Article

Further Theory of Neutrosophic Triplet Topology and Applications

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Abstract: In this paper we study and develop the Neutrosophic Triplet Topology (NTT) that was recently introduced by Sahin et al. Like classical topology, the NTT tells how the elements of a set relate spatially to each other in a more comprehensive way using the idea of Neutrosophic Triplet Sets. This article is important because it opens new ways of research resulting in many applications in different disciplines, such as Biology, Computer Science, Physics, Robotics, Games and Puzzles and Fiber Art etc. Herein we study the application of NTT in Biology. The Neutrosophic Triplet Set (NTS) has a natural symmetric form, since this is a set of symmetric triplets of the form <A>, <anti(A)>, where <A> and <anti(A)> are opposites of each other, while <neuti(A)>, being in the middle, is their axis of symmetry. Further on, we obtain in this paper several properties of NTT, like bases, closure and subspace. As an application, we give a multicriteria decision making for the combining effects of certain enzymes on chosen DNA using the developed theory of NTT.

Keywords: neutrosophic triplet set; neutrosophic triplet topolgy; decision making; application

1. Introduction

The main aim of the paper is to introduce the Neutrosophic Triplet Topology (NTT) in various fields of research, due to its great potential of applicability. However, in order to do so, we first study its theoretical properties, such as open and closed sets, base and subspace, all extended from classical topology and neutrosophic topology to (NTT). In daily life we are witnessing many situations in which the role of neutralities is very important. To control neutralities Smarandache initiated the theme of neutrosophic logic in 1995, which later on proved to be a very handy tool to capture uncertainty. Thus Smarandache [1], generalizes almost all the existing logics like, fuzzy logic, intutionistic fuzzy logic etc. After this many reserchers used neutrosophic sets and logic in algebra, such as Kandasamy et al. [2–4], Agboola et al. [5–8], Ali et al. [9–12], Gulistan et al. [13–15]. More recently Smarandache et al. [16,17] introduced the idea of NT group which open a new research direction. Zhang et al. [18], Bal et al. [19], Jaiyeola el al. [20], Gulistan et al. [21] used NT set in different directions.

On the other hand Munkres [22], studied topology in detail. Chang [23] gave the concept of fuzzy topology in 1968. After this further study at fuzzy topology has been done by Thivagar [24], Lowen [25], Sarkar [26] and Palaniappan [27] , Onasanya et al. [28], Shumrani et al. [29]. Sahin et al. [30] presented the fresh idea of NTT.

Thus in this aricle, we further extended the theory of NT topology. We study some basic properties of NTT where we introduce NT base, NT closure and NT subspace and investigate these topological

notions. Moreover, as an application, we give a multicriteria decision making for the combining effects of certain enzymes on chosen DNA.

2. Preliminaries

In this section we recall some helpful material from [1,16] and for basics of topology we refer the reader [22].

Definition 1. *[1] A neutrosophic set is of the form*

$$\mathbb{H} = \{(\flat, T(\flat), I(\flat), F(\flat)) :: \flat \in U\}$$

where $T, I, F : U \longmapsto]0^-, 1^+[$.

Definition 2. *[16] "Let $\mathbb{H}$ be a set together with a binary operation $\star$. Then $\mathbb{H}_T$ is called a NT set if for any $\flat \in \mathbb{H}$, there exist a neutral of "$\flat$" called $neut(\flat)$, different from the classical algebraic unitary element, and an opposite of "$\flat$" called $anti(\flat)$, with $neut(\flat)$ and $anti(\flat)$ belonging to $\mathbb{H}$, such that:*
$\flat \star neut(\flat) = neut(\flat) \star \flat = \flat$
and
$\flat \star anti(\flat) = anti(\flat) \star \flat = neut(\flat)$."

3. Neutrosophic Triplet Topology (NTT)

In this section, we study NTT in detail.

Definition 3. *[30] Let $\mathbb{H}_T$ be a NT set and let $\mathbb{H}_\tau$ be a non-empty subset of $\mathcal{P}(\mathbb{H}_T)$. If $\mathbb{H}_\tau$ satisfy the following conditions:*

- $\varnothing, \mathbb{H}_T$ *in $\mathbb{H}_\tau$,*
- *The intersection of a finite number of sets in $\mathbb{H}_\tau$ is also in $\mathbb{H}_\tau$,*
- *The union of an arbitrary number of sets in $\mathbb{H}_\tau$ is also in $\mathbb{H}_\tau$.*

then $\mathbb{H}_\tau$ is called a NTT.

Remark 1. *The pair $(\mathbb{H}_T, \mathbb{H}_\tau)$ is called a NT topological space. The elements of $\mathbb{H}_\tau$ which are subsets of $\mathbb{H}_T$ are called NT open sets of NT topological space $(\mathbb{H}_T, \mathbb{H}_\tau)$.*

Example 1. *Let $\mathbb{H}_T$ be a NT set of $\mathbb{H}$ and $\mathbb{H}_\tau = \{\varnothing, \mathbb{H}_T\}$. Then $\mathbb{H}_\tau$ is a topology for $\mathbb{H}_T$ and it is called the NT trivial (or indiscrete) topology.*

Example 2. *Let $\mathbb{H}_T$ be a NT set of $\mathbb{H}$ and $\mathbb{H}_\tau = \mathcal{P}(\mathbb{H}_T)$. Then τ is a topology for $\mathbb{H}_T$ and it is called the NT discrete topology.*

Example 3. *Let $\mathbb{H}_T$ be a NT set and $\mathbb{H}_\tau$ be the collection of $\varnothing$ and those subsets of $\mathbb{H}_T$ whose complements are finite. Then $\mathbb{H}_\tau$ is called the neutrosophic triplet cofinite topology.*

Example 4. *Let $\mathbb{H} = \{\flat_1, \flat_2, \flat_3\}$ with the binary operation defined by the following table*

$*$	$\flat_1$	$\flat_2$	$\flat_3$
$\flat_1$	$\flat_3$	$\flat_2$	$\flat_1$
$\flat_2$	$\flat_2$	$\flat_2$	$\flat_3$
$\flat_3$	$\flat_1$	$\flat_3$	$\flat_2$

Then $(\flat_1, \flat_3, \flat_1), (\flat_2, \flat_2, \flat_2)$ and $(\flat_3, \flat_2, \flat_3)$ are neutrosophic triplets of $\hbar$. Let $\hbar_T = \{(\flat_1, \flat_3, \flat_1), (\flat_2, \flat_2, \flat_2), (\flat_3, \flat_2, \flat_3)\}$ be the set of triplets of $\hbar$. Then

$$\mathcal{P}(\hbar_T) = \{\varnothing, \{(\flat_1, \flat_3, \flat_1)\}, \{(\flat_2, \flat_2, \flat_2)\}, \{(\flat_3, \flat_2, \flat_3)\}, \{(\flat_1, \flat_3, \flat_1), (\flat_2, \flat_2, \flat_2)\},$$
$$\{(\flat_2, \flat_2, \flat_2), (\flat_3, \flat_2, \flat_3)\}, \{(\flat_1, \flat_3, \flat_1), (\flat_3, \flat_2, \flat_3)\}, \hbar_T\}.$$

Consider the following subsets

$$\hbar_{\tau 1} = \{\varnothing, \{(\flat_1, \flat_3, \flat_1)\}, \hbar_T\},$$
$$\hbar_{\tau 2} = \{\varnothing, \{(\flat_1, \flat_3, \flat_1)\}, \{(\flat_2, \flat_2, \flat_2)\}, \hbar_T\},$$
$$\hbar_{\tau 3} = \{\varnothing, \{(\flat_3, \flat_2, \flat_3)\}, \{(\flat_1, \flat_3, \flat_1)\}, \{(\flat_3, \flat_2, \flat_3), (\flat_1, \flat_3, \flat_1)\}, \hbar_T\}$$

then $\hbar_{\tau 1}$ and $\hbar_{\tau 3}$ are NT topologies while $\hbar_{\tau 2}$ is not NTT.

Definition 4. *Let $(\hbar_T, \hbar_\tau)$ be a topological space. A subset $F \subseteq \hbar_T$ is said to be NT closed if and only if its complement $\hbar_T \setminus F$ is NT open.*

Example 5. *Let $\hbar_T = \{(\flat_1, \flat_3, \flat_1), (\flat_2, \flat_2, \flat_2), (\flat_3, \flat_2, \flat_3)\}$ be as in Example 4 with the NTT $\hbar_\tau = \{\varnothing, \hbar_T, \{(\flat_3, \flat_2, \flat_3)\}, \{(\flat_1, \flat_3, \flat_1)\}, \{(\flat_3, \flat_2, \flat_3), (\flat_1, \flat_3, \flat_1)\}\}$. Then the NT closed subsets of $\hbar_T$ are*

$$\hbar_T, \varnothing, \{(\flat_1, \flat_3, \flat_1), (\flat_2, \flat_2, \flat_2)\}, \{(\flat_2, \flat_2, \flat_2), (\flat_3, \flat_2, \flat_3)\}, \{(\flat_2, \flat_2, \flat_2)\}.$$

Remark 2. *The NT closed sets of a NT topological space $(\hbar_T, \hbar_\tau)$ has the following properties,*

1. $\varnothing, \hbar_T$ *are NT closed.*
2. *Finite union of NT closed sets is NT closed set.*
3. *The arbitrary intersection of NT closed sets is a NT closed set.*

Definition 5. *Two NT topologies $\hbar_{\tau 1}$ and $\hbar_{\tau 2}$ of the NT set $\hbar_T$ are said to be comparable if $\hbar_{\tau 1} \subset \hbar_{\tau 2}$ or $\hbar_{\tau 2} \subset \hbar_{\tau 1}$. Further $\hbar_{\tau 1}$ and $\hbar_{\tau 2}$ are said to be equal if $\hbar_{\tau 1} \subset \hbar_{\tau 2}$ and $\hbar_{\tau 2} \subset \hbar_{\tau 1}$. If $\hbar_{\tau 1} \subset \hbar_{\tau 2}$ holds, then we say that $\hbar_{\tau 2}$ is finer than $\hbar_{\tau 1}$ and $\hbar_{\tau 1}$ is coarser than $\hbar_{\tau 2}$.*

Example 6. *Let $\hbar_T$ be a NT set having more than one element as a triplet element then any topology on $\hbar_T$ is finer than the NT indiscrete topology on $\hbar_T$ and coarser than the NT discrete topology on $\hbar_T$.*

The intersection of two NT topologies is always a NTT while the union of two NT topologies is not in general a NTT as shown in the following example.

Example 7. *Let $\hbar_T = \{(\flat_1, \flat_3, \flat_1), (\flat_2, \flat_2, \flat_2), (\flat_3, \flat_2, \flat_3)\}$ be as in Example 4. Consider the two NT topologies*

$$\hbar_{\tau 1} = \{\varnothing, \{(\flat_1, \flat_3, \flat_1)\}, \hbar_T\}$$

$$\hbar_{\tau 2} = \{\varnothing, \{(\flat_2, \flat_2, \flat_2)\}, \hbar_T\}.$$

Then

$$\hbar_{\tau 1} \cup \hbar_\tau = \{\varnothing, \{(\flat_1, \flat_3, \flat_1)\}, \{(\flat_2, \flat_2, \flat_2)\}, \hbar_T\}$$

is not a NTT.

Example 8. *Let $(\hbar_T, \hbar_\tau)$ be a NT topological space. If for some $(\flat_1, \flat_2, \flat_3) \in \hbar_T$ and $M \in \hbar_\tau$, we have $(\flat_1, \flat_2, \flat_3) \in M$, we say that M is a neighborhood of $(\flat_1, \flat_2, \flat_3)$. A set $L \subseteq \hbar_T$ is open if and only if for each $(\flat_1, \flat_2, \flat_3) \in L$ there exists a neighborhood $M_{(\flat_1, \flat_2, \flat_3)}$ of $(\flat_1, \flat_2, \flat_3)$ contained in L.*

Example 9. *Let* $\hbar_T = \{(\flat_1, \flat_3, \flat_1), (\flat_2, \flat_2, \flat_2), (\flat_3, \flat_2, \flat_3)\}$ *be as in Example* 4. *Consider the following NTT*

$$\hbar_{\tau 1} = \{\varnothing, \{(\flat_1, \flat_3, \flat_1)\}, \hbar_T\}$$

Note that the NT $(\flat_1, \flat_3, \flat_1)$ *has two neighborhoods, namely* $\{(\flat_1, \flat_3, \flat_1)\}$ *and* $\hbar_T$ *while* $\hbar_T$ *is the only neighborhood for both* $(\flat_2, \flat_2, \flat_2)$ *and* $(\flat_3, \flat_2, \flat_3)$.

4. Neutrosophic Triplet Bases of Neutrosophic Triplet Topology (NTT)

In this section, we define and study bases of a NTT for generating NT topologies.

Definition 6. *Let* $(\hbar_T, \hbar_\tau)$ *be a NT topological space. A family* $\hbar(\beta) \subset \hbar_\tau$ *is called a NT basis (or NT base) for* $\hbar_\tau$ *if each NT open subset of* $\hbar_T$ *is the union of members of* $\hbar(\beta)$. *The members of* $\hbar(\beta)$ *are called basis open sets of the topology* $\hbar_\tau$.

Example 10. *Let* $\hbar_T$ *be any NT set. Then the collection of all NT subsets of* $\hbar_T$ *is a basis for the NT discrete topology on* $\hbar_T$.

Example 11. *Let* $\hbar_T = \{(\flat_1, \flat_3, \flat_1), (\flat_2, \flat_2, \flat_2), (\flat_3, \flat_2, \flat_3)\}$ *be as in Example* 4 *with the NTT*

$$\hbar_\tau = \{\varnothing, \{(\flat_3, \flat_2, \flat_3)\}, \{(\flat_1, \flat_3, \flat_1)\}, \{(\flat_3, \flat_2, \flat_3), (\flat_1, \flat_3, \flat_1)\}, \hbar_T\}.$$

Then $\hbar(\beta) = \{\{(\flat_3, \flat_2, \flat_3)\}, \{(\flat_1, \flat_3, \flat_1)\}, \hbar_T\}$ *is a NT basis for* $(\hbar_T, \hbar_\tau)$.

Theorem 1. *Let* $(\hbar_T, \hbar_\tau)$ *be a NT topological space. A family*

$$\hbar(\beta) \subseteq \hbar_\tau$$

is a NT basis for $\hbar_\tau$ *if and only if, for each*

$$\hbar(O) \in \hbar_\tau$$

and

$$(\flat_o, \flat_o, \flat_o) \in \hbar(O),$$

there is a

$$\hbar(\Im) \in \hbar(\beta)$$

such that

$$(\flat_o, \flat_o, \flat_o) \in \hbar(\Im) \subseteq \hbar(O).$$

Proof. Suppose that $\hbar(\beta)$ is a NT base for NTT τ. By definition each $\hbar(O) \in \hbar_\tau$ is a union of members of $\hbar_\tau$. Let

$$\hbar(O) = \cup \{\hbar(\Im_\alpha) : \hbar(\Im_\alpha) \in \hbar(\beta)\}.$$

If $(\flat_o, \flat_o, \flat_o)$ is an arbitrary NT point of $\hbar(O)$, then $(\flat_o, \flat_o, \flat_o)$ belongs to at least one $\hbar(\Im_\alpha)$ in the union

$$\cup_\alpha \hbar(\Im_\alpha) = \hbar(O).$$

Hence

$$(\flat_o, \flat_o, \flat_o) \in \hbar(\Im_\alpha) \subseteq \cup_\alpha \hbar(\Im_{2\alpha}) = \hbar(O).$$

Thus

$$(\flat_o, \flat_o, \flat_o) \in \hbar(\Im_\alpha) \subseteq \hbar(O).$$

Conversely, suppose that for each

$$(\flat_o, \flat_o, \flat_o) \in \hbar(O),$$

there is a

$$\hbar\left(\Im_{(\flat_o,\flat_o,\flat_o)}\right) \in \hbar(\beta)$$

such that

$$(\flat_o, \flat_o, \flat_o) \in \hbar\left(\Im_{(\flat_o,\flat_o,\flat_o)}\right) \subseteq \hbar(O).$$

Thus

$$
\begin{aligned}
\hbar(O) &= \cup\left\{\{(\flat_o, \flat_o, \flat_o)\} : (\flat_o, \flat_o, \flat_o) \in \hbar(O)\right\} \\
&\subseteq \cup\left\{\hbar\left(\Im_{(\flat_o,\flat_o,\flat_o)}\right) : (\flat_o, \flat_o, \flat_o) \in \hbar(O)\right\} \subseteq \hbar(O).
\end{aligned}
$$

Therefore

$$\hbar(O) = \cup\left\{\hbar\left(\Im_{(\flat_o,\flat_o,\flat_o)}\right) : (\flat_o, \flat_o, \flat_o) \in \hbar(O)\right\}.$$

Thus $\hbar(O)$ is a union of members of $\hbar(\beta)$ and therefore $\hbar(\beta)$ is a NT bases for τ. □

Theorem 2. *A family $\hbar(\beta)$ of NT subsets of a neutrosophic triplet set(NTS) $\hbar_T$ is a NT bases for some NTT on $\hbar_T$ if and only if the following conditions are satisfied:*

(1) *Each $(\flat_o, \flat_o, \flat_o)$ in $\hbar_T$ is contained in some*

$$\hbar(\Im) \in \hbar(\beta)$$

   i.e.,

$$\hbar_T = \cup\{\hbar(\Im) : \hbar(\Im) \in \hbar(\beta)\}.$$

(2) *For any $\hbar(\Im_1), \hbar(\Im_2)$ belonging to $\hbar(\beta)$ the intersection*

$$\hbar(\Im_1) \cap \hbar(\Im_2)$$

   is a union of members of $\hbar(\beta)$. Equivalently, for each

$$(\flat_o, \flat_o, \flat_o) \in \hbar(\Im_1) \cap \hbar(\Im_2)$$

   there exist a

$$\hbar(\Im_3) \in \hbar(\beta)$$

   such that

$$(\flat_o, \flat_o, \flat_o) \in \hbar(\Im_3) \subseteq \hbar(\Im_1) \cap \hbar(\Im_2).$$

Proof. Suppose that a family $\hbar(\beta)$ of a NT subsets of NT set $\hbar_T$ is a NT basis for some NTT on $\hbar_T$. Since $\hbar_T \in \hbar_\tau$ (is open), then by definition of NT basis, $\hbar_T$ can be written as union of members of $\hbar(\beta)$. Now let $\hbar(\Im_1)$, $\hbar(\Im_2)$ be members of $\hbar(\beta)$. Then $\hbar(\Im_1)$, $\hbar(\Im_2)$ are NT sets and so is $\hbar(\Im_1) \cap \hbar(\Im_2)$. By Theorem 1, for each

$$(\flat_o, \flat_o, \flat_o) \in \hbar(\Im_1) \cap \hbar(\Im_2)$$

there is a

$$\hbar(\Im_3) \in \hbar(\beta)$$

such that

$$(\flat_o, \flat_o, \flat_o) \in \hbar(\Im_3) \subseteq \hbar(\Im_1) \cap \hbar(\Im_2).$$

Conversly, Suppose that both conditions (1) and (2) hold. Let $\mathbb{H}_\tau$ be the family of NT subsets of $\mathbb{H}_T$. Which are obtained by taking union of members of $\mathbb{H}(\beta)$. We claim that $\mathbb{H}_\tau$ is a NTT on $\mathbb{H}_T$. We need to show that the conditions of NTT are satisfied by the member of $\mathbb{H}_\tau$. Let

$$\{\mathbb{H}(O_\alpha) : \alpha \in \Omega\}$$

be a class of members of $\mathbb{H}_\tau$. Each $\mathbb{H}(O_\alpha)$ is a union of members of $\mathbb{H}(\beta)$ and so

$$\cup\{\mathbb{H}(O_\alpha) : \alpha \in \Omega\}$$

is also a union of members of $\mathbb{H}(\beta)$. Hence

$$\cup_{\alpha \in \Omega}\mathbb{H}(O_\alpha) \in \mathbb{H}_\tau.$$

Next suppose that
$$\mathbb{H}(O_1), \mathbb{H}(O_2) \in \mathbb{H}_\tau.$$

We shall show that
$$N(O_1) \cap \mathbb{H}(O_2) \in \mathbb{H}_\tau.$$

Let
$$(\flat_o, \flat_o, \flat_o) \in \mathbb{H}(O_1) \cap \mathbb{H}(O_2).$$

There are sets $\mathbb{H}(\Im_1), \mathbb{H}(\Im_2)$ in $\mathbb{H}(\beta)$ such that

$$(\flat_o, \flat_o, \flat_o) \in \mathbb{H}(\Im_1) \subset \mathbb{H}(O_1)$$

and
$$(\flat_o, \flat_o, \flat_o) \in \mathbb{H}(\Im_2) \subset \mathbb{H}(O_2).$$

Let $\mathbb{H}(\Im_{23}) \in \mathbb{H}(\beta)$ be such that

$$(\flat_o, \flat_o, \flat_o) \in \mathbb{H}(\Im_3) \subset \mathbb{H}(\Im_1) \cap \mathbb{H}(\Im_{22}).$$

Then
$$(\flat_o, \flat_o, \flat_o) \in \mathbb{H}(\Im_3) \subset \mathbb{H}(\Im_1) \cap \mathbb{H}(\Im_2) \subset \mathbb{H}(O_1) \cap \mathbb{H}(O_2)$$

which means that
$$\mathbb{H}(O_1) \cap \mathbb{H}(O_2)$$

belong to τ. By (1)
$$\mathbb{H}_T = \cup\{\mathbb{H}(\Im) : \mathbb{H}(\Im) \in \mathbb{H}(\beta)\}$$

So $\mathbb{H}_T \in \mathbb{H}_\tau$. Also, if we take the union of empty class of members of $\mathbb{H}(\beta)$ we note that $\phi \in \mathbb{H}_\tau$. Hence $\mathbb{H}_\tau$ is a topology on $\mathbb{H}_T$. Since each member of $\mathbb{H}_\tau$ is a union of members of $\mathbb{H}(\beta)$ by definition, $\mathbb{H}(\beta)$ is a NT basis for $\mathbb{H}_\tau$. □

5. Neutrosophic Triplet Closure

In this section, we define NT closure of neutrosophic triplet topological space.

Definition 7. *Let $(\mathbb{H}_T, \tau)$ be a NT topological space and let $\mathbb{H}(\Im)$ be any NT subset of $\mathbb{H}_T$. A NT $(\flat_o, \flat_o, \flat_o) \in \mathbb{H}_T$ is said to be NT adherent to $\mathbb{H}(\Im)$ if each NT neighbourhood of $(\flat_o, \flat_o, \flat_o)$ contain a NT point of $\mathbb{H}(\Im)$*

(which may be $(\flat_o, \flat_o, \flat_o)$ itself). The NT set of all NT points of $\hbar_T$ adherent to $\hbar(\Im)$ is called the NT closure of $\hbar(\Im)$ and is denoted by $\hbar\left(\overline{\Im}\right)$ in symbols,

$$\hbar\left(\overline{\Im}\right) = \left\{ (\flat_o, \flat_o, \flat_o) \in \hbar_T : \text{ for all } \hbar_{(\flat_o, \flat_o, \flat_o)}, \hbar_{(\flat_o, \flat_o, \flat_o)} \cap \hbar(\Im) \right\} \neq \phi.$$

Equivalently, NT closure of $\hbar(\Im)$ is the smallest NT closed super set of $\hbar(\Im)$. Neutrosophic triplet closure of $\hbar(\Im)$ is denoted by $\overline{\hbar(\Im)}$ or $\hbar\left(\overline{\Im}\right)$.

Remark 3. *It is clear from the definition that $\hbar(\Im) \subset \hbar\left(\overline{\Im}\right)$.*

Example 12. *Let $\hbar_T = \{(\flat_1, \flat_3, \flat_1), (\flat_2, \flat_2, \flat_2), (\flat_3, \flat_2, \flat_3)\}$ be as in Example 4 with the NTT $\tau = \{\phi, \{(\flat_1, \flat_3, \flat_1)\}, \hbar_T\}$. Let $\hbar(\Im_1) = \{(\flat_1, \flat_3, \flat_1)\}$ and $\hbar(\Im_2) = \{(\flat_2, \flat_2, \flat_2)\}$. We will find $\hbar\left(\overline{\Im_1}\right)$ and $\hbar\left(\overline{\Im_2}\right)$. Since $\hbar(\Im_1) \subset \hbar(\overline{\Im_1})$, we have $(\flat_1, \flat_3, \flat_1) \in \hbar\left(\overline{\Im_1}\right)$.*
   Now

$$(\flat_2, \flat_2, \flat_2) \in \hbar_T.$$

Since the only neighborhood of $(\flat_2, \flat_2, \flat_2)$ is $\hbar_T$ and $\hbar_T \cap \hbar(\Im_1) \neq \phi$, we have that $(\flat_2, \flat_2, \flat_2) \in \hbar\left(\overline{\Im_1}\right)$. Similarly, we have that $(\flat_3, \flat_2, \flat_3) \in \hbar(\overline{\Im_1})$. Therefore, $\hbar\left(\overline{\Im_1}\right) = \hbar_T$.
   Next we will find $\hbar(\overline{\Im_2})$. Since $\{(\flat_1, \flat_3, \flat_1)\}$ is a neighborhood of $(\flat_1, \flat_3, \flat_1)$ and $\{(\flat_1, \flat_3, \flat_1)\} \cap \hbar(\Im_2) = \phi$, we have that $(\flat_1, \flat_3, \flat_1) \notin \hbar\left(\overline{\Im_2}\right)$. Since the only neighborhood of $(\flat_2, \flat_2, \flat_2)$ is $\hbar_T$ and $\hbar_T \cap \hbar(\Im_2) \neq \phi$, we have $(\flat_2, \flat_2, \flat_2) \in \hbar\left(\overline{\Im_2}\right)$. Similarly, we have that $(\flat_3, \flat_2, \flat_3) \in \hbar\left(\overline{\Im_2}\right)$. Hence, $\hbar\left(\overline{\Im_2}\right) = \{(\flat_2, \flat_2, \flat_2), (\flat_3, \flat_2, \flat_3)\}$.

Theorem 3. *$\hbar(\Im)$ is NT closed if and only if $\hbar(\Im) = \hbar\left(\overline{\Im}\right)$.*

Proof. Assume that $\hbar(\Im)$ is a NT closed. Then $\hbar(\Im)$ is a closed set containing $\hbar(\Im)$. Therefore, $\hbar\left(\overline{\Im}\right) \subset \hbar(\Im)$. However, by definition $\hbar(\Im) \subset \hbar\left(\overline{\Im}\right)$. Hence, $\hbar(\Im) = \hbar\left(\overline{\Im}\right)$. Conversely, assume that $\hbar(\Im) = \hbar\left(\overline{\Im}\right)$. Since $\hbar\left(\overline{\Im}\right)$ is the smallest NT superset of $\hbar(\Im)$, so $\hbar\left(\overline{\Im}\right)$ is NT closed, which implies that $\hbar(\Im)$ is NT closed. $\square$

Theorem 4. *Let $(\hbar_T, \hbar_\tau)$ be a NT topological space and let $\hbar(\Im_1)$ and $\hbar(\Im_2)$ be arbitrary NT subsets of $\hbar_T$. Then*

- $\overline{\phi} = \phi$
- $\overline{\hbar_T} = \hbar_T$
- $\overline{\hbar(\Im_1) \cup \hbar(\Im_2)} = \overline{\hbar(\Im_1)} \cup \overline{\hbar(\Im_2)}$
- $\overline{\hbar(\Im_1) \cap \hbar(\Im_2)} \subset \overline{\hbar(\Im_1)} \cap \overline{\hbar(\Im_2)}$
- $\overline{\overline{\hbar(\Im_1)}} = \overline{\hbar(\Im_1)}$
- *If $\hbar(\Im_1) \subset \hbar(\Im_2)$, then $\overline{\hbar(\Im_1)} \subset \overline{\hbar(\Im_2)}$.*

Proof.

(1)   It is trivial.

(2) $\hbar_T$ and $\overline{\hbar_T}$ are both closed sets and therefore $\hbar_T = \overline{\hbar_T}$ by Theorem 3.

(3)   Let $(\flat_o,\flat_o,\flat_o) \in \overline{\text{Ħ}(\Im_1)}$. Then each NT neighbourhood $\text{Ħ}_{(\flat_o,\flat_o,\flat_o)}$ of $(\flat_o,\flat_o,\flat_o)$ contains some point of $\text{Ħ}(\Im_1)$ and hence $\text{Ħ}_{(\flat_o,\flat_o,\flat_o)}$ contains some point of $\text{Ħ}(\Im_1 \cup \Im_2)$. Thus $(\flat_o,\flat_o,\flat_o) \in \overline{\text{Ħ}(\Im_1 \cup \Im_2)}$. Therefore, $\overline{\text{Ħ}(\Im_1)} \subset \overline{\text{Ħ}(\Im_1 \cup \Im_2)}$. Similarly, $\overline{\text{Ħ}(\Im_2)} \subset \overline{\text{Ħ}(\Im_1 \cup \Im_2)}$. Thus

$$\overline{\text{Ħ}(\Im_1)} \cup \overline{\text{Ħ}(\Im_2)} \subset \overline{\text{Ħ}(\Im_1 \cup \Im_2)}.$$

For the converse inclusion, we have , by definition $\text{Ħ}(\Im_1) \subset \overline{\text{Ħ}(\Im_1)}$ and $\text{Ħ}(\Im_2) \subset \overline{\text{Ħ}(\Im_2)}$. Therefore

$$\text{Ħ}(\Im_1 \cup \Im_2) \subset \overline{\text{Ħ}(\Im_1)} \cup \overline{\text{Ħ}(\Im_2)}.$$

However, $\overline{\text{Ħ}(\Im_1)} \cup \overline{\text{Ħ}(\Im_2)}$ is a NT closed set containing $\text{Ħ}(\Im_1 \cup \Im_2)$. Hence by Theorem 3 we have

$$\overline{\text{Ħ}(\Im_1)} \cup \overline{\text{Ħ}(\Im_2)} = \overline{\text{Ħ}(\Im_1)} \cup \overline{\text{Ħ}(\Im_2)}.$$

(4)   Since $\text{Ħ}(\Im_1) \subset \overline{\text{Ħ}(\Im_1)}$, and $\text{Ħ}(\Im_2) \subset \overline{\text{Ħ}(\Im_2)}$ we have

$$\text{Ħ}(\Im_1) \cap \text{Ħ}(\Im_2) \subset \overline{\text{Ħ}(\Im_1)} \cap \overline{\text{Ħ}(\Im_2)}.$$

However, $\overline{\text{Ħ}(\Im_1)} \cap \overline{\text{Ħ}(\Im_2)}$ is a NT closed set and therefore by Theorem 3

$$\begin{aligned}
\text{Ħ}(\Im_1) \cap \text{Ħ}(\Im_2) &\subset \overline{\text{Ħ}(\Im_1) \cap \text{Ħ}(\Im_2)} \\
&\subset \overline{\text{Ħ}(\Im_1)} \cap \overline{\text{Ħ}(\Im_2)}.
\end{aligned}$$

Implies that

$$\overline{\text{Ħ}(\Im_1) \cap \text{Ħ}(\Im_2)} \subset \overline{\text{Ħ}(\Im_1)} \cap \overline{\text{Ħ}(\Im_2)}.$$

(5)   We apply Theorem 3 to the NT closed set $\overline{N(\Im_1)}$ to obtain

$$\overline{\overline{\text{Ħ}(\Im_1)}} = \overline{\text{Ħ}(\Im_1)}.$$

(6)   If $\text{Ħ}(\Im_1) \subset \text{Ħ}(\Im_2)$ then $\text{Ħ}(\Im_1) \cup \text{Ħ}(\Im_2) = \text{Ħ}(\Im_2)$. Taking closures on both sides and applying (3) we have

$$\overline{\text{Ħ}(\Im_1)} \cup \overline{\text{Ħ}(\Im_2)} = \overline{\text{Ħ}(\Im_2)}.$$

Hence, $\overline{\text{Ħ}(\Im_1)} \subset \overline{\text{Ħ}(\Im_2)}$.

☐

Remark 4. *The equality*

$$\overline{\text{Ħ}(\Im_1) \cap \text{Ħ}(\Im_2)} = \overline{\text{Ħ}(\Im_1)} \cap \overline{\text{Ħ}(\Im_2)}$$

does not hold in general.

6. Neutrosophic Triplet Subspace

In this section, we define the NT subspace.

Definition 8. *Let $(\text{Ħ}_T, \text{Ħ}_\tau)$ be a NT topological space and $\text{Ħ}(Y) \subset \text{Ħ}_T$, where $\text{Ħ}(Y) \neq \phi$. Then*

$$\tau_{\text{Ħ}(Y)} = \{\text{Ħ}(V) \cap \text{Ħ}(Y) : \text{Ħ}(V) \in \text{Ħ}_\tau\}$$

is a NTT on $\text{Ħ}(Y)$, called NT subspace topology. Open sets in $\text{Ħ}(Y)$ consist of all intersections of open sets of Ħ_T with $\text{Ħ}(Y)$.

Let us check that the collection $Ħ_{\tau Ħ(Y)}$ is a NTT on $Ħ(Y)$.

We shall show that $Ħ_{\tau Ħ(Y)}$ satisfies the three properties of a NT topology on $Ħ(Y)$.

T_1: Suppose that

$$Ħ(O_1), Ħ(O_2), ..., Ħ(O_Ħ)$$

belong to $Ħ_{\tau Ħ(Y)}$ then, there are subsets $Ħ(U_1), Ħ(U_2), ..., Ħ(U_Ħ)$ of $Ħ_T$ belonging to $Ħ_\tau$ such that

$$Ħ(O_i) = Ħ(Y) \cap Ħ(U_i), \quad i = 1, 2, ..., n.$$

Now $Ħ(O_1) \cap Ħ(O_2) ...$

$$Ħ(O_n) = (Ħ(Y) \cap Ħ(U_1)) \cap (Ħ(Y) \cap Ħ(U_2)) ... \cap (Ħ(Y) \cap Ħ(U_n))$$
$$= Ħ(Y) \cap (Ħ(U_1) \cap Ħ(U_2) .... \cap Ħ(U_n))$$

A NT open set in $Ħ(Y)$, since

$$Ħ(U_1) \cap Ħ(U_2) .... \cap Ħ(U_n) \in Ħ_\tau$$

Hence

$$Ħ(O_1) \cap Ħ(O_2) .... Ħ(O_n) \in \tau_{Ħ(Y)}.$$

This finite intersection of members of $Ħ_{\tau Ħ(Y)}$ is again in $\tau_{Ħ(Y)}$.

T_2: Let $\{Ħ(O_\alpha) : \alpha \in \Omega\}$ be an arbitrary family of members of $Ħ_{\tau Ħ(Y)}$. Then there exist a family $\{U_\alpha : \alpha \in \Omega\}$ of member of $Ħ_\tau$ such that $Ħ(O_\alpha) = Ħ(Y) \cap Ħ(U_\alpha)$ for all $\alpha \in \Omega$. Therefore,

$$\cup_{\alpha \in \Omega} Ħ(O_\alpha) = \cup_{\alpha \in \Omega} (Ħ(Y) \cap Ħ(U_\alpha)) = Ħ(Y) \cap (\cup_{\alpha \in \Omega} U_\alpha)$$

Since $Ħ_\tau$ is a NTT on $Ħ(Y)$.

$T, \cup \{Ħ(U_\alpha) : \alpha \in \Omega\}$ is in τ. Hence

$$Ħ(Y) \cap (\cup_{\alpha \in \Omega} U_\alpha) \in Ħ_{\tau Ħ(Y)}.$$

Thus, $\cup_{\alpha \in \Omega} Ħ(O_\alpha)$ belongs to $\tau_{Ħ(Y)}$. Hence arbitrary union of members of $Ħ_{\tau Ħ(Y)}$ is also in $Ħ_{\tau Ħ(Y)}$.

T_3: $Ħ(Y)$ and ϕ belong to $Ħ_{\tau Ħ(Y)}$ since

$$Ħ(Y) \cap Ħ_T = Ħ(Y)$$

and

$$Ħ(Y) \cap \phi = \phi$$

Hence, $Ħ_{\tau Ħ(Y)}$ is a NTT on $Ħ(Y)$.

Example 13. *Let $Ħ_T = \{(\flat_1, \flat_3, \flat_1), (\flat_2, \flat_2, \flat_2), (\flat_3, \flat_2, \flat_3)\}$ be as in Example 4 with the NTT*

$$Ħ_\tau = \{\phi, \{(\flat_1, \flat_3, \flat_1)\}, \{(\flat_2, \flat_2, \flat_2)\}, \{(\flat_1, \flat_3, \flat_1), (\flat_2, \flat_2, \flat_2)\}, Ħ_T\}$$

and $Ħ(Y) = \{(\flat_1, \flat_3, \flat_1), (\flat_3, \flat_2, \flat_3)\}$
Taking intersection of each member of τ with $Ħ(Y)$. Then

$$\phi \cap \hbar(Y) = \phi$$
$$\{(\flat_1, \flat_3, \flat_1)\} \cap \hbar(Y) = \{(\flat_1, \flat_3, \flat_1)\}$$
$$\{(\flat_2, \flat_2, \flat_2)\} \cap \hbar(Y) = \phi$$
$$\{(\flat_1, \flat_3, \flat_1), (\flat_2, \flat_2, \flat_2)\} \cap \hbar(Y) = \{(\flat_1, \flat_3, \flat_1)\}$$
$$\hbar_T \cap \hbar(Y) = \hbar(Y)$$
$$\tau_{\hbar(Y)} = \{\phi, \{(\flat_1, \flat_3, \flat_1)\}, \hbar(Y)\}.$$

7. Applications

In Mathematics, topology is concerned with the properties of space that are preserved under continuous deformations, such as stretching, twisting, crumpling and bending, but not tearing or gluing. Like topology, the NTT tells how elements of a set relate spatially to each other in a more comprehensive way using the idea of Neutrosophic triplet sets. It has many application in different disciplines, Biology, Computer science, Physics, Robotics, Games and Puzzles and Fiber art etc. Here we study the application of NTT in Biology.

Suppose that we have a certain type of DNA and we are going to discuss the combine effects of certain enzymes like, $\Im_1, \Im_2, \Im_3$ on chosen DNA using the idea of NT sets. These enzymes cut, twist, and reconnect the DNA, causing knotting with observable effects. Assume the set $\hbar = \{\Im_1, \Im_2, \Im_3\}$ and assume that their mutual effect on each other is shown in the following table

$*$	$\Im_1$	$\Im_2$	$\Im_3$
$\Im_1$	$\Im_3$	$\Im_2$	$\Im_1$
$\Im_2$	$\Im_2$	$\Im_2$	$\Im_3$
$\Im_3$	$\Im_1$	$\Im_3$	$\Im_2$

Then $(\Im_1, \Im_3, \Im_1), (\Im_2, \Im_2, \Im_2)$ and $(\Im_3, \Im_2, \Im_3)$ are neutrosophic triplets of $\hbar$. Here $(\Im_1, \Im_3, \Im_1)$ means that the enzymes $\Im_1, \Im_3$ play the role of anti and neut of each other, $(\Im_2, \Im_2, \Im_2)$ means that the enzyme $\Im_2$ has no neut and anti and $\Im_1, \Im_3$ are anti and neut of each other in different situations. Let $\hbar_T = \{(\Im_1, \Im_3, \Im_1), (\Im_2, \Im_2, \Im_2), (\Im_3, \Im_2, \Im_3)\}$ be the set of triplets of $\hbar$. Then

$$\mathcal{P}(\hbar_T) = \{\varnothing, \{(\Im_{11}, \Im_3, \Im_1)\}, \{(\Im_2, \Im_2, \Im_2)\}, \{(\Im_3, \Im_2, \Im_3)\}, \{(\Im_1, \Im_3, \Im_1), (\Im_2, \Im_2, \Im_2)\},$$
$$\{(\Im_2, \Im_2, \Im_2), (\Im_3, \Im_2, \Im_3)\}, \{(\Im_1, \Im_3, \Im_1), (\Im_3, \Im_2, \Im_3)\}, \hbar_T\}.$$

Here $\mathcal{P}(\hbar_T)$ discuss the all possible outcomes of anti and neut. Consider the following two subsets of $\mathcal{P}(\hbar_T)$. $\tau_1 = \{\varnothing, \{(\Im_1, \Im_3, \Im_1)\}, \hbar_T\}$ and $\tau_2 = \{\varnothing, \{(\Im_3, \Im_2, \Im_3)\}, \{(\Im_1, \Im_3, \Im_1)\}, \{(\Im_3, \Im_2, \Im_3), (\Im_1, \Im_3, \Im_1)\}, \hbar_T\}$. Then τ_1 and τ_2 are NT topologies and stand for the combination of enzymes that effect the DNA. While $\tau_3 = \{\varnothing, \{(\Im_3, \Im_2, \Im_3)\}, \{(\Im_2, \Im_2, \Im_2)\}, \hbar_T\}$ is not NTT and stands for the combination of enzymes that does not effect the DNA as union of $\{(\Im_3, \Im_2, \Im_3)\}, \{(\Im_2, \Im_2, \Im_2)\}$ does not belongs to τ_3. As τ_1 and τ_2 neutrosophic triplet topologies so $\tau_1 \cap \tau_2 = \tau_1$ and $\tau_1 \cup \tau_2 = \tau_2$ is again a neutrosophic triplets topology which effects the DNA. The NTT $\varnothing$ stands for the combination of enzymes where we can not have any answer while neutrosophic triplet topology $\mathcal{P}(\hbar_T)$ stands for the strongest case of combination of enzymes which effects the DNA. Now if we want more insight of this problem we may use other concepts like, NT neighborhoods etc.

On the other hand Leonhard Euler demonstrated problem that it was impossible to find a route through the town that would cross each of its seven bridges exactly once. This problem leads us towards the NT graph theory using the concept of NTT as the route does not depend upon the any physical scenario, but it depends upon the spatially connectivity between the bridges.

Similarly to classify the letters correctly and the hairy ball theorem of algebraic topology can be discussed in a more practical way using the concept of NT.

8. Conclusions

In this article, we used the idea of NTT and introduced some of their properties, such as NT base, NT closure and NT subspace. At the end we discuss an application of multicriteria decision making problem with the help of NTT.

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