

Article

Historical Introduction to Chiral Quark Models

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Abstract: Chiral symmetry, and its dynamical breaking, has become a cornerstone in the description of the hadron's phenomenology at low energy. The present manuscript gives a historical survey on how the quark model of hadrons has been implemented along the last decades trying to incorporate, among other important non-perturbative features of quantum chromodynamics (QCD), the dynamical chiral symmetry breaking mechanism. This effort has delivered different models such as the chiral bag model, the cloudy bag model, the chiral quark model or the chiral constituent quark model. Our main aim herein is to provide a brief introduction of the Special Issue "Advances in Chiral Quark Models" in *Symmetry* and contribute to the clarification of the differences among the above-mentioned models that include the adjective chiral in their nomenclature.

Keywords: quantum chromodynamics; hadron physics; quark models



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Shortly after the notion of quarks was introduced by Gell-Man and Zweig [1,2] several models were developed to describe the properties of hadrons in terms of these new degrees of freedom. Greenberg [3] was the first who tried to understand the structure of baryons in a kind of potential model. To avoid evident issues with the Pauli principle, Greenberg introduced the hypothesis that quarks are parafermions of order three, which later became the $SU(3)_C$ color degree of freedom [4,5].

The strong interaction was better understood after the introduction of quantum chromodynamics (QCD), which is a Poincaré-invariant quantum non-Abelian gauge field theory based on quark and gluon degrees of freedom [6–8]. That progress was mostly due to the fact that quarks, and gluons, interact weakly at high energies (asymptotic freedom); thus, the hadronic processes can be described perturbatively. At the QCD's low-energy regime, where hadrons live, the quark and gluon interactions become strong and lead to many non-perturbative phenomena, such as dynamical chiral symmetry breaking and confinement, which make the description of hadron properties in terms of quarks and gluons degrees of freedom very complicated. This is the reason why effective models of QCD at low energies were developed over time that incorporated as many QCD properties as their formulation allowed.

De Rújula et al. [9] and Eichten et al. [10] were the first to develop potential quark models, which imitated two of the most important properties of QCD, namely asymptotic freedom and confinement. Both potential quark models incorporate an effective quark mass with a value around one-third of the nucleon mass. This was already suggested by Greenberg [3] to explain the nucleon magnetic moments, but it was also discussed by Politzer [11] within the framework of a field theoretical description of inclusive lepton-hadron scattering. The idea of the potential quark model was then further developed in its non-relativistic and semirelativistic versions by Isgur and Karl in Refs. [12–14].

At the same time, researches from the Massachusetts Institute of Technology (MIT) pursued a completely different approach, more quantum field theory based, to the same problem: the bag model [15]. Within this formalism, massless quarks are confined in a

spherical region with radius R and satisfy the Dirac equation inside it. However, the sharp bag's boundary alone is not enough to confine the quarks because the quark energy decreases as the cavity radius increases. To introduce confinement, it is necessary to include a repulsive energy density, B , which stabilizes the bag. Then, the model has two adjustable parameters R and B :

$$\mathcal{L} = (\bar{\psi} i \partial_\mu \gamma^\mu \psi - B) \vartheta_V - \frac{1}{2} \bar{\psi} \psi \delta_V, \quad (1)$$

where ϑ_V is the step function which is equal to one inside the bag and zero otherwise, and it ensures quark confinement. The surface delta function $\delta_V = \delta(r - R)$ guarantees that the vector current vanishes at the bag surface, and thus no baryon number leakage exists from inside to outside the bag.

Both branches of modeling triggered a huge number of research works, not only describing the hadron spectra [14,16,17] but also the short range of the nucleon–nucleon interaction [18–21]. Reviews of these two kinds of models can be found in, for instance, References [22,23]. Their theoretical output is still used by experimentalists as a template to assign quantum numbers to the new discovered resonances, considering as exotic states those that do not fit in the model predictions. Moreover, such exotic phenomena are always under discussion by the scientific community and lead to new quark model based development [24]. However, it is important to note that despite the success of both the potential quark model and the bag model, an important QCD's property, chiral symmetry and its dynamical breaking, was still missing in their original versions.

The QCD lagrangian for N_f massless quarks posses a symmetry under $U(N_f) \times U(N_f)$ independent rotations of the left- and right-handed quark fields. This symmetry group factorizes according to $SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$. The $U(1)_V$ is responsible for the baryon number conservation, whereas the $U(1)_A$ cannot be realized in a quantum theory, and it is known as the axial anomaly. The remaining $SU(N_f)_L \times SU(N_f)_R$ is the chiral symmetry. If this symmetry was exact, all hadron states with same quantum numbers but opposite parity would be degenerate. However, in nature the mass splittings between the parity partners are big and difficult to explain due to the fact of having current quark masses in the QCD Lagrangian. For example, the mass splitting between the ρ meson and its chiral partner, the a_1 meson, is about 300 MeV. In the baryon sector, the mass splitting between the nucleon and its parity partner is even larger, namely about 600 MeV. The conclusion that one can draw is that chiral symmetry is dynamically broken in QCD. It is also important to notice herein that there are two generic consequences of the dynamical chiral symmetry breaking: (i) the appearance of an octet of pseudo-Goldstone bosons, and (ii) the acquisition by the valence quarks of a constituent (dynamical) mass.

The importance of the chiral symmetry, and its dynamical breaking pattern, was already emphasized by Skyrme [25]. Preliminary investigations on how to incorporate them were performed by, for instance, Gross and Neveu [26] using two-dimensional model field theories, involving massless fermions with quartic interactions, and they were expanded in powers of $1/N$, where N is the number of components of the fermion field (see also [27,28], and references therein, for recent developments on this formalism). At the same time, the bag model was developed in Reference [29] to deal with the dynamical breaking of chiral symmetry, and its related features. In such an article the authors point out that the conservation of axial current, a consequence of chiral symmetry, is violated on the surface of the bag because the quarks change their momentum but not the spin on the surface. To solve this problem, Chodos and Thorn [29] take into account outside the bag a $SU(2) \times SU(2)$ multiplet, $\{\sigma, \pi\}$, that is coupled to the degrees of quarks on the bag surface:

$$\mathcal{L} = (\bar{\psi} i \partial_\mu \gamma^\mu \psi - B) \vartheta_V - \frac{\lambda}{2} \bar{\psi} (\sigma + i\tau \cdot \pi \gamma_5) \psi \delta_V - \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi), \quad (2)$$

where λ is a Lagrange multiplier. These authors attempted to find exact classical solutions of the pion field; the only feasible way was a highly idealized baryon called the hedgehog ansatz for spherically symmetric systems, which reduces the field equations to single scalar equations. Within this educated guess, an S -wave spinor with mixed spin-isospin structure describes a quark, while the pion field at a point r is proportional to the radial unit vector. Therefore, the pion fields point radially outwards, like the spines of an afraid hedgehog, whose configuration provides the lowest-energy, self-consistent solution of the equation of motion.

In the bag interior the axial vector current is carried by the massless quarks, whereas outside the bag the pion field is introduced as to ensure the continuity of this current. The existence of the pion field is important not only to guarantee the axial vector current conservation but also in the possible description of nuclear forces from these models.

In the original description of the MIT chiral bag, the effect of the pion field is assumed to be small and then treated perturbatively. However, when nonlinear effects are summed up to all orders in the pion field the pressure exerted by the pion cloud squeezes the chiral bag to a little bag [30], which has a radius considerably smaller than the prediction of the MIT bag. Moreover, one of the major triumphs of the MIT bag model was the correct prediction of the axial charge of the nucleon g_a . Once the pion field was introduced, additional contributions to g_a destroy the former agreement, with the situation becoming worse as the bag radius decreases.

The question of the size of a baryon, in particular the nucleon, is relevant for several reasons. On one hand, the average distance between nucleons in nuclear matter is 1.8 fm, and thus a bag radius of about 1.1 fm is large enough to consider overlap between bags in the nuclear medium, and that explicit quark degrees of freedom must be relevant for the description of nuclear phenomena. On the other hand, considering the nucleons as very small, almost point-like, objects leads essentially to the conventional nuclear system.

To avoid the issues found with the implementation of the pion field outside the bag and its coupling with the quark field on the surface, Thomas, Theberg and Miller developed the so-called cloudy bag model. These authors argue that there is no reason to exclude pions from the bag's interior, and that such a simplification is not only unreasonable but may be wrong. They succeed to stabilize the nucleon bag without messing up the agreement with the experimental data. A complete and clear review of the cloudy bag model can be found in Thomas [31] (see also Miller [32]).

Despite their success in describing the hadron spectra, the static bag models are necessarily limited in scope because of their non-covariant formulation of the bag surface. Moreover, the static bag surface hinders the treatment of the center-of-mass, which affects the description of the hadron observables. To improve it, several Lagrangian models with explicit quark degrees of freedom appeared. Examples of them are the Friedberg–Lee model [33], the Nambu–Jona-Lasinio model [34] and the chiral quark models proposed by Diakonov and Petrov [35], as well as by Manohar and Georgi [36]. The motivation for all these models resides in the idea that QCD produces very strong forces between quarks and antiquarks at low energy, and this results in quark–antiquark condensates in the vacuum.

A novel way to confine quarks, which avoids the difficulties found with the static bag surface, is provided by the soliton bag model (the concept of soliton will be used in a broad sense, referring to any static or non-dispersive solution of a set of non-linear equations). The idea behind this model is to look for non-topological soliton solutions of a Dirac Hamiltonian based on the Lagrangian proposed by Friedberg and Lee, which includes a scalar field χ . This field is a phenomenological representation of the self-interacting gluon field. The energy of the system $U(\chi)$ as a function of χ has two minima. The value of the χ -field coincides with the vacuum one in the absence of quarks; when quarks are introduced, the (expectation) value of the χ -field moves to the second minimum. In the last situation quarks become confined and, therefore, confinement is given dynamically by the scalar field rather than imposed as a boundary condition at the bag surface.

Goldflam and Wilets developed from the ideas above the soliton bag model [37]. Although the model does not incorporate chiral dynamics, the proposed quark confining mechanism is more suitable to include such a property of QCD. However, their model had no mechanism to split the N and Δ masses, and it also had difficulties to reproduce the nucleon's magnetic moment and axial coupling.

Attempts to include chirality, and its dynamical breaking, in the soliton bag model were done by Birse and Banerjee [38], Kahana and Ripka [39], and Seki, Ohta and Shigemitsu [40,41]. The last authors developed a chiral soliton bag model based on the soliton solution of a Lagrangian, which combines the soliton bag model's idea and the old non-linear σ model from Gell-Mann–Levy [42]:

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi + \frac{1}{2} (\partial_\mu \chi)^2 - U(\chi) - \bar{\psi} \chi (\sigma + i\pi \cdot \tau \gamma_5) \psi + \frac{1}{2} [(\partial_\mu \sigma)^2 + (\partial_\mu \pi)^2], \quad (3)$$

with the condition

$$\sigma^2 + \pi^2 = f_\pi^2. \quad (4)$$

The equations obtained from the Lagrangian above have two soliton solutions with winding numbers $Z = 0$ and $Z = 1$. The solution with $Z = 0$ coincides basically with the solution of the cloudy bag model, while the $Z = 1$ turns out to be a chiral soliton solution.

Both Birse–Banerjee [38] and Kahana–Ripka [39] solve the mean field equations for a hedgehog baryon and calculate the observables corresponding to N and Δ baryons. In contrast with the Seki–Ohta–Shigemitsu's work, the Birse–Banerjee and Kahana–Ripka Lagrangians do not include a χ -field, and thus the chiral quartet must be responsible of both chiral invariance and quark confinement, although there was no evidence that confinement and chiral symmetry breaking have the same origin.

Continuing with the idea of describing baryons as solitons of a certain Lagrangian, one of the most successful models was the one proposed by Nambu and Jona-Lasinio [34,43]. The NJL model is a quark model with a chiral invariant quartic quark interaction:

$$\mathcal{L} = \bar{\psi} (i \not{\partial} - m_0) \psi + \frac{G}{2} [(\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \tau \psi)^2], \quad (5)$$

where ψ is the quark field and m_0 the average u - and d -quark current mass. The local four-fermion interaction with a dimension full coupling constant is not renormalizable and lacks confinement. The non-renormalizability can be avoided with a suitable ultraviolet cut-off. Usually, the model is semi-bosonized by introducing two auxiliary fields, σ and π [44]:

$$\mathcal{L} = \bar{\psi} (i \not{\partial} - \sigma - i\pi \cdot \tau \gamma_5) \psi + \frac{1}{2G} [(\sigma)^2 + (\pi)^2] + \frac{m_0}{G} \sigma. \quad (6)$$

The mean field only exists if the classical meson fields are subject to the condition

$$\sigma^2 + \pi^2 = M^2, \quad (7)$$

where M is the constituent quark mass, generated by dynamical chiral symmetry breaking. Imposing this condition, the Nambu–Jona-Lasinio's Lagrangian reduces to an effective one (note here that from now on we keep only the contributions bilinear in the fermion):

$$\mathcal{L} = \bar{\psi} (i \not{\partial} - m_0 - MU^{\gamma_5}) \psi, \quad (8)$$

where $U^{\gamma_5} = e^{i\lambda_a \phi^a \gamma_5 / f_\pi}$ is the matrix of Goldstone-boson fields.

At this level, the NJL Lagrangian describes a theory of quarks which interact between themselves through boson fields. One can, however, integrate out the quark degrees of freedom (bosonization) and arrives to an effective meson theory in which baryons appear as soliton solutions of the meson fields. This model is usually referred as either non-linear NJL model or chiral quark soliton model (this last notation when applied to baryons).

The simplicity of the NJL model allows to describe a wide variety of baryon observables such as mass splittings, magnetic moments, mass and electromagnetic radii, as well as Dirac and Pauli, axial, induced-pseudoscalar and pion-nucleon form factors. A thorough review of the achievements made by the different versions of the NJL model can be found in References [45–47] and references therein.

Among various attempts to derive the NJL model as some low-energy limit of QCD, we shall refer to the quark-soliton model based on the instanton liquid model developed by Diakonov and Petrov [35].

Instantons are gluon-field fluctuations of a non-perturbative nature, and Shuryak [48] demonstrated that the QCD vacuum can be described as an instanton medium relatively dilute, with average distances $\bar{R} \approx (200 \text{ MeV})^{-1}$ and average size $\bar{\rho} \approx (600 \text{ MeV})^{-1}$. Using these ideas, Diakonov and Petrov [35] studied the instanton vacuum by means of a variational methods using, as a trial ansatz, a superposition of instantons and anti-instantons. They show that the picture of QCD vacuum as a dilute medium of instantons nicely explains the dynamical braking of chiral symmetry. Moreover, the quark propagator in the instanton vacuum acquires the form of a massive propagator

$$S(p) = \frac{\not{p} + iM(p^2)}{p^2 + M^2(p^2)}, \quad (9)$$

where $M(p)$ is a momentum-dependent mass, usually called dynamical or constituent quark mass. The momentum-dependent point-wise behavior of $M(p)$ is the key to understand why the notion of constituent quark has worked so well over the last 30 years in hadron physics [49]. Once the dynamical or constituent quark mass is introduced, a simple Lagrangian invariant under chiral transformation can be formulated as

$$\mathcal{L} = \bar{\psi}(i \not{\partial} - M(p)U^{\gamma_5})\psi, \quad (10)$$

where $U^{\gamma_5} = \exp(i\pi^a \lambda^a \gamma_5 / f_\pi)$ is again the matrix of Goldstone boson fields.

The effective Lagrangian induced by instantons phenomenology is not local, i.e., the constituent quark mass depends on the quark's momentum in contrast to the NJL model. However, one can assume that the dynamical quark mass $M(p)$ is a constant at momenta $p \ll 1/\bar{\rho}$, but it vanishes when the momenta $p \gg 1/\bar{\rho}$. Interesting too, such momentum-dependent quark mass makes the instanton liquid model be ultraviolet finite.

As in the NJL model, the chiral Lagrangian introduced by Diakonov and Petrov is bilinear in the quark fields. They can be integrated out to obtain an effective meson action. The soliton solution of this action connects the idea of having pseudo massive particles as quarks and the nucleon bound state made of them, giving place to the chiral quark soliton model of the nucleon. Within this approach, excited baryons appear as excitations of the static classical solution of the nucleon when quantizing the slow rotations of the soliton field [50,51]. In fact, taking the hedgehog ansatz, the baryon octet and decuplet are obtained.

Besides the chiral quark-soliton model, others can be derived from the Diakonov Lagrangian. An example is the chiral constituent quark model [52,53] (see also [54]), which can be derived from the Diakonov Hamiltonian by expanding the Goldstone-boson fields as

$$U^{\gamma_5} = 1 + \frac{i}{f_\pi} \gamma_5 \lambda^a \pi^a - \frac{1}{2f_\pi^2} \pi^a \pi^a + \dots \quad (11)$$

Moreover, although an expression of the dynamical constituent quark mass can be obtained from Diakonov's effective theory [49], it can be parametrized as

$$M(p) = m_q F(p^2), \text{ with } F(p^2) = \left[\frac{\Lambda^2}{\Lambda^2 + p^2} \right]^{1/2}, \quad (12)$$

where the parameter Λ is related with the scale at which the chiral symmetry is broken.

Using the above expression for the dynamical mass, the first term in Equation (11) generates the constituent quark mass, the second gives rise to a one pion exchange between quarks with a form factor $F(p^2) = \left[\frac{\Lambda^2}{\Lambda^2 + p^2} \right]$, and the main contribution of the third term comes from the two-pion exchange interaction, which can be simulated by a σ -potential term between quarks.

With these definitions one arrives at an effective Lagrangian that is invariant under chiral transformations and, at second order in the pion field, contains a constituent quark mass as well as the one-pion and one-sigma exchange interactions modulated by the form factor $F(p^2)$. The non-relativistic reduction in this Lagrangian constitutes the basis of the chiral quark constituent model [52,53], where hadrons are described as clusters of constituent quarks (antiquarks) interacting through, at least, one-pion and one-sigma potentials.

In the heavy quark sector, chiral symmetry is explicitly broken and the Goldstone-boson exchanges are not active. Therefore, one cannot reproduce the hyperfine splitting for heavy mesons. Then, the quark-(anti-)quark interaction is complemented with a one-gluon exchange potential generated from the vertex Lagrangian

$$\mathcal{L}_{qqg} = i\sqrt{4\pi\alpha_s} \bar{\psi} \gamma_\mu G_c^\mu \lambda^c \psi, \quad (13)$$

where λ^c are the $SU(3)$ color matrices, G_c^μ is the gluon field and α_s is the strong coupling constant. The scale dependence of α_s can be found in, for instance, Reference [53]; it allows a consistent description of light, strange and heavy mesons.

The chiral constituent quark model lacks a quark confining mechanism, which must be implemented by means of an additional confinement potential. It is well known that multi-gluon exchanges produce an attractive linearly rising potential proportional to the distance between infinite-heavy quarks [55,56]. However, sea quarks are also important ingredients of the strong interaction dynamics that contribute to the screening of the rising potential at low momenta and constitute the basis eventually to the breaking of the quark-antiquark binding string [57]. The chiral quark model of Fernández et al. [52,53] tries to mimic this behavior using the following expression:

$$V_{\text{CON}}(\vec{r}) = [-a_c(1 - e^{-\mu_c r}) + \Delta] (\vec{\lambda}_q^c \cdot \vec{\lambda}_{\bar{q}}^c), \quad (14)$$

where a_c and μ_c are model parameters. At short distances this potential presents a linear behavior with an effective confinement strength, $\sigma = -a_c \mu_c (\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)$, while it becomes constant at large distances.

Obviously, these models do not constitute a relativistic field-theoretical treatment of hadrons. However, they keep most of the non-perturbative characteristics of QCD and have enough flexibility to describe a large amount of hadron [53,58–66], hadron-hadron [52,67–70] and multi-quark [71–76] phenomenology.

It is fair to mention herein that similar approaches have been proposed by other authors like Manohar and Georgi [77] who assume that the scale of the chiral symmetry breaking Λ_χ is greater than the confinement scale Λ_{QCD} . Then, an effective Lagrangian including quarks, gluons and Goldstone boson fields is characteristic of the energy region between Λ_χ and Λ_{QCD} .

Summarizing, the aim of this manuscript has been to provide, on one side, an introduction to the Special Issue “Advances in Chiral Quark Models” in *Symmetry* and, on the other side, a brief historical development of the chiral quark model with the main purpose of clarifying the differences between those models that include the word chiral in their names. We have described how chiral symmetry can be implemented in the bag model (chiral bag model, little bag model, cloudy bag model) and how the requirement of more flexibility, avoiding the static bag surfaces, gives place to models based on phenomenological Lagrangians such as the Friedberg–Lee model (soliton bag model) and the Nambu–Jona-Lasinio model (chiral quark soliton model). As an improvement of the NJL Hamiltonian, we have introduced the ideas developed by Diakonov based on the instanton

liquid mode,] which led to soliton type-solutions of baryons and then becoming a variant of the chiral quark soliton model; but it also constitutes a foundation of the constituent quark model. A non-relativistic reduction of Diakonov's Lagrangian gives rise to a chiral constituent quark model, which includes gluons and Goldstone-boson exchanges between constituent valence quarks. Because of its flexibility, the model has been very popular in describing a large amount of hadron phenomena like meson and baryon spectra, their decays and reactions, exotic matter as the so-called XYZ states recently discovered and even meson-meson, meson-baryon and baryon-baryon interactions.

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References

- Gell-Mann, M. A schematic model of baryons and mesons. *Phys. Lett.* **1964**, *8*, 214–215. [[CrossRef](#)]
- Zweig, G. *An SU(3) Model for Strong Interaction Symmetry and Its Breaking*; Version 1; Hadronic Press: Nonantum, MA, USA, 1964.
- Greenberg, O. Spin and Unitary Spin Independence in a Paraquark Model of Baryons and Mesons. *Phys. Rev. Lett.* **1964**, *13*, 598–602. [[CrossRef](#)]
- Han, M.Y.; Nambu, Y. Three-Triplet Model with Double SU(3) Symmetry. *Phys. Rev.* **1965**, *139*, B1006–B1010. [[CrossRef](#)]
- Fritzsch, H.; Gell-Mann, M.; Leutwyler, H. Advantages of the Color Octet Gluon Picture. *Phys. Lett. B* **1973**, *47*, 365–368. [[CrossRef](#)]
- Politzer, H. Reliable Perturbative Results for Strong Interactions? *Phys. Rev. Lett.* **1973**, *30*, 1346–1349. [[CrossRef](#)]
- Gross, D.J.; Wilczek, F. Ultraviolet Behavior of Non-Abelian Gauge Theories. *Phys. Rev. Lett.* **1973**, *30*, 1343–1346. [[CrossRef](#)]
- Marciano, W.J.; Pagels, H. Quantum Chromodynamics: A Review. *Phys. Rept.* **1978**, *36*, 137. [[CrossRef](#)]
- De Rújula, A.; Georgi, H.; Glashow, S.L. Hadron masses in a gauge theory. *Phys. Rev. D* **1975**, *12*, 147–162. [[CrossRef](#)]
- Eichten, E.; Gottfried, K.; Kinoshita, T.; Lane, K.D.; Yan, T.M. Charmonium: The model. *Phys. Rev. D* **1978**, *17*, 3090–3117. [[CrossRef](#)]
- Politzer, H. Effective quark masses in the chiral limit. *Nucl. Phys. B* **1976**, *117*, 397–406. [[CrossRef](#)]
- Isgur, N.; Karl, G. P-wave baryons in the quark model. *Phys. Rev. D* **1978**, *18*, 4187–4205. [[CrossRef](#)]
- Isgur, N.; Karl, G. Positive-parity excited baryons in a quark model with hyperfine interactions. *Phys. Rev. D* **1979**, *19*, 2653–2677. [[CrossRef](#)]
- Capstick, S.; Isgur, N. Baryons in a relativized quark model with chromodynamics. *Phys. Rev. D* **1986**, *34*, 2809–2835. [[CrossRef](#)] [[PubMed](#)]
- Chodos, A.; Jaffe, R.L.; Johnson, K.; Thorn, C.B.; Weisskopf, V.F. New extended model of hadrons. *Phys. Rev. D* **1974**, *9*, 3471–3495. [[CrossRef](#)]
- Godfrey, S.; Isgur, N. Mesons in a Relativized Quark Model with Chromodynamics. *Phys. Rev. D* **1985**, *32*, 189–231. [[CrossRef](#)] [[PubMed](#)]
- Chodos, A.; Jaffe, R.; Johnson, K.; Thorn, C.B. Baryon Structure in the Bag Theory. *Phys. Rev. D* **1974**, *10*, 2599. [[CrossRef](#)]
- Oka, M.; Yazaki, K. Nuclear Force in a Quark Model. *Phys. Lett. B* **1980**, *90*, 41–44. [[CrossRef](#)]
- Faessler, A.; Fernandez, F.; Lübeck, G.; Shimizu, K. The quark model and the nature of the repulsive core of the nucleon-nucleon interaction. *Phys. Lett. B* **1982**, *112*, 201–205. [[CrossRef](#)]
- Faessler, A.; Fernandez, F.; Lübeck, G.; Shimizu, K. The nucleon-nucleon interaction and the role of the [42] orbital six-quark symmetry. *Nuclear Phys. A* **1983**, *402*, 555–568. [[CrossRef](#)]
- DeTar, C. Hadronic deformation energy. II. Two-nucleon interaction. *Phys. Rev. D* **1978**, *17*, 323–339. [[CrossRef](#)]
- Isgur, N. An Introduction to the quark model for baryons. *Int. J. Mod. Phys. E* **1992**, *1*, 465–490. [[CrossRef](#)]
- DeTar, C.E.; Donoghue, J.F. BAG MODELS OF HADRONS. *Ann. Rev. Nucl. Part. Sci.* **1983**, *33*, 235–264. doi:10.1146/annurev.ns.33.120183.001315. [[CrossRef](#)]
- Giannini, M.; Santopinto, E. The hypercentral Constituent Quark Model and its application to baryon properties. *Chin. J. Phys.* **2015**, *53*, 020301. [[CrossRef](#)]
- Skyrme, T. A Nonlinear field theory. *Proc. R. Soc. Lond. A* **1961**, *260*, 127–138. [[CrossRef](#)]

26. Gross, D.J.; Neveu, A. Dynamical Symmetry Breaking in Asymptotically Free Field Theories. *Phys. Rev. D* **1974**, *10*, 3235. [[CrossRef](#)]
27. Karbstein, F.; Thies, M. Integrating out the Dirac sea: Effective field theory approach to exactly solvable four-fermion models. *Phys. Rev. D* **2008**, *77*, 025008. [[CrossRef](#)]
28. Boehmer, C.; Thies, M. Competing mechanisms of chiral symmetry breaking in a generalized Gross-Neveu model. *Phys. Rev. D* **2010**, *81*, 105027. [[CrossRef](#)]
29. Chodos, A.; Thorn, C.B. Chiral invariance in a bag theory. *Phys. Rev. D* **1975**, *12*, 2733–2743. [[CrossRef](#)]
30. Brown, G.; Rho, M. The Little Bag. *Phys. Lett. B* **1979**, *82*, 177–180. [[CrossRef](#)]
31. Thomas, A.W. Chiral Symmetry and the Bag Model: A New Starting Point for Nuclear Physics. *Adv. Nucl. Phys.* **1984**, *13*, 1–137. [[CrossRef](#)]
32. Miller, G.A. Building the Nucleus From Quarks: the Cloudy bag Model and the Quark Description of the Nucleon-nucleon Wave Functions. *Int. Rev. Nucl. Phys.* **1984**, *1*, 189–323. [[CrossRef](#)]
33. Friedberg, R.; Lee, T.D. Fermion-field nontopological solitons. *Phys. Rev. D* **1977**, *15*, 1694–1711. [[CrossRef](#)]
34. Nambu, Y.; Jona-Lasinio, G. Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I. *Phys. Rev.* **1961**, *122*, 345–358. [[CrossRef](#)]
35. Diakonov, D.; Petrov, V. A theory of light quarks in the instanton vacuum. *Nucl. Phys. B* **1986**, *272*, 457–489. [[CrossRef](#)]
36. Manohar, A.; Georgi, H. Chiral quarks and the non-relativistic quark model. *Nucl. Phys. B* **1984**, *234*, 189–212. [[CrossRef](#)]
37. Goldflam, R.; Wilets, L. Soliton bag model. *Phys. Rev. D* **1982**, *25*, 1951–1963. [[CrossRef](#)]
38. Birse, M.C.; Banerjee, M.K. A chiral soliton model of nucleon and delta. *Phys. Lett. B* **1984**, *136*, 284–288. [[CrossRef](#)]
39. Kahana, S.; Ripka, G.; Soni, V. Soliton with Valence Quarks in the Chiral Invariant Sigma Model. *Nucl. Phys. A* **1984**, *415*, 351–364. [[CrossRef](#)]
40. Seki, R.; Ohta, S. Nontopological and topological chiral soliton bags as a model of the nucleon. *Nucl. Phys. A* **1985**, *437*, 541–566. [[CrossRef](#)]
41. Shigemi, O.; Seki, R. Spin-isospin projection of hedgehog chiral soliton bags using collective coordinates. *Nucl. Phys. A* **1986**, *458*, 669–688. [[CrossRef](#)]
42. Gell-Mann, M.; Levy, M. The axial vector current in beta decay. *Nuovo Cim.* **1960**, *16*, 705. [[CrossRef](#)]
43. Nambu, Y.; Jona-Lasinio, G. Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. II. *Phys. Rev.* **1961**, *124*, 246–254. [[CrossRef](#)]
44. Eguchi, T. New approach to collective phenomena in superconductivity models. *Phys. Rev. D* **1976**, *14*, 2755–2763. [[CrossRef](#)]
45. Alkofer, R.; Reinhardt, H.; Weigel, H. Baryons as chiral solitons in the Nambu-Jona-Lasinio model. *Phys. Rep.* **1996**, *265*, 139–252. [[CrossRef](#)]
46. Christov, C.; Blotz, A.; Kim, H.C.; Pobylitsa, P.; Watabe, T.; Meissner, T.; Ruiz Arriola, E.; Goeke, K. Baryons as nontopological chiral solitons. *Prog. Part. Nucl. Phys.* **1996**, *37*, 91–191. [[CrossRef](#)]
47. Weigel, H. *Chiral Soliton Models for Baryons*; Springer: Berlin/Heidelberg, Germany, 2008; Volume 743.
48. Shuryak, E.V. The Role of Instantons in Quantum Chromodynamics. 1. Physical Vacuum. *Nucl. Phys. B* **1982**, *203*, 93. [[CrossRef](#)]
49. Diakonov, D. Foundations of the constituent quark model. *Prog. Part. Nucl. Phys.* **1996**, *36*, 1–18. [[CrossRef](#)]
50. Diakonov, D.; Petrov, V.Y. Nucleons as chiral solitons. In *At The Frontier of Particle Physics: Handbook of QCD*; World Scientific Publishing Company: Singapore, 2000; pp. 359–415. [[CrossRef](#)]
51. Diakonov, D. Instantons at work. *Prog. Part. Nucl. Phys.* **2003**, *51*, 173–222. [[CrossRef](#)]
52. Fernandez, F.; Valcarce, A.; Straub, U.; Faessler, A. The Nucleon-nucleon interaction in terms of quark degrees of freedom. *J. Phys. G* **1993**, *19*, 2013–2026. [[CrossRef](#)]
53. Vijande, J.; Fernandez, F.; Valcarce, A. Constituent quark model study of the meson spectra. *J. Phys. G* **2005**, *31*, 481. [[CrossRef](#)]
54. Glozman, L.; Riska, D. The spectrum of the nucleons and the strange hyperons and chiral dynamics. *Phys. Rep.* **1996**, *268*, 263–303. [[CrossRef](#)]
55. Brown, L.S.; Weisberger, W.I. Remarks on the Static Potential in Quantum Chromodynamics. *Phys. Rev. D* **1979**, *20*, 3239. [[CrossRef](#)]
56. Billoire, A. How Heavy Must Be Quarks in Order to Build Coulombic q anti- q Bound States. *Phys. Lett. B* **1980**, *92*, 343–347. [[CrossRef](#)]
57. Bali, G.S.; Neff, H.; Duessel, T.; Lippert, T.; Schilling, K. Observation of string breaking in QCD. *Phys. Rev. D* **2005**, *71*, 114513. [[CrossRef](#)]
58. Valcarce, A.; Fernandez, F.; Gonzalez, P.; Vento, V. Chiral quark cluster model study of the low-energy baryon spectrum. *Phys. Lett. B* **1996**, *367*, 35–39. [[CrossRef](#)]
59. Segovia, J.; Entem, D.; Fernandez, F. Is chiral symmetry restored in the excited meson spectrum? *Phys. Lett. B* **2008**, *662*, 33–36. [[CrossRef](#)]
60. Segovia, J.; Yasser, A.; Entem, D.; Fernandez, F. JPC=1– hidden charm resonances. *Phys. Rev. D* **2008**, *78*, 114033. [[CrossRef](#)]
61. Segovia, J.; Yasser, A.; Entem, D.; Fernandez, F. Ds-1 (2536) + decays and the properties of P-wave charmed strange mesons. *Phys. Rev. D* **2009**, *80*, 054017. [[CrossRef](#)]
62. Segovia, J.; Entem, D.; Fernandez, F. Charmonium resonances in $e^+ e^-$ exclusive reactions around the $\psi(4415)$ region. *Phys. Rev. D* **2011**, *83*, 114018. [[CrossRef](#)]

63. Segovia, J.; Entem, D.R.; Fernandez, F. Charmed-strange Meson Spectrum: Old and New Problems. *Phys. Rev. D* **2015**, *91*, 094020. [[CrossRef](#)]
64. Ortega, P.G.; Segovia, J.; Entem, D.R.; Fernández, F. Canonical description of the new LHCb resonances. *Phys. Rev. D* **2016**, *94*, 114018. [[CrossRef](#)]
65. Yang, G.; Ping, J.; Segovia, J. The S- and P-Wave Low-Lying Baryons in the Chiral Quark Model. *Few Body Syst.* **2018**, *59*, 113. [[CrossRef](#)]
66. Yang, G.; Ping, J.; Ortega, P.G.; Segovia, J. Triply heavy baryons in the constituent quark model. *Chin. Phys. C* **2020**, *44*, 023102. [[CrossRef](#)]
67. Valcarce, A.; Fernandez, F.; Buchmann, A.; Faessler, A. Can one simultaneously describe the deuteron properties and the nucleon-nucleon phase shifts in the quark cluster model? *Phys. Rev. C* **1994**, *50*, 2246–2249. [[CrossRef](#)]
68. Ortega, P.G.; Segovia, J.; Entem, D.R.; Fernandez, F. Molecular components in P-wave charmed-strange mesons. *Phys. Rev. D* **2016**, *94*, 074037. [[CrossRef](#)]
69. Ortega, P.G.; Segovia, J.; Entem, D.R.; Fernández, F. Threshold effects in P-wave bottom-strange mesons. *Phys. Rev. D* **2017**, *95*, 034010. [[CrossRef](#)]
70. Ortega, P.G.; Segovia, J.; Entem, D.R.; Fernández, F. Charmonium resonances in the 3.9 GeV/ c^2 energy region and the X(3915)/X(3930) puzzle. *Phys. Lett. B* **2018**, *778*, 1–5. [[CrossRef](#)]
71. Vijande, J.; Valcarce, A.; Tsushima, K. Dynamical study of bf QQ - anti-u anti-d mesons. *Phys. Rev. D* **2006**, *74*, 054018. [[CrossRef](#)]
72. Yang, G.; Ping, J. The structure of pentaquarks P_c^+ in the chiral quark model. *Phys. Rev. D* **2017**, *95*, 014010. [[CrossRef](#)]
73. Yang, G.; Ping, J.; Segovia, J. Hidden-bottom pentaquarks. *Phys. Rev. D* **2019**, *99*, 014035. [[CrossRef](#)]
74. Yang, G.; Ping, J.; Segovia, J. Doubly-heavy tetraquarks. *Phys. Rev. D* **2020**, *101*, 014001. [[CrossRef](#)]
75. Yang, G.; Ping, J.; Segovia, J. Doubly Charmed Pentaquarks. *Phys. Rev. D* **2020**, *101*, 074030. [[CrossRef](#)]
76. Yang, G.; Ping, J.; Segovia, J. QQ $\bar{s}\bar{s}$ tetraquarks in the chiral quark model. *Phys. Rev. D* **2020**, *102*, 054023. [[CrossRef](#)]
77. Manohar, A.V. Equivalence of the Chiral Soliton and Quark Models in Large N. *Nucl. Phys. B* **1984**, *248*, 19. [[CrossRef](#)]