

## Article

# Computational Intelligent Paradigms to Solve the Nonlinear SIR System for Spreading Infection and Treatment Using Levenberg–Marquardt Backpropagation

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**Abstract:** The current study aims to design an integrated numerical computing-based scheme by applying the Levenberg–Marquardt backpropagation (LMB) neural network to solve the nonlinear susceptible (*S*), infected (*I*) and recovered (*R*) (SIR) system of differential equations, representing the spreading of infection along with its treatment. The solutions of both the categories of spreading infection and its treatment are presented by taking six different cases of SIR models using the designed LMB neural network. A reference dataset of the designed LMB neural network is established with the Adam numerical scheme for each case of the spreading infection and its treatment. The approximate outcomes of the SIR system based on the spreading infection and its treatment are presented in the training, authentication and testing procedures to adapt the neural network by reducing the mean square error (MSE) function using the LMB. Studies based on the proportional performance and inquiries based on correlation, error histograms, regression and MSE results establish the efficiency, correctness and effectiveness of the proposed LMB neural network scheme.

**Keywords:** SIR nonlinear systems; numerical computing; spreading infection; neural networks; Levenberg–Marquardt backpropagation

## 1. Introduction

Mathematical models play a fundamental, powerful and significant role in understanding infectious disease dynamics and recovery practice in infection control in epidemiology. An epidemic system is known as an intelligent formulation to predict any promising manifestation of the disease [1–3]. The mathematical epidemic systems are categorized into a different set of states. The simple form of the model to estimate the theoretical form of those infected people who are suffering from a contagious virus was defined by Kermack et al. in 1927. This epidemic model has susceptible (*S*), infected (*I*) and recovered (*R*) states—i.e., SIR, where *S*, *I* and *R* denote those individuals who are not diseased, infected and recovered. The SIR systems and their simplified versions are broadly implemented using mathematical systems, diseases spread analysis, and control and counter treatment actions,

whereas the purpose of the current work is to solve both the categories of the SIR system using the Levenberg–Marquardt backpropagation (LMB) neural network; we aim to find and examine the dynamics for two nonlinear types of systems signified with modified SIR (M.SIR) and SIR treatment (T.SIR). These two M.SIR and T.SIR models have been discussed in the literature [4]:

$$\begin{cases} \frac{dS}{d\tau} = B - (\beta I(\tau) + \alpha)S(\tau), & S_0 = i_1, \\ \frac{dI}{d\tau} = (\beta S(\tau) - \alpha - \mu)I(\tau), & I_0 = i_2, \\ \frac{dR}{d\tau} = \mu I(\tau) - \alpha R(\tau), & R_0 = i_3. \end{cases} \quad (1)$$

$$\begin{cases} \frac{dS}{d\tau} = B - \beta(S(\tau)I(\tau) + \delta T(\tau)) - \alpha S(\tau), & S_0 = i_4, \\ \frac{dI}{d\tau} = \beta(S(\tau)I(\tau) + \delta T(\tau)) - (\mu + \alpha)I(\tau), & I_0 = i_5, \\ \frac{dT}{d\tau} = \mu I(\tau) - (\rho + \alpha)T(\tau), & T_0 = i_6, \\ \frac{dR}{d\tau} = \rho T(\tau) - \alpha R(\tau), & R_0 = i_7. \end{cases} \quad (2)$$

The above systems (1) and (2) represent the M.SIR and T.SIR models. The terms  $\mu$ ,  $\beta$ ,  $B$ ,  $\alpha$ ,  $\delta$ , and  $\rho$  represent the recovery rate, contact rate, natural birth rate, natural rate of death, reduction in infection through treatment and rate of infection, respectively. The values of the constants and initial conditions are the same as reported in [4].

The SIR mathematical systems are normally implemented to analyze the infection, control proposal, treatment and recovery of infections. These mathematical models are formulated to incorporate the appropriate assumptions, decision variables and parameter settings. To examine the infection diseases, safeguard and spread, many investigations have been performed on these epidemic models together with the valuations of theoretical advances [5–8]. Some possible examples are the work Ogren et al. [9], who discussed the optimal control in the SIR-based biological system using Newton’s approach, as well as Goufo et al. [10], who presented the epidemic fractional SEIR model for the progressive spread of spatial and measles and in met populations. Mickens [11] observed the consequences of vaccination for the spread of episodic viruses and many other epidemic systems [12–17]. The above SIR biological systems have been analytically/numerically solved and every scheme has its own limitations. However, the stochastic Levenberg–Marquardt backpropagation (LMB) neural network has never been applied to solve both the above-mentioned categories of the SIR systems in the set of Equations (1) and (2).

Stochastic approaches are efficient to solve many complex models using the swarming/evolutionary approaches such as singular higher order models [18–21], dusty plasma models [22], functional singular differential systems [23,24], biological models [25–28], fluid dynamic problems [29–31], singular Lane–Emden model [32,33], electric circuits [34,35], Thomas–Fermi singular model [36], singular three-point model [37] and periodic differential model [38]. The potential visualizations of the proposed LMB neural network are provided as:

- A novel integrated design of an intelligent computing scheme is introduced via modeling competency of Levenberg–Marquardt backpropagation neural network applied to scrutinize the dynamics of both the categories of SIR systems represented with set of nonlinear ordinary differential equations.
- The designed LMB neural networks operate effectively on a dataset generated from numerical Adam method for different variants of the nonlinear modeling and treatment-based SIR systems.
- The performance via comparative investigations from reference results of Adam method on correlation, error histograms, regression and mean square error (MSE) metrics establish the worth of designed Levenberg–Marquardt backpropagation neural networks.
- Advantage of the proposed LMB neural network methodology is the smooth implementation, simplicity of the concept, stability and exhaustive applicability.

The rest of the paper is structured as: the solution of the LMB neural network approach to solve the M.SIR and T.SIR is provided in Section 2, the designed LMB neural network

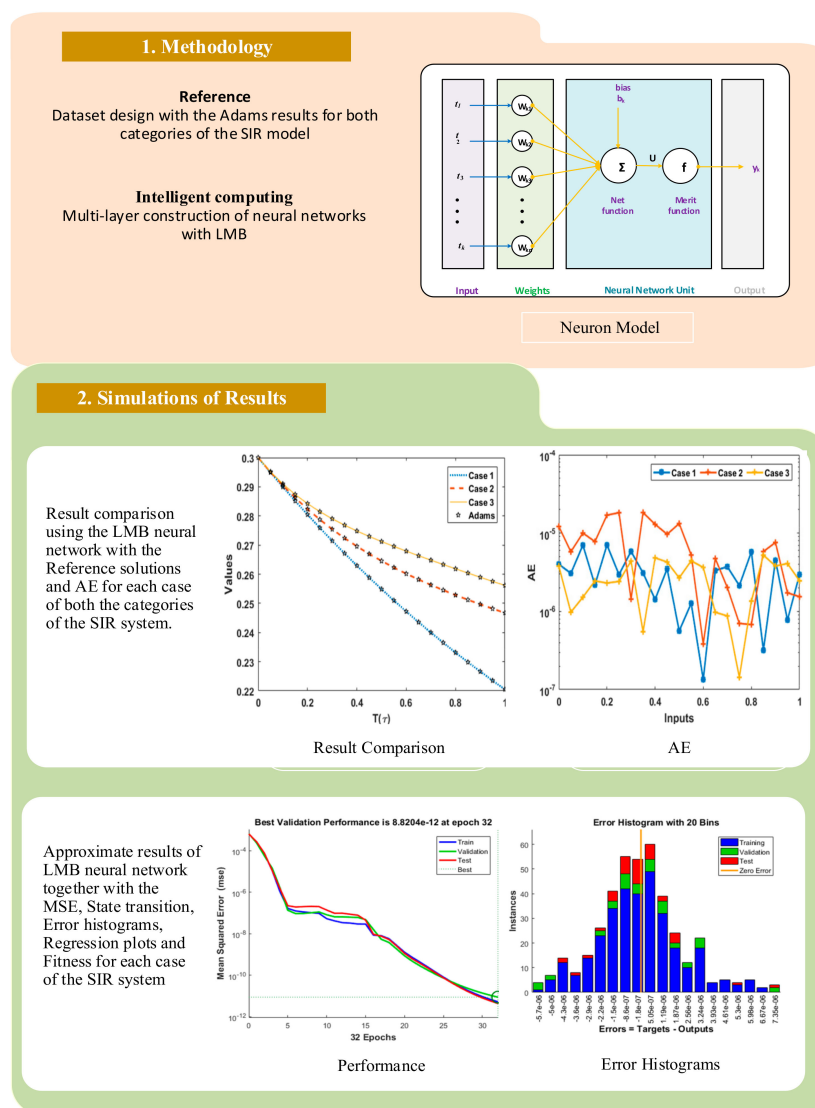
with essential explanations are given in Section 3, the concluding statements with latent connected studies together with future research clarifications are stated in the final section.

## 2. Methodology

The novel design of the LMB neural network approach is provided in two steps as:

- Essential descriptions are given to make or formulate the LMB neural networks dataset by the used of standard numerical methods—i.e., Runge–Kutta or Adam numerical solvers.
- Implementation procedure approved for LMB neural networks is introduced to find the approximate solution of both modified SIR (M.SIR) and SIR treatment (T.SIR) models presented in set of Equations (1) and (2).

The design of workflow of the LMB neural network approach is plotted in Figure 1, which is a mixture of multilayer construction of neural networks under the optimization of the LMB method. A system for a single neuron based on the neural network methodology is available in Figure 2 in the of input, output and hidden layers structure. The proposed LMB neural networks are implemented using the “nftool” routine in the “Matlab” package for the proper sets of validation data, hidden neurons, testing data and learning schemes.



**Figure 1.** Workflow diagram of the Levenberg–Marquardt backpropagation (LMB) proposed neural network for the nonlinear susceptible (S), infected (I) and recovered (R) (SIR) system.

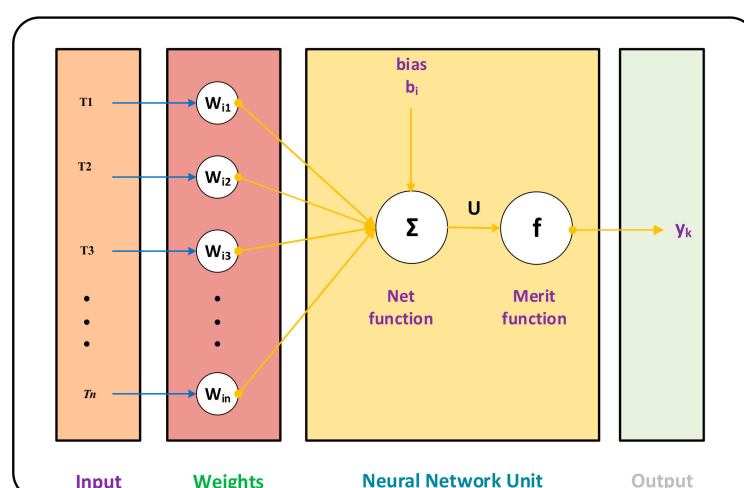


Figure 2. Designed model system based on the single neuron.

### 3. Numerical Measures with Analysis

The present section is related to solve two different categories based on the non-linear M.SIR and T.SIR systems numerically using the LMB neural network. Three different cases using the contact rates of 0.1, 2.1 and 4.1 in the M. SIR and T.SIR given in Equations (1) and (2) have been numerically solved by the LMB neural network process. The proposed numerical result has been obtained using the LMB neural network with the step size of 0.01 in the interval [1] for both nonlinear M.SIR and T.SIR systems. The LMB designed neural network approach is capable of solving both of the SIR systems given in the Equations (1) and (2) using “nftool” with 10 hidden neurons, 70% values of data training, 15% testing and 15% validation of the LMB optimization process. The LMB designed neural network is presented in Figure 3, and the LMB neural network approach was implemented to solve each case of both the categories of the SIR system.

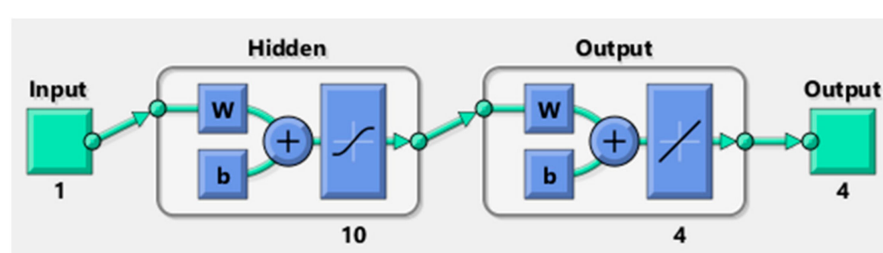


Figure 3. LMB designed neural network structure of the nonlinear SIR system.

The LMB designed neural network for both the categories of the nonlinear SIR systems are plotted in Figures 4–17. The outcomes for both M.SIR and T.SIR models using the performance and transition states are provided in Figures 4 and 5. The values of the mean square error (MSE) training, testing, best curve and validation are presented for each case of the M.SIR and T.SIR models in Figure 4.

The network’s best performances at epochs 25, 36, 89, 32, 551 and 265 lie around  $10^{-12}$ ,  $10^{-10}$ ,  $10^{-12}$ ,  $10^{-12}$ ,  $10^{-10}$  and  $10^{-10}$ , respectively. The gradient-based values using the step size ( $\mu$ ) of the designed LMB process given for the M.SIR and T.SIR systems are  $[9.48 \times 10^{-8}, 9.75 \times 10^{-08}, 9.83 \times 10^{-08}, 9.97 \times 10^{-08}, 9.83 \times 10^{-08}]$  and  $[10^{-13}, 10^{-11}, 10^{-11}, 10^{-12}, 10^{-10}, 10^{-09}]$  and are shown in Figure 5. These plots designate the specificity as well as the convergence of the LMB neural network for each case of the M.SIR and T.SIR systems.

Figures 6–11 validate the fitting plots for each category that shows the result comparisons obtained by the LMB neural network and the numerical Adam results for all the

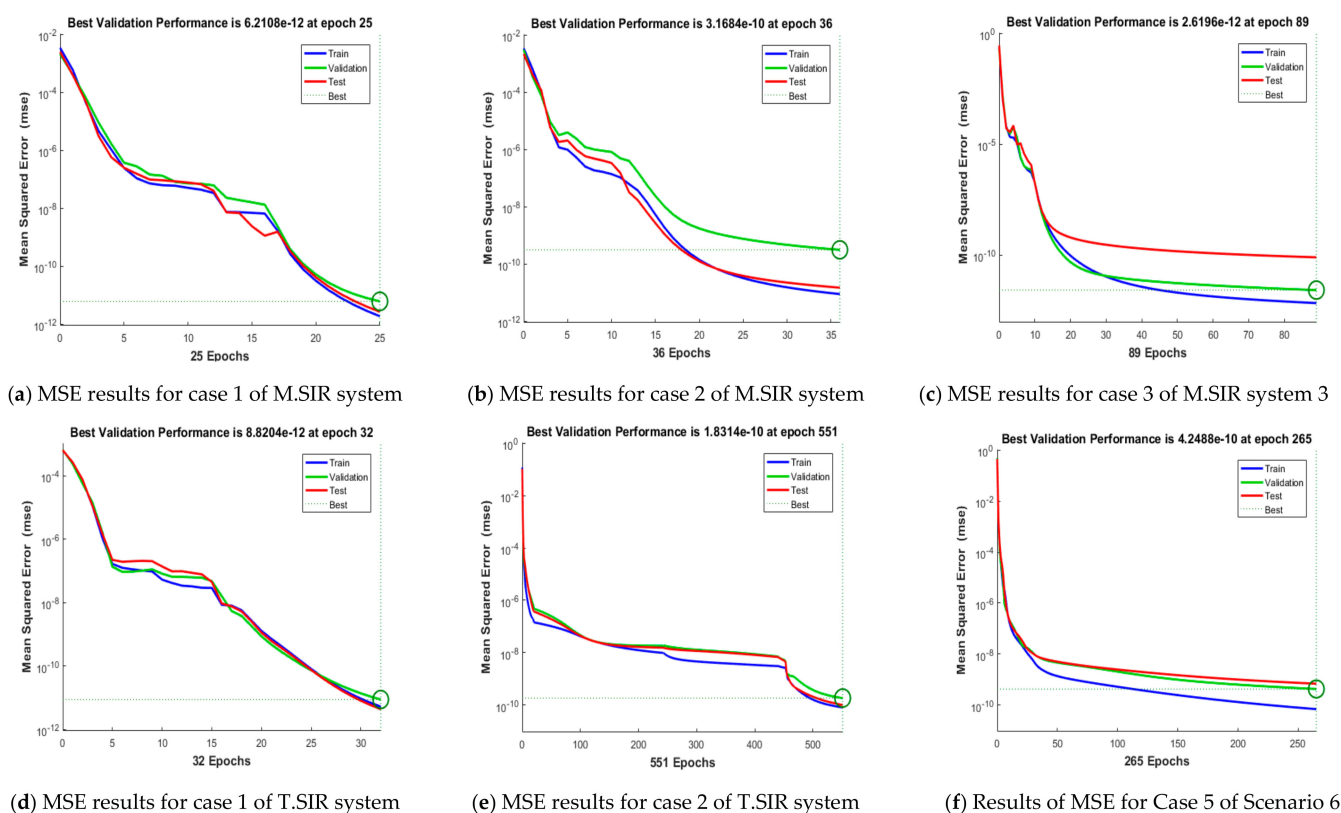
cases of the M.SIR and T.SIR. The maximum values of the error using training, testing and validation inputs of the LMB neural network lie around  $10^{-6}$  to  $10^{-7}$  for each case of the M.SIR and T.SIR. The values of the error histograms (EHs) are plotted in Figure 12.

The regression values can be observed in Figure 13 for each case of the M.SIR and T.SIR systems. The correlation investigations were applied to form the regression analysis study. It was perceived that the correlation values of “R” lie close to 1, showing the perfect form of the modeling, validation, testing, and training. The regression analysis generally showed the correctness of the LMB neural network in order to solve the SIR systems.

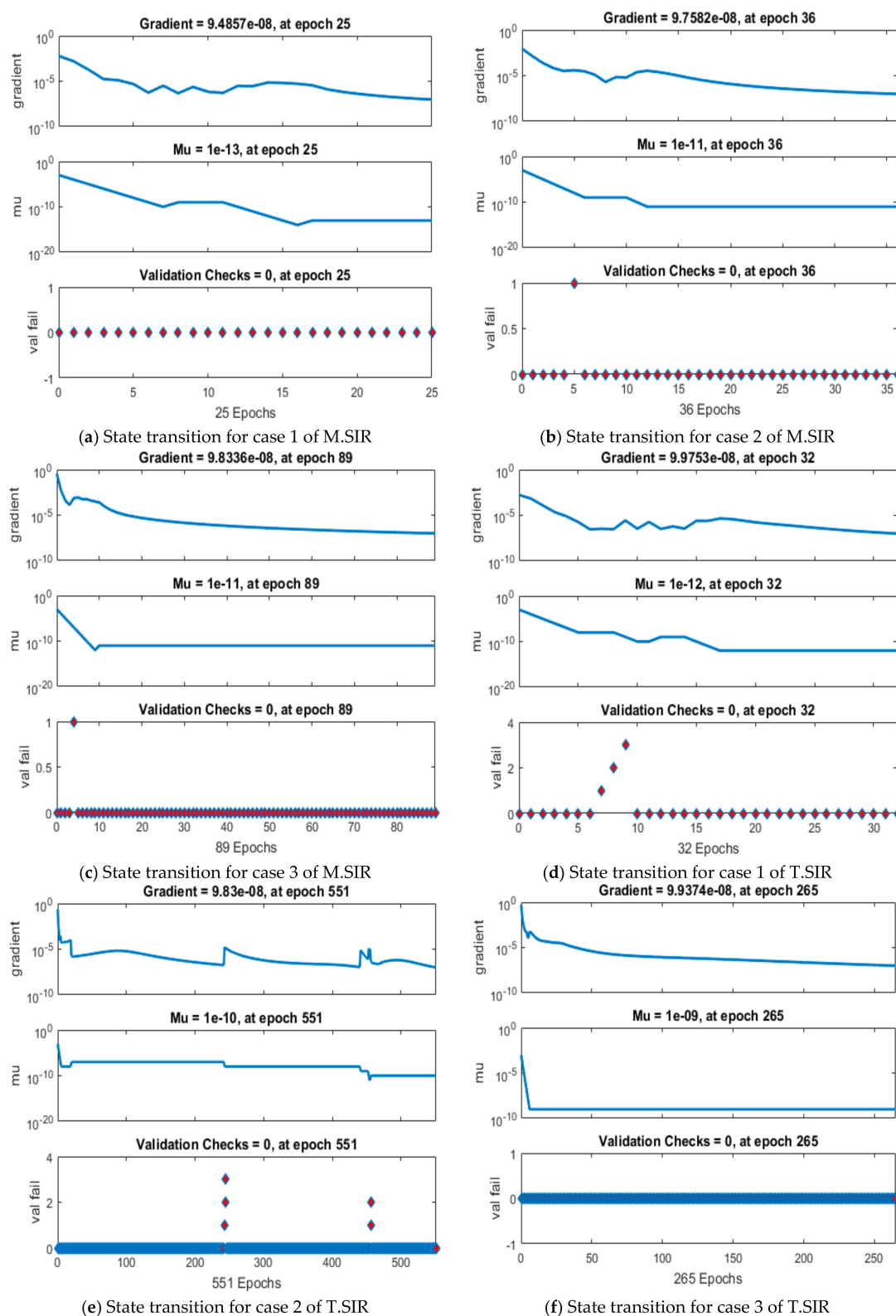
Moreover, convergence values in MSE sense were achieved for training, testing, validation, performances, backpropagation, executed epochs and complexity, tabulated in Tables 1 and 2 for the M.SIR and T.SIR systems.

Result comparison of the LMB neural network and the Adam method for each case of both the categories of SIR system is plotted in Figures 14 and 15. The results of each case of M.SIR are provided in Figure 14, while the comparison of T.SIR is provided in Figure 15.

The plots of  $S(\tau)$ ,  $I(\tau)$  and  $R(\tau)$  in the M.SIR system are provided in Figure 14a–c, while the plots of  $S(\tau)$ ,  $I(\tau)$ ,  $T(\tau)$  and  $R(\tau)$  in the T.SIR system are provided in Figure 15a–d. It was observed that the obtained form of the results overlapped with the Adam results for each case of the M.SIR and T.SIR. This complete overlapping of the outcomes designates the perfection and correctness of the proposed approach. The absolute error (AE) plots of each case of both the categories of the SIR system can be seen in Figures 16 and 17. In the M.SIR system, the AE values for the parameters  $S(\tau)$  and  $I(\tau)$  lie around  $[10^{-5}, 10^{-7}]$ , while for  $R(\tau)$  these values lie around  $[10^{-6}, 10^{-8}]$ . In the T.SIR system, the AE values for the all the parameters lie around  $[10^{-5}, 10^{-7}]$ . These outcomes indicate the exactness of the LMB designed neural network scheme.



**Figure 4.** Performance curves based on the mean square error (MSE) using the LMB designed neural networks for both of the SIR systems.



**Figure 5.** State transition for the LMB designed neural networks for each case of modified SIR (M.SIR) and SIR treatment (T.SIR) systems.

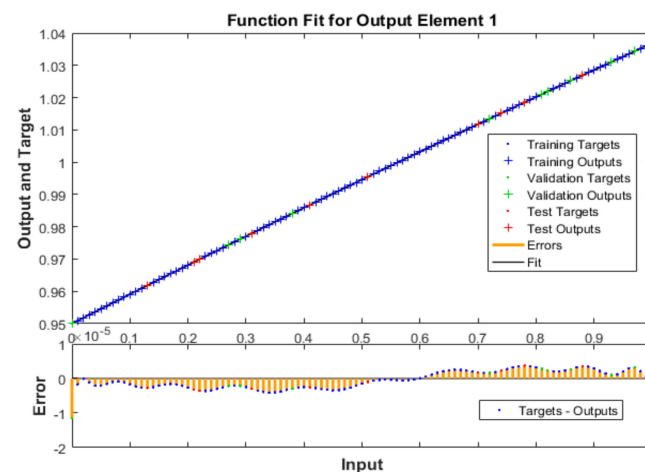


Figure 6. Comparison based on LMB neural network and exact solutions for case 1 of M.SIR.

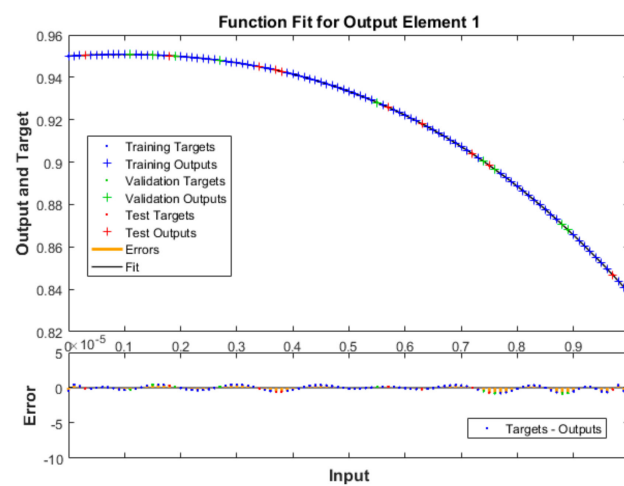


Figure 7. Comparison based on LMB neural network and exact solutions for case 2 of M.SIR system.

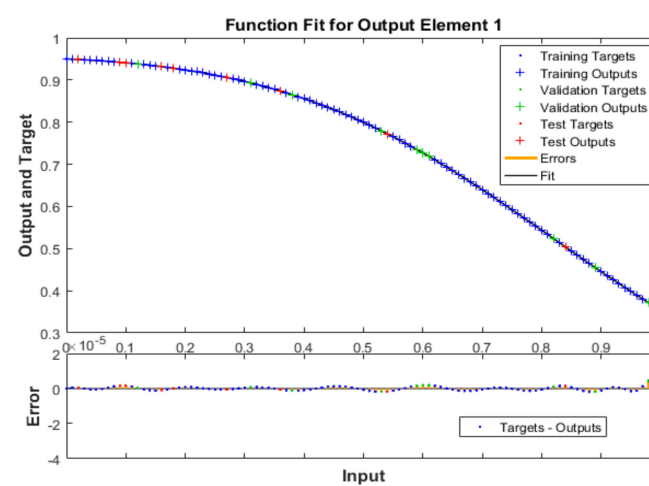


Figure 8. Comparison based on LMB neural network and exact solutions for case 3 of M.SIR system.

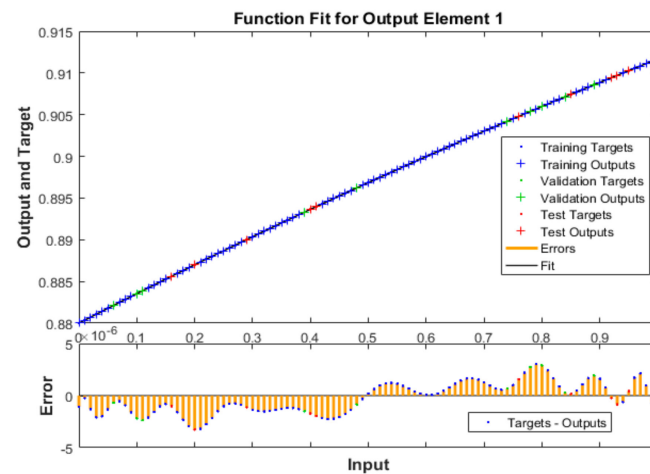


Figure 9. Comparison based on LMB neural network and exact solutions for case 1 of T.SIR system.

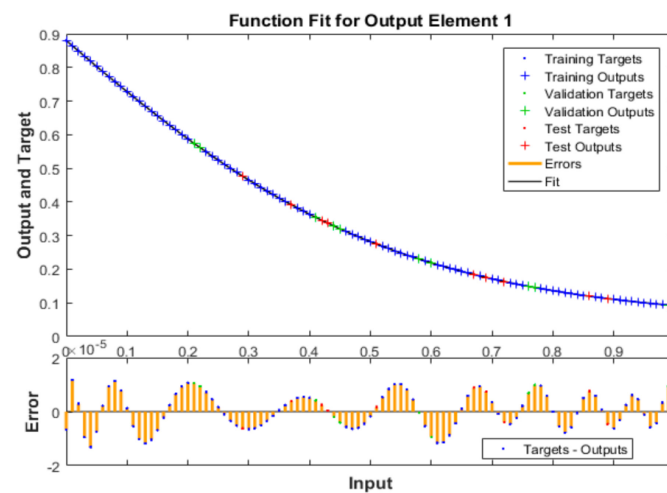


Figure 10. Comparison based on LMB neural network and exact solutions for case 2 of T.SIR system.

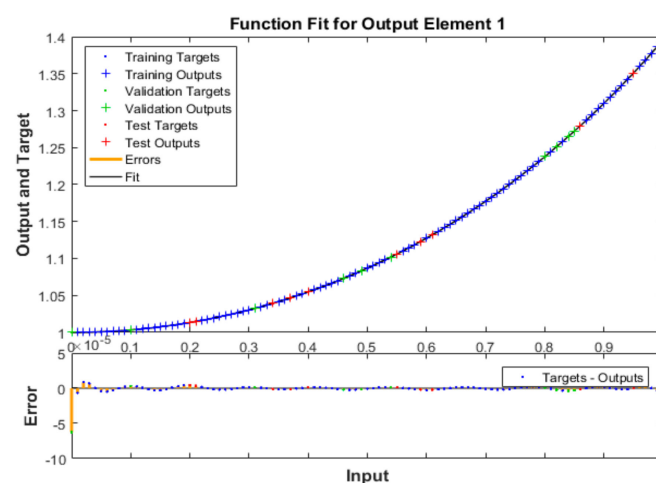


Figure 11. Comparison based on LMB neural network and exact solutions for case 3 of T.SIR system.

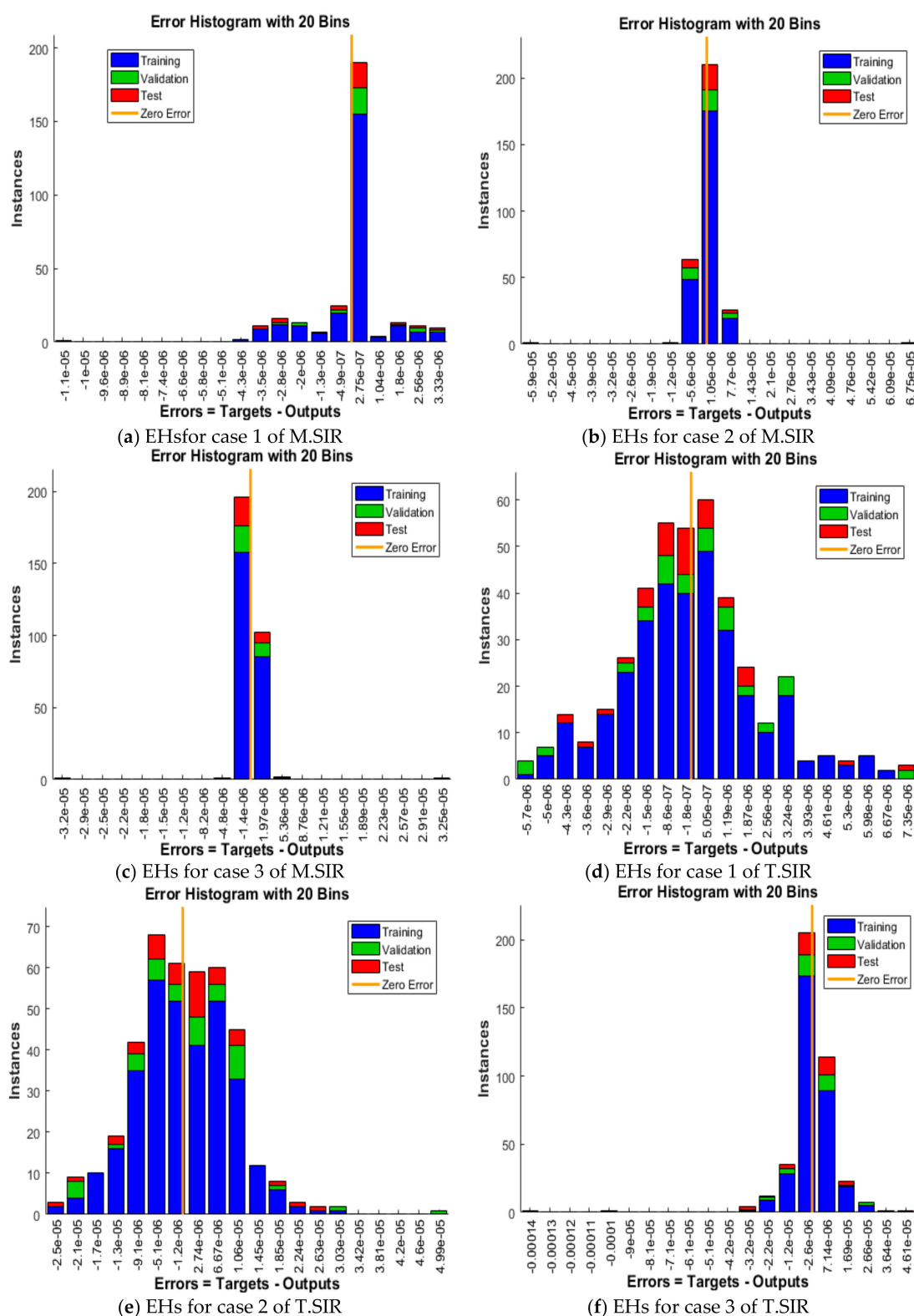


Figure 12. Plots of the error histograms (EHs) for LMB designed neural networks for all cases of the M.SIR and T.SIR systems.

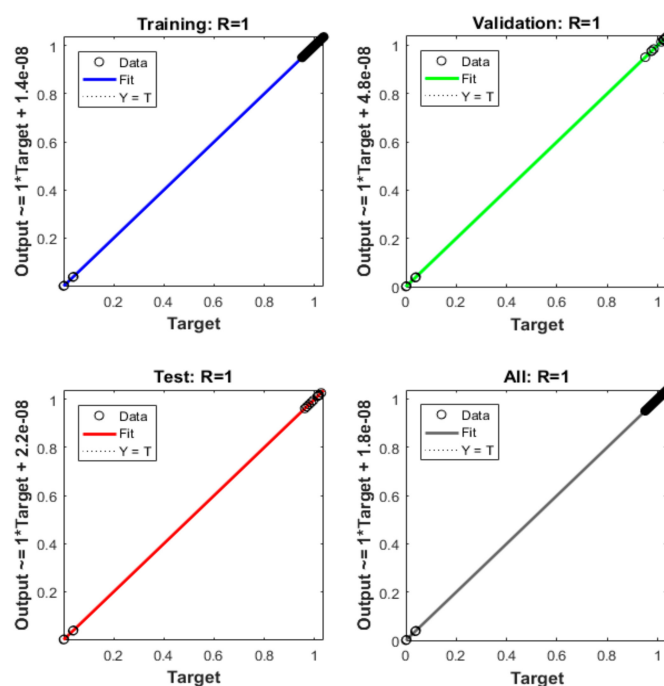
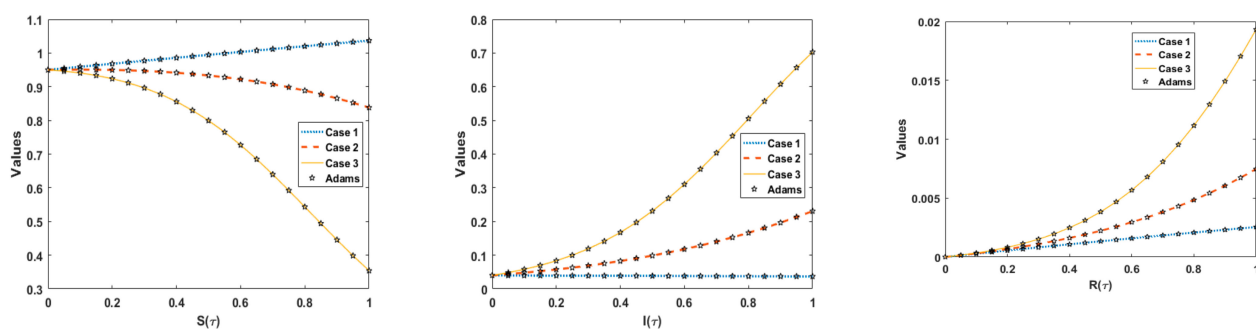
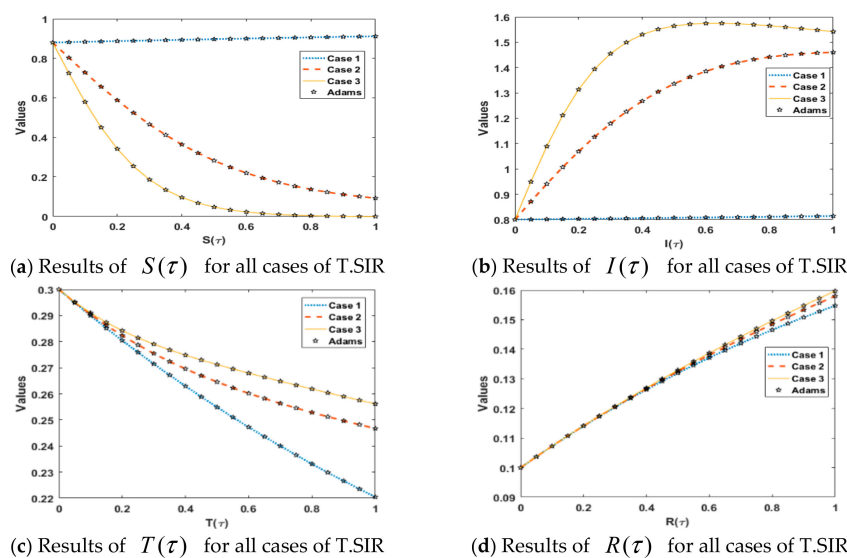


Figure 13. Regression values using the LMB neural network for case 1 of M.SIR.



(a) Results of  $S(\tau)$  for all cases of M.SIR (b) Results of  $I(\tau)$  for all cases of M.SIR (c) Results of  $R(\tau)$  for all cases of M.SIR

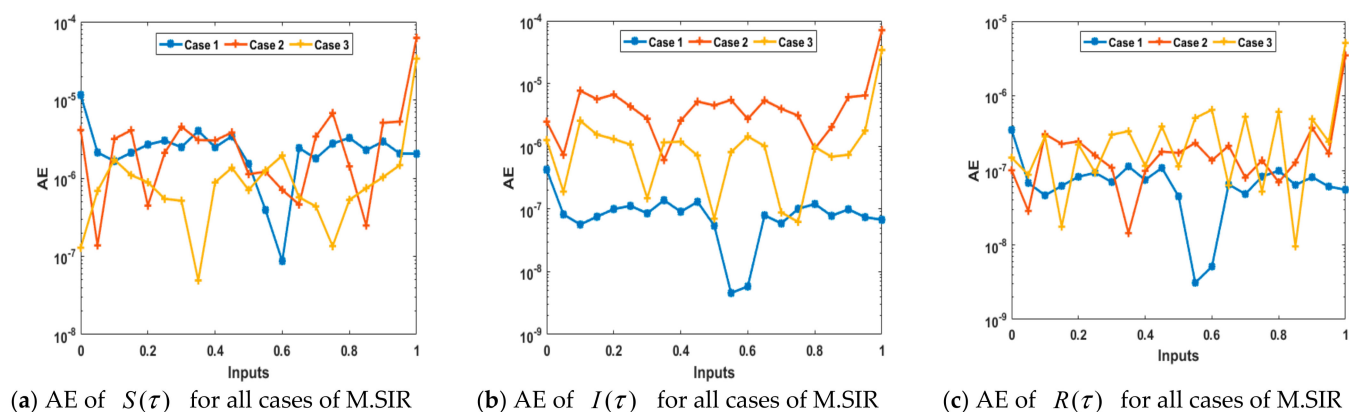
Figure 14. Comparison of the obtained results through LMB neural network for each case of the M.SIR.



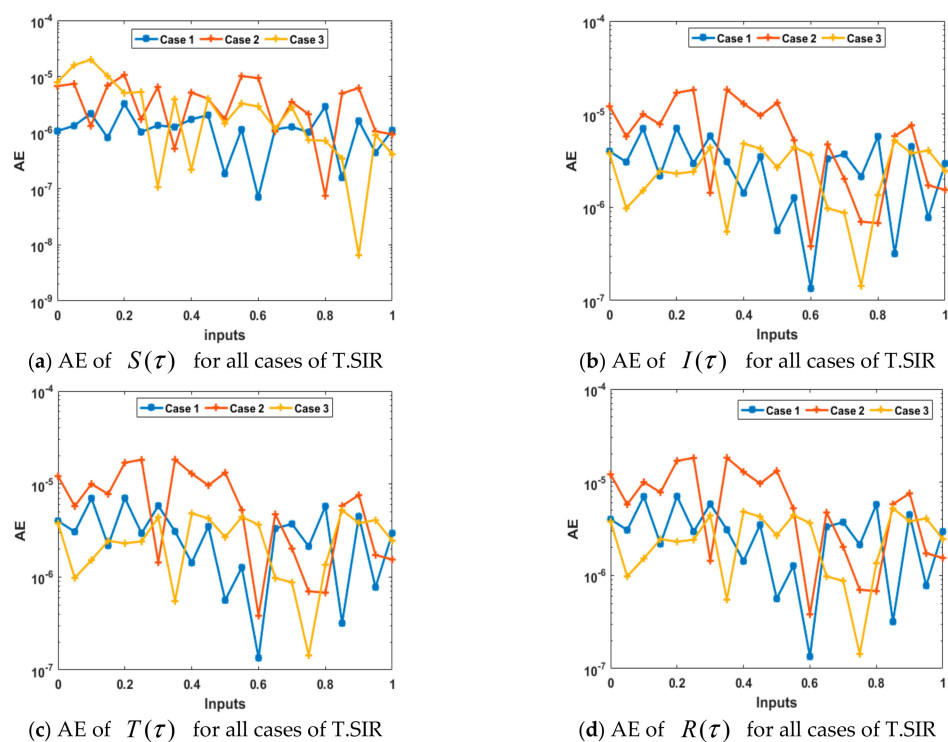
(c) Results of  $T(\tau)$  for all cases of T.SIR

(d) Results of  $R(\tau)$  for all cases of T.SIR

Figure 15. Comparison of the obtained results through LMB neural network for each case of the T.SIR.



**Figure 16.** Absolute error (AE) based on the obtained results and Adam results via LMB neural network for each case of the M.SIR.



**Figure 17.** AE based on the obtained results and Adam results via LMB neural network for each case of the T.SIR.

**Table 1.** SNN-LMB neural network results for each case of the M.SIR system.

| Case | MSE                    |                        |                        | Performance            | Gradient               | Mu                     | Epoch | Time |
|------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|-------|------|
|      | Training               | Validation             | Testing                |                        |                        |                        |       |      |
| 1    | $1.90 \times 10^{-12}$ | $6.21 \times 10^{-12}$ | $2.71 \times 10^{-12}$ | $1.90 \times 10^{-12}$ | $9.49 \times 10^{-08}$ | $1.00 \times 10^{-13}$ | 25    | 1    |
| 2    | $9.04 \times 10^{-12}$ | $3.16 \times 10^{-10}$ | $1.50 \times 10^{-11}$ | $9.04 \times 10^{-12}$ | $9.76 \times 10^{-08}$ | $1.00 \times 10^{-11}$ | 36    | 1    |
| 3    | $6.80 \times 10^{-13}$ | $2.61 \times 10^{-12}$ | $7.86 \times 10^{-11}$ | $6.80 \times 10^{-08}$ | $9.83 \times 10^{-08}$ | $1.00 \times 10^{-11}$ | 89    | 1    |

**Table 2.** LMB neural network results for each case of the T.SIR system.

| Case | MSE                    |                        |                        | Performance            | Gradient               | Mu                     | Epoch | Time |
|------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|-------|------|
|      | Training               | Validation             | Testing                |                        |                        |                        |       |      |
| 1    | $5.26 \times 10^{-12}$ | $8.82 \times 10^{-12}$ | $4.36 \times 10^{-12}$ | $5.26 \times 10^{-12}$ | $9.98 \times 10^{-08}$ | $1.00 \times 10^{-12}$ | 32    | 1    |
| 2    | $8.05 \times 10^{-11}$ | $1.83 \times 10^{-10}$ | $1.00 \times 10^{-10}$ | $8.50 \times 10^{-12}$ | $9.83 \times 10^{-08}$ | $1.00 \times 10^{-10}$ | 551   | 3    |
| 3    | $6.89 \times 10^{-11}$ | $4.24 \times 10^{-10}$ | $6.81 \times 10^{-10}$ | $6.90 \times 10^{-11}$ | $9.94 \times 10^{-08}$ | $1.00 \times 10^{-09}$ | 265   | 4    |

#### 4. Conclusions

In this work, the biological nonlinear modified SIR system and treatment SIR system were solved by using the combinations of the integrated intelligent computing numerical Levenberg–Marquardt backpropagation (LMB) approach and neural networks. The arbitrary data of 70% were used for training, while the 15% and 15% of validation and testing data were used, respectively, to adjust the designed LMB neural network with 10 hidden neurons. In order to assess the perfection and correctness, the overlapping of the obtained outcomes from the LMB designed neural network approach with the Adam reference results was drawn. To assess the performance through convergence, the values based on mean square error were used for the testing, training, best curve and validation for all cases of both categories of the SIR system. The performance of correlation is provided to observe the regression analysis. The values of the gradient together with the step size were performed for all cases of both the SIR systems. Furthermore, the exactness and precision were certified using the numerical as well as graphical configurations of convergence plots, regression dynamics on the error histograms and MSE index, respectively.

In future, the LMB designed neural network may be explored for the bioinformatics systems [39–41], fractional order systems [42,43], system of nanofluid models [44–47] and higher order system of equations [48–52]. Additionally, the proposed LMB neural network methodology can be implemented for SIR-based infection spread dynamical models involving high nonlinearity, stiffness, singularities, and delay differentials for the analysis of the models, which still remain challenging traditional/conferential numerical methodologies.

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