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Fixed-Time Formation Control for Second-Order Disturbed Multi-Agent Systems under Directed Graph

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Abstract: This paper investigates the fixed-time formation (FixF) control problem for second-order multi-agent systems (MASs), where each agent is subject to disturbance and the communication network is general directed. First, a FixF protocol is presented based on the backstepping technique, where the distributed cooperative variable structure control method is utilized to handle the bounded disturbances. Then, to remove the dependence of control gains on the global information, a practical adaptive FixF control is presented, where the MASs can achieve formation with a bounded error within fixed time. Finally, a numerical example is presented to validate the theoretical result.

Keywords: multi-agent systems; formation control; adaptive control; cooperative variable structure control; fixed-time convergence

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1. Introduction

Formation control is one of the most actively investigated topics on the cooperation of multi-agent systems (MASs), such as multiple vehicle [1] and quadrotor aircraft [2]. Generally speaking, each agent is driven to satisfy predefined relative state constraints with respect to its neighbors in a formation control task.

A key point in the study of the formation control problem is the convergence rate, which indicates how efficient a proposed control protocol could be. The formation tracking problem for the general linear MASs was studied in [3] over fixed and switching topologies, where the formation can be achieved asymptotically. Compared to the asymptotic controllers, designing a formation controller with finite-time convergence is more desirable as it indeed guarantees faster convergence rate as well as improved system performance [4–6]. The formation control problems with finite-time convergence rate for linear and nonlinear first-order MASs were studied in [6,7] over undirected and directed graphs, respectively, while in [1,8], such problems were studied for multiple vehicles and multiple nonholonomic mobile robots, where the dynamics of the systems in [1,8] were second-order. The authors of [9] proposed the adaptive formation control protocol for double integrator disturbed MASs with a general directed networks, which is fully distributed.

Having evolved from finite-time stability, fixed-time stability (FixS) [10] inherits the advantage of finite-time convergence. Meanwhile, the convergence time is uniformly bounded with respect to the initial conditions. After that, FixS was well studied for single systems [11], and extended to single integrator [12–15] and double integrator MASs [16–26]. Specifically, the works in [23–26] all studied the leader–follower FixF control problem. Compared with the leader–follower case, it is more challenging to study the leaderless formation control problem because the agreed trajectory is not known in advance, and thus is not available to any agent. Furthermore, disturbances usually exist in real control networked systems [27,28].

Note that the communication topology discussed in most of the above-mentioned works is undirected. It is nontrivial to generalize these results to the formation problem with general directed topologies. The main challenge lies in the asymmetry property of directed topologies. In addition, global information is usually utilized in the control design in most existing works. Bearing the observation in mind, we will discover the FixF control problem for a kind of disturbed second-order MASs, where the communication graph is directed. We summarized the main contribution as follows. First, a FixF control protocol is proposed by utilizing the backstepping technique. The distributed cooperative variable structure control is utilized to handle the bounded disturbances. Then, to remove the dependence of control gains on the global information, a practical adaptive FixF control is presented, where the MASs can achieve formation with a bounded error within fixed time.

The rest of the paper consists of seven parts in total. Some concepts in graph theory and problem statement are provided in Section 2. Main results are given in Section 3. Section 4 provides a numerical example to verify the effectiveness of the theoretical results, and the paper is concluded in Section 5.

2. Preliminaries and Problem Statement

Before continuing, necessary notations used in this paper are introduced below.

Notations: The transposition of a vector or a matrix is marked by the superscript T; the eigenvalues of the symmetric and real matrix Q are arranged as $\lambda_1(Q) \leq \lambda_2(Q) \leq \cdots \leq \lambda_N(Q)$. For any $s \in \mathbb{R}$, $\mathrm{sign}(s)$ is the signum function. We further define $s^{[k]} = \mathrm{sign}(s)|s|^k$ and $A^{[k]} = (a_{ij}^{[k]})_{n\times m}$, where $A = (a_{ij})_{n\times m}$. Symbol $\|x\|$ denotes the 2-norm of $x \in \mathbb{R}^n$; denote $\mathcal{I}_N = \{1, 2, \cdots, N\}$.

2.1. Graph Theory

The concepts for graph are given below, which can be found in [29].

A directed graph is denoted as $\mathcal{G} = (\mathbb{V}, \mathbb{E}, \mathbb{A})$ with $\mathbb{V} = \{\nu_1, \nu_2, \cdots, \nu_N\}$, $\mathbb{E} \in \mathbb{V} \times \mathbb{V}$ being the node set and edge set, respectively, and $\mathbb{A} = (a_{ij})_{N\times N}$ is the adjacency matrix of the graph $\mathcal{G}$ with non-negative elements. An edge e_{ij} rooted at spacecraft j and ending at spacecraft i is denoted by (i, j). The weight $a_{ij} > 0 \iff (i, j) \in \mathbb{E}$. Furthermore, it is assumed that no self-loop exists. The corresponding Laplacian matrix $L = (l_{ij})_{N\times N}$ is defined by $l_{ij} = -a_{ij}$ for $i \neq j$ and $l_{ii} = \sum_{j=1}^{N} a_{ij}$ for $i \in \mathcal{I}_N$. Denote $\mathcal{N}_i = \{j | a_{ij} > 0, j \in \mathcal{I}_N\}$ and $\mathcal{N}_i^- = \{j | a_{ji} > 0, j \in \mathcal{I}_N\}$ the inner and outer neighbors of agent i, respectively, and $|\mathcal{N}_i|$ is the number of the inner neighbors of agent i.

Assumption 1. *The graph $\mathcal{G}$ is strongly connected.*

2.2. Supporting Lemmas

First, we record the following lemma about the properties of directed graphs.

Lemma 1. *[30,31] Suppose Assumption 1 holds. Denote $L \in \mathbb{R}^{N\times N}$ the Laplacian matrix associated with the graph $\mathcal{G}$. Then, $L1_N = 0$, and there exists a vector $\gamma = (\gamma_1, \cdots, \gamma_N)^T$ with $\sum_{i=1}^{N} \gamma_i = 1$ and $\gamma_i > 0, i \in \mathcal{I}_N$ such that $\gamma^T L = 0$. Furthermore, let the matrix $\hat{L} = \Gamma L + L^T \Gamma$ with $\Gamma = \mathrm{diag}(\gamma)$. Let $\vartheta \in \mathbb{R}^N$ be any positive vector. Then, $\min_{z^T\vartheta=0,\, z^T z=1} z^T \hat{L} z > \frac{\lambda_2(\hat{L})}{N}$, where $\lambda_2(\hat{L}) > 0$ is the second smallest eigenvalue of $\hat{L}$.*

One lemma for FixS and several lemmas about some inequalities are introduced, which will be used to prove the main result.

Lemma 2. *[10] We consider the system*

$$\dot{z} = g(t, z(t)), z(0) = z_0, \tag{1}$$

with its equilibrium point being the origin. If there exists a continuous radially unbounded function (CRUF) $W(z(t))$ such that $W(z(t)) = 0 \Leftrightarrow z(t) = 0$ and $\dot{W}(z(t)) \leq -c_1 W^{r_1} - c_2 W^{r_2}$ for some *positive constants c_1, c_2, r_1, and r_2 satisfying $0 < r_1 < 1 < r_2$. Then, the system (1) is said FixS with the settling time estimated by $T(z_0) \leq \frac{1}{c_1(1-r_1)} + \frac{1}{c_2(r_2-1)}$.*

Lemma 3. *[32] Consider the system (1). If there exists a CRUF $W(z(t))$ such that $\dot{W}(z(t)) \leq -c_1 W^{r_1} - c_2 W^{r_2} + s$ for some positive constants c_1, c_2, s, r_1, and r_2 satisfying $0 < r_1 < 1 < r_2$. Then, the system is practical FixS, and the residual set of the solution of system (1) is given by $\{\lim_{t\to T} z | W(z) \leq \min\{c_1^{-1/r_1}(s/(1-\theta))^{1/r_1}, c_2^{-1/r_2}(s/(1-\theta))^{1/r_2}\}\}$ with $\theta \in (0,1)$. The time needed to reach the residual set is bounded as $T(z_0) \leq \frac{1}{c_1\theta(1-r_1)} + \frac{1}{c_2\theta(r_2-1)}$.*

Lemma 4. *[33] For $z_1, z_2 \in \mathbb{R}, 0 < r \leq 1$, it always holds that $|z_1^{[r]} - z_2^{[r]}| \leq 2^{1-r}|z_1 - z_2|^r$.*

Lemma 5. *[34] For $z_1, z_2, \cdots, z_n \geq 0$ and $l > s \geq 1$, it holds that $\left(\sum_{i=1}^n z_i^l\right)^{\frac{1}{l}} \leq \left(\sum_{i=1}^n z_i^s\right)^{\frac{1}{s}} \leq n^{\frac{1}{s}-\frac{1}{l}}\left(\sum_{i=1}^n z_i^l\right)^{\frac{1}{l}}$.*

Lemma 6. *[35] Given $z_1, z_2 \in \mathbb{R}, c, d > 0$, it holds that $|z_1|^c|z_2|^d \leq \frac{c}{c+d}|z_1|^{c+d} + \frac{d}{c+d}|z_2|^{c+d}$.*

Lemma 7. *[36] For $z \in \mathbb{R}^N$, and $\alpha > 1 > \beta > 0$, the following inequalities hold:*

$$\left(\sum_{k=1}^{N}(z_k^{\alpha+1} + z_k^{\beta+1})\right)^{\frac{\alpha+\beta}{\alpha+1}} \leq \sum_{k=1}^{N}(z_k^{\alpha} + z_k^{\beta})^2,$$

$$(2N)^{\frac{1-\alpha}{\alpha+1}}\left(\sum_{k=1}^{N}(z_k^{\alpha+1} + z_k^{\beta+1})\right)^{\frac{2\alpha}{\alpha+1}} \leq \sum_{k=1}^{N}(z_k^{\alpha} + z_k^{\beta})^2.$$

2.3. Problem Formulation

We consider the second-order MASs with disturbances.

$$\begin{aligned} \dot{p}_i &= v_i, \\ \dot{v}_i &= u_i + d_i(t), \quad i \in \mathcal{I}_N \end{aligned} \tag{2}$$

where $p_i, v_i \in \mathbb{R}$ represent the position and the velocity states of the ith agent, $d_i(t) \in \mathbb{R}$ stands for the disturbance, and $u_i \in \mathbb{R}$ is a control input.

Definition 1. *Formation information can be represented by a time-dependent vector $H = (h_1, ..., h_N)^T$. The FixS formation for the second-order MASs (2) is said to be achieved if there exists a settling time $T(p(0), v(0))$, which is uniformly bounded with regard to the initial state, viz, there exists a fixed constant $T_{\max}$ with $T \leq T_{\max}$, such that*

$$\begin{cases} \lim_{t\to T^-}(p_i - h_i) - (p_j - h_j) = 0, \\ \lim_{t\to T^-}(v_i - \dot{h}_i) - (v_j - \dot{h}_j) = 0, \\ p_i - h_i = p_j - h_j,\ v_i - \dot{h}_i = v_j - \dot{h}_j,\ t \geq T, \\ i, j \in \mathcal{I}_N. \end{cases} \tag{3}$$

Here, $T(p(0), v(0))$ is simplified as T.

The following assumptions are introduced which will be used in the proof of the main result.

Assumption 2. *Suppose that the unknown disturbances $d_i(t)$, $i \in \mathcal{I}_N$ are bounded:*

$$|d_i(t)| \leq \bar{d}, \; i \in \mathcal{I}_N, \tag{4}$$

where $\bar{d} > 0$ is a constant.

Assumption 3. *Suppose the second derivative of $H = (h_1, ..., h_N)^T$ is bounded, that is,*

$$|\ddot{h}_i| \leq \bar{h}, \tag{5}$$

where $\bar{h} > 0$ is a constant.

In the following, we will design a distributed control protocol by invoking local information to drive the system (2) to achieve the formation with fixed-time convergence.

3. Main Results

3.1. Fixed-Time Formation Control Protocol

In this subsection, a FixF control protocol will be designed via backstepping method. Let $\tilde{p}_i = p_i - h_i$ and $\tilde{v}_i = v_i - \dot{h}_i$. The error dynamics can be given:

$$\begin{aligned} \dot{\tilde{p}}_i &= \tilde{v}_i, \\ \dot{\tilde{v}}_i &= u_i + \tilde{d}_i(t), \;\; i \in \mathcal{I}_N, \end{aligned} \tag{6}$$

where $\tilde{d}_i = d_i - \ddot{h}_i$, satisfying $|\tilde{d}_i| \leq \bar{d} + \bar{h}$.

Define $\tau_i = \sum_{j=1}^N a_{ij}(\tilde{p}_i - \tilde{p}_j), \sigma_i = \dot{\tau}_i = \sum_{j=1}^N a_{ij}(\tilde{v}_i - \tilde{v}_j), \tau = (\tau_1, \tau_2, \cdots, \tau_N)^T, \sigma = (\sigma_1, \sigma_2, \cdots, \sigma_N)^T, \tilde{p} = (\tilde{p}_1, \tilde{p}_2, \cdots, \tilde{p}_N)^T$ and $\tilde{v} = (\tilde{v}_1, \tilde{v}_2, \cdots, \tilde{v}_N)^T$. Then, we have $\tau = L\tilde{p}$ and $\sigma = \dot{\tau} = L\tilde{v}$.

The formation control protocol design procedure is illustrated as follows.

(1) First, we consider the following systems:

$$\dot{\tau} = L\tilde{v}. \tag{7}$$

A virtual control input for system (7) is designed as

$$v_i^* = \tilde{v}_i + l_1 \tau_i^{[\alpha]}, \;\; i \in \mathcal{I}_N, \tag{8}$$

where $\alpha > 1$ and $l_1 > 0$. Furthermore, a virtual control law for v_i^* is designed as

$$\bar{v}_i = -l_2 \tau_i^{[\beta]}, \;\; i \in \mathcal{I}_N, \tag{9}$$

where $\frac{1}{2} < \beta < 1$ and $l_2 > 0$.

For further analysis, we construct a Lyapunov function candidate as follows:

$$V_0 = \sum_{k=1}^N \left(\frac{l_1 \gamma_k}{1+\alpha} |\tau_k|^{1+\alpha} + \frac{l_2 \gamma_k}{1+\beta} |\tau_k|^{1+\beta} \right),$$

where $\gamma_k, k \in \mathcal{I}_N$ is given in Lemma 1.

The time derivative of V_0 can be computed as follows:

$$\begin{aligned}\dot{V}_0 =& \sum_{k=1}^{N}\left(l_1\gamma_k\tau_k^{[\alpha]} + l_2\gamma_k\tau_k^{[\beta]}\right)\dot{\tau}_k \\ =&(l_1\tau^{[\alpha]} + l_2\tau^{[\beta]})^T\Gamma\dot{\tau} \\ =&(l_1\tau^{[\alpha]} + l_2\tau^{[\beta]})^T\Gamma L\tilde{v} \\ =&(l_1\tau^{[\alpha]} + l_2\tau^{[\beta]})^T(\Gamma L\otimes I_3)(v^* - \bar{v} - l_1\tau^{[\alpha]} - l_2\tau^{[\beta]}) \\ =&(l_1\tau^{[\alpha]} + l_2\tau^{[\beta]})^T(\Gamma L\otimes I_3)(v^* - \bar{v}) - \frac{1}{2}(l_1\tau^{[\alpha]} + l_2\tau^{[\beta]})^T\hat{L}(l_1\tau^{[\alpha]} + l_2\tau^{[\beta]}),\end{aligned} \tag{10}$$

where the definition of $\hat{L}$ can be found in Lemma 1, $v^* = (v_1^*, v_2^*, \cdots, v_N^*)^T$ and $\bar{v} = (\bar{v}_1, \bar{v}_2, \cdots, \bar{v}_N)^T$.

As inspired by the work in [9], we introduce the following indicative function: $\mathbb{I}(x) = \begin{cases} 1, & x = 0, \\ \frac{1}{l_1|x|^{\alpha-1} + l_2|x|^{\beta-1}}, & x \neq 0, \end{cases}$ with $x \in \mathbb{R}$. By Lemma 1, one obtains $\gamma^T\tau = \gamma^T L\tilde{p} = 0$. It follows that

$$0 = \gamma^T\tau = \sum_{k=1}^{N}\gamma_k\tau_k = \sum_{k=1,\tau_k\neq 0}^{N}\gamma_k\frac{(l_1\tau_k^{[\alpha]} + l_2\tau_k^{[\beta]})}{l_1|\tau_k|^{\alpha-1} + l_2|\tau_k|^{\beta-1}} + \sum_{k=1,\tau_k=0}^{N}\gamma_k(l_1\tau_k^{[\alpha]} + l_2\tau_k^{[\beta]}). \tag{11}$$

Thus, $(l_1\tau^{[\alpha]} + l_2\tau^{[\beta]})^T\hat{\gamma} = 0$, where $\hat{\gamma} = \left(\mathbb{I}(\tau_1)\gamma_1, \cdots, \mathbb{I}(\tau_N)\gamma_N\right)^T$. One can observe that $\hat{\gamma}$ is a positive vector. By Lemma 1, we have

$$(l_1\tau^{[\alpha]} + l_2\tau^{[\beta]})^T\hat{L}(l_1\tau^{[\alpha]} + l_2\tau^{[\beta]}) > \frac{\lambda_2(\hat{L})}{N}(l_1\tau^{[\alpha]} + l_2\tau^{[\beta]})^T(l_1\tau^{[\alpha]} + l_2\tau^{[\beta]}). \tag{12}$$

On the other hand,

$$\begin{aligned}&(l_1\tau^{[\alpha]} + l_2\tau^{[\beta]})^T(\Gamma L\otimes I_3)(v^* - \bar{v}) \\ \leq&\frac{\epsilon}{2}(l_1\tau^{[\alpha]} + l_2\tau^{[\beta]})^T(\Gamma LL^T\Gamma\otimes I_3)(l_1\tau^{[\alpha]} + l_2\tau^{[\beta]}) + \frac{1}{2\epsilon}\sum_{k=1}^{N}(v_k^* - \bar{v}_k)^2 \\ \leq&\frac{\epsilon}{2}\lambda_{\max}(\Gamma LL^T\Gamma)||l_1\tau^{[\alpha]} + l_2\tau^{[\beta]}||^2 + \frac{2^{1-2\beta}}{\epsilon}\sum_{k=1}^{N}|\xi_k|^{2\beta},\end{aligned} \tag{13}$$

where $\xi_k = v_k^{*[\frac{1}{\beta}]} - \bar{v}_k^{[\frac{1}{\beta}]}, \epsilon > 0$, and $|v_k^* - \bar{v}_k| \leq 2^{1-\beta}|\xi_k|^{\beta}$ is utilized based on Lemma 4 to derive the last inequality. From (10)–(13), one obtains

$$\dot{V}_0 \leq -\frac{1}{2}\left(\frac{\lambda_2(\hat{L})}{N} - \epsilon\lambda_{\max}(\Gamma LL^T\Gamma)\right)||l_1\tau^{[\alpha]} + l_2\tau^{[\beta]}||^2 + \frac{2^{1-2\beta}}{\epsilon}\sum_{k=1}^{N}|\xi_k|^{2\beta}. \tag{14}$$

(2) Design of control law $u_i, i \in \mathcal{I}_N$.
Consider the dynamics of v_i^*:

$$\dot{v}_i^* = \dot{\bar{v}}_i + \alpha l_1|\tau_i|^{\alpha-1}\sigma_i = u_i + \tilde{d}_i + \alpha l_1|\tau_i|^{\alpha-1}\sigma_i.$$

Then, the control protocol u_i is designed as follows:

$$u_i = -k_1\xi_i^{[2\alpha-1]} - k_2\xi_i^{[2\beta-1]} - k_3\mathrm{sign}(\xi_i) - \alpha l_1|\tau_i|^{\alpha-1}\sigma_i, \tag{15}$$

where $\frac{1}{2} < \beta < 1 < \alpha, l_1 > 0$, and the constant coefficients k_1, k_2, and k_3 will be given in the next Theorem.

To this end, we summarize the main result and provide the corresponding stability analysis.

Theorem 1. *Consider the second-order disturbed MASs (2). If the Assumptions 1–3 hold, then the FixF can be achieved under control protocol (15) with the control gains selected as*

$$\begin{aligned} & k_1 > 0,\ k_2 > 2^{1-2\beta}\epsilon^{-2} + \bar{c}_{2k},\ k_3 \geq \bar{d} + \bar{h}, \\ & 0 < \epsilon < \frac{\lambda_2(\hat{L})}{N(\lambda_{\max}(\Gamma L L^T \Gamma) + 2\bar{c}_{1k})}, \end{aligned} \tag{16}$$

where $\bar{c}_{1k} = \frac{1}{2} l_2^{\frac{1}{\beta}} \Pi_k + \frac{1}{2} l_2^{\frac{1}{\beta}} \sum_{j \in \mathcal{N}_k^-} \tau_j$, $\bar{c}_{2k} = l_2^{\frac{1}{\beta}} (3 \times 2^{1-2\beta} \Pi_k + 2^{2-2\beta} \pi_k |\mathcal{N}_k|) + 2^{1-2\beta} l_2^{\frac{1}{\beta}} \sum_{j \in \mathcal{N}_k^-} \pi_j$ *with* $\Pi_k = \sum_{j \in \mathcal{N}_k} a_{kj}$, $\pi_k = \max_{j \in \mathcal{N}_k}\{a_{kj}\}$, *and* Γ *is given in Lemma 1.*

Proof. Consider the Lyapunov function candidate

$$V = V_0 + \epsilon \sum_{k=1}^{N} V_k, \tag{17}$$

with

$$\begin{aligned} V_0 &= \sum_{k=1}^{N} \left(\frac{l_1 \gamma_k}{1+\alpha} |\tau_k|^{1+\alpha} + \frac{l_2 \gamma_k}{1+\beta} |\tau_k|^{1+\beta} \right), \\ V_k &= \int_{\bar{v}_k}^{v_k^*} \left(s^{[\frac{1}{\beta}]} - \bar{v}_k^{[\frac{1}{\beta}]} \right) \mathrm{d}s,\ \ k \in \mathcal{I}_N, \end{aligned} \tag{18}$$

where γ_k is defined in Lemma 1. It is not difficult to verify that V is differentiable, positive definite, and proper. Denote $v^* = (v_1^*, \cdots, v_N^*)^T$ and $\bar{v} = (\bar{v}_1, \cdots, \bar{v}_N)^T$.

The time derivative of V_k is given as

$$\begin{aligned} \dot{V}_k =& -\frac{d\bar{v}_k^{[\frac{1}{\beta}]}}{dt} (v_k^* - \bar{v}_k) + \xi_k \dot{v}_k^* \\ =& l_2^{\frac{1}{\beta}} \dot{\tau}_k (v_k^* - \bar{v}_k) + \xi_k \dot{v}_k^* \\ =& l_2^{\frac{1}{\beta}} \sum_{j=1}^{N} a_{kj} (\tilde{v}_k - \tilde{v}_j)(v_k^* - \bar{v}_k) + \xi_k \dot{v}_k^*. \end{aligned} \tag{19}$$

Let $\Pi_k = \sum_{j \in \mathcal{N}_k} a_{kj}$ and $\pi_k = \max_{j \in \mathcal{N}_k}\{a_{kj}\}$, then one obtains $\left|\sum_{j=1}^N a_{kj}(\tilde{v}_k - \tilde{v}_j)\right| \leq \Pi_k |\tilde{v}_k| + \pi_k \sum_{j \in \mathcal{N}_k} |\tilde{v}_j|$.

Based on Lemma 4, one obtains

$$\begin{aligned} & |\tilde{v}_k||v_k^* - \bar{v}_k| \\ =& |v_k^* - \bar{v}_k - l_1 \tau_k^{[\alpha]} - l_2 \tau_k^{[\beta]}||v_k^* - \bar{v}_k| \\ \leq& \frac{3}{2}|v_k^* - \bar{v}_k|^2 + \frac{1}{2}|l_1 \tau_k^{[\alpha]} + l_2 \tau_k^{[\beta]}|^2 \\ \leq& 3 \times 2^{1-2\beta}|\xi_k|^{2\beta} + \frac{1}{2}|l_1 \tau_k^{[\alpha]} + l_2 \tau_k^{[\beta]}|^2, \end{aligned} \tag{20}$$

and

$$
\begin{aligned}
&|\tilde{v}_j||v_k^* - \bar{v}_k| \\
=&|v_j^* - \bar{v}_j - l_1\tau_j^{[\alpha]} - l_2\tau_j^{[\beta]}||v_k^* - \bar{v}_k| \\
\leq&|v_j^* - \bar{v}_j||v_k^* - \bar{v}_k| + |l_1\tau_j^{[\alpha]} + l_2\tau_j^{[\beta]}||v_k^* - \bar{v}_k| \\
\leq&|v_k^* - \bar{v}_k|^2 + \frac{1}{2}|v_j^* - \bar{v}_j|^2 + \frac{1}{2}|l_1\tau_j^{[\alpha]} + l_2\tau_j^{[\beta]}|^2 \\
\leq&2^{2-2\beta}|\xi_k|^{2\beta} + 2^{1-2\beta}|\xi_j|^{2\beta} + \frac{1}{2}|l_1\tau_j^{[\alpha]} + l_2\tau_j^{[\beta]}|^2.
\end{aligned} \tag{21}
$$

From (19)–(21), one obtains that

$$
\begin{aligned}
&\left|\sum_{k=1}^{N} a_{kj}(\tilde{v}_k - \tilde{v}_j)\right||v_k^* - \bar{v}_k| \\
\leq&\left(\Pi_k|\tilde{v}_k| + \pi_k\sum_{j\in\mathcal{N}_k}|\tilde{v}_j|\right)|v_k^* - \bar{v}_k| \\
\leq&(3\cdot 2^{1-2\beta}\Pi_k + 2^{2-2\beta}\pi_k|\mathcal{N}_k|)|\xi_k|^{2\beta} + \pi_k\sum_{j\in\mathcal{N}_k}2^{1-2\beta}|\xi_j|^{2\beta} \\
&+\frac{1}{2}\Pi_k|l_1\tau_k^{[\alpha]} + l_2\tau_k^{[\beta]}|^2 + \frac{1}{2}\pi_k\sum_{j\in\mathcal{N}_k}|l_1\tau_j^{[\alpha]} + l_2\tau_j^{[\beta]}|^2.
\end{aligned} \tag{22}
$$

Furthermore,

$$
\begin{aligned}
\sum_{k=1}^{N}\dot{V}_k \leq& \sum_{k=1}^{N}\left(l_2^{\frac{1}{\beta}}\left|\sum_{j=1}^{N}a_{kj}(\tilde{v}_k - \tilde{v}_j)\right||v_k^* - \bar{v}_k| + \xi_k\dot{v}_k^*\right) \\
\leq& \sum_{k=1}^{N}\left(l_2^{\frac{1}{\beta}}(\Pi_k|\tilde{v}_k| + \pi_k\sum_{j\in\mathcal{N}_k}|\tilde{v}_j|)|v_k^* - \bar{v}_k| + \xi_k\dot{v}_k^*\right) \\
\leq& \sum_{k=1}^{N}(\bar{c}_{1k}|l_1\tau_k^{[\alpha]} + l_2\tau_k^{[\beta]}|^2 + \bar{c}_{2k}|\xi_k|^{2\beta} + \xi_k\dot{v}_k^*),
\end{aligned} \tag{23}
$$

where $\bar{c}_{1k} = \frac{1}{2}l_2^{\frac{1}{\beta}}\Pi_k + \frac{1}{2}l_2^{\frac{1}{\beta}}\sum_{j\in\mathcal{N}_k^-}\pi_j$ and $\bar{c}_{2k} = l_2^{\frac{1}{\beta}}(\frac{3\Pi_k}{2^{2\beta-1}} + \frac{\pi_k|\mathcal{N}_k|}{2^{2\beta-2}}) + 2^{1-2\beta}l_2^{\frac{1}{\beta}}\sum_{j\in\mathcal{N}_k^-}\pi_j$.

From (14) and (23), it follows that

$$
\dot{V} \leq -\sum_{k=1}^{N}c_{1k}|l_1\tau_k^{[\alpha]} + l_2\tau_k^{[\beta]}|^2 + \sum_{k=1}^{N}(2^{1-2\beta}\epsilon^{-1} + \epsilon\bar{c}_{2k})|\xi_k|^{2\beta} + \epsilon\xi^T\dot{v}^*, \tag{24}
$$

where $c_{1k} = \frac{\lambda_2(\hat{L})}{2N} - \frac{1}{2}\epsilon\lambda_{\max}(\Gamma LL^T\Gamma) - \epsilon\bar{c}_{1k}$.

Taking the control protocol (15) into consideration and $\dot{v}_i^* = u_i + \tilde{d}_i + \alpha l_1|\tau_i|^{\alpha-1}\sigma_i$, one has

$$
\dot{V} \leq -\sum_{k=1}^{N}c_{1k}|l_1\tau_k^{[\alpha]} + l_2\tau_k^{[\beta]}|^2 - \sum_{k=1}^{N}(\bar{k}_1|\xi_k|^{2\alpha} + \bar{k}_2|\xi_k|^{2\beta}) - \sum_{k=1}^{N}(k_3 - |\tilde{d}_i|)|\xi_i|, \tag{25}
$$

where $\bar{k}_1 = \epsilon k_1$ and $\bar{k}_2 = \epsilon k_2 - 2^{1-2\beta}\epsilon^{-1} - \epsilon\max_{k\in\mathcal{I}_N}\{\bar{c}_{2k}\}$.

Thus, the parameters can be chosen as

$$
\begin{aligned}
0 <\epsilon < & \frac{\lambda_2(\hat{L})}{N(\lambda_{\max}(\Gamma L L^T \Gamma) + 2\bar{c}_{1k})}, \\
k_2 > & 2^{1-2\beta}\epsilon^{-2} + \max_{k\in\mathcal{I}_N}\{\bar{c}_{2k}\}, \\
k_3 \geq & \bar{d} + \bar{h},
\end{aligned}
$$

such that $c_{1k}, \bar{k}_1, \bar{k}_2$ and $k_3 - |\tilde{d}_i|$ are all positive.

Then, from (25), one obtains

$$
\begin{aligned}
\dot{V} \leq & -\sum_{k=1}^{N} c_{1k}(l_1^2|\tau_k|^{2\alpha} + 2l_2 l_2|\tau_k|^{\alpha+\beta} + l_2^2|\tau_k|^{2\beta}) - \sum_{k=1}^{N}(\bar{k}_1|\xi_k|^{2\alpha} + \bar{k}_2|\xi_{ik}|^{2\beta}) \\
\leq & -c_0\sum_{k=1}^{N}(|\tau_k|^{\alpha} + |\tau_k|^{\beta})^2 - k_0\sum_{k=1}^{N}(|\xi_k|^{\alpha} + |\xi_k|^{\beta})^2,
\end{aligned} \tag{26}
$$

where $c_0 = \min_{k\in\mathcal{I}_N}\{c_{1k}l_1^2, c_{1k}l_1 l_2, c_{1k}l_2^2\}$ and $k_0 = \min_{k\in\mathcal{I}_N}\{\bar{k}_1/3, \bar{k}_2/3\}$. The last inequality is because $\bar{k}_1|\xi_k|^{2\alpha} + \bar{k}_2|\xi_k|^{2\beta} \geq k_0(3|\xi_k|^{2\alpha} + 3|\xi_k|^{2\beta}) \geq k_0(|\xi_k|^{2\alpha} + |\xi_k|^{2\beta} + 2|\xi_k|^{\alpha+\beta})$ due to $2\alpha < \alpha + \beta < 2\beta$.

On the other hand, from (18) one has

$$
\begin{aligned}
V_0 \leq & \, l_0\sum_{k=1}^{N}(|\tau_k|^{1+\alpha} + |\tau_k|^{1+\beta}) \\
\bar{V}_0 = & \, \epsilon\sum_{k=1}^{N} V_k \leq \epsilon\sum_{k=1}^{N}|\xi_k||v_k^* - \bar{v}_k| \leq \epsilon\sum_{k=1}^{N}2^{1-\beta}|\xi_k|^{1+\beta} \leq a_0\sum_{k=1}^{N}(|\xi_k|^{1+\alpha} + |\xi_k|^{1+\beta}),
\end{aligned} \tag{27}
$$

with $l_0 = \max_{k\in\mathcal{I}_N}\left\{\frac{l_1\gamma_k}{1+\alpha}, \frac{l_2\gamma_k}{1+\beta}\right\}$ and $a_0 = 2^{1-\beta}\epsilon$.

According to Lemma 7 and from (26), (27), one has

$$
\begin{aligned}
\dot{V} \leq & -\frac{c_0}{2l_0^{r_1}}V_0^{r_1} - \frac{c_0}{2l_0^{r_2}}(2N)^{\frac{1-\alpha}{1+\alpha}}V_0^{r_2} - \frac{k_0}{2a_0^{r_1}}\bar{V}_0^{r_1} - \frac{k_0}{2a_0^{r_2}}(2N)^{\frac{1-\alpha}{1+\alpha}}\bar{V}_0^{r_2} \\
\leq & -\bar{c}_0 V^{r_1} - \bar{k}_0 V^{r_2},
\end{aligned} \tag{28}
$$

where the Lemma 5 is utilized for the last inequality by noting $V = V_0 + \bar{V}_0$, $\bar{c}_0 = \min\left\{\frac{c_0}{2l_0^{r_1}}, \frac{k_0}{2a_0^{r_1}}\right\}$ and $\bar{k}_0 = 2^{1-r_2}(2N)^{\frac{1-\alpha}{1+\alpha}}\min\left\{\frac{c_0}{2l_0^{r_2}}, \frac{k_0}{2a_0^{r_2}}\right\}$ with $r_1 = \frac{\alpha+\beta}{\alpha+1}, r_2 = \frac{2\alpha}{\alpha+1}$.

Because $0 < r_1 < 1 < r_2$, by using Lemma 2, it can be obtained that $V(t)$ converges to 0 within a fixed T satisfying $T \leq \frac{1}{\bar{c}_0(1-r_1)} + \frac{1}{\bar{k}_0(r_2-1)}$. Furthermore, $V(t) = 0 \Rightarrow \tau_k = 0, \tilde{v}_k = 0 \Rightarrow \tilde{x}_k = \tilde{x}_j$, which means the FixF is achieved. The proof is completed. □

Remark 1. *We claim that the control input (15) is globally bounded. In fact, from (26) in the paper, one obtains that τ_k and $\bar{v}_k$ are bounded, which implies that $v_k^*, \xi_k, k \in \mathcal{I}_N$ are bounded. From (8), we have $\bar{v}_k = v_k^* - l_1\tau_k^{[\alpha]}$, which means that $\bar{v}_k$ is bounded. Besides, σ_k is bounded by noting that $\sigma_k = \dot{\tau}_k = \sum_{k=1}^{N} a_{kj}(\bar{v}_k - \bar{v}_j)$. By noting the expression of controller u_k (15), it is known that the control input is globally bounded.*

Remark 2. *From Theorem 1, it can be observed that the parameters l_1, l_2 in (8), (9) and k_1 in (15) can be chosen as any positive numbers. However, from (16), it can be seen that the control gains k_2 relies on the global information, viz., the spectrum of the Laplacian matrix and the outer degree. To remove such constraint, a new adaptive practical fixed-time controller will be proposed in the next subsection.*

3.2. Adaptive Practical Fixed-Time Formation Control Protocol

In this subsection, an adaptive practical FixF controller will be designed to adjust the control gain online.

An adaptive FixF control protocol is designed as follows:

$$u_i = -k_1\xi_i^{[2\alpha-1]} - w_i\xi_i^{[2\beta-1]} - k_3\mathrm{sign}(\xi_i) - \alpha l_1|\tau_i|^{\alpha-1}\sigma_i, \tag{29}$$

where k_1, l_1 are any positive constants, $k_3 \geq \bar{d} + \bar{h}$, $\frac{1}{2} < \beta < 1 < \alpha$, and τ_i, σ_i, ξ_i are defined in the above subsection. Moreover, $w_i, i \in \mathcal{I}_N$ are dynamic control gains, which are defined as

$$\dot{w}_i = |\xi_i|^{2\beta} - \ell w_i,\ w_i(0) = w_{i0}, i \in \mathcal{I}_N. \tag{30}$$

Theorem 2. *Consider the second-order disturbed system* (2)*. Suppose the Assumptions 1–3 hold and $k_3 > \bar{d} + \bar{h}$, then the FixF can be achieved under control protocol* (29)*.*

Proof. Consider the Lyapunov function candidate

$$W = V + \frac{1}{2}\epsilon\sum_{k=1}^{N}\tilde{w}_k^2, \tag{31}$$

in which V is defined in (17) and $\tilde{w}_k = w_k - \hat{w}_k$ with $\hat{w}_k$ being some positive constants to be designed later.

Following the same procedures in the proof of Theorem 1, one obtains

$$\begin{aligned}\dot{V} \leq & -\sum_{k=1}^{N} c_{1k}|l_1\tau_k^{[\alpha]} + l_2\tau_k^{[\beta]}|^2 + \sum_{k=1}^{N}(2^{1-2\beta}\epsilon^{-1} + \epsilon\bar{c}_{2k})|\xi_k|^{2\beta} + \epsilon\xi^T\dot{v}^*, \\ \leq & -\sum_{k=1}^{N} c_{1k}|l_1\tau_k^{[\alpha]} + l_2\tau_k^{[\beta]}|^2 - \sum_{k=1}^{N}((\epsilon w_k - 2^{1-2\beta}\epsilon^{-1} - \epsilon\bar{c}_{2k})|\xi_k|^{2\beta} + \epsilon k_1|\xi_k|^{2\alpha}).\end{aligned} \tag{32}$$

The last inequality is due to $-\sum_{k=1}^{N}(k_3 - |\tilde{d}_i|)|\xi_i| \leq 0$.

One can choose $\hat{w}_k = 2^{1-2\beta}\epsilon^{-2} + \bar{c}_{2k} + \epsilon^{-1}\bar{k}_2$ and let $\bar{k}_1 = \epsilon k_1$. Then, following the same steps in the proof of Theorem 1, one has

$$\begin{aligned}\dot{W} \leq & -\sum_{k=1}^{N} c_{1k}|l_1\tau_k^{[\alpha]} + l_2\tau_k^{[\beta]}|^2 - \sum_{k=1}^{N}(\bar{k}_1|\xi_k|^{2\alpha} + \bar{k}_2|\xi_k|^{2\beta}) - \epsilon\ell\sum_{k=1}^{N}\tilde{w}_k w_k \\ \leq & -\bar{c}_0V^{r_1} - \bar{k}_0V^{r_2} - \epsilon\ell\sum_{k=1}^{N}\tilde{w}_k w_k,\end{aligned} \tag{33}$$

where $\bar{c}_0, \bar{k}_0, r_1$ and r_2 are designed as before.

Because $-\tilde{w}_k w_k = -\tilde{w}_k(\tilde{w}_k + \hat{w}_k) = -\tilde{w}_k^2 - \tilde{w}_k\hat{w}_k \leq -\frac{1}{2}\tilde{w}_k^2 + \frac{1}{2}\hat{w}_k^2$ and $V^{r_1} + V^{r_2} \geq V$, one obtains

$$\begin{aligned}\dot{W} \leq & -\bar{c}_0V^{r_1} - \bar{k}_0V^{r_2} - \epsilon\ell\sum_{k=1}^{N}\tilde{w}_k w_k \\ \leq & -\min\{\bar{c}_0, \bar{k}_0\}V - \frac{1}{2}\epsilon\ell\sum_{k=1}^{N}\tilde{w}_k^2 + \frac{1}{2}\epsilon\ell\sum_{k=1}^{N}\hat{w}_k^2 \\ \leq & -\min\{\bar{c}_0, \bar{k}_0, 1\}W + \frac{1}{2}\epsilon\ell\sum_{k=1}^{N}\hat{w}_k^2.\end{aligned} \tag{34}$$

By noticing that $\hat{w}_k$ are constants, it can be concluded that W is bounded, which implies $\tilde{w}_k$ is bounded, viz., there exists a constant Δ such that $\sum_{k=1}^N \tilde{w}_k^2 \leq \Delta$.

Moreover,

$$
\begin{aligned}
\dot{W} \leq & -\bar{c}_0 V^{r_1} - \bar{k}_0 V^{r_2} - \epsilon\ell \sum_{k=1}^N \tilde{w}_k w_k \\
\leq & -\bar{c}_0 V^{r_1} - \bar{k}_0 V^{r_2} - \frac{1}{2}\epsilon\ell \sum_{k=1}^N \tilde{w}_k^2 + \frac{1}{2}\epsilon\ell \sum_{k=1}^N \hat{w}_k^2 \\
\leq & -\bar{c}_0 V^{r_1} - \bar{c}_0 \left(\frac{1}{2}\epsilon \sum_{k=1}^N \tilde{w}_k^2\right)^{r_1} - \bar{k}_0 V^{r_2} - \bar{k}_0 \left(\frac{1}{2}\epsilon \sum_{k=1}^N \tilde{w}_k^2\right)^{r_2} \\
& + \bar{c}_0 \left(\frac{1}{2}\epsilon \sum_{k=1}^N \tilde{w}_k^2\right)^{r_1} + \bar{k}_0 \left(\frac{1}{2}\epsilon \sum_{k=1}^N \tilde{w}_k^2\right)^{r_2} - \frac{1}{2}\epsilon\ell \sum_{k=1}^N \tilde{w}_k^2 + \frac{1}{2}\epsilon\ell \sum_{k=1}^N \hat{w}_k^2 \\
\leq & -\bar{c}_0 W^{r_1} - 2^{1-r_2}\bar{k}_0 W^{r_2} + s,
\end{aligned}
\tag{35}
$$

where

$$
s = \begin{cases} \frac{1}{2}\epsilon\ell \sum_{k=1}^N \hat{w}_k^2, & \text{if } \bar{c}_0 \left(\frac{1}{2}\epsilon \sum_{k=1}^N \tilde{w}_k^2\right)^{r_1} + \bar{k}_0 \left(\frac{1}{2}\epsilon \sum_{k=1}^N \tilde{w}_k^2\right)^{r_2} - \frac{1}{2}\epsilon\ell \sum_{k=1}^N \tilde{w}_k^2 \leq 0 \\ \bar{c}_0 \epsilon^{r_1} \Delta^{r_1} + \bar{k}_0 \epsilon^{r_2} \Delta^{r_2} + \frac{1}{2}\epsilon\ell \sum_{k=1}^N \hat{w}_k^2, & \text{otherwise} \end{cases}.
$$

Based on Lemma 3, we can conclude that the system in (6) is practical FixS. Furthermore, we calculate the residual set as $\{\lim_{t\to T} z | W(z) \leq \min\{\bar{c}_0^{-1/r_1}(s/(1-\theta))^{1/r_1}, (2^{1-r_2}\bar{k}_0)^{-1/r_2}(s/(1-\theta))^{1/r_2}\}$ with $\theta \in (0,1)$. The time needed to reach the residual set is bounded as $T \leq \frac{1}{\bar{c}_0\theta(1-r_1)} + \frac{1}{2^{1-r_2}\bar{k}_0\theta(r_2-1)}$. The proof is completed. □

Remark 3. *By invoking the tools of Kronecker product, the theoretical result obtained here can be extended to MASs with multidimensional dynamics directly.*

Remark 4. *In this work, some practical factors are considered such as disturbance, convergence time, and the dependence of global information. In fact, there are many other practical but important factors worth considering. For example, actuator failure, communication switching, collision, and deadlock [37–40]. We will consider these interesting problems in our future research.*

4. Simulations

In this section, a numerical simulation is provided to show the performance of the proposed adaptive control protocol.

Consider a MASs with six agents, where $x_i = (x_{i1}, x_{i2})^T, v_i = (v_{i1}, v_{i2})^T$ and $d_i = (0.5\cos(it+1), 0.2 - 0.5\sin(it))^T, i \in \mathcal{I}_6$. The interaction topology is represented by the strongly connected directed graph shown in Figure 1. The desired formation is a regular hexagon and the desired trajectory of the formation center is "O": $(10\sin(\frac{\pi}{4}t), 10\cos(\frac{\pi}{4}t))^T$. Furthermore, in the desired formation, the relative displacements of each agent to the formation center are given as $(-2,0)^T$, $(-1,\sqrt{3})^T$, $(1,\sqrt{3})^T$, $(2,0)^T$, $(1,-\sqrt{3})^T$, $(-1,-\sqrt{3})^T$. Control parameters are chosen as $\alpha = 1.3, \beta = 0.7, k_1 = 5, k_3 = 6.2, l_1 = 5, l_2 = 2, \ell = 0.5$. Figure 2 depicts the adaptive gains. It can be observed that all the adaptive gains will be bounded. The trajectories of the 6 agents are depicted in Figure 3. From Figure 3, it is seen that when $t = 0.5s$, the formation is not formed; when $t = 4s$, the formation is achieved and maintained. The formation trajectory tracking errors $p_i - h_i$ and $v_i - \dot{h}_i$ are shown in Figures 4 and 5, where $p_i - h_i$ and $v_i - \dot{h}_i$ reach the same value very fast. Another interesting thing is that $v_i - \dot{h}_i$ converges to zero finally, while $x_i - h_i$ converges to some steady but nonzero value. In fact, what matters most is $(x_i - h_i) - (x_j - h_j)$. $x_i - h_i$ is not required to reach zero.

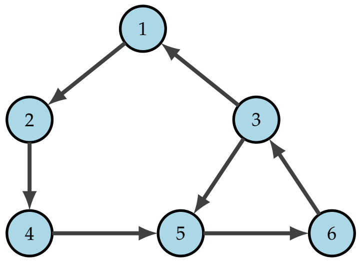

Figure 1. The communication graph $\mathcal{G}$.

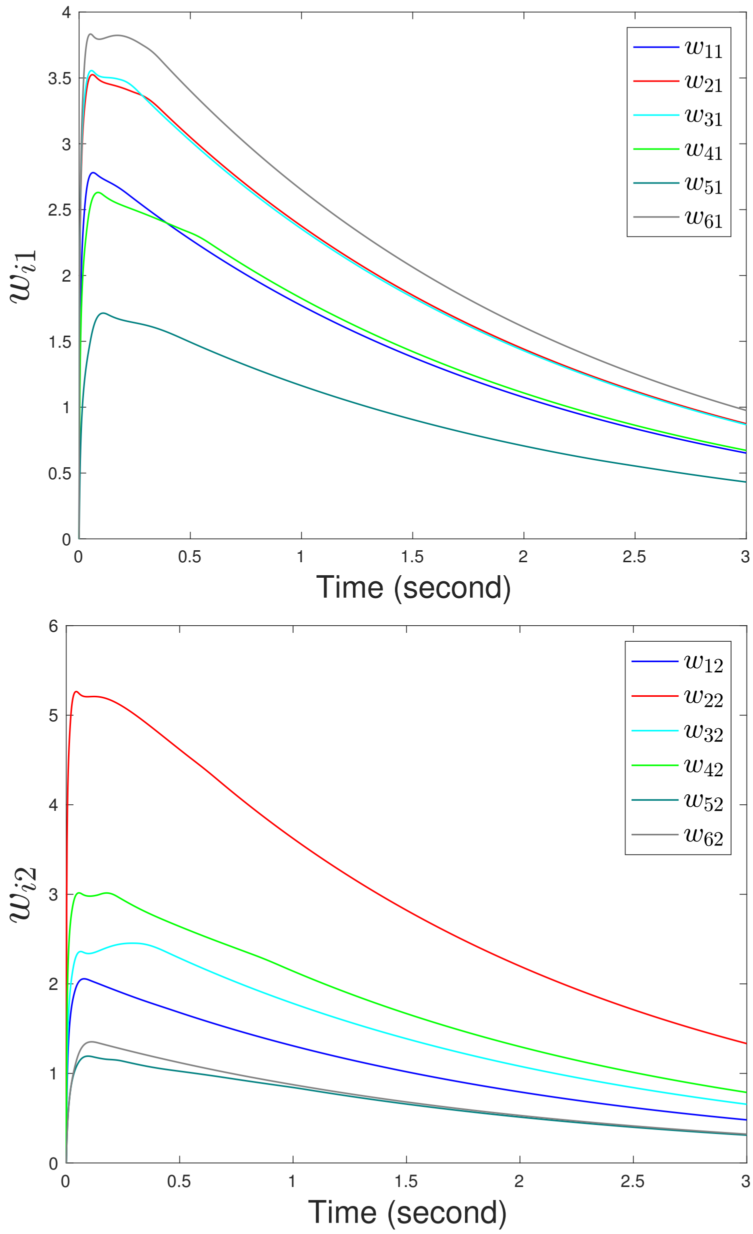

Figure 2. The trajectory of adaptive control gain. **Above**: the first dimension ω_{i1}; **below**: the second dimension ω_{i2}.

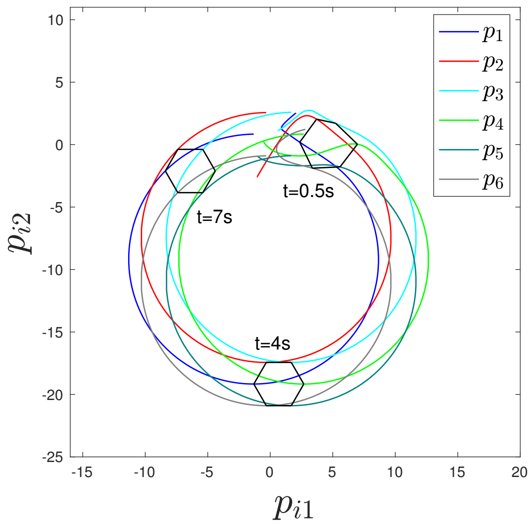

Figure 3. The trajectories of agents.

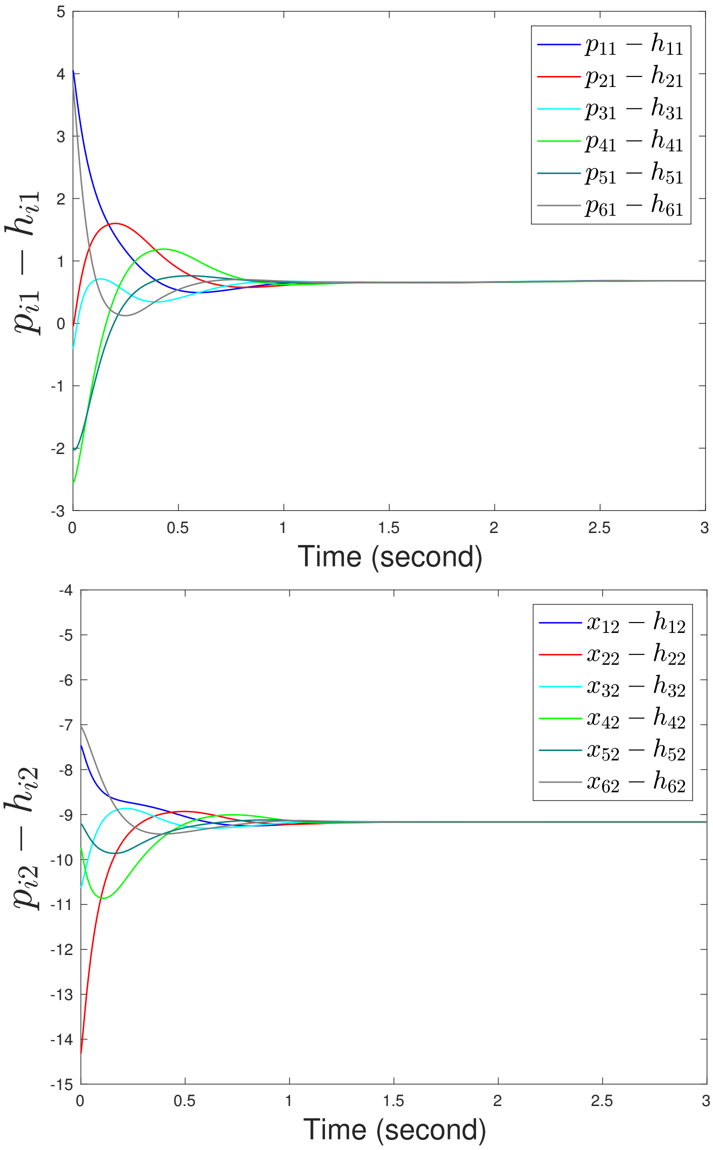

Figure 4. Formation trajectory tracking error: $p_i - h_i, i \in \mathcal{I}_6$. **Above**: the first dimension $p_{i1} - h_{i1}$; **below**: the second dimension $p_{i2} - h_{i2}$.

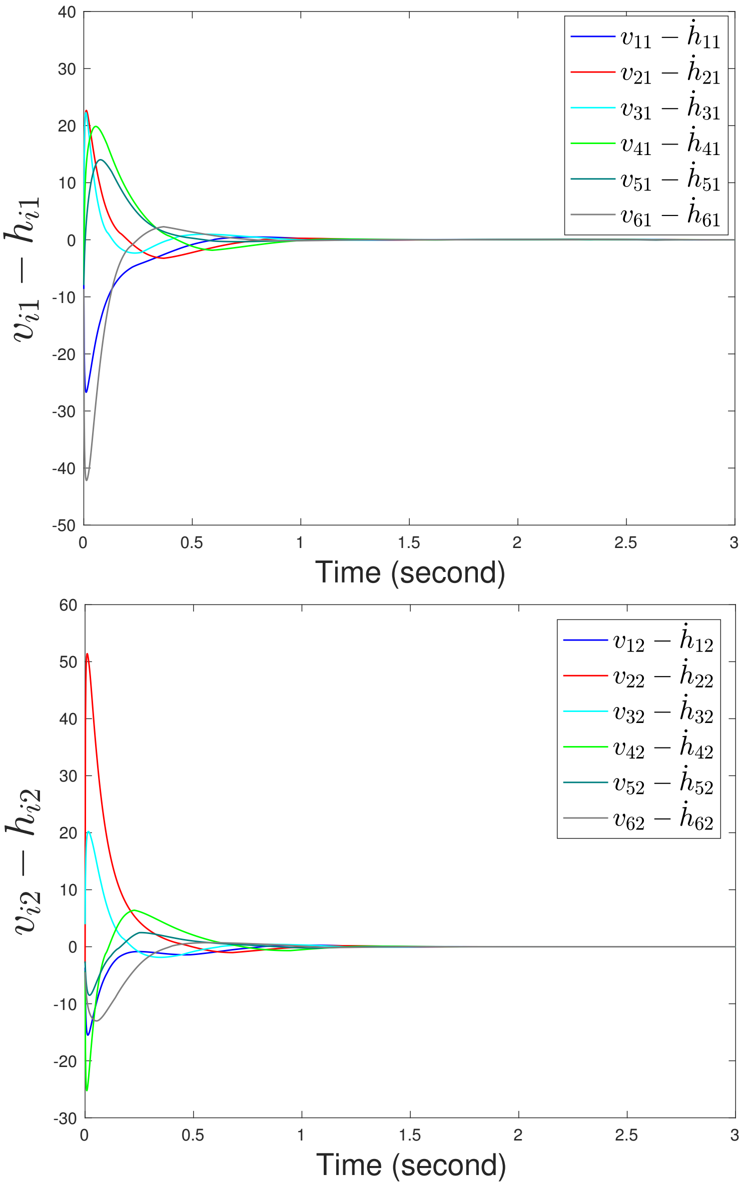

Figure 5. Formation trajectory tracking error: $v_i - h_i, i \in \mathcal{I}_6$. **Above**: the first dimension $v_{i1} - h_{i1}$; **below**: the second dimension $v_{i2} - h_{i2}$.

5. Conclusions

The FixF problem for a kind of second-order MASs with disturbances over general directed graphs has been resolved in this paper. A novel FixF control protocol has been designed based on backstepping method, and the convergence time can be estimated directly, which is uniformly bounded. Then, to remove the dependence of control gains on global conditions, an adaptive practical FixF control protocol has been presented, which is fully distributed. In fact, the results can be generalized to more general directed graphs that just contain a directed spanning tree [9]. Future work includes considering FixF of general nonlinear MASs with directed communication topologies. Another research direction is to design a fully distributed FixF control protocol to achieve an accurate convergence for second-order or higher-order MASs by noticing that the errors converge to a small region in this paper.

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