

## Article

# A General Fixed Point Theorem for Mappings Satisfying a Cyclical Contractive Condition in S-Metric Spaces

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**Abstract:** In this paper we introduce a new type of implicit relation in S-metric spaces. Our aim is to prove a general fixed point theorem for mappings satisfying the cyclical contractive condition, which extends several results from the literature.

**Keywords:** fixed point; S-metric space; compatible mappings; cyclical contractive condition; implicit relation

**MSC:** 54H25; 47H10



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## 1. Introduction

Banach’s contraction principle [1] was extended to a case of cyclical mappings by Kirk et al. [2].

Many fundamental metrical fixed point theorems are extended to cyclic contractive mappings in [3], and other results are presented in [4–9] and in other papers.

A new class of generalized metric space, named *D*-metric spaces, is introduced in [10,11], by Dhage.

It has been shown, by Mustafa and Sims [12,13], that most of the statements about fundamental topological structures on *D*-metric spaces are incorrect. Thus, the authors introduced the appropriate notion of generalized metrics space, called *G*-metric space. In fact, Mustafa, Sims and other authors obtained numerous fixed point results for self mappings in *G*-metric spaces. Fixed points for cyclical contractive mappings in *G*-metric spaces are stated in [14–16] and in other papers.

Recently, in [17], the authors introduced a new type of generalized metric spaces, named *S*-metric spaces, as a “generalization” of *G*-metric spaces.

In [18], the authors proved that the notion of *S*-metric space is not a generalization of *G*-metric space or vice versa. Hence, the notions of *G*-metric space and *S*-metric space are independent.

The study of cyclical contractive mapping in *S*-metric spaces is introduced in [19].

By considering a general condition given by an implicit relation, various classical (common) fixed point theorems in metric spaces have been unified into [20,21]. In [22,23] begins the study of fixed points for mappings that satisfy an implicit relation in *G*-metric spaces.

The study of cyclical implicit contractive conditions is initiated in [24] and in *G*-metric spaces in [25].

The study of fixed points for mappings satisfying implicit relation in *S*-metric spaces is initiated in [26,27].

The purpose of this paper is to prove, using a new type of implicit relation, one general fixed point theorem for mappings which satisfy a cyclical contractive condition, thus extending some results from [26,28–31].

## 2. Preliminaries

**Definition 1** ([17,26]). Let  $X$  be a nonempty set. An  $S$ -metric on  $X$  is a function  $S : X^3 \mapsto \mathbb{R}_+$  such that:

- ( $S_1$ ):  $S(x, y, z) = 0$  if and only if  $x = y = z$ ;  
 ( $S_2$ ):  $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$  for all  $x, y, z, a \in X$ .

The pair  $(X, S)$  is called an  $S$ -metric space.

**Example 1.** Let  $X = \mathbb{R}$  and  $S(x, y, z) = |x - z| + |y - z|$ . Then,  $S(x, y, z)$  is an  $S$ -metric on  $\mathbb{R}$ , which is named the usual  $S$ -metric on  $X$ .

**Lemma 1** ([17]). If  $S$  is an  $S$ -metric on a nonempty set  $X$ , then

$$S(x, x, y) = S(y, y, x) \text{ for all } x, y \in X.$$

**Definition 2.** Let  $(X, S)$  be a  $S$ -metric space. For  $r > 0$  and  $x \in X$  we define the open ball with center in  $x$  and radius  $r$ , the set

$$B_S(x, r) = \{y \in X : S(x, x, y) < r\}.$$

The topology induced by the  $S$ -metric is the topology determined by the base of all open balls in  $X$ .

**Definition 3.**

- (a) A sequence  $\{x_n\}$  in  $(X, S)$  is convergent to  $x$ , denoted  $\lim_{n \rightarrow \infty} x_n = x$  or  $x_n \rightarrow x$ , if  $S(x_n, x_n, x) \rightarrow 0$  as  $n \rightarrow \infty$ .  
 (b) A sequence  $\{x_n\}$  in  $(X, S)$  is a Cauchy sequence if  $S(x_n, x_n, x_m) \rightarrow 0$  as  $n, m \rightarrow \infty$ .  
 (c)  $(X, S)$  is complete if every Cauchy sequence is convergent.

**Example 2.**  $(X, S)$  from Example 1 is complete.

**Lemma 2** ([17,26]). Let  $(X, S)$  be a  $S$ -metric space. If  $x_n \rightarrow x$  and  $y_n \rightarrow y$ , then  $S(x_n, x_n, y_n) \rightarrow S(x, x, y)$ .

**Lemma 3** ([17,26]). Let  $(X, S)$  be a  $S$ -metric space and  $x_n \rightarrow x$ . Then  $\lim_{n \rightarrow \infty} x_n$  is unique.

**Lemma 4** ([19]). Let  $(X, S)$  be a  $S$ -metric space and  $A$  is a nonempty subset of  $X$ . If  $A$  is  $S$ -closed, then for all convergent sequence  $x_n$  to  $x$ ,  $x \in A$ .

**Definition 4** ([2]). Let  $(X, d)$  be a metric space. Let  $p > 1$ ,  $p \in \mathbb{N}$  be,  $T$  a self mapping on  $X$  and  $\{A_i\}_{i=1}^p$  nonempty closed subsets of  $X$ . The mapping  $T$  is said to be cyclical if

$$T(A_i) \subset A_{i+1}, i = 1, 2, \dots, p, \text{ where } A_{p+1} = A_1. \quad (1)$$

The following theorems are proved in [3,29].

**Theorem 1** ([3]). Let  $\{A_i\}_{i=1}^p$  be nonempty closed subsets of a complete metric space  $(X, d)$  and  $f : \bigcup_{i=1}^p A_i \rightarrow \bigcup_{i=1}^p A_i$  satisfying (1) such that

$$d(fx, fy) \leq \alpha \max\{d(x, y), d(x, fx), d(y, fy), d(y, fx), d(x, fy)\} \quad (2)$$

for all  $x \in A_i$ ,  $y \in A_{i+1}$ ,  $i = 1, 2, \dots, p$  and some  $\alpha \in \left(0, \frac{1}{2}\right)$ . Then  $f$  has a unique fixed point in  $\bigcap_{i=1}^p A_i$ .

**Theorem 2** ([29]). Let  $(X, d)$  be a complete metric space,  $p \in \mathbb{N}$ ,  $\{A_i\}_{i=1}^p$  a finite family of nonempty closed subsets of  $X$  and  $T : \bigcup_{i=1}^p A_i \rightarrow \bigcup_{i=1}^p A_i$  satisfying (1) and

$$d(Tx, Ty) \leq k \max \left\{ d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2} \right\}, \quad (3)$$

where  $k \in (0, 1)$ , for all  $x \in A_i$ ,  $y \in A_{i+1}$ ,  $i = 1, 2, \dots, p$ . Then  $T$  has a unique fixed point in  $\bigcap_{i=1}^p A_i$ .

### 3. S-Implicit Relations

Let  $\mathcal{F}_S$  be the set of all lower semi-continuous functions  $F : \mathbb{R}_+^6 \mapsto \mathbb{R}$  satisfying the following conditions:

- $(F_1)$ :  $F$  is nondecreasing in  $t_6$ ;
- $(F_2)$ : there exists  $h \in [0, 1)$  such that for all  $u, v \geq 0$ ,  $F(u, v, v, u, 0, 2u + v) \leq 0$  implies  $u \leq hv$ ;
- $(F_3)$ :  $F(t, t, 0, 0, t, t) > 0$ ,  $\forall t > 0$ .

**Example 3.**  $F(t_1, \dots, t_6) = t_1 - at_2 - b \max\{t_3, t_4, t_5, t_6\}$ , where  $a, b \geq 0$  and  $a + 3b < 1$ .

- $(F_1)$ : obvious.
- $(F_2)$ : Let  $u, v \geq 0$  and  $F(u, v, v, u, 0, 2u + v) = u - av - b \max\{u, v, 2u + v\} \leq 0$ . If  $u > v$ , then  $u[1 - (a + 3b)] \leq 0$ , a contradiction. Hence,  $u \leq v$ , which implies  $u \leq hv$ , where  $0 \leq h = a + 3b < 1$ .
- $(F_3)$ :  $F(t, t, 0, 0, t, t) = t[1 - (a + 3b)] > 0$ ,  $\forall t > 0$ .

In the following examples, since the proofs are similar, we will omit them.

**Example 4.**  $F(t_1, \dots, t_6) = t_1 - at_2 - bt_3 - ct_4 - d \max\{t_5, t_6\}$ , where  $a, b, c, d \geq 0$  and  $a + b + c + 3d < 1$ .

**Example 5.**  $F(t_1, \dots, t_6) = t_1 - at_2 - d \max\{t_3, t_4\} - bt_5 - ct_6$ , where  $a, b, c, d \geq 0$ ,  $a + 3c + d < 1$  and  $a + b + c < 1$ .

**Example 6.**  $F(t_1, \dots, t_6) = t_1 - at_2 - bt_3 - et_4 - ct_5 - dt_6 - f \max\{t_2, t_3, t_4, t_5, t_6\}$ , where  $a, b, c, d, e, f \geq 0$ ,  $a + b + e + 3d + 3f < 1$  and  $a + d + c + f < 1$ .

**Example 7.**  $F(t_1, \dots, t_6) = t_1 - a(t_5 + t_6) - bt_2 - c \max\{t_3, t_4\}$ , where  $a, b, c \geq 0$  and  $3a + b + c < 1$ .

**Example 8.**  $F(t_1, \dots, t_6) = t_1 - a(t_3 + t_4) - bt_2 - c \max\{t_5, t_6\}$ , where  $a, b, c \geq 0$  and  $2a + b + 3c < 1$ .

**Example 9.**  $F(t_1, \dots, t_6) = t_1 - a \max\{t_4 + t_5, t_3 + t_6\} - bt_2$ , where  $a, b \geq 0$  and  $4a + b < 1$ .

**Example 10.**  $F(t_1, \dots, t_6) = t_1^2 - t_1(at_2 + bt_3 + ct_4) - dt_5t_6$ , where  $a, b, c \geq 0$ ,  $a + b + c < 1$  and  $a + d < 1$ .

**Example 11.**  $F(t_1, \dots, t_6) = t_1^2 - at_1t_2 - bt_3t_4 - ct_5t_6$ , where  $a, b, c \geq 0$ ,  $a + b < 1$  and  $a + c < 1$ .

**Example 12.**  $F(t_1, \dots, t_6) = t_1^3 - at_1t_2t_3 - bt_2t_3t_4 - ct_3t_4t_5 - dt_4t_5t_6$ , where  $a, b, c \geq 0$  and  $a + b < 1$ .

**Example 13.**  $F(t_1, \dots, t_6) = t_1 - at_2 - bt_3 - ct_4 - dt_5 - et_6$ , where  $a, b, c, d, e \geq 0$ ,  $a + b + c + 3e < 1$  and  $a + d + e < 1$ .

**Example 14.**  $F(t_1, \dots, t_6) = t_1 - k \max\{t_2, t_3, t_4, t_5, t_6\}$ , where  $k \in \left[0, \frac{1}{3}\right)$ .

**Example 15.**  $F(t_1, \dots, t_6) = t_1 - k \max\left\{t_2, t_3, t_4, \frac{t_5 + t_6}{3}\right\}$ , where  $k \in [0, 1)$ .

**Remark 1.** Examples 10–12 are not of the type of the examples from [18].

#### 4. Main Result

**Theorem 3.** Let  $(X, S)$  be a complete  $S$ -metric space and  $\{A_i\}_{i=1}^p$  be a family of nonempty closed subsets of  $X$ . Let  $Y = \bigcup_{i=1}^p A_i$  and let  $T : Y \mapsto Y$  satisfying

$$T(A_i) \subset A_{i+1}, \quad i = 1, 2, \dots, p, \quad (4)$$

where  $A_{p+1} = A_1$ .

If the inequality

$$F \left( \begin{array}{c} S(Tx, Ty, Ty), S(x, y, y), S(x, Tx, Tx), \\ S(y, Ty, Ty), S(y, Tx, Tx), S(x, Ty, Ty) \end{array} \right) \leq 0 \quad (5)$$

holds for all  $x \in A_i, y \in A_{i+1}, i = 1, 2, \dots, p$  and  $F \in \mathcal{F}_S$ , then  $T$  has a unique fixed point in  $\bigcap_{i=1}^p A_i$ .

**Proof.** Let  $x_0$  be an arbitrary point of  $A_1$ . We define  $x_n = Tx_{n-1}$ ,  $n = 1, 2, \dots$ . By (5) for  $x_0 \in A_1$  and  $x_1 \in A_2, x_{p-1} = Tx_{p-2} \in A_p, x_p = Tx_{p-1} \in A_{p+1} = A_1, x_{p+1} = Tx_p \in A_2$ , we have

$$\begin{aligned} F \left( \begin{array}{c} S(Tx_0, Tx_1, Tx_1), S(x_0, x_1, x_1), S(x_0, Tx_0, Tx_0), \\ S(x_1, Tx_1, Tx_1), S(x_1, Tx_0, Tx_0), S(x_0, Tx_1, Tx_1) \end{array} \right) &\leq 0, \\ F \left( \begin{array}{c} S(x_1, x_2, x_2), S(x_0, x_1, x_1), S(x_0, x_1, x_1), \\ S(x_1, x_2, x_2), 0, S(x_0, x_2, x_2) \end{array} \right) &\leq 0. \end{aligned} \quad (6)$$

By  $(S_2)$  and Lemma 1 we have

$$\begin{aligned} S(x_0, x_2, x_2) &= S(x_2, x_2, x_0) \leq 2S(x_2, x_2, x_1) + S(x_0, x_0, x_1) \\ &= 2S(x_1, x_2, x_2) + S(x_0, x_1, x_1). \end{aligned}$$

By (6) and  $(F_1)$  we obtain

$$F \left( \begin{array}{c} S(x_1, x_2, x_2), S(x_0, x_1, x_1), S(x_0, x_1, x_1), \\ S(x_1, x_2, x_2), 0, 2S(x_1, x_2, x_2) + S(x_0, x_1, x_1) \end{array} \right) \leq 0.$$

By  $(F_2)$  we obtain

$$S(x_1, x_2, x_2) \leq hS(x_0, x_1, x_1).$$

Similarly, we obtain

$$S(x_{p-2}, x_{p-1}, x_{p-1}) \leq hS(x_{p-3}, x_{p-2}, x_{p-2}).$$

Again, we get

$$S(x_{p-1}, x_p, x_p) \leq hS(x_{p-2}, x_{p-1}, x_{p-1}).$$

Hence, we have

$$S(x_n, x_{n+1}, x_{n+1}) \leq hS(x_{n-1}, x_n, x_n) \leq \dots \leq h^n S(x_0, x_1, x_1).$$

Using  $(S_2)$  it follows that  $\{x_n\}$  is a Cauchy sequence in  $X$ . Since  $(X, S)$  is complete, it follows that  $x_n$  is convergent to a point  $z$ . Then, the sequence

$$\{x_{np+1} \in A_1, x_{np+2} \in A_2, \dots, x_{np+p} \in A_p, n = 0, 1, 2, \dots\}$$

also converges to  $z$  and  $z \in \bigcap_{i=1}^p A_i$ , because by Lemma 4,  $z \in A_i$ ,  $i = 1, 2, \dots, p$ .

We have to prove that  $z$  is a fixed point of  $T$ .

For  $x = x_n$  and  $y = z$ , by (5) we obtain

$$F \left( \begin{array}{c} S(Tx_n, Tz, Tz), S(x_n, z, z), S(x_n, Tx_n, Tx_n), \\ S(z, Tz, Tz), S(z, Tx_n, Tx_n), S(x_n, Tz, Tz) \end{array} \right) \leq 0,$$

$$F \left( \begin{array}{c} S(x_{n+1}, Tz, Tz), S(x_n, z, z), S(x_n, x_{n+1}, x_{n+1}), \\ S(z, Tz, Tz), S(z, x_{n+1}, x_{n+1}), S(x_n, Tz, Tz) \end{array} \right) \leq 0.$$

Letting  $n$  tend to infinity, by Lemma 2 we obtain

$$F(S(z, Tz, Tz), 0, 0, S(z, Tz, Tz), 0, S(z, Tz, Tz)) \leq 0.$$

By  $(F_1)$  we get

$$F(S(z, z, Tz), 0, 0, S(z, z, Tz), 0, 2S(z, Tz, Tz)) \leq 0.$$

By  $(F_2)$  we obtain  $S(z, Tz, Tz) = 0$  and by  $(S_1)$  it follows that  $z = Tz$ . Hence,  $z$  is a fixed point and  $z \in \bigcap_{i=1}^p A_i$ .

Suppose that there exists another fixed point  $z' \in \bigcap_{i=1}^p A_i$ . By (5) for  $x = z$  and  $y = z'$  we obtain

$$F \left( \begin{array}{c} S(Tz, Tz', Tz'), S(z, z', z'), S(z, Tz, Tz), \\ S(z', Tz', Tz'), S(z', Tz, Tz), S(z, Tz', Tz') \end{array} \right) \leq 0,$$

$$F(S(z, z', z'), S(z, z', z'), 0, 0, S(z', z, z), S(z, z', z')) \leq 0.$$

By Lemma 1,  $S(z', z, z) = S(z, z', z')$ . Hence,

$$F(S(z, z', z'), S(z, z', z'), 0, 0, S(z, z', z'), S(z, z', z')) \leq 0,$$

a contradiction of  $(F_3)$  if  $S(z, z', z') > 0$ . Hence,  $S(z, z', z') = 0$  and by  $(S_1)$ ,  $z = z'$ . Hence,  $z$  is the unique fixed point of  $T$  and  $z \in \bigcap_{i=1}^p A_i$ .  $\square$

## 5. Final Remarks

In this paper, using a new type of implicit relation, we have proved a general fixed point theorem for mappings that satisfy a cyclical contractive condition. Our result extends certain results from [26,28–31]:

- (i) Theorem 3 and Example 3 extend Corollary 2.21 [26] to cyclical form;
- (ii) Theorem 3 and Examples 3–9 extend Theorem 3.1 [28] to cyclical form;
- (iii) Theorem 3 and Example 15 extend Theorem 2 to cyclical form in  $S$ -metric spaces;
- (iv) Theorem 3 and Example 13 extend Corollary 2.19 [26], Theorems 2.3 and 2.4 [30], Theorems 3.2–3.4 [31] to cyclical form.

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