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Data Analysis Approach for Incomplete Interval-Valued Intuitionistic Fuzzy Soft Sets

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Abstract: The model of interval-valued intuitionistic fuzzy soft sets is a novel excellent solution which can manage the uncertainty and fuzziness of data. However, when we apply this model into practical applications, it is an indisputable fact that there are some missing data in many cases for a variety of reasons. For the purpose of handling this problem, this paper presents new data processing approaches for an incomplete interval-valued intuitionistic fuzzy soft set. The missing data will be ignored if percentages of missing degree of membership and nonmember ship in total degree of membership and nonmember ship for both the related parameter and object are below the threshold values; otherwise, it will be filled. The proposed filling method fully considers and employs the characteristics of the interval-valued intuitionistic fuzzy soft set itself. A case is shown in order to display the proposed method. From the results of experiments on all thirty randomly generated datasets, we can discover that the overall accuracy rate is up to 80.1% by our filling method. Finally, we give one real-life application to illustrate our proposed method.

Keywords: soft set; interval-valued intuitionistic fuzzy soft sets; incomplete information; data filling

1. Introduction

It is true that we are drowning in uncertain and fuzzy data that are ubiquitous in fields such as business management, banking, environmental governance, industrial engineering, evaluation systems and so on. Soft set theory [1,2] is an excellent solution designed for uncertainty. Since soft sets do not have the problem of setting membership functions, it has been extensively use Dina lot of fields as diverse as information system data analysis, decision making [3–10], resource discovery [11], text classification, data mining [12,13], medical diagnosis and so on.

Soft sets combining with other mathematical models and then initiating new, more powerful tools which deal with uncertainty are the main research trends of soft set. There are many integrated types, such as combining the 2-tuple linguistic representation and soft set [14], the belief interval-valued soft set [15], confidence soft sets [16], the linguistic value soft set [17], dual hesitant fuzzy soft sets [18], the Z-soft fuzzy rough set [19], trapezoidal interval type-2 fuzzy soft sets[20,21], bell-shaped fuzzy soft sets [22], possibility neutrosophic soft sets [23], soft-set-based VIKOR approach [24], hesitant linguistic expression soft sets [25], hesitant N-soft sets [26], interval-valued picture fuzzy soft set [27], Q-neutrosophic soft set [28] , totally dependent-neutrosophic soft sets[29], and soft rough sets [30,31] so on. There are some mentionable and notable extended directions besides the above combined models. In the first place, the fuzzy soft set is a combination of fuzzy set and soft set [32]. Object recognition for inexact data from multiple observers is focused [33], providing a decision scheme on account of the model of fuzzy soft sets. A decision making scheme is given by integrating a grey relational analysis and Dempster–Shafer (D–S) theory of evidence and fuzzy soft set [34]. Literature [35] provided a decision solution for treatment of disease by means of ambiguity measure and Dempster–Shafer theory of evidence and the model of fuzzy soft set. Literature [36] proposed a chest X-ray medical diagnosis approach aiming at pneumonia deformity by the model of fuzzy soft set and D–S evidence theory. In order to identify the cognitive differences of different decision makers, Chen, W et al. [37] proposed a group decision scheme by means of the extended fuzzy soft set. There were some parameter reduction approaches [38–40] for the model of the fuzzy soft set in order to get the most efficient information. Furthermore, the model of the soft set to is expanded into an intuitionistic fuzzy soft set [41,42]. Jiang et al. [43] defined a distance measurements scheme for intuitionistic fuzzy soft sets. Literature [44,45] proposed a group medical diagnosis model on account of an intuitionistic fuzzy soft set. The interval-valued fuzzy soft set [46] is also one of the extension models of soft sets. Literature [47] had conducted in-depth research on the decision problem with an interval-valued fuzzy soft set as the data representation, and proposed three algorithms based on weighted distance approximation, combined distance evaluation and similarity measure for the interval-valued fuzzy soft decision problem. Then, four different parameter reduction algorithms for interval value fuzzy soft set were expressed [48]. However, in the practical application of this model, some incomplete data should be processed. An interval-valued fuzzy soft set data analysis method for incomplete information was given [49]. We also [50] proposed an entire decision-making and evaluation system described by an interval-valued fuzzy soft set. Jiang et al. [51] combined an interval-valued intuitionistic fuzzy set with a soft set and depicted the theory of the interval-valued intuitionistic fuzzy soft set (IVIFSS). Literature [51] defined the operations of complement, union and intersection on this model, and discussed its basic properties. The interval-valued intuitionistic fuzzy soft set is an effective tool to deal with uncertainty. In a lot of practical scenarios, only membership expression cannot fully describe the fuzziness of data and nonmember ship functions that should be provided. The membership and the nonmember ship expressions are very individual; it is difficult to determine their value accurately. A more reasonable approach is to use interval values to describe membership and nonmember ship level. The interval-valued intuitionistic fuzzy soft set inherits the advantages of the soft set, interval-valued fuzzy soft set and intuitionistic fuzzy set. Zhang et al. [52] proposed a decision-making algorithm for an interval-valued intuitionistic fuzzy soft set using a level soft set.

It is certain that the vast majority of studies on the above mathematical models involve complete datasets. In practice, however, the information is incomplete in most cases, which can lead to inappropriate and unreasonable decisions. For example, when a questionnaire needs to be filled out, people who take questionnaires always ignore the income item, because they are reluctant to disclose private information and then drop this question. Consequently, dealing with incomplete information is essential. Researchers have investigated the issues of data analysis methods under the incomplete information on soft set [53–56], fuzzy soft set [57] and interval-valued fuzzy soft set [49]. However, the model of the interval-valued intuitionistic fuzzy soft set is different from the above models, which has its own specific features. Hence, a new data analysis method under incomplete information for this model is necessary, rather than indiscriminately imitating the methods of the above-mentioned models. The contributions of this paper are as follows:

(1) In this paper, we propose the related data analysis approaches for incomplete interval-valued intuitionistic fuzzy soft sets. In detail, the missing data will be ignored, if percentages of missing degree of membership and nonmember ship in total degree of membership and nonmember ship for the related parameter and object are below the threshold values; otherwise, it will be filled. The proposed filling method fully considers and employs the characteristics of this model itself.

(2) From the results of experiments on all of thirty randomly generated datasets, we can discover that the overall accuracy rate is high and acceptable by our filling method.

(3) One real-life application demonstrates the intended outcome of our proposed methods.

The rest of this paper is as follows. In section II, we review the necessary theoretical background of IVIFSS theory. In section III, we give some related definitions and the detailed data analysis approaches for incomplete interval-valued intuitionistic fuzzy soft sets, which are illustrated by an example. In section IV, aiming to verify our method, we generate randomly thirty datasets which are described by IVIFSS. In section V, one real-life application is given to illustrate our contribution. Finally, section VI draws the conclusion for this article.

2. Preliminaries

Some basic notions about interval-valued intuitionistic fuzzy soft set theory are described in retrospect.

Relevant Definitions

Definition 1. ([58,59]). *An interval-valued intuitionistic fuzzy set on a universe X is an object of the form*

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle | x \in X\} \quad (1)$$

where $\mu_A(x): X \to Int([0,1])$ *and* $\gamma_A(x): X \to Int([0,1])$ *(* $Int([0,1])$ *stands for the set of all closed subintervals of [0,1]) ,which are defined as the degree of membership and nonmember ship respectively of the element x to the set A, satisfy the following condition:* $\forall x \in X, \sup \mu_A(x) + \sup \gamma_A(x) \le 1$.

Let U be an initial universe of objects, E be a set of parameters in relation to objects in U, $\zeta(U)$ be the set of all interval-valued intuitionistic fuzzy sets of U. The definition of the interval-valued intuitionistic fuzzy soft set is given as follows.

Definition 2. ([51]). *A pair* $(\widetilde{\varphi}, E)$ *is called an interval-valued intuitionistic fuzzy soft set over* $\zeta(U)$, *where* $\widetilde{\varphi}$ *is a mapping given by*

$$\widetilde{\varphi}: E \to \zeta(U) \quad (2)$$

The following example is able to illustrate this theory.

Example 1. *Suppose that* $U = \{h_1, h_2, h_3, h_4, h_5\}$ *is a set of five mobile phones,* $E = \{e_1, e_2, e_3, e_4\}$ *is a set of parameters, where* e_i *stands for "expensive", "fashionable", "voluminous" "comfortable", respectively. We apply this model to fully represent the popularity of mobile phones. The interval-valued intuitionistic fuzzy soft set* $(\widetilde{\theta}, E)$ *is illustrated by a tabular representation in Table 1.*

Table 1. An interval-valued intuitionistic fuzzy soft set $(\widetilde{\varphi}, E)$.

U/E	e_1	e_2	e_3	e_4
h_1	[0.32,0.41],[0.39,0.52]	[0.70,0.80],[0.10,0.20]	[0.75,0.85],[0.00,0.10]	[0.65,0.75],[0.10,0.20]
h_2	[0.50,0.82],[0.05,0.13]	[0.60,0.80],[0.05,0.20]	[0.45,0.55],[0.20,0.40]	[0.60,0.70],[0.15,0.25]
h_3	[0.30,0.50],[0.20,0.40]	[0.55,0.65],[0.20,0.30]	[0.28,0.60],[0.22,0.35]	[0.60,0.80],[0.05,0.15]
h_4	[0.73,0.85],[0.03,0.12]	[0.75,0.85],[0.05,0.15]	[0.65,0.85],[0.05,0.15]	[0.68,0.88],[0.01,0.12]
h_5	[0.60,0.70],[0.10,0.20]	[0.55,0.73],[0.11,0.23]	[0.62,0.75],[0.15,0.23]	[0.40,0.50],[0.20,0.35]

In this case, only membership functions cannot fully describe the popularity of mobile phones, and then nonmember ship functions should be offered. The membership and the nonmember ship expressions are very individual; it is not easy to determine the accurate value. This theory solves this conflict. For instance, we should observe that mobile phone h_1 is least expensive on the membership degree of 0.32 and it is most expensive on the membership degree of 0.41; mobile phone h_1 is not least expensive on the nonmember ship degree of 0.39 and it is not most expensive on the nonmember ship degree of 0.52. Table 1 is a complete interval-valued intuitionistic fuzzy soft set. However, it is an indisputable fact that there are some missing data represented by this model in many cases for a variety of reasons. So we propose data analysis method for an incomplete data representation by means of this model as follows.

3. Data Analysis Approaches for Incomplete Interval-Valued Intuitionistic Fuzzy Soft Sets

First of all, we give some new related definitions. According to these definitions, we propose the data analysis approaches for incomplete data representation by means of this model. An example is given to illustrate it.

3.1. Relevant Definitions

Definition 3. *Let* $x \in U$, $\varepsilon \in E$ *; a pair* $(\tilde{\vartheta}, E)$ *is called an interval-valued intuitionistic fuzzy soft set over* $\Re(U)$, $U = \{h_1, h_2, \cdots, h_n\}$, $E = \{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_m\}$, $\tilde{\vartheta}(\varepsilon) = \{\langle x, \mu_{\tilde{\vartheta}(\varepsilon)}(x), v_{\tilde{\vartheta}(\varepsilon)}(x)\rangle\}$, $u_{\tilde{\vartheta}(\varepsilon)}(x) = [u^-_{\tilde{\vartheta}(\varepsilon)}(x), u^+_{\tilde{\vartheta}(\varepsilon)}(x)]$, $v_{\tilde{\vartheta}(\varepsilon)}(x) = [v^-_{\tilde{\vartheta}(\varepsilon)}(x), v^+_{\tilde{\vartheta}(\varepsilon)}(x)]$ *.Let* $u^*_{\tilde{\vartheta}(\varepsilon)}(x) = [u^{-*}_{\tilde{\vartheta}(\varepsilon)}(x), u^{+*}_{\tilde{\vartheta}(\varepsilon)}(x)]$, $v^*_{\tilde{\vartheta}(\varepsilon)}(x) = [v^{-*}_{\tilde{\vartheta}(\varepsilon)}(x), v^{+*}_{\tilde{\vartheta}(\varepsilon)}(x)]$ *be missing degree of membership and nonmember ship of elements* h_b *to* $\tilde{\vartheta}$ (ε_a) *, respectively. We denote* $p^*_{\tilde{\vartheta}(\varepsilon_a)(u(\varepsilon))}$ *and* $p^*_{\tilde{\vartheta}(\varepsilon_a)(v(\varepsilon))}$ *as percentage of missing degree of membership and nonmember ship in total degree of membership and nonmember ship for the parameter* ε_a *respectively as follows:*

$$p^*_{\tilde{\vartheta}(\varepsilon_a)(u(\varepsilon))} = \frac{\sum_{k=1}^{|U|} \left|u^{-*}_{\tilde{\vartheta}(\varepsilon_a)}(h_k)\right| + \sum_{k=1}^{|U|} \left|u^{+*}_{\tilde{\vartheta}(\varepsilon_a)}(h_k)\right|}{2n} \tag{3}$$

$$p^*_{\tilde{\vartheta}(\varepsilon_a)(v(\varepsilon))} = \frac{\sum_{k=1}^{|U|} \left|v^{-*}_{\tilde{\vartheta}(\varepsilon_a)}(h_k)\right| + \sum_{k=1}^{|U|} \left|v^{+*}_{\tilde{\vartheta}(\varepsilon_a)}(h_k)\right|}{2n} \tag{4}$$

where $|U|$ *denotes the number of objects.* $\left|u^{-*}_{\tilde{\vartheta}(\varepsilon_a)}(h_k)\right|$ *and* $\left|v^{-*}_{\tilde{\vartheta}(\varepsilon_a)}(h_k)\right|$ *are the number of the missing lower degrees of membership and nonmember ship of an element* h_k *to* $\tilde{\vartheta}(\varepsilon_a)$*, respectively.* $\left|u^{+*}_{\tilde{\vartheta}(\varepsilon_a)}(h_k)\right|$ *and* $\left|v^{+*}_{\tilde{\vartheta}(\varepsilon_a)}(h_k)\right|$ *are the number of the missing upper degrees of membership and nonmember ship of an element* h_k *to* $\tilde{\vartheta}(\varepsilon_a)$*, respectively.* n *is the number of objects.*

Definition 4. *Let* $x \in U, \varepsilon \in E$ *,a pair* $(\tilde{\vartheta}, E)$ *is called an interval-valued intuitionistic fuzzy soft set over* $\Re(U)$, $U = \{h_1, h_2, \cdots, h_n\}$, $E = \{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_m\}$, $\tilde{\vartheta}(\varepsilon) = \{\langle x, \mu_{\tilde{\vartheta}(\varepsilon)}(x), v_{\tilde{\vartheta}(\varepsilon)}(x)\rangle\}$, $u_{\tilde{\vartheta}(\varepsilon)}(x) = [u^-_{\tilde{\vartheta}(\varepsilon)}(x), u^+_{\tilde{\vartheta}(\varepsilon)}(x)]$, $v_{\tilde{\vartheta}(\varepsilon)}(x) = [v^-_{\tilde{\vartheta}(\varepsilon)}(x), v^+_{\tilde{\vartheta}(\varepsilon)}(x)]$ *. Let* $u^*_{\tilde{\vartheta}(\varepsilon)}(x) = [u^{-*}_{\tilde{\vartheta}(\varepsilon)}(x), u^{+*}_{\tilde{\vartheta}(\varepsilon)}(x)]$, $v^*_{\tilde{\vartheta}(\varepsilon)}(x) = [v^{-*}_{\tilde{\vartheta}(\varepsilon)}(x), v^{+*}_{\tilde{\vartheta}(\varepsilon)}(x)]$ *be missing degree of membership and nonmember ship of elements* h_b *to* $\tilde{\vartheta}$ (ε_a) *, respectively. We denote* $p^*_{\tilde{\vartheta}(h_b)(u(\varepsilon))}$ *and* $p^*_{\tilde{\vartheta}(h_b)(v(\varepsilon))}$ *as percentage of missing degree of membership and nonmember ship in total degree of membership and nonmember ship for the object* h_b *respectively as follows:*

$$p^{*}_{\tilde{\vartheta}(h_b)(u(\varepsilon))} = \frac{\sum_{k=1}^{|E|} \left| u^{-*}_{\tilde{\vartheta}(\varepsilon_k)}(h_b) \right| + \sum_{k=1}^{|E|} \left| u^{+*}_{\tilde{\vartheta}(\varepsilon_k)}(h_b) \right|}{2m} \tag{5}$$

$$p^{*}_{\tilde{\vartheta}(h_b)(v(\varepsilon))} = \frac{\sum_{k=1}^{|E|} \left| v^{-*}_{\tilde{\vartheta}(\varepsilon_k)}(h_b) \right| + \sum_{k=1}^{|E|} \left| v^{+*}_{\tilde{\vartheta}(\varepsilon_k)}(h_b) \right|}{2m} \tag{6}$$

where $|E|$ *denotes the number of parameters.* $\left| u^{-*}_{\tilde{\vartheta}(\varepsilon_k)}(h_b) \right|$ *and* $\left| v^{-*}_{\tilde{\vartheta}(\varepsilon_k)}(h_b) \right|$ *are the number of the missing lower degrees of membership and nonmember ship of an element* h_b *to* $\tilde{\vartheta}(\varepsilon_k)$ *,respectively.* $\left| u^{+*}_{\tilde{\vartheta}(\varepsilon_k)}(h_b) \right|$ *and* $\left| v^{+*}_{\tilde{\vartheta}(\varepsilon_k)}(h_b) \right|$ *are the number of the missing upper degrees of membership and nonmember ship of an element* h_b *to* $\tilde{\vartheta}(\varepsilon_k)$ *,respectively.* m *is the number of parameters.*

Definition 5. *Let* $x \in U, \varepsilon \in E$ *; a pair* $(\tilde{\vartheta}, E)$ *is called an interval-valued intuitionistic fuzzy soft set over* $\Re(U)$, $U = \{h_1, h_2, \cdots, h_n\}$, $E = \{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_m\}$, $\tilde{\vartheta}(\varepsilon) = \{\langle x, \mu_{\tilde{\vartheta}(\varepsilon)}(x), v_{\tilde{\vartheta}(\varepsilon)}(x) \rangle\}$, $u_{\tilde{\vartheta}(\varepsilon)}(x) = [u^{-}_{\tilde{\vartheta}(\varepsilon)}(x), u^{+}_{\tilde{\vartheta}(\varepsilon)}(x)]$, $v_{\tilde{\vartheta}(\varepsilon)}(x) = [v^{-}_{\tilde{\vartheta}(\varepsilon)}(x), v^{+}_{\tilde{\vartheta}(\varepsilon)}(x)]$ *. Let* $u^{*}_{\tilde{\vartheta}(\varepsilon)}(x) = [u^{-*}_{\tilde{\vartheta}(\varepsilon)}(x), u^{+*}_{\tilde{\vartheta}(\varepsilon)}(x)]$, $v^{*}_{\tilde{\vartheta}(\varepsilon)}(x) = [v^{-*}_{\tilde{\vartheta}(\varepsilon)}(x), v^{+*}_{\tilde{\vartheta}(\varepsilon)}(x)]$ *be the missing degree of membership and nonmember ship of elements* h_b *to* $\tilde{\vartheta}$ (ε_a)*, respectively. We denote* $E_{avg^{-*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(u(\varepsilon))}$ *,* $E_{avg^{-*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(v(\varepsilon))}$ *,* $E_{avg^{+*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(u(\varepsilon))}$ *and* $E_{avg^{+*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(v(\varepsilon))}$ *as lower and upper membership degree and nonmember ship degree for parameter* ε_a *respectively, where they are formulated as*

$$E_{avg^{-*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(u(\varepsilon))} = \frac{\sum_{k=1}^{|q_1|} u^{-}_{\tilde{\vartheta}(\varepsilon_a)}(h_k)}{|q_1|} \tag{7}$$

$$E_{avg^{+*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(u(\varepsilon))} = \frac{\sum_{k=1}^{|q_1|} u^{+}_{\tilde{\vartheta}(\varepsilon_a)}(h_k)}{|q_1|} \tag{8}$$

$$E_{avg^{-*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(v(\varepsilon))} = \frac{\sum_{k=1}^{|q_1|} v^{-}_{\tilde{\vartheta}(\varepsilon_a)}(h_k)}{|q_1|} \tag{9}$$

$$E_{avg^{+*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(v(\varepsilon))} = \frac{\sum_{k=1}^{|q_1|} v^{+}_{\tilde{\vartheta}(\varepsilon_a)}(h_k)}{|q_1|} \tag{10}$$

where $|q_1|$ *is the number of existing membership degrees of the objects.*

Definition 6. *Let* $x \in U, \varepsilon \in E$ *; a pair* $(\tilde{\vartheta}, E)$ *is called an interval-valued intuitionistic fuzzy soft sets over* $\Re(U)$, $U = \{h_1, h_2, \cdots, h_n\}$, $E = \{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_m\}$, $\tilde{\vartheta}(\varepsilon) = \{\langle x, \mu_{\tilde{\vartheta}(\varepsilon)}(x), v_{\tilde{\vartheta}(\varepsilon)}(x)\rangle\}$, $u_{\tilde{\vartheta}(\varepsilon)}(x) = [u^-_{\tilde{\vartheta}(\varepsilon)}(x), u^+_{\tilde{\vartheta}(\varepsilon)}(x)]$, $v_{\tilde{\vartheta}(\varepsilon)}(x) = [v^-_{\tilde{\vartheta}(\varepsilon)}(x), v^+_{\tilde{\vartheta}(\varepsilon)}(x)]$.

Let $u^*_{\tilde{\vartheta}(\varepsilon)}(x) = [u^{-*}_{\tilde{\vartheta}(\varepsilon)}(x), u^{+*}_{\tilde{\vartheta}(\varepsilon)}(x)]$, $v^*_{\tilde{\vartheta}(\varepsilon)}(x) = [v^{-*}_{\tilde{\vartheta}(\varepsilon)}(x), v^{+*}_{\tilde{\vartheta}(\varepsilon)}(x)]$ *be the missing degree of membership and nonmember ship of elements* h_b *to* $\tilde{\vartheta}$ (ε_a)*, respectively. We denote* $\mathrm{H}_{avg^{-*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(u(\varepsilon))}$*,* $\mathrm{H}_{avg^{-*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(v(\varepsilon))}$*)* *,* $\mathrm{H}_{avg^{+*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(u(\varepsilon))}$ *and* $\mathrm{H}_{avg^{+*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(v(\varepsilon))}$*) as the lower and upper membership degree and nonmember ship degrees for the object* h_b*, where it is formulated as*

$$\mathrm{H}_{avg^{-*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(u(\varepsilon))} = \frac{\sum_{k=1}^{|q_2|} u^-_{\tilde{\vartheta}(\varepsilon_k)}(h_b)}{|q_2|} \tag{11}$$

$$\mathrm{H}_{avg^{+*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(u(\varepsilon))} = \frac{\sum_{k=1}^{|q_2|} u^+_{\tilde{\vartheta}(\varepsilon_k)}(h_b)}{|q_2|} \tag{12}$$

$$\mathrm{H}_{avg^{-*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(v(\varepsilon))} = \frac{\sum_{k=1}^{|q_2|} v^-_{\tilde{\vartheta}(\varepsilon_k)}(h_b)}{|q_2|} \tag{13}$$

$$\mathrm{H}_{avg^{+*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(v(\varepsilon))} = \frac{\sum_{k=1}^{|q_2|} v^+_{\tilde{\vartheta}(\varepsilon_k)}(h_b)}{|q_2|} \tag{14}$$

where $|q_2|$ *is the number of existing membership degrees of the parameters.*

3.2. Data Analysis Approaches for Incomplete Interval-Valued Intuitionistic Fuzzy Soft Sets

Based on the above definitions, we give our algorithm as follows:

(a) Input the incomplete interval-valued intuitionistic fuzzy soft sets $(\tilde{\vartheta}, E)$ and the parameter set E.

(b) Find the missing degree of membership and nonmember ship of elements h_b to $\tilde{\vartheta}$ (ε_a) as $u^*_{\tilde{\vartheta}(\varepsilon)}(x) = [u^{-*}_{\tilde{\vartheta}(\varepsilon)}(x), u^{+*}_{\tilde{\vartheta}(\varepsilon)}(x)]$ ($v^*_{\tilde{\vartheta}(\varepsilon)}(x) = [v^{-*}_{\tilde{\vartheta}(\varepsilon)}(x), v^{+*}_{\tilde{\vartheta}(\varepsilon)}(x)]$).

(c) Compute $p^*_{\tilde{\vartheta}(\varepsilon_a)(u(\varepsilon))}$, $p^*_{\tilde{\vartheta}(\varepsilon_a)(v(\varepsilon))}$, $p^*_{\tilde{\vartheta}(h_b)(u(\varepsilon))}$ and $p^*_{\tilde{\vartheta}(h_b)(v(\varepsilon))}$. If $p^*_{\tilde{\vartheta}(\varepsilon_a)(u(\varepsilon))} \le 40\%$ $\left(p^*_{\tilde{\vartheta}(\varepsilon_a)(v(\varepsilon))} \le 40\%\right)$, the remainder data which belong to the same column with the missing data are reliable; if $p^*_{\tilde{\vartheta}(h_b)(u(\varepsilon))} \le 40\%$ $(p^*_{\tilde{\vartheta}(h_b)(v(\varepsilon))} \le 40\%)$, the remainder data which belong to the same row with the missing data are reliable; otherwise, this missing data should be ignored.

(d) When the missing value is one of membership degree or nonmember ship degree, for $\forall x \in X$, $\sup u_A(x) + \sup v_A(x) \le 1$, we fill the missing data by the following equations:

$$\begin{cases} u^{+*}_{\tilde{\vartheta}(\varepsilon)}(x) = 1 - v^{+}_{\tilde{\vartheta}(\varepsilon)}(x), u^{-*}_{\tilde{\vartheta}(\varepsilon)}(x) = u^{+*}_{\tilde{\vartheta}(\varepsilon)}(x) - [v^{+}_{\tilde{\vartheta}(\varepsilon)}(x) - v^{-}_{\tilde{\vartheta}(\varepsilon)}(x)] \\ v^{+*}_{\tilde{\vartheta}(\varepsilon)}(x) = 1 - u^{+}_{\tilde{\vartheta}(\varepsilon)}(x), v^{-*}_{\tilde{\vartheta}(\varepsilon)}(x) = v^{+*}_{\tilde{\vartheta}(\varepsilon)}(x) - [u^{+}_{\tilde{\vartheta}(\varepsilon)}(x) - u^{-}_{\tilde{\vartheta}(\varepsilon)}(x)] \end{cases} \tag{15}$$

(e) When both membership degree and nonmember ship degree are missing, we calculate $u^{-*}_{\tilde{\vartheta}(\varepsilon_a)}(h_b)$ ($v^{-*}_{\tilde{\vartheta}(\varepsilon_a)}(h_b)$) and $u^{+*}_{\tilde{\vartheta}(a)}(h_b)$ ($v^{+*}_{\tilde{\vartheta}(a)}(h_b)$) as lower and upper (non-) membership degrees of an element h_n to $\tilde{\vartheta}(\varepsilon_a)$, where

$$\begin{cases} u^{-*}_{\tilde{\vartheta}(\varepsilon_a)}(h_b) = \left(E_{avg^{-*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(u(\varepsilon))} + \mathrm{H}_{avg^{-*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(u(\varepsilon))}\right)/2 \\ u^{+*}_{\tilde{\vartheta}(a)}(h_b) = (E_{avg^{+*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(u(\varepsilon))} + \mathrm{H}_{avg^{+*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(u(\varepsilon))})/2 \\ v^{-*}_{\tilde{\vartheta}(\varepsilon_a)}(h_b) = (E_{avg^{-*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(v(\varepsilon))} + \mathrm{H}_{avg^{-*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(v(\varepsilon))})/2 \\ v^{+*}_{\tilde{\vartheta}(a)}(h_b) = (E_{avg^{+*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(v(\varepsilon))} + \mathrm{H}_{avg^{+*}\tilde{\vartheta}(\varepsilon_a)}(h_b)_{(v(\varepsilon))})/2 \end{cases} \tag{16}$$

(f) Finally, we can get a complete interval-valued intuitionistic fuzzy soft set.

3.3. One Example for the Proposed Approaches

Suppose that $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}\}$ is a set of ten objects and $E = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6\}$ is a set of parameters. The interval-valued intuitionistic fuzzy soft set $(\tilde{\theta}, E)$ is depicted by a tabular representation in Table 2, which has missing values denoted by "*". We apply our proposed method to convert the incomplete interval-valued intuitionistic fuzzy soft set into a complete one.

Step 1: Input the incomplete interval-valued intuitionistic fuzzy soft set $(\tilde{\vartheta}, E)$ and the parameter set E.

Step2: Find $u^{*}_{\tilde{\vartheta}(\varepsilon_1)}(h_8) = [u^{-*}_{\tilde{\vartheta}(\varepsilon_1)}(h_8), u^{+*}_{\tilde{\vartheta}(\varepsilon_1)}(h_8)]$; $u^{*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5) = [u^{-*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5), u^{+*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5)]$; $v^{*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5) = [v^{-*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5), v^{+*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5)]$; $u^{*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8) = [u^{-*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8), u^{+*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8)]$; $v^{*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8) = [v^{-*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8), v^{+*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8)]$; $u^{*}_{\tilde{\vartheta}(\varepsilon_5)}(h_1) = [u^{-*}_{\tilde{\vartheta}(\varepsilon_5)}(h_1), u^{+*}_{\tilde{\vartheta}(\varepsilon_5)}(h_1)]$; $u^{*}_{\tilde{\vartheta}(\varepsilon_6)}(h_5) = [v^{-*}_{\tilde{\vartheta}(\varepsilon_6)}(h_5), v^{+*}_{\tilde{\vartheta}(\varepsilon_6)}(h_5)]$; $v^{*}_{\tilde{\vartheta}(\varepsilon_6)}(h_7) = [v^{-*}_{\tilde{\vartheta}(\varepsilon_6)}(h_7), v^{+*}_{\tilde{\vartheta}(\varepsilon_6)}(h_7)]$ as missing degrees of membership and nonmember ship of elements h_b to $\tilde{\vartheta}(\varepsilon_a)$.

Step3: Compute $p^{*}_{\tilde{\vartheta}(\varepsilon_1)(u(\varepsilon))} = p^{*}_{\tilde{\vartheta}(\varepsilon_3)(u(\varepsilon))} = p^{*}_{\tilde{\vartheta}(\varepsilon 4)(u(\varepsilon))} = p^{*}_{\tilde{\vartheta}(\varepsilon 5)(u(\varepsilon))} = p^{*}_{\tilde{\vartheta}(\varepsilon_6)(u(\varepsilon))} = \frac{2}{2 \times 10} = 10\% \le 40\%$

$p^{*}_{\tilde{\vartheta}(\varepsilon_1)(v(\varepsilon))} = p^{*}_{\tilde{\vartheta}(\varepsilon_4)(v(\varepsilon))} = p^{*}_{\tilde{\vartheta}(\varepsilon_6)(v(\varepsilon))} = \frac{2}{2 \times 10} = 10\% \le 40\%$, the remainder data which belong to the same column with the missing data are reliable; and compute $p^{*}_{\tilde{\vartheta}(h_1)(u(\varepsilon))} = \frac{2}{2 \times 6} = 16.7\% \le 40\%$, $p^{*}_{\tilde{\vartheta}(h_5)(u(\varepsilon))} = p^{*}_{\tilde{\vartheta}(h_8)(u(\varepsilon))} = \frac{4}{2 \times 6} = 33.3\% \le 40\%$, $p^{*}_{\tilde{\vartheta}(h_5)(v(\varepsilon))} = p^{*}_{\tilde{\vartheta}(h_7)(v(\varepsilon))} = p^{*}_{\tilde{\vartheta}(h_8)(v(\varepsilon))} = \frac{2}{2 \times 6} = 16.7\% \le 40\%$. The remainder data which belong to the same row with the missing data are reliable.

Step4: There are four groups of missing data which involve one of membership degree or nonmember ship degree, hence we calculate $u^{*}_{\tilde{\vartheta}(\varepsilon_1)}(h_8)$, $u^{*}_{\tilde{\vartheta}(\varepsilon_5)}(h_1)$, $u^{*}_{\tilde{\vartheta}(\varepsilon_6)}(h_5)$ and $v^{*}_{\tilde{\vartheta}(\varepsilon_6)}(h_7)$ by Equation (15) as the related lower and upper membership degrees and nonmember ship degrees:

$u^{*}_{\tilde{\vartheta}(\varepsilon_1)}(h_8)$ = [1-0.73-(0.73-0.6),1-0.73] = [0.14,0.27], $u^{*}_{\tilde{\vartheta}(\varepsilon_5)}(h_1)$ = [1-0.67-(0.67- 0.6),1-0.67] = [0.26,0.33], $u^{*}_{\tilde{\vartheta}(\varepsilon_6)}(h_5)$ = [1-0.2-(0.2-0.0),1-0.20]=[0.60,0.8], $v^{*}_{\tilde{\vartheta}(\varepsilon_6)}(h_7)$ = [1-0.8-(0.8-0.69),1-0.80] = [0.09,0.2];

Step5: Because there are eight groups of missing data which involve both membership degree and nonmember ship degree, we calculate $u^{-*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5)$ ($v^{-*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5)$), $u^{+*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5)$ ($v^{+*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5)$) and $u^{-*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8)$ ($v^{-*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8)$), $u^{+*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8)$ ($v^{+*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8)$) by Equation (16) as related lower and upper membership degrees and nonmember ship degrees, where

$$E_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(u(\varepsilon))} = \left(0.35+0.48+0.55+0.75+0.38+0.60+0.35+0.65+0.71\right)/9 = 0.54$$

$$E_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(u(\varepsilon))} = \left(0.81+0.58+0.68+0.90+0.69+0.70+0.55+0.85+0.84\right)/9 = 0.73$$

$$E_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(v(\varepsilon))} = \left(0.05+0.20+0.20+0.01+0.01+0.15+0.30+0.05+0.05\right)/9 = 0.14$$

$$E_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(v(\varepsilon))} = \left(0.15+0.30+0.30+0.10+0.20+0.25+0.40+0.15+0.15\right)/9 = 0.22$$

$$E_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(u(\varepsilon))} = \left(0.14+0.38+0.76+0.41+0.39+0.70+0.70+0.23+0.32\right)/9 = 0.45;$$

$$E_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(u(\varepsilon))} = \left(0.40+0.69+0.87+0.88+0.71+0.80+0.89+0.71+0.55\right)/9 = 0.72;$$

$$E_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(v(\varepsilon))} = \left(0.45+0.01+0.03+0.05+0.11+0.08+0.05+0.18+0.33\right)/9 = 0.14;$$

$$E_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(v(\varepsilon))} = \left(0.55+0.20+0.13+0.12+0.21+0.18+0.11+0.28+0.44\right)/9 = 0.25;$$

And,

$$\mathrm{H}_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(u(\varepsilon))} = \left(0.61+0.65+0.39+0.73+0.80\right)/5 = 0.64;$$

$$\mathrm{H}_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(u(\varepsilon))} = \left(0.78+0.75+0.71+0.94+1.00\right)/5 = 0.84$$

$$\mathrm{H}_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(v(\varepsilon))} = \left(0.11+0.10+0.11+0.01+0.00\right)/5 = 0.07;$$

$$\mathrm{H}_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(v(\varepsilon))} = \left(0.19+0.20+0.21+0.06+0.20\right)/5 = 0.17;$$

$$\mathrm{H}_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(u(\varepsilon))} = \left(0.27+0.72+0.35+0.78+0.70\right)/5 = 0.56;$$

$$\mathrm{H}_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(u(\varepsilon))} = \left(0.40+0.82+0.55+0.90+0.95\right)/5 = 0.72;$$

$$\mathrm{H}_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(v(\varepsilon))} = \left(0.60+0.06+0.30+0.01+0.00\right)/5 = 0.19;$$

$$\mathrm{H}_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(v(\varepsilon))} = \left(0.73+0.17+0.40+0.10+0.05\right)/5 = 0.29;$$

So, we can get

$$u^{-*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5)= (E_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(u(\varepsilon))} + \mathrm{H}_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(u(\varepsilon))})/2 = \left(0.54+0.64\right)/2 = 0.59;$$

$$u^{+*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5)= (E_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(u(\varepsilon))} + \mathrm{H}_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(u(\varepsilon))})/2 = \left(0.73+0.84\right)/2 = 0.79$$

$$v^{-*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5)= (E_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(v(\varepsilon))} + \mathrm{H}_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(v(\varepsilon))})/2 = \left(0.14+0.07\right)/2 = 0.11;$$

$$v^{+*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5)= (E_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(v(\varepsilon))} + \mathrm{H}_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(v(\varepsilon))})/2 = \left(0.22+0.17\right)/2 = 0.20;$$

$$u^{-*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8)= (E_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(u(\varepsilon))} + \mathrm{H}_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(u(\varepsilon))})/2 = \left(0.45+0.56\right)/2 = 0.51;$$

$$u^{+*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8)= (E_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(u(\varepsilon))} + \mathrm{H}_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(u(\varepsilon))})/2 = \left(0.72+0.72\right)/2 = 0.72;$$

$$v^{-*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8)= (E_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(v(\varepsilon))} + \mathrm{H}_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(v(\varepsilon))})/2 = \left(0.14+0.19\right)/2 = 0.17;$$

$$v^{+*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8)= (E_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(v(\varepsilon))} + \mathrm{H}_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_8)_{(v(\varepsilon))})/2 = \left(0.25+0.29\right)/2 = 0.27;$$

Finally, we convert this incomplete IVIFSS into one complete IVIFSS which is shown in Table 3.

Table 2. The incomplete interval-valued intuitionistic fuzzy soft set $(\widetilde{\vartheta}, E)$.

U/E	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6
h_1	[0.13,0.27] [0.60,0.67]	[0.77,0.88] [0.02,0.12]	[0.35,0.81] [0.05,0.15]	[0.14,0.40] [0.45,0.55]	[*, *] [0.60,0.67]	[0.70,0.80] [0.10,0.17]
h_2	[0.53,0.80] [0.00,0.20]	[0.60,0.80],[0.05,0.20]	[0.48,0.58] [0.20,0.30]	[0.38,0.69] [0.01,0.20]	[0.60,0.81] [0.09,0.19]	[0.38,0.69] [0.01,0.20]
h_3	[0.70,0.85] [0.05,0.15]	[0.70,0.89] [0.05,0.11]	[0.55,0.68] [0.20,0.30]	[0.76,0.87] [0.03,0.13]	[0.70,0.95] [0.01,0.05]	[0.67,0.80] [0.00,0.20]
h_4	[0.07,0.27] [0.60,0.67]	[0.69,0.95] [0.00,0.05]	[0.75,0.90] [0.01,0.10]	[0.41,0.88] [0.05,0.12]	[0.20,0.40] [0.00,0.40]	[0.75,0.85] [0.05,0.13]
h_5	[0.61,0.78] [0.11,0.19]	[0.65,0.75] [0.10,0.20]	[*, *] [*, *]	[0.39,0.71] [0.11,0.21]	[0.73,0.94] [0.01,0.06]	[*, *] [0.00,0.20]
h_6	[0.27,0.40] [0.00,0.53]	[0.65,0.85] [0.05,0.15]	[0.38,0.69] [0.01,0.20]	[0.70,0.80] [0.08,0.18]	[0.35,0.50] [0.20,0.40]	[0.23,0.26] [0.60,0.71]
h_7	[0.67,0.80] [0.00,0.20]	[0.74,1.00] [0.00,0.00]	[0.60,0.70] [0.15,0.25]	[0.70,0.89] [0.05,0.11]	[0.65,0.69] [0.21,0.31]	[0.69,0.80] [*, *]
h_8	[*, *] [0.60,0.73]	[0.72,0.82] [0.06,0.17]	[0.35,0.55] [0.30,0.40]	[*,*][*,*]	[0.78,0.90] [0.01,0.10]	[0.70,0.95] [0.00,0.05]
h_9	[0.00,0.13] [0.60,0.80]	[0.71,0.81] [0.09,0.19]	[0.65,0.85] [0.05,0.15]	[0.23,0.71] [0.18,0.28]	[0.68,0.81] [0.08,0.18]	[0.20,0.40] [0.40,0.60]
h_{10}	[0.07,0.27] [0.33,0.60]	[0.58,0.68] [0.15,0.25]	[0.71,0.84] [0.05,0.15]	[0.32,0.55] [0.33,0.44]	[0.70,0.95] [0.00,0.05]	[0.23,0.26] [0.60,0.71]

Table 3. A complete interval-valued intuitionistic fuzzy soft set for Table 2.

U/E	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6
h_1	[0.13,0.27] [0.60,0.67]	[0.77,0.88] [0.02,0.12]	[0.35,0.81] [0.05,0.15]	[0.14,0.40] [0.45,0.55]	[0.26,0.33] [0.60,0.67]	[0.70,0.80] [0.10,0.17]
h_2	[0.53,0.80] [0.00,0.20]	[0.60,0.80],[0.05,0.20]	[0.48,0.58] [0.20,0.30]	[0.38,0.69] [0.01,0.20]	[0.60,0.81] [0.09,0.19]	[0.38,0.69] [0.01,0.20]
h_3	[0.70,0.85] [0.05,0.15]	[0.70,0.89] [0.05,0.11]	[0.55,0.68] [0.20,0.30]	[0.76,0.87] [0.03,0.13]	[0.70,0.95] [0.01,0.05]	[0.67,0.80] [0.00,0.20]
h_4	[0.07,0.27] [0.60,0.67]	[0.69,0.95] [0.00,0.05]	[0.75,0.90] [0.01,0.10]	[0.41,0.88] [0.05,0.12]	[0.20,0.40] [0.00,0.40]	[0.75,0.85] [0.05,0.13]
h_5	[0.61,0.78] [0.11,0.19]	[0.65,0.75] [0.10,0.20]	[0.59,0.79] [0.11,0.20]	[0.39,0.71] [0.11,0.21]	[0.73,0.94] [0.01,0.06]	[0.6,0.80] [0.00,0.20]
h_6	[0.27,0.40]	[0.65,0.85] [0.05,0.15]	[0.38,0.69]	[0.70,0.80]	[0.35,0.50]	[0.23,0.26]

	[0.00,0.53]		[0.01,0.20]	[0.08,0.18]	[0.20,0.40]	[0.60,0.71]
h_7	[0.67,0.80] [0.00,0.20]	[0.74,1.00] [0.00,0.00]	[0.60,0.70] [0.15,0.25]	[0.70,0.89] [0.05,0.11]	[0.65,0.69] [0.21,0.31]	[0.69,0.80] [0.09,0.20]
h_8	[0.14,0.27] [0.60,0.73]	[0.72,0.82] [0.06,0.17]	[0.35,0.55] [0.30,0.40]	[0.51,0.72] [0.17,0.27]	[0.78,0.90] [0.01,0.10]	[0.70,0.95] [0.00,0.05]
h_9	[0.00,0.13] [0.60,0.80]	[0.71,0.81] [0.09,0.19]	[0.65,0.85] [0.05,0.15]	[0.23,0.71] [0.18,0.28]	[0.68,0.81] [0.08,0.18]	[0.20,0.40] [0.40,0.60]
h_{10}	[0.07,0.27] [0.33,0.60]	[0.58,0.68] [0.15,0.25]	[0.71,0.84] [0.05,0.15]	[0.32,0.55] [0.33,0.44]	[0.70,0.95] [0.00,0.05]	[0.23,0.26] [0.60,0.71]

4. Experimental Results

In this part, we use specific examples to prove the effectiveness and accuracy of our data filling method. Firstly, we define the deviation rate as follows:

$$\tilde{p}_i = \frac{|q_i - \tilde{q}_i|}{q_i} \tag{17}$$

where $\tilde{p}_i$ is the deviation rate, q_i is the actual data value, and $\tilde{q}_i$ is the filled data value.

Therefore, the accuracy rate of our filling algorithm is:

$$p_i = 1 - \tilde{p}_i \tag{18}$$

The average accuracy rate is:

$$p_{avg} = \frac{\sum_{i=1}^{n} p_i}{n} \tag{19}$$

where n is the number of data we have filled in.

Aiming to verify our method, we generate randomly thirty datasets which are described by IVIFSS. The first ten datasets involve ten objects and five parameters. The next ten datasets have 50 objects and ten parameters. The last ten datasets have 100 objects and 15 parameters. For every dataset, we set the number of the test missing data as four groups, among which two groups miss the related lower and upper membership degrees and nonmember ship degrees, the other two groups miss one of membership degree and nonmember ship degree. For example, there is one dataset which has ten objects and five parameters displayed in Table 4. The test data are randomly chosen from the initial data set. We randomly choose $[u^{-*}_{\tilde{\vartheta}(\varepsilon_1)}(h_1), u^{+*}_{\tilde{\vartheta}(\varepsilon_1)}(h_1)][v^{-*}_{\tilde{\vartheta}(\varepsilon_1)}(h_1), v^{+*}_{\tilde{\vartheta}(\varepsilon_1)}(h_1)]$, $[u^{-*}_{\tilde{\vartheta}(\varepsilon_2)}(h_3), u^{+*}_{\tilde{\vartheta}(\varepsilon_2)}(h_3)][v^{-*}_{\tilde{\vartheta}(\varepsilon_2)}(h_3), v^{+*}_{\tilde{\vartheta}(\varepsilon_2)}(h_3)]$, $[u^{-*}_{\tilde{\vartheta}(\varepsilon_3)}(h_{10}), u^{+*}_{\tilde{\vartheta}(\varepsilon_3)}(h_{10})]$ and $[u^{-*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8), u^{+*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8)]$ as the test missing data.

Next, we use our method to fill in the missing data and get the following results.

$$[u^{-*}_{\tilde{\vartheta}(\varepsilon_1)}(h_1), u^{+*}_{\tilde{\vartheta}(\varepsilon_1)}(h_1)][v^{-*}_{\tilde{\vartheta}(\varepsilon_1)}(h_1), v^{+*}_{\tilde{\vartheta}(\varepsilon_1)}(h_1)] = [0.54, 0.71][0.11, 0.23];$$

$$[u^{-*}_{\tilde{\vartheta}(\varepsilon_2)}(h_3), u^{+*}_{\tilde{\vartheta}(\varepsilon_2)}(h_3)][v^{-*}_{\tilde{\vartheta}(\varepsilon_2)}(h_3), v^{+*}_{\tilde{\vartheta}(\varepsilon_2)}(h_3)] = [0.58, 0.75][0.09, 0.21];$$

$$[u^{-*}_{\tilde{\vartheta}(\varepsilon_3)}(h_{10}), u^{+*}_{\tilde{\vartheta}(\varepsilon_3)}(h_{10})] = [1 - 0.2 - (0.8 - 0.7), 1 - 0.2] = [0.7, 0.8];$$

$$[u^{-*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8), u^{+*}_{\tilde{\vartheta}(\varepsilon_4)}(h_8)] = [1 - 0.1 - (0.9 - 0.8), 1 - 0.1] = [0.8, 0.9];$$

In order to evaluate the accuracy of the predicted data for the missing data, we compute the accuracy rate for all of missing data, respectively.

$$\widetilde{p}_1 = \frac{|q-\widetilde{q}|}{q} = \frac{|0.5 - 0.54|}{0.5} = 0.08, \ p_1 = 1 - \widetilde{p}_1 = 1 - 0.08 = 0.92;$$

$$\widetilde{p}_2 = \frac{|q-\widetilde{q}|}{q} = \frac{|0.7 - 0.71|}{0.7} = 0.01, \ p_2 = 1 - \widetilde{p}_2 = 1 - 0.01 = 0.99;$$

$$\widetilde{p}_3 = \frac{|q-\widetilde{q}|}{q} = \frac{|0.1 - 0.11|}{0.1} = 0.1, \ p_3 = 1 - \widetilde{p}_3 = 1 - 0.1 = 0.90;$$

$$\widetilde{p}_4 = \frac{|q-\widetilde{q}|}{q} = \frac{|0.2 - 0.23|}{0.2} = 0.15, \ p_4 = 1 - \widetilde{p}_4 = 1 - 0.15 = 0.85;$$

$$\widetilde{p}_5 = \frac{|q-\widetilde{q}|}{q} = \frac{|0.6 - 0.58|}{0.6} = 0.03, \ p_5 = 1 - \widetilde{p}_5 = 1 - 0.03 = 0.97;$$

$$\widetilde{p}_6 = \frac{|q-\widetilde{q}|}{q} = \frac{|0.7 - 0.75|}{0.7} = 0.07, \ p_6 = 1 - \widetilde{p}_6 = 1 - 0.07 = 0.93$$

$$\widetilde{p}_7 = \frac{|q-\widetilde{q}|}{q} = \frac{|0.1 - 0.09|}{0.1} = 0.1, \ p_7 = 1 - \widetilde{p}_7 = 1 - 0.10 = 0.90;$$

$$\widetilde{p}_8 = \frac{|q-\widetilde{q}|}{q} = \frac{|0.2 - 0.21|}{0.2} = 0.05, \ p_8 = 1 - \widetilde{p}_8 = 1 - 0.05 = 0.95;$$

$$\widetilde{p}_9 = \frac{|q-\widetilde{q}|}{q} = \frac{|0.7 - 0.7|}{0.7} = 0, \quad p_9 = 1 - \widetilde{p}_9 = 1 - 0 = 1;$$

$$\widetilde{p}_{10} = \frac{|q-\widetilde{q}|}{q} = \frac{|0.8 - 0.8|}{0.8} = 0, \quad p_{10} = 1 - \widetilde{p}_{10} = 1 - 0 = 1;$$

$$\widetilde{p}_{11} = \frac{|q-\widetilde{q}|}{q} = \frac{|0.8 - 0.8|}{0.8} = 0.0, \ p_{11} = 1 - \widetilde{p}_{11} = 1 - 0 = 1;$$

$$\widetilde{p}_{12} = \frac{|q-\widetilde{q}|}{q} = \frac{|0.9 - 0.9|}{0.9} = 0.0, \ p_{12} = 1 - \widetilde{p}_{12} = 1 - 0 = 1;$$

$$p_{avg} = \frac{\sum_{i=1}^{n} p_i}{n} = (0.92 + 0.99 + 0.90 + 0.85 + 0.97 + 0.93 + 0.90 + 0.95 + 1 + 1 + 1 + 1)$$

As a result, we obtain that the average accuracy is 95.1%. We repeat this process 10 times on this dataset, in which the missing data is randomly chosen. Finally, we get the average accuracy for this dataset as 93.2%. This above verifying process is made on the thirty datasets. We find that the lowest accuracy rate is 58.8% and the highest one is up to 96.1% on these datasets. In summary, the overall accuracy rate on all of thirty datasets is 80.1%.

Table 4. One-test interval-valued intuitionistic fuzzy soft set (IVIFSS).

U /	ε_1	ε_2	ε_3	ε_4	ε_5
h_1	[0.5,0.7] [0.1,0.2]	[0.8,0.9] [0.0,0.1]	[0.4,0.6] [0.2,0.3]	[0.5,0.6] [0.2,0.3]	[0.7,0.9] [0.0,0.1]
h_2	[0.6,0.7] [0.1,0.2]	[0.4,0.5] [0.2,0.4]	[0.6,0.8] [0.1,0.2]	[0.6,0.8] [0.0,0.2]	[0.6,0.8] [0.1,0.2]
h_3	[0.2,0.5] [0.1,0.3]	[0.6,0.7] [0.1,0.2]	[0.4,0.6] [0.3,0.4]	[0.7,0.9] [0.0,0.1]	[0.7,0.8] [0.1,0.2]
h_4	[0.5,0.6] [0.1,0.4]	[0.6,0.8] [0.0,0.1]	[0.7,0.8] [0.1,0.2]	[0.7,0.8] [0.1,0.2]	[0.6,0.9] [0.0,0.1]
h_5	[0.5,0.7] [0.1,0.2]	[0.7,0.9] [0.0,0.1]	[0.5,0.6] [0.1,0.2]	[0.6,0.8] [0.1,0.2]	[0.5,0.8] [0.1,0.2]
h_6	[0.7,0.8] [0.1,0.2]	[0.7,0.9] [0.0,0.1]	[0.6,0.8] [0.1,0.2]	[0.7,0.8] [0.1,0.2]	[0.6,0.8] [0.0,0.1]
h_7	[0.4,0.8] [0.1,0.2]	[0.6,0.7] [0.1,0.2]	[0.7,0.8] [0.1,0.2]	[0.6,0.8] [0.1,0.2]	[0.6,0.7] [0.1,0.2]
h_8	[0.3,0.7] [0.1,0.3]	[0.6,0.8] [0.1,0.2]	[0.5,0.7] [0.2,0.3]	[0.8,0.9] [0.0,0.1]	[0.8,0.9] [0.0,0.1]
h_9	[0.5,0.6] [0.2,0.3]	[0.8,0.9] [0.0,0.1]	[0.7,0.9] [0.0,0.1]	[0.7,0.8] [0.1,0.2]	[0.6,0.8] [0.1,0.2]
h_{10}	[0.6,0.7] [0.2,0.3]	[0.7,0.8] [0.1,0.2]	[0.7,0.8] [0.1,0.2]	[0.7,0.9] [0.0,0.1]	[0.6,0.9] [0.0,0.1]

5. One Real-Life Application

In this section, we apply our proposed data analysis methods for incomplete interval-valued intuitionistic fuzzy soft set into one real-life application as follows.

One university held a competition to create the university website for students, aiming to inspire the students' positive regard for the university. The competition committee will reward the best design. Five evaluation indexes are used, such as ''distinct purpose'', ''effective communication'', "good navigation'', ''excellent layouts'' and ''acceptable load time''. The feelings of different evaluators about the seven designed websites from the above five aspects are fuzzy and unclear. So we use the model of the interval-valued intuitionistic fuzzy soft set to express this fuzziness.

Let $U=\{h_1,h_2,h_3,h_4,h_5,h_6,h_7\}$ be a set of seven website designs, $E=\{e_1,e_2,e_3,e_4,e_5\}$ be a set of parameters, where e_i stand for ''clear purpose'', ''effective communication'', "good navigation'', ''excellent layouts'', and ''acceptable load time'', respectively. We apply an interval-valued intuitionistic fuzzy soft set to fully represent the seven candidates. The interval-valued intuitionistic fuzzy soft set $(\widetilde{\theta},E)$ is illustrated by a tabular representation in Table 5. However, because of some reasons, there are some missing data which are not recorded. We have to apply our proposed methods to complete this data set. The process is shown as follows:

Table 5. The Incomplete interval valued intuitionistic Fuzzy Soft Set $(\widetilde{\vartheta},E)$ for Website Design Competition.

U/E	ε_1	ε_2	ε_3	ε_4	ε_5
h_1	[0.25,0.34] [0.50,0.66]	[0.17,0.25] [0.50,0.70]	[0.20,0.41] [0.30,0.50]	[*,*] [*,*]	[0.11, 0.29] [0.60,0.70]

h_2	[0.55,0.68]	[0.61,0.80]	[0.42,0.61]	[0.75,0.80]	[0.20,0.40]
	[0.10,0.30]	[0.10,0.20]	[0.10,0.33]	[0.01,0.20]	[0.40,0.60]
h_3	[0.15,0.45]	[0.30,0.40]	[0.25,0.60]	[0.36,0.50]	[0.70,0.85]
	[0.22,0.55]	[0.20,0.50]	[0.22,0.40]	[0.21,0.50]	[0.00,0.10]
h_4	[0.07,0.27]	[0.20,0.35]	[0.60,0.71]	[0.40,0.80]	[0.30,0.40]
	[0.60,0.67]	[0.40,0.60]	[0.01,0.20]	[0.02,0.12]	[0.30,0.50]
h_5	[0.31,0.48]	[0.70,0.90]	[0.32, 0.42]	[0.61,0.71]	[0.30,0.60]
	[0.22,0.50]	[0.00,0.10]	[0.20, 0.50]	[0.10,0.25]	[0.11,0.32]
h_6	[0.40,0.60]	[0.41,0.72]	[0.10,0.32]	[0.72,0.80]	[0.40,0.55]
	[0.11,0.33]	[0.01,0.22]	[0.42,0.60]	[0.10,0.20]	[0.20,0.45]
h_7	[0.60,0.70]	[0.50,0.071	[0.50,1.00]	[0.70,0.90]	[0.50,0.70]
	[*,*]	[0.06,0.22]	[0.00,0.00]	[0.00,0.10]	[0.11,0.30]

Step1: Input the incomplete interval-valued intuitionistic fuzzy soft set $(\tilde{\vartheta}, E)$ and the parameter set E.

Step2: Find $u^{*}_{\tilde{\vartheta}(\varepsilon_4)}(h_1)=[u^{-*}_{\tilde{\vartheta}(\varepsilon 4)}(h_1),u^{+*}_{\tilde{\vartheta}(\varepsilon_4)}(h_1)]$; $v^{*}_{\tilde{\vartheta}(\varepsilon_4)}(h_1)=[v^{-*}_{\tilde{\vartheta}(\varepsilon 4)}(h_1),v^{+*}_{\tilde{\vartheta}(\varepsilon_4)}(h_1)]$; $v^{*}_{\tilde{\vartheta}(\varepsilon_1)}(h_7)=[v^{-*}_{\tilde{\vartheta}(\varepsilon_1)}(h_7),v^{+*}_{\tilde{\vartheta}(\varepsilon 1)}(h_7)]$ as missing degrees of membership and nonmember ship of elements h_b to $\tilde{\vartheta}$ (ε_a).

Step3: Compute $p^{*}_{\tilde{\vartheta}(\varepsilon_1)(v(\varepsilon))}=\frac{2}{2\times 7}=14.3\%\le 40\%$, $p^{*}_{\tilde{\vartheta}(\varepsilon_4)(v(\varepsilon))}=p^{*}_{\tilde{\vartheta}(\varepsilon_4)(u(\varepsilon))}=\frac{2}{2\times 7}=14.3\%\le 40\%$; the remainder data which belong to the same column with the missing data are reliable. Compute $p^{*}_{\tilde{\vartheta}(h_1)(u(\varepsilon))}=p^{*}_{\tilde{\vartheta}(h_1)(v(\varepsilon))}=\frac{2}{2\times 5}=20\%\le 40\%$, $p^{*}_{\tilde{\vartheta}(h_7)(v(\varepsilon))}=\frac{2}{2\times 5}=20\%\le 40\%$; the remainder data which belong to the same row with the missing data are reliable.

Step 4: There is one group of missing data which involves one of membership degree or nonmember ship degree, hence we calculate $v^{-*}_{\tilde{\vartheta}(\varepsilon_1)}(h_7)$, and $v^{+*}_{\tilde{\vartheta}(\varepsilon 1)}(h_7)$ by Equation (16) as the related lower and upper membership degrees and nonmember ship degrees:

$v^{-*}_{\tilde{\vartheta}(\varepsilon_1)}(h_7)$ =1-0.7-(0.7-0.6)=0.2, $v^{+*}_{\tilde{\vartheta}(\varepsilon 1)}(h_7)$ =1-0.7=0.3.

Step5: There are four groups of missing data which involve both membership degree and nonmember ship degree, we calculate $u^{-*}_{\tilde{\vartheta}(\varepsilon_4)}(h_1)$ ($v^{-*}_{\tilde{\vartheta}(\varepsilon_4)}(h_1)$), $u^{+*}_{\tilde{\vartheta}(\varepsilon_4)}(h_1)$ ($v^{+*}_{\tilde{\vartheta}(\varepsilon_4)}(h_1)$)by Equation (17) as related lower and upper membership degrees and nonmember ship degrees, where

$$E_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_1)_{(u(\varepsilon))}=\left(0.75+0.36+0.40+0.61+0.72+0.70\right)/6=0.59;$$

$$E_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_1)_{(u(\varepsilon))}=\left(0.8+0.5+0.8+0.71+0.8+0.90\right)/6=0.75;$$

$$E_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_1)_{(v(\varepsilon))}=\left(0.01+0.21+0.02+0.10+0.10+0.00\right)/6=0.09;$$

$$E_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_1)_{(v(\varepsilon))}=\left(0.2+0.5+0.12+0.25+0.20+0.1\right)/6=0.23;$$

$$\mathrm{H}_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_1)_{(u(\varepsilon))}=\left(0.25+0.17+0.2+0.11\right)/4=0.19;$$

$$\mathrm{H}_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_1)_{(u(\varepsilon))}=\left(0.34+0.25+0.41+0.29\right)/4=0.32;$$

$$\mathrm{H}_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_1)_{(v(\varepsilon))}=\left(0.5+0.5+0.3+0.6\right)/4=0.475;$$

$$\mathrm{H}_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_1)_{(v(\varepsilon))}=\left(0.66+0.70+0.5+0.7\right)/4=0.64;$$

So, we can get

$$u^{-*}_{\tilde{\vartheta}(\varepsilon_4)}(h_1) = (E_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_4)}}(h_1)_{(u(\varepsilon))} + \mathrm{H}_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(u(\varepsilon))})/2 = \left(0.59+0.19\right)/2 = 0.39;$$

$$u^{+*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5) = (E_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(u(\varepsilon))} + \mathrm{H}_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(u(\varepsilon))})/2 = \left(0.75+0.32\right)/2 = 0.54;$$

$$v^{-*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5) = (E_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(v(\varepsilon))} + \mathrm{H}_{avg^{-*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(v(\varepsilon))})/2 = \left(0.09+0.475\right)/2 = 0.28;$$

$$v^{+*}_{\tilde{\vartheta}(\varepsilon_3)}(h_5) = (E_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(v(\varepsilon))} + \mathrm{H}_{avg^{+*}_{\tilde{\vartheta}(\varepsilon_3)}}(h_5)_{(v(\varepsilon))})/2 = \left(0.23+0.64\right)/2 = 0.44;$$

Finally, we get a complete data set represented by the interval-valued intuitionistic fuzzy soft set shown in Table 6. Based on this complete data set, we can make decision to get the best website for this university.

Table 6. The complete interval-valued intuitionistic fuzzy soft set $(\widetilde{\vartheta}, E)$ for a website design competition.

U/E	ε_1	ε_2	ε_3	ε_4	ε_5
h_1	[0.25,0.34] [0.50,0.66]	[0.17,0.25] [0.50,0.70]	[0.20,0.41] [0.30,0.50]	[0.39,0.54] [0.28,0.44]	[0.11, 0.29] [0.60,0.70]
h_2	[0.55,0.68] [0.10,0.30]	[0.61,0.80] [0.10,0.20]	[0.42,0.61] [0.10,0.33]	[0.75,0.80] [0.01,0.20]	[0.20,0.40] [0.40,0.60]
h_3	[0.15,0.45] [0.22,0.55]	[0.30,0.40] [0.20,0.50]	[0.25,0.60] [0.22,0.40]	[0.36,0.50] [0.21,0.50]	[0.70,0.85] [0.00,0.10]
h_4	[0.07,0.27] [0.60,0.67]	[0.20,0.35] [0.40,0.60]	[0.60,0.71] [0.01,0.20]	[0.40,0.80] [0.02,0.12]	[0.30,0.40] [0.30,0.50]
h_5	[0.31,0.48] [0.22,0.50]	[0.70,0.90] [0.00,0.10]	[0.32, 0.42] [0.20, 0.50]	[0.61,0.71] [0.10,0.25]	[0.30,0.60] [0.11,0.32]
h_6	[0.40,0.60] [0.11,0.33]	[0.41,0.72] [0.01,0.22]	[0.10,0.32] [0.42,0.60]	[0.72,0.80] [0.10,0.20]	[0.40,0.55] [0.20,0.45]
h_7	[0.60,0.70] [0.20,0.30]	[0.50,0.071 [0.06,0.22]	[0.50,1.00] [0.00,0.00]	[0.70,0.90] [0.00,0.10]	[0.50,0.70] [0.11,0.30]

6. Conclusions

The model of the interval-valued intuitionistic fuzzy soft set has been widely used since it was proposed. In actual data processing, we have to face up to missing data, which leads to unsuccessful and improper applications based on the model of the interval-valued intuitionistic fuzzy soft set. This paper focuses on data analysis methods for an incomplete interval-valued intuitionistic fuzzy soft set. The related filling idea fully considers and employs the characteristics of this model itself. The experimental results verify that the overall accuracy rate on all of thirty randomly generated datasets is up to 80.1% by our filling method. One real-life application illustrates our contribution.

Author Contributions: For research articles with several authors, a short paragraph specifying their individual contributions must be provided. The following statements should be used "Conceptualization, H.Q. and X.M.; methodology, H.L.; software, Z.G.; validation, Y.C.; formal analysis, Q.F.; investigation, H.L.; resources, Z.G.; data curation, Y.C.; writing—original draft preparation, H.L.; writing—review and editing, X.M.; visualization, H.Q.; supervision, H.Q.; project administration, H.Q.; funding acquisition, X.M. All authors have read and agreed to the published version of the manuscript.

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