Article

Racetrack Microtron—Pushing the Limits

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Abstract: We consider three types of electron accelerators that can be used for various applications, such as industrial, medical, cargo inspection, and isotope production applications, and that require small- and medium-sized machines, namely classical microtron (CM), race-track microtron (RTM), and multisection linac. We review the principles of their operation, the specific features of the beam dynamics in these machines, discuss their advantages and weak points, and compare their technical characteristics. In particular, we emphasize the intrinsic symmetry of the stability region of microtrons. We argue that RTMs can be a preferable choice for medium energies (up to 100 MeV) and that the range of their potential applications can be widened, provided that the beam current losses are significantly reduced. In the article, we analyze two possible solutions in detail, namely increasing the longitudinal acceptance of an RTM using a higher-order harmonic accelerating structure and improving beam matching at the injection.

Keywords: electron accelerator; classical microtron; racetrack microtron; multisection LINAC; longitudinal phase space

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1. Introduction

Since its invention, the pulsed racetrack microtron (RTM) has been the attracting attention of potential users due to the possibility of its use resulting in obtaining an accelerated electron beam of sufficiently high energy using a seemingly simple, compact, and quite economic machine. However, the number of RTMs built and that are in operation is incomparably less than the number of linear accelerators, which have an average accelerated beam current that is too low for many potential applications. In the article, we analyze the problems that led to this situation and outline solutions that are aimed at increasing the average current of RTMs.

The idea of a classical microtron (CM) (Figure 1a) was first proposed in 1944 in the pioneering article by V.I. Veksler that was devoted to the principle of phase stability in cyclic accelerators [1]. The idea of an RTM (Figure 1b), as indicated in Ref. [2], belongs to J.S. Schwinger. The main period of CM and RTM development fell within the 1960s–1990s of the last century. There is a number of books and review articles in which various versions of these types of accelerators are described in detail and where references to original articles are given, see, for example, Refs. [3–7]. The majority of CM and RTM applications are in the fields of nuclear physics research, radiation therapy, radiography, injection into synchrotrons, gamma-activation analysis, research on the production of medical isotopes, and the detection of explosives. The pinnacle of microtron development, which is unlikely to be surpassed because of the development of the technology of superconducting accelerating

structures, is the MAMI cascade of three RTMs and one double-sided continuous wave (CW) microtron with normal conducting accelerating structures, which have a final energy of 1.5 GeV and an average current of 100 μA for fundamental research in the field of nuclear physics [8–11].

Figure 1. (a) Classical microtron. (b) Racetrack microtron. (c) Multisection linear accelerator. (1)—electron gun (electron source), (2)—accelerating structure (resonator), (3)—RF energy source (magnetron, klystron) with a high-voltage power supply (modulator), (4)—bending magnet(s), (5)—beam extraction magnet (or system), (6)—injection magnet, (7)—quadrupole lenses.

In this article, we compare CMs, RTMs, and multisection linear accelerators (LINACs), such as those shown schematically in Figure 1, in terms of design complexity, dimensions, cost, and capability of achieving the main beam parameters such as energy, energy spread, pulse and average current, and beam stability. The ultimate goal of this analysis is to formulate lines of improvement for pulsed RTMs that would allow them to produce beams with an average beam current that is higher than 100 μA and with an energy of several tens of MeV; these parameters are necessary for the production of medical isotopes through the use of photonuclear reactions [12–14]. Particular emphasis can be placed on methods for reducing beam current losses during acceleration by increasing the longitudinal acceptance and the better matching of the injected longitudinal beam emittance with the RTM acceptance.

This paper is based on the experience of developing and constructing various types of RTM at the Skobeltsyn Institute of Nuclear Physics at Lomonosov Moscow State University in cooperation with research groups from Russia and other countries [10,15–20].

2. Comparison of Classical and Racetrack Microtrons

2.1. RTM Advantages

The following features are usually indicated when comparing CM and RTM: Because of the design geometry in a CM, only one resonator with a moderate electric field strength can be installed (see Figure 1a); therefore, the synchronous energy gain per turn ΔE_s is quite low and is typically equal to the electron rest mass, $\Delta E_s \approx 0.5 \text{ MeV}$.

In the case of the CMs and RTMs for the resonance conditions of acceleration to be fulfilled, the magnetic field B_0 of the bending magnet and the synchronous energy gain per turn must be related through the following formula [3–5]:

$$B_0 = \frac{2\pi\Delta E_s}{evc\lambda}, \quad (1)$$

where e is the elementary electric charge, λ is the wavelength of the accelerating field in free space, ν is the increase of the harmonic number per turn, and c is the speed of light. We would like to note that ν is essentially the number of the accelerating field wavelengths

by which the length of the orbit increases from turn to turn. The maximum width of the region of stable phase oscillations is achieved for $\nu = 1$ [21]. Thus, for a typical value of $\lambda = 0.1$ m, the field in the CM magnet is $B_0 \approx 0.1$ T. For an energy of, for example, 50 MeV, the number of orbits will be close to 100, the last orbit radius will be approximately 1.7 m, and the outer diameter of the magnet will be about 4 m.

In practice, CMs are operated at larger value of the synchronous energy gain, which makes it possible to decrease the dimensions of the accelerator slightly. However, this also reduces the stability of the resonator operation, and for an energy of tens of MeV, the dimensions of the accelerator and the number of orbits are still large.

In an RTM, a multi-cavity accelerating structure can be placed in the drift space between the bending magnets (see Figure 1b) so that the energy gain per turn can be increased considerably. Therefore, if, for example, $\Delta E_s = 5$ MeV, $\nu = 1$, $\lambda = 0.1$ m, and the final energy is 50 MeV, then the magnetic field will increase to 1 T, and consequently, the maximum orbit radius will decrease to 0.17 m. Thus, the RTM design makes it possible to build significantly more compact accelerators that are able to produce higher energy beams.

Another advantage of RTMs is the possibility of implementing various beam focusing schemes. In CMs, the problem of focusing in the vertical plane (a plane perpendicular to the plane of the orbits) is partially solved by introducing RF quadrupole focusing by means of elliptical resonator beam holes [22,23]. The main disadvantages of this approach are the dependence of the optical power on the phase of the particle relative to the accelerating field that leads to the coupling of the longitudinal and transverse planes and the appearance of defocusing in the horizontal plane.

RTM design creates different possibilities for electron injection, thus being more flexible than the CM design. In fact, a large number of injection schemes for both low-energy beams from an electron gun and relativistic beams from a linear pre-accelerator can be implemented in RTMs. To stress the importance of optimal injection, we would like to note that CMs became widespread in various applications after the invention of new injection schemes in the 1960s and 1970s that made it possible to increase the accelerated beam current significantly [24].

Finally, it is straightforward to obtain electrons of different energies in an RTM by simply extracting the beam from the different orbits; a review of these extraction schemes can be seen in Ref. [4].

2.2. Common Problems of CM and RTM

CM and RTM share a number of common problems. The most serious one is the narrow stable phase oscillation region. For $\nu = 1$, the region of stability is at its maximum for the synchronous phase $\varphi_s = 16^\circ$ and is of only 32° width in phase, whereas its size in energy is given by $|\delta E_n / \Delta E_s| < 0.05$. Here, $\delta E_n = E_n - E_{s,n}$ is the deviation of the beam energy at the n th orbit from the synchronous particle energy $E_{s,n} = E_{inj} + n\Delta E_s$, and E_{inj} is the injection energy.

The problem that is caused by the stable phase oscillation region being narrow is aggravated by the presence of nonlinear resonances. The most dangerous nonlinear resonances causing beam losses are the third-order and forth-order resonances situated for $\nu = 1$ at the synchronous phase values $\varphi_s = 25.5^\circ$ and $\varphi_s = 17.7^\circ$, respectively. If the number of orbits is not small, 10 or more, and if φ_s is close to the resonance, then the stable phase oscillation region narrows sharply [25–27]. Previous studies have also revealed quite non-trivial properties of the stable phase oscillation region in RTMs. It turns out to be symmetrical with respect to certain symmetry lines, whose shape is determined by the accelerating voltage as a function of the phase. Additionally, the stability region includes stable elliptic islands that are separated from the longitudinal acceptance which, in terms of the theory of dynamical systems, is a connected to a local stability domain that includes the synchronous trajectory (see Ref. [27] for details).

Within the framework of the present work, we studied a possibility of expanding the region of stable phase oscillations by linearizing the time dependence of the accelerating field by adding a second harmonic (see Section 4.1).

Another problem that is common for both types of accelerators is a high-current longitudinal instability that can occur for a high total load of the resonator or accelerating structure by the beam current [28–30]. To a certain extent, this instability is similar to the Robinson instability in synchrotrons [31], but there are also significant differences: first, in CMs and RTMs the accelerating structure is simultaneously loaded by the beam current from all of the orbits and second, the stable phase oscillation region in the microtrons is considerably smaller than that of the synchrotrons.

In Ref. [30], a model describing the behavior of a beam current-loaded accelerating structure fed by a klystron and longitudinal beam dynamics in an RTM is considered. Computer simulations show that the threshold current is 20–30 mA with an instability development time of the order of 1 μ s for a 70 MeV pulsed RTM with 14 orbits. The threshold current can be increased by reducing the effective shunt impedance of the accelerating structure, diminishing the number of orbits, optimizing the coupling factor of the accelerating structure with the feeding waveguide, and increasing the size of the effective aperture (i.e., the aperture that takes into account field errors and the misalignments of machine elements) in the horizontal plane on the return orbits.

Finally, the acceleration of bunches with a large charge and a small phase length both in CMs and RTMs leads to an increase of the beam emittance due to the coherent synchrotron radiation in the bending magnets [32]. According to results obtained in Refs. [16,32] for the wavelengths of $\lambda \sim 0.1$ m, if no special measures to suppress this radiation are taken, then the upper limit of the bunch charge for which no significant increase of the emittance occurs is about 10 pC. This corresponds to a pulse current of approximately 30 mA. The suppression of the coherent synchrotron radiation can be achieved by reducing the height of the vacuum chamber section inside of the bending magnets to a value close to the bunch length [33]. However, in this case, the vertical aperture may decrease, leading to a decrease of the transverse acceptance of the accelerator.

2.3. RTM Problems

As it has been explained above, dividing the CM magnet into two 180° dipoles and separating them by a drift space allows a multi-cavity accelerating structure to be placed within a common orbit. However, this also leads to new problems that complicate the beam dynamics and that limit the value of the accelerated beam current in RTMs. We consider these problems below.

2.3.1. Beam Defocusing and Distortion by the Magnet Fringe Field

In contrast to the CM magnet, the RTM magnets have a fringe field that exerts a strong defocusing effect in the vertical plane on the beam. To compensate it, an additional narrow dipole magnet is installed at the entrance of the main bending magnet that has a field of polarization that is opposite to that of the main field [34]. The focal length f_0 of this magnetic system is determined by the expression [8]:

$$\frac{1}{f_0} = \frac{\int_0^l (B(z)/B_0)^2 dz - \int_0^l (B(z)/B_0) dz}{R^2} \quad (2)$$

where $B(z)$ is the magnetic field profile as a function of the distance $z = 0 \dots l$ in the direction orthogonal to the edge of the magnet, and B_0 and R are the value of the field and bending radius in the region of the uniform field of the main dipole, respectively.

By properly selecting the position, shape, and magnitude of the inverse field, it is possible to adjust the focal length of the 2-dipole magnetic system over a wide range. It is also possible to achieve the closure of the orbit and reflection of the beam back to the accelerating structure after the first acceleration, and in this way, it is possible solve

the problem of the beam bypassing the accelerating structure in RTMs; see Ref. [4] for more details.

The beam defocusing the fringe field also causes the nonlinear distortion of the beam phase space due to aberrations and an increase in the vertical effective emittance. One of the manifestations of this effect is the contribution to the deviation of the quadratic focal length from the median plane given by the formula [8]:

$$\frac{1}{f_1} = \frac{2y^2 \int_0^l (B'(z)/B_0)^2 dz}{3R^2} \quad (3)$$

where y is the particle vertical displacement and $B'(z)$ is derivative of the magnetic field profile. It is clear that generally, the aberrations in the magnetic system with an inverse dipole will be much stronger than in an ordinary main bending magnet. Indeed, the integrand is positive, and in the former case, there are two regions with varying magnetic fields and therefore a non-zero derivative, namely the main magnet fringe field region and the inverse field one, whereas in the latter case, there is just one fringe field region.

The contribution of aberrations to the emittance growth is determined by the ratio of the vertical beam size σ_y to the pole gap height h . For example, in the cascade of high energy CW RTMs [8–11], $\sigma_y \ll h$, and as a consequence, the aberrations are small. However, an increase in the pole gap height leads to a proportional increase of the current in the magnet coils and an increase in the size and mass of the magnet that is not acceptable in the case of accelerators for applied purposes because it results in a significant increase in the size and cost of the whole machine. Therefore, for energies of up to 100 MeV in pulsed RTMs, a usual choice is $h = 2\text{--}3\text{ cm}$ with the vertical beam size in the region of the fringe field reaching $\sigma_y \approx 1\text{ cm}$, leading to strong nonlinear distortions of the phase space and noticeable losses of the beam current.

As an example, in Figure 2, the results of the numerical simulations of the beam dynamics in the 55 MeV RTM exit are shown [18]. In this case, the pole gap height of the magnets is $h = 2\text{ cm}$. Nonlinear distortions of the phase space are clearly seen in the vertical plane.

Figure 2. Results of numerical simulations of the beam dynamics of the 55 MeV RTM [18]; (a) phase space in the vertical plane, (b) beam spot at the RTM exit. The pole gap height of the magnet is 2 cm.

The results discussed here show that when choosing the shape of the fringe field, one should minimize the aberrations given by Equation (3). Moreover, the pole gap height should be at least 3 cm, and the optics of the RTM should be optimized with the

minimization criteria for the vertical beam size and should be in the region of the fringe field of the magnets.

2.3.2. Phase Slip

Because of the presence of the drift space between the magnets, electrons with even a slight deviation in their velocity from the speed of light experience a significant phase slip with respect to the synchronous phase. The slip decreases from orbit to orbit, as the electron energy increases and approaches ultrarelativistic values. In addition, the fringe field of the bending magnets causes a deviation in the electron trajectory from the ideal circular field, which also contributes to the phase shift that changes from orbit to orbit. The variable phase slip in the drift space and phase shift in the fringe field limit the minimum allowable energy of electrons in the first orbit, which is determined by the length of the drift space and the shape of the fringe field. As shown in Refs. [35,36], if the energy of the electrons in the first orbit exceeds this threshold value, then it is possible to provide an asymptotic approach of the phase of a particle to the synchronous phase φ_s through the correct choice of the injection phase.

Mitigating the phase slip effect of a low-energy beam injected from an electron gun can be achieved in a scheme with reflection back to the beam in the bending magnet after the first acceleration and the subsequent acceleration of that beam to an energy that is approximately equal to twice the synchronous energy gain. The beam of this energy will be further accelerated with less phase slip. The exact value of the energy and injection phase of bunches into the microtron acceleration regime are controlled by the choice of the position of the bending magnet that reflects the beam (see, for example, [4] and references therein).

A phase shifter installed in the first return orbit can be used to correct the phase slip effect. Since insufficient beam energy leads to a lag of the bunch phase with respect to the optimal one, such a device should change the phase by more than 360° . A simple chicane cannot provide a phase shift that is this large. In Ref. [37], a phase shifter that was able to control the bunch phase over a large range was described, and its successful implementation in the 70 MeV RTM was reported in Ref. [17].

2.3.3. Errors in Magnets Manufacturing and Magnet Poles Positioning

The poles of a CM magnet can be manufactured and positioned relative to each other with high accuracy that, in principle, allows high field uniformity to be obtained in the entire working area and a high degree of symmetry with respect to the vertical plane passing through the center of the resonator gap and the center of the magnet. Nevertheless, in practice, a problem related to the non-uniformity of the field exists due to inaccuracies in the manufacturing and assembling of the magnet, the effects of steel saturation, the inhomogeneity of its magnetic properties, and the deformation of the poles under the influence of atmospheric pressure as well as other factors that require the use of correcting coils or the introduction of shims [7].

In the case of RTMs, the situation is noticeably more complicated. In the presence of a drift space of about 1 m, an error in the bending angle of 1 mrad caused by the first magnet of an electron after crossing the drift space arrives at the entrance of the second magnet with a displacement of about 1 mm. Such displacements grow from turn to turn, resulting with a complete beam loss. It is important to keep in mind that in a 180° bending magnet, an error of 1 mrad means a relative difference of the field integrals along the two halves of the trajectory in the magnet at a level of just 3×10^{-4} . Usually, in RTMs for applied purposes, the field inhomogeneity that is primarily caused by edge effects and a strayed field in the drift space is noticeably higher [38]. In addition to errors in the bending angle, other errors that are caused by imprecisions in the magnet positioning are of great importance.

In RTMs, beam steering along the orbits is often provided by pairs of steering coils that displace the beam in both the horizontal and vertical planes. To correct both the offset and slope of the beam trajectory, two pairs of coils need to be installed in each orbit. This

inevitably leads to the appearance of apertures in the drift space between the magnets, on which the beam current can be lost. Keep in mind that these are sections with a significant increase in the horizontal beam size due to the dispersion.

The mitigation of the effects of these errors is achieved by increasing the accuracy of manufacturing and assembling the magnets and the supports on which they are positioned, the better alignment of the magnets, improving the field uniformity, and expanding the uniform field region by shimming. Active and passive magnetic screens to control the strayed field in the drift space are also used. Other important aspects of the design, building, and adjusting of an RTM magnetic system include high accuracy field measurements in manufactured magnets and beam tracing in the measured fields that is conducted using existing programs for numerical simulations of the beam dynamics that take into account the position of the axis of the accelerating structure [38]. If necessary, the pole shimming is used as a local field correction technique. Finally, the design of the beam steering coils and their placement should only limit the aperture in the horizontal and vertical planes minimally.

2.3.4. Beam Blow-Up

Along with the aforementioned longitudinal instability, which can arise in both CMs and RTMs, the RTM may also exhibit transverse instability, namely a beam blow-up that is associated with an excitation of parasitic modes with a transverse magnetic field on the axis in the accelerating structure. The threshold current for this instability depends on the effective shunt impedance of the parasitic modes, the optics of the RTM, and a number of other parameters [39,40].

Accurate estimates of the threshold current and for the time in which instability develops for specific characteristics of the RTM and its accelerating structure can be obtained by numerical simulations using codes such as, for example, HBBU (stationary case) [40] and TDBBU (time evolution of the process) [41]. In Ref. [40], the following formula is given for the conservative estimate of the threshold current for the stationary case obtained under the assumption of the resonance between the parasitic mode and transverse beam oscillations:

$$I_{thr} = \frac{4\Delta E_s \lambda_{tr}}{r_{tr} \pi L \beta_f} \times \frac{1}{N \ln(E_f/E_i)} \quad (4)$$

where λ_{tr} is the wavelength of the parasitic mode with a transverse magnetic field on the axis, r_{tr} is its effective shunt impedance per unit length, L is the length of the accelerating structure, β_f is the value of the RTM beta-function at the center of the accelerating structure, N is the number of orbits, E_i is the injection energy, and E_f is the energy at the exit of the accelerator. The beta function depends on the RTM's optical scheme, which should have the relationship $\beta_f \cong (0.5 - 2)C$, where C is the orbit length, which can be used for estimates.

As an example, let us consider the 55 MeV pulsed RTM described in Ref. [18]. Its parameters are $\Delta E_s = 5$ MeV, $L = 0.4$ m, $N = 9$, $\beta_f = 3$ m, $\lambda_{tr} = 0.06$ m, $r_{tr} = 18$ M Ω /m, $E_f = 55$ MeV, and $E_i = 10$ MeV in the RTM mode. In this case, after the first acceleration, a 5 MeV beam is reflected back into the accelerating structure by the bending magnet and is consequently accelerated to 10 MeV energy. Equation (4) provides the estimate $I_{thr} \approx 7$ mA. A more accurate result for the threshold current obtained in Ref. [40] through the numerical simulations with the real optics parameters for the RTM produces a value that exceeds the conservative estimate by about an order of magnitude. Hence, in the design of a concrete RTM project, in order to achieve estimates for the current and time of instability development thresholds, it is advisable to rely on numerical simulations of the beam blow-up phenomenon.

3. Comparison of RTM and LINAC

An RTM can be regarded as a LINAC with non-isochronous beam recirculation in many aspects. In this respect, an RTM with N orbits replaces a LINAC with an accelerating structure with N sections, each section being fed by a separate klystron (see Figure 1b,c). Thus, there is an obvious gain in the size of the accelerator and, possibly, in its cost, since the cost of a klystron, its power supply, a modulator, and the accelerating structure make a decisive contribution to the cost budget of the entire machine. Of course, normally conducting electron LINACs for energies up to 100 MeV, as a rule, are built following a different scheme, namely less but more powerful klystrons and longer accelerating sections are used. Moreover, most LINACs operate in a traveling wave mode; however, this makes little difference on the results in the analysis of RF power consumption by these two types of accelerators given below.

Beam blow-up caused by the excitation of the hybrid modes with a transverse magnetic field on the axis can also occur in LINACs [42]. However, for the range of energies of the accelerated beam that is discussed here and, accordingly, for a small number of relatively short accelerating sections, the threshold current turns out to be a few hundreds of milliamperes, especially if dedicated suppression measures are taken for the parasitic modes.

An important advantage of RTMs over linear accelerators is high energy stability and the energy spectrum of the accelerated beam, which is quite narrow ($\sim 10^{-3}$), which are due to specific features of the longitudinal dynamics of the RMT. Achieving similar values in a LINAC requires a complex injection system that forms short bunches and an expensive RF system with high parameter stability. We would like to note that energy stability and a narrow energy spectrum are important for accelerators that are dedicated to the production of isotopes where a magnetic system for switching the beam between targets is required.

When estimating the gain in the cost of the accelerator, one must take into account the total consumption of the RF power for the beam acceleration. In the case of RTM, the RF power consumption that is required to build-up the accelerating field is given by the formula:

$$P_w^{RTM} = \frac{\left(\frac{\Delta E_s}{e \cos \varphi_s}\right)^2}{Z_e L} \quad (5)$$

where Z_e is the effective shunt impedance of the fundamental mode per unit length.

For a LINAC, which shown in Figure 1c, with an acceleration that is in the phase of the maximum field

$$P_w^{Linac} = N \frac{\Delta E_s^2}{e^2 Z_e L} \quad (6)$$

The total RF power consumption for the accelerating field that is building-up and the beam acceleration neglecting current losses is given by

$$P_\Sigma^{RTM} = \frac{\left(\frac{\Delta E_s}{\cos \varphi_s}\right)^2}{e^2 Z_e L} + P_b \quad (7)$$

$$P_\Sigma^{Linac} = N \frac{(\Delta E_s)^2}{e^2 Z_e L} + P_b \quad (8)$$

where $P_b = (N \Delta E_s I_b) / e$ is the pulse beam power, and I_b is the pulse beam current.

The electronic efficiencies of the RTM and LINAC are determined by the expressions:

$$\eta_e^{RTM} = P_b / P_\Sigma^{RTM} \quad (9)$$

$$\eta_e^{linac} = P_b / P_\Sigma^{linac} \quad (10)$$

If we take the following typical RTM values as an example: $\Delta E_s = 5$ MeV, $N = 10$, $Z_e = 80$ M Ω /m, $L = 0.4$ m, and $\varphi_s = 16^\circ$, then we obtain the plots of η_e^{RTM} , η_e^{Linac} and P_Σ^{RTM} , $P_\Sigma^{Linac} / P_\Sigma^{RTM}$ as functions of the beam current, as shown in Figure 3a,b.

Figure 3. (a) Electronic efficiency of the RTM and LINAC as functions of the pulse beam current I_b ; (b) ratio $P_\Sigma^{Linac} / P_\Sigma^{RTM}$ of the LINAC and RTM total RF power consumption and the RTM total RF power consumption P_Σ^{RTM} as functions of I_b .

As it can be seen from Figure 3, for a pulse beam current $I_b < 100$ mA in the LINAC, most of the consumed RF power is spent on building-up an accelerating field so that the RTM obviously has advantages in terms of efficiency and total RF power consumption. For example, for $I_b = 20$ mA and beam energy 50 MeV, the RTM electronic efficiency is 54% and the total RF pulse power consumption is about 1.9 MW, whereas in the case of the LINAC, these parameters are 11% and 8.8 MW, respectively.

Taking into account these estimates and the effects limiting the pulse beam current discussed above, it can be argued that RTMs have undoubted advantages over LINACs in those applications for which the required beam energy is of tens of MeV and for which a pulse current about 20 mA is sufficient.

One potential application of RTMs is the production of medical isotopes, which requires an average current of at least 100 μ A. To achieve this value with a pulse current of 20 mA, the RTM RF system must operate with a duty cycle $D > 0.5\%$. Thus, to produce a beam with the energy 50 MeV, the pulse current $I_b = 20$ mA and the average current 100 μ A of the RTM RF power source must provide a pulse and an average RF power that is about 2 MW and 10 kW, respectively.

However, in the most of the previously built pulsed RTMs with a low-energy injection from an electron gun, large beam current losses take place at the orbits. This not only leads to an increase in the RF power consumption, but it also leads to a high radiation background from the accelerator and to the appearance of significantly induced activity on the accelerator elements. In the subsequent parts of this article, possible approaches for reducing beam current losses that have not been previously tested in RTMs are discussed.

As a conclusion of this section, we would like to note that to accelerate a significantly larger average beam current where both a high duty cycle and a large pulse current are required, there is certainly no alternative to LINACs.

Another observation is pertinent here. An attractive option is to accelerate a beam with a moderately high average current using a superconducting LINAC. In this case, the losses of the RF power in the walls of the accelerating structure are negligible, and the electronic efficiency will be larger than that of the RTM. However, the complexity and cost of the injection system, which should form short bunches of electrons with a sufficiently high energy to allow acceleration in a structure with $\beta = 1$, as well as the high cost of an accelerating structure with a cryostat, result in this LINAC design not being competitive with RTM.

4. Approaches for Minimizing Beam Current Losses in RTM

In the case, the beam current in an RTM is lower than the threshold value determined by the longitudinal and transverse instabilities; the main reason for the loss of the beam current along the orbits is the small value of the longitudinal and transverse acceptances. Of course, if an injected beam is relativistic and if its transverse and longitudinal emittances are much smaller than the acceptance values, then the current loss can be reduced to almost zero. A good example is the above-mentioned cascade of CW microtrons with the final beam energy of 1.5 GeV [8–11]. It includes a low-energy line for the formation of short bunches and has a small transverse emittance as well as a 3.5 MeV CW LINAC injector. Another example is the 35 MeV pulsed RTM with bunches formed by an RF gun that has a photocathode and a 5 MeV booster-accelerating structure [16]. However, because of the design complexity and high cost of such approaches, their use is not appropriate when using pulsed RTM for applied purposes.

From previous vast experience in the design, construction, and operation of RTMs the following lines of improvement for the machine beam dynamics and for the optimization of its elements with the aim of minimizing current losses at the orbits and increasing the electronic efficiency can be suggested: (1) increasing the threshold of the current due to the development of longitudinal and transverse instabilities; (2) increasing the longitudinal and transverse acceptances; (3) matching the longitudinal and transverse emittances of the injected beam with the corresponding acceptances; and (4) setting limits on the accuracy of the machining, tuning, and positioning of RTM elements and the stability of the accelerating and magnetic fields to ensure the passage of the beam along the orbits with the minimal trajectory corrections.

Here, we discuss two of these approaches, namely increasing the longitudinal acceptance of the RTM through the linearization of the accelerating field time dependence and through the matching of the longitudinal emittance of the injected beam with the acceptance by optimizing the initial part of the accelerating structure.

4.1. Increasing the Longitudinal Acceptance of the RTM by Linearizing the Accelerating Field

The width of the stable phase oscillation region in a cyclic accelerator is limited by the harmonic law of variation of the accelerating field. In early works devoted to the theory of synchrotrons [43,44], it was shown that the linearization of the time dependence of the accelerating field can widen the stable phase oscillation region significantly with the addition of higher harmonics.

In the case of CMs, a theoretical proof for the possibility of expanding the stable phase oscillation region with the addition of a second or third harmonic to the field was given in Ref. [45]. An experiment with a third harmonic resonator added to the main resonator that was reported in Ref. [46] did not produce tangible results, which was primarily due to the complexity of placing this additional resonator. Nevertheless, it was shown that the excitation of the third harmonic resonator by the beam leads to a certain increase of the accelerated current.

Using its drift space, an RTM offers significantly more options for placing a higher harmonic resonator. To assess this possibility of increasing the longitudinal acceptance in RTMs, we performed computer simulations of the phase motion for the following two cases: (1) with the fundamental harmonic field only and (2) with the fundamental and second harmonics. The voltage across the infinitely narrow gap of the equivalent resonator in the presence of the second harmonic is given by

$$U(t) = U_1 \cos \omega t + U_2 \cos(2\omega t + \theta) \quad (11)$$

where U_1 and U_2 are the amplitudes of the fundamental and second harmonics, ω is the angular frequency of the fundamental harmonic, and θ is the phase of the second harmonic with respect to the fundamental one.

The simulations were conducted for a fundamental frequency of 2856 MHz. To calculate the acceptance in the presence of the fundamental harmonic alone, the value

$U_1 = 5.2$ MV was chosen so that the synchronous energy gain of 5 MeV would be achieved for the synchronous phase $\varphi_s = 16^\circ$. In the case of two harmonics the parameters U_1 , U_2 , and θ were optimized with the condition of obtaining the maximum acceptance, while the magnetic field remained the same as it did in the case for the fundamental harmonic on its own.

The results of the computer simulations are shown in Figure 4. In Figure 4a plots of the voltage across the equivalent resonator gap as a function of the phase $\varphi = \omega t$ for one and two harmonics is given. In the case of two harmonics, the maximum acceptance is obtained for $U_1 = 7.17$ MV, $U_2 = 2.3$ MV, $\theta = 155^\circ$, and $\varphi_s = 2.8^\circ$.

Figure 4. (a) Voltage across the resonator gap as a function of the phase $\varphi = \omega t$ given by Equation (11) for one ($U_2 = 0$) and two harmonics; (b) longitudinal acceptance as functions of $\delta\varphi$; the deviation of the phase from the synchronous phase φ_s for the cases of harmonics one ($\varphi_s = 16^\circ$) and two ($\varphi_s = 2.8^\circ$) in the accelerating field.

In Figure 4b, the longitudinal acceptances for the case of one (orange region) and two (blue region) harmonics are shown. The vertical axis is the ratio $\delta E_{inj} / \Delta E_s$ of the deviation of the particle energy at the injection from the synchronous particle energy to the synchronous energy gain. One can see that the addition of the second harmonic leads to a significant, approximately 20-fold, increase in the area of the acceptance region. From the comparison of the plots in Figure 4a,b, it can be concluded that this increase in the stable phase oscillation region is due to the flat part in the voltage plot in the in phase interval between -50° and 70° .

The installation of an additional active accelerating structure with its own RF power supply system operating at the second harmonic would significantly complicate the design of the RTM. In cases where the microtron would operate at a fixed beam current, a passive accelerating structure mounted on the axis of the main structure can be used to solve the problem. The voltage in the passive cavities was excited by the beam itself; thus, no external RF source is required. In the absence of the structure coupling with the external loads, it is equal to [47]:

$$U_2 = -I_b^\Sigma Z_e^{(2)} L^{(2)} \cos \psi^{(2)} \quad (12)$$

where I_b^Σ is the sum of the beam currents from all of the RTM orbits, $Z_e^{(2)}$ and $L^{(2)}$ are the effective shunt impedance per unit length and length of the second harmonic accelerating structure, respectively, and $\psi^{(2)}$ is the detuning angle determined by the difference between the second harmonic of the bunch repetition rate and the operating frequency of the structure. The negative sign in the right-hand side of Equation (12) is due to the fact that the accelerating field in the structure is induced by the beam in the decelerating phase. To

ensure the required phase ratio of the first and second harmonics, the detuning angle in Formula (11) should be

$$\psi^{(2)} = \theta - \pi \quad (13)$$

For the example considered above, the second harmonic frequency is 5712 MHz, and the accelerating structure is described in Refs. [48,49], which allows the use of an effective shunt impedance of 98 M Ω /m. For an accelerated current of 20 mA and for neglecting beam losses in 10 beam passages through the accelerating structure, the total current $I_b^\Sigma = 0.2$ A. For the second harmonic phase $\theta = 155^\circ$, the detuning angle is $\psi^{(2)} = 25^\circ$, and for $U_2 = 2.3$ MV, we obtain the length $L^{(2)} = 0.13$ m. The period of the accelerating structure is 2.625 cm; thus, it must contain five accelerating cells. Since it is the beam that excites the field in these cells, the coupling between them is not necessary and the accelerating structure is composed of six properly detuned and uncoupled cavities.

As we have shown, the addition of the second harmonic to the accelerating field enlarges the longitudinal acceptance; in particular, the RTM accepts a relative deviation $\Delta E/\Delta E_s$ of the energy from the synchronous one by up to 30% with respect to the synchronous energy gain (see Figure 4b). However, such a large deviation in energy can lead to a loss of particles because of the limited size of the aperture in the horizontal plane in the return orbits. This limitation is due to installed steering coils, quadrupole lenses, or beam current monitors.

The deviation Δx of the particle trajectory from the synchronous one due to the particle momentum offset Δp with respect to the synchronous value p_s is equal to $\Delta x = 2R_s \Delta p p_s$, where R_s is the radius of the synchronous orbit. If the relative energy deviation in energy is within the range $|\Delta E/\Delta E_s| < 0.3$, then for the shift of the entire beam to be within the acceptance, the aperture radius must satisfy the inequality $a > \frac{\nu\lambda}{\pi} \times \frac{\Delta p}{\Delta E_s} \approx \frac{\nu\lambda}{\pi} \times \frac{\Delta E}{\Delta E_s}$. Thus, for $\nu = 1$, $a > \frac{0.3\lambda}{\pi}$ in the presence of the second harmonic. For the RF field wavelength $\lambda = 10$ cm, the aperture radius $a > 0.95$ cm, which is a quite feasible condition. Notice that $\frac{\nu\lambda}{\pi}$ is the distance between the RTM orbits.

4.2. Matching the Longitudinal Emittance of the Injected Beam with the RTM Acceptance by Optimizing the Initial Part of the Accelerating Structure

When developing modern electron LINACs for applied purposes with beam injection from an electron gun without intermediate systems for beam bunching and focusing, the initial part of the accelerating structure is optimized with conditions to provide a high capture efficiency up to 60–90% (see, for example, [49–52]). As a rule, for this aim, it is sufficient to choose the correct lengths and field amplitudes of the first three accelerating cells. Their function is beam bunching and focusing, and they also provide a sufficient increase in the energy of the particles in order to further their acceleration in subsequent regular $\beta = 1$ cells.

However, this optimization principle has not been applied to the accelerating structures of the existing RTMs. Usually, the field amplitude of the first accelerating cell is set close to that of the regular cells, and its length is chosen to be approximately equal to $\lambda/4$. With this design, the beam current at the exit of the accelerating structure is 30–40% of the injected current and, according to numerical simulations, only 20% of the injected particles reach the RTM beam exit. Taking into account the magnetic field errors and errors in the positioning of the accelerator elements discussed above, the fraction of the electrons at the end of acceleration turns out to be even lower.

In the 70 MeV RTM described in Ref. [17], a pre-buncher was installed at the exit of the electron gun in order to increase the capture efficiency. In Figure 5, theoretical and measured values of the ratio I_{b1}/I_{gun} , where I_{b1} is the current at the exit of the accelerating structure after the first beam passage, and I_{gun} is the electron gun current, is shown [53]. It can be seen that in principle, the capture efficiency can be increased by up to 60–70%. However, in this case, a significant number of particles have energy deviations and phases from the synchronous particle values that are too large and that are outside of RTM acceptance.

In addition, schemes with an external pre-buncher complicate the accelerator RF system significantly and require a dispersion-free injection system.

Figure 5. Theoretical and measured capture efficiency for the first passage of the beam through the accelerating structure as functions of the pre-buncher field phase [53].

We have studied the possibility of increasing the RTM capture efficiency by optimizing the initial part of the accelerating structure with the aim of maximizing the fraction of electrons injected from the gun within the longitudinal acceptance. In Figure 6a, the location of the longitudinal beam emittance after the first acceleration for a beam with zero transverse emittance in both planes with respect to the acceptance of the 55 MeV RTM with its accelerating structure described in Ref. [18] is shown. Figure 6b shows the emittance in the case of the optimized accelerating structure. The accelerating field amplitudes for both variants of the accelerating structure are shown in Figure 7. Finally, plots depicting the decay to the beam current along the orbits for the considered variants of the accelerating structure are given in Figure 8. As one can clearly see, the optimization of the initial cells allows the RTM capture efficiency to increase from 21 to 81%.

Figure 6. Longitudinal phase space (green and red dotted line) of the beam after the first acceleration superimposed on the RTM acceptance (a) for the accelerating structure of the 55 MeV RTM [18]; (b) for the structure with optimized initial part.

Figure 7. On-axis distribution of the accelerating field for the accelerating structure of the 55 MeV RTM [18] (blue line) and for the structure with the optimized initial part (orange line).

Figure 8. Decay of the beam current along the orbits for the two variants of the accelerating structure considered in Section 4.2.

With a nonzero transverse beam emittance and in the presence of magnetic field errors and errors in the RTM elements, positioning particles with parameters near the acceptance boundary can be lost during the acceleration. The addition of the second harmonic of the field, as discussed in Section 4.1, moves the acceptance boundary farther away, allowing a high capture efficiency to be maintained and reducing the particle current loss that may take place during the acceleration.

5. Conclusions

The analysis presented in this article shows that for applications requiring relatively small pulse currents (20–30 mA) and energies up to tens of MeV, RTMs have clear advantages over LINACs in terms of efficiency, size, and cost. An average current of about 100 μ A can be achieved by operating an RTM RF system with a high duty cycle. However, the significant beam current losses that can take place during the acceleration that occurs in pulsed RTMs lead to a significant radiation background and induced activity on the accelerator elements. The article describes a number of approaches that aim to reduce the losses of the beam current. These include decreasing the fringe field aberrations of the magnets; minimizing the inhomogeneity of the magnetic field in the area of the orbits;

reducing the strayed magnetic field in the drift space; increasing the accuracy of the machining and positioning of the magnets; expanding the aperture dimensions on the return orbits; increasing the current thresholds due to the longitudinal and transverse instability; enlarging the RTM longitudinal acceptance by adding the second harmonic of the accelerating field; and matching the emittance of the injected beam with the RTM acceptance better by optimizing the initial part of the accelerating structure. We also explored the possibility of enlarging the stability of the phase oscillation region using its intrinsic symmetry, which was mentioned in Section 2.2.

We plan to implement the approaches for reducing beam current losses that occur during the acceleration process that were outlined here in the development of a specific RTM project that is intended for the production of medical isotopes.

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