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Control Charts for Joint Monitoring of the Lognormal Mean and Standard Deviation

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Abstract: The Shewhart $\overline{X}$- and S-charts are most commonly used for monitoring the process mean and variability based on the assumption of normality. However, many process distributions may follow a positively skewed distribution, such as the lognormal distribution. In this study, we discuss the construction of three combined $\overline{X}$- and S-charts for jointly monitoring the lognormal mean and the standard deviation. The simulation results show that the combined lognormal $\overline{X}$- and S-charts are more effective when the lognormal distribution is more skewed. A real example is used to demonstrate how the combined lognormal $\overline{X}$- and S-charts can be applied in practice.

Keywords: average run length; lognormal distribution; phase II monitoring; S-chart; $\overline{X}$-chart

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Citation: Huang, W.H. Control Charts for Joint Monitoring of the Lognormal Mean and Standard Deviation. *Symmetry* **2021**, *13*, 549. https://doi.org/10.3390/sym13040549

Academic Editors: Olgierd Hryniewicz, Fadel Megahed, Alejandro F Villaverde and Sergei D. Odintsov

Received: 27 January 2021
Accepted: 23 March 2021
Published: 26 March 2021

1. Introduction

Control charts are widely used in statistical process control (SPC) for monitoring and detecting out-of-control processes. The research on constructing control charts for monitoring normal processes has been extensively studied. Most control charts are designed to monitor either the process mean or the process variability, but it is usually desirable to simultaneously monitor the process mean and the process variability because both may change at the same time. A change in the standard deviation usually leads to out-of-control signals on the mean chart. When the distribution of quality characteristics is normal, the Shewhart $\overline{X}$-chart [1] is one of the most commonly used control charting techniques for monitoring the process mean, while the Shewhart S-chart is commonly used to monitor the process variability. However, in many manufacturing applications, the quality variable typically follows a positively skewed distribution, such as the lognormal distribution. For example, the percent viscosity increase (PVI) of an engine oil after it has been put to an accelerated aging test for a specific period of time is a critical quality dimension of engine oil in the automotive industry. Engineering experience indicates that the PVI follows a lognormal distribution. In this case, it is very important to simultaneously monitor the mean and the standard deviation of the PVI based on a lognormal distribution.

In general, the implementation of a control chart is done in two stages, also known as Phase I control and Phase II monitoring. In Phase I control, in order to evaluate the variation of the process over time, assess the process stability, and estimate the in-control process parameters, one collects and analyzes certain amounts of historical data. In Phase II monitoring, one collects data sequentially and monitors the process in real time to quickly detect changes in the process parameters.

In the literature, there have been several studies on constructing control charts for monitoring the lognormal mean or the lognormal standard deviation. In monitoring the lognormal mean, a modified control chart using the sample ratio was proposed by Morrison [2]. A control chart for monitoring the "geometric midrange" of a lognormal distribution was developed by Ferrell [3]. A control chart for sequentially testing the arithmetic mean of a lognormal distribution was constructed by Joffe and Sichel [4]. A simple heuristic method for constructing the $\overline{X}$- and R- charts using the weighted variance (WV) method with no assumption on the form of the distribution was proposed by Bai and Choi [5]. Castagliola [6] proposed a new $\bar{X}$ control chart devoted to the monitoring of skewed populations. Huang et al. [7] discussed the control charts for the lognormal mean based on the confidence intervals of the lognormal mean. In monitoring the standard deviation, Abu-Shawiesh [8] presented a simple approach for robustly estimating the process standard deviation based on the median absolute deviation. Adekeye and Azubuike [9] derived the limits for control charts using the median absolute deviation for monitoring non-normal processes. Adekeye [10] proposed modified control limits based on the median absolute deviation. Huang et al. [11] proposed a control chart for monitoring the standard deviation of a lognormal process based on an approximate confidence interval of the lognormal standard deviation. Karagöz [12] proposed an asymmetric control limit for a range chart under a non-normal distributed process. Liao and Pearm [13] presented a modified weighted standard deviation index for the capability of a lognormal process. Shaheen et al. [14] presented a monitoring control chart based on lognormal process variation using a repetitive sampling scheme. Omar et al. [15] proposed an efficient approach for monitoring a positively skewed process. The control charts for jointly monitoring the mean and the standard deviation of a lognormal distribution are not as well established as those for a normal distribution. McCracken and Chakraborti [16] gave an overview of control charts for joint monitoring of the mean and variance. Yang [17] proposed a single-average loss control chart to monitor a process's mean and variability. Chen and Lu [18] proposed a new sum-of-squares exponentially weighted moving average (SSEWMA) chart using auxiliary information—called the AIB-SSEWMA chart—for jointly monitoring the process mean and variability .

In this study, we discuss three combined $\overline{X}$- and S-charts for jointly monitoring the mean and the standard deviation of a lognormal process: (1) The first combined charts are the conventional combined Shewhart $\overline{X}$- and S-charts. (2) The second combined charts are constructed based on the median absolute deviation method. (3) The third combined charts are the combined lognormal $\overline{X}$- and S-charts based on the methodologies studied in Huang et al. [7] and Huang et al. [11], respectively. The performances of these combined control charts are evaluated and compared in terms of the average run length (ARL), where the run length is defined as the number of samples taken before the first out-of-control signal alerts on a control chart [19].

The rest of this paper is organized as follows. The aforementioned combined $\overline{X}$- and S-charts for jointly monitoring the lognormal mean and standard deviation are discussed in Section 2. Section 3 is devoted to assessing the performance of the combined $\overline{X}$- and S-charts. A real example from the automotive industry is given in Section 4 to demonstrate how the aforementioned combined $\overline{X}$- and S-charts can be used in practice. Concluding remarks are given in Section 5.

2. The Methodologies

In this section, we discuss three combined $\overline{X}$- and S-charts for jointly monitoring the lognormal mean and the standard deviation. Let $X_{i1}, X_{i2}, \ldots, X_{in}$, $i = 1, 2, \ldots, m$, be m samples, each with size n, following the lognormal distribution with parameters μ and σ, with a probability density function

$$f(x) = \frac{1}{x\sqrt{2\pi}\sigma} e^{-\frac{(\log x-\mu)^2}{2\sigma^2}}, \quad x > 0, \quad -\infty < \mu < \infty, \quad \sigma > 0$$

and, consequently, $Y_{ij} = \log(X_{ij}), i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n$, which follow a normal distribution with the mean, μ, and variance, σ^2. Let θ and ξ denote the mean and the standard deviation of the lognormal distribution such that $\theta = e^{\mu+\sigma^2/2}$ and $\xi = \sqrt{(e^{\sigma^2} - 1)e^{2\mu+\sigma^2}}$.

2.1. The Combined Shewhart $\overline{X}$- and S-Charts

The Shewhart $\overline{X}$- and S-charts are based on the assumption that the distribution of the quality characteristic is normal. The upper control limit (UCL) and lower control limit (LCL) of the combined Shewhart $\overline{X}$- and S-charts are given by

$$\begin{cases} \mathrm{UCL}_{SW_x} = & \theta + L_x \frac{\xi}{\sqrt{n}} \\ \mathrm{LCL}_{SW_x} = & \theta - L_x \frac{\xi}{\sqrt{n}} \end{cases}$$

and

$$\begin{cases} \mathrm{UCL}_{SW_s} & = c_4\xi + L_s\xi\sqrt{1-c_4^2} \\ \mathrm{LCL}_{SW_s} & = c_4\xi - L_s\xi\sqrt{1-c_4^2}, \end{cases}$$

respectively, where L_x and L_s are multipliers chosen to satisfy a specific in-control chart performance and $c_4 = (2/(n-1))^{1/2}[\Gamma(n/2)/\Gamma((n-1)/2)]$ [1,19].

If the parameters θ and ξ are unknown, they can be estimated by $\hat{\theta}$ and $\hat{\xi}$ using data obtained from Phase I control data. Let $\overline{X}_i$ and $S_X(i)$ be the sample mean and sample standard deviation of the ith sample, $i = 1, 2, \ldots, m$, that is, $\overline{X}_i = (\sum_{j=1}^{n} X_{ij})/n$ and $S_X(i) = \sqrt{\sum_{j=1}^{n}(X_{ij} - \overline{X}_i)^2/(n-1)}$, respectively. The sample grand mean is $\overline{\overline{X}} = \sum_{i=1}^{m} \overline{X}_i/m$, and the average of the m standard deviations is $\bar{S}_X = \sum_{i=1}^{m} S_X(i)/m$. Then, the parameters θ and ξ are estimated by $\hat{\theta}_{SW} = \overline{\overline{X}}$ and $\hat{\xi}_{SW} = \bar{S}_X/c_4$, respectively. Therefore, the control limits of the combined Shewhart $\overline{X}$- and S-charts are estimated by

$$\begin{cases} \mathrm{UCL}_{SW_x} = \overline{\overline{X}} + L_{SW_x}\frac{\bar{S}_X}{c_4\sqrt{n}} \\ \mathrm{LCL}_{SW_x} = \overline{\overline{X}} - L_{SW_x}\frac{\bar{S}_X}{c_4\sqrt{n}}, \end{cases}$$

and

$$\begin{cases} \mathrm{UCL}_{SW_s} & = \bar{S}_X + L_{SW_s}\frac{\bar{S}_X}{c_4}\sqrt{1-c_4^2} \\ \mathrm{LCL}_{SW_s} & = \bar{S}_X - L_{SW_s}\frac{\bar{S}_X}{c_4}\sqrt{1-c_4^2}, \end{cases}$$

respectively, where L_{SW_x} and L_{SW_s} are multipliers that depend on n and the desired in-control average run length (ARL_0).

When Phase II joint monitoring begins, independent samples, each of size n, are repeatedly taken from the process. Assume that the true in-control parameter θ_0 and ξ_0 are equal to $\hat{\theta}_{SW}$ and $\hat{\xi}_{SW}$, respectively, for each sample, $X_1, X_2, \ldots, X_n$; one calculates $\overline{X} = (\sum_{j=1}^{n} X_j)/n$ and $S_X = \sqrt{\sum_{j=1}^{n}(X_j - \overline{X})^2/(n-1)}$ and plots $\overline{X}$ and S_X against the sampling sequence, respectively. An out-of-control signal is detected when $\overline{X}$ is below LCL_{SW_x} or above UCL_{SW_x}, or when S_X is below LCL_{SW_s} or above UCL_{SW_s}.

2.2. The Combined Median Absolute Deviation $\overline{X}$- and S-Charts

The median absolute deviation (MAD), which measures the deviation of the data from the sample median, was first studied by Hampel [20]. It is a more robust scale estimator than the sample standard deviation and is often used as an initial value for computing more efficient and robust estimators. The MAD for a random sample, $X_1, X_2, \ldots, X_n$, is defined as follows:

$$\mathrm{MAD} = b \times Median\{|X_i - \mathrm{MD}|\},$$

where b is a constant used to make the estimator consistent for the parameter of interest and $\mathrm{MD} = Median\{X_i\}$ is the sample median of $X_1, X_2, \ldots, X_n$. If the sample observations are normally distributed, the constant b is equal to 1.4826, and the statistic $b_n\mathrm{MAD}$ is an unbiased estimator of the standard deviation (Rousseeuw and Croux) [21], where

b_n is a function of the sample size n; the values of b_n were derived and tabulated in Abu-Shawiesh [8].

Based on the conventional Shewhart principle, when the parameters θ and ξ are unknown, they can be estimated by $\hat{\theta}$ and $\hat{\xi}$ using data obtained from Phase I control data. Let $\overline{\text{MAD}} = \sum_{i=1}^{m} \text{MAD}_i/m$ be the average median absolute deviation, where MAD_i is the median absolute deviation of the ith sample, $i = 1, 2, \ldots, m$. The parameter θ and ξ can be estimated by $\hat{\theta}_{MAD} = \overline{\overline{X}}$ and $\hat{\xi}_{MAD} = b_n\overline{\text{MAD}}$, respectively. Hence, the control limits of the combined $\overline{X}$- and S-charts based on MAD are estimated by

$$\begin{cases} \text{UCL}_{MAD_x} = \overline{\overline{X}} + L_{MAD_x}\dfrac{b_n\overline{\text{MAD}}}{\sqrt{n}} \\ \text{LCL}_{MAD_x} = \overline{\overline{X}} - L_{MAD_x}\dfrac{b_n\overline{\text{MAD}}}{\sqrt{n}} \end{cases}$$

and

$$\begin{cases} \text{UCL}_{MAD_s} &= c_4 b_n\overline{\text{MAD}} + L_{MAD_s} b_n\overline{\text{MAD}}\sqrt{1-c_4^2} \\ \text{LCL}_{MAD_s} &= c_4 b_n\overline{\text{MAD}} - L_{MAD_s} b_n\overline{\text{MAD}}\sqrt{1-c_4^2}, \end{cases}$$

respectively, where L_{MAD_x} and L_{MAD_s} are multipliers that depend on n and the desired ARL_0.

When Phase II joint monitoring begins, independent samples, each of size n, are repeatedly taken from the process. For each sample, $X_1, X_2, \ldots, X_n$, one calculates $\overline{X} = (\sum_{j=1}^{n} X_j)/n$ and $b_n\text{MAD}$ and plots $\overline{X}$ and $b_n\text{MAD}$ against the sampling sequence, respectively. An out-of-control signal is detected when $\overline{X}$ is below LCL_{MAD_x} or above UCL_{MAD_x}, or when $b_n\text{MAD}$ is below LCL_{MAD_s} or above UCL_{MAD_s}.

2.3. The Combined Lognormal $\overline{X}$- and S-Charts

Based on the conventional Shewhart $\overline{X}$-chart, if the parameters θ and ξ are unknown, they can be replaced by the estimators $\hat{\theta}$ and $\hat{\xi}$ obtained from Phase I in-control data. According to Huang et al. [7], let $\overline{Y}_i = \sum_{j=1}^{n} Y_{ij}/n$ and $S_Y(i) = \sqrt{\sum_{j=1}^{n}(Y_{ij} - \overline{Y}_i)^2/(n-1)}$ be the sample mean and the sample standard deviation of the ith sample, $i = 1, 2, \ldots, m$, respectively. The grand sample mean of Y is $\overline{\overline{Y}} = \sum_{i=1}^{m} \overline{Y}_i/m$, and the average of the m standard deviations is $\bar{S}_Y = \sum_{i=1}^{m} S_Y(i)/m$. The parameters θ and ξ can be estimated by $\hat{\theta}_{Log} = e^{\overline{\overline{Y}} + \bar{S}_Y^2/2}$ and $\hat{\xi}_{Log} = \sqrt{(e^{\bar{S}_Y^2} - 1)e^{2\overline{\overline{Y}} + \bar{S}_Y^2}}$, respectively. Therefore, the control limits of the lognormal $\overline{X}$-chart are estimated using

$$\begin{cases} \text{UCL}_{Log_x} = & \hat{\theta}_{Log} + L_{Log_x}\dfrac{\hat{\xi}_{Log}}{\sqrt{n}} \\ \text{LCL}_{Log_x} = & \hat{\theta}_{Log} - L_{Log_x}\dfrac{\hat{\xi}_{Log}}{\sqrt{n}}, \end{cases}$$

where L_{Log_x} is set to satisfy a desired ARL_0.

According to Huang, et al. [11], there are two cases for the standard deviation. For the first case of $\sigma < 1$, the control limits of the lognormal S-chart are estimated using

$$\begin{cases} \text{UCL}_{Log_s} &= \left(\overline{\overline{Y}} + \frac{\bar{S}_Y^2}{2} + \log \bar{S}_Y\right) + L_{Log_s}\sqrt{\frac{\bar{S}_Y^2}{n} + \frac{\bar{S}_Y^4}{2(n+1)} + \frac{(n-4)}{2(n-3)^2} + \frac{\bar{S}_Y^2}{(n-1)}} \\ \text{LCL}_{Log_s} &= \left(\overline{\overline{Y}} + \frac{\bar{S}_Y^2}{2} + \log \bar{S}_Y\right) - L_{Log_s}\sqrt{\frac{\bar{S}_Y^2}{n} + \frac{\bar{S}_Y^4}{2(n+1)} + \frac{(n-4)}{2(n-3)^2} + \frac{\bar{S}_Y^2}{(n-1)}}, \end{cases} \quad (1)$$

where L_{Log_s} is a multiplier that depends on n and the desired ARL_0. For the second case of $\sigma > 1$, the control limits of the lognormal S-chart are estimated using

$$\begin{cases} \mathrm{UCL}_{Log_s} &= \left(\overline{\overline{Y}} + \bar{S}_Y^2\right) + L_{Log_s}\sqrt{\frac{\bar{S}_Y^2}{n} + \frac{2\bar{S}_Y^4}{n+1}} \\ \mathrm{LCL}_{Log_s} &= \left(\overline{\overline{Y}} + \bar{S}_Y^2\right) - L_{Log_s}\sqrt{\frac{\bar{S}_Y^2}{n} + \frac{2\bar{S}_Y^4}{n+1}}. \end{cases} \tag{2}$$

When Phase II joint monitoring begins, independent samples, each of size n, are repeatedly taken from the process. For each sample, $Y_1, Y_2, \ldots, Y_n$, in the case of $\sigma < 1$, one calculates and plots $e^{\overline{Y}+S_Y^2/2}$ and $\overline{Y} + S_Y^2/2 + \log(S_Y)$ against the sampling sequence, where $\overline{Y} = (\sum_{j=1}^n Y_j)/n$ and $S_Y = \sqrt{\sum_{j=1}^n (Y_j - \overline{Y})^2/(n-1)}$. An out-of-control signal is detected if the plotting statistic $e^{\overline{Y}+S_Y^2/2}$ is below LCL_{Log_x} or above UCL_{Log_x}, or when the plotting statistic $\overline{Y} + S_Y^2/2 + \log(S_Y)$ falls below LCL_{Log_s} or above UCL_{Log_s}. For the case of $\sigma > 1$, one computes and plots $e^{\overline{Y}+S_Y^2/2}$ and $\overline{Y} + S_Y^2/2$ against the sampling sequence. An out-of-control signal is detected if the plotting statistic $e^{\overline{Y}+S_Y^2/2}$ is below LCL_{Log_x} or above UCL_{Log_x}, or when the plotting statistic $\overline{Y} + S_Y^2/2$ falls below LCL_{Log_s} or above UCL_{Log_s}.

Remark 1. *The population standard deviation σ is usually unknown and needs to be estimated in practice. It can be estimated by utilizing data collected from Phase I control, when the process was in control. Based on the estimate, one can then decide whether to use Case I or Case II to construct the lognormal S-chart.*

3. Chart Performance Evaluations and Comparisons

In this section, we conduct a simulation study to compare the performance of the combined lognormal $\overline{X}$- and S-charts with the combined MAD $\overline{X}$- and S-charts and the conventional Shewhart $\overline{X}$- and S-charts in terms of the ARL.

3.1. Simulated Settings

Since $E(X) = e^{\mu+\sigma^2/2}$, we set $\mu = -\sigma^2/2$ such that the mean of the lognormal distribution $E(X) = 1$ remains unchanged. Let $\xi_0 = \sqrt{e^{\sigma_0^2} - 1}$ be the value of the in-control parameter. As discussed earlier, the control limits need to be determined using Phase I observations and will depend on σ_0, the subgroup size n, and the number of Phase I samples m. Therefore, we approximate the control limits of the three combined $\overline{X}$- and S-charts using simulations for various values of σ_0 and combinations of $m = 50$ and 100, as well as $n = 5$ and 10. The value of σ_0 is set to be between 0.2 and 2.0, with an increment of 0.2. The multipliers of the three combined control charts are calibrated to have an overall ARL_0 that is approximately equal to 370. Note that the considered ARL for the three combined control charts is conditional on the estimated UCL and LCL. Here, we describe how the simulation is conducted for the combined lognormal $\overline{X}$- and S-charts, as it is similar for the other two combined charts. Given σ_0, m, and n, the following steps are carried out:

Step 1: Choose a value of L_{Log_x} and a value of L_{Log_s}, and generate m independent samples of n observations each from a lognormal distribution with a mean of 1 and a standard deviation of ξ_0. Compute the UCL_{Log_x} and LCL_{Log_x}, and calculate the UCL_{Log_s} and LCL_{Log_s} using either Equation (1) if $\sigma_0 < 1$ or Equation (2) if $\sigma_0 \geq 1$.

Step 2: Repeatedly generate samples of n observations each from a lognormal distribution with a mean of 1 and standard deviation of ξ_0. For each sample, calculate the two plotting statistics for the $\overline{X}$-chart and S-chart, respectively. Then, evaluate whether the plotting statistic for the $\overline{X}$-chart exceeds UCL_{Log_x} or goes below LCL_{Log_x}; next, evaluate whether the plotting statistic for the S-chart exceeds UCL_{Log_s} or goes below LCL_{Log_s}. Stop when it does, and denote the number of samples generated by RL_i.

Step 3: Repeat Step 2 5000 times, resulting in RL_i, $i = 1, 2, \ldots, 5000$. Calculate the simulated in-control average run length $\text{ARL}_{\text{sim}} = \sum_{i=1}^{5000} \text{RL}_i / 5000$.

Step 4: If $\text{ARL}_{\text{sim}} \approx 370$, stop. If $\text{ARL}_{\text{sim}} \neq 370$, return to Step 1. Choose a larger L_S if $\text{ARL}_{\text{sim}} < 370$ and a smaller L_S if $\text{ARL}_{\text{sim}} > 370$.

The multipliers of the three combined $\overline{X}$- and S-charts are given in Tables 1 and 2 for different values of ξ_0. Note that the multipliers of these three combined $\overline{X}$- and S-charts increase when $\xi_0(\sigma_0)$ increases.

Denote the out-of-control process parameters by $\theta_1 = \theta_0 + a\xi_0$ and $\xi_1 = b\xi_0$, where $a > 0,\ b > 1$. Given $\xi_0 = \sqrt{e^{\sigma_0^2} - 1}$, we simulated the ARL_1 for the three combined charts in the same way as in Steps 2 and 3, which were mentioned earlier. The out-of-control ARLs (ARL_1s) of the three combined $\overline{X}$- and S-charts are summarized in Tables 3 and 4 ($m = 50$) and Tables 5 and 6 ($m = 100$).

Table 1. The multipliers of the three combined $\overline{X}$- and S-charts based on different values of ξ_0 when the sample $m = 50$ with the subgroup size $n = 5$ and 10. ($\text{ARL}_0 \approx 370$).

n	σ_0	ξ_0	Shewhart		MAD		Lognormal	
			$\overline{X}$-Chart	S-Chart	$\overline{X}$-Chart	S-Chart	$\overline{X}$-Chart	S-Chart
5	0.2	0.20	3.18	3.92	3.62	4.89	3.42	4.31
	0.4	0.41	3.64	6.56	4.47	7.55	4.14	3.92
	0.6	0.65	4.41	10.43	5.86	12.68	5.31	3.58
	0.7	0.79	4.97	11.53	5.93	16.97	6.22	3.54
	0.8	0.95	5.51	12.41	7.19	22.76	7.43	3.49
	0.9	1.12	6.27	14.67	10.49	26.48	8.85	3.51
	1.0	1.31	7.10	17.30	13.13	33.17	9.04	4.07
	1.2	1.79	9.37	23.99	20.59	56.47	10.45	3.88
	1.4	2.47	11.70	34.12	34.33	93.66	11.23	3.74
	1.6	3.45	14.53	44.21	54.21	150.35	13.06	3.63
	1.8	4.95	19.82	52.31	85.53	256.64	24.56	3.51
	2.0	7.32	21.98	64.76	148.05	427.10	30.88	3.41
10	0.2	0.20	3.28	4.33	3.45	4.50	3.44	3.78
	0.4	0.41	3.66	6.62	4.12	7.97	3.86	3.60
	0.6	0.65	4.14	10.90	5.38	14.11	4.48	3.39
	0.7	0.79	4.28	11.20	5.92	16.88	5.17	3.56
	0.8	0.95	5.77	14.24	7.84	25.05	5.27	3.38
	0.9	1.12	6.88	16.74	9.98	32.49	6.00	3.31
	1.0	1.31	7.70	19.66	12.06	44.17	6.12	4.13
	1.2	1.79	8.31	32.31	22.67	73.78	9.75	3.25
	1.4	2.47	10.49	38.49	34.00	95.11	10.86	3.26
	1.6	3.45	15.70	48.70	59.27	161.38	13.18	4.48
	1.8	4.95	20.48	64.48	97.42	277.53	23.99	3.09
	2.0	7.32	24.28	71.28	175.14	485.25	29.96	2.96

Table 2. The multipliers of the three combined $\overline{X}$- and S-charts based on different values of ξ_0 when the sample $m = 100$ with the subgroup size $n = 5$ and 10. ($ARL_0 \approx 370$).

n	σ_0	ξ_0	Shewhart		MAD		Lognormal	
			$\overline{X}$-Chart	S-Chart	$\overline{X}$-Chart	S-Chart	$\overline{X}$-Chart	S-Chart
5	0.2	0.20	3.43	3.98	3.65	5.19	3.62	4.31
	0.4	0.41	3.84	6.67	4.68	8.64	4.25	3.72
	0.6	0.65	4.61	10.53	6.16	14.28	5.81	3.68
	0.7	0.79	5.07	11.63	6.83	18.77	6.62	3.56
	0.8	0.95	5.61	12.81	7.49	26.56	7.63	3.69
	0.9	1.12	6.73	14.77	11.39	28.38	8.75	3.61
	1.0	1.31	7.20	17.50	13.43	43.17	9.04	4.02
	1.2	1.79	9.77	24.21	21.59	62.47	10.25	3.95
	1.4	2.47	11.80	35.12	35.33	98.66	11.45	3.76
	1.6	3.45	14.62	45.21	56.41	155.75	14.62	3.66
	1.8	4.95	19.89	53.51	86.73	266.46	24.21	3.44
	2.0	7.32	22.13	65.86	150.05	447.23	30.26	3.23
10	0.2	0.20	3.48	4.67	3.25	5.52	3.64	3.48
	0.4	0.41	3.97	6.94	4.32	8.17	3.84	3.67
	0.6	0.65	4.45	11.24	5.78	15.11	4.88	3.46
	0.7	0.79	4.97	12.40	6.72	17.88	5.47	3.23
	0.8	0.95	5.67	14.54	7.82	26.05	6.27	3.28
	0.9	1.12	6.98	17.24	11.78	33.49	7.00	3.51
	1.0	1.31	7.60	20.56	16.06	46.17	8.12	4.04
	1.2	1.79	8.21	31.31	32.67	75.78	11.45	3.63
	1.4	2.47	10.38	37.59	44.00	107.11	12.76	3.36
	1.6	3.45	15.60	47.60	63.27	159.38	18.38	4.25
	1.8	4.95	20.28	63.78	102.32	272.53	25.89	3.45
	2.0	7.32	25.38	72.38	165.14	485.25	31.66	3.99

3.2. Discussion of Results

According to the assessment of the numerical results summarized in Tables 3–6, the combined lognormal $\overline{X}$- and S-charts perform better than the other two combined $\overline{X}$- and S-charts when $\sigma_0 > 0.6$. Nevertheless, the ARL_1s of the combined lognormal $\overline{X}$- and S-charts are larger than those of the other two combined $\overline{X}$- and S-charts when $\sigma_0 < 0.6$. Note that smaller values of ξ_0 correspond to smaller values of σ_0, under which the lognormal distribution is more symmetric. Therefore, when σ_0 is small, which means that the data are more symmetric, the combined lognormal $\overline{X}$- and S-charts are less effective than the combined Shewhart $\overline{X}$- and S-charts and the combined MAD $\overline{X}$- and S-charts. On the other hand, as σ_0 becomes larger, the lognormal distribution becomes more skewed, thus making the combined lognormal $\overline{X}$- and S-charts more effective in detecting changes in θ and ξ than the other two combined control charts.

Table 3. The ARL_1s of the three combined $\overline{X}$- and S-charts for different shift sizes a, b when the sample $m = 50$ with the subgroup size $n = 5$ under various values of in-control σ_0 ($\theta_1 = \theta_0 + a\xi_0$, $\xi_1 = b\xi_0$).

σ_0	b	Method	a = 0	0.5	1.0	1.5	2.0
0.2	1	Shewhart	370.5876	33.1389	5.4409	1.7323	1.0933
		MAD	370.6514	33.0567	5.4974	1.7392	1.0953
		Lognormal	370.6674	34.0549	5.5621	1.7687	1.1041
	1.5	Shewhart	9.4106	6.8289	3.3947	1.8161	1.2299
		MAD	9.8232	6.8452	3.4130	1.8256	1.2240
		Lognormal	25.4409	9.4809	3.6692	1.9173	1.2532
	2	Shewhart	3.2328	3.1404	2.4388	1.7480	1.3173
		MAD	3.2727	3.1149	2.4467	1.7439	1.3165
		Lognormal	8.9818	5.6787	3.0896	1.8960	1.3359
	2.5	Shewhart	2.0104	2.0738	1.8742	1.6014	1.3234
		MAD	2.0080	2.0718	1.9071	1.5913	1.3477
		Lognormal	4.9118	4.0865	2.7400	1.8789	1.4099
	3	Shewhart	1.5383	1.6315	1.5952	1.4578	1.3135
		MAD	1.5413	1.6484	1.5961	1.4643	1.3094
		Lognormal	3.2998	3.2510	2.5333	1.9262	1.4989
0.4	1	Shewhart	370.8525	57.4500	9.4631	2.3675	1.1967
		MAD	370.7704	59.3796	9.7455	2.4411	1.2127
		Lognormal	370.4578	64.7503	12.6992	3.0439	1.3340
	1.5	Shewhart	23.2063	13.0698	5.3308	2.3498	1.3732
		MAD	17.6266	11.9182	5.2832	2.3658	1.3923
		Lognormal	30.8691	14.3416	5.8628	2.6144	1.4868
	2	Shewhart	9.0517	6.7347	4.0695	2.3476	1.5085
		MAD	7.4380	5.9136	3.8270	2.2987	1.5223
		Lognormal	12.3345	7.6288	4.2017	2.4229	1.5861
	2.5	Shewhart	5.6074	4.7608	3.4066	2.3225	1.6056
		MAD	4.8728	4.2014	3.1759	2.2572	1.6220
		Lognormal	7.6414	5.4351	3.6175	2.3523	1.6571
	3	Shewhart	4.2780	3.8354	3.0153	2.2537	1.7097
		MAD	3.7098	3.4302	2.8059	2.1490	1.6486
		Lognormal	5.5752	4.4662	3.2416	2.3127	1.7068

σ_0	b	Method	a = 0	0.5	1.0	1.5	2.0
0.7	1	Shewhart	370.3207	155.7740	33.7742	6.5182	1.9279
		MAD	370.3010	151.8658	32.7340	5.6943	1.2404
		Lognormal	370.9756	155.4663	34.5119	6.3932	6.0618
	1.5	Shewhart	44.9486	27.2042	16.4369	4.3916	2.0714
		MAD	32.0282	25.7571	11.1332	3.5526	1.8292
		Lognormal	34.7829	33.3733	19.9915	9.1296	3.8278
	2	Shewhart	22.2764	14.2369	7.7026	3.9024	2.1394
		MAD	17.8550	9.8207	5.0002	2.6065	1.5831
		Lognormal	**14.8546**	13.7376	9.5722	5.6798	3.2390
	2.5	Shewhart	15.8194	10.5024	6.2064	3.6811	2.2267
		MAD	13.6498	8.0096	6.4987	2.6282	1.7091
		Lognormal	**9.7387**	**7.9411**	**5.8226**	**2.5524**	1.9664
	3	Shewhart	13.4376	8.6236	5.5912	3.5186	2.3480
		MAD	11.5635	7.0947	5.3955	3.7033	1.8276
		Lognormal	**7.3832**	**6.9745**	**5.2248**	**3.4183**	**1.7068**
0.8	1	Shewhart	370.9561	200.6194	51.7492	39.7897	5.5356
		MAD	370.1949	172.5236	45.8093	33.6037	4.4000
		Lognormal	370.3930	**89.4831**	**38.7071**	**28.6250**	**3.3813**
	1.5	Shewhart	48.5403	33.1507	24.7707	5.8389	2.4189
		MAD	40.1210	29.4552	17.8604	5.1750	2.6354
		Lognormal	**37.7890**	**19.6181**	**12.0974**	**3.6312**	**1.6426**
	2	Shewhart	25.9967	17.1290	9.1585	4.6981	2.4421
		MAD	23.7717	12.3919	8.1207	4.0455	1.9666
		Lognormal	**16.6343**	**11.2391**	**7.7895**	**3.5212**	**1.4357**
	2.5	Shewhart	19.3779	12.3220	7.4559	4.2395	2.5540
		MAD	18.1812	10.0862	5.3710	3.0231	1.8996
		Lognormal	**10.8657**	**9.3758**	**4.2406**	**2.5652**	**1.3031**
	3	Shewhart	16.6609	10.2463	6.6053	4.0743	2.4744
		MAD	15.8463	8.9725	5.1200	3.0391	1.7997
		Lognormal	**8.2814**	**8.4022**	**3.9498**	**2.3421**	**1.1035**

Table 3. *Cont.*

σ_0	b	Method	a					σ_0	b	Method	a				
			0	**0.5**	**1.0**	**1.5**	**2.0**				**0**	**0.5**	**1.0**	**1.5**	**2.0**
0.6	1	Shewhart	370.7858	107.6300	20.1744	4.1764	1.5040	0.9	1	Shewhart	370.8647	272.1883	86.9446	17.1402	3.7232
		MAD	370.0903	98.9225	18.3133	3.9410	1.4588			MAD	370.9004	267.3391	82.7822	16.2753	3.6143
		Lognormal	370.9223	163.3991	48.2872	10.0419	2.6444			Lognormal	370.5228	**254.0149**	**73.1266**	**14.1121**	**2.7163**
	1.5	Shewhart	39.0469	21.9195	8.3676	3.4159	1.7065		1.5	Shewhart	57.4917	42.1448	20.2266	7.8431	3.1752
		MAD	39.9318	20.3995	8.0336	3.3291	1.6461			MAD	58.5613	41.3647	19.5978	7.5968	3.1091
		Lognormal	42.2877	23.4891	12.0946	5.1926	2.3812			Lognormal	**41.2614**	**40.0169**	**12.3985**	**6.2638**	**2.8720**
	2	Shewhart	18.1314	11.7363	6.0349	3.2057	1.8352		2	Shewhart	32.8299	21.0701	11.7676	5.8344	3.0573
		MAD	14.8473	10.3234	5.7272	3.0329	1.8043			MAD	33.0775	21.3152	11.6221	5.8441	2.9709
		Lognormal	**13.7309**	**10.0470**	6.8059	3.9859	2.3085			Lognormal	**18.7614**	**19.2499**	**9.6325**	**4.6376**	**2.4841**
	2.5	Shewhart	12.9515	8.4981	5.2365	3.1303	1.9703		2.5	Shewhart	24.9555	15.4599	9.0535	5.0748	2.9942
		MAD	10.4468	7.4428	4.7616	2.9565	1.9278			MAD	24.7510	15.3980	8.9365	5.1398	2.9378
		Lognormal	**8.8325**	**7.2478**	**4.1635**	3.4308	2.2837			Lognormal	**12.3057**	**14.1052**	**7.7054**	**4.2312**	**2.1702**
	3	Shewhart	10.5233	7.2291	4.6484	3.0989	2.0432		3	Shewhart	21.5519	13.0571	8.0724	4.8944	3.0179
		MAD	8.6382	6.3010	4.3445	2.9296	1.9929			MAD	21.7011	12.8185	7.8447	4.8321	2.9723
		Lognormal	**6.5468**	**5.8288**	**4.1018**	**2.8463**	2.2316			Lognormal	**9.3971**	**10.4718**	**6.0702**	**3.4203**	**1.4492**
1.0	1	Shewhart	370.1082	352.5607	140.7904	29.4373	5.8323	1.6	1	Shewhart	370.6627	197.5850	137.9188	124.4284	35.2713
		MAD	370.8587	340.7272	138.9174	28.9253	5.7747			MAD	370.8194	125.8934	112.2759	106.4028	39.2523
		Lognormal	370.8260	**305.0791**	**108.3718**	**23.2703**	**5.1347**			Lognormal	370.6201	**96.0210**	**85.5027**	**77.2155**	**28.8118**
	1.5	Shewhart	82.0592	53.1252	27.2081	10.5922	4.1476		1.5	Shewhart	188.0067	91.9555	57.2729	29.2764	19.8352
		MAD	79.1992	51.8306	26.2795	10.6095	4.2097			MAD	168.6240	95.4633	61.9889	30.7608	18.8336
		Lognormal	**62.2954**	**50.2297**	**24.6471**	**10.3423**	**4.0879**			Lognormal	**124.0627**	**61.1622**	**55.8386**	**28.3368**	**17.0085**
	2	Shewhart	42.7011	26.3384	14.4086	7.3259	3.7441		2	Shewhart	139.7935	54.3841	28.7977	15.0900	14.5243
		MAD	41.1443	26.4623	14.6683	7.3509	3.6969			MAD	126.9600	55.0958	30.5782	16.3860	14.0639
		Lognormal	**35.1545**	**24.4176**	**13.6976**	**6.7383**	**3.5356**			Lognormal	**76.9825**	**44.5238**	**29.4728**	**13.2595**	**12.6282**
	2.5	Shewhart	32.8655	19.6279	11.2450	6.2599	3.5027		2.5	Shewhart	120.0266	40.7964	21.0882	11.5571	9.2596
		MAD	31.7344	18.6792	11.1421	6.1192	3.5155			MAD	109.6311	40.6920	22.2004	12.2555	8.7899
		Lognormal	**26.7705**	**17.1480**	**10.6884**	**5.1928**	**3.4647**			Lognormal	**33.7981**	**19.2101**	**17.7653**	**10.9052**	**8.2451**
	3	Shewhart	28.3724	16.4578	9.4951	5.6448	3.4626		3	Shewhart	108.8337	35.4507	17.7070	10.8780	7.8112
		MAD	27.0354	15.7492	9.4560	5.6625	3.4579			MAD	99.7151	34.6422	18.3294	10.5207	7.1688
		Lognormal	**22.8247**	**13.7855**	**8.2770**	**4.4195**	**3.3431**			Lognormal	**21.6028**	**14.6141**	**12.5705**	**9.0014**	**5.9512**

Table 3. *Cont.*

σ_0	b	Method	a					σ_0	b	Method	a				
			0	**0.5**	**1.0**	**1.5**	**2.0**				**0**	**0.5**	**1.0**	**1.5**	**2.0**
1.2	1	Shewhart	370.2070	127.1685	93.0347	58.4919	28.2132	1.8	1	Shewhart	370.0281	211.7618	194.6514	122.9319	44.5595
		MAD	370.1579	112.9836	85.5007	50.8073	25.0635			MAD	370.7778	171.3655	114.8130	97.4993	46.4308
		Lognormal	370.5541	**73.5001**	**69.0082**	**42.9366**	**18.9995**			Lognormal	370.4372	**104.2771**	**96.3862**	**84.3398**	**36.3830**
	1.5	Shewhart	107.6791	80.8439	48.1836	31.0348	17.9942		1.5	Shewhart	228.2614	105.2041	69.0107	36.8851	25.1169
		MAD	101.6682	74.3598	43.5487	28.9886	17.1623			MAD	215.5399	96.2680	63.2132	39.6925	27.9185
		Lognormal	**91.8313**	**58.8524**	**41.3862**	**26.2482**	**15.6045**			Lognormal	**121.6934**	**67.5094**	**58.6162**	**35.5343**	**17.2989**
	2	Shewhart	68.6448	39.7361	23.2139	15.1838	13.8145		2	Shewhart	179.7938	61.0501	35.1689	28.9061	19.3550
		MAD	66.6215	38.4405	21.9905	14.2946	11.6379			MAD	170.6680	59.1682	34.5734	27.2855	16.0695
		Lognormal	**58.8728**	**33.6829**	**18.0457**	**12.3029**	**10.8368**			Lognormal	**76.5758**	**44.8745**	**32.3269**	**22.3175**	**14.5751**
	2.5	Shewhart	55.0992	28.8660	16.6221	13.2518	11.1436		2.5	Shewhart	161.9149	47.6491	25.1014	18.8123	9.5373
		MAD	53.7369	27.4828	15.9406	12.8614	10.9320			MAD	151.0164	41.1177	23.1699	19.8495	10.2423
		Lognormal	**43.8543**	**24.9547**	**14.9882**	**10.7131**	**9.4383**			Lognormal	**33.8813**	**26.5738**	**19.9272**	**17.5985**	**9.3929**
	3	Shewhart	48.3724	23.9408	13.7399	8.1559	7.7497		3	Shewhart	146.2088	40.9759	20.7838	12.6363	6.7109
		MAD	46.2084	23.3242	13.2061	7.7780	7.3648			MAD	137.6635	38.5105	16.2812	11.8535	6.9841
		Lognormal	**31.2842**	**21.5299**	**10.7722**	**6.0039**	**5.1424**			Lognormal	**22.2045**	**17.4965**	**13.5542**	**10.9552**	**5.1279**
1.4	1	Shewhart	370.2804	234.3393	150.2702	96.7646	51.6362	2.0	1	Shewhart	370.9529	207.0595	192.4963	127.9295	56.2271
		MAD	370.1446	186.6619	114.4979	92.8002	45.9106			MAD	370.7339	195.0250	139.9612	86.9680	50.1305
		Lognormal	370.3465	**141.0920**	**93.7749**	**81.3092**	**34.1376**			Lognormal	370.8155	**101.2070**	**94.8611**	**82.6824**	**43.7312**
	1.5	Shewhart	148.6589	88.8568	53.2927	24.8184	9.6825		1.5	Shewhart	244.1069	108.3417	67.3953	38.4738	27.4324
		MAD	141.2181	98.1891	66.6576	32.9896	13.2180			MAD	254.0395	95.2738	64.1288	35.1422	25.9771
		Lognormal	**96.1163**	**48.5422**	**33.8262**	**22.7247**	**8.5101**			Lognormal	**119.3429**	**65.0821**	**58.8626**	**33.8535**	**19.2322**
	2	Shewhart	102.3473	48.4194	26.6202	13.7468	7.8379		2	Shewhart	206.3446	60.0536	37.8583	28.4061	18.0862
		MAD	97.6310	51.9425	30.9965	17.0774	8.2794			MAD	215.9862	59.2243	33.1267	27.2283	16.9412
		Lognormal	**57.3688**	**32.2964**	**25.0057**	**11.7694**	**6.9322**			Lognormal	**75.1197**	**45.7710**	**30.5755**	**22.2057**	**14.2517**
	2.5	Shewhart	85.2871	35.6013	19.5332	10.6402	5.8483		2.5	Shewhart	187.0434	48.1952	24.8348	18.3137	9.7941
		MAD	80.4928	37.7911	21.6191	12.3122	6.8970			MAD	195.5457	41.9829	22.0425	18.6588	9.5545
		Lognormal	**43.4382**	**24.6000**	**14.8537**	**9.9589**	**5.2384**			Lognormal	**33.1885**	**25.7326**	**17.9497**	**16.0139**	**8.2033**
	3	Shewhart	75.8918	30.5052	16.1662	9.0808	5.4607		3	Shewhart	174.7410	40.4570	21.2539	12.8095	6.8517
		MAD	72.6004	31.5639	17.6427	10.3472	6.0633			MAD	186.5626	36.9426	15.0864	11.9275	6.4438
		Lognormal	**31.3230**	**21.7687**	**10.8915**	**8.3807**	**4.8195**			Lognormal	**21.7899**	**16.6963**	**13.8620**	**10.0169**	**5.4363**

Table 4. The ARL_1s of the three combined $\overline{X}$- and S-charts for different shift sizes a, b when the sample $m = 50$ with the subgroup size $n = 10$ under various values of in-control σ_0 ($\theta_1 = \theta_0 + a\xi_0$, $\xi_1 = b\xi_0$).

σ_0	b	Method	a = 0	0.5	1.0	1.5	2.0
0.2	1	Shewhart	370.9081	19.3222	3.0885	1.1424	1.0014
		MAD	370.4250	19.2560	2.4100	1.0870	1.0004
		Lognormal	370.7297	20.7056	3.2961	1.1897	1.0015
	1.5	Shewhart	7.4076	5.1928	2.0826	1.2732	1.0185
		MAD	7.3296	5.0890	2.0758	1.2035	1.0163
		Lognormal	30.4634	7.4018	2.1568	1.2972	1.0273
	2	Shewhart	2.1828	2.2355	1.7981	1.3080	1.0793
		MAD	2.1541	2.1362	1.6753	1.2546	1.0509
		Lognormal	5.7311	4.0887	2.0713	1.3100	1.0886
	2.5	Shewhart	1.3869	1.4733	1.3829	1.2361	1.0973
		MAD	1.3181	1.4417	1.3563	1.2009	1.0765
		Lognormal	2.4695	2.4231	1.8401	1.3491	1.1048
	3	Shewhart	1.1758	1.2133	1.2073	1.1481	1.0861
		MAD	1.1713	1.2126	1.1908	1.1385	1.0701
		Lognormal	1.6297	1.7096	1.5474	1.3299	1.1476
0.4	1	Shewhart	370.2767	33.4037	3.2156	1.1499	1.0016
		MAD	370.9081	32.7884	3.0868	1.1257	1.0015
		Lognormal	370.3675	34.3551	3.3507	1.1621	1.0017
	1.5	Shewhart	16.9536	8.8801	2.7310	1.3036	1.0224
		MAD	16.5707	8.7360	2.7237	1.2894	1.0218
		Lognormal	33.9817	9.7627	2.7814	1.3231	1.0265
	2	Shewhart	5.5229	4.5005	2.4701	1.4245	1.0749
		MAD	5.4136	4.4632	2.4676	1.4111	1.0748
		Lognormal	8.3369	5.4050	2.5674	1.4385	1.0772
	2.5	Shewhart	3.3396	2.9814	2.1863	1.5088	1.1405
		MAD	3.2846	2.9530	2.1720	1.4740	1.1401
		Lognormal	4.1471	3.6042	2.3501	1.5247	1.1536
	3	Shewhart	2.4718	2.4012	1.9537	1.5250	1.1983
		MAD	2.4456	2.3429	1.9471	1.5123	1.1909
		Lognormal	2.8474	2.7256	2.1264	1.5519	1.1984
0.7	1	Shewhart	370.2767	49.7335	4.6505	1.2710	1.0036
		MAD	370.4789	48.7692	4.7311	1.2712	1.0051
		Lognormal	370.3414	180.6116	15.8844	2.2847	1.0673
	1.5	Shewhart	28.7398	13.5346	3.6539	1.4543	1.0426
		MAD	28.3007	13.4169	3.6603	1.4607	1.0384
		Lognormal	45.6241	23.1712	6.6534	2.1842	1.1719
	2	Shewhart	13.4718	7.9900	3.3861	1.6153	1.1009
		MAD	13.1096	7.9343	3.3007	1.6141	1.1088
		Lognormal	**12.5832**	**6.6929**	**3.1316**	2.1587	1.2745
	2.5	Shewhart	9.3689	6.9686	3.1551	1.7195	1.1860
		MAD	9.3583	6.9769	3.1826	1.7160	1.1855
		Lognormal	**8.5567**	**6.0140**	**3.0690**	**1.4102**	**1.1690**
	3	Shewhart	7.6901	5.1861	3.0773	1.8125	1.2640
		MAD	7.6831	5.1278	3.0448	1.7879	1.2610
		Lognormal	**6.1854**	**4.9091**	**2.4611**	**1.4053**	**1.1318**
0.8	1	Shewhart	370.2878	219.6113	27.2480	3.0930	1.1442
		MAD	370.9130	231.9436	11.7455	1.8489	1.0291
		Lognormal	370.8994	**126.8635**	**10.5180**	**1.4161**	**1.0111**
	1.5	Shewhart	47.6360	34.7741	9.8071	2.7904	1.2881
		MAD	46.3061	23.9643	6.2360	1.9596	1.1315
		Lognormal	**38.3655**	**22.4835**	**6.1672**	**1.2322**	**1.1139**
	2	Shewhart	23.3160	14.9901	7.0482	2.7521	1.3842
		MAD	22.1100	12.6533	4.9237	2.0922	1.2473
		Lognormal	**14.3060**	**11.3461**	**4.7131**	**2.0561**	**1.1392**
	2.5	Shewhart	15.3962	9.8542	5.5461	2.7333	1.5371
		MAD	15.2581	9.0520	4.4284	2.1882	1.3474
		Lognormal	**8.8853**	**7.4473**	**3.9501**	**2.1392**	**1.3141**
	3	Shewhart	13.1704	8.1301	4.7085	2.6973	1.6861
		MAD	12.2926	7.4219	4.1621	2.2708	1.4252
		Lognormal	**7.0435**	**5.7202**	**3.6646**	**2.0742**	**1.4081**

Table 4. *Cont.*

σ_0	b	Method	a				
			0	**0.5**	**1.0**	**1.5**	**2.0**
0.6	1	Shewhart	370.5135	52.0465	4.6072	1.2627	1.0034
		MAD	370.4250	57.4244	4.9432	1.2930	1.0045
		Lognormal	370.4296	75.3478	6.7514	1.4739	1.0129
	1.5	Shewhart	31.5803	13.3133	3.5249	1.4411	1.0423
		MAD	31.6280	14.1203	3.7293	1.4882	1.0489
		Lognormal	33.7391	14.7977	4.1932	1.6408	1.0699
	2	Shewhart	13.2349	7.6655	3.2550	1.5879	1.1067
		MAD	11.5893	7.6865	3.2483	1.5213	1.1011
		Lognormal	**10.5323**	**7.6301**	3.3955	1.7064	1.1516
	2.5	Shewhart	8.5239	5.6841	3.0766	1.6909	1.1764
		MAD	7.7860	5.5123	3.0867	1.7394	1.1980
		Lognormal	**6.0973**	**5.0923**	**3.0757**	1.7682	1.2296
	3	Shewhart	6.7565	4.8221	2.8942	1.8193	1.2609
		MAD	6.2130	4.5108	2.8719	1.8126	1.2415
		Lognormal	**4.5294**	**3.8448**	**2.7636**	**1.8035**	1.2935
1.0	1	Shewhart	370.6166	332.8367	188.9562	14.3727	2.0394
		MAD	370.0342	322.5611	71.5062	8.4427	1.6568
		Lognormal	370.0850	**302.2157**	**50.4446**	**5.3961**	**1.3435**
	1.5	Shewhart	67.3707	68.0598	27.8241	6.7689	2.1004
		MAD	63.4231	39.5850	21.1149	4.0292	1.7226
		Lognormal	**50.4176**	**32.7854**	**12.1184**	**3.6601**	**1.4980**
	2	Shewhart	38.0298	26.5652	13.7881	5.2555	2.1974
		MAD	36.9797	19.9437	8.5620	2.9163	1.8728
		Lognormal	**22.8857**	**14.9519**	**7.1471**	**2.1647**	**1.5914**
	2.5	Shewhart	28.7747	17.2229	9.6787	4.6791	2.2443
		MAD	27.0439	14.2056	6.4149	3.9089	1.8873
		Lognormal	**15.3440**	**9.8909**	**5.5149**	**2.8982**	**1.6562**
	3	Shewhart	23.8529	13.3245	8.0833	4.2880	2.3511
		MAD	22.6212	12.0043	5.7307	3.8915	1.9994
		Lognormal	**12.0215**	**7.8531**	**4.6714**	**2.7470**	**1.7187**

σ_0	b	Method	a				
			0	**0.5**	**1.0**	**1.5**	**2.0**
0.9	1	Shewhart	370.0321	249.0215	33.4006	4.9541	3.5103
		MAD	370.8518	240.4069	15.0331	4.8766	3.3986
		Lognormal	370.9942	**206.4453**	**12.7513**	**3.7410**	**2.2861**
	1.5	Shewhart	58.7436	39.4739	16.5238	4.7206	3.6789
		MAD	61.1703	38.9699	15.7333	4.6883	3.5494
		Lognormal	**50.6196**	**23.7521**	**11.4119**	**3.4189**	**2.4456**
	2	Shewhart	27.2146	20.7901	10.3342	4.9975	3.8407
		MAD	31.9813	18.8158	8.7983	4.4567	3.6840
		Lognormal	**25.4479**	**12.2603**	**6.7844**	**3.9861**	**2.5404**
	2.5	Shewhart	23.1256	13.5208	7.8576	3.7030	2.9191
		MAD	21.5171	11.9236	6.6845	3.9491	2.7285
		Lognormal	**10.0244**	**9.0275**	**5.1902**	**2.7617**	**1.6205**
	3	Shewhart	18.6660	10.5044	6.4344	3.4826	2.0193
		MAD	16.4025	9.7620	5.1470	3.9268	1.8118
		Lognormal	**8.4741**	**7.6326**	**6.4502**	**2.6404**	**1.6675**
1.6	1	Shewhart	370.3005	238.0582	115.4427	33.4474	13.5167
		MAD	370.0476	214.9566	105.5952	21.8773	12.6838
		Lognormal	370.8943	**118.1608**	**75.0212**	**17.6984**	**7.3762**
	1.5	Shewhart	154.0701	94.8094	42.0174	21.1098	11.0633
		MAD	149.5153	84.0922	34.7502	18.9286	8.5982
		Lognormal	**86.6869**	**67.1434**	**24.2182**	**15.6860**	**7.9335**
	2	Shewhart	106.3674	47.4599	20.4541	17.5517	9.9238
		MAD	99.3679	44.0346	17.9171	16.5748	7.5864
		Lognormal	**79.8444**	**35.4288**	**14.6199**	**12.7701**	**5.8412**
	2.5	Shewhart	83.6302	33.5085	15.1198	13.4812	8.8672
		MAD	82.9371	31.3471	13.2152	11.7166	6.6122
		Lognormal	**60.3090**	**26.7887**	**11.0548**	**8.7066**	**4.0178**
	3	Shewhart	74.5579	28.9458	12.4605	9.8249	7.8962
		MAD	71.5010	26.2747	11.4387	8.2252	5.6417
		Lognormal	**49.8441**	**20.0574**	**7.1552**	**5.0035**	**2.2546**

Table 4. *Cont.*

σ_0	b	Method	a					σ_0	b	Method	a				
			0	**0.5**	**1.0**	**1.5**	**2.0**				**0**	**0.5**	**1.0**	**1.5**	**2.0**
1.2	1	Shewhart	370.2590	119.4423	95.7040	46.7686	23.6387	1.8	1	Shewhart	370.7331	267.5101	139.7311	99.8356	81.5189
		MAD	370.4981	102.2915	88.0626	39.5021	21.6952			MAD	370.9340	218.7282	111.2793	86.8288	73.7733
		Lognormal	370.1986	**96.9861**	**75.1701**	**24.9985**	**15.2376**			Lognormal	370.3471	**152.8028**	**89.2766**	**76.3249**	**63.4091**
	1.5	Shewhart	111.2461	96.7831	44.8211	31.5984	23.0701		1.5	Shewhart	169.2932	138.5298	105.3594	85.5992	78.5866
		MAD	121.0704	85.5627	42.3401	25.6643	15.8411			MAD	162.7079	97.1963	83.1813	71.9104	63.1722
		Lognormal	**100.0704**	**75.3427**	**35.5807**	**19.0272**	**11.5984**			Lognormal	**97.7623**	**86.3125**	**73.4812**	**61.2627**	**56.4756**
	2	Shewhart	65.4779	39.9694	20.3181	7.6031	2.8969		2	Shewhart	121.7872	82.2942	79.4858	56.7287	36.0529
		MAD	68.8580	31.5779	12.6512	4.5810	2.9623			MAD	116.4741	79.3296	61.7585	48.1303	21.0153
		Lognormal	**49.8265**	**29.9561**	**11.1236**	**3.6387**	**1.9623**			Lognormal	**76.7656**	**51.3154**	**41.3677**	**31.4657**	**18.7674**
	2.5	Shewhart	49.7859	25.9561	13.7870	6.1883	2.8980		2.5	Shewhart	98.8758	52.8025	35.0286	21.7636	15.1767
		MAD	54.3320	22.5779	9.8930	4.2648	2.0798			MAD	95.6819	45.5839	25.5422	16.6144	12.9906
		Lognormal	**37.6345**	**18.6019**	**8.0398**	**3.0701**	**1.6952**			Lognormal	**56.1755**	**39.4338**	**18.0370**	**9.2918**	**3.0296**
	3	Shewhart	42.0431	20.3735	11.1236	5.5226	2.9128		3	Shewhart	87.5328	33.5427	18.7233	9.8025	4.7402
		MAD	47.3320	18.7271	8.3936	4.0592	2.1918			MAD	83.1324	29.5000	13.0807	6.0418	3.9906
		Lognormal	**32.5708**	**12.5413**	**7.9663**	**3.0015**	**1.2411**			Lognormal	**28.7711**	**17.0698**	**10.7923**	**4.6909**	**2.0893**
1.4	1	Shewhart	370.0141	224.6411	140.2278	93.7023	48.8145	2.0	1	Shewhart	370.6166	219.3423	125.3890	89.5021	61.6925
		MAD	370.4412	176.7376	104.9514	90.5071	42.5477			MAD	370.0342	202.2915	108.0626	86.7686	52.6387
		Lognormal	370.6708	**139.4238**	**95.2345**	**80.1542**	**32.1139**			Lognormal	370.0850	**149.8508**	**82.1701**	**64.9985**	**45.8041**
	1.5	Shewhart	131.4391	88.0244	35.3004	19.1651	9.6493		1.5	Shewhart	111.2461	96.9861	74.8211	65.6643	51.8411
		MAD	133.9885	82.2780	33.1199	18.5865	8.4798			MAD	100.0704	85.5627	68.3401	58.5984	43.0801
		Lognormal	**125.4576**	**75.7924**	**30.3423**	**16.7329**	**7.9722**			Lognormal	**89.8256**	**60.9705**	**45.9802**	**39.3296**	**24.2376**
	2	Shewhart	83.4962	42.6921	18.4730	6.7087	5.6159		2	Shewhart	65.4779	39.9694	20.3181	17.6031	8.9623
		MAD	84.3445	41.3564	17.2405	6.2115	5.4942			MAD	58.8508	31.5779	12.6517	9.5810	6.8969
		Lognormal	**49.2699**	**27.6760**	**16.0602**	**5.7510**	**4.4475**			Lognormal	**37.7009**	**22.5708**	**11.1236**	**5.0272**	**4.8957**
	2.5	Shewhart	67.1804	29.8370	13.4529	5.6988	4.6192		2.5	Shewhart	49.7859	25.9561	13.7870	7.6031	6.0798
		MAD	68.0016	28.4279	12.8530	5.3873	4.5376			MAD	44.3320	22.5779	9.8930	4.2648	3.9128
		Lognormal	**38.8654**	**26.3912**	**11.1561**	**4.8850**	**3.9722**			Lognormal	**22.7507**	**18.6019**	**8.0398**	**3.9340**	**2.6015**
	3	Shewhart	57.7019	24.3296	11.2959	5.2408	2.7001		3	Shewhart	42.0431	20.3735	11.1236	5.5226	4.1918
		MAD	59.0947	23.7750	10.7698	5.0668	2.6163			MAD	37.6385	18.7271	9.3936	4.0592	2.9238
		Lognormal	**17.4289**	**14.4565**	**8.1651**	**3.2356**	**2.1152**			Lognormal	**12.3872**	**10.5413**	**6.1883**	**3.0592**	**1.6532**

Table 5. The ARL_1s of the three combined $\overline{X}$- and S-charts for different shift sizes a, b when the sample $m = 100$ with the subgroup size $n = 5$ under various values of in-control σ_0 ($\theta_1 = \theta_0 + a\xi_0$, $\xi_1 = b\xi_0$).

σ_0	b	Method	a					σ_0	b	Method	a				
			0	**0.5**	**1.0**	**1.5**	**2.0**				**0**	**0.5**	**1.0**	**1.5**	**2.0**
0.2	1	Shewhart	370.9016	55.3650	7.9903	2.1497	1.1600	0.7	1	Shewhart	370.5071	190.4507	40.8053	7.8322	2.1362
		MAD	370.8395	36.4077	5.8171	1.7933	1.1031			MAD	370.5397	121.7739	25.1164	5.1980	1.6975
		Lognormal	370.9886	45.0588	7.0902	2.0323	1.1483			Lognormal	370.6082	136.8815	44.3188	7.5337	2.3633
	1.5	Shewhart	10.6802	8.0056	4.0935	2.0777	1.3079		1.5	Shewhart	46.4995	30.4028	12.7228	4.8621	2.2102
		MAD	11.1129	7.6972	3.6433	1.8904	1.2454			MAD	44.7339	24.7369	9.8757	3.8546	1.8598
		Lognormal	33.4088	11.6091	4.2257	1.9996	1.2921			Lognormal	39.6129	41.5392	26.6182	12.3581	5.2618
	2	Shewhart	3.4710	3.3237	2.6542	1.9040	1.4031		2	Shewhart	23.2218	15.1449	8.4345	4.1378	2.2363
		MAD	3.6531	3.4362	2.5617	1.7970	1.3341			MAD	23.1127	13.6708	7.0070	3.5382	1.9994
		Lognormal	10.3974	6.5554	3.4482	2.0129	1.3937			Lognormal	**16.0156**	15.3495	11.7225	6.8655	3.9728
	2.5	Shewhart	2.1017	2.1687	1.9707	1.6746	1.3802		2.5	Shewhart	16.7204	10.8594	6.5921	3.7849	2.3460
		MAD	2.1433	2.2185	1.9741	1.6567	1.3601			MAD	16.5863	10.2585	5.8961	3.3682	2.1043
		Lognormal	5.5567	4.5314	3.0373	2.0122	1.4773			Lognormal	**10.1440**	**9.1308**	**5.6368**	**2.2542**	2.4445
	3	Shewhart	1.6306	1.7165	1.6569	1.5019	1.3519		3	Shewhart	13.5188	9.2179	5.8614	3.6914	2.4272
		MAD	1.6248	1.7218	1.6921	1.5281	1.3418			MAD	13.6560	8.6257	5.4303	3.3449	2.1905
		Lognormal	3.6375	3.5284	2.7008	2.0385	1.5424			Lognormal	**7.7164**	**7.2359**	**5.2297**	**3.2255**	**1.1370**
0.4	1	Shewhart	370.9314	79.6328	12.2728	2.8266	1.2917	0.8	1	Shewhart	370.6176	226.2076	57.9894	10.9907	2.7017
		MAD	370.6151	80.4118	12.2418	2.8465	1.2809			MAD	370.4988	190.5745	45.4455	8.5614	2.5818
		Lognormal	370.4528	69.9835	13.9338	3.3536	1.3719			Lognormal	370.0727	**151.7156**	**41.9022**	**7.1019**	**2.3817**
	1.5	Shewhart	24.6988	15.0483	6.0797	2.6283	1.4666		1.5	Shewhart	50.3043	35.4730	15.8658	6.0561	2.5915
		MAD	24.3265	14.7810	6.1379	2.6162	1.4618			MAD	46.0280	22.7365	12.0570	5.6448	1.7684
		Lognormal	31.5471	15.2285	6.1915	2.7723	1.5222			Lognormal	**41.8124**	**19.4677**	**10.9022**	**3.7974**	**1.5586**
	2	Shewhart	9.5795	7.2927	4.4470	2.5435	1.6030		2	Shewhart	27.2826	18.0042	9.8194	4.9193	2.6158
		MAD	9.5019	7.2179	4.3881	2.5176	1.6043			MAD	26.9406	13.8408	6.7650	3.3538	1.9191
		Lognormal	12.2790	7.8405	4.3608	2.5241	1.6365			Lognormal	**16.6343**	**11.2391**	**7.7895**	**3.5212**	**1.4357**
	2.5	Shewhart	5.9749	5.0762	3.5773	2.4551	1.6784		2.5	Shewhart	20.1774	12.9157	7.7880	4.4182	2.5975
		MAD	5.9351	4.9181	3.5855	2.4407	1.6972			MAD	21.3340	11.2423	5.8643	3.2989	1.9985
		Lognormal	7.2373	5.5649	3.6287	2.4376	1.6918			Lognormal	**10.8657**	**9.3758**	**4.2406**	**2.5652**	**1.3031**
	3	Shewhart	4.5362	4.0379	3.1549	2.3704	1.7642		3	Shewhart	17.2413	10.7990	6.7531	4.1698	2.6189
		MAD	4.4915	3.9163	3.1128	2.3344	1.7743			MAD	18.0876	9.7964	5.5839	3.2915	2.1099
		Lognormal	5.1366	4.4069	3.2418	2.3835	1.7602			Lognormal	**9.1491**	**8.9575**	**4.4075**	**2.5652**	**1.9797**

Table 5. *Cont.*

σ_0	b	Method	a					σ_0	b	Method	a				
			0	**0.5**	**1.0**	**1.5**	**2.0**				**0**	**0.5**	**1.0**	**1.5**	**2.0**
0.6	1	Shewhart	370.5410	145.9308	27.0365	5.2673	1.7298	0.9	1	Shewhart	370.1778	217.3113	155.4664	30.3790	5.9278
		MAD	370.1480	131.0920	24.6761	4.8723	1.6460			MAD	370.0360	244.3086	162.0598	31.4455	6.0325
		Lognormal	370.8481	277.0049	98.0339	19.4298	4.1632			Lognormal	370.0608	**154.6369**	**73.1460**	**14.5291**	**4.2504**
	1.5	Shewhart	42.4780	25.2664	10.0290	3.9044	1.8515		1.5	Shewhart	63.6406	52.4173	26.8709	10.7647	4.1776
		MAD	38.4577	23.3232	9.4520	3.7450	1.8142			MAD	69.1315	54.5196	28.3967	11.0939	4.2536
		Lognormal	39.6396	32.7611	17.0642	7.1942	3.1687			Lognormal	**41.7785**	**50.1331**	**22.4749**	**8.3712**	**2.5544**
	2	Shewhart	19.2155	12.9257	6.7823	3.5195	1.9833		2	Shewhart	35.1643	24.4080	14.3996	7.2639	3.6934
		MAD	17.6217	11.6009	6.5569	3.3824	1.9531			MAD	38.3506	25.2972	14.8797	7.5030	3.7330
		Lognormal	**15.961**	**10.2909**	8.5428	4.9031	2.8063			Lognormal	**28.9982**	**19.3626**	**9.4104**	**4.6376**	**2.4841**
	2.5	Shewhart	13.3861	9.1854	5.6161	3.2664	2.1017		2.5	Shewhart	26.1606	16.9973	10.4319	6.1394	3.4947
		MAD	12.1451	8.5403	5.3802	3.2315	2.0240			MAD	28.1136	18.1060	10.8646	6.1737	3.5424
		Lognormal	**9.4419**	**8.2474**	**4.1635**	3.4308	2.2837			Lognormal	**12.3057**	**14.1052**	**7.3149**	**4.4452**	**2.4151**
	3	Shewhart	10.9107	7.6134	4.9626	3.2367	2.1613		3	Shewhart	22.3136	14.0040	8.8461	5.5431	3.3768
		MAD	9.9914	7.0673	4.7658	3.1513	2.1580			MAD	24.1638	14.8069	9.2924	5.6140	3.4644
		Lognormal	**6.9849**	**6.4789**	**4.1755**	**2.8635**	2.5278			Lognormal	**9.9144**	**10.3277**	**6.0999**	**3.0726**	**2.3151**
1.0	1	Shewhart	370.7898	265.2932	150.5660	71.5345	39.2700	1.6	1	Shewhart	370.2438	195.3295	132.0411	122.9400	33.3969
		MAD	370.6102	222.7192	133.8872	70.0540	38.1148			MAD	370.1108	145.6593	110.9581	109.2558	45.6967
		Lognormal	370.6248	**156.4460**	**110.8452**	**65.8481**	**30.0903**			Lognormal	370.6363	**101.4220**	**87.1537**	**70.8551**	**35.8838**
	1.5	Shewhart	83.8116	65.2542	57.4986	41.1276	24.3316		1.5	Shewhart	181.9572	105.9072	62.0206	31.9808	19.8443
		MAD	73.6919	61.5702	50.5961	42.1865	24.7968			MAD	179.8638	103.9975	61.2664	36.6212	18.8419
		Lognormal	**64.0985**	**57.9965**	**44.4665**	**34.8421**	**21.9756**			Lognormal	**126.1263**	**69.1888**	**55.7130**	**30.7788**	**18.2854**
	2	Shewhart	52.8113	36.6474	25.2491	17.5267	16.7747		2	Shewhart	135.7049	60.9735	31.2527	29.1956	14.8769
		MAD	55.0756	30.6472	26.4545	18.1266	17.9899			MAD	136.4367	57.6048	33.2191	28.3719	14.3049
		Lognormal	**47.5736**	**21.7852**	**20.9081**	**16.0748**	**15.9452**			Lognormal	**78.4343**	**47.1673**	**27.6061**	**25.7279**	**12.6375**
	2.5	Shewhart	32.6805	19.9120	17.4336	14.3709	12.5456		2.5	Shewhart	119.0752	49.2554	25.7809	18.8969	9.9685
		MAD	33.3603	22.3566	16.1030	14.6474	12.8234			MAD	115.8748	43.7291	24.4997	17.4337	8.4911
		Lognormal	**31.3085**	**13.6388**	**12.4264**	**11.6088**	**9.7954**			Lognormal	**35.0446**	**27.7994**	**21.0268**	**20.8416**	**7.5749**
	3	Shewhart	28.8769	16.4279	10.7290	8.8103	6.5993		3	Shewhart	104.2608	38.1536	16.2970	11.4921	6.5133
		MAD	26.4679	18.7657	10.4680	8.0109	6.7217			MAD	104.6651	36.5088	16.4926	11.5669	6.6382
		Lognormal	**19.8212**	**10.3306**	**9.0798**	**7.2923**	**5.5481**			Lognormal	**22.8515**	**18.7209**	**14.2866**	**10.7094**	**5.0796**

Table 5. *Cont.*

σ_0	b	Method	a					σ_0	b	Method	a				
			0	**0.5**	**1.0**	**1.5**	**2.0**				**0**	**0.5**	**1.0**	**1.5**	**2.0**
1.2	1	Shewhart	370.6953	125.9365	94.8224	57.3222	28.7561	1.8	1	Shewhart	370.5018	210.4511	189.5219	125.1391	52.1819
		MAD	370.2559	126.2396	85.0125	47.5747	23.0003			MAD	370.7394	178.2020	135.8119	86.9995	48.8297
		Lognormal	370.3140	**85.5085**	**71.6081**	**41.9300**	**20.3709**			Lognormal	370.2277	**106.4689**	**95.0906**	**82.0501**	**39.9855**
	1.5	Shewhart	119.6618	89.3746	57.6442	36.4644	19.9191		1.5	Shewhart	232.4151	104.7997	67.8061	37.1922	27.8138
		MAD	112.9011	90.2821	54.0151	33.9604	19.1557			MAD	219.6655	94.7731	65.6926	36.1519	28.2529
		Lognormal	**90.8496**	**55.5713**	**42.7798**	**25.4305**	**17.6912**			Lognormal	**122.4212**	**68.7156**	**57.9764**	**33.0755**	**18.5322**
	2	Shewhart	71.7199	43.1612	26.2276	18.5499	16.8099		2	Shewhart	186.2658	62.6488	34.2403	28.6898	19.2206
		MAD	76.0665	44.2589	25.7059	17.9980	15.4376			MAD	178.9823	60.7335	35.4172	26.9040	16.2140
		Lognormal	**58.0324**	**32.5689**	**24.0822**	**16.6669**	**14.0492**			Lognormal	**76.6721**	**44.4219**	**32.8877**	**24.7624**	**14.3078**
	2.5	Shewhart	55.7860	30.9711	18.1577	10.4691	9.7772		2.5	Shewhart	164.6413	47.4567	24.6731	18.4738	9.4022
		MAD	61.5567	31.8417	17.9295	10.9719	9.6001			MAD	153.0848	40.1385	21.4908	19.2426	9.5480
		Lognormal	**43.5375**	**24.6017**	**14.0966**	**9.7004**	**8.5674**			Lognormal	**33.8951**	**26.7475**	**18.9042**	**17.9590**	**8.7293**
	3	Shewhart	48.5523	25.0586	14.8890	8.9893	7.4467		3	Shewhart	150.8591	40.6577	20.9346	12.7442	6.5993
		MAD	52.8461	26.1823	14.8473	8.6069	7.1718			MAD	142.6608	36.4662	16.5238	11.0332	6.1295
		Lognormal	**31.1436**	**21.1622**	**10.2786**	**7.5985**	**6.5434**			Lognormal	**22.3327**	**17.4478**	**13.6536**	**10.4880**	**5.7888**
1.4	1	Shewhart	370.4344	223.8190	147.8086	96.4420	60.0654	2.0	1	Shewhart	370.6947	203.4288	191.9267	128.0003	56.1778
		MAD	370.6994	166.2400	112.8536	92.0444	58.0154			MAD	370.6296	196.2946	137.0089	89.6702	52.0158
		Lognormal	370.8020	**141.1156**	**94.1306**	**84.9857**	**42.8454**			Lognormal	370.4300	**105.8645**	**94.6337**	**82.1692**	**44.5951**
	1.5	Shewhart	155.6119	93.9078	77.6937	57.8283	43.8482		1.5	Shewhart	241.0581	107.7327	67.7341	39.3981	29.4795
		MAD	154.8240	110.4558	74.9751	58.4268	45.2447			MAD	261.1836	99.4651	66.5260	36.2654	26.2014
		Lognormal	**96.5051**	**49.2596**	**75.3531**	**49.2179**	**33.8839**			Lognormal	**120.1582**	**65.4292**	**58.1263**	**32.4077**	**19.1157**
	2	Shewhart	109.5455	50.4418	38.0207	24.7842	17.3182		2	Shewhart	208.6734	60.9465	37.8257	28.3495	19.0270
		MAD	106.9295	55.7462	34.1242	28.4956	19.4324			MAD	225.2464	57.0337	34.8920	30.7169	15.1717
		Lognormal	**57.2207**	**32.1005**	**25.3369**	**22.6862**	**17.2442**			Lognormal	**75.6851**	**43.7940**	**32.8101**	**24.6983**	**14.8541**
	2.5	Shewhart	89.4818	37.2984	29.9738	16.0140	12.2008		2.5	Shewhart	186.1588	49.4534	24.7675	17.2946	9.8132
		MAD	86.3784	40.6085	26.2499	16.1167	11.4734			MAD	205.6977	43.7180	21.3371	19.0919	9.7325
		Lognormal	**43.5073**	**24.8701**	**19.0727**	**15.1669**	**10.3011**			Lognormal	**33.3589**	**26.6950**	**18.5956**	**17.6108**	**8.0072**
	3	Shewhart	80.3441	32.0909	19.8886	9.4655	7.6233		3	Shewhart	178.3930	40.3065	23.1181	13.8196	6.8294
		MAD	76.7373	33.6398	19.0677	9.0836	7.6451			MAD	191.8933	35.6211	16.3589	11.0241	6.5674
		Lognormal	**31.6475**	**21.5948**	**15.8064**	**8.7031**	**5.5612**			Lognormal	**22.1292**	**17.3515**	**13.8677**	**10.8121**	**5.5185**

Table 6. The ARL_1s of the three combined $\overline{X}$- and S-charts for different shift sizes a, b when the sample $m = 100$ with the subgroup size $n = 10$ under various values of in-control σ_0 ($\theta_1 = \theta_0 + a\xi_0$, $\xi_1 = b\xi_0$).

σ_0	b	Method	a					σ_0	b	Method	a				
			0	**0.5**	**1.0**	**1.5**	**2.0**				**0**	**0.5**	**1.0**	**1.5**	**2.0**
0.2	1	Shewhart	370.6249	19.2723	2.2109	1.0684	1.0008	0.7	1	Shewhart	370.3365	51.1671	4.7968	1.2913	1.0028
		MAD	370.3850	21.2739	2.3487	1.0784	1.0006			MAD	370.0587	49.3659	4.7428	1.2640	1.0024
		Lognormal	370.4292	19.7756	2.2602	1.0735	1.0005			Lognormal	370.9749	180.5644	16.0972	2.2685	1.0657
	1.5	Shewhart	7.7160	5.0717	2.0934	1.1898	1.0170		1.5	Shewhart	28.5080	13.6460	3.7622	1.4479	1.0384
		MAD	7.2093	4.9979	2.1467	1.2010	1.0173			MAD	28.1371	13.4143	3.6648	1.4611	1.0406
		Lognormal	30.1436	7.3406	2.2109	1.1998	1.0169			Lognormal	45.7943	23.3459	6.6040	2.1862	1.1875
	2	Shewhart	2.2345	2.1682	1.6763	1.2378	1.0501		2	Shewhart	13.7286	8.0436	3.3637	1.6053	1.1107
		MAD	2.1743	2.0958	1.6699	1.2558	1.0532			MAD	13.5956	7.8584	3.3185	1.6046	1.1128
		Lognormal	5.7847	4.0258	2.0827	1.2928	1.0538			Lognormal	**12.8804**	**6.9918**	**3.1759**	2.1416	1.2866
	2.5	Shewhart	1.4276	1.4677	1.3677	1.2079	1.0731		2.5	Shewhart	9.5516	6.0713	3.2189	1.7216	1.1954
		MAD	1.3995	1.4502	1.3642	1.1998	1.0730			MAD	9.2834	5.9171	3.2314	1.7164	1.1830
		Lognormal	2.4174	2.4373	1.8450	1.3555	1.1086			Lognormal	**8.4591**	**5.5918**	**3.0770**	**1.4487**	**1.1772**
	3	Shewhart	1.1857	1.2403	1.2088	1.1424	1.0741		3	Shewhart	7.8404	5.1923	3.0570	1.8395	1.2752
		MAD	1.1632	1.2081	1.1947	1.1369	1.0713			MAD	7.7873	5.1356	3.0186	1.8081	1.2748
		Lognormal	1.6068	1.7071	1.5625	1.3151	1.1410			Lognormal	**6.0719**	**4.2301**	**2.4887**	**1.4318**	**1.1206**
0.4	1	Shewhart	370.8823	31.1820	3.0210	1.1239	1.0016	0.8	1	Shewhart	370.0325	314.4624	28.2737	3.0755	1.1245
		MAD	370.7238	31.8305	3.0978	1.1324	1.0018			MAD	370.8596	234.6784	11.7298	1.8416	1.0301
		Lognormal	370.6207	33.7048	3.3326	1.1633	1.0013			Lognormal	370.9259	**165.4269**	**10.3596**	**1.3742**	**1.0721**
	1.5	Shewhart	16.4781	8.6895	2.7061	1.2929	1.0212		1.5	Shewhart	45.3517	34.4682	9.9748	2.8761	1.2806
		MAD	16.8106	8.8397	2.7370	1.2925	1.0233			MAD	47.0147	23.9918	6.2954	2.0153	1.1324
		Lognormal	32.7500	9.7097	2.7532	1.3071	1.0288			Lognormal	**39.3592**	**22.7961**	**6.1386**	**1.2325**	**1.1107**
	2	Shewhart	5.3745	4.3586	2.4642	1.4207	1.0804		2	Shewhart	21.2088	15.2076	6.7767	2.7338	1.4012
		MAD	5.4550	4.3563	2.4816	1.4267	1.0788			MAD	21.8900	12.8129	4.9566	2.0659	1.2372
		Lognormal	8.0022	5.4040	2.5522	1.4012	1.0767			Lognormal	**14.7824**	**10.8008**	**4.7131**	**2.0559**	**1.1397**
	2.5	Shewhart	3.2714	2.9347	2.1543	1.4769	1.1337		2.5	Shewhart	14.6923	10.0534	5.4729	2.6409	1.5307
		MAD	3.2665	3.0137	2.1695	1.4869	1.1417			MAD	15.0995	8.9875	4.5171	2.1702	1.3516
		Lognormal	4.0856	3.5677	2.3484	1.4954	1.1398			Lognormal	**8.7293**	**7.2087**	**4.0410**	**2.1091**	**1.3169**
	3	Shewhart	2.4250	2.3500	1.9627	1.4890	1.1919		3	Shewhart	12.1179	8.0784	4.8306	2.7187	1.6479
		MAD	2.4673	2.3708	1.9495	1.5013	1.2025			MAD	12.6244	7.6972	4.1163	2.2483	1.4278
		Lognormal	2.8058	2.6781	2.0716	1.5476	1.1946			Lognormal	**6.5449**	**5.5764**	**3.5752**	**2.1666**	**1.4626**

Table 6. *Cont.*

σ_0	b	Method	a = 0	0.5	1.0	1.5	2.0	σ_0	b	Method	a = 0	0.5	1.0	1.5	2.0
0.6	1	Shewhart	370.8767	49.1099	4.4552	1.2516	1.0043	0.9	1	Shewhart	370.1426	247.2320	32.1964	4.4373	3.6033
		MAD	370.5655	56.6009	4.9419	1.2811	1.0047			MAD	370.4514	236.1081	12.8284	4.3373	3.6567
		Lognormal	370.9765	71.2961	6.4950	1.4460	1.0111			Lognormal	370.2022	**206.9857**	**11.8998**	**3.9667**	**2.0657**
	1.5	Shewhart	31.2774	13.1573	3.4987	1.4228	1.0377		1.5	Shewhart	59.3125	39.0145	11.6738	4.1813	3.7827
		MAD	28.1526	13.6570	3.6870	1.4858	1.0456			MAD	66.0847	32.5375	11.9919	4.4183	3.7991
		Lognormal	32.1493	14.4554	4.0255	1.5842	1.0644			Lognormal	**58.9318**	**22.6306**	**10.5491**	**3.8511**	**2.4525**
	2	Shewhart	13.0624	7.5976	3.1838	1.5703	1.1053		2	Shewhart	28.6545	19.3920	9.2341	4.2331	3.9003
		MAD	11.4052	7.5459	3.3309	1.6058	1.1172			MAD	31.4294	18.0718	9.8214	4.3866	3.9219
		Lognormal	**10.3026**	**7.3274**	3.3851	1.6939	1.1417			Lognormal	**20.4988**	**12.7808**	**9.0811**	**3.1637**	**2.3557**
	2.5	Shewhart	8.4986	5.6831	3.0432	1.6855	1.1817		2.5	Shewhart	20.3691	12.0838	8.6351	3.8659	2.0091
		MAD	7.6801	5.5103	3.0827	1.7170	1.1947			MAD	22.0304	12.2530	8.4283	3.9274	2.0054
		Lognormal	**5.9665**	**4.9054**	**2.9833**	1.7595	1.2095			Lognormal	**11.9950**	**10.1109**	**8.1045**	**3.1948**	**1.2632**
	3	Shewhart	6.7745	4.6449	2.8974	1.7641	1.2630		3	Shewhart	16.7530	10.9342	6.9511	3.6762	2.0894
		MAD	6.2363	4.4583	2.9600	1.8247	1.2862			MAD	17.8226	10.8734	7.0597	3.7613	2.1485
		Lognormal	**4.4606**	**3.7618**	**2.7133**	**1.7707**	1.2931			Lognormal	**8.7775**	**7.4708**	**6.2498**	**2.7167**	**1.2442**
1.0	1	Shewhart	370.5876	332.1389	188.4409	14.7323	2.0933	1.6	1	Shewhart	370.0288	242.4147	116.2533	34.8732	14.7930
		MAD	370.6514	322.0567	71.4794	8.7392	1.2240			MAD	370.9365	218.1352	106.3100	22.9605	12.2699
		Lognormal	370.6674	**302.0549**	**50.3621**	**5.7687**	**1.0941**			Lognormal	370.7202	**116.2428**	**73.0415**	**18.7435**	**9.1231**
	1.5	Shewhart	68.4106	65.0598	28.8241	7.7689	2.8004		1.5	Shewhart	157.7624	95.8841	44.0340	22.7117	12.1041
		MAD	64.8232	41.5850	22.1149	5.0292	2.1226			MAD	147.0321	85.7616	34.5001	19.1279	9.7572
		Lognormal	**50.4409**	**32.4809**	**12.2501**	**3.7400**	**1.5421**			Lognormal	**87.6566**	**68.0508**	**22.8179**	**15.8359**	**8.2282**
	2	Shewhart	39.0298	28.5452	15.7581	6.2355	2.5974		2	Shewhart	108.3985	48.8425	21.4229	16.1369	9.8969
		MAD	37.9797	21.9467	11.5620	4.9163	1.9728			MAD	98.0217	44.1004	18.0898	15.7391	8.0819
		Lognormal	**22.8857**	**13.9519**	**8.1471**	**3.1647**	**1.6924**			Lognormal	**76.1466**	**34.0286**	**15.0956**	**12.7628**	**7.9336**
	2.5	Shewhart	28.7747	18.2229	10.6787	4.6441	2.2333		2.5	Shewhart	81.6768	32.0144	14.7143	14.3718	8.0394
		MAD	27.0439	15.2056	7.4149	3.9239	1.2373			MAD	82.7323	33.9494	14.1184	12.2187	7.7398
		Lognormal	**15.3440**	**9.8569**	**5.5459**	**2.7882**	**1.2532**			Lognormal	**60.8896**	**25.5633**	**12.7188**	**9.2443**	**6.9013**
	3	Shewhart	24.8429	15.1245	8.6733	4.3480	2.3341		3	Shewhart	72.1803	27.5704	13.5167	10.4651	7.6024
		MAD	23.6412	13.0043	5.7356	3.3415	1.9344			MAD	71.8043	26.5740	12.1619	9.0030	6.6019
		Lognormal	**12.0215**	**8.4531**	**4.6314**	**2.7450**	**1.7123**			Lognormal	**48.9878**	**21.6615**	**8.0125**	**7.9284**	**5.4146**

Table 6. *Cont.*

σ_0	b	Method	a					σ_0	b	Method	a				
			0	**0.5**	**1.0**	**1.5**	**2.0**				**0**	**0.5**	**1.0**	**1.5**	**2.0**
1.2	1	Shewhart	370.7639	118.7621	96.0941	44.2281	24.8636	1.8	1	Shewhart	370.9133	270.6285	138.2420	100.5837	80.5336
		MAD	370.6381	102.5473	89.0725	40.7686	22.5156			MAD	370.7464	220.9564	116.6627	86.4779	73.0828
		Lognormal	370.6125	**93.1757**	**78.2418**	**23.3026**	**16.5286**			Lognormal	370.0479	**155.7008**	**91.8479**	**76.8506**	**65.5048**
	1.5	Shewhart	125.5238	93.3553	43.2413	32.4830	23.6877		1.5	Shewhart	167.7768	139.3215	106.4592	86.2866	78.3906
		MAD	120.6421	87.5886	42.5784	26.3124	16.4305			MAD	160.5609	98.8299	83.9650	70.1705	62.8137
		Lognormal	**107.2715**	**76.4650**	**36.2722**	**19.0092**	**11.2555**			Lognormal	**93.3113**	**87.8621**	**72.2333**	**63.2680**	**57.9860**
	2	Shewhart	66.7972	38.4446	21.0246	8.8095	3.2671		2	Shewhart	121.4682	83.4297	78.8565	56.8938	36.0268
		MAD	64.5466	30.3624	14.0514	5.9877	3.4222			MAD	117.3275	79.8040	62.9010	47.3086	22.7954
		Lognormal	**47.4302**	**29.2963**	**12.5475**	**3.8293**	**2.0900**			Lognormal	**78.7948**	**52.8935**	**42.4785**	**30.0127**	**17.3813**
	2.5	Shewhart	45.3974	26.7339	14.0651	6.8869	2.8287		2.5	Shewhart	97.1103	53.7600	35.3494	22.8285	15.2460
		MAD	44.1852	23.6704	10.3006	4.8876	2.4922			MAD	96.3726	46.9754	26.9029	16.0954	12.7808
		Lognormal	**38.5118**	**18.5447**	**9.3648**	**3.1981**	**1.9206**			Lognormal	**57.1573**	**40.2238**	**17.3344**	**10.7588**	**4.8566**
	3	Shewhart	38.2804	20.5603	10.8400	5.0462	2.1198		3	Shewhart	87.3199	34.4839	19.6351	11.8637	5.7989
		MAD	37.4674	18.1985	9.3016	4.2840	2.2265			MAD	82.1231	29.6127	14.5544	7.5768	4.8026
		Lognormal	**30.9059**	**12.0981**	**8.0468**	**3.0413**	**1.4839**			Lognormal	**27.2949**	**18.7639**	**12.9951**	**6.1110**	**3.8682**
1.4	1	Shewhart	370.4334	226.5348	142.8182	92.3615	47.5565	2.0	1	Shewhart	370.4574	216.7288	126.8258	88.7438	59.8440
		MAD	370.6755	179.8435	106.0189	89.3577	43.1553			MAD	370.4761	203.4134	109.9837	85.4514	52.3166
		Lognormal	370.2762	**135.9040**	**92.6311**	**82.8181**	**33.1501**			Lognormal	370.6554	**150.4155**	**85.0482**	**65.0028**	**46.2182**
	1.5	Shewhart	135.4340	87.6885	36.7662	18.4473	9.4632		1.5	Shewhart	113.6621	97.8254	76.3943	66.1226	52.6319
		MAD	130.8541	82.6396	34.9382	17.5380	8.1937			MAD	100.7551	88.0618	66.7885	57.2075	44.5197
		Lognormal	**120.2205**	**74.7948**	**31.7334**	**15.8090**	**7.3406**			Lognormal	**86.6885**	**61.9292**	**46.1987**	**40.1851**	**26.1696**
	2	Shewhart	83.6016	43.1796	18.9066	7.2501	6.4774		2	Shewhart	66.0187	39.6510	21.1841	18.4362	9.6132
		MAD	82.6732	42.4938	17.8597	7.8359	6.7944			MAD	59.8590	32.3301	13.6341	10.8040	7.6654
		Lognormal	**49.8582**	**28.0273**	**16.6549**	**6.3655**	**5.6999**			Lognormal	**37.9804**	**23.7264**	**11.6771**	**8.9980**	**6.1990**
	2.5	Shewhart	65.2459	29.6119	13.6478	6.4716	5.5629		2.5	Shewhart	48.3473	27.3936	13.8589	9.5331	7.2107
		MAD	64.6933	28.3466	12.0279	6.0511	5.3396			MAD	45.2306	23.5541	10.0507	8.7218	4.8000
		Lognormal	**35.6569**	**26.0764**	**11.1939**	**5.0032**	**4.4837**			Lognormal	**24.7504**	**19.7866**	**8.9662**	**7.5446**	**2.7688**
	3	Shewhart	56.5420	24.3761	11.9033	5.0789	4.5688		3	Shewhart	41.6550	21.1342	11.3953	7.1948	3.0252
		MAD	53.7485	23.0139	11.4725	5.1401	4.4483			MAD	36.2235	17.8810	8.9821	6.7534	2.9346
		Lognormal	**19.3947**	**17.4814**	**9.0136**	**4.7861**	**3.9554**			Lognormal	**14.7591**	**11.8353**	**7.7627**	**3.5694**	**1.9153**

4. Example from the Automotive Industry

In this section, we present a real example to illustrate the applicability of the combined lognormal $\overline{X}$- and S-charts. The ASTM D7320 Ref Oil Data were provided by the Test Monitoring Center [22]. In order to assess the engine oil quality, especially for new vehicles, the percent viscosity increase (PVI), which follows a lognormal distribution, needs to be tested. In this dataset, the quality of three reference oils—Ref Oils 434, 435, and 438—needs to be tested. We collected 50 samples, each of size 10, for each of these three reference oils, and used them to construct the Phase I control charts.

The multipliers of the three combined $\overline{X}$- and S-charts were calibrated to have a Type I error approximately equal to 0.0027 in order to have a fair comparison. Summarized in Table 7 are the upper and lower control limits of the three combined $\overline{X}$- and S-charts, the estimated means, and the estimated standard deviations of the PVI for the three reference oils. Figures A1–A3 show the three combined $\overline{X}$- and S-charts for Ref Oils 434, 435, and 438, respectively.

Table 7. The control limits, estimated means, and estimated standard deviations of percent viscosity increase (PVI) for the three reference oils in the Phase I control.

Reference	$(\hat{\theta}_0, \hat{\xi}_0)$	Method	$\overline{X}$-Chart		S-Chart	
			LCL	UCL	LCL	UCL
Oil-434	(154.747, 127.376)	Shewhart	45.526	263.967	0.000	282.495
		MAD	46.980	262.513	0.000	285.250
		Lognormal	23.457	271.014	3.166	5.679
Oil-435	(175.846, 66.963)	Shewhart	105.654	246.038	0.000	146.391
		MAD	105.935	245.757	0.000	146.785
		Lognormal	100.771	248.680	3.025	5.121
Oil-438	(95.088, 20.828)	Shewhart	73.406	116.771	0.000	41.155
		MAD	73.228	116.948	0.608	40.910
		Lognormal	73.229	116.897	2.003	3.985

Neither the combined Shewhart $\overline{X}$- and S-charts nor the combined MAD $\overline{X}$- and S-charts showed any out-of-control samples for Ref Oil 434 (Figure A1). On the other hand, sample 9 was outside the control limits of the lognormal S-chart. Consequently, the control limits were recalculated without sample 9 for the Phase II joint monitoring of the combined lognormal $\overline{X}$- and S-charts for Ref Oil 434. Similarly, there were no out-of-control samples in the combined Shewhart $\overline{X}$- and S-charts or the combined MAD $\overline{X}$- and S-charts for Ref Oil 435 (Figure A2), while sample 15 was out of control on the lognormal S-chart. Hence, the control limits of the combined lognormal $\overline{X}$- and S-charts for Ref Oil 435 were recalculated without sample 15 for the Phase II joint monitoring. As for Ref Oil 438 (Figure A3), none of the three combined $\overline{X}$-charts and S-charts showed any out-of-control samples.

The sample mean and the sample standard deviation of the PVI calculated based on the Phase I samples were used as the "true mean" and the "true standard deviation", respectively, in the simulation process because the true population mean and population standard deviation were unknown. The mean and the standard deviation of the PVI were respectively assumed to be 154.747 and 127.376, 175.846 and 66.963, and 94.647 and 20.828 for Ref Oils 434, 435, and 438, respectively. For Phase II monitoring, 20 new samples with a subgroup size of 10 were generated when θ changed from θ_0 to $\theta_1 = \theta_0 + \xi_0$ and ξ changed from ξ_0 to $\xi_1 = 2.5\xi_0$. Note that the 20 new samples were computer simulated, not obtained from additional samples using the ASTM test method. All combined $\overline{X}$- and S-charts were tuned to produce an overall ARL_0 that was approximately equal to 370. The resulting control charts for Ref Oils 434, 435, and 438 are shown in Figures A4–A6, respectively.

For Ref Oil 434 (Figure A4), the combined lognormal $\overline{X}$- and S-charts detected out-of-control signals at samples 5 and 4 in the $\overline{X}$- and S-charts, respectively, while the combined Shewhart $\overline{X}$- and S-charts triggered at samples 8 and 11 in the $\overline{X}$- and S-charts, respectively.

In addition, out-of-control signals were detected at sample 8 in both the MAD $\overline{X}$- and S-charts. For Ref Oil 435 (Figure A5), out-of-control signals were detected on sample 6 in both the Shewhart $\overline{X}$- and S-charts and both the MAD $\overline{X}$- and S-charts. At the same time, out-of-control signals also appeared at sample 6 in both the lognormal $\overline{X}$- and S-charts. As for Ref Oil 438 (Figure A6), the combined Shewhart $\overline{X}$- and S-charts detected out-of-control signals at samples 6 and 2 in the $\overline{X}$- and S-charts, respectively, while the combined MAD $\overline{X}$- and S-charts detected at sample 6 in both charts. In addition, out-of-control signals were detected at sample 6 in both the lognormal $\overline{X}$- and S-charts.

5. Conclusions

In this study, we discuss the construction of three combined $\overline{X}$- and S-charts for jointly monitoring the mean and the standard deviation of the lognormal distribution. The simulation studies show that the combined lognormal $\overline{X}$- and S-charts have good performance when the underlying lognormal distribution is more skewed. The practical application of the combined lognormal $\overline{X}$- and S-charts is also demonstrated in a real example.

The numerical results of the current work indicate that for skewed non-normal processes, it is possible to construct more effective control charts for monitoring the process mean, process variability, or both based on the actual process distribution. This is at least the case for lognormal processes. It would be worth it to investigate how to construct more effective control charts for other skewed non-normal processes.

Remark 2. *As for other skewed non-normal processes, the process mean or process variability may need to be approximated first, and these can be estimated using Phase I in-control data. Based on the conventional Shewhart $\overline{X}$- and S-charts, one can construct the control limits of $\overline{X}$- and S-charts for monitoring the process mean or process variability under other skewed non-normal processes.*

Funding: This research was funded by by Ministry of Science and Technology, MOST 108-2118-M-035-005-MY3, Taiwan.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data presented in this study are available on request from the corresponding author.

Acknowledgments: The author would like to thank the editor and the reviewers for their valuable suggestions and constructive comments. The author also wants to thank Arthur B. Yeh at Bowling Green State University for his help and insightful suggestions.

Conflicts of Interest: The author declares no conflict of interest.

Appendix A

For Phase I, the three combined $\overline{X}$- and S-charts are shown in Figures A1–A3 for Ref Oils 434, 435, and 438, respectively. For Phase II monitoring, the resulting control charts for Ref Oils 434, 435, and 438 are shown in Figures A4–A6, respectively.

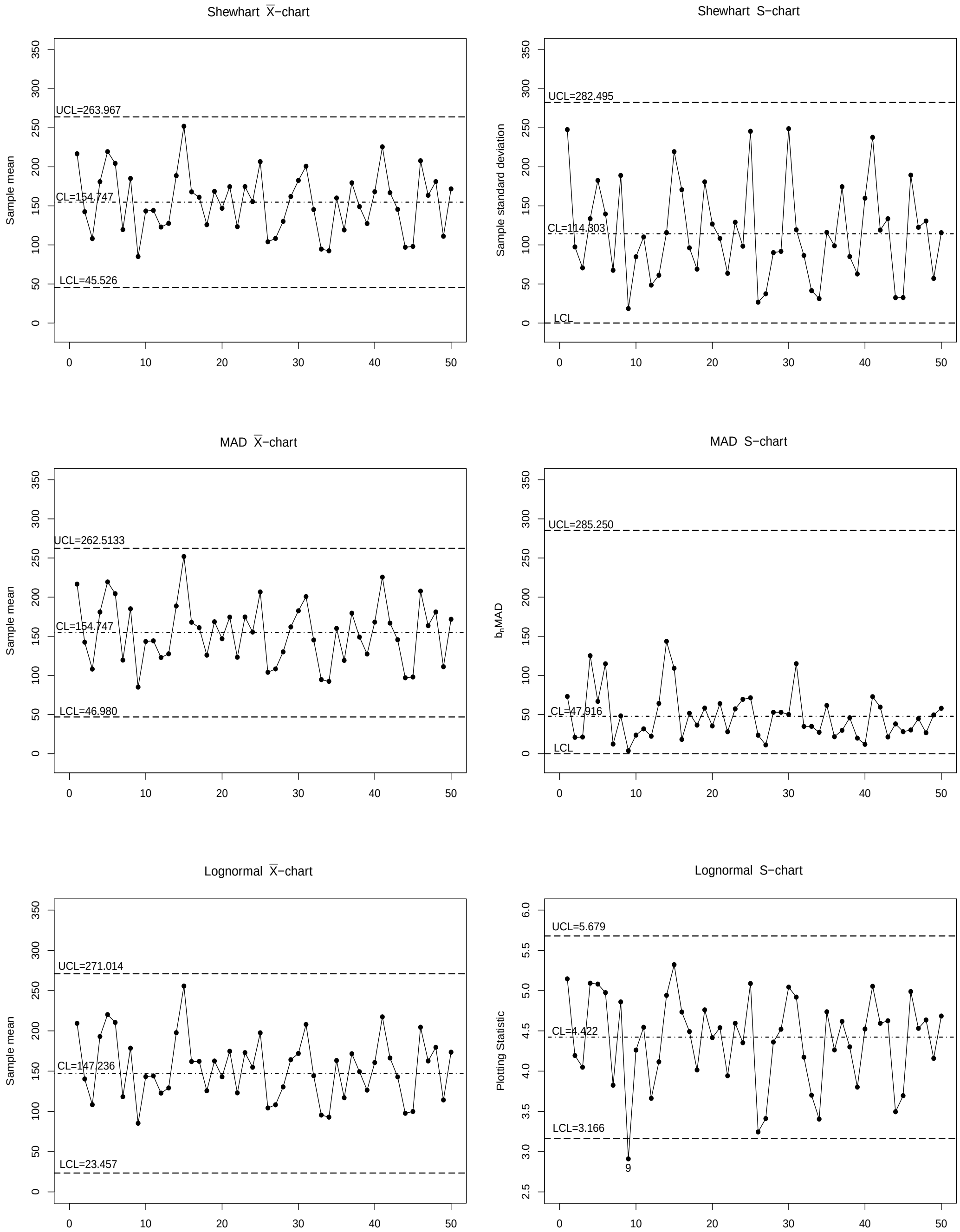

Figure A1. The three combined $\overline{X}$- and S-charts for the PVI of Ref Oil 434 in Phase I.

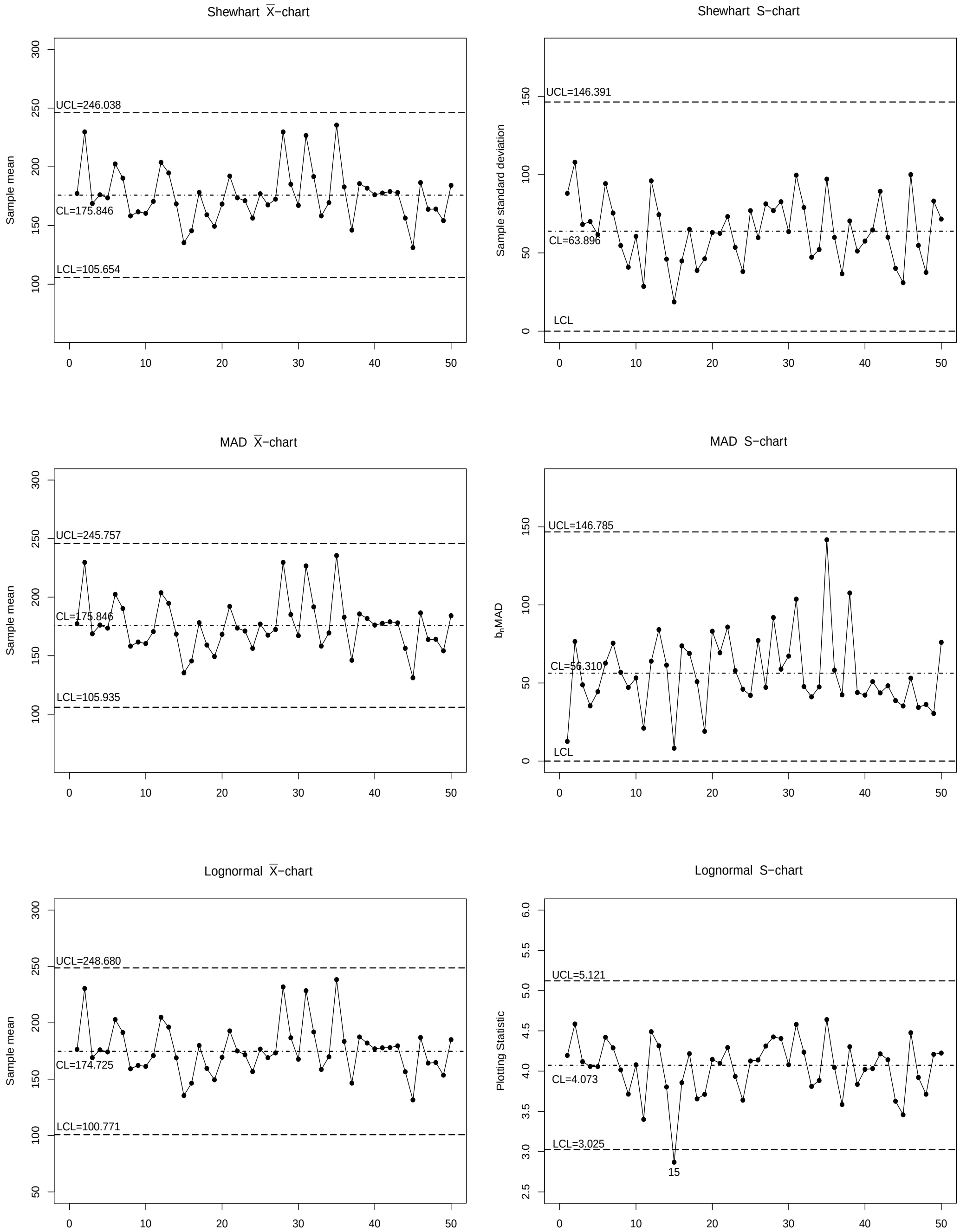

Figure A2. The three combined $\overline{X}$- and S-charts for the PVI of Ref Oil 435 in Phase I.

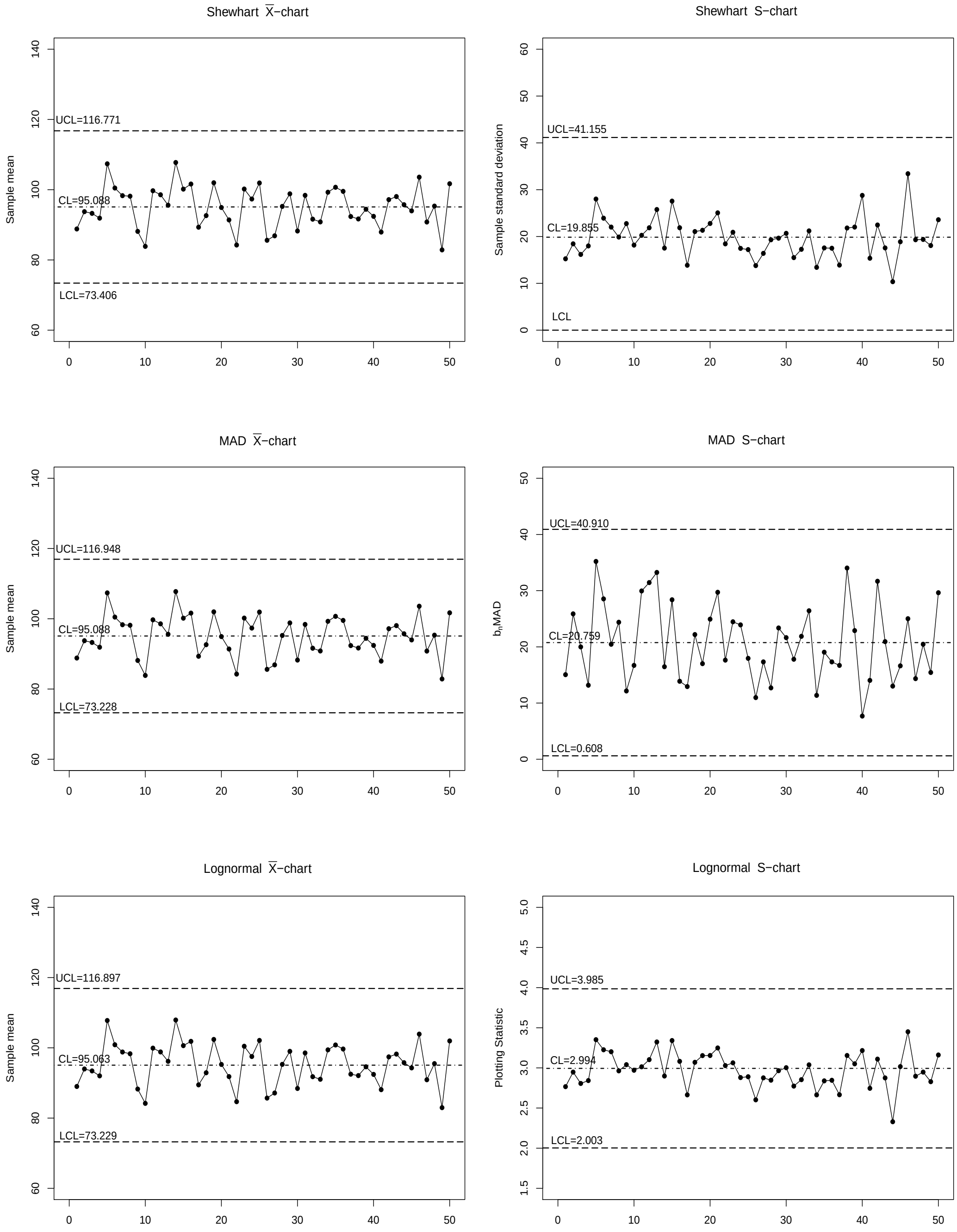

Figure A3. The three combined $\overline{X}$- and S-charts for the PVI of Ref Oil 438 in Phase I.

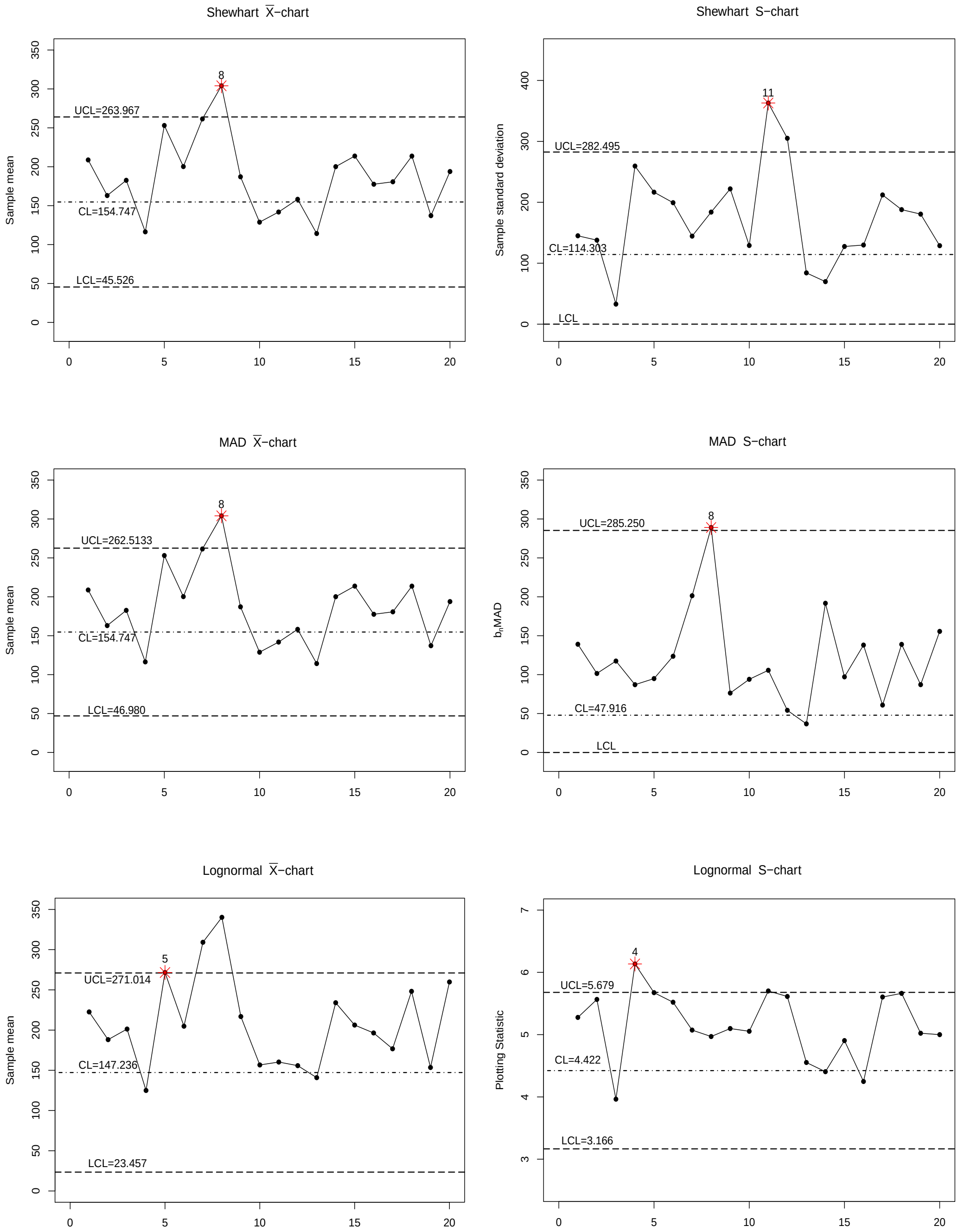

Figure A4. The three combined $\overline{X}$- and S-charts for the PVI of Ref Oil 434 in Phase II.

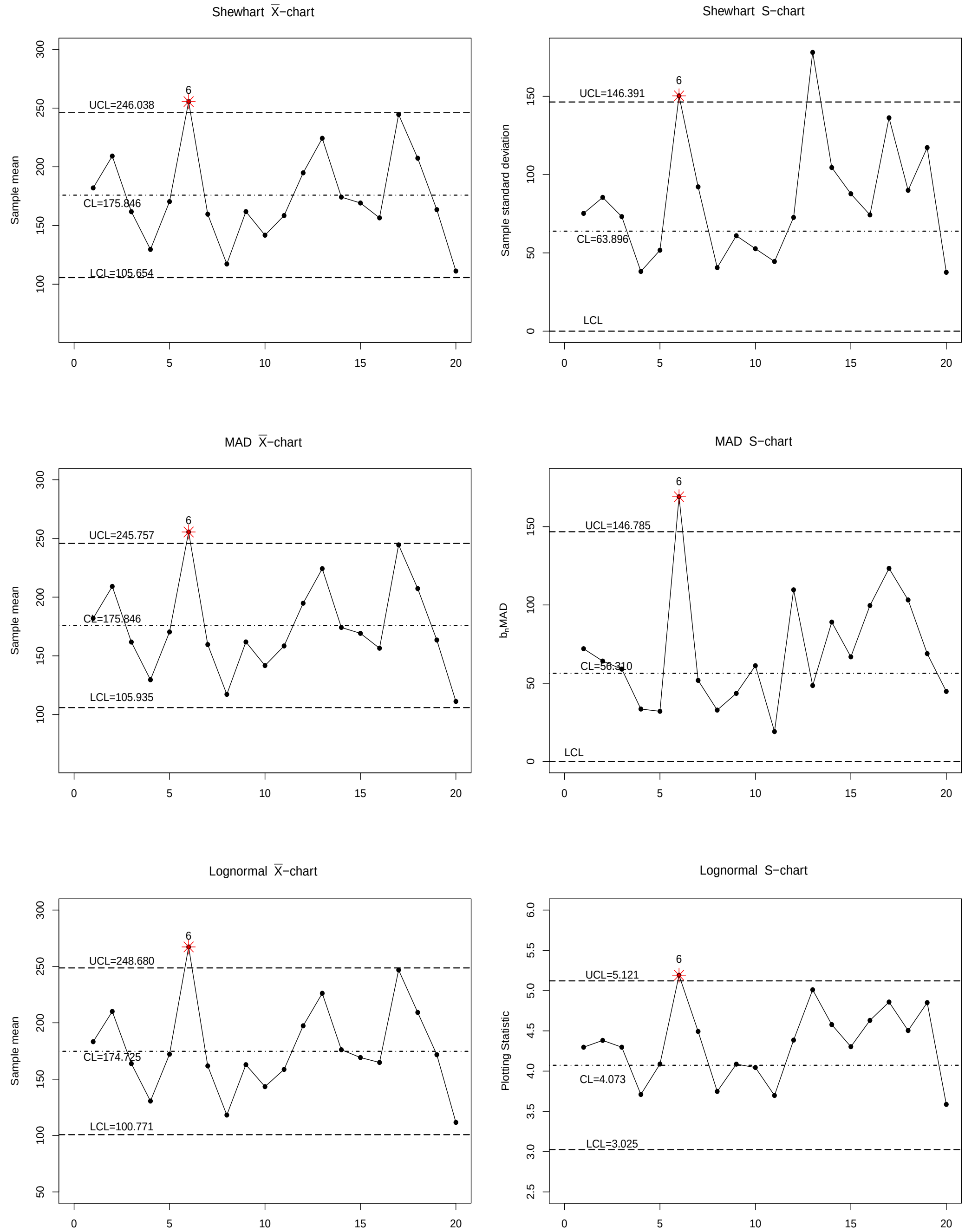

Figure A5. The three combined $\overline{X}$- and S-charts for the PVI of Ref Oil 435 in Phase II.

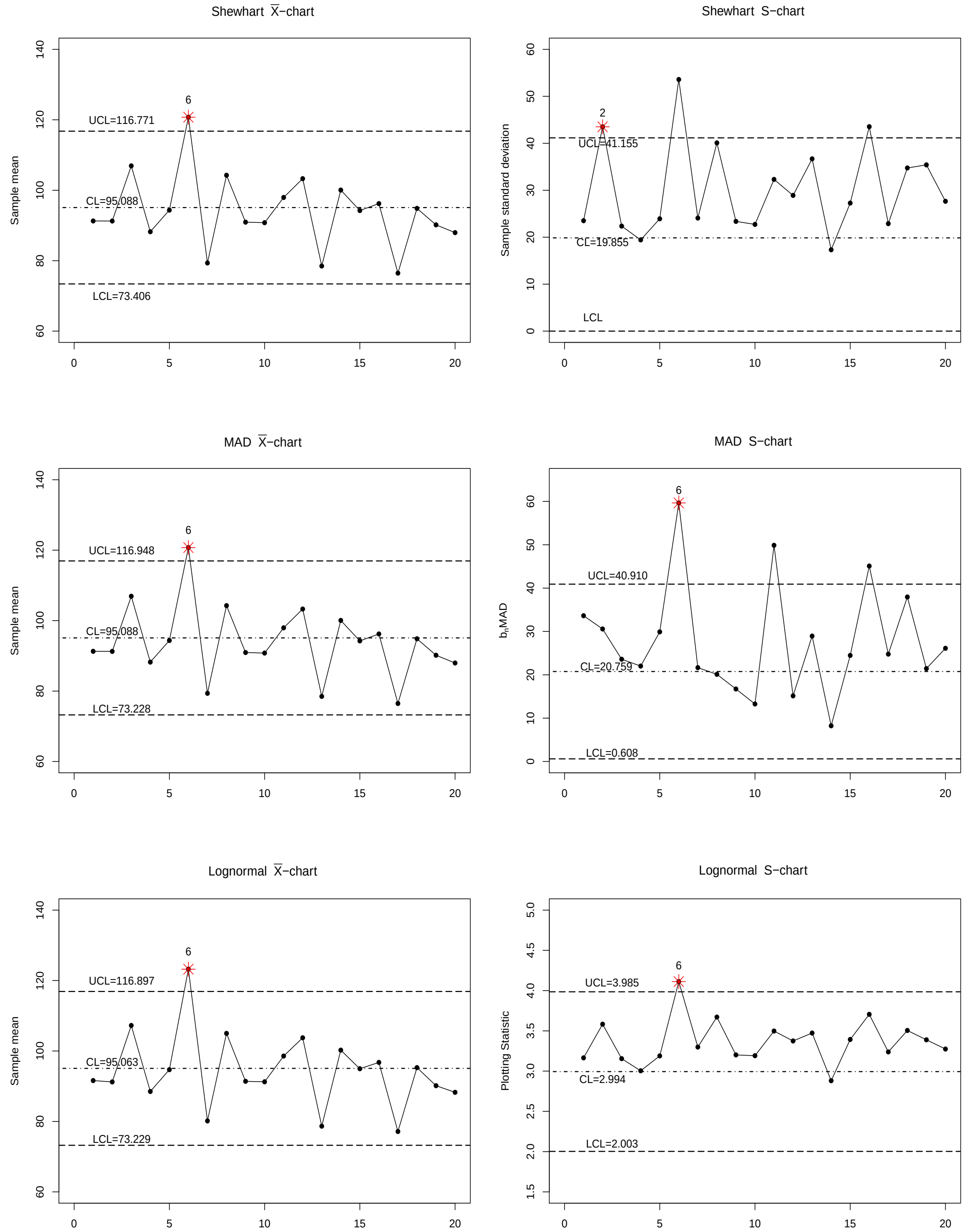

Figure A6. The three combined $\overline{X}$- and S-charts for the PVI of Ref Oil 438 in Phase II.

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