}^{M_1}, \breve{\mathfrak{T}})$ be a MPNS-topological space and $\Lambda_{N_2}^{M_2}, \Lambda_{N_3}^{M_3} \tilde{\sqsubseteq} \Lambda_{N_1}^{M_1}$. Then*

(1) $\Big((\Lambda_{N_2}^{M_2})^\circ\Big)^\circ = (\Lambda_{N_2}^{M_2})^\circ$
(2) $\Lambda_{N_2}^{M_2} \tilde{\sqsubseteq} \Lambda_{N_3}^{M_3} \Rightarrow (\Lambda_{N_2}^{M_2})^\circ \tilde{\sqsubseteq} (\Lambda_{N_3}^{M_3})^\circ$
(3) $(\Lambda_{N_2}^{M_2})^\circ \tilde{\sqcap} (\Lambda_{N_3}^{M_3})^\circ = (\Lambda_{N_2}^{M_2} \tilde{\sqcap} \Lambda_{N_3}^{M_3})^\circ$
(4) $(\Lambda_{N_2}^{M_2})^\circ \tilde{\sqcup} (\Lambda_{N_3}^{M_3})^\circ \tilde{\sqsubseteq} (\Lambda_{N_2}^{M_2} \tilde{\sqcup} \Lambda_{N_3}^{M_3})^\circ$.

Proof. (1) Let $(\Lambda_{N_2}^{M_2})^\circ = \Lambda_{N_4}^{M_4}$, then $\Lambda_{N_4}^{M_4} \in \breve{\mathfrak{T}}$ if and only if $\Lambda_{N_4}^{M_4} = (\Lambda_{N_4}^{M_4})^\circ$. Therefore, $\Big((\Lambda_{N_2}^{M_2})^\circ\Big)^\circ = (\Lambda_{N_2}^{M_2})^\circ$.

(2) Let $\Lambda_{N_2}^{M_2} \tilde{\sqsubseteq} \Lambda_{N_3}^{M_3}$. From the definition of a MPNS-interior, $(\Lambda_{N_2}^{M_2})^\circ \tilde{\sqsubseteq} \Lambda_{N_2}^{M_2}$ and $(\Lambda_{N_3}^{M_3})^\circ \tilde{\sqsubseteq} \Lambda_{N_3}^{M_3}$. $(\Lambda_{N_3}^{M_3})^\circ$ is the biggest MPNS open set that is contained by $\Lambda_{N_3}^{M_3}$. Hence, $\Lambda_{N_2}^{M_2} \tilde{\sqsubseteq} \Lambda_{N_3}^{M_3} \Rightarrow (\Lambda_{N_2}^{M_2})^\circ \tilde{\sqsubseteq} (\Lambda_{N_3}^{M_3})^\circ$.

(3) By definition of a MPNS interior, $(\Lambda_{N_2}^{M_2})^\circ \tilde{\sqsubseteq} \Lambda_{N_2}^{M_2}$ and $(\Lambda_{N_3}^{M_3})^\circ \tilde{\sqsubseteq} \Lambda_{N_3}^{M_3}$.
Then, $(\Lambda_{N_2}^{M_2})^\circ \tilde{\sqcap} (\Lambda_{N_3}^{M_3})^\circ \tilde{\sqsubseteq} \Lambda_{N_2}^{M_2} \tilde{\sqcap} \Lambda_{N_3}^{M_3}$.
$(\Lambda_{N_2}^{M_2} \tilde{\sqcap} \Lambda_{N_3}^{M_3})^\circ$ is the biggest MPNS open set that is contained by $\Lambda_{N_2}^{M_2} \tilde{\sqcap} \Lambda_{N_3}^{M_3}$.
Hence, $(\Lambda_{N_2}^{M_2})^\circ \tilde{\sqcap} (\Lambda_{N_3}^{M_3})^\circ \tilde{\sqsubseteq} (\Lambda_{N_2}^{M_2} \tilde{\sqcap} \Lambda_{N_3}^{M_3})^\circ$. Conversely, consider $(\Lambda_{N_2}^{M_2})^\circ \tilde{\sqcap} \Lambda_{N_3}^{M_3} \tilde{\sqsubseteq} \Lambda_{N_2}^{M_2}$ and $(\Lambda_{N_2}^{M_2})^\circ \tilde{\sqcap} \Lambda_{N_3}^{M_3} \tilde{\sqsubseteq} \Lambda_{N_3}^{M_3}$. Then, $(\Lambda_{N_2}^{M_2} \tilde{\sqcap} \Lambda_{N_3}^{M_3})^\circ \tilde{\sqsubseteq} (\Lambda_{N_2}^{M_2})^\circ$ and $(\Lambda_{N_2}^{M_2} \tilde{\sqcap} \Lambda_{N_3}^{M_3})^\circ \tilde{\sqsubseteq} (\Lambda_{N_3}^{M_3})^\circ$.
Therefore, $(\Lambda_{N_2}^{M_2} \tilde{\sqcap} \Lambda_{N_3}^{M_3})^\circ \tilde{\sqsubseteq} (\Lambda_{N_2}^{M_2})^\circ \tilde{\sqsubseteq} (\Lambda_{N_3}^{M_3})^\circ$.

(4) $(\Lambda_{N_2}^{M_2})^\circ \tilde{\sqsubseteq} \Lambda_{N_2}^{M_2}$ and $(\Lambda_{N_3}^{M_3})^\circ \tilde{\sqsubseteq} \Lambda_{N_3}^{M_3}$.
Then, $(\Lambda_{N_2}^{M_2})^\circ \tilde{\sqcup} (\Lambda_{N_3}^{M_3})^\circ \tilde{\sqsubseteq} \Lambda_{N_2}^{M_2} \tilde{\sqcup} \Lambda_{N_3}^{M_3}$.
$(\Lambda_{N_2}^{M_2} \tilde{\sqcup} \Lambda_{N_3}^{M_3})^\circ$ is the biggest MPNS open set that is contained by $(\Lambda_{N_2}^{M_2} \tilde{\sqcap} \Lambda_{N_3}^{M_3})$.
Hence, $(\Lambda_{N_2}^{M_2})^\circ \tilde{\sqcup} (\Lambda_{N_3}^{M_3})^\circ \tilde{\sqsubseteq} (\Lambda_{N_2}^{M_2} \tilde{\sqcup} \Lambda_{N_2}^{M_2})^\circ$.

□

Definition 21. *Let* $(\Lambda_{N_1}^{M_1}, \breve{\mathfrak{T}})$ *be a MPNS-topological space and* $\Lambda_{N_2}^{M_2} \widetilde{\sqsubseteq} \Lambda_{N_1}^{M_1}$*. The MPNS-closure of* $\Lambda_{N_2}^{M_2}$*, indicated as,* $\overline{\Lambda_{N_2}^{M_2}}$ *, is the MPNS-intersection of all MPNS-closed super sets of* $\Lambda_{N_2}^{M_2}$*.*

Remark 2. *It should be emphasized that* $\overline{\Lambda_{N_2}^{M_2}}$ *is the smallest closed super MPNSS of* $\Lambda_{N_2}^{M_2}$ *and* $\overline{\Lambda_{N_2}^{M_2}}$ *is MPNS-closed being the MPNS-intersection of MPNS-closed sets.*

Example 14. *Consider 8P8S-set* Λ_8^8 *and 8P8S-topology* $\breve{\mathfrak{T}}_1$ *as given in Example 8.*
$\Lambda_{(11)} = \left\{\left\langle \frac{6}{\delta_2}, \{(\xi_1, 3), (\xi_2, 2)\}\right\rangle\right\} \widetilde{\sqsubseteq} \Lambda_8^8$*. The closed sets can be calculated as*

$$
\begin{aligned}
\Lambda_{(1)}^{\hat{b}} &= \mathfrak{S}_{N-1}^{M-1},\\
\Lambda_{(2)}^{\hat{b}} &= \left\{\left\langle \frac{0}{\delta_1}, \{(\xi_1,0),(\xi_2,7),(\xi_3,7)\}\right\rangle, \left\langle \frac{7}{\delta_2}, \{(\xi_1,7),(\xi_2,7)\}\right\rangle\right\},\\
\Lambda_{(7)}^{\hat{b}} &= \left\{\left\langle \frac{0}{\delta_1}, \{(\xi_1,0),(\xi_2,0),(\xi_3,7)\}\right\rangle, \left\langle \frac{7}{\delta_2}, \{(\xi_1,7),(\xi_2,7)\}\right\rangle\right\},\\
\Lambda_{(10)}^{\hat{b}} &= \left\{\left\langle \frac{0}{\delta_1}, \{(\xi_1,0),(\xi_2,0),(\xi_3,0)\}\right\rangle, \left\langle \frac{7}{\delta_2}, \{(\xi_1,7),(\xi_2,7)\}\right\rangle\right\},\\
\Lambda_{(32)}^{\hat{b}} &= \mathfrak{P}_0^0.
\end{aligned}
$$

The closed supersets of $\Lambda_{(18)}$ *are* $\Lambda_{(1)}^{\hat{b}}, \Lambda_{(2)}^{\hat{b}}, \Lambda_{(7)}^{\hat{b}}, \Lambda_{(10)}^{\hat{b}}$*. Hence*

$$
\begin{aligned}
\overline{\Lambda_{(11)}} &= \Lambda_{(1)}^{\hat{b}} \widetilde{\sqcap} \Lambda_{(2)}^{\hat{b}} \widetilde{\sqcap} \Lambda_{(7)}^{\hat{b}} \widetilde{\sqcap} \Lambda_{(10)}^{\hat{b}}\\
\overline{\Lambda_{(11)}} &= \left\{\left\langle \frac{0}{\delta_1}, \{(\xi_1,0),(\xi_2,0),(\xi_3,0)\}\right\rangle, \left\langle \frac{7}{\delta_2}, \{(\xi_1,7),(\xi_2,7)\}\right\rangle\right\}.
\end{aligned}
$$

Theorem 5. *Let* $(\Lambda_{N_1}^{M_1}, \breve{\mathfrak{T}})$ *be a MPNS-topological space and* $\Lambda_{N_2}^{M_2} \widetilde{\sqsubseteq} \Lambda_{N_1}^{M_1}$*.* $\Lambda_{N_2}^{M_2}$ *is a MPNS-closed set iff* $\Lambda_{N_2}^{M_2} = \overline{\Lambda_{N_2}^{M_2}}$*.*

Proof. The proof is obvious. □

Theorem 6. *Let* $(\Lambda_{N_1}^{M_1}, \breve{\mathfrak{T}})$ *is a MPNS-topological space and* $\Lambda_{N_2}^{M_2} \widetilde{\sqsubseteq} \Lambda_{N_1}^{M_1}$*. Then* $(\Lambda_{N_2}^{M_2})^\circ \widetilde{\sqsubseteq} \Lambda_{N_2}^{M_2} \widetilde{\sqsubseteq} \overline{\Lambda_{N_2}^{M_2}}$*.*

Proof. Indeed, $(\Lambda_{N_2}^{M_2})^\circ = \widetilde{\sqcup}\Lambda_{(\mathrm{i})} = \{\Lambda_{(\mathrm{i})} \in \breve{\mathfrak{T}}, \Lambda_{(\mathrm{i})} \widetilde{\sqsubseteq} \Lambda_{N_2}^{M_2}, \mathrm{i} \in \widehat{\mathcal{I}} \widetilde{\sqsubseteq} \aleph\}$. Then, $\Lambda_{(\mathrm{i})}(\xi) \widetilde{\sqsubseteq} \Lambda_{N_2}^{M_2}$ (ξ) and $\widetilde{\sqcup}_{\mathrm{i}\in\mathrm{i}}\Lambda_{(i)}(\xi) \widetilde{\sqsubseteq} \Lambda_N^M(\xi)$ for all $\xi \in \Lambda$. So $(\Lambda_{N_2}^{M_2})^\circ \widetilde{\sqsubseteq} \Lambda_{N_2}^{M_2}$. $\overline{\Lambda_{N_2}^{M_2}} = \widetilde{\sqcap}\{\Lambda_{(\mathrm{i})} : \Lambda_{(\mathrm{i})}^{\hat{b}} \in \breve{\mathfrak{T}}, \Lambda_{N_2}^{M_2} \widetilde{\sqsubseteq} \{\Lambda_{(\mathrm{i})}, \mathrm{i} \in \mathcal{J} \widetilde{\sqsubseteq} \aleph\}$. Then, $\Lambda_{N_2}^{M_2}(\xi) \widetilde{\sqsubseteq} \Lambda_{(\mathrm{i})}(\xi)$ and $\Lambda_{N_2}^{M_2}(\xi) \widetilde{\sqsubseteq} \widetilde{\sqcap}_{\mathrm{i}\in\mathcal{J}}\Lambda_{(\mathrm{i})}(\xi)$ for all $\xi \in \Lambda$. So $\Lambda_{N_2}^{M_2} \widetilde{\sqsubseteq} \overline{\Lambda_{N_2}^{M_2}}$. Hence, $(\Lambda_{N_2}^{M_2})^\circ \widetilde{\sqsubseteq} \Lambda_{N_2}^{M_2} \widetilde{\sqsubseteq} \overline{\Lambda_{N_2}^{M_2}}$.
□

Theorem 7. *Let* $(\Lambda_{N_1}^{M_1}, \breve{\mathfrak{T}})$ *be MPNS-topological space and* $\Lambda_{N_2}^{M_2}, \Lambda_{N_3}^{M_3} \widetilde{\sqsubseteq} \Lambda_{N_1}^{M_1}$*. Then,*

(1) $\overline{\left(\overline{\Lambda_{N_2}^{M_2}}\right)} = \overline{\Lambda_{N_2}^{M_2}}$

(2) $\Lambda_{N_3}^{M_3} \widetilde{\sqsubseteq} \Lambda_{N_2}^{M_2} \Rightarrow \overline{\Lambda_{N_3}^{M_3}} \widetilde{\sqsubseteq} \overline{\Lambda_{N_2}^{M_2}}$

Proof. (1) Let $\overline{\Lambda_{N_2}^{M_2}} = \Lambda_{N_4}^{M_4}$. Then, $\Lambda_{N_4}^{M_4}$ is a MPNS closed set. Therefore, $\Lambda_{N_4}^{M_4}$ and $\overline{\Lambda_{N_4}^{M_4}}$ are equal. Hence $\overline{\left(\overline{\Lambda_{N_2}^{M_2}}\right)} = \overline{\Lambda_{N_2}^{M_2}}$.

(2) Let $\Lambda^{M_3}_{N_3} \tilde{\sqsubseteq} \Lambda^{M_2}_{N_2}$. By the definition of a MPNS-closure, $\Lambda^{M_2}_{N_2} \tilde{\sqsubseteq} \overline{\Lambda^{M_2}_{N_2}}$ and $\Lambda^{M_3}_{N_3} \tilde{\sqsubseteq} \overline{\Lambda^{M_3}_{N_3}}$. $\overline{\Lambda^{M_2}_{N_2}}$ is the smallest MPNS-closed set that containing $\Lambda^{M_3}_{N_3}$. Then $\overline{\Lambda^{M_3}_{N_3}} \tilde{\sqsubseteq} \overline{\Lambda^{M_2}_{N_2}}$.
□

Corollary 1. *Let $\Lambda_{(\mathrm{i})}$ is any subset of MPNSS Λ^M_N then,*

(1) $\left((\Lambda_{(\mathrm{i})})^\circ\right)^{\hat{b}} = \overline{(\Lambda_{(\mathrm{i})})^{\hat{b}}}$

(2) $(\Lambda_{(\mathrm{i})})^\circ = (\Lambda_{(\mathrm{i})}) \backslash \overline{(\Lambda_{(\mathrm{i})})^{\hat{b}}}$

Definition 22. *Let $(\Lambda^{M_1}_{N_1}, \breve{\mathfrak{T}})$ be MPNS-topological space and $\Lambda^{M_2}_{N_2} \tilde{\sqsubseteq} \Lambda^{M_1}_{N_1}$, the frontier or boundary of $\Lambda^{M_2}_{N_2}$ is represented by $Fr(\Lambda^{M_2}_{N_2})$ and determined as,*

$$Fr(\Lambda^{M_2}_{N_2}) = \overline{\Lambda^{M_2}_{N_2}} \tilde{\sqcap} \overline{((\Lambda^{M_2}_{N_2})^{\hat{b}})}.$$

Example 15. *Consider 8P8SS(Λ^8_8) and 8P8S-topology $(\breve{\mathfrak{T}}_1)$ as given in Example 8. Let $\Lambda_{(11)} \tilde{\sqsubseteq} \Lambda^8_8$. Then*

$$\begin{aligned}
\overline{\Lambda_{(11)}} &= \left\{ \left\langle \frac{0}{\delta_1}, \{(\xi_1,0),(\xi_2,0),(\xi_3,0)\} \right\rangle, \left\langle \frac{7}{\delta_2}, \{(\xi_1,7),(\xi_2,7)\} \right\rangle \right\}, \\
(\Lambda_{(11)})^{\hat{b}} &= \left\{ \left\langle \frac{7}{\delta_1}, \{(\xi_1,7),(\xi_2,7),(\xi_3,7)\} \right\rangle, \left\langle \frac{0}{\delta_2}, \{(\xi_1,0),(\xi_2,0)\} \right\rangle \right\}, \\
\overline{(\Lambda_{(11)})^{\hat{b}}} &= \mathfrak{S}^{M-1}_{N-1}. \\
\textit{Thus} \ \ Fr(\Lambda_{(11)}) &= \overline{\Lambda_{(11)}} \tilde{\sqcap} \overline{\Lambda^{\hat{b}}_{(11)}} \\
&= \left\{ \left\langle \frac{0}{\delta_1}, \{(\xi_1,0),(\xi_2,0),(\xi_3,0)\} \right\rangle, \left\langle \frac{7}{\delta_2}, \{(\xi_1,7),(\xi_2,7)\} \right\rangle \right\} \tilde{\sqcap} \mathfrak{S}^{M-1}_{N-1} \\
&= \left\{ \left\langle \frac{0}{\delta_1}, \{(\xi_1,0),(\xi_2,0),(\xi_3,0)\} \right\rangle, \left\langle \frac{7}{\delta_2}, \{(\xi_1,7),(\xi_2,7)\} \right\rangle \right\}.
\end{aligned}$$

Theorem 8. *Let $\Lambda_{(\mathrm{i})}$ be a subset of MPNS-topological space $(\Lambda^M_N, \breve{\mathfrak{T}})$. Then*

(1) $\Lambda_{(\mathrm{i})} \tilde{\sqcup} Fr(\Lambda_{(\mathrm{i})}) = \overline{\Lambda_{(\mathrm{i})}}$

(2) $\Lambda_{(\mathrm{i})} \backslash Fr(\Lambda_{(\mathrm{i})}) = (\Lambda_{(\mathrm{i})})^\circ$

(3) $\Lambda_{(\mathrm{i})}$ *is open* $\Leftrightarrow \Lambda_{(\mathrm{i})} \tilde{\sqcap} Fr(\Lambda_{(\mathrm{i})}) = \mathfrak{P}^0_0$

(4) $\Lambda_{(\mathrm{i})}$ *is closed* $\Leftrightarrow Fr(\Lambda_{(\mathrm{i})}) \tilde{\sqsubseteq} \Lambda_{(\mathrm{i})}$

(5) $\Lambda_{(\mathrm{i})}$ *is both open and closed* $\Leftrightarrow Fr(\Lambda_{(\mathrm{i})}) = \mathfrak{P}^0_0$

Proof. It can be proved by using Definitions 20–22. □

Definition 23. *Let $(\Lambda^{M_1}_{N_1}, \breve{\mathfrak{T}})$ be MPNS-topological space and $\Lambda^{M_2}_{N_2} \tilde{\sqsubseteq} \Lambda^{M_1}_{N_1}$. The exterior of $\Lambda^{M_2}_{N_2}$ is indicated by $Ext(\Lambda^{M_2}_{N_2})$ and characterized as,*

$$Ext(\Lambda^{M_2}_{N_2}) = \left(\overline{(\Lambda^{M_2}_{N_2})}\right)^{\hat{b}}.$$

Theorem 9. *Let Λ^M_N be a MPNSS, Then*

(1) $Ext\left((\Lambda^M_N)^{\hat{b}}\right) = (\Lambda^M_N)^\circ$

(2) $Ext(\Lambda^M_N) = \left((\Lambda^M_N)^{\hat{b}}\right)^\circ$

Proof. (1) $\mathrm{Ext}((\Lambda_N^M)^{\hat{b}}) = \left(\overline{((\Lambda_N^M)^{\hat{b}})}\right)^{\hat{b}}$. Then $\mathrm{Ext}((\Lambda_N^M)^{\hat{b}}) = [((\Lambda_N^M)^{\hat{b}})^{\hat{b}}]^{\circ}$. Thus, $\mathrm{Ext}((\Lambda_N^M)^{\hat{b}}) = (\Lambda_N^M)^{\circ}$.

(2) Clearly $\mathrm{Ext}(\Lambda_N^M) = \left(\overline{(\Lambda_N^M)}\right)^{\hat{b}}$. Then $\mathrm{Ext}(\Lambda_N^M) = ((\Lambda_N^M)^{\hat{b}})^{\circ}$. □

5. MPNS-Topology Based MADM

MPNS topology is the generalization of soft topology and NS-topology. In this section, we execute the MPNS topology towards MADM to make a robust optimal decision. MPNS topology provides strong mathematical modeling towards uncertainty. The eminent characteristic of MPNS topology-based MADM is that the attributes and alternatives are analyzed by the decision makers (say) $\mathcal{D}_1, \mathcal{D}_2, ..., \mathcal{D}_n$ and their evaluations are represented in terms of MPNS-open sets (say) $\Lambda_{T_1}, \Lambda_{T_2}, \cdots \Lambda_{T_n}$. To meet these objectives, we present two algorithms named as Algorithms 1 and 2 and their corresponding real life applications. The flow chart of MPNS topology based method 1 is expressed by Algorithm 1 as follows.

Algorithm 1: (MPNS topology based method 1).

Step 1: Input $\Lambda = \{\xi_1, \xi_2, \cdots, \xi_n\}$ as a collection of objects, $Y = \{\delta_1, \delta_2, \cdots, \delta_k\}$ as a collection of attributes, and a team of decision makers

Step 2: Compute MPNSSs according to opinion of each decision expert with the help of information systems which assign attributes with feasible number of $\diamondsuit$, non-zero grading with † and zero grading with • to the alternatives.

Step 3: Construct MPNS topology $\tilde{\mathfrak{T}}$, where $\Lambda_{T_1}, \Lambda_{T_2}, \cdots \Lambda_{T_n}$ are MPNS-open sets of $\tilde{\mathfrak{T}}$ over the universal MPNSS (Λ_N^M).

Step 4: Compute the aggregate MPNSSs of all MPNS-open sets by using the formula,

$$\Lambda_{T_n}^{\star} = \left[\frac{\mathcal{L}(\xi_i)}{\xi_i} : \xi_i \in \Lambda\right], \text{ where } \mathcal{L}(\xi_i) = \sum_{j \in \mathcal{J} \tilde{\subseteq} \aleph} \mathcal{I}(\delta_j)\mathcal{I}(\xi_{ij}) \tag{1}$$

Step 5: Compute the sum of $\Lambda_{T_1}^{\star}, \Lambda_{T_2}^{\star}, \cdots \Lambda_{T_n}^{\star}$.

Step 6: Final the optimal alternative with maximum of aggregated values

$$\max \Lambda_{T_1 \oplus T_2 \cdots \oplus T_n}^{\star}(\xi_i).$$

The flow chart of Algorithm 1 is given in Figure 1.

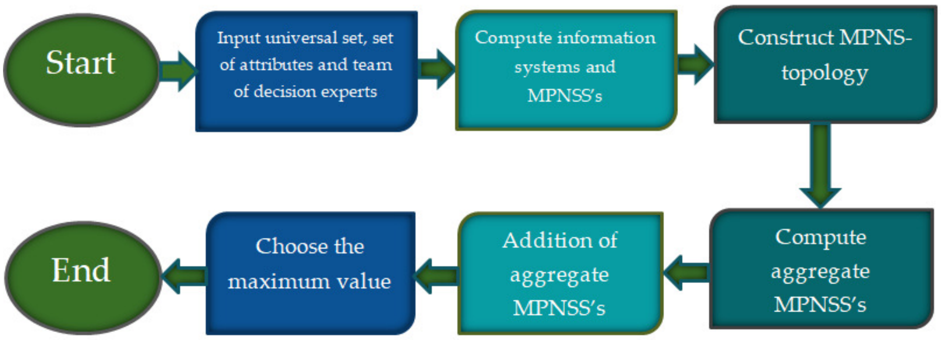

Figure 1. Flow chart of Algorithm 1.

MPNS topology based method 2 is expressed by Algorithm 2 as follows.

Algorithm 2: (MPNS topology based method 2).

Step 1: Input $\Lambda = \{\xi_1, \xi_2, \cdots, \xi_n\}$ as a collection of objects, $\Upsilon = \{\delta_1, \delta_2, \cdots, \delta_k\}$ as a collection of attributes, and a team of decision makers.
Step 2: Compute information systems with feasible number of $\diamondsuit$ to attributes, † for non-zero grade and • for zero grade to alternatives corresponding to the opinion of each decision expert and compute MPNSSs.
Step 3: Construct MPNS topology $\breve{\mathfrak{T}}$, where $\Lambda_{T_1}, \Lambda_{T_2}, \cdots \Lambda_{T_n}$ are MPNS-open sets of $\breve{\mathfrak{T}}$ over the universal MPNSS(Λ_N^M).
Step 4: Find the cardinal MPNSS of all MPNS-open sets by using the formula,

$$c\Lambda_{T_n} = \left[\frac{\mathfrak{o}(\delta_j)}{\delta_j} : \delta_j \in \Upsilon\right], \text{ where } \mathfrak{o}(\delta_j) = \sum_{\mathfrak{i} \in \widehat{\mathcal{I}} \sqsubseteq \aleph} \mathcal{I}(\delta_j)\mathcal{I}(\xi_{ij}). \quad (2)$$

Step 5: Find the aggregate MPNSSs by using the formula,

$$\mathcal{M}_{\Lambda^{\star}_{T_n}} = \mathcal{M}_{\Lambda_{T_n}} * \mathcal{M}^t_{c\Lambda_{T_n}}, \quad (3)$$

where $\mathcal{M}_{\Lambda_{T_n}}$, $\mathcal{M}^t_{c\Lambda_{T_n}}$ and $\mathcal{M}_{\Lambda^{\star}_{T_n}}$ are the matrices corresponding Λ_{T_n}, $c\Lambda_{T_n}$ and $\Lambda^{\star}_{T_n}$, respectively. The matrix $\mathcal{M}^t_{c\Lambda_{T_n}}$ represent transpose of the matrix $\mathcal{M}_{c\Lambda_{T_n}}$.
Step 6: Add $\Lambda^{\star}_{T_1}, \Lambda^{\star}_{T_2}, \cdots \Lambda^{\star}_{T_n}$ to find decision of MPNSS.
Step 7: Find the optimal decision by using max $\Lambda^{\star}_{T_1 \oplus T_2 \cdots \oplus T_n}(\xi_{\mathfrak{i}})$.

Flow chart of Algorithm 2 is given in Figure 2.

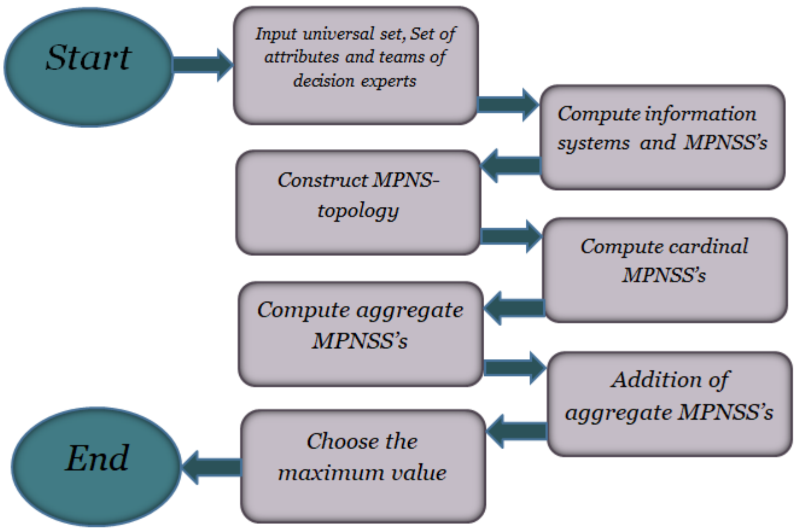

Figure 2. Flow chart of Algorithm 2.

Numerical Example

Floods normally are short-lived and local incidences that can occur all of sudden, often with no alerts. They are generally occur due to exquisite storms that develop more drain than a region can stream or store may carry inside its normal channel. Floods can also occur when ice jams, when dams fail or landslides provisionally obstruct a channel or when snow melts swiftly. In a more comprehensive manner, usually floods occur in dry lands by high tides, by high levels of lakes or by waves directed in the ground by stiff breeze. Some floods occur seasonally due to monsoon rains, fill river basins, along with melting snows. Pakistan continued to face crisis situation due to the destructive flood of 2011. In Sindh province, a disastrous flood entered in August 2011, presumed as the most severe in the history, molded extensive devastation and crowd out thousands of people and millions were badly affected. The province persisted disabled over the end of 2011, as the affected communities and government accomplished with overburdened funds and enormous financial damage. About 4.8 million people, in which children were half of the number were badly affected by the floods in Sindh and according to estimates that some 72,000 people inhabited in relief camps. Sindh was the most affected province where monsoon rains, swamped 22 districts. According to, National Disaster Management Authority (NDMA), the floods in Sindh have caused 756 injuries and 466 deaths, 1.5 million houses were damaged and 6.6 million acres of land was affected. Provincial Disaster Management Authority (PDMA) and Pakistan Red Crescent Society (PRCS) had been dispatched evaluation teams to the area, which illustrated a sketch of huge destruction, although because of unavailability of roads, had problems to carry out a comprehensive evaluation. The provincial branch of PRCS in Sindh aligned with PDMA. Specifically, due to Pakistan's dreadful economic condition, is conviction that total devastation of standing crops was about 10 million acres of land. An evaluated loss of 7 billion to Pakistan's land economy was caused by floods in 2011.

Step 1: Let $\Lambda = \{\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6, \xi_7, \xi_8\}$ be the collection of the worst affected districts of Sindh, where ξ_1 = Badin, ξ_2 = Dadu, ξ_3 = Khairpur, ξ_4 = Mirpurkhas, ξ_5 = Sh. Banazirabad, ξ_6 = Tharparkar, ξ_7 = Sanghar, ξ_8 = T. M. Khan. Let $\Upsilon = \{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_7, \delta_8\}$ be the set of decision variables, where

$$\begin{aligned}
\delta_1 &= \text{affected people(AP)}, \\
\delta_2 &= \text{damaged house (DH)}, \\
\delta_3 &= \text{died people(DP)}, \\
\delta_4 &= \text{damaged crop area (DCA)}, \\
\delta_5 &= \text{affected area (AA)}, \\
\delta_6 &= \text{affected villages (AV)}, \\
\delta_7 &= \text{affected taluka's (AT)} \\
\delta_8 &= \text{cattle head perished (CHP)}.
\end{aligned}$$

The great challenge of this problem is to get estimation of most affected area on the basis of grading assessment of decision experts in two teams, in order to distribute the resources and funding according to the damage level. Let $\mathcal{H} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $\Re = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be two grading sets.

Step 2: PRCS Sindh branch sent two rapid evaluation teams to get estimation of immediate requirements to make urgent progressive scheme in most affected district firstly. We consider two decision-makers (DMs) and made two separate teams for assessment and analysis of consequences of flood. Both teams gave the report about the situation of badly affected districts in accordance with chosen subsets by team-T_1 and team-T_2 in terms of sets, in which grades are given to the attributes. i.e $\mathcal{A}_{T_1} = \left\{\frac{4}{\delta_1}, \frac{8}{\delta_2}, \frac{6}{\delta_3}, \frac{7}{\delta_4}\right\}$ and $\mathcal{B}_{T_2} = \left\{\frac{3}{\delta_1}, \frac{2}{\delta_2}, \frac{4}{\delta_4}\right\}$, respectively. After a complete research both teams construct 10P10SS's, Λ_{T_1} and Λ_{T_2} over Λ. First we construct a 10P10SS over Λ namely Λ_{10}^{10} on the assessment of other departments of different institutions, feed back of people of affected areas and according to demand of assessment teams of PRCS. The information system corresponding to collected data from other resources and people of affected areas is given in Table 10 and its matrix form is given in Table 11.

$$\begin{aligned}
\Lambda_{10}^{10} = \Big\{ & \Big\langle \frac{4}{\delta_1}, \{(\xi_1,5),(\xi_2,7),(\xi_3,4),(\xi_4,6),(\xi_5,7),(\xi_6,3),(\xi_7,1),(\xi_8,2)\} \Big\rangle, \\
& \Big\langle \frac{8}{\delta_2}, \{(\xi_1,4),(\xi_2,4),(\xi_3,5),(\xi_4,3),(\xi_5,6),(\xi_6,2),(\xi_7,5),(\xi_8,7)\} \Big\rangle, \\
& \Big\langle \frac{6}{\delta_3}, \{(\xi_1,5),(\xi_2,2),(\xi_3,3),(\xi_4,6),(\xi_5,5),(\xi_6,4),(\xi_7,4),(\xi_8,6)\} \Big\rangle, \\
& \Big\langle \frac{7}{\delta_4}, \{(\xi_1,3),(\xi_2,3),(\xi_3,2),(\xi_4,5),(\xi_5,6),(\xi_6,5),(\xi_7,1),(\xi_8,5)\} \Big\rangle, \\
& \Big\langle \frac{9}{\delta_5}, \{(\xi_1,3),(\xi_2,1),(\xi_3,3),(\xi_4,7),(\xi_5,5),(\xi_6,6),(\xi_7,4),(\xi_8,9)\} \Big\rangle, \\
& \Big\langle \frac{5}{\delta_6}, \{(\xi_1,6),(\xi_2,2),(\xi_3,4),(\xi_4,8),(\xi_5,5),(\xi_6,4),(\xi_7,3),(\xi_8,7)\} \Big\rangle, \\
& \Big\langle \frac{3}{\delta_7}, \{(\xi_1,6),(\xi_2,2),(\xi_3,4),(\xi_4,1),(\xi_5,3),(\xi_6,4),(\xi_7,3),(\xi_8,7)\} \Big\rangle, \\
& \Big\langle \frac{2}{\delta_8}, \{(\xi_1,5),(\xi_2,3),(\xi_3,1),(\xi_4,8),(\xi_5,4),(\xi_6,2),(\xi_7,5),(\xi_8,6)\} \Big\rangle \Big\}
\end{aligned}$$

Table 10. Information system obtained by different resouces in terms of 10P10SS.

Y → **Λ** ↓	$\boldsymbol{\delta_1}$ ◇◇◇◇	$\boldsymbol{\delta_2}$ ◇◇◇◇◇◇◇◇	$\boldsymbol{\delta_3}$ ◇◇◇◇◇◇	$\boldsymbol{\delta_4}$ ◇◇◇◇◇◇◇
ξ_1	†††††	††††	†††††	†††
ξ_2	†††††††	††††	††	†††
ξ_3	††††	†††††	†††	††
ξ_4	††††††	†††	††††††	†††††
ξ_5	†††††††	††††††	†††††	††††††
ξ_6	†††	††	††††	†††††
ξ_7	†	†††††	††††	†
ξ_8	††	†††††††	††††††	†††††
	δ_5 ◇◇◇◇◇◇◇◇◇	δ_6 ◇◇◇◇◇	δ_7 ◇◇◇	δ_8 ◇◇
ξ_1	†††	††††††	††††††	†††††
ξ_2	†	††	††	†††
ξ_3	†††	††††	††††	†
ξ_4	†††††††	††††††††	†	††††††††
ξ_5	†††††	†††††	†††	††††
ξ_6	††††††	††††	††††	††
ξ_7	††††	†††	†††	†††††
ξ_8	†††††††††	†††††††	†††††††	††††††

Table 11. Tabular representation of 10P10SS Λ_{10}^{10}.

$\boldsymbol{\Lambda_{10}^{10}}$	$\frac{4}{\delta_1}$	$\frac{8}{\delta_2}$	$\frac{6}{\delta_3}$	$\frac{7}{\delta_4}$	$\frac{9}{\delta_5}$	$\frac{5}{\delta_6}$	$\frac{3}{\delta_7}$	$\frac{2}{\delta_8}$
ξ_1	5	4	5	3	3	6	6	5
ξ_2	7	4	2	3	1	2	2	3
ξ_3	4	5	3	2	3	4	4	1
ξ_4	6	3	6	5	7	8	1	8
ξ_5	7	6	5	6	5	5	3	4
ξ_6	3	2	4	5	6	4	4	2
ξ_7	1	5	4	1	4	3	3	5
ξ_8	2	7	6	5	9	7	7	6

The information system corresponding to team of decision experts, T_1 is shown in Table 12 and its matrix form is shown in Table 13.

Table 12. Information system provided by decision team T_1 in terms of 10P10SS.

$\mathbf{Y} \rightarrow$ $\mathbf{\Lambda} \downarrow$	δ_1 ◇◇◇◇	δ_2 ◇◇◇◇◇◇◇◇	δ_3 ◇◇◇◇◇◇	δ_4 ◇◇◇◇◇◇◇
ξ_1	†††††	††††	†††	†††
ξ_2	†††††††	††††	†††	•
ξ_3	††††	•	††	•
ξ_4	†††††	•	†††††	†††††
ξ_5	•	•	††††††	•
ξ_6	†††	††	•	•
ξ_7	•	†††††	•	†
ξ_8	•	•	•	•

Table 13. Tabular representation of Λ_{T_1}.

$\mathbf{\Lambda}_{T_1}$	$\frac{4}{\delta_1}$	$\frac{8}{\delta_2}$	$\frac{6}{\delta_3}$	$\frac{7}{\delta_4}$
ξ_1	5	4	3	3
ξ_2	7	4	3	0
ξ_3	4	0	2	0
ξ_4	5	0	5	5
ξ_5	0	0	6	0
ξ_6	3	2	0	0
ξ_7	0	5	0	1
ξ_8	0	0	0	0

The information system corresponding to team of decision experts T_2 is shown in Table 14 and its matrix form is given in Table 15.

Table 14. Information system provided by decision team T_2 in terms of 10P0SS.

$\mathbf{Y} \rightarrow$ $\mathbf{\Lambda} \downarrow$	δ_1 ◇◇◇	δ_2 ◇◇	δ_4 ◇◇◇◇
ξ_1	††††	†††	††
ξ_2	†††††	††	•
ξ_3	††	•	•
ξ_4	•	•	†††
ξ_5	•	•	•
ξ_6	•	†	•
ξ_7	•	•	†
ξ_8	•	•	•

Table 15. Tabular representation of Λ_{T_2}.

$\mathbf{\Lambda}_{T_2}$	$\frac{3}{\delta_1}$	$\frac{2}{\delta_2}$	$\frac{4}{\delta_4}$
ξ_1	4	3	2
ξ_2	5	2	0
ξ_3	2	0	0
ξ_4	0	0	3
ξ_5	0	0	0
ξ_6	0	1	0
ξ_7	0	0	1
ξ_8	0	0	0

Step 3: Now we construct a 10P10S-topology as

$$\breve{\mathfrak{T}} = \left\{ \mathfrak{P}_0^0, \Lambda_{T_1}, \Lambda_{T_2}, \Lambda_{10}^{10} \right\},$$

Step 4: Computing aggregate 10P10SS's of all 10P10S-open sets by using Equation (1), given by

$$
\begin{aligned}
(\mathfrak{P}_0^0)^\star &= \left\{\frac{0}{\xi_1}, \frac{0}{\xi_2}, \frac{0}{\xi_3}, \frac{0}{\xi_4}, \frac{0}{\xi_5}, \frac{0}{\xi_6}, \frac{0}{\xi_7}\right\} \\
\Lambda_{T_1}^\star &= \left\{\frac{91}{\xi_1}, \frac{78}{\xi_2}, \frac{28}{\xi_3}, \frac{85}{\xi_4}, \frac{36}{\xi_5}, \frac{35}{\xi_6}, \frac{47}{\xi_7}, \frac{0}{\xi_8}\right\}, \\
\Lambda_{T_2}^\star &= \left\{\frac{26}{\xi_1}, \frac{19}{\xi_2}, \frac{6}{\xi_3}, \frac{12}{\xi_4}, \frac{0}{\xi_5}, \frac{2}{\xi_6}, \frac{4}{\xi_7}, \frac{0}{\xi_8}\right\}, \\
(\Lambda_{10}^{10})^\star &= \left\{\frac{188}{\xi_1}, \frac{124}{\xi_2}, \frac{149}{\xi_3}, \frac{241}{\xi_4}, \frac{235}{\xi_5}, \frac{177}{\xi_6}, \frac{145}{\xi_7}, \frac{284}{\xi_8}\right\}.
\end{aligned}
$$

Step 5: By adding $\Lambda_{T_1}^\star$ and $\Lambda_{T_2}^\star$, we obtain the final decision. There is unnecessary to incorporate the aggregate 10P10S-sets of $\mathfrak{P}_0^0$ and Λ_{10}^{10}. By adding the aggregate 10P10SS's, $(\mathfrak{P}_0^0)^\star$ and $(\Lambda_{10}^{10})^\star$ to the sum of $\Lambda_{T_1}^\star$ and $\Lambda_{T_2}^\star$, we get the same ranking. Hence there is no need to include these two sets. We have

$$\Lambda_{T_1 \oplus T_2}^\star(\xi_i) = \Lambda_{T_1}^\star(\xi_i) + \Lambda_{T_2}^\star(\xi_i), \ \forall \xi_i \in \Lambda.$$

This shows that

$$\Lambda_{T_1}^\star(\xi_i) \oplus \Lambda_{T_2}^\star(\xi_i) = \left\{\frac{117}{\xi_1}, \frac{97}{\xi_2}, \frac{34}{\xi_3}, \frac{97}{\xi_4}, \frac{36}{\xi_5}, \frac{37}{\xi_6}, \frac{51}{\xi_7}, \frac{0}{\xi_8}\right\}.$$

Step 6: By taking maximum of grading values, we obtain the optimal decision as,

$$\max \Lambda_{T_1 \oplus T_2}^\star(\xi_i) = 117$$

The greatest aggregated value is 117. This shows that ξ_1 = Badin is most affected district than others. PRCS, Sindh Branch responded rapidly through its district branches. Teams comprising volunteers and trained staff in emergency relief, first assistance were posted to the badly affected areas within 24 h to implement quick requirement evaluations and deliver humanitarian assistance. Now we solve the same problem by using proposed Algorithm 2. First 3 steps are same as calculated in Algorithm 1. In Algorithm 2, we proceed from step 4.

Step 7: Now finding the cardinal 10P10SS's of all 10P10SS's by using Equation (2), given by

$$
\begin{aligned}
c\Lambda_{T_1} &= \left\{\frac{96}{\delta_1}, \frac{120}{\delta_2}, \frac{114}{\delta_3}, \frac{63}{\delta_4}\right\}, \\
c\Lambda_{T_2} &= \left\{\frac{33}{\delta_1}, \frac{12}{\delta_2}, \frac{24}{s_3}\right\}, \\
c\mathfrak{P}_0^0 &= \left\{\frac{0}{\delta_1}, \frac{0}{\delta_2}, \frac{0}{\delta_3}, \frac{0}{\delta_4}, \frac{0}{\delta_5}, \frac{0}{\delta_6}, \frac{0}{\delta_7}\right\}
\end{aligned}
$$

$$c\Lambda_{10}^{10} = \left\{\frac{140}{\delta_1}, \frac{288}{\delta_2}, \frac{210}{\delta_3}, \frac{210}{\delta_4}, \frac{342}{\delta_5}, \frac{195}{\delta_6}, \frac{90}{\delta_7}, \frac{68}{\delta_8}\right\}.$$

Step 8: Then we find out the matrix of $\Lambda_{T_1}^\star$ by using Equation (3).

$$\mathcal{M}_{\Lambda^{\star}_{T_1}} = \begin{bmatrix} 5 & 4 & 3 & 3 \\ 7 & 4 & 3 & 0 \\ 4 & 0 & 2 & 3 \\ 5 & 0 & 5 & 5 \\ 0 & 0 & 6 & 0 \\ 3 & 2 & 0 & 0 \\ 0 & 5 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 96 \\ 120 \\ 114 \\ 63 \end{bmatrix} = \begin{bmatrix} 1491 \\ 1494 \\ 801 \\ 1365 \\ 684 \\ 528 \\ 663 \\ 0 \end{bmatrix}$$

that means, $\Lambda^{\star}_{T_1} = \left\{ \frac{1491}{\xi_1}, \frac{1494}{\xi_2}, \frac{801}{\xi_3}, \frac{1365}{\xi_4}, \frac{684}{\xi_5}, \frac{528}{\xi_6}, \frac{663}{\xi_7}, \frac{0}{\xi_7} \right\}$. Similarly, we can find the aggregate 10P10SS for Λ_{T_2} given as,

$$\mathcal{M}_{\Lambda^{\star}_{T_2}} = \begin{bmatrix} 4 & 3 & 2 \\ 5 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 33 \\ 12 \\ 24 \end{bmatrix} = \begin{bmatrix} 216 \\ 189 \\ 66 \\ 72 \\ 0 \\ 12 \\ 24 \\ 0 \end{bmatrix}$$

that means, $\Lambda^{\star}_{T_2} = \left\{ \frac{216}{\sigma_1}, \frac{189}{\xi_2}, \frac{66}{\xi_3}, \frac{72}{\sigma_4}, \frac{0}{\xi_5}, \frac{12}{\xi_6}, \frac{24}{\xi_7}, \frac{0}{\xi_8} \right\}$.

Step 9: Now we find the final decision 10P10SS by adding $\Lambda^{\star}_{T_1}$ and $\Lambda^{\star}_{T_2}$ only because there is no need to add $(\mathfrak{P}^0_0)^{\star}$ and $(\Lambda^{10}_{10})^{\star}$.

$$\Lambda^{\star}_{T_1 \oplus T_2}(\xi_i) = \Lambda^{\star}_{T_1}(\xi_i) + \Lambda^{\star}_{T_2}(\xi_i), \ \ \forall \xi_i \in \Lambda.$$
$$\Lambda^{\star}_{T_1} \oplus \Lambda^{\star}_{T_2}(\xi_i) = \left\{ \frac{1707}{\xi_1}, \frac{1683}{\xi_2}, \frac{867}{\xi_3}, \frac{1437}{\xi_4}, \frac{684}{\xi_5}, \frac{540}{\xi_6}, \frac{687}{\xi_7}, \frac{0}{\xi_8} \right\}.$$

Step 10: The optimal decision is obtained by taking maximum of final aggregated values as,

$$\max \Lambda^{\star}_{T_1 \oplus T_2}(\xi_i) = 1707$$

This implies that the district ξ_1 = Badin has highest grading value and according to Algorithms 1 and 2 ξ_1 = Badin is the badly affected district. 2011 floods mobile health units morbidity surveillance is shown in Figure 3.

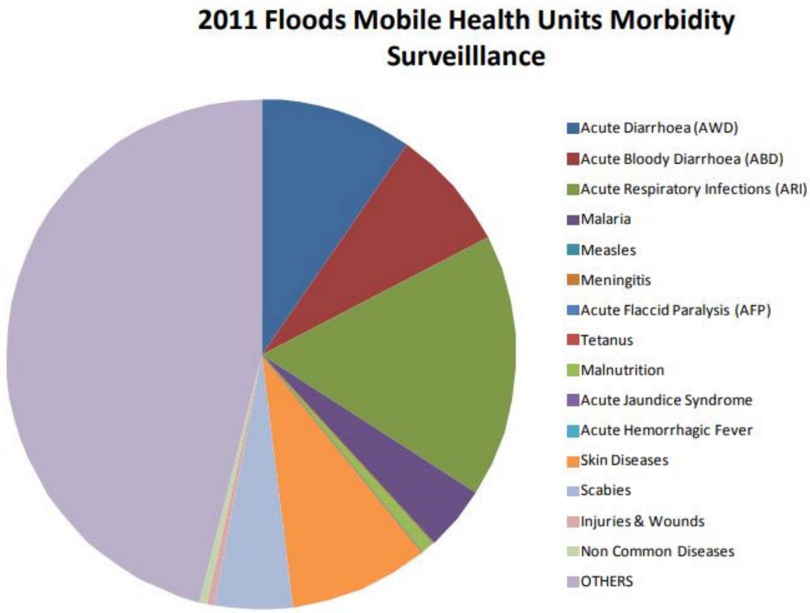

Figure 3. Source: www.ifrc.org.pk or www.ifrc.org/docs/Appeals/11/MDRPK007FR.pdf.

PRCS had intensified the efforts of healthcare by conducting the sessions of health education of mobile health units in Dadu, Badin, and Benazirabad. The most essential food items (FI's) and non-food items (NFI's), as contemplated by PRCS, guided the arrangement of the items supplied to affected families, as given in Figure 4.

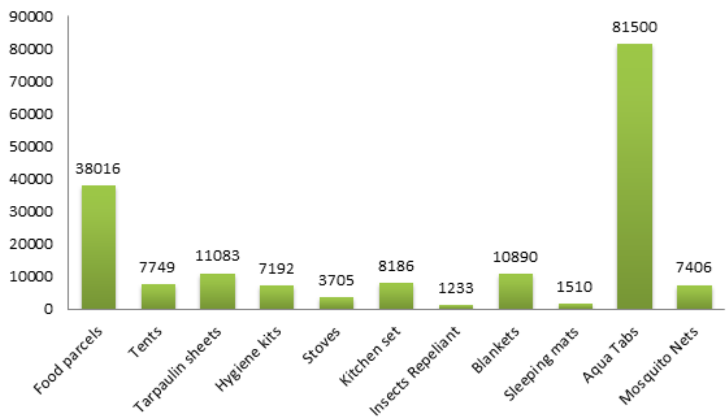

Figure 4. Assistance provided by PRCS, Sindh Branch from 19 August 2011 to 24 November 2011 (Source: www.prcs.org.pk, accessed on 1 January 2021.)

6. TOPSIS Method under M-Parameterized N-Soft Topology

Many researchers thoroughly investigated the multi-attribute decision making (MADM). The established methods particularly relay on the nature of problem under consideration. There are large number of vague, imperfect and uncertain realistic issues. In this section, we discuss how MPNS topology is useful in MADM, to cope with such real life circumstances. We develop TOPSIS method under MPNSSs and MPNS topology for MADM. TOPSIS method is strong and powerful approach for critical decision analysis to estimate the losses, constructed extensive damage and moving thousands of people and millions of people in worst affected districts in Sindh province in the course of flooding of August 2011. The linguistic variables, according to importance of attribute and the condition of most affected areas/alternatives are given below.

Step 1: Identification of decision problem:
Consider $\mathfrak{T} = \{T_i, i \in I_n\}$ is a collection of teams of decision experts, $\Lambda = \{\xi_j, j \in$

$\mathcal{J}\}$ is a collection of alternatives, $\mathcal{H} = \{0, 1, 2, 3, ...N - 1\}$, $\Re = \{0, 1, 2, 3, ...M - 1\}$ be two grading sets, $\Upsilon = \{\delta_k : k \in K_m\}$ is the set of evaluation attrbiutes.

Step 2: By choosing linguistic variables from Table 16, construct weighted parameterized matrix,

$$
\pounds = \begin{bmatrix} \check{\mathbf{V}}_\mathbf{1} & \check{\mathbf{V}}_\mathbf{2} & \cdots & \check{\mathbf{V}}_\mathbf{m} \\ \mathcal{I}_{11} & \mathcal{I}_{12} & \cdots & \mathcal{I}_{1m} \\ \mathcal{I}_{21} & \mathcal{I}_{12} & \cdots & \mathcal{I}_{2m} \\ \vdots & \vdots & & \vdots \\ \mathcal{I}_{\mathrm{i}1} & \mathcal{I}_{\mathrm{i}2} & \cdots & \mathcal{I}_{im} \\ \vdots & \vdots & & \vdots \\ \mathcal{I}_{n1} & \mathcal{I}_{n2} & \cdots & \mathcal{I}_{nm} \end{bmatrix}
$$

Decision experts $(\mathcal{D}_\mathrm{i})$ assigned grades, row-wise to each parameter, represented by $\mathcal{I}_{ik}$ by using the linguistic variables. In all matrices, the first row (in bold letters) represents the grading values, assigned to parameters by chairman of PRCS according to the surveyed data of teams of other departments, by using linguistic variables from Table 17.

Table 16. Linguistic terms for alternatives.

Linguistic Terms	Grading Values
Worst (W)	9, 8
Very Bad (VB)	7
Bad (B)	6
Intermediate (I)	5, 4
Safe (S)	3
Very safe (VS)	2
Completely safe (CS)	1, 0

Table 17. Linguistic terms for attributes.

Linguistic Terms	Grading Values
Very Important (VI)	8, 9
Important (I)	6, 7
Medium (M)	3, 4, 5
Less important (LI)	1, 2
Not importantt (NI)	0

Step 3: Creating normalized weighted parameterized matrix $\mathfrak{U}$,

$$
\mathfrak{U} = \begin{bmatrix} \varpi_{11} & \varpi_{12} & \cdots & \varpi_{1m} \\ \varpi_{21} & \varpi_{12} & \cdots & \varpi_{2m} \\ \vdots & \vdots & & \vdots \\ \varpi_{\mathrm{i}1} & \varpi_{\mathrm{i}2} & \cdots & \varpi_{im} \\ \vdots & \vdots & & \vdots \\ \varpi_{n1} & \varpi_{n2} & \cdots & \varpi_{nm} \end{bmatrix} = [\varpi_{ik}]_{n \times m}
$$

where

$$
\varpi_{ik} = \frac{\mathcal{I}_{ik} \times \nabla_k}{\sqrt{\sum_{a=1}^{n} \mathcal{I}_{ak}^2}} \tag{4}
$$

Step 4: Creating weight vector $\mathfrak{W} = (\mathbb{W}_1, \mathbb{W}_2, \mathbb{W}_3, \cdots, \mathbb{W}_m)$ by using the expression

$$
\mathbb{W}_k = \frac{\mathfrak{w}_k}{\sum_{a=1}^{n} \mathfrak{w}_a}, \quad \mathfrak{w}_k = \frac{1}{n} \sum_{\mathrm{i}=1}^{n} \varpi_{ik} \tag{5}
$$

Step 5: Constructing MPNS-decision matrices $T_{\mathfrak{i}}$ for each team such that all $T_{\mathfrak{i}}$ make MPNS topology,

$$T_{\mathfrak{i}} = \begin{bmatrix} \check{\mathbf{V}}_\mathbf{1} & \check{\mathbf{V}}_\mathbf{2} & \check{\mathbf{V}}_\mathbf{3} & \dots & \check{\mathbf{V}}_\mathbf{m} \\ \varrho_{11} & \varrho_{12} & \varrho_{13} & \cdots & \varrho_{1m} \\ \varrho_{21} & \varrho_{22} & \varrho_{23} & \cdots & \varrho_{2m} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \varrho_{j1} & \varrho_{j2} & \varrho_{j3} & \cdots & \varrho_{jm} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \varrho_{\ell m} & \varrho_{\ell m} & \varrho_{\ell m} & \cdots & \varrho_{\ell m} \end{bmatrix}$$

Here ϱ_{jk} are MPNS-elements.

Step 6: The aggregated matrix can be calculated as,

$$\begin{aligned} ¥ &= T_1 \oplus T_2 \oplus T_3, ..., T_n \\ &= \begin{bmatrix} \widehat{\nabla}_1 & \widehat{\nabla}_2 & \widehat{\nabla}_3 & \dots & \widehat{\nabla}_m \\ \dot{\zeta}_{11} & \dot{\zeta}_{12} & \dot{\zeta}_{13} & \cdots & \dot{\zeta}_{1m} \\ \dot{\zeta}_{21} & \dot{\zeta}_{22} & \dot{\zeta}_{23} & \cdots & \dot{\zeta}_{2m} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \dot{\zeta}_{j1} & \dot{\zeta}_{j2} & \dot{\zeta}_{j3} & \cdots & \dot{\zeta}_{jm} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \dot{\zeta}_{\ell m} & \dot{\zeta}_{\ell m} & \dot{\zeta}_{\ell m} & \cdots & \dot{\zeta}_{\ell m} \end{bmatrix} \\ &= [\dot{\zeta}_{jk}]_{\ell \times m} \end{aligned}$$

Step 7: Constructing the final weighted decision matrix,

$$\Omega = \begin{bmatrix} \vartheta_{11} & \vartheta_{12} & \cdots & \vartheta_{1n} \\ \vartheta_{21} & \vartheta_{12} & \cdots & \vartheta_{2n} \\ \vdots & \vdots & & \vdots \\ \vartheta_{j1} & \vartheta_{j2} & \cdots & \vartheta_{jn} \\ \vdots & \vdots & & \vdots \\ \vartheta_{l1} & \vartheta_{l2} & \cdots & \vartheta_{lm} \end{bmatrix} = [\vartheta_{jk}]_{l \times m}$$

where

$$\vartheta_{jk} = \mathbb{W}_k \dot{\zeta}_{jk} \widehat{\nabla}_k \tag{6}$$

Step 8: Now finding positive ideal solution (PIS) and negative ideal solution (NIS).

$$\text{PIS} = \{\vartheta_1^+, \vartheta_2^+, \vartheta_3^+, \cdots, \vartheta_j^+ \cdots, \vartheta_l^+\} = \{\max(\vartheta_{jk}) : j \in \ell\} \tag{7}$$

$$\text{NIS} = \{\vartheta_1^-, \vartheta_2^-, \vartheta_3^-, \cdots, \vartheta_j^- \cdots, \vartheta_l^-\} = \{\min(\vartheta_{jk}) : j \in \ell\} \tag{8}$$

Step 9: Calulating separation measurements $\check{\mathbb{S}}^+$ and $\check{\mathbb{S}}^-$ of PIS and NIS, respectively, for each parameter by making use of

$$\check{\mathbb{S}}_j^+ = \sqrt{\sum_{k=1}^{m} (\vartheta_{jk} - \vartheta_j^+)^2}, \quad \forall\, j \in \ell \tag{9}$$

and

$$\check{\mathbb{S}}_j^- = \sqrt{\sum_{k=1}^{m} (\vartheta_{jk} - \vartheta_j^-)^2}, \quad \forall\, j \in \ell \tag{10}$$

Step 10: Calculating the relative closeness,

$$\mathfrak{R}_j^+ = \frac{\check{\mathbb{S}}_j^-}{\check{\mathbb{S}}_j^- + \check{\mathbb{S}}_j^+}, \quad 0 \leq \mathfrak{R}_j^+ \leq 1, \quad \forall j \in \ell \tag{11}$$

Step 11: Ranking the alternatives in descending order. The optimal choice would be the alternative with largest value of $\mathfrak{R}_j^+$.

Figure 5 shows the the flow chart of MPNS topology based TOPSIS.

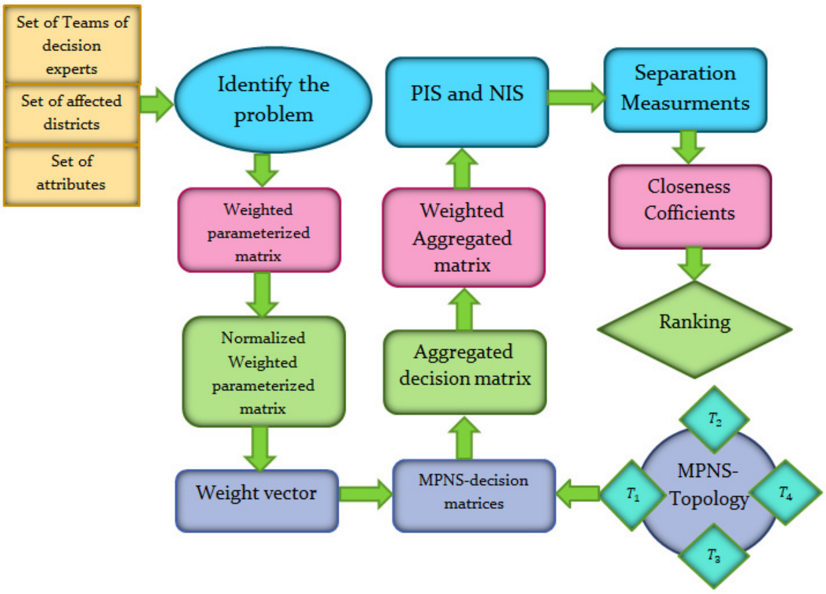

Figure 5. Flow chart of TOPSIS method under MPNS topology.

6.1. Numerical Example

Step 1: Let $\Lambda = \{\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6, \xi_7, \xi_8\}$ is a collection of the badly affected districts of Sindh, where ξ_1 = Badin, ξ_2 = Dadu, ξ_3 = Khairpur, ξ_4 = Mirpurkhas, ξ_5 = Sh.Banazirabad, ξ_6 = Tharparkar, ξ_7 = Sanghar, ξ_8 = T.M Khan. Let $Y = \{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_7, \delta_8\}$ be the set of evaluation attributes, where

$$\begin{aligned}
\delta_1 &= (\text{Affected People (AP)}, 4), \\
\delta_2 &= (\text{Damaged House (DH)}, 8), \\
\delta_3 &= (\text{Died People (DP)}, 6), \\
\delta_4 &= (\text{Damaged Crop Area (DCA)}, 7), \\
\delta_5 &= (\text{Affected Area (AA)}, 9), \\
\delta_6 &= (\text{Affected Villages (AV)}, 5), \\
\delta_7 &= (\text{Affected Taluka's (AT)}, 3), \\
\delta_8 &= (\text{Cattle Head Perished (CHP)}, 2).
\end{aligned}$$

The major challenge is to estimate which district/area is most affected on the basis of grading values of decision experts in two teams, so as to allocate the funds accordingly to the level of damage. Let $\mathcal{H} = \{0,1,2,3,4,5,6,7,8,9\}$ and $\Re = \{0,1,2,3,4,5,6,7,8,9\}$ be two grading sets.

Step 2: By choosing linguistic terms from Tables 17 and 16, constructing weighted parameterized matrix

$$£ = \begin{bmatrix} \mathbf{M} & \mathbf{VI} & \mathbf{I} & \mathbf{I} & \mathbf{VI} & \mathbf{M} & \mathbf{M} & \mathbf{LI} \\ M & M & M & M & M & I & I & M \\ I & M & LI & M & LI & LI & LI & M \\ M & M & M & LI & M & M & M & LI \\ I & M & I & M & I & VI & LI & VI \\ I & I & M & I & M & M & M & M \\ M & LI & M & M & I & M & M & LI \\ LI & M & M & LI & M & M & M & M \\ LI & I & I & M & VI & I & I & I \end{bmatrix}$$

$$£ = \begin{bmatrix} \mathbf{4} & \mathbf{8} & \mathbf{6} & \mathbf{7} & \mathbf{9} & \mathbf{5} & \mathbf{3} & \mathbf{2} \\ 5 & 4 & 5 & 3 & 3 & 6 & 6 & 5 \\ 7 & 4 & 2 & 3 & 1 & 2 & 2 & 3 \\ 4 & 5 & 3 & 2 & 3 & 4 & 4 & 1 \\ 6 & 3 & 6 & 5 & 7 & 8 & 1 & 8 \\ 7 & 6 & 5 & 6 & 5 & 5 & 3 & 4 \\ 3 & 2 & 4 & 5 & 6 & 4 & 4 & 2 \\ 1 & 5 & 4 & 1 & 4 & 3 & 3 & 5 \\ 2 & 7 & 6 & 5 & 9 & 7 & 7 & 6 \end{bmatrix}$$

Decision experts $(\mathcal{D}_i)$ of assessment teams of PRCS, assigned grades to each evaluation attribute, represented by $\mathcal{I}_{ik}$ by using the linguistic variables. In all matrices, first row (in bold letters) represents the grading values, assigned to evaluation attributes by chairman of PRCS according to the information of teams of other departments, by using linguistic variables given in Table 17.

Step 3: The normalized weighted parameterized matrix $\mathfrak{U}$, by using Equation (4) is given as,

$$\mathfrak{U} = \begin{bmatrix} 1.4547 & 0.3851 & 2.3214 & 1.8141 & 1.7960 & 2.0272 & 1.5212 & 0.7453 \\ 2.0367 & 0.3851 & 0.9285 & 1.8141 & 0.5986 & 0.6757 & 0.5070 & 0.4472 \\ 1.1638 & 2.9814 & 1.3928 & 1.2094 & 1.7960 & 1.3514 & 1.0141 & 0.1490 \\ 1.7457 & 1.7888 & 2.7857 & 3.0235 & 4.1906 & 2.7029 & 0.2535 & 1.1925 \\ 2.0367 & 3.5777 & 2.3214 & 3.6282 & 2.9933 & 1.6893 & 0.7606 & 0.5962 \\ 0.8728 & 1.1925 & 1.8571 & 3.0235 & 3.5920 & 1.3514 & 1.0141 & 0.2981 \\ 0.2909 & 2.9814 & 1.8571 & 0.6047 & 2.3946 & 1.0136 & 0.7606 & 0.7453 \\ 0.5819 & 4.1739 & 2.7857 & 3.0235 & 5.3880 & 2.3650 & 1.7748 & 0.8944 \end{bmatrix}$$

Step 4: The weight vector by using Equation (5) is given as,
$\mathfrak{W} = (0.0920, 0.1578, 0.1468, 0.1639, 0.2056, 0.1190, 0.0687, 0.0458)$

Step 5: The 10P10S-decision matrices T_i of two teams are given in which each row represents alternatives and each column represents evaluation attributes and all T_i make 10P10S-topology. There is no need to write null matrix and universal matrix for 10P10S-topology.

$$T_1 = \begin{bmatrix} \mathbf{4} & \mathbf{8} & \mathbf{6} & \mathbf{7} \\ 5 & 4 & 3 & 3 \\ 7 & 4 & 3 & 0 \\ 4 & 0 & 2 & 0 \\ 5 & 0 & 5 & 5 \\ 0 & 0 & 6 & 0 \\ 3 & 2 & 0 & 0 \\ 0 & 5 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} \mathbf{3} & \mathbf{2} & \mathbf{4} \\ 4 & 3 & 2 \\ 5 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Step 6: The Aggregated matrix ¥ obtained as,

$$¥ = \begin{bmatrix} \mathbf{9} & \mathbf{9} & \mathbf{9} & \mathbf{9} & \mathbf{9} & \mathbf{5} & \mathbf{3} & \mathbf{2} \\ 9 & 9 & 8 & 9 & 3 & 6 & 6 & 5 \\ 9 & 9 & 5 & 3 & 1 & 2 & 2 & 3 \\ 9 & 5 & 5 & 2 & 3 & 4 & 4 & 1 \\ 9 & 3 & 9 & 9 & 7 & 8 & 1 & 8 \\ 7 & 6 & 9 & 6 & 5 & 5 & 3 & 4 \\ 6 & 5 & 4 & 5 & 6 & 4 & 4 & 2 \\ 1 & 9 & 4 & 3 & 4 & 3 & 3 & 5 \\ 2 & 7 & 6 & 5 & 9 & 7 & 7 & 6 \end{bmatrix}$$

Step 7: Constructing final weighted decision matrix Ω as,

$$\Omega = \begin{bmatrix} 7.452 & 12.781 & 10.569 & 13.275 & 5.551 & 3.57 & 1.236 & 0.458 \\ 7.452 & 14.202 & 6.606 & 4.425 & 1.850 & 1.19 & 0.412 & 0.274 \\ 7.452 & 7.101 & 6.606 & 2.950 & 5.551 & 2.38 & 0.824 & 0.091 \\ 7.452 & 4.260 & 11.890 & 13.275 & 12.952 & 4.76 & 0.206 & 0.732 \\ 5.796 & 8.521 & 11.890 & 8.850 & 9.252 & 2.975 & 0.618 & 0.366 \\ 4.968 & 7.101 & 5.284 & 7.375 & 11.102 & 2.38 & 0.824 & 0.183 \\ 0.828 & 12.781 & 5.284 & 4.425 & 7.401 & 1.785 & 0.618 & 0.458 \\ 1.656 & 9.941 & 7.927 & 7.375 & 16.653 & 4.165 & 1.442 & 0.549 \end{bmatrix}$$

Step 8: The positive ideal solution (PIS) and negative ideal solution (NIS) are given below

$$\text{PIS} = \{7.452,\ 14.202,\ 11.890,\ 13.275,\ 16.653,\ 4.76,\ 1.442,\ 0.732\}$$

and

$$\text{NIS} = \{0.828,\ 4.260,\ 5.284,\ 2.950,\ 1.850,\ 1.19,\ 0.206,\ 0.091\}$$

Step 9: The separation measurements of PIS and NIS for each parameter by using the Equations (9) and (10) are given in Table 18.

Table 18. Separation measurements.

$\breve{\mathbb{S}}_1^{+}$	$\breve{\mathbb{S}}_2^{+}$	$\breve{\mathbb{S}}_3^{+}$	$\breve{\mathbb{S}}_4^{+}$	$\breve{\mathbb{S}}_5^{+}$	$\breve{\mathbb{S}}_6^{+}$	$\breve{\mathbb{S}}_7^{+}$	$\breve{\mathbb{S}}_8^{+}$
13.351	30.081	23.297	22.755	32.122	9.744	13.779	21.150
$\breve{\mathbb{S}}_1^{-}$	$\breve{\mathbb{S}}_2^{-}$	$\breve{\mathbb{S}}_3^{-}$	$\breve{\mathbb{S}}_4^{-}$	$\breve{\mathbb{S}}_5^{-}$	$\breve{\mathbb{S}}_6^{-}$	$\breve{\mathbb{S}}_7^{-}$	$\breve{\mathbb{S}}_8^{-}$
21.603	12.668	8.400	18.007	16.421	14.319	16.344	22.513

Step 10: The relative closeness to alternatives are given in Table 19 as follows,

Table 19. Relative clossness.

$\mathfrak{R}_1^{+}$	$\mathfrak{R}_2^{+}$	$\mathfrak{R}_3^{+}$	$\mathfrak{R}_4^{+}$	$\mathfrak{R}_5^{+}$	$\mathfrak{R}_6^{+}$	$\mathfrak{R}_7^{+}$	$\mathfrak{R}_8^{+}$
0.618	0.296	0.265	0.441	0.338	0.595	0.542	0.515

Step 11: The ranking order is $\xi_1 \succ \xi_6 \succ \xi_7 \succ \xi_8 \succ \xi_4 \succ \xi_5 \succ \xi_2 \succ \xi_3$. This shows, Badin is the most affected district.

Figure 6 shows the ranking of alternatives obtained by TOPSIS method.

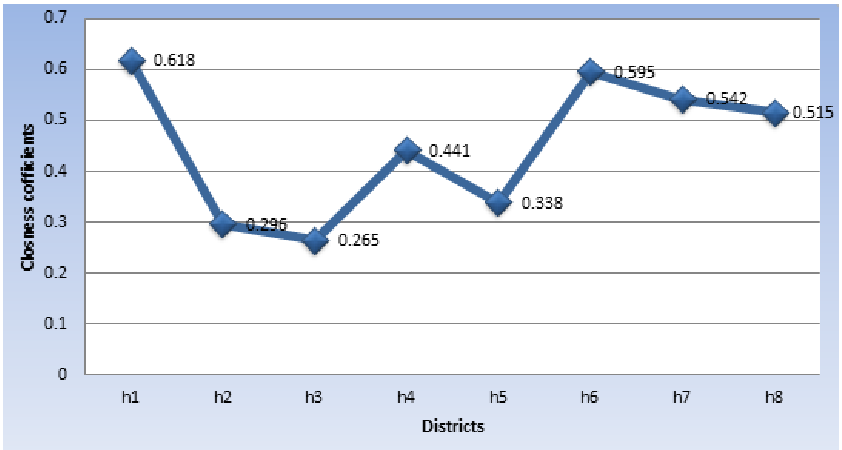

Figure 6. Ranking of alternative by TOPSIS method.

Pakistan Red Crescent Society (PRCS), supported by the International Federation of Red Cross and Red Crescent Societies (IFRC) and other Red Cross Red Crescent movement partners, reached 65,406 families (457,842 poeople) with food and non-food items, 208,600 people with water, 140,112 people with health services, as given in Figure 7.

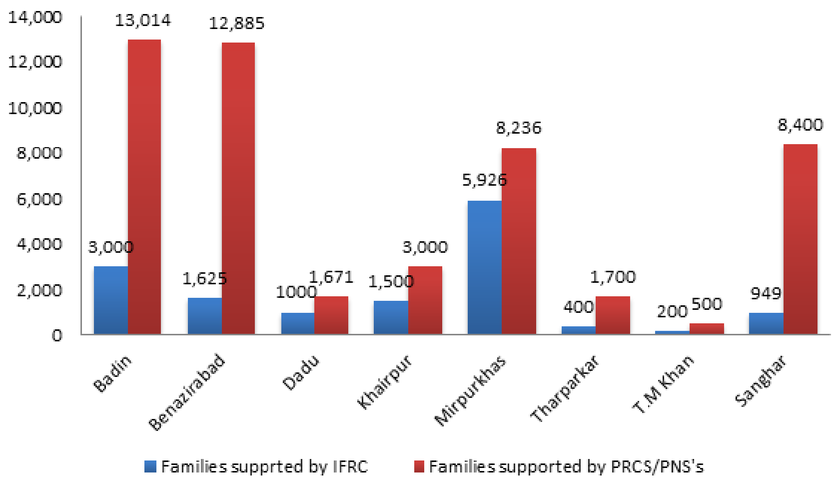

Figure 7. Assistance provided by PRCS and IFRC (source: www.ifrc.org.pk).

Provincial disaster management authority (PDMA) provide data about the losses in most affected districts in Sindh which is approximately same as we evaluate from Algorithms and MPNS-TOPSIS technique, as given in Figure 8. According to this data, Badin was the badly affected district. The bad condition of districts measured according to number of cattle head perished (CHP), affected villages (AV), affected people (AP), damaged houses (DH), affected area in acres (AA) and damaged crop area in acres (DCA).

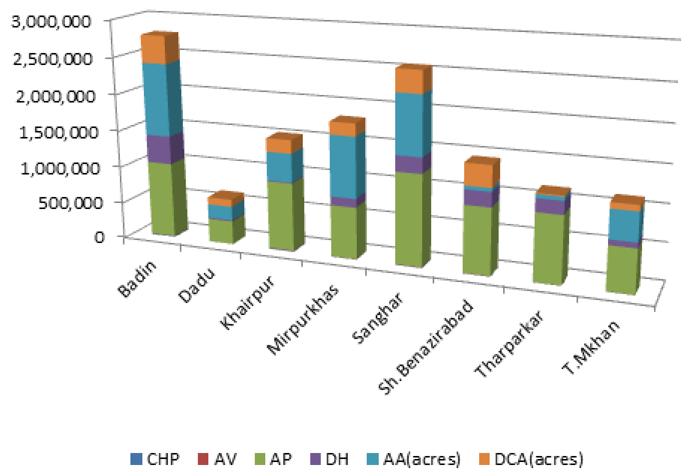

Figure 8. Summary of losses due to flood-2011 Dated:15 November 2011. (Source: www.pdma.gos.pk, accessed on 1 January 2021).

6.2. Comparison Analysis

The proposed MPNS topology-based Algorithms 1 and 2 and TOPSIS are compared as indicated in Table 20. In the comparison analysis, it can be noted that the suitable alternative obtained by any one proposed technique endorses the authenticity and effectiveness of the proposed algorithms. The comparison analysis of final ranking is also shown by multiple bar chart in the Figure 9.

Table 20. Comparison analysis of final ranking with existing methods in given numerical example.

Method	Ranking of Alternatives	Optimal Alternative
Algorithm 1 (Proposed)	$\xi_1 \succ \xi_2 = \xi_4 \succ \xi_7 \succ \xi_6 \succ \xi_5 \succ \xi_3 \succ \xi_8$	ξ_1
Algorithm 2 (Proposed)	$\xi_1 \succ \xi_2 \succ \xi_4 \succ \xi_3 \succ \xi_5 = \xi_7 \succ \xi_6 \succ \xi_8$	ξ_1
MPNS-TOPSIS (Proposed)	$\xi_1 \succ \xi_6 \succ \xi_7 \succ \xi_8 \succ \xi_4 \succ \xi_5 \succ \xi_2 \succ \xi_3$	ξ_1
Algorithm (Eraslan and Karaaslan [22])	$\xi_1 \succ \xi_4 \succ \xi_2 \succ \xi_5 \succ \xi_7 \succ \xi_6 \succ \xi_3 \succ \xi_8$	ξ_1
Algorithm (Cagman et al. [40])	$\xi_1 \succ \xi_4 \succ \xi_2 \succ \xi_7 \succ \xi_3 \succ \xi_6 \succ \xi_5 \succ \xi_8$	ξ_1
Algorithm (Tehrim and Riaz [41])	$\xi_1 \succ \xi_4 \succ \xi_2 \succ \xi_5 \succ \xi_7 \succ \xi_6 \succ \xi_3 \succ \xi_8$	ξ_1

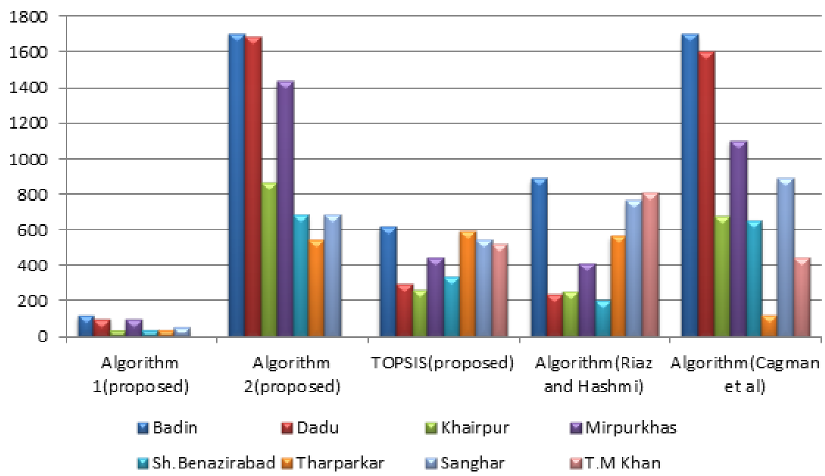

Figure 9. Comparison of final ranking by TOPSIS and other MADM techniques.

7. Conclusions

We deal with vague, ambiguous, unclear, and imprecise data in various real world issues. Existing models of soft sets, fuzzy sets, intuitionistic fuzzy sets, and neutrosophic sets are helpful in capturing these uncertainties. However, all of these models have some limitations on membership and non-membership grades. Existing mathematical frameworks are unable to address realistic issues when non-binary assessments are required while modeling uncertainty. Non-binary assessments are absolutely essential in ranking, grading or rating systems. The ranking may be specified in terms of grades, dots, stars or any notation. To deal with the real situation in life when the grading/rating of both parameters and alternatives is desired, we have introduced the novel concept of the M-parameterized N-soft set (MPNSS). Various concepts including MPNS-empty, MPNS-universal, MPNS-weak compliment, MPNS-top weak compliment, MPNS-bottom weak compliment, extended union, and restricted intersection of MPNSSs are defined. On the basis of these concepts, the idea of MPNS topology is established and various properties of MPNS topology are well established. MPNS topology is the extension of soft topology and N-soft topology. MPNS topology is a strong mathematical model of uncertainties that has a large number of applications in many fields like image processing, artificial intelligence, computational intelligence, forecasting, medical diagnosis. We developed algorithms for MADM applications of MPNSSs and MPNS topology. We established the TOPSIS method for multi attribute decision making by using MPNSSs and MPNS topology. The symmetry of the optimal decision is illustrated by interesting applications of proposed models and new MADM techniques. The viability and flexibility of the proposed MADM techniques are justified by comparison analysis them with existing MADM techniques.

Author Contributions: M.R., A.R. and M.A., conceived and worked together to achieve this manuscript, D.P. and M.A. construct the ideas and algorithms for data analysis and design the model of the manuscript, M.R., A.R. and D.P., processed the data collection and wrote the paper. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors extend their appreciation to the Deanship of Scientific Research at King Khalid University, Abha 61413, Saudi Arabia for funding this work through research groups program under grant number R.G. P-2/29/42.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Zadeh, L.A. Information and Control. *Fuzzy Sets* **1965**, *8*, 338–353.
2. Pawlak, Z. Rough sets. *Int. J. Inf. Comput. Sci.* **1982**, *11*, 341–356. [CrossRef]
3. Molodtsov, D. Soft set theory-first results. *Comput. Math. Appl.* **1999**, *37*, 19–31. [CrossRef]
4. Atanassov, K.T. Fuzzy Sets and Systems. *Intuit. Fuzzy Sets* **1896**, *20*, 87–96. [CrossRef]
5. Yager, R.R. Pythagorean fuzzy subsets. In *2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS)*; IEEE: New York, NY, USA, 2013; pp. 57–61.
6. Yager, R.R.; Abbasov, A.M. Pythagorean membership grades, complex numbers, and decision making. *Int. J. Intell. Syst.* **2013**, *28*, 436–452. [CrossRef]
7. Fatimah, F.; Rosadi, D.; Hakim, R.B.F.; Alcantud, J.C.R. N-soft sets and their decision-making algoritms. *Soft Comput.* **2018**, *22*, 3829–3842. [CrossRef]
8. Riaz, M.; Çağman, N.; Zareef, I.; Aslam, M. N-Soft Topology and its Applications to Multi-Criteria Group Decision Making. *J. Ournal Intell. Fuzzy Syst.* **2018**, *36*, 6521–6536. [CrossRef]
9. Akram, M.; Adeel, A.; Alcantud, J.C.R. Group decision-making methods based on hesitant N-soft sets. *Expert Syst. Appl.* **2019**, 115, 95–105. [CrossRef]
10. Akram, M.; Adeel, A.; Alcantud, J.C.R. Fuzzy N-soft sets: A novel model with applications. *J. Intell. Fuzzy Syst.* **2018**, *35*, 4757–4771. [CrossRef]
11. Akram, M.; Adeel, A. TOPSIS Approach for MAGDM Based on Interval-Valued Hesitant Fuzzy N-Soft Environment. *Int. J. Fuzzy Syst.* **2019**, *21*, 993–1009. [CrossRef]
12. Ashraf, S.; Abdullah, S.; Mahmood, T. Spherical fuzzy Dombi aggregation operators and their application in group decision making problems. *J. Ambient. Intell. Humaniz. Comput.* **2019**. [CrossRef]

13. Ali, M.I.; Feng, F.; Liu, X.Y.; Min, W.K.; Shabir, M. On some new operations in soft set theory. *Comput. Math. Appl.* **2009**, *57*, 1547–1553. [CrossRef]
14. Ali, M.I. A note on soft sets, rough soft sets and fuzzy soft sets. *Appl. Soft Comput.* **2011**, *11*, 3329–3332.
15. Karaaslan, F.; Hunu, F. Type-2 single-valued neutrosophic sets and their applications in multi-criteria group decision making based on TOPSIS method. *J. Ambient. Intell. Humaniz. Comput.* **2020**. [CrossRef]
16. Kumar, K.; Garg, H. TOPSIS method based on the connection number of set pair analysis under interval-valued intuitionistic fuzzy set environment. *Comput. Appl. Math.* **2018**, *37*, 1319–1329. [CrossRef]
17. Maji, P.K.; Biswas, R.; Roy, A.R. Fuzzy Soft sets. *J. Fuzzy Math.* **2001**, *9*, 589–602.
18. Maji, P.K.; Biswas, R.; Roy, A.R. Intuitionistic fuzzy soft sets. *J. Fuzzy Math.* **2001**, *9*, 677–691.
19. Çağman, N.; Karataş, S.; Enginoglu, S. Soft topology. *Comput. Math. Appl.* **2011**, *62*, 351–358. [CrossRef]
20. Shabir, M.; Naz, M. On soft topological spaces. *Comput. Math. Appl.* **2011**, *61*, 1786–1799. [CrossRef]
21. Riaz, M.; Tehrim, S.T. On Bipolar Fuzzy Soft Topology with Application. *Soft Comput.* **2011**, *24*, 18259–18272. [CrossRef]
22. Eraslan, S.; Karaaslan, F. A group decision making method based on TOPSIS under fuzzy soft environment. *J. New Theory* **2015**, 3, 30–40.
23. Feng, F.; Jun, Y.B.; Liu, X.; Li, L. An adjustable approach to fuzzy soft set based decision making. *J. Comput. Appl. Math.* **2010**, *234*, 10–20. [CrossRef]
24. Feng, F.; Li, C.; Davvaz, B.; Ali, M.I. Soft sets combined with fuzzy sets and rough sets, a tentative approach. *Soft Comput.* **2010**, *14*, 899–911. [CrossRef]
25. Peng, X.D.; Yang, Y. Some results for Pythagorean fuzzy sets. *Int. J. Intell. Syst.* **2015**, *30*, 1133–1160. [CrossRef]
26. Peng, X.D.; Yuan, H.Y.; Yang, Y. Pythagorean fuzzy information measures and their applications. *Int. J. Intell. Syst.* **2017**, *32*, 991–1029. [CrossRef]
27. Peng, X.D.; Selvachandran, G. Pythagorean fuzzy set: state of the art and future directions. *Artif. Intell. Rev.* **2019**, *52*, 1873–1927. [CrossRef]
28. Peng, X.D.; Liu, L. Information measures for q-rung orthopair fuzzy sets. *Int. J. Intell. Syst.* **2019**, *34*, 1795–1834. [CrossRef]
29. Zhang, X.L.; Xu, Z.S. Extension of TOPSIS to multiple criteria decision making with pythagorean fuzzy sets. *Int. J. Intell. Syst.* **2014**, *29*, 1061–1078. [CrossRef]
30. Zhang, L.; Zhan, J. Fuzzy soft β-covering based fuzzy rough sets and corresponding decision-making applications. *Int. J. Mach. Learn. Cybernatics* **2019**, *10*, 1487–1502. [CrossRef]
31. Zhang, L.; Zhan, J.; Alcantud, J.C.R. Novel classes of fuzzy soft β-coverings-based fuzzy rough sets with applications to multi-criteria fuzzy group decision making. *Soft Comput.* **2019**, *23*, 5327–5351. [CrossRef]
32. Garg, H.; Arora, R. Generalized intuitionistic fuzzy soft power aggregation operator based on t-norm and their application in multicriteria decision-making. *Int. J. Intell. Syst.* **2019**, *34*, 215–246. [CrossRef]
33. Garg, H.; Arora, R. Dual hesitant fuzzy soft aggregation operators and their application in decision-making. *Cogn. Comput.* **2018**, *10*, 769–789. [CrossRef]
34. Pamucar, D.; Jankovic, A. The application of the hybrid interval rough weighted Power-Heronian operator in multi-criteria decision making. *Oper. Res. Eng. Sci. Theory Appl.* **2020**, *3*, 54–73. [CrossRef]
35. Riaz, M.; Davvaz, B.; Fakhar, A.; Firdous, A. Hesitant fuzzy soft topology and its applications to multi-attribute group decision-making. *Soft Comput.* **2020**. [CrossRef]
36. Riaz, M.; Smarandache, F.; Firdous, A.; Fakhar, A. On soft rough topology with multi-attribute group decision making. *Mathematics* **2019**, *7*, 67. [CrossRef]
37. Riaz, M.; agman, N.; Wali, N.; Mushtaq, A. Certain properties of soft multi-set topology with applications in multi-criteria decision making. *Decis. Making: Appl. Manag. Eng.* **2020**, *3*, 70–96. [CrossRef]
38. Riaz, M.; Hashmi, M.R. Linear Diophantine fuzzy set and its applications towards multi-attribute decision making problems. *J. Intell. Fuzzy Syst.* **2019**, *37*, 5417–5439. [CrossRef]
39. Kamaci, H. Linear Diophantine fuzzy algebraic structures. *J. Ambient. Intell. Humaniz. Comput.* **2021**. [CrossRef]
40. Çağman, N.; Enginoglu, S.; Çitak, F. Fuzzy soft set theory and its applications. *Iran. J. Fuzzy Syst.* **2011**, *8*, 137–147.
41. Tehrim, S.T.; Riaz, M. A novel extension of TOPSIS to MCGDM with bipolar neutrosophic soft topology. *J. Intell. Fuzzy Syst.* **2019**, *37*, 5531–5549. [CrossRef]