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Invariance Analysis, Exact Solution and Conservation Laws of $(2 + 1)$ Dim Fractional Kadomtsev-Petviashvili (KP) System

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Abstract: In this work, a Lie group reduction for a $(2 + 1)$ dimensional fractional Kadomtsev-Petviashvili (KP) system is determined by using the Lie symmetry method with Riemann Liouville derivative. After reducing the system into a two-dimensional nonlinear fractional partial differential system (NLFPDEs), the power series (PS) method is applied to obtain the exact solution. Further the obtained power series solution is analyzed for convergence. Then, using the new conservation theorem with a generalized Noether's operator, the conservation laws of the KP system are obtained.

Keywords: fractional Kadomtsev-Petviashvili system; lie group analysis; power series solutions; convergence analysis; conservation laws

MSC: 35R11; 34A05; 34C14; 70S10

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1. Introduction

In the field of fractional order differential equations, prevalent advancement is currently speculated. The dominant use of multifarious projects which are masked by fractional differential equations (FDEs), lies in the field of nano-technology, bio-informatics, control system, chemical engineering, heat conduction, ion-acoustic wave, mechanical engineering, diffusion equations and, additionally, several other sciences. Because of its prodigious scope and applications in the various area of science and technology, congruent consideration has been given to the exact solutions of FDEs. There are many techniques that can be used to analyze NLFPDEs [1–14]. The exact solution provides a proper understanding of the physical phenomena modeled by NLFPDEs. Finding exact solutions to NLFPDEs are quite difficult as compared to approximate solutions. The Lie symmetry method is one of the most powerful methods used to find the exact solution of NLFPDEs [15–23]. This technique is used to reduce the NLFPDEs into a lower dimension. The conservation laws can be investigated for nonlinear FPDEs, which are very important tool for the study of differential equations. Noether's theorem involves a methodology for constructing conservation laws, using symmetries associated with Noether's operator [19–22,24–29]. In general, there is no technique that provides specific solutions for the system. In recent years, many researchers have concentrated on the approximate analytical solutions to the FDE system and some methods have been developed. One of the most useful techniques for solving the linear system and non-linear system of fractional differential equations with a quick convergence rate and small calculation error is the fractional power series method.

Another major benefit is that this approach can be used directly, without requiring linearization, discretization, Adomian polynomials, etc., to the non-linear fractional PDE system. The power series method is applied to finding an exact solution in the form of a power series of a fractional differential equation. The $(2 + 1)$ dimensional Kadomtsev-Petviashvili (KP) system [30,31] is given by

$$u_{tx} - uu_{xx} - u_x^2 - u_{xxx} = u_{yy},$$

which can also be written as the system

$$\begin{aligned} u_t - uu_x - u_{xxx} - w_y &= 0, \\ w_x - u_y &= 0. \end{aligned} \quad (1)$$

In nonlinear wave theory, the KP system is one of the most universal models which arises as a reduction in the system with quadratic nonlinearity. This system has been broadly studied in terms of its mathematical association in recent years. The KP equation was originated by the two Soviet physicists, Boris Kadomtsev and Vladimir Petviashvili in [32]. The KP equation has been studied by many authors for integer-order or fraction-order derivatives by different methods in recent years. Exact traveling wave solutions have been analyzed in [31]. In [30], KP equation is studied for symmetry reduction using a loop algebra. In [33], KP solitary waves has been studied. Symmetries of the integer order KP equation have been studied in [34]. In [35], the Cauchy problem for the fractional KP equations has been discussed.

The main goal of this work is to analyze the fractional order KP system with arbitrary constant coefficients as

$$\begin{aligned} \partial_t^\alpha u - A_1 u \partial_x^\beta u - A_2 \partial_y^\gamma w - A_3 u_{xxx} &= 0, \\ \partial_x^\beta w - A_4 \partial_y^\gamma u &= 0. \end{aligned} \quad (2)$$

This is a system of NLPDEs of fractional order, which depicts the evolution of nonlinear long waves with small amplitude. Here, u and w are dependent functions of x, y, t , and A_1, A_2, A_3, A_4 are arbitrary constants. x and y are the longitudinal and transverse spatial coordinates, respectively.

In this work, the KP system (2) is considered for symmetry reduction. The exact solutions, in the form of power series, are obtained, and the conservation laws are investigated.

To find some new exact solutions to the system (2), we apply the Lie symmetry method to reduce the system into lower dimensions. The system is also studied for conservation laws by using the new conservation theorem [27]. The preliminary material is given in Section 2. In Section 3, the symmetry of system (2) is obtained via the classical Lie method. Through the corresponding generators, we reduce system (2) to lower-dimensional NLPDEs. Some exact solutions are obtained, corresponding to the reduced equation, by using the power series method in Section 4. In Section 5 the obtained power series solutions are analyzed for convergence. Some conservation laws are investigated in Section 6. In the last section, the conclusion to the study is presented.

2. Preliminaries

In this section, we will discuss basic definitions and theories for Lie symmetry analysis.

Definition 1. Riemann-Liouville fractional derivative [36,37]

Let $f : [a, b] \subseteq \mathcal{R} \rightarrow \mathcal{R}$, such that $\frac{\partial^n f}{\partial t^n}$ is continuous and integrable for all $n \in \mathbb{N} \cup \{0\}$ and $n - 1 < \alpha < n$, then the Riemann-Liouville fractional derivative of order $\alpha > 0$ is defined by

$${}_0D_t^\alpha f(x, y, t) = \frac{\partial^\alpha f(x, y, t)}{\partial t^\alpha} = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \frac{\partial^n}{\partial t^n} \int_0^t (t-s)^{n-\alpha-1} f(x, y, s) ds, & t > 0, \quad n-1 < \alpha < n, \\ \frac{\partial^n f(x, y, t)}{\partial t^n}, & \alpha = n \in \mathbb{N}, \end{cases} \quad (3)$$

where $\Gamma(\alpha)$ is the Euler's gamma function.

Definition 2. Erdélyi-Kober operator

The left-hand-side Erdélyi-Kober fractional differential operator $(\mathcal{P}_{q_1, q_2}^{\vartheta, \alpha})$ is defined as

$$(\mathcal{P}_{q_1, q_2}^{\vartheta, \alpha})(y_1, y_2) = \prod_{k=0}^{r-1} \left(\vartheta + k - \frac{1}{q_1} y_1 \frac{d}{dy_1} - \frac{1}{q_2} y_2 \frac{d}{dy_2} \right) (\mathcal{M}_{q_1, q_2}^{\vartheta+\alpha, r-\alpha} g)(y_1, y_2), \quad y_i > 0, q_i > 0, \alpha > 0, \quad i = 1, 2, \quad (4)$$

$$r = \begin{cases} [\alpha] + 1 & \text{if } \alpha \notin \mathbb{N}, \\ \alpha & \text{if } \alpha \in \mathbb{N}, \end{cases}$$

where

$$(\mathcal{M}_{q_1, q_2}^{\vartheta, \alpha} g)(y_1, y_2) = \begin{cases} \frac{1}{\Gamma(\alpha)} \int_1^\infty (\rho-1)^{\alpha-1} \rho^{-(\vartheta+\alpha)} g(y_1 \rho^{\frac{1}{q_1}}, y_2 \rho^{\frac{1}{q_2}}) d\rho & \text{if } \alpha > 0, \\ g(y_1, y_2) & \text{if } \alpha = 0, \end{cases} \quad (5)$$

is the left-hand-side Erdélyi-Kober fractional integral operator.

The right-hand-side Erdélyi-Kober fractional differential operator $(\mathcal{D}_{q_1, q_2}^{\vartheta, \beta})$ is defined as

$$(\mathcal{D}_{q_1, q_2}^{\vartheta, \beta})(y_1, y_2) = \prod_{k=1}^r \left(\vartheta + k + \frac{1}{q_1} y_1 \frac{d}{dy_1} + \frac{1}{q_2} y_2 \frac{d}{dy_2} \right) (\mathcal{I}_q^{\vartheta+\beta, r-\beta} g)(y_1, y_2), \quad y_i > 0, q_i > 0, \beta > 0, \quad i = 1, 2, \quad (6)$$

$$r = \begin{cases} [\beta] + 1 & \text{if } \beta \notin \mathbb{N}, \\ \beta & \text{if } \beta \in \mathbb{N}, \end{cases}$$

where

$$(\mathcal{I}_q^{\vartheta, \beta} g)(y_1, y_2) = \begin{cases} \frac{1}{\Gamma(\beta)} \int_0^1 (1-\rho)^{\beta-1} \rho^\vartheta g(y_1 \rho^{\frac{1}{q_1}}, y_2 \rho^{\frac{1}{q_2}}) d\rho & \text{if } \beta > 0, \\ g(y_1, y_2) & \text{if } \beta = 0, \end{cases} \quad (7)$$

is the right-hand-side Erdélyi-Kober fractional integral operator.

Symmetry Analysis

Consider the system of NLFPDEs as follows

$$\Delta_h = F_h \left(x, y, t, \mathbf{v}, \frac{\partial^\alpha \mathbf{v}}{\partial t^\alpha}, \frac{\partial^\beta \mathbf{v}}{\partial x^\beta}, \frac{\partial^\gamma \mathbf{v}}{\partial y^\gamma}, \frac{\partial \mathbf{v}}{\partial x}, \frac{\partial^2 \mathbf{v}}{\partial x^2}, \dots \right), \quad h = 1, 2, \dots, \quad (8)$$

where $\frac{\partial^\alpha \mathbf{v}}{\partial t^\alpha}$, $\frac{\partial^\beta \mathbf{v}}{\partial x^\beta}$ and $\frac{\partial^\gamma \mathbf{v}}{\partial y^\gamma}$ are the fractional derivatives of Riemann-Liouville (RL) type. Suppose that the Lie group of transformations are given by

$$\begin{aligned} x^* &= x + \epsilon \xi(x, y, t, \mathbf{v}) + O(\epsilon^2), \\ t^* &= t + \epsilon \tau(x, y, t, \mathbf{v}) + O(\epsilon^2), \\ y^* &= t + \epsilon \mu(x, y, t, \mathbf{v}) + O(\epsilon^2), \\ v^{r*} &= v^r + \epsilon \eta^{(r)}(x, t, v^r) + O(\epsilon^2), \\ \frac{\partial^\alpha v^{r*}}{\partial t^{\alpha}} &= \frac{\partial^\alpha v^r}{\partial t^\alpha} + \epsilon \eta^{(r)\alpha, t} + O(\epsilon^2), \\ \frac{\partial^\beta v^{r*}}{\partial x^\beta} &= \frac{\partial^\beta v^r}{\partial x^\beta} + \epsilon \eta^{(r)\beta, x} + O(\epsilon^2), \\ \frac{\partial^\gamma v^{r*}}{\partial y^\gamma} &= \frac{\partial^\gamma v^r}{\partial y^\gamma} + \epsilon \eta^{(r)\gamma, y} + O(\epsilon^2), \\ \frac{\partial v^{r*}}{\partial x} &= \frac{\partial v^r}{\partial x} + \epsilon \eta^{(r)x} + O(\epsilon^2), \\ \frac{\partial^2 v^{r*}}{\partial x^2} &= \frac{\partial^2 v^r}{\partial x^2} + \epsilon \eta^{(r)xx} + O(\epsilon^2), \\ &\vdots \end{aligned} \quad (9)$$

where ϵ being the group parameter and $\xi, \tau, \mu, \eta^{(r)}$ are the infinitesimals,

$$\begin{aligned} \eta^{(k),x} &= D_x(\eta^{(k)}) - v_x^k D_x(\xi) - v_t^k(\tau) - v_y^k D_y(\mu), \\ \eta^{(k)\alpha, t} &= D_t^\alpha(\eta^{(k)}) + \xi D_t^\alpha(v_x^k) - D_t^\alpha(\xi v_x^k) + \tau D_t^\alpha(v_t^k) - D_t^\alpha(\tau v_t^k) + \mu D_t^\alpha(v_y^k) \\ &\quad - D_t^\alpha(\mu v_y^k), \\ \eta^{(k)\beta, x} &= D_x^\beta(\eta^{(k)}) + \xi D_x^\beta(v_x^k) - D_x^\beta(\xi v_x^k) + \tau D_x^\beta(v_t^k) - D_x^\beta(\tau v_t^k) + \mu D_x^\beta(v_y^k) \\ &\quad - D_x^\beta(\mu v_y^k), \\ \eta^{(k)\gamma, y} &= D_y^\gamma(\eta^{(k)}) + \xi D_y^\gamma(v_x^k) - D_y^\gamma(\xi v_x^k) + \tau D_y^\gamma(v_t^k) - D_y^\gamma(\tau v_t^k) + \mu D_y^\gamma(v_y^k) \\ &\quad - D_y^\gamma(\mu v_y^k), \end{aligned} \quad (10)$$

are extended infinitesimals. In (10), D_x and D_t are total derivative operators. The α^{th} , β^{th} and γ^{th} extended infinitesimals related to the RL fractional derivative are given in [38].

The associated vector field is

$$X = \xi(x, y, t, \mathbf{v}) \frac{\partial}{\partial x} + \mu(x, y, t, \mathbf{v}) \frac{\partial}{\partial y} + \tau(x, y, t, \mathbf{v}) \frac{\partial}{\partial t} + \sum_{r=1}^p \eta^{(r)}(x, y, t, \mathbf{v}) \frac{\partial}{\partial v^r}. \quad (11)$$

The corresponding extended symmetry generator is as follows

$$\begin{aligned} pr^{(\alpha, \beta, \gamma)} X &= X + \sum_r \eta^{(r)\alpha, t} \partial_{\partial_t^\alpha v^r} + \sum_r \eta^{(r)\beta, x} \partial_{\partial_x^\beta v^r} + \sum_r \eta^{(r)\gamma, y} \partial_{\partial_y^\gamma v^r} + \sum_r \eta^{(r)x} \partial_{v_x^r} \\ &\quad + \sum_r \eta^{(r)xx} \partial_{v_{xx}^r} + \dots, \end{aligned} \quad (12)$$

As the lower limit of RL fractional derivative [36,37,39] is fixed, we have

$$\xi(x, y, t, u, w)|_{x=0} = 0, \quad \tau(x, y, t, u, w)|_{t=0} = 0, \quad \mu(x, y, t, u, w)|_{y=0} = 0. \quad (13)$$

3. Symmetry Analysis of (2 + 1)-Dimensional Fractional Kadomtsev-Petviashvili System

Let us assume that the system (2) is invariant under group of transformations (9), then we have

$$\begin{aligned}\partial_t^\alpha u^* - A_1 u^* \partial_x^\beta u^* - A_2 \partial_y^\gamma w^* - A_3 u_{x^* x^* x^*}^* &= 0, \\ \partial_x^\beta w^* - A_4 \partial_y^\gamma u^* &= 0.\end{aligned}\quad (14)$$

Therefore, using (9) in (14) the invariance criteria for (2) are obtained as

$$\begin{aligned}\eta^{\alpha,t} - A_1 \eta \partial_x^\beta u - A_1 u \eta^{\beta,x} - A_2 \phi^{\gamma,y} - A_3 \eta^{xxx} &= 0, \\ \phi^{\beta,x} - A_4 \eta^{\gamma,y} &= 0.\end{aligned}\quad (15)$$

Using the value of extended infinitesimals and collecting the coefficients of various powers of u and partial derivatives of u and w , we have

$$\begin{aligned}\xi_t = \xi_u = \xi_w &= 0, \\ \tau_x = \tau_u = \tau_v &= 0, \\ \eta_w = \phi_u &= 0, \\ \eta_{uu} = \phi_{ww} &= 0, \\ \eta_u - \phi_w - \alpha D_t \tau + \gamma D_y \mu &= 0, \\ 3\xi_x - \alpha D_t \tau &= 0, \\ \eta_u - \phi_w + \beta D_x \xi - 9\gamma D_y \mu &= 0, \\ \eta - u\beta D_x \xi + u\alpha D_t \tau &= 0, \\ \partial_t^\alpha \eta - A_1 u \partial_t^\alpha \eta_u - A_1 u (\partial_x^\beta \eta - u \partial_x^\beta \eta_u) - A_2 (\partial_y^\gamma \phi - w \partial_y^\gamma \phi_w) - A_3 \eta_{xxx} &= 0, \\ \partial_x^\beta \phi - w \partial_x^\beta \phi_w - A_4 \partial_y^\gamma \eta + A_4 y \partial_y^\gamma \eta_u &= 0, \\ \binom{\alpha}{n} \partial_t^n \eta_u - \binom{\alpha}{n+1} D_t^{n+1} \tau &= 0, \\ \binom{\beta}{n} \partial_x^n \eta_u - \binom{\beta}{n+1} D_x^{n+1} \xi &= 0, \\ \binom{\gamma}{n} \partial_y^n \eta_u - \binom{\gamma}{n+1} D_y^{n+1} \mu &= 0, \\ \binom{\beta}{n} \partial_x^n \phi_w - \binom{\beta}{n+1} D_x^{n+1} \xi &= 0, \\ \binom{\gamma}{n} \partial_y^n \phi_w - \binom{\gamma}{n+1} D_y^{n+1} \xi &= 0,\end{aligned}\quad (16)$$

where $n \in \mathbb{N}$.

Solving these equations simultaneously, we get the infinitesimals

$$\begin{aligned}\xi &= C_1 x, \quad \tau = \frac{3}{\alpha} C_1 t, \quad \mu = \frac{3+\beta}{2\gamma} C_1 y, \\ \eta &= (\beta-3) C_1 u, \quad \phi = \frac{3(\beta-3)}{2} C_1 w,\end{aligned}\quad (17)$$

where C_1 is the arbitrary constant.

Thus, the corresponding vector field is

$$V = x \partial_x + \frac{3}{\alpha} t \partial_t + \frac{3+\beta}{2\gamma} y \partial_y + (\beta-3) u \partial_u + \frac{3(\beta-3)}{2} w \partial_w. \quad (18)$$

Corresponding to vector field V , the characterisitic equation is written as

$$\frac{dx}{x} = \frac{dt}{\frac{3}{\alpha}t} = \frac{dy}{\frac{3+\beta}{2\gamma}y} = \frac{du}{(\beta-3)u} = \frac{dw}{\frac{3(\beta-3)}{2}w}.$$

After solving these equations, we get the symmetry variables

$$z_1 = xt^{-\frac{\alpha}{3}}, \quad z_2 = yt^{-\frac{\alpha(\beta+3)}{6\gamma}}, \quad (19)$$

and symmetry transformations

$$u = t^{\frac{\alpha(\beta-3)}{3}} f(z_1, z_2), \quad w = t^{\frac{\alpha(\beta-3)}{2}} g(z_1, z_2), \quad (20)$$

where f and g are arbitrary functions.

Theorem 1. *The symmetry reduction in the system (2) corresponding to the symmetry variables (19) and symmetry transformations (20) is*

$$\begin{aligned} & \left(P_{\frac{3}{\alpha}, \frac{6\gamma}{\alpha(\beta+3)}}^{\frac{\alpha(\beta-6)}{3}+1, \alpha} f \right) (z_1, z_2) - A_1 z_1^{-\beta} f(z_1, z_2) \left(D_{1,\infty}^{-\beta, \beta} f \right) (z_1, z_2) - A_2 z_2^{-\gamma} \left(D_{\infty,1}^{-\gamma, \gamma} g \right) (z_1, z_2) \\ & - A_3 \frac{\partial^3 f(z_1, z_2)}{\partial z_1^3} = 0, \\ & z_1^{-\beta} \left(D_{1,\infty}^{-\beta, \beta} g \right) (z_1, z_2) - A_4 z_2^{-\gamma} \left(D_{\infty,1}^{-\gamma, \gamma} f \right) (z_1, z_2) = 0, \end{aligned} \quad (21)$$

where $(\mathcal{P}_{\varrho_1, \varrho_2}^{\theta, \alpha})$ and $(\mathcal{D}_{\varrho_1, \varrho_2}^{\theta, \beta})$ are the left- and right-hand-side Erdélyi-Kober fractional differential operators, respectively.

Proof. Let us assume $n-1 < \alpha < n$, where $n \in \mathbb{N}$; then, using the definition of RL fractional differentiation, we have

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^n}{\partial t^n} \left(\frac{1}{\Gamma(n-\alpha)} \int_0^t (t-s)^{n-\alpha-1} s^{\frac{\alpha(\beta-3)}{3}} f\left(xs^{-\frac{\alpha}{3}}, ys^{-\frac{\alpha(\beta+3)}{6\gamma}}\right) ds \right).$$

Let $s = \frac{t}{\rho}$; then, we get

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^n}{\partial t^n} \left(\frac{t^{n+\frac{\alpha(\beta-6)}{3}}}{\Gamma(n-\alpha)} \int_1^\infty (\rho-1)^{n-\alpha-1} \rho^{-(n-\alpha+\frac{\alpha(\beta-3)}{3})} f(z_1 \rho^{\frac{\alpha}{3}}, z_2 \rho^{\frac{\alpha(\beta+3)}{6\gamma}}) d\rho \right).$$

□

By using the definition of the left-hand-side EK fractional integral operator $(\mathcal{M}_{\varrho}^{\theta, \alpha} f)(z_1, z_2)$, defined in [36,37,39], we have

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^n}{\partial t^n} \left(t^{n+\frac{\alpha(\beta-6)}{3}} (\mathcal{M}_{\frac{3}{\alpha}, \frac{6\gamma}{\alpha(\beta+3)}}^{1+\frac{\alpha(\beta-3)}{3}, n-\alpha} f)(z_1, z_2) \right), \quad (22)$$

where

$$\left(\mathcal{M}_{\frac{3}{\alpha}, \frac{6\gamma}{\alpha(\beta+3)}}^{1+\frac{\alpha(\beta-3)}{3}, n-\alpha} f \right) (z_1, z_2) = \frac{1}{\Gamma(n-\alpha)} \int_1^\infty (\rho-1)^{n-\alpha-1} \rho^{-(n-\alpha+\frac{\alpha(\beta-3)}{3})} f(z_1 \rho^{\frac{\alpha}{3}}, z_2 \rho^{\frac{\alpha(\beta+3)}{6\gamma}}) d\rho.$$

For further simplification, let us assume that $\mu(z_1, z_2)$ is a continuously differential function for z_1, z_2 given in (19); then

$$t \frac{\partial}{\partial t} \mu(z_1, z_2) = -\frac{\alpha}{3} z_1 \frac{\partial \mu}{\partial z_1} - \frac{\alpha(\beta+3)}{6\gamma} z_2 \frac{\partial \mu}{\partial z_2}.$$

Thus, Equation (22) becomes

$$\begin{aligned} \frac{\partial^\alpha u}{\partial t^\alpha} &= \frac{\partial^n}{\partial t^n} \left(t^{n+\frac{\alpha(\beta-6)}{3}} \left(\mathcal{M}_{\frac{3}{\alpha}, \frac{6\gamma}{\alpha(\beta+3)}}^{1+\frac{\alpha(\beta-3)}{3}, n-\alpha} f \right)(z_1, z_2) \right) \\ &= \frac{\partial^{n-1}}{\partial t^{n-1}} \left(t^{n+\frac{\alpha(\beta-6)}{3}} \left(\mathcal{M}_{\frac{3}{\alpha}, \frac{6\gamma}{\alpha(\beta+3)}}^{1+\frac{\alpha(\beta-3)}{3}, n-\alpha} f \right)(z_1, z_2) \right) \\ &= \frac{\partial^{n-1}}{\partial t^{n-1}} \left(t^{n+\frac{\alpha(\beta-6)}{3}-1} \left(n-\alpha + \frac{\alpha(\beta-3)}{3} - \frac{\alpha}{3} z_1 \frac{\partial}{\partial z_1} - \frac{\alpha(\beta+3)}{6\gamma} z_2 \frac{\partial}{\partial z_2} \right) \left(\mathcal{M}_{\frac{3}{\alpha}, \frac{6\gamma}{\alpha(\beta+3)}}^{1+\frac{\alpha(\beta-3)}{3}, n-\alpha} f \right)(z_1, z_2) \right). \end{aligned}$$

Continuing in this way, we get

$$\frac{\partial^\gamma u}{\partial t^\gamma} = \left(t^{\frac{\alpha(\beta-6)}{3}} \prod_{j=0}^{n-1} \left(1+j + \frac{\alpha(\beta-6)}{3} - \frac{\alpha}{3} z_1 \frac{\partial}{\partial z_1} - \frac{\alpha(\beta+3)}{6\gamma} z_2 \frac{\partial}{\partial z_2} \right) \left(\mathcal{M}_{\frac{3}{\alpha}, \frac{6\gamma}{\alpha(\beta+3)}}^{1+\frac{\alpha(\beta-3)}{3}, n-\alpha} f \right)(z_1, z_2) \right). \quad (23)$$

By using the right-hand-side EK fractional differential operator $(\mathcal{P}_\epsilon^{\theta, \alpha} f)(z_1, z_2)$ [36,37,39] into (23), we have

$$\frac{\partial^\alpha u}{\partial t^\alpha} = t^{\frac{\alpha(\beta-6)}{3}} \left(\mathcal{P}_{\frac{3}{\alpha}, \frac{6\gamma}{\alpha(\beta+3)}}^{1+\frac{\alpha(\beta-6)+1}{3}, \alpha} f \right)(z_1, z_2). \quad (24)$$

In similar manner, the RL fractional derivatives of order $\beta, \gamma > 0$ are obtained as

$$\begin{aligned} \frac{\partial^\beta u}{\partial x^\beta} &= t^{-\alpha} z_1^{-\beta} (\mathcal{D}_{1,\infty}^{-\beta, \beta} f)(z_1, z_2), \\ \frac{\partial^\beta w}{\partial x^\beta} &= t^{\frac{\alpha(\beta-9)}{6}} z_1^{-\beta} (\mathcal{D}_{1,\infty}^{-\beta, \beta} g)(z_1, z_2), \\ \frac{\partial^\gamma u}{\partial y^\beta} &= t^{\frac{\alpha(\beta-9)}{6}} z_2^{-\beta} (\mathcal{D}_{\infty,1}^{-\gamma, \gamma} f)(z_1, z_2), \\ \frac{\partial^\gamma w}{\partial y^\gamma} &= t^{\frac{\alpha(\beta-6)}{3}} z_2^{-\gamma} (\mathcal{D}_{\infty,1}^{-\gamma, \gamma} g)(z_1, z_2), \end{aligned} \quad (25)$$

where $(\mathcal{D}_{1,\infty}^{-\beta, \beta})$ and $(\mathcal{D}_{\infty,1}^{-\gamma, \gamma})$ are the differential operators defined in [36,37,39].

By using (24) and (25), the symmetry reduction in KP system (2) is obtained as

$$\begin{aligned} \left(\mathcal{P}_{\frac{3}{\alpha}, \frac{6\gamma}{\alpha(\beta+3)}}^{\frac{\alpha(\beta-6)}{3}+1, \alpha} f \right)(z_1, z_2) - A_1 z_1^{-\beta} f(z_1, z_2) (\mathcal{D}_{1,\infty}^{-\beta, \beta} f)(z_1, z_2) - A_2 z_2^{-\gamma} (\mathcal{D}_{\infty,1}^{-\gamma, \gamma} g)(z_1, z_2) - A_3 \frac{\partial^3 f(z_1, z_2)}{\partial z_1^3} &= 0, \\ z_1^{-\beta} (\mathcal{D}_{1,\infty}^{-\beta, \beta} g)(z_1, z_2) - A_4 z_2^{-\gamma} (\mathcal{D}_{\infty,1}^{-\gamma, \gamma} f)(z_1, z_2) &= 0. \end{aligned} \quad (26)$$

4. Power Series Solution

In this section, we will obtain the power series solutions of NLFPEs (26) [18,28,40].

Let us consider two double power series

$$f(z_1, z_2) = \sum_{n,m=0}^{\infty} a_{n,m} z_1^n z_2^m, \quad g(z_1, z_2) = \sum_{n,m=0}^{\infty} b_{n,m} z_1^n z_2^m. \quad (27)$$

Therefore, from (27), we have

$$\begin{aligned}\frac{\partial f}{\partial z_1} &= \sum_{n,m=0}^{\infty} (n+1)a_{n+1,m}z_1^n z_2^m, \\ \frac{\partial^2 f}{\partial z_1^2} &= \sum_{n,m=0}^{\infty} (n+2)(n+1)a_{n+2,m}z_1^n z_2^m, \\ \frac{\partial^3 f}{\partial z_1^3} &= \sum_{n,m=0}^{\infty} (n+3)(n+2)(n+1)a_{n+3,m}z_1^n z_2^m\end{aligned}\quad (28)$$

Inserting (27) and (28) into (26), we have

$$\begin{aligned}&\sum_{n,m=0}^{\infty} \frac{\Gamma\left(\frac{\alpha(\beta-3)}{3} + 1 - \frac{n\alpha}{3} - \frac{m\alpha(\beta+3)}{6\gamma}\right)}{\Gamma\left(\frac{\alpha(\beta-6)}{3} + 1 - \frac{n\alpha}{3} - \frac{m\alpha(\beta+3)}{6\gamma}\right)} a_{n,m} z_1^n z_2^m - A_1 z_1^{-\beta} \sum_{n,m=0}^{\infty} \sum_{k=0}^n \sum_{j=0}^m \left(\frac{\Gamma(1+k)}{\Gamma(1+k-\beta)} a_{n-k,m-j} a_{k,j} \right) \\ &- A_2 z_2^{-\gamma} \sum_{n,m=0}^{\infty} \frac{\Gamma(1+m)}{\Gamma(1+m-\gamma)} b_{n,m} z_1^n z_2^m - A_3 \sum_{n,m=0}^{\infty} \left((n+3)(n+2)(n+1)a_{n+3,m} z_1^n z_2^m \right) = 0, \\ &z_1^{-\beta} \sum_{n,m=0}^{\infty} \frac{\Gamma(1+n)}{\Gamma(1+n-\beta)} b_{n,m} z_1^n z_2^m - A_4 z_2^{-\gamma} \sum_{n,m=0}^{\infty} \frac{\Gamma(1+m)}{\Gamma(1+m-\gamma)} a_{n,m} z_1^n z_2^m = 0.\end{aligned}\quad (29)$$

Comparing coefficients for $n = m = 0$, we have

$$\begin{aligned}a_{3,0} &= \frac{1}{6A_3} \left\{ \frac{\Gamma\left(\frac{\alpha(\beta-3)}{3} + 1\right)}{\Gamma\left(\frac{\alpha(\beta-6)}{3} + 1\right)} a_{0,0} - A_1 z_1^{-\beta} \frac{1}{\Gamma(1-\beta)} a_{0,0}^2 - A_2 z_2^{-\gamma} \frac{1}{\Gamma(1-\gamma)} b_{0,0} \right\}, \\ b_{0,0} &= A_4 z_1^{\beta} z_2^{-\gamma} \frac{\Gamma(1-\beta)}{\Gamma(1-\gamma)} a_{0,0}.\end{aligned}\quad (30)$$

When $n \geq 0, m \geq 0$, but both are not simultaneously zero, we have

$$\begin{aligned}a_{n+3,m} &= \frac{1}{A_3(n+3)(n+2)(n+1)} \left\{ \frac{\Gamma\left(\frac{\alpha(\beta-3)}{3} + 1 - \frac{n\alpha}{3} - \frac{m\alpha(\beta+3)}{6\gamma}\right)}{\Gamma\left(\frac{\alpha(\beta-6)}{3} + 1 - \frac{n\alpha}{3} - \frac{m\alpha(\beta+3)}{6\gamma}\right)} a_{n,m} \right. \\ &\quad \left. - A_1 z_1^{-\beta} \sum_{k=0}^n \sum_{j=0}^m \frac{\Gamma(1+k)}{\Gamma(1+k-\beta)} a_{n-k,m-j} a_{k,j} - A_2 z_2^{-\gamma} \frac{\Gamma(1+m)}{\Gamma(1+m-\gamma)} b_{n,m} \right\}, \\ b_{n,m} &= A_4 z_1^{\beta} z_2^{-\gamma} \frac{\Gamma(1+n-\beta)\Gamma(1+m)}{\Gamma(1+n)\Gamma(1+m-\gamma)} a_{n,m}.\end{aligned}\quad (31)$$

In view of (31), we can obtain all coefficients $a_{n,m}$ ($n \geq 3, m \geq 0$) and $b_{n,m}$ ($n, m \geq 0$) of the power series (27) for arbitrary chosen series $\sum_{m=0}^{\infty} a_{i,m}$ ($i = 0, 1, 2$).

Therefore, the system (2) has the exact power series solution, and the coefficient of the series depends on (31). Hence, we can write the power series (27) as

$$\begin{aligned}f(z_1, z_2) &= a_{0,0} + a_{1,0}z_1 + a_{2,0}z_1^2 + \frac{1}{6A_3} \left(\frac{\Gamma\left(\frac{\alpha(\beta-3)}{3} + 1\right)}{\Gamma\left(\frac{\alpha(\beta-6)}{3} + 1\right)} a_{0,0} - A_1 z_1^{-\beta} \frac{1}{\Gamma(1-\beta)} a_{0,0}^2 - A_2 z_2^{-\gamma} \frac{1}{\Gamma(1-\gamma)} b_{0,0} \right) z_1^3 \\ &\quad + \sum_{n=1,m=0}^{\infty} \frac{1}{A_3(n+3)(n+2)(n+1)} \left(\frac{\Gamma\left(\frac{\alpha(\beta-3)}{3} + 1 - \frac{n\alpha}{3} - \frac{m\alpha(\beta+3)}{6\gamma}\right)}{\Gamma\left(\frac{\alpha(\beta-6)}{3} + 1 - \frac{n\alpha}{3} - \frac{m\alpha(\beta+3)}{6\gamma}\right)} a_{n,m} \right. \\ &\quad \left. - A_1 z_1^{-\beta} \sum_{k=0}^n \sum_{j=0}^m \frac{\Gamma(1+k)}{\Gamma(1+k-\beta)} a_{n-k,m-j} a_{k,j} - A_2 z_2^{-\gamma} \frac{\Gamma(1+m)}{\Gamma(1+m-\gamma)} b_{n,m} \right) z_1^n z_2^m, \\ g(z_1, z_2) &= \sum_{n,m=0}^{\infty} A_4 z_1^{\beta} z_2^{-\gamma} \frac{\Gamma(1+n-\beta)\Gamma(1+m)}{\Gamma(1+n)\Gamma(1+m-\gamma)} a_{n,m} z_1^n z_2^m.\end{aligned}\quad (32)$$

Therefore, the required exact solution of the reduced form (26) is

$$\begin{aligned}
 u(x, t) = & t^{\frac{\alpha(\beta-3)}{3}} a_{0,0} + x t^{\frac{\alpha(\beta-4)}{3}} a_{1,0} + x^2 t^{\frac{\alpha(\beta-5)}{3}} a_{2,0} + \frac{1}{6A_3} \left(\frac{\Gamma(\frac{\alpha(\beta-3)}{3} + 1)}{\Gamma(\frac{\alpha(\beta-6)}{3} + 1)} a_{0,0} - A_1 x^{-\beta} t^{\frac{\alpha\beta}{3}} \frac{1}{\Gamma(1-\beta)} a_{0,0}^2 \right. \\
 & \left. - A_2 y^{-\gamma} t^{\frac{\alpha(\beta+3)}{6}} \frac{1}{\Gamma(1-\gamma)} b_{0,0} \right) x^3 t^{\frac{\alpha(\beta-6)}{3}} + \sum_{n=1, m=0}^{\infty} \frac{1}{A_3(n+3)(n+2)(n+1)} \\
 & \times \left(\frac{\Gamma(\frac{\alpha(\beta-3)}{3} + 1 - \frac{n\alpha}{3} - \frac{m\alpha(\beta+3)}{6\gamma})}{\Gamma(\frac{\alpha(\beta-6)}{3} + 1 - \frac{n\alpha}{3} - \frac{m\alpha(\beta+3)}{6\gamma})} a_{n,m} - A_1 x^{-\beta} t^{\frac{\alpha\beta}{3}} \sum_{k=0}^n \sum_{j=0}^m \frac{\Gamma(1+k)}{\Gamma(1+k-\beta)} a_{n-k, m-j} a_{k,j} \right. \\
 & \left. - A_2 y^{-\gamma} t^{\frac{\alpha(\beta+3)}{6}} \frac{\Gamma(1+m)}{\Gamma(1+m-\gamma)} b_{n,m} \right) x^n y^m t^{\left(\frac{\alpha(\beta-3)}{3} - \frac{n\alpha}{3} - \frac{m\alpha(\beta+3)}{6\gamma}\right)}, \quad (33)
 \end{aligned}$$

$$w(x, t) = \sum_{n, m=0}^{\infty} A_4 \frac{\Gamma(1+n-\beta)\Gamma(1+m)}{\Gamma(1+n)\Gamma(1+m-\gamma)} a_{n,m} x^{n+\beta} y^{m-\gamma} t^{\left(\frac{\alpha(\beta-3)}{3} - \frac{n\alpha}{3} - \frac{m\alpha(\beta+3)}{6\gamma}\right)}. \quad (34)$$

5. Analysis of the Convergence

In this section, we will analyze the convergence of the power series solution (33) and (34).

Theorem 2. The power series of the solutions (33) and (34) converges.

Proof. From (31), we have

$$|a_{n+3, m}| \leq M \left\{ |a_{n, m}| + \sum_{k=0}^n \sum_{j=0}^m |a_{n-k, m-j}| |a_{k, j}| + |b_{n, m}| \right\}, \quad (35)$$

$$\text{where } M = \max \left(\frac{1}{A_3} \left\{ \frac{\Gamma(\frac{\alpha(\beta-3)}{3} + 1 - \frac{n\alpha}{3} - \frac{m\alpha(\beta+3)}{6\gamma})}{\Gamma(\frac{\alpha(\beta-6)}{3} + 1 - \frac{n\alpha}{3} - \frac{m\alpha(\beta+3)}{6\gamma})} \right\}, \sum_{k=0}^n \frac{\Gamma(1+k)}{\Gamma(1+k-\beta)} \left(\frac{A_1 z_1^{-\beta}}{A_3}, \frac{A_2 z_2^{-\gamma}}{A_3} \right) \right).$$

and

$$|b_{n, m}| \leq N(|a_{n, m}|), \quad (36)$$

$$\text{where } N = \max \left(1, A_4 z_1^{\beta} z_2^{-\gamma} \frac{\Gamma(1+n-\beta)\Gamma(1+m)}{\Gamma(1+n)\Gamma(1+m-\gamma)} \right).$$

Let us consider two double power series

$$\begin{aligned}
 P &= P(z_1, z_2) = \sum_{n, m=0}^{\infty} p_{n, m} z_1^n z_2^m, \\
 R &= R(z_1, z_2) = \sum_{n, m=0}^{\infty} r_{n, m} z_1^n z_2^m, \quad (37)
 \end{aligned}$$

by

$$p_{n, m} = |a_{n, m}|, \quad r_{i, j} = |b_{i, j}|, \quad n = 0, 1, 2, \quad m = 0, \quad i, j = 0, \quad (38)$$

and

$$\begin{aligned}
 p_{n+3, m} &= M \left(p_{n, m} + \sum_{k=0}^n \sum_{j=0}^m p_{n-k, m-j} p_{k, j} + r_{n, m} \right), \\
 r_{n, m} &= N(p_{n, m}), \quad (39)
 \end{aligned}$$

where $n = 0, 1, 2, 3, \dots$. Therefore, one can easily check that

$$|a_{n, m}| \leq p_{n, m}, \quad |b_{n, m}| \leq r_{n, m}, \quad n = 0, 1, 2, \dots \quad (40)$$

Therefore, the series

$$P = P(z_1, z_2) = \sum_{n,m=0}^{\infty} p_{n,m} z_1^n z_2^m$$

and

$$R = R(z_1, z_2) = \sum_{n,m=0}^{\infty} r_{n,m} z_1^n z_2^m$$

are the majorant series of the series $f(z_1, z_2)$ and $g(z_1, z_2)$, respectively. $\square$

Let us consider one particular case,

$$A_i = \sum_{m=0}^{\infty} p_{i,m} z_1^i z_2^m, \quad i = 0, 1, 2.$$

Next, we investigate the convergence of the series $P = P(z_1, z_2)$ and $R = R(z_1, z_2)$.

$$\begin{aligned} P &= \sum_{m=0}^{\infty} \left\{ p_{0,m} + p_{1,m} z_1 + p_{2,m} z_1^2 + p_{3,m} z_1^3 + \sum_{n=1}^{\infty} p_{n+3,m} z_1^{n+3} \right\} z_2^m \\ &= \sum_{m=0}^{\infty} p_{0,m} z_2^m + \sum_{m=0}^{\infty} p_{1,m} z_1 z_2^m + \sum_{m=0}^{\infty} p_{2,m} z_1^2 z_2^m + \sum_{m=0}^{\infty} p_{3,m} z_1^3 z_2^m \\ &\quad + \sum_{n=1, m=0}^{\infty} M \left(p_{n,m} + \sum_{k=0}^n \sum_{j=0}^m p_{n-k, m-j} p_{k,j} + r_{n,m} \right) z_1^{n+3} z_2^m \end{aligned} \quad (41)$$

$$= A_0 + A_1 z_1 + A_2 z_1^2 + A_3 z_1^3 + M z_1^3 \left[(P - A_0) + (P^2 - A_0^2) + N(P - A_0) \right], \quad (42)$$

and

$$\begin{aligned} R &= \sum_{n,m=0}^{\infty} r_{n,m} z_1^n z_2^m = \sum_{n,m=0}^{\infty} N p_{n,m} z_1^n z_2^m \\ &= N \left[A_0 + A_1 z_1 + A_2 z_1^2 + A_3 z_1^3 + M z_1^3 \left((P - A_0) + (P^2 - A_0^2) + N(P - A_0) \right) \right]. \end{aligned} \quad (43)$$

Consider the implicit functional system as follows

$$\begin{aligned} F(z_1, z_2, P, R) &= P - A_0 - A_1 z_1 - A_2 z_1^2 - A_3 z_1^3 - M z_1^3 \left((P - A_0) + (P^2 - A_0^2) + N(P - A_0) \right), \\ H(z_1, z_2, P, R) &= R - N \left[A_0 + A_1 z_1 + A_2 z_1^2 + A_3 z_1^3 + M z_1^3 \left((P - A_0) + (P^2 - A_0^2) + N(P - A_0) \right) \right]. \end{aligned}$$

F, H are analytics in the neighbourhood of $(0, 0, A_0, N A_0)$. $F(0, 0, A_0, N A_0) = 0$, $G(0, 0, A_0, N A_0) = 0$, and the Jacobian determinant is

$$\left. \frac{\partial(F, H)}{\partial(P, R)} \right|_{(0,0,A_0,NA_0)} = 1 \neq 0.$$

Then, by the implicit function theorem [41], both power series are convergent. Hence, an exact solution of KP system (2) exists.

6. Conservation Laws

In this section, conservation laws of (2) will be constructed by using the new conservation theorem and the nonlinear self adjointness [27,29].

The conservation laws for (2) are introduced as

$$D_t(C^t) + D_x(C^x) + D_y(C^y) = 0, \quad (44)$$

where $C^t(x, y, t, u, w)$, $C^x(x, y, t, u, w)$ and $C^y(x, y, t, u, w)$ are conserved vectors of (2).

The Euler–Lagrange operators given by

$$\begin{aligned} \frac{\delta}{\delta u^j} = & \frac{\partial}{\partial u^j} + (D_t^\alpha)^* \frac{\partial}{\partial (D_t^\alpha u^j)} + (D_x^\beta)^* \frac{\partial}{\partial (D_x^\beta u^j)} + (D_y^\gamma)^* \frac{\partial}{\partial (D_y^\gamma u^j)} \\ & + \sum_{k=1}^{\infty} (-1)^k D_{i_1} D_{i_2} \dots D_{i_k} \frac{\partial}{\partial (u^j)_{i_1, i_2, \dots, i_k}}, \end{aligned} \quad (45)$$

where D_{i_k} represents the total derivative operator. $(D_t^\alpha)^*$, $(D_x^\beta)^*$ and $(D_y^\gamma)^*$ are also the adjoint operators of the RL derivative operators [36,39] D_t^γ and D_x^β , respectively, given as follows

$$\begin{aligned} (D_t^\alpha)^* &= (-1)^n I_p^{n-\alpha} (D_t^n) = {}^C D_p^\alpha, \\ (D_x^\beta)^* &= (-1)^m I_q^{m-\beta} (D_x^m) = {}^C D_q^\beta, \\ (D_y^\gamma)^* &= (-1)^k I_r^{k-\gamma} (D_y^r) = {}^C D_r^\gamma, \end{aligned} \quad (46)$$

where $I_p^{n-\alpha}$, $I_q^{m-\beta}$ and $I_r^{k-\gamma}$ are the right-hand-side fractional integral operators of order $n - \alpha$, $m - \beta$ and $k - \gamma$, respectively, defined as follows

$$I_p^{n-\alpha} f(x, t) = \frac{1}{\Gamma(n-\alpha)} \int_t^p \frac{f(x, y, s)}{(s-t)^{1+\alpha-n}} ds, \quad (47)$$

where $n = [\alpha] + 1$

$$I_q^{m-\beta} f(x, t) = \frac{1}{\Gamma(m-\beta)} \int_x^q \frac{f(s, y, t)}{(s-x)^{1+\beta-m}} ds, \quad (48)$$

where $m = [\beta] + 1$

$$I_r^{k-\gamma} f(x, t) = \frac{1}{\Gamma(k-\gamma)} \int_y^r \frac{f(x, s, t)}{(s-y)^{1+\gamma-r}} ds, \quad (49)$$

where $k = [\gamma] + 1$

The formal Lagrangian of the system (2) is given by

$$\mathcal{L} = T(\partial_t^\alpha u - A_1 u \partial_x^\beta u - A_2 \partial_y^\gamma w - A_3 u_{xxx}) + Q(\partial_x^\beta w - A_4 \partial_y^\gamma u), \quad (50)$$

where T and Q are new dependent variables.

The adjoint equations are defined by

$$F_j^* \equiv \frac{\delta \mathcal{L}}{\delta u^j} = 0, \quad j = 1, 2. \quad (51)$$

From (50) and (51), the adjoint equations are

$$\begin{aligned} \frac{\delta \mathcal{L}}{\delta u} = F_1^* &= (D_t^\alpha)^* T - A_1 u (D_x^\beta)^* T - A_4 (D_y^\gamma)^* Q + A_3 D_x^3 T = 0, \\ \frac{\delta \mathcal{L}}{\delta w} = F_2^* &= (D_x^\beta)^* Q - A_2 (D_y^\gamma)^* T = 0. \end{aligned} \quad (52)$$

If, by substituting the values

$$T = \varphi(x, y, t, u, w), \quad Q = \psi(x, y, t, u, w), \quad (53)$$

Equation (52) satisfies, with at least one of T, Q variable being non-zero, the system (2) is called the nonlinear self adjoint. Now, the derivative(s) of $T = \varphi(x, y, t, u, w)$ with respect to x , are

$$\begin{aligned} T_x &= \varphi_x + \varphi_u u_x + \varphi_w w_x, \\ T_{xx} &= \varphi_{xx} + 2\varphi_{xu} u_x + 2\varphi_{xw} w_x + \varphi_{uu} u_x^2 + 2\varphi_{uw} u_x w_x + \varphi_{ww} w_x^2 + \varphi_{uu} u_{xx} + \varphi_{uw} u_{xw} + \varphi_{ww} w_{xx}, \\ T_{xxx} &= \varphi_{xxx} + 6\varphi_{xuw} u_x w_x + 3\varphi_{uuw} u_x^2 w_x + 3\varphi_{uu} u_x w_{xx} + 3\varphi_{uww} u_x w_x^2 + 3\varphi_{xu} u_{xx} \\ &\quad + 3\varphi_{uw} (u_x w_{xx} + w_x u_{xx}) + 3\varphi_{ww} w_x w_{xx} + 3\varphi_{xxw} w_x + 3\varphi_{xwu} u_x + 3\varphi_{xw} w_{xx} \\ &\quad + \varphi_{uu} u_{xxx} + \varphi_{uw} w_{xxx} + 3\varphi_{xuu} u_x^2 + 3\varphi_{xw} w_x^2 + \varphi_{uuu} u_x^3 + \varphi_{uww} w_x^3. \end{aligned} \quad (54)$$

Thus, the nonlinear self adjointness conditions are

$$\begin{aligned} \frac{\delta \mathcal{L}}{\delta u} &= \lambda_1 (\partial_t^\alpha u - A_1 u \partial_x^\beta u - A_2 \partial_y^\gamma w - A_3 u_{xxx}) + \lambda_2 (\partial_x^\beta w - A_4 \partial_y^\gamma u) \\ \frac{\delta \mathcal{L}}{\delta w} &= \lambda_3 (\partial_t^\alpha u - A_1 u \partial_x^\beta u - A_2 \partial_y^\gamma w - A_3 u_{xxx}) + \lambda_4 (\partial_x^\beta w - A_4 \partial_y^\gamma u), \end{aligned} \quad (55)$$

where λ_i ($i = 1, 2, 3, 4$) are to be determined.

Therefore, we have

$$\begin{aligned} &(D_t^\alpha)^* \varphi + -A_1 u (D_x^\beta)^* \varphi - A_4 (D_y^\gamma)^* \psi + A_3 \left(\varphi_{xxx} + 6\varphi_{xuw} u_x w_x + 3\varphi_{uuw} u_x^2 w_x + 3\varphi_{uu} u_x w_{xx} \right. \\ &\quad + 3\varphi_{ww} w_x w_{xx} + 3\varphi_{uw} (u_x w_{xx} + w_x u_{xx}) + 3\varphi_{xxw} w_x + 3\varphi_{xwu} u_x + 3\varphi_{xw} w_{xx} + \varphi_{uu} u_{xxx} + 3\varphi_{xuu} u_x^2 \\ &\quad \left. + 3\varphi_{xw} w_x^2 + \varphi_{uw} w_{xxx} + 3\varphi_{xu} u_{xx} + 3\varphi_{uww} u_x w_x^2 + \varphi_{uuu} u_x^3 + \varphi_{uww} w_x^3 \right) \\ &= \lambda_1 (\partial_t^\alpha u - A_1 u \partial_x^\beta u - A_2 \partial_y^\gamma w - A_3 u_{xxx}) + \lambda_2 (\partial_x^\beta w - A_4 \partial_y^\gamma u), \\ &(D_x^\beta)^* \psi - A_2 (D_y^\gamma)^* \varphi \\ &= \lambda_3 (\partial_t^\alpha u - A_1 u \partial_x^\beta u - A_2 \partial_y^\gamma w - A_3 u_{xxx}) + \lambda_4 (\partial_x^\beta w - A_4 \partial_y^\gamma u). \end{aligned} \quad (56)$$

Collecting the coefficients of various powers of u, w and their derivatives on both sides of (56), and solving them simultaneously, we have $\lambda_i = 0$, $i = 1, 2, 3, 4$, and

$$\varphi = a(t, y)x^2 + b(t, y)x + c(t, y), \quad \psi = \psi(x, y, t, u, w), \quad (57)$$

where $a(t, y)$, $b(t, y)$ and $c(t, y)$ are functions of t, y .

Corresponding with symmetry generators, the characteristic functions W^1 and W^2 , are defined by

$$W^1 = \eta - \xi u_x - \mu u_y - \tau u_t, \quad W^2 = \phi - \xi w_x - \mu w_y - \tau w_t. \quad (58)$$

The fractional Noether's operator [29] is defined as

$$C^t = \sum_{j=1}^2 \left[\sum_{k=0}^{m-1} (-1)^k D_t^{\alpha-1-k} (W^j) D_t^k \left(\frac{\partial \mathcal{L}}{\partial (D_t^\alpha u_j)} \right) - (-1)^m \mathcal{J}_1 \left(W^j, D_t^m \left(\frac{\partial \mathcal{L}}{\partial (D_t^\alpha u_j)} \right) \right) \right], \quad (59)$$

where $m = [\alpha] + 1$, and W^j , ($j = 1, 2$) are defined in (58) and u_1, u_2 are dependent variables. Additionally, $\mathcal{J}_1(h_1, h_2)$ is the integral

$$\mathcal{J}_1(h_1, h_2) = \frac{1}{\Gamma(m - \alpha)} \int_0^t \int_t^q \frac{h_1(x, y, s) h_2(x, y, r)}{(r - s)^\alpha + 1 - m} dr ds,$$

for any two functions $h_1(x, y, t)$ and $h_2(x, y, t)$.

In a similar way, other fractional Noether's operators C^x and C^y are defined.

By using (58) and vector field (18), the characteristic functions are

$$W^1 = (\beta - 3)u - xu_x - \frac{3 + \beta}{2\gamma} yu_y - \frac{3}{\alpha} t, \quad W^2 = \frac{3(\beta - 3)}{2} w - xu_x - \frac{3 + \beta}{2\gamma} yu_y - \frac{3}{\alpha} t. \quad (60)$$

Now, we will obtain the conserved vectors of the system (2) as follows.

Case 1. For $0 < \alpha < 1$, we have

$$C^t = I_t^{1-\alpha}(W^1)\varphi + \mathcal{J}_1(W^1, \varphi_t).$$

Case 2. For $1 < \alpha < 2$, we have

$$C^t = D_t^{\alpha-1}(W^1)\varphi - I_t^{2-\alpha}(W^1)\varphi + \mathcal{J}_1(W^1, \varphi_{tt}).$$

Case 3. Similarly, for $0 < \beta < 1$, we have

$$C^x = -I_x^{1-\beta}(W^1)(A_1 u \varphi) + I_x^{1-\beta}(W^2)(\varphi) + \mathcal{J}_2(W^1, D_x(-A_1 u \varphi)) + \mathcal{J}_2(W^2, D_x(\varphi)).$$

Case 4. For $1 < \beta < 2$, we have

$$\begin{aligned} C^x = & -D_x^{\beta-1}(W^1)(A_1 u \varphi) + D_x^{\beta-1}(W^2)(\varphi) + I_x^{2-\beta}(W^1)D_x(A_1 u \varphi) - I_x^{2-\beta}(W^2)D_x(\varphi) \\ & - \mathcal{J}_2(W^1, D_x^2(-A_1 u \varphi)) - \mathcal{J}_2(W^2, D_x^2(\varphi)). \end{aligned}$$

Case 5. For $0 < \gamma < 1$, we have

$$C^y = -I_y^{1-\beta}(W^1)(A_4 \varphi) - I_y^{1-\beta}(W^2)(A_2 \varphi) + \mathcal{J}_3(W^1, D_x(-A_4 \varphi)) + \mathcal{J}_3(W^2, D_x(-A_2 \varphi)).$$

Case 6. For $1 < \gamma < 2$, we have

$$\begin{aligned} C^y = & -D_x^{\beta-1}(W^1)(A_4 \varphi) - D_x^{\beta-1}(W^2)(A_2 \varphi) + I_x^{2-\beta}(W^1)D_x(A_4 \varphi) + I_x^{2-\beta}(W^2)D_x(A_2 \varphi) \\ & - \mathcal{J}_3(W^1, D_x^2(-A_4 \varphi)) - \mathcal{J}_3(W^2, D_x^2(-A_2 \varphi)). \end{aligned}$$

7. Concluding Remarks

In this work, we have studied a $(2 + 1)$ -dimensional fractional Kadomtsev-Petviashvili system (2) by Lie symmetry analysis and power series expansion techniques, via. an RL fractional derivative. First, we obtained the Lie point symmetries, and then the similarity transformations were successfully presented. Using the similarity transformations, we were able to reduce the system of NLFPDEs (2) of three dimensions into a system of NLFPDEs of two dimensions. Further, the explicit exact solution for the reduced NLFPDEs was obtained using the power series expansion method. The analysis of convergence for the power series solution was also performed. Using the new conservation theorem [27], the conservation laws of the system are successfully obtained. The obtained solutions might be of substantial consequence in the corresponding physical phenomena of science and applied mathematics.

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