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# The Renewable Energy Source Selection by Remoteness Index-Based VIKOR Method for Generalized Intuitionistic Fuzzy Soft Sets 

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#### Abstract

In this paper, we introduce the Euclidean, Hamming, and generalized distance measures for the generalized intuitionistic fuzzy soft sets (GIFSSs). We discuss the properties of the presented distance measures. The numerical example of decision making and pattern recognition is discussed based on the proposed distance measures. We develop a remoteness index-based VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method for GIFSSs. The displaced and fixed ideals intuitionistic fuzzy values (IFVs) are defined. The novel concept of displaced and fixed remoteness indexes for IFVs are discussed. We discuss the methods to obtain the precise and intuitionistic fuzzy (IF) weights. The several displaced and fixed ranking indexes are defined based on the precise and IF weights. The remoteness indexes based VIKOR methods are proposed in the form of four algorithms. In the end, the selection of renewable energy sources problem is solved by using the four remoteness index-based VIKOR methods.


Keywords: VIKOR method; generalized intuitionistic fuzzy soft set; distance measures; multi-attribute decision making; pattern recognition; renewable energy source

## 1. Introduction

In the fuzzy sets theory, the membership function is used to represents the information [1]. Real-life uncertainties handle effectively by fuzzy set theory. In Reference [2], Molodtsov defines the
soft set which is a new logical instrument for dealing with uncertainties. Molodtsov soft set theory deals with uncertainties effectively by considering the parametric point of view, that is, each element is judged by some criteria of attributes. Atanassove defined the intuitionistic fuzzy set (IFS), which is the generalization of the fuzzy set theory [3]. The information in IFS is represented in the form of membership (favor) and non-membership functions (against). The membership and non-membership functions assign the values from the unit interval $[0,1]$ with the condition that their sum is less than or equal to one, i.e., if we represent the membership and non-membership functions by $\xi$ and $v$, respectively, than $0 \leq \xi+v \leq 1$. This condition specifies a range of $\xi$ and $v$. The range of membership and non-membership functions increases in Yager's Pythagorean fuzzy sets [4], i.e., the experts make their judgments more freely in Pythagorean fuzzy environment. The condition $0 \leq \xi^{2}+v^{2} \leq 1$ specifies the range of membership and non-membership functions. Further, improvement continues by defining the more general environment, the q-rung orthopair fuzzy sets by Yager [5]. The condition $0 \leq \xi^{q}+v^{q} \leq 1$, where $q>1$ is any real number, specifies the range of membership and non-membership functions.

Many researchers work on the hybridization of soft sets with fuzzy sets and extensions of fuzzy sets. In References [6,7], the fuzzy soft set, and intuitionistic fuzzy soft set were defined, respectively. The vague soft set [8], the interval-valued fuzzy soft set [9], the trapezoidal fuzzy soft set [10], the soft rough set [11], the neutrosophic soft set [12], and the q-rung orthopair fuzzy soft sets [13] were defined. Feng et al. [14] clarified and redefined the concept of Agarwal model of GIFSS [15] and apply to decision-making problems. For more about decision making, we refer to [16-21].

Keeping in mind the importance of distance measure and application in data mining, medical diagnosis, decision making, and pattern recognition many authors work on this topic. A wide theory of distance measures of fuzzy sets and intuitionistic fuzzy sets is presented in the literature [22-24].

## Review of VIKOR Method

For determining the compromise solution and ameliorate the standard of the decision-making process, the VlseKriterijumska Optimizacija I Kompromisno Resenje (i.e., multicriteria optimization and compromise solution) (VIKOR) method is used [25-27]. The VIKOR method balancing the majority's maximum group utility and opponent's individual regret. Therefore, the VIKOR method has been widely used in different areas [28-30].

A literature review paper about the VIKOR method by Gul et al. [31], which described the application of the VIKOR method in different areas until 2016. Many authors continuing to use VIKOR methodology for decision making and it's applications in different areas. Hafezalkotob et al. [32] discussed the machine selection problem by using target-based VIKOR in an interval environment. Wang et al. [33] used a projection-based VIKOR method for risk evaluation of construction projects using picture fuzzy set (PFS). Zhao et al. [34] used extended VIKOR method for supplier selection using IFSs and combination weights. Li et al. [35] proposed a decision making procedure based on the VIKOR method and dynamic IFSs with time preferences and discussed its applications in innovation alliance partner selection. The potential evaluation of emerging technology commercialization was discussed by Wang et al. [36] using the VIKOR method in triangular fuzzy neutrosophic numbers. An application of multi-criteria decision-making (MCDM) process in health care service was discussed by Chen using the novel VIKOR method [37]. Meksavang et al. [38] explored the VIKOR method for sustainable supplier selection using PFS and discuss its application in the beef industry.

The VIKOR method based on the fuzzy entropy for linguistic D numbers was extended by Liu et al. [39]. Zhou [40] discussed the extended VIKOR method for the health-care industry. Li et al. explained the VIKOR method for linguistic intuitionistic fuzzy numbers based on entropy and operational laws [41]. The decision-making problem was discussed by Wei [42], using the 2-tuple linguistic neutrosophic VIKOR method. Kaya et al. [43] defined the VIKOR method for renewable energy planning. The failure risk assessment problem was discussed by Mohsen et al. [44] with a case study of the geothermal power plant.

The remoteness index-based VIKOR method for Pythagorean fuzzy sets was proposed by Chen. Chen applies the proposed method to evaluation of service quality among domestic airlines, investment in Internet stocks, practical applications in the criteria satisfaction problem, evaluation of Internet stock performance, and the investment in R and D projects [45].

Feng et al. [14] redefined and clarified the concept of GIFSS. This higher model of IFS is important in decision making because it further allows the experts to check or evaluate the process of decision making or evaluation of alternatives against criterion and express their preferences in the form of an extra IFS called primary IFS. Feng redefined the operators and operations of GIFSS. He applies it to the university appointment problem which is the multi-attribute decision making (MADM) problems. Khan et al. [46] used the discernibility matrix approach for GIFSS and apply it to the decision-making problems. The discernibility matrix approach is important when you have to start the process with predefined conditions on membership functions, i.e., threshold values.

The motivation of this research is to develop another technique to solve MADM problems using GIFSS by using the VIKOR method. The VIKOR method is important because it used to determine the compromise solution and ameliorate the standard of the decision-making process. The VIKOR method is balancing the majority's maximum group utility and opponent's minimum individual regret. In the VIKOR method, the best and worst solutions serve as the point of reference and the distance between the best solution and evaluative ratings don't have an upper bound. The novel concept of remoteness index is important because it provides the upper bounds by dividing the distance between the best solution and evaluative ratings by the distance between the best solution and the worst solution. The new ranking indexes based on the novel concept of remoteness indexes are discussed to provide the more effective compromise rankings. Therefore, a remoteness index-based VIKOR method for GIFSS is developed and apply it to the selection of renewable energy source problem.

This paper aims to discuss the selection of renewable energy sources for under developing countries using the remoteness index-based VIKOR method for GIFSS.

Major contributions of our work are:

1. The Hamming distance measures are defined for GIFSS.
2. The Euclidean distance measures are defined for GIFSS.
3. The generalized distance measures are defined for GIFSS.
4. The pattern recognition and decision-making problem is discussed by using the proposed distance measures for GIFSS.
5. The displaced and fixed ideal are defined for intuitionistic fuzzy values (IFVs) which are helpful to move towards ideal alternative and move away from an undesired alternative.
6. The displaced and fixed remoteness indexes are defined for IFVs.
7. Two types of weights called precise weights and intuitionistic fuzzy (IF) weights are defined and discuss their methods of generation.
8. Four groups of ranking indexes are defined based on displaced and fixed ideals and remoteness indexes.
9. Four algorithms are proposed which representing the VIKOR procedures with four different environments.
10. The problem of selection of renewable energy sources is discussed with the proposed remoteness based VIKOR method.

Rest of the paper is designed as follows: Section 2 contains the basic definitions. In Section 3, the Euclidean, Hamming, and the generalized distance measures for GIFSS are defined and their application in pattern recognition problem and decision making are discussed. The displaced and fixed ideal, the displaced and fixed remoteness indexes, precise and IF weights, displaced and fixed ranking indexes, and VIKOR procedures are defined in Section 3. An application of the proposed method in renewable energy source selection is discussed in Section 5. The comparison analysis and conclusion are discussed in Sections 6 and 7.

## 2. Preliminaries

The definitions of IFS, soft set, IFSs, and GIFSS are written in this section.
Definition 1 ([3]). An IFS R on a universal set $\hat{Y}$ is defined as

$$
R=\left\{\left(\xi_{R}(y), v_{R}(y)\right) \mid y \in \hat{Y}\right\}
$$

where $\xi_{R}$ and $v_{R}$ are the membership and non-membership functions from the universal set $\hat{Y}$ to the unit interval $[0,1]$, respectively. For IFSs, the sum of the membership and non-membership functions is less than or equal to one, i.e., $\xi_{R}(y)+v_{R}(y) \leq 1$. The quantity $\pi_{R}(y)=1-\left(\xi_{R}(y)+v_{R}(y)\right)$ is called the hesitancy degree of the element $y \in \hat{Y}$. For any $y \in \hat{Y}$, the value $\left(\xi_{R}(y), v_{R}(y)\right)$ is called is the intuitionistic fuzzy value (IFV) or intuitionistic fuzzy number (IFN).

Definition 2 ([3]). For two IFSs $R$ and S in $\hat{Y}$, the following notions are defined as follows:

1. $R \cap S=\left\{\left(y, \min \left\{\xi_{R}(y), \xi_{S}(y)\right\}, \max \left\{v_{R}(y), v_{S}(y)\right\}\right) \mid y \in \hat{Y}\right\}$
2. $R \cup S=\left\{\left(y, \max \left\{\xi_{R}(y), \xi_{S}(y)\right\}, \min \left\{v_{R}(y), v_{S}(y)\right\}\right) \mid y \in \hat{Y}\right\}$
3. $R \subset S \Leftrightarrow \xi_{R}(y) \leq \xi_{S}(y)$ and $v_{R}(y) \geq v_{S}(y), \forall y \in \hat{Y}$
4. $\quad R^{c}=\left\{\left(y, v_{R}(y), \xi_{R}(y)\right) \mid y \in \hat{Y}\right\}$.

Molodtsov soft set theory deals uncertainty effectively by considering parametric point of view [2], that is, each element is judged by some criteria of attributes (characteristics).

Definition 3 ([2]). Let universal space and parametric space are represented by $\hat{Y}$ and $C$, respectively. Let $\hat{A} \subset C$ be a parametric set and power set of $\hat{Y}$ is represented by $P(\hat{Y})$. A pair $(\hat{F}, \hat{A})$ is called a soft set over $\hat{Y}$, where $\hat{F}$ is a set valued mapping given by $\hat{F}: \hat{A} \rightarrow P(\hat{Y})$.

In [7], Maji defines the IFSs as follows.
Definition 4 ([7]). Let universal space and parametric space are represented by $\hat{Y}$ and $C$, respectively. Let $\hat{A} \subset C$ be a parametric set and $\operatorname{IF}(\hat{Y})$ the set of all IFSs of $\hat{Y}$. A pair $(\hat{F}, \hat{A})$ is called an IFSs over $\hat{Y}$, where $\hat{F}$ is a set valued mapping given by $\hat{F}: \hat{A} \rightarrow \operatorname{IF}(\hat{Y})$.

Wang and He [47], as well as, Deschrijver and Kerre [48] showed that IFSs can be viewed as L-fuzzy sets with respect to complete lattice $\left(K^{*}, \preceq_{K^{*}}\right)$, where $K^{*}=\left\{\left(r_{1}, r_{2}\right) \in[0,1]^{2} \mid r_{1}+r_{2} \leq 1\right\}$, and the corresponding partial order $\preceq_{K^{*}}$ is defined as

$$
\left(r_{1}, r_{2}\right) \preceq_{K^{*}}\left(s_{1}, s_{2}\right) \Leftrightarrow\left(r_{1} \leq s_{1}\right) \wedge\left(r_{2} \geq s_{2}\right), \quad \forall\left(r_{1}, r_{2}\right),\left(s_{1}, s_{2}\right) \in K^{*}
$$

Any ordered pair $\left(r_{1}, r_{2}\right) \in K^{*}$ is called an IFV in [49-51]. According to this point of view, the IFS

$$
R=\left\{\left(\xi_{R}(y), v_{R}(y)\right) \mid y \in \hat{Y}\right\}
$$

can be identified with the L-fuzzy set $R: \hat{Y} \rightarrow K^{*}$ such that for all $y \in \hat{Y}$

$$
R(y)=\left(\xi_{R}(y), v_{R}(y)\right)
$$

The elements $(1,0)$ and $(0,1)$ are considered as the top and bottom or largest and smallest IFVs, respectively, in ( $K^{*}, \preceq_{K^{*}}$ ).

The idea of GIFSS is constructive in decision-making since it considers how to take advantage of an extra intuitionistic fuzzy input from the director to decrease any possible distortion in the data provided by evaluating experts.

Definition 5 ([14]). Let universal space and parametric space are represented by $\hat{Y}$ and $C$, respectively. Let $\hat{A} \subset C$ be a parametric subset. We call $(\hat{F}, \hat{A}, \hat{\rho})$ a GIFSS, where $(\hat{F}, \hat{A})$ is an IFSs over $\hat{Y}$ and $\hat{\rho}: \hat{A} \rightarrow K^{*}$ is an IFS in $\hat{A}$.

Where $(\hat{F}, \hat{A})$ is called basic intuitionistic fuzzy soft set (BIFSS) and $\hat{\rho}$ is called the parametric intuitionistic fuzzy set (PIFS).

Example 1. Suppose $\hat{Y}=\left\{y_{1}, y_{2}, y_{3}, y_{4}\right\}$ be a set of universe and $C=\left\{e_{1}, e_{2}, e_{3}, e_{4}, e_{5}\right\}$ represents the parametric space. Let $\hat{A}=\left\{e_{1}, e_{2}, e_{3}\right\} \subset C$ be a parametric set and $\hat{F}: \hat{A} \rightarrow I F(\hat{Y})$ be a mapping such that

$$
\begin{aligned}
& \hat{F}\left(e_{1}\right)=\left\{(0.7,0.2) / y_{1},(0.9,0.0) / y_{2},(0.6,0.2) / y_{3},(0.7,0.1) / y_{4}\right\} \\
& \hat{F}\left(e_{2}\right)=\left\{(0.5,0.4) / y_{1},(0.7,0.2) / y_{2},(0.4,0.5) / y_{3},(0.6,0.3) / y_{4}\right\}
\end{aligned}
$$

and

$$
\hat{F}\left(e_{3}\right)=\left\{(0.2,0.7) / y_{1},(0.4,0.6) / y_{2},(0.5,0.3) / y_{3},(0.7,0.2) / y_{4}\right\} .
$$

This constitute the BIFSS $(\hat{F}, \hat{A})$, where each $\hat{F}\left(e_{j}\right)$ represents an IFS. In addition, suppose that the PIFS $\hat{\rho}$ is given by

$$
\hat{\rho}=\left\{(0.3,0.5) / e_{1},(0.5,0.3) / e_{2},(0.3,0.4) / e_{3}\right\} .
$$

All the above information in BIFSS and PIFS can be summarized in $\operatorname{GIFSS}(\hat{F}, \hat{A}, \hat{\rho})$, whose tabular representation is shown in Table 1.

Table 1. A GIFSS ( $\hat{F}, \hat{A}, \hat{\rho})$.

| $\hat{\gamma}$ | $e_{1}$ | $e_{2}$ | $e_{3}$ |
| :---: | :---: | :---: | :---: |
| $y_{1}$ | $(0.7,0.2)$ | $(0.5,0.4)$ | $(0.2,0.7)$ |
| $y_{2}$ | $(0.9,0.0)$ | $(0.7,0.2)$ | $(0.4,0.6)$ |
| $y_{3}$ | $(0.6,0.2)$ | $(0.4,0.5)$ | $(0.5,0.3)$ |
| $y_{4}$ | $(0.7,0.1)$ | $(0.6,0.3)$ | $(0.7,0.2)$ |
| $\hat{\rho}$ | $(0.3,0.5)$ | $(0.5,0.3)$ | $(0.3,0.4)$ |

Definition 6 ([14]). The expectation score function $\delta$ for an IFV $a=\left(\xi_{a}, v_{a}\right)$ is defined as follows:

$$
\begin{equation*}
\delta(a)=\frac{1+\xi_{a}-v_{a}}{2} \in[0,1] . \tag{1}
\end{equation*}
$$

## 3. Distance and Similarity Measures

This section contains the Hamming, Euclidean and generalized distance measures for GIFSSs. Some properties of distance measures are discussed. A numerical example of decision making and pattern recognition is discussed in this section.

Definition 7. A distance measure between two GIFSSs $\Gamma_{1}$ and $\Gamma_{2}$ is a mapping D : GIFSS $\times$ GIFSS $\rightarrow[0,1]$, which satisfies the following properties:
(D1) $0 \leq D\left(\Gamma_{1}, \Gamma_{2}\right) \leq 1$
(D2) $D\left(\Gamma_{1}, \Gamma_{2}\right)=0 \Longleftrightarrow \Gamma_{1}=\Gamma_{2}$
(D3) $D\left(\Gamma_{1}, \Gamma_{2}\right)=D\left(\Gamma_{2}, \Gamma_{1}\right)$
(D4) If $\Gamma_{1} \subseteq \Gamma_{2} \subseteq \Gamma_{3}$ then $D\left(\Gamma_{1}, \Gamma_{3}\right) \geq D\left(\Gamma_{1}, \Gamma_{2}\right)$ and $D\left(\Gamma_{1}, \Gamma_{3}\right) \geq D\left(\Gamma_{2}, \Gamma_{3}\right)$.
Definition 8. A similarity measure between two GIFSSs $\Gamma_{1}$ and $\Gamma_{2}$ is a mapping $D: G I F S S \times G I F S S \rightarrow[0,1]$, which satisfies the following properties:
(S1) $0 \leq S\left(\Gamma_{1}, \Gamma_{2}\right) \leq 1$
(S2) $S\left(\Gamma_{1}, \Gamma_{2}\right)=1 \Longleftrightarrow \Gamma_{1}=\Gamma_{2}$
(S3) $S\left(\Gamma_{1}, \Gamma_{2}\right)=S\left(\Gamma_{2}, \Gamma_{1}\right)$
(S4) If $\Gamma_{1} \subseteq \Gamma_{2} \subseteq \Gamma_{3}$ then $S\left(\Gamma_{1}, \Gamma_{3}\right) \leq S\left(\Gamma_{1}, \Gamma_{2}\right)$ and $S\left(\Gamma_{1}, \Gamma_{3}\right) \leq S\left(\Gamma_{2}, \Gamma_{3}\right)$.
Definition 9. For two GIFSSs $\Gamma_{1}=(\hat{F}, \hat{A}, \hat{\rho})$ and $\Gamma_{2}=(\hat{G}, \hat{B}, \hat{\sigma})$ in $\hat{Y}$, the Hamming distance measures between $\Gamma_{1}$ and $\Gamma_{2}$ are defined as follows:

$$
\begin{align*}
& D_{\hat{h}}^{\star}\left(\Gamma_{1}, \Gamma_{2}\right)=\frac{1}{4 m n} \sum_{j=1}^{n} \sum_{i=1}^{m} {\left[\left(\left|\xi_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\xi_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|+\left|v_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-v_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|\right)\right.} \\
&\left.+\left(\left|\xi_{\hat{\rho}\left(e_{j}\right)}-\xi_{\hat{\mathcal{O}}\left(e_{j}\right)}\right|+\left|v_{\hat{\rho}\left(e_{j}\right)}-v_{\hat{\sigma}\left(e_{j}\right)}\right|\right)\right]  \tag{2}\\
& D_{h}^{\circ}\left(\Gamma_{1}, \Gamma_{2}\right)=\frac{1}{4 m n} \sum_{j=1}^{n} \sum_{i=1}^{m} {\left[\left(\left|\xi_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\xi_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|+\left|v_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-v_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|\right)\right.} \\
&\left.+\left(\left|\xi_{\hat{\rho}\left(e_{j}\right)}-\xi_{\hat{\sigma}\left(e_{j}\right)}\right|+\mid v_{\hat{\rho}^{( }\left(e_{j}\right)}\right)-v_{\hat{\sigma}\left(e_{j}\right)} \mid\right) \\
&\left.+\left(\left|\hbar_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\hbar_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|+\left|\hbar_{\hat{\rho}\left(e_{j}\right)}-\hbar_{\hat{\sigma}\left(e_{j}\right)}\right|\right)\right] \tag{3}
\end{align*}
$$

Definition 10. Let $\Gamma_{1}=(\hat{F}, \hat{A}, \hat{\rho})$ and $\Gamma_{2}=(\hat{G}, \hat{B}, \hat{\sigma})$ be two GIFSSs in $\hat{Y}$, the Euclidean distance measures between $\Gamma_{1}$ and $\Gamma_{2}$ are defined as follows:

$$
\begin{align*}
& D_{e}^{\star}\left(\Gamma_{1}, \Gamma_{2}\right)=\left(\frac{1}{4 m n} \sum_{j=1}^{n}\right. \sum_{i=1}^{m}\left[\left(\xi_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\xi_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right)^{2}+\left(v_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-v_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right)^{2}\right. \\
&\left.\left.+\left(\xi_{\hat{\rho}\left(e_{j}\right)}-\xi_{\hat{\sigma}\left(e_{j}\right)}\right)^{2}+\left(v_{\hat{\rho}\left(e_{j}\right)}-v_{\hat{\sigma}\left(e_{j}\right)}\right)^{2}\right]\right)^{\frac{1}{2}}  \tag{4}\\
& D_{e}^{\circ}\left(\Gamma_{1}, \Gamma_{2}\right)=\left(\frac { 1 } { 4 m n } \sum _ { j = 1 } ^ { n } \sum _ { i = 1 } ^ { m } \left[\left(\tilde{\xi}_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\tilde{\zeta}_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right)^{2}+\left(v_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-v_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right)^{2}\right.\right. \\
&+\left(\xi_{\hat{\rho}\left(e_{j}\right)}-\xi_{\hat{\sigma}\left(e_{j}\right)}\right)^{2}+\left(v_{\hat{\rho}\left(e_{j}\right)}-v_{\hat{\sigma}\left(e_{j}\right)}\right)^{2} \\
&\left.\left.+\left(\hbar_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\hbar_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right)^{2}+\left(\hbar_{\hat{\rho}\left(e_{j}\right)}-\hbar_{\hat{\sigma}\left(e_{j}\right)}\right)^{2}\right]\right)^{\frac{1}{2}} . \tag{5}
\end{align*}
$$

Definition 11. For two GIFSSs $\Gamma_{1}=(\hat{F}, \hat{A}, \hat{\rho})$ and $\Gamma_{2}=(\hat{G}, \hat{B}, \hat{\sigma})$ in $\hat{Y}$, the generalized distance measures between $\Gamma_{1}$ and $\Gamma_{2}$ are defined as follows:

$$
\begin{align*}
& D_{p}^{\star}\left(\Gamma_{1}, \Gamma_{2}\right)=\left(\frac{1}{4 m n} \sum_{j=1}^{n} \sum_{i=1}^{m}\right. {\left[\left(\left|\xi_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\xi_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|^{p}+\left|v_{\hat{\mathcal{F}}\left(e_{j}\right)}\left(y_{i}\right)-v_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|^{p}\right)\right.} \\
&\left.\left.+\left(\left|\xi_{\hat{\rho}\left(e_{j}\right)}-\xi_{\hat{\sigma}\left(e_{j}\right)}\right|^{p}+\left|v_{\hat{\rho}\left(e_{j}\right)}-v_{\hat{\sigma}\left(e_{j}\right)}\right|^{p}\right)\right]\right)^{\frac{1}{p}}  \tag{6}\\
& D_{p}^{\circ}\left(\Gamma_{1}, \Gamma_{2}\right)=\left(\frac{1}{4 m n} \sum_{j=1}^{n} \sum_{i=1}^{m}\right. {\left[\left(\left|\xi_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\xi_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|^{p}+\left|v_{\hat{\mathcal{F}}\left(e_{j}\right)}\left(y_{i}\right)-v_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|^{p}\right)\right.} \\
&+\left(\left|\xi_{\hat{\rho}\left(e_{j}\right)}-\xi_{\hat{\sigma}\left(e_{j}\right)}\right|^{p}+\left|v_{\hat{\rho}\left(e_{j}\right)}-v_{\hat{\sigma}\left(e_{j}\right)}\right|^{p}\right) \\
&\left.\left.+\left(\left|\hbar_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\hbar_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|^{p}+\left|\hbar_{\hat{\rho}\left(e_{j}\right)}-\hbar_{\hat{\sigma}\left(e_{j}\right)}\right|^{p}\right)\right]\right)^{\frac{1}{p}} \tag{7}
\end{align*}
$$

Remark 1. The generalized distance measures $D_{p}^{\star}$ and $D_{p}^{\circ}$ are reduced to Hamming distances $D_{h}^{\star}$ and $D_{h}^{\circ}$, respectively, for $p=1$. Also, the Euclidean distances $D_{e}^{\star}$ and $D_{e}^{\circ}$ are obtained from $D_{p}^{\star}$ and $D_{p}^{\circ}$, respectively, for $p=2$.

Theorem 1. The mappings in Equations (2)-(7) satisfies the axioms of distance measures (Definition 7).
Proof. The proof is straightforward from the [22,52,53].

Theorem 2. The distance measures in Equations (2)-(7) between two GIFSSs $\Gamma_{1}=(\hat{F}, \hat{A}, \hat{\rho})$ and $\Gamma_{2}=$ $(\hat{G}, \hat{B}, \hat{\sigma})$ holds the following properties:
(1) $D\left(\Gamma_{1}, \Gamma_{2}\right)=D\left(\Gamma_{1} \cup \Gamma_{2}, \Gamma_{1} \cap \Gamma_{2}\right)$, where $D=D_{h}^{\star}, D_{e}^{\star}, D_{p}^{\star}$
(2) $D\left(\Gamma_{1}, \Gamma_{2}\right)=D\left(\Gamma_{1}^{c}, \Gamma_{2}^{c}\right), \quad$ where $D=D_{h}^{\star}, D_{h}^{\circ}, D_{e}^{\star}, D_{e}^{\circ}, D_{p}^{\star}, D_{p}^{\circ}$

Proof. (1) The distance between two GIFSSs $\Gamma_{1}=(\hat{F}, \hat{A}, \hat{\rho})$ and $\Gamma_{2}=(\hat{G}, \hat{B}, \hat{\sigma})$ can be written as follows:

$$
\begin{equation*}
D\left(\Gamma_{1}, \Gamma_{2}\right)=D\left(\left(\xi_{\hat{F}\left(e_{j}\right)}, v_{\hat{F}\left(e_{j}\right)}, \xi_{\hat{\rho}\left(e_{j}\right)}, v_{\hat{\rho}\left(e_{j}\right)}\right),\left(\xi_{\hat{G}\left(e_{j}\right)}, v_{\hat{G}\left(e_{j}\right)}, \xi_{\hat{\sigma}\left(e_{j}\right)}, v_{\hat{\sigma}\left(e_{j}\right)}\right)\right) \tag{8}
\end{equation*}
$$

From Definition 2, we have

$$
\begin{gather*}
\Gamma_{1} \cup \Gamma_{2}=\left(\max \left(\xi_{\hat{F}\left(e_{j}\right)}, \xi_{\hat{G}\left(e_{j}\right)}\right), \min \left(v_{\hat{F}\left(e_{j}\right)}, v_{\hat{G}\left(e_{j}\right)}\right), \max \left(\xi_{\hat{\rho}\left(e_{j}\right)}, \xi_{\hat{\sigma}\left(e_{j}\right)}\right), \min \left(v_{\hat{\rho}\left(e_{j}\right)}, v_{\hat{\sigma}\left(e_{j}\right)}\right)\right)  \tag{9}\\
\left.\Gamma_{1} \cap \Gamma_{2}=\left(\min \left(\xi_{\hat{F}\left(e_{j}\right)}\right) \xi_{\hat{G}\left(e_{j}\right)}\right), \max \left(v_{\hat{F}\left(e_{j}\right)}, v_{\hat{G}\left(e_{j}\right)}\right), \min \left(\xi_{\hat{\rho}\left(e_{j}\right)}, \xi_{\hat{\sigma}\left(e_{j}\right)}\right), \max \left(v_{\hat{\rho}\left(e_{j}\right)}, v_{\hat{\sigma}\left(e_{j}\right)}\right)\right) \tag{10}
\end{gather*}
$$

From Equations (9) and (10), we have seen that it may possible that the positions of the memberships and non-memberships degrees alters but the values remains unchanged. Therefore, the distance between $\Gamma_{1} \cup \Gamma_{2}$ and $\Gamma_{1} \cap \Gamma_{2}$ is same as the distance between $\Gamma_{1}$ and $\Gamma_{2}$.
(2) The distance between two GIFSSs $\Gamma_{1}=(\hat{F}, \hat{A}, \hat{\rho})$ and $\Gamma_{2}=(\hat{G}, \hat{B}, \hat{\sigma})$ can be written as follows:

$$
\begin{equation*}
D\left(\Gamma_{1}, \Gamma_{2}\right)=D\left(\left(\xi_{\hat{F}\left(e_{j}\right)}, v_{\hat{F}\left(e_{j}\right)}, \xi_{\hat{\rho}\left(e_{j}\right)}, v_{\hat{\rho}\left(e_{j}\right)}\right),\left(\xi_{\hat{G}\left(e_{j}\right)}, v_{\hat{G}\left(e_{j}\right)}, \xi_{\hat{\sigma}\left(e_{j}\right)}, v_{\hat{\sigma}\left(e_{j}\right)}\right)\right) \tag{11}
\end{equation*}
$$

From Definition 2, we have

$$
\begin{equation*}
D\left(\Gamma_{1}^{c}, \Gamma_{2}^{c}\right)=\left(\left(v_{\hat{F}\left(e_{j}\right)}, \xi_{\hat{F}\left(e_{j}\right)}, v_{\hat{\rho}\left(e_{j}\right)}, \xi_{\hat{\rho}\left(e_{j}\right)}\right),\left(v_{\hat{G}\left(e_{j}\right)}, \xi_{\hat{G}\left(e_{j}\right)}, v_{\hat{\sigma}\left(e_{j}\right)}, \xi_{\hat{G}\left(e_{j}\right)}\right)\right) \tag{12}
\end{equation*}
$$

From Equation (12), we have seen that the position of the membership and non-membership degrees have changed but the corresponding between the membership and non-membership degrees remain same, i.e., the membership degrees of $\Gamma_{1}^{c}$ relates with the membership degrees of $\Gamma_{2}^{c}$ and the non-membership degrees of $\Gamma_{1}^{c}$ relates with the non-membership degrees of $\Gamma_{2}^{c}$. Therefore, the distance between $\Gamma_{1}$ and $\Gamma_{2}$ is same as the distance between $\Gamma_{1}^{c}$ and $\Gamma_{2}^{c}$.

Example 2. Suppose two GIFSSs $\Gamma_{1}=(\hat{F}, \hat{A}, \hat{\rho})$ and $\Gamma_{2}=(\hat{G}, \hat{B}, \hat{\sigma})$ be given in Table 2. We find the distance between $\Gamma_{1}$ and $\Gamma_{2}$ by using above mentioned distance measures.

Table 2. Two generalized intuitionistic fuzzy soft sets (GIFSSs) $\Gamma_{1}$ and $\Gamma_{1}$.

| $\Gamma_{\mathbf{1}}=(\hat{\boldsymbol{F}}, \hat{A}, \hat{\boldsymbol{\rho}})$ |  |  |  |  | $\Gamma_{\mathbf{2}}=(\hat{\boldsymbol{G}}, \hat{\mathbf{B}}, \hat{\sigma})$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\hat{\boldsymbol{\gamma}}$ | $e_{\mathbf{1}}$ | $e_{\mathbf{2}}$ | $e_{\mathbf{3}}$ | $\hat{\gamma}$ | $e_{\mathbf{1}}$ | $e_{\mathbf{2}}$ | $e_{\mathbf{3}}$ |
| $y_{1}$ | $(0.7,0.2)$ | $(0.5,0.4)$ | $(0.2,0.7)$ | $y_{1}$ | $(0.3,0.5)$ | $(0.7,0.2)$ | $(0.5,0.4)$ |
| $y_{2}$ | $(0.9,0.0)$ | $(0.7,0.2)$ | $(0.4,0.6)$ | $y_{2}$ | $(0.4,0.5)$ | $(0.6,0.3)$ | $(0.4,0.5)$ |
| $\hat{\rho}$ | $(0.3,0.5)$ | $(0.5,0.3)$ | $(0.3,0.4)$ | $\hat{\sigma}$ | $(0.5,0.2)$ | $(0.7,0.3)$ | $(0.8,0.1)$ |

$$
\begin{aligned}
D_{h}^{\star}\left(\Gamma_{1}, \Gamma_{2}\right)= & \frac{1}{24}[(|0.7-0.3|+|0.2-0.5|+|0.9-0.4|+|0.0-0.5|+|0.3-0.5|+|0.5-0.2|) \\
& +(|0.5-0.7|+|0.4-0.2|+|0.7-0.6|+|0.2-0.3|+|0.5-0.7|+|0.3-0.3|) \\
& +(|0.2-0.5|+|0.7-0.4|+|0.4-0.4|+|0.6-0.5|+|0.3-0.8|+|0.4-0.1|)] \\
& =0.1875
\end{aligned}
$$

Similarly, we can find distance by remaining distance measures and the results are $D_{h}^{\circ}\left(\Gamma_{1}, \Gamma_{2}\right)=0.216667$, $D_{e}^{\star}\left(\Gamma_{1}, \Gamma_{2}\right)=0.254133$ and $D_{e}^{\circ}\left(\Gamma_{1}, \Gamma_{2}\right)=0.262996$. The distance by using generalized distances for $p=3$ are $D_{p}^{\star}\left(\Gamma_{1}, \Gamma_{2}\right)=0.293857$ and $D_{p}^{\circ}\left(\Gamma_{1}, \Gamma_{2}\right)=0.296881$.

Definition 12. For two GIFSSs $\Gamma_{1}=(\hat{F}, \hat{A}, \hat{\rho})$ and $\Gamma_{2}=(\hat{G}, \hat{B}, \hat{\sigma})$ in $\hat{Y}$, the weighted Hamming distance measures between $\Gamma_{1}$ and $\Gamma_{2}$ are defined as follows

$$
\begin{align*}
& D_{h}^{\star \omega}\left(\Gamma_{1}, \Gamma_{2}\right)=\frac{1}{4 m n} \sum_{j=1}^{n} \sum_{i=1}^{m} \omega_{j}[ {\left[\left|\xi_{\hat{\mathcal{F}}\left(e_{j}\right)}\left(y_{i}\right)-\xi_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|+\left|v_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-v_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|\right) } \\
&\left.\quad+\left(\left|\xi_{\hat{\rho}\left(e_{j}\right)}-\xi_{\hat{\sigma}\left(e_{j}\right)}\right|+\left|v_{\hat{\rho}\left(e_{j}\right)}-v_{\hat{\sigma}\left(e_{j}\right)}\right|\right)\right]  \tag{13}\\
& D_{h}^{o \omega}\left(\Gamma_{1}, \Gamma_{2}\right)=\frac{1}{4 m n} \sum_{j=1}^{n} \sum_{i=1}^{m} \omega_{j}\left[\left(\left|\xi_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\xi_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|+\left|v_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-v_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|\right)\right. \\
&+\left(\left|\xi_{\hat{\rho}\left(e_{j}\right)}-\xi_{\hat{\sigma}\left(e_{j}\right)}\right|+\left|v_{\hat{\rho}\left(e_{j}\right)}-v_{\hat{\sigma}\left(e_{j}\right)}\right|\right) \\
&\left.+\left(\left|\hbar_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\hbar_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|+\left|\hbar_{\hat{\hat{\rho}}\left(e_{j}\right)}-\hbar_{\hat{\sigma}\left(e_{j}\right)}\right|\right)\right] \tag{14}
\end{align*}
$$

where $\omega=\left\{\omega_{1}, \omega_{2}, \ldots, \omega_{n}\right\}^{T}$ is the weight vector with $0 \leq \omega_{j} \leq 1$ and $\sum_{j=1}^{n} \omega_{j}=1$.
Definition 13. Let $\Gamma_{1}=(\hat{F}, \hat{A}, \hat{\rho})$ and $\Gamma_{2}=(\hat{G}, \hat{B}, \hat{\sigma})$ be two GIFSSs in $\hat{Y}$, the weighted Euclidean distances between $\Gamma_{1}$ and $\Gamma_{2}$ are defined as follows:

$$
\begin{align*}
D_{e}^{\star \omega}\left(\Gamma_{1}, \Gamma_{2}\right)=\left(\frac{1}{4 m n} \sum_{j=1}^{n}\right. & \sum_{i=1}^{m} \omega_{j}\left[\left(\xi_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\xi_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right)^{2}+\left(v_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-v_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right)^{2}\right. \\
& \left.\left.+\left(\xi_{\hat{\rho}\left(e_{j}\right)}-\xi_{\hat{\sigma}\left(e_{j}\right)}\right)^{2}+\left(v_{\hat{\rho}\left(e_{j}\right)}-v_{\hat{\sigma}\left(e_{j}\right)}\right)^{2}\right]\right)^{\frac{1}{2}}  \tag{15}\\
D_{e}^{o \omega}\left(\Gamma_{1}, \Gamma_{2}\right)=\left(\frac{1}{4 m n} \sum_{j=1}^{n}\right. & \sum_{i=1}^{m} \omega_{j}\left[\left(\tilde{\zeta}_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\xi_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right)^{2}+\left(v_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-v_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right)^{2}\right. \\
& +\left(\xi_{\hat{\rho}\left(e_{j}\right)}-\xi_{\hat{\sigma}\left(e_{j}\right)}\right)^{2}+\left(v_{\hat{\rho}\left(e_{j}\right)}-v_{\hat{\sigma}\left(e_{j}\right)}\right)^{2} \\
& \left.\left.+\left(\hbar_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\hbar_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right)^{2}+\left(\hbar_{\hat{\rho}\left(e_{j}\right)}-\hbar_{\hat{\sigma}\left(e_{j}\right)}\right)^{2}\right]\right)^{\frac{1}{2}} \tag{16}
\end{align*}
$$

where $\omega=\left\{\omega_{1}, \omega_{2}, \ldots, \omega_{n}\right\}^{T}$ is the weight vector with $0 \leq \omega_{j} \leq 1$ and $\sum_{j=1}^{n} \omega_{j}=1$.

Definition 14. For two GIFSSs $\Gamma_{1}=(\hat{F}, \hat{A}, \hat{\rho})$ and $\Gamma_{2}=(\hat{G}, \hat{B}, \hat{\sigma})$ in $\hat{Y}$, the weighted generalized distances between $\Gamma_{1}$ and $\Gamma_{2}$ are defined as follows

$$
\begin{align*}
& D_{p}^{\star \omega}\left(\Gamma_{1}, \Gamma_{2}\right)=\left(\frac { 1 } { 4 m n } \sum _ { j = 1 } ^ { n } \sum _ { i = 1 } ^ { m } \omega _ { j } \left[\left(\left|\xi_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\xi_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|^{p}+\left|v_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-v_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|^{p}\right)\right.\right. \\
&\left.\left.+\left(\left|\xi_{\hat{\rho}\left(e_{j}\right)}-\xi_{\hat{\sigma}\left(e_{j}\right)}\right|^{p}+\left|v_{\hat{\rho}\left(e_{j}\right)}-v_{\hat{\sigma}\left(e_{j}\right)}\right|^{p}\right)\right]\right)^{\frac{1}{p}}  \tag{17}\\
& D_{p}^{\circ \omega}\left(\Gamma_{1}, \Gamma_{2}\right)=\left(\frac{1}{4 m n} \sum_{j=1}^{n} \sum_{i=1}^{m} \omega_{j}\right. {\left[\left(\left|\xi_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\xi_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|^{p}+\left|v_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-v_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|^{p}\right)\right.} \\
&+\left(\left|\xi_{\hat{\rho}\left(e_{j}\right)}-\xi_{\hat{\sigma}\left(e_{j}\right)}\right|^{p}+\left|v_{\hat{\rho}\left(e_{j}\right)}-v_{\hat{\sigma}\left(e_{j}\right)}\right|^{p}\right) \\
&+\left.\left.\left(\left|\hbar_{\hat{F}\left(e_{j}\right)}\left(y_{i}\right)-\hbar_{\hat{G}\left(e_{j}\right)}\left(y_{i}\right)\right|^{p}+\left|\hbar_{\hat{\rho}\left(e_{j}\right)}-\hbar_{\hat{\sigma}\left(e_{j}\right)}\right|^{p}\right)\right]\right)^{\frac{1}{p}} \tag{18}
\end{align*}
$$

where $\omega=\left\{\omega_{1}, \omega_{2}, \ldots, \omega_{n}\right\}^{T}$ is the weight vector with $0 \leq \omega_{j} \leq 1$ and $\sum_{j=1}^{n} \omega_{j}=1$.
Theorem 3. The similarity measures for two GIFSSs $\Gamma_{1}$ and $\Gamma_{2}$ are obtained from above distance measures by $S\left(\Gamma_{1}, \Gamma_{2}\right)=1-D\left(\Gamma_{1}, \Gamma_{2}\right)$.

Proof. The proof is straightforward from the proof of Theorem 1.

## Application in Decision Making and Pattern Recognition

In this subsection, the selection of a brilliant student for scholarship in a technological institute, which is classical decision making and pattern recognition problem.

A technological institute have some scholarships for their students. The scholarships awarded to the best students. To specify a criteria for selecting students. A committee is established, which decide the criteria for evaluation of student performance in the institute. The director of the institute is the head of the committee. For example, five students are shortlisted for further evaluation. The committee set up the criteria for evaluation which consist of: $e_{1}, e_{2}, e_{3}$ and $e_{4}$ which stand for knowledge and understanding, practical applications of their work, lab work and performance in the test, respectively. The committee evaluate the students according to the predefined criteria and their evaluation are saved in the form of BIFSS. The head of the committee judged the evaluation of the students by the committee and carefully monitored the whole procedure of evaluation and give their preferences in the form of PIFS which completes the formation of GIFSS. For comparison, an ideal student performance according to the prescribed criteria is set by the committee and all the data is presented in the Table 3.

To select best candidate for scholarship, the distance measures is used. The distance between the evaluated values and the ideal performance set by the committee is calculated by using above proposed distance measures.

The candidate $y_{2}$ has the minimum distance with the ideal candidate (for Hamming and Euclidean distances). Therefore, $y_{2}$ selected for scholarship. But for higher values of $p$, the $y_{3}$ alternative chosen for scholarship. It means, the values of the parameter $p$ effect the order of alternatives. The calculations are summarized in Table 4.

Table 3. The GIFSS.

| $\hat{\boldsymbol{\gamma}}$ | $\boldsymbol{e}_{\mathbf{1}}$ | $\boldsymbol{e}_{\mathbf{2}}$ | $\boldsymbol{e}_{\mathbf{3}}$ | $\boldsymbol{e}_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $y_{1}$ | $(0.7,0.2)$ | $(0.5,0.4)$ | $(0.2,0.7)$ | $(0.8,0.1)$ |
| $y_{2}$ | $(0.9,0.0)$ | $(0.7,0.2)$ | $(0.4,0.6)$ | $(0.6,0.3)$ |
| $y_{3}$ | $(0.5,0.5)$ | $(0.8,0.2)$ | $(0.5,0.4)$ | $(0.5,0.1)$ |
| $y_{4}$ | $(0.4,0.5)$ | $(0.6,0.3)$ | $(0.4,0.5)$ | $(0.7,0.3)$ |
| $\hat{\rho}$ | $(0.3,0.5)$ | $(0.5,0.3)$ | $(0.3,0.4)$ | $(0.7,0.1)$ |
| $y_{\text {ideal }}$ | $(0.9,0.1)$ | $(0.8,0.1)$ | $(0.9,0.0)$ | $(0.8,0.2)$ |
| $\hat{\rho}_{\text {ideal }}$ | $(0.9,0.1)$ | $(0.8,0.1)$ | $(0.9,0.0)$ | $(0.8,0.2)$ |

Table 4. The Distance Between GIFSSs.

|  | $\boldsymbol{D}\left(\boldsymbol{y}_{1}, \boldsymbol{y}_{\text {ideal }}\right)$ | $\boldsymbol{D}\left(\boldsymbol{y}_{2}, \boldsymbol{y}_{\text {ideal }}\right)$ | $\boldsymbol{D}\left(y_{3}, \boldsymbol{y}_{\text {ideal }}\right)$ | $\boldsymbol{D}\left(y_{4}, \boldsymbol{y}_{\text {ideal }}\right)$ | Ranking |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $D_{h}^{\star}$ | 0.31875 | 0.275 | 0.3 | 0.325 | $y_{2} \succ y_{3} \succ y_{1} \succ y_{4}$ |
| $D_{h}^{\circ}$ | 0.375 | 0.3375 | 0.38125 | 0.375 | $y_{2} \succ y_{1} \succ y_{4} \succ y_{3}$ |
| $D_{e}^{\star}$ | 0.388104 | 0.342783 | 0.34821 | 0.37081 | $y_{2} \succ y_{3} \succ y_{4} \succ y_{1}$ |
| $D_{e}^{\circ}$ | 0.4 | 0.357071 | 0.376663 | 0.382426 | $y_{2} \succ y_{3} \succ y_{4} \succ y_{1}$ |
| $D_{3}^{\star}$ | 0.438278 | 0.390123 | 0.380583 | 0.403869 | $y_{3} \succ y_{2} \succ y_{4} \succ y_{1}$ |
| $D_{3}^{\circ}$ | 0.441187 | 0.393918 | 0.394053 | 0.407163 | $y_{2} \succ y_{3} \succ y_{4} \succ y_{1}$ |
| $D_{4}^{\star}$ | 0.475581 | 0.423443 | 0.405125 | 0.42798 | $y_{3} \succ y_{2} \succ y_{4} \succ y_{1}$ |
| $D_{4}^{\circ}$ | 0.47632 | 0.424509 | 0.412467 | 0.428973 | $y_{3} \succ y_{2} \succ y_{4} \succ y_{1}$ |
| $D_{5}^{\star}$ | 0.503959 | 0.447813 | 0.425125 | 0.446336 | $y_{3} \succ y_{4} \succ y_{2} \succ y_{1}$ |
| $D_{5}^{\circ}$ | 0.504151 | 0.448123 | 0.429449 | 0.446644 | $y_{3} \succ y_{4} \succ y_{2} \succ y_{1}$ |
| $D_{10}^{\star}$ | 0.057991 | 0.051076 | 0.048988 | 0.049909 | $y_{3} \succ y_{4} \succ y_{2} \succ y_{1}$ |
| $D_{10}^{\circ}$ | 0.057991 | 0.051076 | 0.049028 | 0.049909 | $y_{3} \succ y_{4} \succ y_{2} \succ y_{1}$ |

## 4. The Remoteness Index-Based VIKOR Method for GIFSSs

In this section, we define the displaced positive ideal IFV (dpi-IFV) and the displaced negative ideal IFV (dni-IFV) which help to define the concept of displaced remoteness index. The fixed remoteness index is defined based on the fixed positive ideal IFV (fpi-IFV) and the fixed negative ideal IFV (fni-IFV). The displaced and fixed group utility, individual regret and compromise indexes are defined as a new multiple criteria ranking indexes. These indexes based on the two types of weights: the precise importance and intuitionistic fuzzy (IF) importance weights. The extra parameter PIFS in GIFSS, which given by the head or director of the decision-making committee, is used to define the precise importance and IF importance weights. The four algorithms are proposed in this section. Each algorithm shows the complete procedure of the remoteness based VIKOR method based on the displaced and fixed terminologies.

### 4.1. The Displaced and Fixed Ideal IF Values

The dpi-IFV $a_{+j}$ and the dni-IFV $a_{-j}$ are the largest and smallest IFVs in the given data, respectively. Similarly, the fixed positive ideal IFV (fpi-IFV) $a_{\hat{+} j}$ and the fixed negative ideal IFV (fni-IFV) $a_{\hat{-}}$ are are the largest and smallest IFVs on the given lattice, respectively. The dpi-IFV and dni-IFV provided as the reference points because, in subjective decision making, it can easily form anchored judgments. The dpi-IFV and dni-IFV are dependent on the given data and can be displaced easily by changing the evaluation values. The dpi-IFV is the largest value and helps to meet our target alternative (favorite), while dni-IFV is the smallest value and helps to avoid the unwanted alternative. The dpi-IFV and dni-IFV for the IFV decision matrix are defined as follows.

Definition 15. The displaced positive ideal IFV (dpi-IFV) $a_{+j}$ and the displaced negative ideal IFV (dni-IFV) $a_{-j}$ for a IFV decision matrix $a=\left[a_{i j}\right]_{m \times n}$ with respect to each criteria $e_{j} \in C\left(C=C_{b} \cup C_{c}\right.$, where $\left.C_{b} \cap C_{c}=\Phi\right)$ are defined as follows:

$$
\begin{align*}
& a_{+j}=\left(\xi_{+j}, v_{+j}\right)= \begin{cases}\left(\max _{i=1}^{m} \xi_{i j}, \min _{i=1}^{m} v_{i j}\right), & \text { if } e_{j} \in C_{b} \\
\left(\min _{i=1}^{m} \xi_{i j}, \max _{i=1}^{m} v_{i j}\right), & \text { if } e_{j} \in C_{c}\end{cases}  \tag{19}\\
& a_{-j}=\left(\xi_{-j}, v_{-j}\right)= \begin{cases}\left(\min _{i=1}^{m} \xi_{i j}, \max _{i=1}^{m} v_{i j}\right), & \text { if } e_{j} \in C_{b} \\
\left(\max _{i=1}^{m} \xi_{i j}, \min _{i=1}^{m} v_{i j}\right), & \text { if } e_{j} \in C_{c}\end{cases} \tag{20}
\end{align*}
$$

Moreover, $\hbar_{+j}$ and $\hbar_{-j}$ represents the hesitancy degrees of dpi-IFV and dni-IFV, respectively and defined as follows:

$$
\begin{gather*}
\hbar_{+j}=1-\left(\xi_{+j}+v_{+j}\right)  \tag{21}\\
\hbar_{-j}=1-\left(\xi_{-j}+v_{-j}\right) \tag{22}
\end{gather*}
$$

The fpi-IFV and fni-IFV are also formed the anchored judgments for subjective decision making. The $(1,0)$ and $(0,1)$ are specified for fpi-IFV and fni-IFV, respectively, for benefit criteria. While, for cost criteria, $(0,1)$ and $(1,0)$ are used for fpi-IFV and fni-IFV, respectively. These are bounds on the lattice, i.e., the highest and lowest values on the lattice.

Definition 16. The fixed positive ideal IFV (fpi-IFV) $a_{\hat{+} j}$ and the fixed negative ideal IFV (fni-IFV) $a_{\wedge j}$ for $a$ IFV decision matrix $a=\left[a_{i j}\right]_{m \times n}$ with respect to each criteria $e_{j} \in C\left(C=C_{b} \cup C_{c}\right.$, where $\left.C_{b} \cap C_{c}=\Phi\right)$ are defined as follows:

$$
\begin{align*}
& a_{+j}=\left(\xi_{\neq j}, v_{\dot{+} j}\right)= \begin{cases}(1,0), & \text { if } e_{j} \in C_{b} \\
(0,1), & \text { if } e_{j} \in C_{c}\end{cases}  \tag{23}\\
& a_{\hat{-}_{j}}=\left(\xi_{\wedge_{j}}, v_{\hat{\mu}^{j}}\right)= \begin{cases}(0,1), & \text { if } e_{j} \in C_{b} \\
(1,0), & \text { if } e_{j} \in C_{c}\end{cases} \tag{24}
\end{align*}
$$

Moreover, $\hbar_{\hat{+} j}$ and $\hbar_{\hat{-j}}$ represents the hesitancy degrees of fpi-IFV and fni-IFV, respectively and $\hbar_{\hat{+} j}=\hbar_{\hat{-} j}=0$.

### 4.2. The Displaced and Fixed Remoteness Indexes

In general, the dpi-IFV is the favorable ideal alternative and dni-IFV is the avoidable alternative. So, if the distance between evaluative value $a_{i j}$ and dpi-IFV $a_{+j}$, i.e., $D\left(a_{i j}, a_{+j}\right)$ is decreases, the favorability of $a_{i j}$ increases and vice versa. Similarly, for fpi-IFV. But $D\left(a_{i j}, a_{+j}\right)$ don't have an upper bound because dpi-IFVs are the largest values in the evaluating data for each criterion and thus frequently changed among criterion. The distance between the dpi-IFV $a_{+j}$ and dni-IFV $a_{-j} D\left(a_{+j}, a_{-j}\right)$ provides the upper bound for $D\left(a_{i j}, a_{+j}\right)$ for each criteria. So instead of considering $D\left(a_{i j}, a_{+j}\right)$, we consider the ratio of $D\left(a_{i j}, a_{+j}\right)$ to $D\left(a_{+j}, a_{-j}\right)$. But when we consider fpi-IFVs, the lacking of upper bound problem is insignificant because $D\left(a_{\hat{+},}, a_{\hat{-}}\right)=1$ for all criterion.

Now, we define the displaced remoteness index $R I^{d}$ as follows:
Definition 17. The displaced remoteness index $R I^{d}\left(a_{i j}\right)$ of $\left(a_{i j}\right)$ based on the above proposed distance measures $D$ is defined as follows:

$$
\begin{equation*}
R I^{d}\left(a_{i j}\right)=\frac{D\left(a_{i j}, a_{+j}\right)}{D\left(a_{-j}, a_{+j}\right)}, \tag{25}
\end{equation*}
$$

where $a_{i j}, a_{+j}$ and $a_{-j}$ are the evaluative value, dpi-IFV and dni-IFVs, respectively, in the decision matrix $a=\left[a_{i j}\right]_{m \times n}$.

Theorem 4. Let $a_{i j}, a_{i 1 j}$ and $a_{i 2 j}$ be three assessment values in the IF decision matrix $a$. The RI ${ }^{d}$ satisfies the following properties:

1. $R I^{d}\left(a_{i j}\right)=0 \quad \Leftrightarrow \quad a_{i j}=a_{+j}$
2. $R I^{d}\left(a_{i j}\right)=1 \quad \Leftrightarrow \quad a_{i j}=a_{-j}$
3. $0 \leq R I^{d}\left(a_{i j}\right) \leq 1$
4. For each $e_{j} \in C_{b}, R I^{d}\left(a_{i^{\prime \prime} j}\right) \leq R I^{d}\left(a_{i^{\prime} j}\right)$ if $a_{i^{\prime} j} \subseteq a_{i^{\prime \prime} j}$
5. For each $e_{j} \in C_{c}, R I^{d}\left(a_{i^{\prime} j}\right) \leq R I^{d}\left(a_{i^{\prime \prime} j}\right)$ if $a_{i^{\prime} j} \subseteq a_{i^{\prime \prime} j}$

Proof. The proof of the theorem is based on the properties of distance measures (Definition 7) and Equation (25).

1. From Equation (25), we have $R I^{d}\left(a_{i j}\right)=0$ iff $D\left(a_{i j}, a_{+j}\right)=0$, where $D$ is distance measure. The distance $D\left(a_{i j}, a_{+j}\right)=0$ iff $a_{i j}=a_{+j}$.
2. From Equation (25), if $R I^{d}\left(a_{i j}\right)=1$ then $D\left(a_{i j}, a_{+j}\right)=D\left(a_{-j}, a_{+j}\right)$. This implies that $a_{i j}=a_{-j}$.
3. Since $a_{-j}$ and $a_{+j}$ are the smallest and largest elements in the given data, i.e., $a_{-j} \subseteq a_{i j} \subseteq a_{+j}$. Therefore, the distance between $a_{-j}$ and $a_{+j}$ is greater than the distance between $a_{+j}$ and $a_{i j}$, i.e., $D\left(a_{i j}, a_{+j}\right) \leq D\left(a_{-j}, a_{+j}\right)$. This implies that $R I^{d}\left(a_{i j}\right) \leq 1$. The non negativity of the displaced remoteness index is trivial.
4. For each $e_{j} \in C_{b}, a_{i^{\prime} j} \subseteq a_{i^{\prime \prime} j}$ iff $\xi_{a_{i^{\prime} j}} \leq \xi_{a_{i^{\prime \prime} j}}$ and $v_{a_{i^{\prime} j}} \geq v_{a_{i^{\prime \prime}}}$. Since $a_{+j}$ is the largest element in the given data, therefore, $\xi_{a_{i^{\prime} j}} \leq \xi_{a_{i^{\prime \prime}}} \leq \xi_{a_{+j}}$ and $v_{a_{i^{\prime} j}} \geq v_{a_{i^{\prime \prime} j}} \geq v_{a_{+j}}$. This implies $D\left(a_{i^{\prime} j}, a_{+j}\right) \geq$ $D\left(a_{i^{\prime \prime} j}, a_{+j}\right)$ and hence $R I^{d}\left(a_{i^{\prime \prime} j}\right) \leq R I^{d}\left(a_{i^{\prime} j}\right)$.
5. The proof is analogous to the proof of part 4.

Example 3. Consider $\hat{Y}=\left\{y_{1}, y_{2}, y_{3}\right\}$ be the set of alternatives to be assessed under the criteria $\hat{C}=\left\{e_{1}, e_{2}\right\}$. This is the classical MADM problem, where $e_{1} \in C_{b}$ and $e_{2} \in C_{c}$. Assume that the IF decision matrix is given by

$$
a=\left[a_{i j}\right]_{3 \times 2}=\begin{gathered}
e_{1} \\
y_{1} \\
y_{2} \\
y_{3}
\end{gathered}\left(\begin{array}{cc}
(0.7,0.3) & (0.4,0.5) \\
(0.5,0.4) & (0.3,0.5) \\
(0.6,0.2) & (0.6,0.2)
\end{array}\right)
$$

1. According to the Definition 15, the dpi-IFVs are $a_{+1}=(0.7,0.2)$ and $a_{+2}=(0.3,0.5)$. Moreover, the dni-IFVs are $a_{-1}=(0.5,0.4)$ and $a_{-2}=(0.6,0.2)$.
2. Since, we consider here the BIFSS and calculate the distance between two IFVs therefore the Formula (5) takes the form

$$
\begin{equation*}
D_{e}^{\circ}\left(a_{i j}, a_{+j}\right)=\left(\frac{1}{2}\left(\left|\xi_{a_{i j}}-\xi_{a_{+j}}\right|^{2}+\left|v_{a_{i j}}-v_{a_{+j}}\right|^{2}+\left|\hbar_{a_{i j}}-\hbar_{a_{+j}}\right|^{2}\right)\right)^{\frac{1}{2}} \tag{26}
\end{equation*}
$$

We obtain $D_{e}^{\circ}\left(a_{+1}, a_{-1}\right)=0.244949$ and $D_{e}^{\circ}\left(a_{+2}, a_{-2}\right)=0.367423$. The displaced remoteness index are obtain by using Definition 17 as follows: $R I^{d}\left(a_{11}\right)=D_{e}^{\circ}\left(a_{11}, a_{+1}\right) / D_{e}^{\circ}\left(a_{+1}, a_{-1}\right)=$ $0.141421 / 0.244949=0.57735, R I^{d}\left(a_{21}\right)=1, R I^{d}\left(a_{31}\right)=0.5, R I^{d}\left(a_{12}\right)=0.333333, R I^{d}\left(a_{22}\right)=$ 0.544331 and $R I^{d}\left(a_{32}\right)=0.638285$.
3. We observe that $a_{11} \subseteq a_{21}$ and $a_{12} \subseteq a_{32}$. From above calculations, we have $R I^{d}\left(a_{11}\right) \leq R I^{d}\left(a_{21}\right)$ and $R I^{d}\left(a_{12}\right) \leq R I^{d}\left(a_{32}\right)$, which are consistent with fourth and fifth property of Theorem 4.

Now, we define the fixed remoteness index $R I^{f}$ as follows:
Definition 18. The fixed remoteness index $R I^{f}\left(a_{i j}\right)$ of $\left(a_{i j}\right)$ based on the above proposed distance measures $D$ is defined as follows:

$$
\begin{align*}
\operatorname{RI}^{f}\left(a_{i j}\right) & =\frac{D\left(a_{i j}, a_{\hat{+} j}\right)}{D\left(a_{\hat{-j}}, a_{\hat{+} j}\right)} \\
& =D\left(a_{i j}, a_{\hat{+} j}\right), \tag{27}
\end{align*}
$$

because $D\left(a_{\hat{-} j}, a_{\hat{+} j}\right)=1$ for fpi-IFV and fni-IFVs.
Theorem 5. Let $a_{i j}, a_{i 1 j}$ and $a_{i 2 j}$ be three assessment values in the IF decision matrix $a$. The RI ${ }^{f}$ satisfies the following properties:

1. $R I^{f}\left(a_{i j}\right)=0 \quad \Leftrightarrow \quad a_{i j}=a_{\hat{+} j}$
2. $R I^{f}\left(a_{i j}\right)=1 \quad \Leftrightarrow \quad a_{i j}=a_{\hat{-} j}$
3. $0 \leq R I^{f}\left(a_{i j}\right) \leq 1$
4. For each $e_{j} \in C_{b}$, RI $I^{f}\left(a_{i^{\prime \prime} j}\right) \leq R I^{f}\left(a_{i^{\prime} j}\right)$ if $a_{i^{\prime} j} \subseteq a_{i^{\prime \prime} j}$
5. For each $e_{j} \in C_{c}, R I^{f}\left(a_{i^{\prime} j}\right) \leq R I^{f}\left(a_{i^{\prime \prime}}\right)$ if $a_{i^{\prime} j} \subseteq a_{i^{\prime \prime} j}$

Proof. The proof of the theorem is based on the properties of distance measures (Definition 7) and Equation (27).

1. From Equation (27), we have $R I^{f}\left(a_{i j}\right)=0$ iff $D\left(a_{i j}, a_{\hat{+} j}\right)=0$, where $D$ is distance measure. The distance $D\left(a_{i j}, a_{\hat{+} j}\right)=0$ iff $a_{i j}=a_{\hat{+} j}$.
2. From Equation (27), if $R I^{f}\left(a_{i j}\right)=1$ then $D\left(a_{i j}, a_{\hat{+} j}\right)=D\left(a_{\hat{-j}}, a_{\hat{+} j}\right)$. This implies that $a_{i j}=a_{\hat{-}}$.
3. Since $a_{\hat{-j}}$ and $a_{\hat{+} j}$ are the smallest and largest elements in the given data, i.e., $a_{\hat{-} j} \subseteq a_{i j} \subseteq a_{\hat{+} j}$. Therefore, the distance between $a_{\hat{-} j}$ and $a_{\hat{+} j}$ is greater than the distance between $a_{\hat{+} j}$ and $a_{i j}$, i.e., $D\left(a_{i j}, a_{\hat{+} j}\right) \leq D\left(a_{\hat{-} j}, a_{\hat{+} j}\right)$. This implies that $R I^{f}\left(a_{i j}\right) \leq 1$. The non negativity of the fixed remoteness index is trivial.
4. For each $e_{j} \in C_{b}, a_{i^{\prime} j} \subseteq a_{i^{\prime \prime} j}$ iff $\xi_{a_{i^{\prime} j}} \leq \xi_{a_{i^{\prime \prime} j}}$ and $v_{a_{i^{\prime} j}} \geq v_{a_{i^{\prime \prime}} j}$. Since $a_{\hat{+} j}$ is the largest element in the given data, therefore, $\xi_{a_{i^{\prime} j}} \leq \xi_{a_{i^{\prime \prime}}} \leq \xi a_{\hat{+} j}$ and $v_{a_{i^{\prime} j}} \geq v_{a_{i^{\prime \prime}}} \geq v_{a_{\hat{\gamma}}}$. This implies $D\left(a_{i^{\prime} j}, a_{\hat{+} j}\right) \geq$ $D\left(a_{i^{\prime \prime} j}, a_{\hat{+} j}\right)$ and hence $R I^{f}\left(a_{i^{\prime \prime} j}\right) \leq R I^{f}\left(a_{i^{\prime} j}\right)$.
5. The proof is analogous to the proof of part 4.

Example 4. We continues Example 3 for fixed ideal and remote index.

1. Since $e_{1} \in C_{b}$ and $e_{2} \in C_{c}$. Therefore, according to the Definition 16, the dpi-IFVs are $a_{+1}=(1,0)$ and $a_{+2}=(0,1)$. Moreover, the dni-IFVs are $a_{-1}=(0,1)$ and $a_{-2}=(1,0)$.
2. We use Formula (26) for calculating distance between IFVs. The distance between $a_{\hat{+} j}$ and $a_{\hat{-j}}$, $(j=1,2)$ is 1. The fixed remoteness index are obtain by using Definition 18 as follows: RI ${ }^{f}\left(a_{11}\right)=$ $D_{e}^{\circ}\left(a_{11}, a_{+1}\right) / D_{e}^{\circ}\left(a_{\hat{+} 1}, a_{-1}\right)=0.367423 / 1=0.367423, \operatorname{RI}^{f}\left(a_{21}\right)=0.543139, \operatorname{RI}^{f}\left(a_{31}\right)=0.4$, $\operatorname{RI}^{f}\left(a_{12}\right)=0.583095, \operatorname{RI}^{f}\left(a_{22}\right)=0.578792$ and $\operatorname{RI}^{f}\left(a_{32}\right)=0.927362$.
3. We observe that $a_{11} \subseteq a_{21}$ and $a_{12} \subseteq a_{32}$. From above calculations, we have $\operatorname{RI}^{f}\left(a_{11}\right) \leq \operatorname{RI} I^{f}\left(a_{21}\right)$ and $R I^{f}\left(a_{12}\right) \leq R I^{f}\left(a_{32}\right)$, which are consistent with fourth and fifth property of Theorem 5.

### 4.3. Precise and IF Importance Weights

In real-life situations, all the attributes are not of equal importance in the decision making (DM) process. Some are more important than others. In the DM process, we handle this issue by assigning the
weights to the attributes. In this paper, we use two types of weights. The precise importance weights and IF importance weights. We calculate the importance weights by PIFS given by the director or head of the DM committee. The two methods are discussed to obtained precise importance weights. In the first method, the expectation score function is used to calculate the weights [14]. In the second method, we use already proposed entropy measures to calculates the precise weights. The IF importance weights are directly assigned to the attributes. The IFVs in PIFS are used as an IF importance weights.

In the DM process, let $\hat{Y}=\left\{y_{1}, y_{2}, \ldots, y_{m}\right\}$ be the $m$ alternatives which are assessed against $n$ attributes represented as $C=\left\{e_{1}, e_{2}, \ldots, e_{n}\right\}$. Each alternative evaluated with respect to each criteria and the aim of DM process to select the optimal alternative on the basis of the criterion.

1. Precise Importance Weights by Expectation Score Function [14]:

In this method for obtaining precise importance weights are based on the expectation score function and PIFS in the GIFSS. Let $\hat{\rho}=\left\{\left(e_{j}, \xi_{\hat{\rho}\left(e_{j}\right)}, v_{\hat{\rho}\left(e_{j}\right)}\right) \mid e_{j} \in \hat{C}\right\}$ be the PIFS and $a_{j}=$ $\left(\xi_{\hat{\rho}\left(e_{j}\right)}, v_{\hat{\rho}\left(e_{j}\right)}\right)$ represents the IFVs in PIFS. Then the expectation score value of IFV is calculated by using Equation (1) as follows:

$$
\begin{equation*}
S\left(a_{j}\right)=\frac{1+\xi_{\hat{\rho}\left(e_{j}\right)}-v_{\hat{\rho}\left(e_{j}\right)}}{2} \tag{28}
\end{equation*}
$$

If we represents the sum of all expected score values by $d=\sum_{j=1}^{n} S\left(a_{j}\right)$ then the precise importance weights are calculated as follows:

$$
\begin{equation*}
\omega_{j}=\frac{S\left(a_{j}\right)}{d} \tag{29}
\end{equation*}
$$

Example 5. Let $\hat{\rho}=\left\{a_{1}=\left(e_{1}, 0.3,0.5\right), a_{2}=\left(e_{2}, 0.5,0.3\right), a_{3}=\left(e_{3}, 0.3,0.4\right), a_{4}=\left(e_{4}, 0.7,0.1\right)\right\}$ be the PIFS. The expectation scores of IFVs are calculated by using Equation (28). The results are: $S\left(a_{1}\right)=0.4$, $S\left(a_{2}\right)=0.6, S\left(a_{3}\right)=0.45$ and $S\left(a_{4}\right)=0.8$. The sum of expectation scores is $d=\sum_{j=1}^{4} S\left(a_{j}\right)=2.25$. The precise importance weights are: $\omega_{1}=0.177778, \omega_{2}=0.266667, \omega_{3}=0.2$ and $\omega_{4}=0.355556$.
2. Precise Importance Weights by Entropy Measures for IFSs:

Motivated by Chen's technique of getting precise importance weights by using entropy measures for IFSs [54]. The Burillo and Bustince [52] entropy measure for IFSs is used. Many authors defined the entropy measure for IFSs and one can use any entropy measure for getting precise weights. The one of the entropy measure from the Burillo and Bustince paper for IFS $R$ in $\hat{Y}$ is defined as

$$
\begin{equation*}
\hat{E}(R)=\sum_{i=1}^{m}\left(1-\left(\left(\xi_{R}\left(y_{i}\right)+v_{R}\left(y_{i}\right)\right) \times \sin \left(\frac{\pi}{2}\left(\xi_{R}\left(y_{i}\right)+v_{R}\left(y_{i}\right)\right)\right)\right) .\right. \tag{30}
\end{equation*}
$$

Let $\hat{\rho}=\left\{\left(e_{j}, \xi_{\hat{\rho}\left(e_{j}\right)}, v_{\hat{\rho}\left(e_{j}\right)}\right) \mid e_{j} \in \hat{C}\right\}$ be the PIFS and $a_{j}=\left(\xi_{\hat{\rho}\left(e_{j}\right)}, v_{\hat{\rho}\left(e_{j}\right)}\right)$ represents the IFVs in PIFS. Then the entropy measure of IFV is calculated by using Equation (30) as follows:

$$
\begin{equation*}
\hat{E}\left(a_{j}\right)=1-\left(\left(\xi_{\hat{\rho}\left(e_{j}\right)}+v_{\hat{\rho}\left(e_{j}\right)}\right) \times \sin \left(\frac{\pi}{2}\left(\xi_{\hat{\rho}\left(e_{j}\right)}+v_{\hat{\rho}\left(e_{j}\right)}\right)\right)\right) . \tag{31}
\end{equation*}
$$

If we represents the sum of all entropy measures by $L$, i.e., $L=\sum_{j=1}^{n} \hat{E}\left(a_{j}\right)$, then the precise importance weights are calculated as follows:

$$
\begin{equation*}
\omega_{j}=\frac{1}{n-L}\left(1-\hat{E}\left(a_{j}\right)\right) \tag{32}
\end{equation*}
$$

The weights obtained by the Equation (32) satisfies the condition of normalization, i.e., $\omega_{j} \in[0,1]$ and $\sum_{j=1}^{n} \omega_{j}=1$.

Example 6. We continues the Example 5 for calculating weights by using the entropy method. The entropy measures of IFVs are calculated by using Equation (31). The results are: $\hat{E}\left(a_{1}\right)=0.239155, \hat{E}\left(a_{2}\right)=0.239155$, $\hat{E}\left(a_{3}\right)=0.376295$ and $\hat{E}\left(a_{4}\right)=0.239155$. The sum of the entropy measures is $L=\sum_{j=1}^{4} \hat{E}\left(a_{j}\right)=1.09376$. The precise importance weights are calculating by Equation (32) and the results are: $\omega_{1}=0.261797$, $\omega_{2}=0.261797, \omega_{3}=0.214609$ and $\omega_{4}=0.261797$.
3. The IF Importance Weights:

The IF importance weights of criteria $e_{j} \in \hat{C}$ is displaced as the IFV $\omega_{j}=\left(\bar{\omega}_{j}, \varrho_{j}\right)$, where $\bar{\omega}_{j}$ and $\underline{\omega}_{j}$ represents the importance and unimportance degrees of the criteria $e_{j} \in \hat{C}$, respectively. In the DM process, the head or director of the committee is seeking the whole process keenly. At the end of the process, he gave his evaluation in the form of PIFS. These PIFS provided by the head are serving as an IF importance weights.

The proposed VIKOR methods, consider both the precise and IF importance weights in decision making process. The new ranking indexes $\hat{S}^{d}, \hat{R}^{d}$ and $\hat{Q}^{d}$ based on the displaced ideals and precise importance weights of attributes are presented as follows.

Definition 19. For each alternative $y_{i}$ the displaced remoteness based (DRB) group utility index $\hat{\mathrm{S}}^{d}\left(y_{i}\right)$ is defined as follows:

$$
\begin{equation*}
\hat{\mathrm{S}}^{d}\left(y_{i}\right)=\sum_{j=1}^{n}\left(R I^{d}\left(a_{i j}\right) \cdot \omega_{j}\right), \tag{33}
\end{equation*}
$$

where $a_{i j} \in\left[a_{i j}\right]_{m \times n}$ and $\omega_{j}$ are the IFVs and precise importance weights, respectively.
The DRB individual regret index $\hat{\mathrm{R}}^{d}\left(y_{i}\right)$ of $y_{i}$ is defined as follows:

$$
\begin{equation*}
\hat{\mathrm{R}}^{d}\left(y_{i}\right)=\max _{j=1}^{n}\left\{R I^{d}\left(a_{i j}\right) \cdot \omega_{j}\right\} . \tag{34}
\end{equation*}
$$

The DRB compromise index $\hat{Q}^{d}\left(y_{i}\right)$ of $y_{i}$ is defined as follows:

$$
\begin{equation*}
\hat{\mathrm{Q}}^{d}\left(y_{i}\right)=\lambda \cdot \frac{\hat{\mathrm{S}}^{d}\left(y_{i}\right)-\min _{i^{\prime}=1}^{m} \hat{\mathrm{~S}}^{d}\left(y_{i^{\prime}}\right)}{\max _{i^{\prime}=1}^{m} \hat{\mathrm{~S}}^{d}\left(y_{i^{\prime}}\right)-\min _{i^{\prime}=1}^{m} \hat{\mathrm{~S}}^{d}\left(y_{i^{\prime}}\right)}+(1-\lambda) \cdot \frac{\hat{\mathrm{R}}^{d}\left(y_{i}\right)-\min _{i^{\prime}=1}^{m} \hat{\mathrm{R}}^{d}\left(y_{i^{\prime}}\right)}{\max _{i^{\prime}=1}^{m} \hat{\mathrm{R}}^{d}\left(y_{i^{\prime}}\right)-\min _{i^{\prime}=1}^{m} \hat{\mathrm{R}}^{d}\left(y_{i^{\prime}}\right)}, \tag{35}
\end{equation*}
$$

where $\lambda \in[0,1]$.
The new ranking indexes $\hat{\mathrm{S}}^{f}, \hat{\mathrm{R}}^{f}$ and $\hat{\mathrm{Q}}^{f}$ based on the fixed ideals and precise weights of attributes are presented as follows.

Definition 20. For each alternative $y_{i}$ the fixed remoteness based FRB group utility index $\hat{\mathrm{S}}^{f}\left(y_{i}\right)$ is defined as follows:

$$
\begin{equation*}
\hat{S}^{f}\left(y_{i}\right)=\sum_{j=1}^{n}\left(R I^{f}\left(a_{i j}\right) \cdot \omega_{j}\right) \tag{36}
\end{equation*}
$$

where $a_{i j} \in\left[a_{i j}\right]_{m \times n}$ and $\omega_{j}$ are the IFVs and precise weights, respectively.
The FRB individual regret index $\hat{\mathrm{R}}^{f}\left(y_{i}\right)$ of $y_{i}$ is defined as follows:

$$
\begin{equation*}
\hat{\mathrm{R}}^{f}\left(y_{i}\right)=\max _{j=1}^{n}\left\{R I^{f}\left(a_{i j}\right) \cdot \omega_{j}\right\} \tag{37}
\end{equation*}
$$

The FRB compromise index $\hat{Q}^{f}\left(y_{i}\right)$ of $y_{i}$ is defined as follows:

$$
\begin{equation*}
\hat{Q}^{f}\left(y_{i}\right)=\lambda \cdot \frac{\hat{\mathrm{S}}^{f}\left(y_{i}\right)-\min _{i^{\prime}=1}^{m} \hat{\mathrm{~S}}^{f}\left(y_{i^{\prime}}\right)}{\max _{i^{\prime}=1}^{m} \hat{\mathrm{~S}}^{f}\left(y_{i^{\prime}}\right)-\min _{i^{\prime}=1}^{m} \hat{\mathrm{~S}}^{f}\left(y_{i^{\prime}}\right)}+(1-\lambda) \cdot \frac{\hat{\mathrm{R}}^{f}\left(y_{i}\right)-\min _{i^{\prime}=1}^{m} \hat{\mathrm{R}}^{f}\left(y_{i^{\prime}}\right)}{\max _{i^{\prime}=1}^{m} \hat{\mathrm{R}}^{f}\left(y_{i^{\prime}}\right)-\min _{i^{\prime}=1}^{m} \hat{\mathrm{R}}^{f}\left(y_{i^{\prime}}\right)} \tag{38}
\end{equation*}
$$

where $\lambda \in[0,1]$.
The decision mechanism coefficient is represented by the parameter $\lambda$. One can modify the decision making strategy by changing the value of the parameter $\lambda$. The value of the parameter $\lambda$ represents the importance of maximum group utility while $1-\lambda$ represents the importance of individual regrets. In the classical VIKOR method, the higher the value of the parameter $\lambda$ (when $\lambda>0.5$ ), the compromise ranking procedure is categorized as the procedure with "voting by majority". The compromise ranking procedure is categorized as the procedure with "veto" when $\lambda<0.5$. The consensus is achieved in the compromise ranking procedure at $\lambda=0.5$.

The new ranking indexes $\mathrm{S}^{d}, \mathrm{R}^{d}$ and $\mathrm{Q}^{d}$ based on the displaced ideals and IF importance weights of attributes are presented.

Definition 21. For each alternative $y_{i}$ the DRB group utility index $\mathrm{S}^{d}\left(y_{i}\right)$ with a set of IF importance weights $\omega_{j}=\left(\bar{\omega}_{j}, \underline{\omega}_{j}\right)$ for all $c_{j} \in C$ is defined as follows:

$$
\begin{align*}
\mathrm{S}^{d}\left(y_{i}\right) & =\sum_{j=1}^{n} \delta\left(R I^{d}\left(a_{i j}\right) \cdot \omega_{j}\right) \\
& =\sum_{j=1}^{n}\left[\frac{1}{2}\left(2-\left(1-\bar{\omega}_{j}\right)^{R I^{d}\left(a_{i j}\right)}-\widehat{\omega}_{j}^{R I^{d}\left(a_{i j}\right)}\right)\right] \tag{39}
\end{align*}
$$

where $a_{i j} \in\left[a_{i j}\right]_{m \times n}$ are the IFVs and $\delta$ is an expectation score function defined in Equation (1).
The DRB individual regret index $\mathrm{R}^{d}\left(y_{i}\right)$ of $\left(y_{i}\right)$ is defined as follows:

$$
\begin{align*}
\mathrm{R}^{d}\left(y_{i}\right) & =\max _{j=1}^{n} \delta\left(R I^{d}\left(a_{i j}\right) \cdot \omega_{j}\right) \\
& =\max _{j=1}^{n}\left\{\frac{1}{2}\left(2-\left(1-\bar{\omega}_{j}\right)^{R I^{d}\left(a_{i j}\right)}-\underline{\omega}_{j}^{R I^{d}\left(a_{i j}\right)}\right)\right\} . \tag{40}
\end{align*}
$$

The DRB compromise index $\mathrm{Q}^{d}\left(y_{i}\right)$ of $\left(y_{i}\right)$ is defined as follows:

$$
\begin{equation*}
\mathrm{Q}^{d}\left(y_{i}\right)=\lambda \cdot \frac{\mathrm{S}^{d}\left(y_{i}\right)-\min _{i^{\prime}=1}^{m} \mathrm{~S}^{d}\left(y_{i^{\prime}}\right)}{\max _{i^{\prime}=1}^{m} \mathrm{~S}^{d}\left(y_{i^{\prime}}\right)-\min _{i^{\prime}=1}^{m} \mathrm{~S}^{d}\left(y_{i^{\prime}}\right)}+(1-\lambda) \cdot \frac{\mathrm{R}^{d}\left(y_{i}\right)-\min _{i^{\prime}=1}^{m} \mathrm{R}^{d}\left(y_{i^{\prime}}\right)}{\max _{i^{\prime}=1}^{m} \mathrm{R}^{d}\left(y_{i^{\prime}}\right)-\min _{i^{\prime}=1}^{m} \mathrm{R}^{d}\left(y_{i^{\prime}}\right)} \tag{41}
\end{equation*}
$$

The new ranking indexes $\mathrm{S}^{f}, \mathrm{R}^{f}$ and $\mathrm{Q}^{f}$ based on the displaced ideals and IF importance weights of attributes are presented as follows.

Definition 22. For each alternative $y_{i}$ the FRB group utility index $\mathrm{S}^{f}\left(y_{i}\right)$ with a set of IF importance weights $\omega_{j}=\left(\bar{\omega}_{j}, \underline{\omega}_{j}\right)$ for all $c_{j} \in C$ is defined as follows:

$$
\begin{align*}
\mathrm{S}^{f}\left(y_{i}\right) & =\sum_{j=1}^{n} \delta\left(R I^{f}\left(a_{i j}\right) \cdot \omega_{j}\right) \\
& =\sum_{j=1}^{n}\left[\frac{1}{2}\left(2-\left(1-\bar{\omega}_{j}\right)^{R I^{f}\left(a_{i j}\right)}-\underline{\omega}_{j}^{R I^{f}\left(a_{i j}\right)}\right)\right] \tag{42}
\end{align*}
$$

where $a_{i j} \in\left[a_{i j}\right]_{m \times n}$ are the IFVs and $\delta$ is an expectation score function defined in Equation (1).

The FRB individual regret index $\mathrm{R}^{f}\left(y_{i}\right)$ of $\left(y_{i}\right)$ is defined as follows:

$$
\begin{align*}
\mathrm{R}^{f}\left(y_{i}\right) & =\max _{j=1}^{n} \delta\left(R I^{f}\left(a_{i j}\right) \cdot \omega_{j}\right) \\
& =\max _{j=1}^{n}\left\{\frac{1}{2}\left(2-\left(1-\bar{\omega}_{j}\right)^{R I^{f}\left(a_{i j}\right)}-\underline{\omega}_{j}^{R I^{f}\left(a_{i j}\right)}\right)\right\} \tag{43}
\end{align*}
$$

The FRB compromise index $\mathrm{Q}^{f}\left(y_{i}\right)$ of $\left(y_{i}\right)$ is defined as follows:

$$
\begin{equation*}
\mathrm{Q}^{f}\left(y_{i}\right)=\lambda \cdot \frac{\mathrm{S}^{f}\left(y_{i}\right)-\min _{i^{\prime}=1}^{m} \mathrm{~S}^{f}\left(y_{i^{\prime}}\right)}{\max _{i^{\prime}=1}^{m} \mathrm{~S}^{f}\left(y_{i^{\prime}}\right)-\min _{i^{\prime}=1}^{m} \mathrm{~S}^{f}\left(y_{i^{\prime}}\right)}+(1-\lambda) \cdot \frac{\mathrm{R}^{f}\left(y_{i}\right)-\min _{i^{\prime}=1}^{m} \mathrm{R}^{f}\left(y_{i^{\prime}}\right)}{\max _{i^{\prime}=1}^{m} \mathrm{R}^{f}\left(y_{i^{\prime}}\right)-\min _{i^{\prime}=1}^{m} \mathrm{R}^{f}\left(y_{i^{\prime}}\right)} \tag{44}
\end{equation*}
$$

## 5. Selection of Renewable Energy Resources in under Developing Countries

Some examples of renewable energy sources are solar energy, wind energy, hydropower, geothermal energy, and biomass energy. These types of energy sources are different from fossil fuels, such as coal, oil, and natural gas. The people are using fossil fuels very quickly. The world will be facing the problem of deficiency of energy because the resources of fossil fuels are decreasing very quickly. Nowadays, most countries depend on electricity generated from fossil fuels. About $64.5 \%$ of the worldwide electricity generated from fossil fuels according to 2017 statistics which is higher than the 1990's statistics when fossil fuels generated $61.9 \%$ of the worldwide electricity. These results are dangerous because the electricity generated from fossil fuels pollute the environment and cost heavily.

Therefore, it's very important to consider renewable energy sources. Comparatively, renewable energy sources are very much less harmful to non-renewable energy resources. For under developing countries, it is much important to consider renewable energy and sustainable sources which are less effective for the environment. For under developing countries, it's important to choose the best renewable source for their country which minimum effects the environment, budget, and economy. Minimum number of peoples are effected from this project. The maintenance, reliability, yields are important parameters to evaluate the suitable energy sources.

In this section, we discuss a factious problems of selecting a renewable energy source for under developing countries. Let $\hat{Y}=\left\{y_{1}, y_{2}, y_{3}, y_{4}, y_{5}\right\}$ represents the set of renewable energy sources (alternative), where $y_{1}, y_{2}, y_{3}, y_{4}$ and $y_{5}$ stands for solar energy, wind energy, geothermal energy, hydro power and biomass energy, respectively. These sources are evaluated against the six parameters (criteria). Let $\hat{C}=\left\{e_{1}, e_{2}, e_{3}, e_{4}, e_{5}, e_{6}\right\}$ represents the set of criterion, where $e_{1}, e_{2}, e_{3}, e_{4}, e_{5}$ and $e_{6}$ stands for cost, environmental friendly, yields, maintenance, reliability and less number of peoples are effected from this project, respectively.

A committee consists of engineers, economists, managers, government servants, and some other policymakers. The committee assessed the proposed five renewable energy resources according to six criteria. The committee give their judgments based on their knowledge and previous statistical measures. The committee preferences are stored in the form of BIFSS. The head of the committee who supervised the whole procedure. He gave their judgments in the form of PIFS. Which completes the formation of GIFSS. The results are summarized in Table 5. This represents the IF decision matrix with $m=5$ rows and $n=6$ columns. Suppose the IF decision matrix represented by $a=\left[a_{i j}\right]_{m \times n}$, where $a_{i j}$ shows the evaluation of $i$ th alternative with respect to $j$ th criteria.

Table 5. The GIFSS ( $\hat{F}, \hat{A}, \hat{\rho})$.

| $\hat{\gamma}$ | $e_{1}$ | $e_{2}$ | $e_{3}$ | $e_{4}$ | $e_{5}$ | $e_{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y_{1}$ | $(0.1,0.8)$ | $(0.6,0.2)$ | $(0.4,0.4)$ | $(0.7,0.2)$ | $(0.6,0.1)$ | $(0.7,0.3)$ |
| $y_{2}$ | $(0.2,0.6)$ | $(0.7,0.2)$ | $(0.4,0.5)$ | $(0.5,0.1)$ | $(0.6,0.2)$ | $(0.5,0.3)$ |
| $y_{3}$ | $(0.3,0.7)$ | $(0.8,0.1)$ | $(0.6,0.3)$ | $(0.7,0.1)$ | $(0.2,0.5)$ | $(0.7,0.1)$ |
| $y_{4}$ | $(0.1,0.8)$ | $(0.9,0.1)$ | $(0.6,0.2)$ | $(0.5,0.3)$ | $(0.7,0.2)$ | $(0.6,0.2)$ |
| $y_{5}$ | $(0.4,0.5)$ | $(0.6,0.3)$ | $(0.7,0.2)$ | $(0.2,0.6)$ | $(0.7,0.1)$ | $(0.3,0.5)$ |
| $\hat{\rho}$ | $(0.4,0.2)$ | $(0.8,0.2)$ | $(0.5,0.3)$ | $(0.6,0.2)$ | $(0.7,0.1)$ | $(0.4,0.4)$ |

Since $e_{1}$ is the cost criteria. We normalize the IF decision matrix by following equation:

$$
a_{i j}= \begin{cases}\left(\xi_{a_{i j}}, v_{a_{i j}}\right) & \text { if } e_{j} \in \hat{C}_{b}  \tag{45}\\ \left(v_{a_{i j}}, \xi \xi_{i j}\right) & \text { if } e_{j} \in \hat{C}_{c}\end{cases}
$$

The normalized IF decision matrix is shown in Table 6 and all criteria are treated as the benefit type.
Table 6. The $\operatorname{GIFSS}(\hat{F}, \hat{A}, \hat{\rho})$.

| $\hat{\boldsymbol{Y}}$ | $e_{1}$ | $e_{2}$ | $e_{3}$ | $e_{4}$ | $e_{5}$ | $e_{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y_{1}$ | $(0.8,0.1)$ | $(0.6,0.2)$ | $(0.4,0.4)$ | $(0.7,0.2)$ | $(0.6,0.1)$ | $(0.7,0.3)$ |
| $y_{2}$ | $(0.6,0.2)$ | $(0.7,0.2)$ | $(0.4,0.5)$ | $(0.5,0.1)$ | $(0.6,0.2)$ | $(0.5,0.3)$ |
| $y_{3}$ | $(0.7,0.3)$ | $(0.8,0.1)$ | $(0.6,0.3)$ | $(0.7,0.1)$ | $(0.2,0.5)$ | $(0.7,0.1)$ |
| $y_{4}$ | $(0.8,0.1)$ | $(0.9,0.1)$ | $(0.6,0.2)$ | $(0.5,0.3)$ | $(0.7,0.2)$ | $(0.6,0.2)$ |
| $y_{5}$ | $(0.5,0.4)$ | $(0.6,0.3)$ | $(0.7,0.2)$ | $(0.2,0.6)$ | $(0.7,0.1)$ | $(0.3,0.5)$ |
| $\hat{\rho}$ | $(0.4,0.2)$ | $(0.8,0.2)$ | $(0.5,0.3)$ | $(0.6,0.2)$ | $(0.7,0.1)$ | $(0.4,0.4)$ |

The problem of selecting renewable energy sources is solved by proposed fourth algorithms, which shows the procedure of remoteness based VIKOR method for different conditions and terminologies.

### 5.1. Solution by Algorithm 1

First two steps of Algorithm 1 have already done. The dpi-IFVs $a_{+j}$ and dni-IFVs $a_{-j}$ are calculating by using Equations (19) and (20), which help us to choose the best alternative and avoidable alternative, respectively.

$$
\begin{align*}
& \text { dpi-IFV } \rightarrow a_{+j}=\left\{\begin{array}{lll}
a_{+1}=(0.8,0.1) & a_{+2}=(0.9,0.1) & a_{+3}=(0.7,0.2) \\
a_{+4}=(0.7,0.1) & a_{+5}=(0.7,0.1) & a_{+6}=(0.7,0.1)
\end{array}\right.  \tag{46}\\
& \text { dni-IFV } \rightarrow a_{-j}=\left\{\begin{array}{lll}
a_{-1}=(0.5,0.4) & a_{-2}=(0.6,0.3) & a_{-3}=(0.4,0.5) \\
a_{-4}=(0.2,0.6) & a_{-5}=(0.2,0.5) & a_{-6}=(0.3,0.5)
\end{array}\right. \tag{47}
\end{align*}
$$

The distance between dpi-IFVs $a_{+j}$ and dni-IFVs $a_{-j}$ are calculating by using Equation (26). We use this formula to find the distance between two IFVs. The results summarized in Equation (48) as follows:

$$
D_{e}^{\circ}\left(a_{+j}, a_{-j}\right)=\left\{\begin{array}{lll}
D_{e}^{\circ}\left(a_{+1}, a_{-1}\right)=0.36742 & D_{e}^{\circ}\left(a_{+2}, a_{-2}\right)=0.30822 & D_{e}^{\circ}\left(a_{+3}, a_{-3}\right)=0.36742  \tag{48}\\
D_{e}^{\circ}\left(a_{+4}, a_{-4}\right)=0.61237 & D_{e}^{\circ}\left(a_{+5}, a_{-5}\right)=0.54314 & D_{e}^{\circ}\left(a_{+6}, a_{-6}\right)=0.48990
\end{array}\right.
$$

Algorithm 1: for scenario 1: IF decision matrix, precise weights and displaced ideals

1. Let $Y=\left\{y_{1}, y_{2}, \ldots, y_{m}\right\}$ represents the alternatives and the set $C=\left\{e_{1}, e_{2}, \ldots, e_{n}\right\}$ represents the criteria.
2. The IFV is given to each alternatives $y_{i}, 1 \leq i \leq m$ according to the each criteria $e_{j}$, $1 \leq j \leq n$ and identified the precise weight by evaluating some data or selecting the appropriate linguistics variables. Which generates the IFV decision matrix $a=\left[a_{i j}\right]_{m \times n}$. One can obtained weights of attributes by using the above mentioned procedures (Section 4.3).
3. Find dpi-IFV and dni-IFV with respect to each criteria by using Equations (19) and (20), respectively.
4. Compute $D\left(a_{-j}, a_{+j}\right)$ and $D\left(a_{i j}, a_{+j}\right)$ for each $1 \leq i \leq m$ and $1 \leq j \leq n$.
5. Compute $R I^{d}\left(a_{i j}\right)$ for each $1 \leq i \leq m$ and $1 \leq j \leq n$ by using Equation (25).
6. Find DRB group utility index $\hat{S}^{d}\left(y_{i}\right)$ and DRB individual regret index $\hat{R}^{d}\left(y_{i}\right)$ by using Equations (33) and (34), respectively. Then compute the DRB compromise index $\hat{Q}^{d}\left(y_{i}\right)$ by using Equation (35).
7. Three ranking lists obtained by sorting the values of $\hat{\mathrm{S}}^{d}\left(y_{i}\right), \hat{\mathrm{R}}^{d}\left(y_{i}\right)$ and $\hat{\mathrm{Q}}^{d}\left(y_{i}\right)$ for each alternative in ascending order.
8. The alternative $y^{\prime}$ with the minimum value in $\hat{\mathrm{Q}}^{d}\left(y_{i}\right)$ ranking list is the compromise solution if the following conditions are satisfied.

C1. Acceptable advantage:

$$
\hat{Q}^{d}\left(y^{\prime \prime}\right)-\hat{Q}^{d}\left(y^{\prime}\right) \geq \frac{1}{m-1}
$$

where $y^{\prime \prime}$ is the alternative have minimum value after $y^{\prime}$ in $\hat{\mathrm{Q}}^{d}\left(y_{i}\right)$ ranking list.
C 2 . Acceptable stability in DM:
The alternative $y^{\prime}$ have the minimum values in $\hat{S}^{d}\left(y_{i}\right)$ and $\hat{R}^{d}\left(y_{i}\right)$ ranking lists.
The set of the ultimate compromise solution is proposed if one of the above condition is not satisfied, which consists of:
a. Alternatives $y^{\prime}$ and $y^{\prime \prime}$ if only C 2 is not satisfied.
b. Alternatives $y^{\prime}, y^{\prime \prime}, \ldots, y^{p}$ if C 1 is not satisfied, where $p$ is the largest $i$ for which $\hat{Q}^{d}\left(y^{p}\right)-\hat{Q}^{d}\left(y^{\prime}\right)<\frac{1}{m-1}$

The distance between the IFVs $a_{i j}$ and the dpi-IFVs $a_{+j}$ are calculating by using Equation (26) and the results are displaced in Equation (49).

|  | $D_{e}^{\circ}\left(a_{i 1}, a_{+1}\right)$ | $D_{e}^{\circ}\left(a_{i 2}, a_{+2}\right)$ | $D_{e}^{\circ}\left(a_{i 3}, a_{+3}\right)$ | $D_{e}^{\circ}\left(a_{i 4}, a_{+4}\right)$ | $D_{e}^{\circ}\left(a_{i 5}, a_{+5}\right)$ | $D_{e}^{\circ}\left(a_{i 6}, a_{+6}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y_{1}$ | ( 0 . | 0.308221 | 0.308221 | 0.141421 | 0.122474 | 0.282843 |
| $y_{2}$ | 0.2 | 0.2 | 0.367423 | 0.244949 | 0.122474 | 0.244949 |
| $y_{3}$ | 0.234521 | 0.122474 | 0.122474 | 0. | 0.543139 | 0. |
| $y_{4}$ | 0. | 0. | 0.122474 | 0.244949 | 0.141421 | 0.122474 |
| $y_{5}$ | 0.367423 | 0.308221 | 0. | 0.612372 | 0. | 0.489898 ) |

The weights of the attributes are calculated by using expectation score function. The PIFS have provided the initial data. Equation (29) is used to calculate the precise weights. After calculation, the weight of the attributes are: $\omega_{1}=0.15, \omega_{2}=0.2, \omega_{3}=0.15, \omega_{4}=0.175, \omega_{5}=0.2$ and $\omega_{6}=0.125$.

The displaced remoteness indexes $R I^{d}\left(a_{i j}\right)$ are calculating by using Definition 17. Then $R I^{d}\left(a_{i j}\right)$ are multiplying by precise weights $\omega_{j}(j=\{1,2, \ldots, 6\})$. The results are presented in Equation (50).

|  | $R I^{d}\left(a_{i 1}\right) \cdot \omega_{1}$ | $R I^{d}\left(a_{i 2}\right) \cdot \omega_{2}$ | $R I^{d}\left(a_{i 3}\right) \cdot \omega_{3}$ | $R I^{d}\left(a_{i 4}\right) \cdot \omega_{4}$ | $R I^{d}\left(a_{i 5}\right) \cdot \omega_{5}$ | $R I^{d}\left(a_{i 6}\right) \cdot \omega_{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y_{1}$ | 0 . | 0.2 | 0.125831 | 0.040415 | 0.045099 | 0.072169 |
| $y_{2}$ | 0.081650 | 0.129777 | 0.15 | 0.07 | 0.045099 | 0.0625 |
| $y_{3}$ | 0.095743 | 0.079471 | 0.05 | 0. | 0.2 | 0. |
| $y_{4}$ | 0. | 0. | 0.05 | 0.07 | 0.052076 | 0.03125 |
| $y_{5}$ | 0.15 | 0.2 | 0. | 0.175 | 0. | 0.125 |

For compromise ranking of the alternatives, the DRB group utility index $\hat{S}^{d}, D R B$ individual regret index $\hat{R}^{d}$ and the DRB compromise index $\hat{Q}^{d}$ are calculated by using Equations (33)-(35), respectively. All the calculations are displaced in Equation (51).

From Equation (51), the three ranking lists $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}, y_{4} \succ y_{2} \succ\left\{y_{3}, y_{5}, y_{1}\right\}$ and $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ are obtained by sorting each $\hat{\mathrm{S}}^{d}\left(y_{i}\right), \hat{\mathrm{R}}^{d}\left(y_{i}\right)$ and $\hat{\mathrm{Q}}^{d}\left(y_{i}\right)$ value in ascending order, respectively. The hydro power among the renewable energy sources is the best option by all ranking lists. Moreover, $\hat{Q}^{d}\left(y^{\prime \prime}\right)-\hat{Q}^{d}\left(y^{\prime}\right)=\hat{Q}^{d}\left(y_{2}\right)-\hat{Q}^{d}\left(y_{4}\right)=0.683469 \geq \frac{1}{5-1}=\frac{1}{4}=0.25$. Both the conditions in step 8 of Algorithm 1 are satisfied for hydro power. Therefore, hydro power is the compromise solution of the selection of renewable energy source problem. The order of the renewable energy sources is $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$.

### 5.2. Solution by Algorithm 2

Since all the criterion are of benefit type after normalization (Table 6), therefore, the fpi-IFV $a_{\hat{+} j}=(1,0)$ and the fni-IFV $a_{\hat{-} j}=(0,1) \forall j=\{1,2, \ldots, 6\}$. We calculate the distance between $a_{i j}$ and $a_{\hat{+} j}$ by using Formula (26) and the result are presented in Equation (52).

|  | $D_{e}^{\circ}\left(a_{i 1}, a_{\hat{+} 1}\right)$ | $D_{e}^{\circ}\left(a_{i 2}, a_{\hat{+} 2}\right)$ | $D_{e}^{\circ}\left(a_{i 3}, a_{\hat{+} 3}\right)$ | $D_{e}^{\circ}\left(a_{i 4}, a_{\hat{+} 4}\right)$ | $D_{e}^{\circ}\left(a_{i 5}, a_{\hat{+} 5}\right)$ | $D_{e}^{\circ}\left(a_{i 6}, a_{\hat{+} 6}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y_{1}$ | ( 0.2 | 0.4 | 0.616441 | 0.308221 | 0.424264 | 0.367423 |
| $y_{2}$ | 0.4 | 0.308221 | 0.663325 | 0.543139 | 0.4 | 0.504975 |
| $y_{3}$ | 0.367423 | 0.2 | 0.424264 | 0.308221 | 0.812404 | 0.308221 |
| $y_{4}$ | 0.2 | 0.122474 | 0.4 | 0.504975 | 0.308221 | 0.4 |
| $y_{5}$ | ( 0.543139 | 0.424264 | 0.308221 | 0.848528 | 0.308221 | 0.731437 ) |

By Definition 18, the fixed remoteness index is equal to the distance, i.e., $R I^{f}\left(a_{i j}\right)=D_{e}^{\circ}\left(a_{i j}, a_{\hat{+} j}\right)$. Now, we calculate $R I^{f}\left(a_{i j}\right) . \omega_{j}$ for FRB group utility index and individual regret index as follows:

| $\mathrm{RI}^{f}\left(a_{i 1}\right) \cdot \omega_{1}$ |
| :--- |
| $y_{1}$ |
| $y_{2} I^{f}\left(a_{i 2}\right) \cdot \omega_{2}$ | $\left.\mathrm{RI}^{f}\left(a_{i 3}\right) \cdot \omega_{3} \quad R I^{f}\left(a_{i 4}\right) \cdot \omega_{4} \quad R I^{f}\left(a_{i 5}\right) \cdot \omega_{5} \quad R I^{f}\left(a_{i 6}\right) \cdot \omega_{6}\right)$

The FRB group utility index $\hat{S}^{f}$, FRB individual regret index $\hat{\mathrm{R}}^{f}$ and FRB compromise index $\hat{Q}^{f}$ are calculating by using Equations (36)-(38) as follows:

## Algorithm 2: for scenario 2: IF decision matrix, precise weights and fixed ideals

1. Steps 1 and 2 are same as Algorithm 1.
2. Find fpi-IFV and fni-IFV with respect to each criteria by using Equations (23) and (24), respectively.
3. Compute $D\left(a_{\hat{-} j}, a_{\hat{+} j}\right)$ and $D\left(a_{i j}, a_{\hat{+} j}\right)$ for each $1 \leq i \leq m$ and $1 \leq j \leq n$.
4. Compute $R I^{f}\left(a_{i j}\right)$ for each $1 \leq i \leq m$ and $1 \leq j \leq n$ by using Equation (27).
5. Find FRB group utility index $\hat{S}^{f}\left(y_{i}\right)$ and FRB individual regret index $\hat{\mathrm{R}}^{f}\left(y_{i}\right)$ by using Equations (36) and (37), respectively. Then compute the FRB compromise index $\hat{Q}^{f}\left(y_{i}\right)$ by using Equation (38).
6. Three ranking lists obtained by sorting the values of $\hat{\mathrm{S}}^{f}\left(y_{i}\right), \hat{\mathrm{R}}^{f}\left(y_{i}\right)$ and $\hat{\mathrm{Q}}^{f}\left(y_{i}\right)$ for each alternative in ascending order.
7. The alternative $y^{\prime}$ with the minimum value in $\hat{Q}^{f}\left(y_{i}\right)$ ranking list is the compromise solution if the following conditions are satisfied.

C1. Acceptable advantage:

$$
\hat{Q}^{f}\left(y^{\prime \prime}\right)-\hat{Q}^{f}\left(y^{\prime}\right) \geq \frac{1}{m-1},
$$

where $y^{\prime \prime}$ is the alternative have minimum value after $y^{\prime}$ in $\hat{Q}^{f}\left(y_{i}\right)$ ranking list.
C 2 . Acceptable stability in DM :
The alternative $y^{\prime}$ have the minimum values in $\hat{S}^{f}\left(y_{i}\right)$ and $\hat{\mathrm{R}}^{f}\left(y_{i}\right)$ ranking lists.
The set of the ultimate compromise solution is proposed if one of the above condition is not satisfied, which consists of:
a. Alternatives $y^{\prime}$ and $y^{\prime \prime}$ if only C 2 is not satisfied.
b. Alternatives $y^{\prime}, y^{\prime \prime}, \ldots, y^{p}$ if C 1 is not satisfied, where $p$ is the largest $i$ for which $\hat{\mathbf{Q}}^{f}\left(y^{p}\right)-\hat{\mathbf{Q}}^{f}\left(y^{\prime}\right)<\frac{1}{m-1}$

From Equation (54), the three ranking lists $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}, y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ and $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ are obtained by sorting each $\hat{S}^{f}\left(y_{i}\right), \hat{\mathrm{R}}^{f}\left(y_{i}\right)$ and $\hat{\mathrm{Q}}^{f}\left(y_{i}\right)$ value in ascending order, respectively. The hydro power among the renewable energy sources is the best option by all ranking lists. Moreover, $\hat{Q}^{f}\left(y^{\prime \prime}\right)-\hat{Q}^{f}\left(y^{\prime}\right)=\hat{Q}^{f}\left(y_{2}\right)-\hat{Q}^{f}\left(y_{4}\right)=0.20967 \leq \frac{1}{5-1}=\frac{1}{4}=0.25$. The first condition (acceptable advantage) in Step 8 of Algorithm 2 is not satisfied. Therefore, the ultimate compromise solution is proposed. The hydro power and wind energy are the ultimate compromise solutions of the selection of renewable energy sources problem. The order of the renewable energy sources is $\left\{y_{4}, y_{2}\right\} \succ y_{3} \succ y_{1} \succ y_{5}$.

### 5.3. Solution by Algorithm 3

We solve this problem by IF importance weights using Algorithm 3. The PIFS, given by the director in GIFSS are served as a IF importance weights. The dpi-IFVs $a_{\hat{+} j}$ and dni-IFVs $a_{\hat{-} j}(j=\{1,2, \ldots, 6\})$ have calculated in Equations (46) and (47), respectively. The distance between $a_{\hat{+} j}$ and $a_{\hat{-} j}$ have calculated in Equation (48). The displaced remoteness index $R I^{d}\left(a_{i j}\right)$ is calculated by using Definition 17 and the results are presented in Equation (55).
$y_{1}$
$y_{1}^{d}\left(a_{i 1}\right)$
$y_{2}$
$y_{3}$
$y_{4}$
$y_{5}$$\left(\begin{array}{cccccc}I^{d}\left(a_{i 2}\right) & R I^{d}\left(a_{i 3}\right) & R I^{d}\left(a_{i 4}\right) & R I^{d}\left(a_{i 5}\right) & R I^{d}\left(a_{i 6}\right) \\ 0.544331 & 0.648886 & 0.83887 & 0.23094 & 0.225494 & 0.57735 \\ 0.638285 & 0.39736 & 0.333333 & 0 . & 0.225494 & 0.5 \\ 0 . & 0 . & 0.333333 & 0.4 & 0.260378 & 0.25 \\ 1 . & 1 . & 0 . & 1 . & 0 . & 1 .\end{array}\right)$

The displaced remoteness indexes $R I^{d}\left(a_{i j}\right)$ are multiplied by IF importance weights $\omega_{j}$ by using Equation (40) and the results are presented in Equation (56) as follows:

|  | $R I^{d}\left(a_{i 1}\right) \cdot \omega_{1}$ | $R I^{d}\left(a_{i 2}\right) \cdot \omega_{2}$ | $R I^{d}\left(a_{i 3}\right) \cdot \omega_{3}$ | $R I^{d}\left(a_{i 4}\right) \cdot \omega_{4}$ | $R I^{d}\left(a_{i 5}\right) \cdot \omega_{5}$ | $R I^{d}\left(a_{i 6}\right) \cdot \omega_{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y_{1}$ | ( 0 . | 0.8 | 0.538346 | 0.250573 | 0.321385 | 0.333116 |
| $y_{2}$ | 0.413165 | 0.648076 | 0.6 | 0.390775 | 0.321385 | 0.296474 |
| $y_{3}$ | 0.460127 | 0.472457 | 0.268433 | 0. | 0.8 | 0. |
| $y_{4}$ | 0. | 0. | 0.268433 | 0.390775 | 0.360022 | 0.162309 |
| $y_{5}$ | ( 0.6 | 0.8 | 0. | 0.7 | 0. | 0.5 ) |

The DRB group utility index $\mathrm{S}^{d}\left(y_{i}\right)$, individual regret index $\mathrm{R}^{d}\left(y_{i}\right)$ and the compromise index $\mathrm{Q}^{d}\left(y_{i}\right)$ with a set of IF importance weights $\omega_{j}=\left(\bar{\omega}_{j}, \underline{\omega}_{j}\right)$ are calculated by using Equations (39)-(41). The results are summarized in Equation (57)

$$
\left.\begin{array}{c} 
 \tag{57}\\
\mathrm{S}^{d}\left(y_{i}\right) \\
\mathrm{R}^{d}\left(y_{i}\right) \\
\mathrm{Q}^{d}\left(y_{i}\right)
\end{array} \begin{array}{cccccc}
2.24342 & y_{2} & y_{3} & y_{4} & y_{5} & \text { Ranking } \\
0.8 & 0.648076 & 2.00102 & 1.18154 & 2.6 & y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2} \\
0.856734 & 0.814377 & 0.8 & 0.390775 & 0.8 & y_{4} \succ y_{2} \succ\left\{y_{3}, y_{5}, y_{1}\right\} \\
0.7753 & 0 . & 0.976526 & y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}
\end{array}\right)
$$

From Equation (57), the three ranking lists $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}, y_{4} \succ y_{2} \succ\left\{y_{3}, y_{5}, y_{1}\right\}$ and $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ are obtained by sorting each $\mathrm{S}^{d}\left(y_{i}\right), \mathrm{R}^{d}\left(y_{i}\right)$ and $\mathrm{Q}^{d}\left(y_{i}\right)$ value in ascending order, respectively. The hydro power among the renewable energy sources is the best option by all ranking lists. Moreover, $\mathrm{Q}^{d}\left(y^{\prime \prime}\right)-\mathrm{Q}^{d}\left(y^{\prime}\right)=\mathrm{Q}^{d}\left(y_{2}\right)-\mathrm{Q}^{d}\left(y_{4}\right)=0.7753 \geq \frac{1}{5-1}=\frac{1}{4}=0.25$. Both the conditions in step 8 of Algorithm 3 are satisfied for hydro power. The conditions of acceptable advantage and acceptable stability are fulfilled. Therefore, hydro power is the compromise solution of the selection of renewable energy source problem. The order of the renewable energy sources is $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$.

### 5.4. Solution by Algorithm 4

We solve this problem by IF importance weights using Algorithm 4. The PIFS, given by the director in GIFSS are served as a IF importance weights. Since all the criterion are of benefit type, therefore, the fpi-IFV $a_{\hat{+} j}=(1,0)$ and the fni-IFV $a_{\hat{-j}_{j}}=(0,1) \forall j=\{1,2, \ldots, 6\}$. The distance between $a_{i j}$ and $a_{\hat{+} j}$ have calculated in Equation (52). By Definition 18, the fixed remoteness index is equal to the distance, i.e., $R I^{f}\left(a_{i j}\right)=D_{e}^{\circ}\left(a_{i j}, a_{\hat{+} j}\right)$. The result of multiplication of fixed remoteness indexes $R I^{f}\left(a_{i j}\right)$ and IF importance weights $\omega_{j}$ are summarized in Equation (58).

Algorithm 3: for scenario 3: IF decision matrix, IF importance weights and displaced ideals

1. Let $X=\left\{x_{1}, x_{2}, \ldots, x_{m}\right\}$ represents the alternatives and the set $C=\left\{e_{1}, e_{2}, \ldots, e_{n}\right\}$ represents the criteria.
2. The IFV is given to each alternatives $y_{i}, 1 \leq i \leq m$ according to the each criteria $e_{j}$, $1 \leq j \leq n$ and identified the IF importance weights by evaluating some data or selecting the appropriate linguistics variables. Which generates the IF decision matrix $a=\left[a_{i j}\right]_{m \times n}$. One can obtained weights of attributes by using the above mentioned procedures (Section 4.3).
3. Steps 3-5 are same as Algorithm 1.
4. Find DRB group utility index $S^{d}\left(y_{i}\right)$ and DRB individual regret index $R^{d}\left(y_{i}\right)$ by using Equations (39) and (40), respectively. Then compute the DRB compromise index $Q^{d}\left(y_{i}\right)$ by using Equation (41).
5. Three ranking lists obtained by sorting the values of $S^{d}\left(y_{i}\right), \mathrm{R}^{d}\left(y_{i}\right)$ and $\mathrm{Q}^{d}\left(y_{i}\right)$ for each alternative in ascending order.
6. The alternative $y^{\prime}$ with the minimum value in $\mathrm{Q}^{d}\left(y_{i}\right)$ ranking list is the compromise solution if the following conditions are satisfied.

C1. Acceptable advantage:

$$
Q^{d}\left(y^{\prime \prime}\right)-Q^{d}\left(y^{\prime}\right) \geq \frac{1}{m-1}
$$

where $y^{\prime \prime}$ is the alternative have minimum value after $y^{\prime}$ in $Q^{d}\left(y_{i}\right)$ ranking list.
C2. Acceptable stability in DM:
The alternative $y^{\prime}$ have the minimum values in $\mathrm{S}^{d}\left(y_{i}\right)$ and $\mathrm{R}^{d}\left(y_{i}\right)$ ranking lists.
The set of the ultimate compromise solution is proposed if one of the above condition is not satisfied, which consists of:
a. Alternatives $y^{\prime}$ and $y^{\prime \prime}$ if only C 2 is not satisfied.
b. Alternatives $y^{\prime}, y^{\prime \prime}, \ldots, y^{p}$ if C 1 is not satisfied, where $p$ is the largest $i$ for which $\mathrm{Q}^{d}\left(y^{p}\right)-\mathrm{Q}^{d}\left(y^{\prime}\right)<\frac{1}{m-1}$

|  | $R I^{f}\left(a_{i 1}\right) \cdot \omega_{1}$ | $R I^{f}\left(a_{i 2}\right) \cdot \omega_{2}$ | $R I^{f}\left(a_{i 3}\right) \cdot \omega_{3}$ | RIf ${ }^{f}\left(a_{i 4}\right) . \omega_{4}$ | $R I^{f}\left(a_{i 5}\right) \cdot \omega_{5}$ | $R I^{f}\left(a_{i 6}\right) \cdot \omega_{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y_{1}$ | ( 0.18617 | 0.47469 | 0.43582 | 0.31856 | 0.511756 | 0.228491 |
| $y_{2}$ | 0.32975 | 0.39108 | 0.45932 | 0.48742 | 0.492046 | 0.298895 |
| $y_{3}$ | 0.30877 | 0.27522 | 0.32738 | 0.31856 | 0.734976 | 0.19586 |
| $y_{4}$ | 0.18617 | 0.17890 | 0.31217 | 0.463387 | 0.409114 | 0.245831 |
| $y_{5}$ | ( 0.41253 | 0.49481 | 0.25119 | 0.642616 | 0.409114 | 0.400086 |

The FRB group utility index $\mathrm{S}^{f}$, FRB individual regret index $\mathrm{R}^{f}$ and $\operatorname{FRB}$ compromise index $\mathrm{Q}^{f}$ are calculating by using Equations (42)-(44) as follows:

$$
\left.\begin{array}{l} 
 \tag{59}\\
\mathrm{S}^{f}\left(y_{i}\right) \\
\mathrm{R}^{f}\left(y_{i}\right) \\
\mathrm{Q}^{f}\left(y_{i}\right)
\end{array} \begin{array}{cccccc}
y_{1} & y_{2} & y_{3} & y_{4} & y_{5} & \text { Ranking } \\
0.15549 & 2.4585 & 2.16077 & 1.79558 & 2.61035 & y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5} \\
0.511756 & 0.492046 & 0.734976 & 0.463387 & 0.642616 & y_{4} \succ y_{2} \succ y_{1} \succ y_{5} \succ y_{3} \\
0.309915 & 0.459576 & 0.724108 & 0 . & 0.829963 & y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}
\end{array}\right)
$$

From Equation (59), the three ranking lists $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}, y_{4} \succ y_{2} \succ y_{1} \succ y_{5} \succ y_{3}$ and $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ are obtained by sorting each $\mathrm{S}^{f}\left(y_{i}\right), \mathrm{R}^{f}\left(y_{i}\right)$ and $\mathrm{Q}^{f}\left(y_{i}\right)$ value in ascending order, respectively. The hydro power among the renewable energy sources is the best option by all ranking lists. Moreover, $Q^{f}\left(y^{\prime \prime}\right)-Q^{f}\left(y^{\prime}\right)=Q^{f}\left(y_{2}\right)-Q^{f}\left(y_{4}\right)=0.309915 \geq \frac{1}{5-1}=\frac{1}{4}=0.25$. Both the conditions (acceptable advantage and acceptable stability) in Step 8 of Algorithm 4 are satisfied for
hydro power. Therefore, hydro power is the compromise solution of the selection of renewable energy source problem. The order of the renewable energy sources is $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$.

The Table 7 shows the ranking results obtained by the above proposed four algorithms. The values of the parameters $p$ and $\lambda$ are mentioned in the table. The effect of the parameters $p$ and $\lambda$ on the ranking of alternatives will be discussed in the sensitivity analysis section.

Table 7. Summary of the Results.

| Algorithm | $\boldsymbol{p}$ | $\boldsymbol{\lambda}$ | Weights | Developed Ranking |
| :---: | :---: | :---: | :---: | :---: |
| Algorithm 1 | 2 | 0.5 | Precise | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ |
| Algorithm 2 | 2 | 0.5 | Precise | $\left\{y_{4}, y_{2}\right\} \succ y_{3} \succ y_{1} \succ y_{5}$ |
| Algorithm 3 | 2 | 0.5 | IF Importance | $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ |
| Algorithm 4 | 2 | 0.5 | IF Importance | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |

```
Algorithm 4: for scenario 4: IF decision matrix, IF importance weights and fixed ideals
    1. Steps 1 and 2 are same as Algorithm 3.
    3. Steps 3-5 are same as Algorithm 2.
    6. Find FRB group utility index \(\mathrm{S}^{f}\left(y_{i}\right)\) and FRB individual regret index \(\mathrm{R}^{f}\left(y_{i}\right)\) by using
        Equations (42) and (43), respectively. Then compute the FRB compromise index \(Q^{f}\left(y_{i}\right)\) by
        using Equation (44).
    7. Three ranking lists obtained by sorting the values of \(\mathrm{S}^{f}\left(y_{i}\right), \mathrm{R}^{f}\left(y_{i}\right)\) and \(\mathrm{Q}^{f}\left(y_{i}\right)\) for each
        alternative in ascending order.
    8. The alternative \(y^{\prime}\) with the minimum value in \(\mathrm{Q}^{f}\left(y_{i}\right)\) ranking list is the compromise
        solution if the following conditions are satisfied.
        C1. Acceptable advantage:
    \(Q^{f}\left(y^{\prime \prime}\right)-Q^{f}\left(y^{\prime}\right) \geq \frac{1}{m-1}\),
    where \(y^{\prime \prime}\) is the alternative have minimum value after \(y^{\prime}\) in \(Q^{f}\left(y_{i}\right)\) ranking list.
    C 2 . Acceptable stability in DM :
    The alternative \(y^{\prime}\) have the minimum values in \(\mathrm{S}^{f}\left(y_{i}\right)\) and \(\mathrm{R}^{f}\left(y_{i}\right)\) ranking lists.
    The set of the ultimate compromise solution is proposed if one of the above condition
    is not satisfied, which consists of:
    a. Alternatives \(y^{\prime}\) and \(y^{\prime \prime}\) if only C 2 is not satisfied.
    b. Alternatives \(y^{\prime}, y^{\prime \prime}, \ldots, y^{p}\) if C 1 is not satisfied, where \(p\) is the largest \(i\) for which
    \(\mathrm{Q}^{f}\left(y^{p}\right)-\mathrm{Q}^{f}\left(y^{\prime}\right)<\frac{1}{m-1}\)
```


### 5.5. Stability Analysis

The effect of parameter $p$ and $\lambda$ is discussed in this section. Table 8 shows the calculations of Algorithms 1 and 2, while Table 9 shows the calculations of Algorithms 3 and 4. Since higher values of the parameter $\lambda$ represents the majority of group utility and when the values are changed from 0 to 1 the maximum group utility is obtained. On the other hand, when the value of parameter $\lambda$ decreases from 1 to 0 , the individual regret get importance. The ranking of alternatives varying when the values of $\lambda$ changes but the alternative $y_{4}$ remain the best option from alternatives. Also, for different values of the parameter $p$ are used for Algorithms 1-4.

The Algorithms 1 and 3 used the displaced terminologies and concepts while Algorithms 2 and 4 based on the fixed terminologies and concepts. The Algorithms 1 and 3 are more sensitive as compared
to the Algorithms 2 and 4, i.e., the ranking of the alternatives changes rapidly by changing the values of the parameter $\lambda$. For more details, we refer to Tables 8 and 9 .

Table 8. Stability Analysis with Different Values of Parameters $p$ and $\lambda$ for the Evaluation of Renewable Energy Sources.

|  |  | $\hat{Q}^{\text {d }}$ |  |  |  |  | Ranking | $\hat{\mathbf{Q}}^{\text {f }}$ |  |  |  |  | Ranking |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $p$ | $\lambda$ | $y_{1}$ | $y_{2}$ | $y_{3}$ | $y_{4}$ | $y_{5}$ |  | $y_{1}$ | $y_{2}$ | $y_{3}$ | $y_{4}$ | $y_{5}$ |  |
| 1 | 0.0 | 1. | 0.618 | 0.745 | 0. | 0.745 | $y_{4} \succ y_{2} \succ\left\{y_{3}, y_{5}\right\} \succ y_{1}$ | 0.081 | 0.081 | 1. | 0. | 0.724 | $y_{4} \succ\left\{y_{1}, y_{2}\right\} \succ y_{5} \succ y_{3}$ |
|  | 0.1 | 0.963 | 0.656 | 0.726 | 0. | 0.769 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ | 0.108 | 0.159 | 0.935 | 0. | 0.752 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.2 | 0.926 | 0.694 | 0.707 | 0. | 0.792 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ | 0.135 | 0.238 | 0.871 | 0. | 0.779 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.3 | 0.889 | 0.732 | 0.689 | 0. | 0.815 | $y_{4} \succ y_{3} \succ y_{2} \succ y_{5} \succ y_{1}$ | 0.162 | 0.317 | 0.806 | 0. | 0.807 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.4 | 0.852 | 0.771 | 0.67 | 0. | 0.838 | $y_{4} \succ y_{3} \succ y_{2} \succ y_{5} \succ y_{1}$ | 0.189 | 0.396 | 0.741 | 0. | 0.834 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.5 | 0.815 | 0.809 | 0.651 | 0. | 0.862 | $y_{4} \succ y_{3} \succ y_{2} \succ y_{1} \succ y_{5}$ | 0.217 | 0.474 | 0.676 | 0. | 0.862 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.6 | 0.779 | 0.847 | 0.632 | 0. | 0.885 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.244 | 0.553 | 0.612 | 0. | 0.89 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.7 | 0.742 | 0.885 | 0.613 | 0. | 0.908 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.271 | 0.632 | 0.547 | 0. | 0.917 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 0.8 | 0.705 | 0.924 | 0.594 | 0. | 0.932 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.298 | 0.711 | 0.482 | 0. | 0.945 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 0.9 | 0.668 | 0.962 | 0.575 | 0. | 0.955 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.325 | 0.79 | 0.417 | 0. | 0.972 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 1.0 | 0.631 | 1. | 0.556 | 0. | 0.978 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.353 | 0.868 | 0.353 | 0. | 1. | $y_{4} \succ\left\{y_{1}, y_{3}\right\} \succ y_{2} \succ y_{5}$ |
| 2 | 0.0 | 1. | 0.615 | 1. | 0. | 1. | $y_{4} \succ y_{2} \succ\left\{y_{3}, y_{1}, y_{5}\right\}$ | 0.055 | 0.15 | 1. | 0. | 0.811 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.1 | 0.963 | 0.629 | 0.95 | 0. | 1. | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.086 | 0.208 | 0.95 | 0. | 0.83 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.2 | 0.925 | 0.643 | 0.899 | 0. | 1. | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.117 | 0.265 | 0.899 | 0. | 0.849 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.3 | 0.888 | 0.656 | 0.849 | 0. | 1. | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.148 | 0.323 | 0.849 | 0. | 0.868 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.4 | 0.851 | 0.67 | 0.799 | 0. | 1. | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.179 | 0.38 | 0.799 | 0. | 0.887 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.5 | 0.814 | 0.683 | 0.748 | 0. | 1. | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.21 | 0.438 | 0.748 | 0. | 0.906 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.6 | 0.776 | 0.697 | 0.698 | 0. | 1. | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.241 | 0.495 | 0.698 | 0. | 0.924 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.7 | 0.739 | 0.711 | 0.648 | 0. | 1. | $y_{4} \succ y_{3} \succ y_{2} \succ y_{1} \succ y_{5}$ | 0.271 | 0.553 | 0.648 | 0. | 0.943 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.8 | 0.702 | 0.724 | 0.597 | 0. | 1. | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.302 | 0.61 | 0.598 | 0. | 0.962 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 0.9 | 0.665 | 0.738 | 0.547 | 0. | 1. | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.333 | 0.668 | 0.547 | 0. | 0.981 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 1.0 | 0.627 | 0.752 | 0.497 | 0. | 1. | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.364 | 0.725 | 0.497 | 0. | 1. | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
| 3 | 0.0 | 0.801 | 0.574 | 1. | 0. | 1. | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.061 | 0.175 | 1. | 0. | 0.833 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.1 | 0.806 | 0.617 | 0.966 | 0. | 0.98 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.091 | 0.227 | 0.951 | 0. | 0.85 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.2 | 0.811 | 0.659 | 0.932 | 0. | 0.961 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.122 | 0.279 | 0.903 | 0. | 0.866 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.3 | 0.816 | 0.702 | 0.898 | 0. | 0.941 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.152 | 0.331 | 0.854 | 0. | 0.883 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.4 | 0.821 | 0.744 | 0.864 | 0. | 0.922 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.182 | 0.383 | 0.805 | 0. | 0.9 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.5 | 0.826 | 0.787 | 0.83 | 0. | 0.902 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.212 | 0.435 | 0.757 | 0. | 0.917 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.6 | 0.83 | 0.83 | 0.795 | 0. | 0.883 | $y_{4} \succ y_{3} \succ\left\{y_{1}, y_{2}\right\} \succ y_{5}$ | 0.242 | 0.487 | 0.708 | 0. | 0.933 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.7 | 0.835 | 0.872 | 0.761 | 0. | 0.863 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.272 | 0.538 | 0.659 | 0. | 0.95 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.8 | 0.84 | 0.915 | 0.727 | 0. | 0.844 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.303 | 0.59 | 0.61 | 0. | 0.967 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.9 | 0.845 | 0.957 | 0.693 | 0. | 0.824 | $y_{4} \succ y_{3} \succ\left\{y_{1}, y_{5}\right\} \succ y_{2}$ | 0.333 | 0.642 | 0.562 | 0. | 0.983 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 1.0 | 0.85 | 1. | 0.659 | 0. | 0.805 | $y_{4} \succ y_{3} \succ y_{5} \succ y_{1} \succ y_{2}$ | 0.363 | 0.694 | 0.513 | 0. | 1. | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
| 5 | 0.0 | 0.787 | 0. | 1. | 0.176 | 1. | $y_{2} \succ y_{4} \succ y_{1} \succ\left\{y_{3}, y_{5}\right\}$ | 0.056 | 0.147 | 1. | 0. | 0.806 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.1 | 0.79 | 0.1 | 0.932 | 0.159 | 0.933 | $y_{2} \succ y_{4} \succ y_{1} \succ y_{3} \succ y_{5}$ | 0.087 | 0.205 | 0.95 | 0. | 0.825 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.2 | 0.793 | 0.2 | 0.864 | 0.141 | 0.867 | $y_{4} \succ y_{2} \succ y_{1} \succ y_{3} \succ y_{5}$ | 0.118 | 0.262 | 0.899 | 0. | 0.845 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.3 | 0.796 | 0.3 | 0.796 | 0.123 | 0.8 | $y_{4} \succ y_{2} \succ\left\{y_{1}, y_{3}\right\} \succ y_{5}$ | 0.149 | 0.32 | 0.849 | 0. | 0.864 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.4 | 0.799 | 0.4 | 0.728 | 0.106 | 0.734 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ | 0.18 | 0.377 | 0.798 | 0. | 0.884 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.5 | 0.802 | 0.5 | 0.66 | 0.0881 | 0.667 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ | 0.212 | 0.435 | 0.748 | 0. | 0.903 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.6 | 0.805 | 0.6 | 0.592 | 0.0705 | 0.601 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ | 0.243 | 0.492 | 0.698 | 0. | 0.922 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.7 | 0.809 | 0.7 | 0.524 | 0.0528 | 0.534 | $y_{4} \succ y_{3} \succ y_{5} \succ y_{2} \succ y_{1}$ | 0.274 | 0.55 | 0.647 | 0. | 0.942 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.8 | 0.812 | 0.8 | 0.456 | 0.0352 | 0.467 | $y_{4} \succ y_{3} \succ y_{5} \succ y_{2} \succ y_{1}$ | 0.305 | 0.607 | 0.597 | 0. | 0.961 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.9 | 0.815 | 0.9 | 0.389 | 0.0176 | 0.401 | $y_{4} \succ y_{3} \succ y_{5} \succ y_{2} \succ y_{1}$ | 0.336 | 0.665 | 0.546 | 0. | 0.981 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 1.0 | 0.818 | 1. | 0.321 | 0. | 0.334 | $y_{4} \succ y_{3} \succ y_{5} \succ y_{2} \succ y_{1}$ | 0.367 | 0.722 | 0.496 | 0. | 1. | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
| 10 | 0.0 | 0.998 | 0.604 | 1. | 0. | 1. | $y_{4} \succ y_{2} \succ y_{1} \succ\left\{y_{3}, y_{5}\right\}$ | 0.045 | 0.068 | 1. | 0. | 0.671 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.1 | 0.976 | 0.644 | 0.96 | 0. | 0.979 | $y_{4} \succ y_{2} \succ y_{1} \succ y_{3} \succ y_{5}$ | 0.076 | 0.135 | 0.952 | 0. | 0.704 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.2 | 0.955 | 0.683 | 0.92 | 0. | 0.957 | $y_{4} \succ y_{2} \succ y_{1} \succ y_{3} \succ y_{5}$ | 0.108 | 0.202 | 0.904 | 0. | 0.737 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.3 | 0.933 | 0.723 | 0.88 | 0. | 0.936 | $y_{4} \succ y_{2} \succ y_{1} \succ y_{3} \succ y_{5}$ | 0.139 | 0.27 | 0.856 | 0. | 0.77 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.4 | 0.911 | 0.763 | 0.84 | 0. | 0.914 | $y_{4} \succ y_{2} \succ y_{1} \succ y_{3} \succ y_{5}$ | 0.17 | 0.337 | 0.808 | 0. | 0.803 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.5 | 0.889 | 0.802 | 0.8 | 0. | 0.893 | $y_{4} \succ y_{3} \succ y_{2} \succ y_{1} \succ y_{5}$ | 0.202 | 0.404 | 0.76 | 0. | 0.835 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.6 | 0.867 | 0.842 | 0.76 | 0. | 0.871 | $y_{4} \succ y_{3} \succ y_{2} \succ y_{1} \succ y_{5}$ | 0.233 | 0.471 | 0.712 | 0. | 0.868 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.7 | 0.845 | 0.881 | 0.72 | 0. | 0.85 | $y_{4} \succ y_{3} \succ y_{2} \succ y_{1} \succ y_{5}$ | 0.264 | 0.538 | 0.664 | 0. | 0.901 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.8 | 0.823 | 0.921 | 0.68 | 0. | 0.829 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.296 | 0.605 | 0.616 | 0. | 0.934 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.9 | 0.802 | 0.96 | 0.64 | 0. | 0.807 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.327 | 0.673 | 0.567 | 0. | 0.967 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 1.0 | 0.78 | 1. | 0.6 | 0. | 0.786 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.358 | 0.74 | 0.519 | 0. | 1. | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |

Table 9. Stability Analysis with Different Values of Parameters $p$ and $\lambda$ for the Evaluation of Renewable Energy Sources.

| $p$ | $\lambda$ | $Q^{\text {d }}$ |  |  |  |  | Ranking | Q ${ }^{\text {f }}$ |  |  |  |  | Ranking |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $y_{1}$ | $y_{2}$ | $y_{3}$ | $y_{4}$ | $y_{5}$ |  | $y_{1}$ | $y_{2}$ | $y_{3}$ | $y_{4}$ | $y_{5}$ |  |
| 1 | 0.0 | 1. | 0.788 | 0.872 | 0. | 0.872 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ | 0.299 | 0.336 | 1. | 0. | 0.641 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.1 | 0.953 | 0.81 | 0.836 | 0. | 0.862 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ | 0.314 | 0.401 | 0.922 | 0. | 0.677 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.2 | 0.906 | 0.831 | 0.8 | 0. | 0.852 | $y_{4} \succ y_{3} \succ y_{2} \succ y_{5} \succ y_{1}$ | 0.329 | 0.467 | 0.845 | 0. | 0.713 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.3 | 0.86 | 0.852 | 0.764 | 0. | 0.843 | $y_{4} \succ y_{3} \succ y_{5} \succ y_{2} \succ y_{1}$ | 0.343 | 0.532 | 0.767 | 0. | 0.749 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.4 | 0.813 | 0.873 | 0.727 | 0. | 0.833 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.358 | 0.597 | 0.689 | 0. | 0.785 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.5 | 0.766 | 0.894 | 0.691 | 0. | 0.823 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.372 | 0.663 | 0.612 | 0. | 0.82 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.6 | 0.719 | 0.915 | 0.655 | 0. | 0.813 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.387 | 0.728 | 0.534 | 0. | 0.856 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.7 | 0.673 | 0.937 | 0.619 | 0. | 0.803 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.401 | 0.793 | 0.456 | 0. | 0.892 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.8 | 0.626 | 0.958 | 0.583 | 0. | 0.794 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.416 | 0.858 | 0.379 | 0. | 0.928 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ |
|  | 0.9 | 0.579 | 0.979 | 0.547 | 0. | 0.784 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.43 | 0.924 | 0.301 | 0. | 0.964 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ |
|  | 1.0 | 0.532 | 1. | 0.511 | 0. | 0.774 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.445 | 0.989 | 0.223 | 0. | 1. | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ |
| 2 | 0.0 | 1. | 0.629 | 1. | 0. | 1. | $y_{4} \succ y_{2} \succ\left\{y_{3}, y_{5}, y_{1}\right\}$ | 0.178 | 0.106 | 1. | 0. | 0.66 | $y_{4} \succ y_{2} \succ y_{1} \succ y_{5} \succ y_{3}$ |
|  | 0.1 | 0.971 | 0.666 | 0.955 | 0. | 0.995 | $y_{4} \succ y_{2} \succ\left\{y_{3}, y_{5}\right\} \succ y_{1}$ | 0.204 | 0.176 | 0.945 | 0. | 0.694 | $y_{4} \succ y_{2} \succ y_{1} \succ y_{5} \succ y_{3}$ |
|  | 0.2 | 0.943 | 0.703 | 0.91 | 0. | 0.991 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ | 0.231 | 0.247 | 0.89 | 0. | 0.728 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.3 | 0.914 | 0.74 | 0.865 | 0. | 0.986 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.257 | 0.318 | 0.834 | 0. | 0.762 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.4 | 0.885 | 0.777 | 0.82 | 0. | 0.981 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.284 | 0.389 | 0.779 | 0. | 0.796 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.5 | 0.857 | 0.814 | 0.775 | 0. | 0.977 | $y_{4} \succ y_{3} \succ y_{2} \succ y_{1} \succ y_{5}$ | 0.31 | 0.46 | 0.724 | 0. | 0.83 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.6 | 0.828 | 0.852 | 0.73 | 0. | 0.972 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.336 | 0.53 | 0.669 | 0. | 0.864 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.7 | 0.799 | 0.889 | 0.685 | 0. | 0.967 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.363 | 0.601 | 0.614 | 0. | 0.898 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.8 | 0.771 | 0.926 | 0.64 | 0. | 0.962 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.389 | 0.672 | 0.559 | 0. | 0.932 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 0.9 | 0.742 | 0.963 | 0.596 | 0. | 0.958 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.415 | 0.743 | 0.503 | 0. | 0.966 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 1.0 | 0.713 | 1. | 0.551 | 0. | 0.953 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.442 | 0.814 | 0.448 | 0. | 1. | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
| 3 | 0.0 | 0.861 | 0.403 | 1. | 0. | 1. | $y_{4} \succ y_{2} \succ y_{1} \succ\left\{y_{3}, y_{5}\right\}$ | 0.189 | 0.106 | 1. | 0. | 0.666 | $y_{4} \succ y_{2} \succ y_{1} \succ y_{5} \succ y_{3}$ |
|  | 0.1 | 0.849 | 0.463 | 0.952 | 0. | 0.995 | $y_{4} \succ y_{2} \succ y_{1} \succ y_{3} \succ y_{5}$ | 0.214 | 0.172 | 0.948 | 0. | 0.7 | $y_{4} \succ y_{2} \succ y_{1} \succ y_{5} \succ y_{3}$ |
|  | 0.2 | 0.837 | 0.523 | 0.904 | 0. | 0.99 | $y_{4} \succ y_{2} \succ y_{1} \succ y_{3} \succ y_{5}$ | 0.238 | 0.239 | 0.896 | 0. | 0.733 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.3 | 0.825 | 0.582 | 0.856 | 0. | 0.985 | $y_{4} \succ y_{2} \succ y_{1} \succ y_{3} \succ y_{5}$ | 0.263 | 0.305 | 0.844 | 0. | 0.766 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.4 | 0.813 | 0.642 | 0.808 | 0. | 0.979 | $y_{4} \succ y_{2} \succ y_{1} \succ y_{3} \succ y_{5}$ | 0.287 | 0.371 | 0.792 | 0. | 0.8 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.5 | 0.801 | 0.702 | 0.76 | 0. | 0.974 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.312 | 0.438 | 0.739 | 0. | 0.833 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.6 | 0.789 | 0.761 | 0.713 | 0. | 0.969 | $y_{4} \succ y_{3} \succ y_{2} \succ y_{1} \succ y_{5}$ | 0.336 | 0.504 | 0.687 | 0. | 0.866 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.7 | 0.777 | 0.821 | 0.665 | 0. | 0.964 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.361 | 0.571 | 0.635 | 0. | 0.9 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.8 | 0.765 | 0.881 | 0.617 | 0. | 0.959 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.385 | 0.637 | 0.583 | 0. | 0.933 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 0.9 | 0.753 | 0.94 | 0.569 | 0. | 0.954 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.41 | 0.704 | 0.531 | 0. | 0.967 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 1.0 | 0.741 | 1. | 0.521 | 0. | 0.949 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.434 | 0.77 | 0.479 | 0. | 1. | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
| 5 | 0.0 | 0.813 | 0. | 1. | 0.263 | 1. | $y_{2} \succ y_{4} \succ y_{1} \succ\left\{y_{3}, y_{5}\right\}$ | 0.173 | 0.0949 | 1. | 0. | 0.647 | $y_{4} \succ y_{2} \succ y_{1} \succ y_{5} \succ y_{3}$ |
|  | 0.1 | 0.802 | 0.1 | 0.924 | 0.237 | 0.959 | $y_{2} \succ y_{4} \succ y_{1} \succ y_{3} \succ y_{5}$ | 0.2 | 0.166 | 0.946 | 0. | 0.682 | $y_{4} \succ y_{2} \succ y_{1} \succ y_{5} \succ y_{3}$ |
|  | 0.2 | 0.79 | 0.2 | 0.848 | 0.211 | 0.919 | $y_{2} \succ y_{4} \succ y_{1} \succ y_{3} \succ y_{5}$ | 0.227 | 0.237 | 0.892 | 0. | 0.717 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.3 | 0.779 | 0.3 | 0.772 | 0.184 | 0.878 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.254 | 0.307 | 0.838 | 0. | 0.753 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.4 | 0.768 | 0.4 | 0.696 | 0.158 | 0.837 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.28 | 0.378 | 0.783 | 0. | 0.788 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.5 | 0.756 | 0.5 | 0.62 | 0.132 | 0.796 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.307 | 0.449 | 0.729 | 0. | 0.823 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.6 | 0.745 | 0.6 | 0.545 | 0.105 | 0.756 | $y_{4} \succ y_{3} \succ y_{2} \succ y_{1} \succ y_{5}$ | 0.334 | 0.52 | 0.675 | 0. | 0.859 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.7 | 0.733 | 0.7 | 0.469 | 0.079 | 0.715 | $y_{4} \succ y_{3} \succ y_{2} \succ y_{5} \succ y_{1}$ | 0.361 | 0.591 | 0.621 | 0. | 0.894 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.8 | 0.722 | 0.8 | 0.393 | 0.0527 | 0.674 | $y_{4} \succ y_{3} \succ y_{2} \succ y_{5} \succ y_{1}$ | 0.388 | 0.661 | 0.567 | 0. | 0.929 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 0.9 | 0.71 | 0.9 | 0.317 | 0.0263 | 0.633 | $y_{4} \succ y_{3} \succ y_{5} \succ y_{1} \succ y_{2}$ | 0.415 | 0.732 | 0.513 | 0. | 0.965 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 1.0 | 0.699 | 1. | 0.241 | 0. | 0.593 | $y_{4} \succ y_{3} \succ y_{5} \succ y_{1} \succ y_{2}$ | 0.442 | 0.803 | 0.459 | 0. | 1. | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
| 10 | 0.0 | 1. | 0.692 | 0.925 | 0. | 0.925 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ | 0.128 | 0.128 | 1. | 0. | 0.605 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.1 | 0.966 | 0.723 | 0.885 | 0. | 0.922 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ | 0.159 | 0.199 | 0.945 | 0. | 0.644 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.2 | 0.932 | 0.754 | 0.844 | 0. | 0.918 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ | 0.19 | 0.269 | 0.89 | 0. | 0.684 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.3 | 0.899 | 0.784 | 0.803 | 0. | 0.914 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.222 | 0.339 | 0.835 | 0. | 0.723 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.4 | 0.865 | 0.815 | 0.763 | 0. | 0.91 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ | 0.253 | 0.41 | 0.78 | 0. | 0.763 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{5} \succ y_{3}$ |
|  | 0.5 | 0.831 | 0.846 | 0.722 | 0. | 0.907 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.284 | 0.48 | 0.725 | 0. | 0.802 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.6 | 0.797 | 0.877 | 0.682 | 0. | 0.903 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.315 | 0.55 | 0.67 | 0. | 0.842 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |
|  | 0.7 | 0.764 | 0.908 | 0.641 | 0. | 0.899 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ | 0.347 | 0.621 | 0.615 | 0. | 0.881 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 0.8 | 0.73 | 0.938 | 0.601 | 0. | 0.896 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.378 | 0.691 | 0.56 | 0. | 0.921 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 0.9 | 0.696 | 0.969 | 0.56 | 0. | 0.892 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.409 | 0.762 | 0.505 | 0. | 0.96 | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
|  | 1.0 | 0.662 | 1. | 0.519 | 0. | 0.888 | $y_{4} \succ y_{3} \succ y_{1} \succ y_{5} \succ y_{2}$ | 0.44 | 0.832 | 0.45 | 0. | 1. | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |

## 6. Comparison Analysis

In this section, we compared the results of our proposed methods with the existing methods.
First, we compare our methods with Feng et al. [14]. Feng et al. [14] proposed the method to solve MADM problems using GIFSS. He used the Xu's weighted averaging operator for IFVs to aggregate the information [50]. The ranking of the alternatives for solving the problem of renewable energy source selection with Feng's model is $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$.

Khan et al. [46] proposed a method to solve the MADM problems by using a soft discernibility matrix for GIFSSs. The ranking of the alternatives for solving the problem of renewable energy source selection with Khan's model is $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$. In both approaches, the alternative $y_{4}$, i.e., the hydro-power remains the best alternative for renewable energy sources. When we solve the renewable energy source selection problem with proposed algorithms, we also obtained the $y_{4}$ as the best option. But the rankings obtained by using the Algorithms 1-4 are different from the proposed methods. The reasons are to choose the different ideal values and weights.

The Table 10 shows the comparison of the proposed method with different already proposed methods. The alternative $y_{4}$ obtained the best option from all methods. The rankings obtained from the proposed methods are slightly different from the already proposed methods.

Table 10. Comparison Analysis.

| Method | Operator/Method Used | Developed Ranking |
| :---: | :---: | :---: |
| Feng et al. [14] | Extended Intersection, IFWA | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
| Khan et al. [46] | Soft Discernibility Matrix | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ |
| Xu [50] | IFWA Operator | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ |
| Xu and Yager [51] | IFGW Operator | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
| Wang and Liu [55] | IFWA Einstein Operator | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ |
| Zhao et al. [56] | GIFWA Operator | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ |
| Garg [57] | PFEWA Operator | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ |
| Yager [58] | PFWA Operator | $y_{4} \succ y_{3} \succ y_{1} \succ y_{2} \succ y_{5}$ |
| Yager [58] | PFWG Operator | $y_{4} \succ y_{1} \succ y_{3} \succ y_{2} \succ y_{5}$ |
| Proposed VIKOR | Algorithm 1 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{1} \succ y_{5}$ |
| Proposed VIKOR | Algorithm 2 | $\left\{y_{4}, y_{2}\right\} \succ y_{3} \succ y_{1} \succ y_{5}$ |
| Proposed VIKOR | Algorithm 3 | $y_{4} \succ y_{2} \succ y_{3} \succ y_{5} \succ y_{1}$ |
| Proposed VIKOR | Algorithm 4 | $y_{4} \succ y_{1} \succ y_{2} \succ y_{3} \succ y_{5}$ |

We obtained the same order by using Algorithm 2 as References [14,51,58] for $p=1, \lambda=0.7-1.0$, $p=2, \lambda=0.8-1.0, p=3, \lambda=0.9-1.0, p=5, \lambda=0.9-1.0$ and $p=10, \lambda=0.9-1.0$ (Table 8). Similarly, we obtained the same order by using Algorithm 4 for $p=2, \lambda=0.8-1.0$, $p=3, \lambda=0.9-1.0, p=5, \lambda=0.8-1.0$ and $p=10, \lambda=0.8-1.0$ (Table 9).

Also, we obtained the same order by using Algorithm 1 as References [46,50,55-58] for $p=$ $1, \lambda=0.6-0.8, p=2, \lambda=0.8-1.0$ and $p=3, \lambda=0.6$ (Table 8 ). Similarly, by using Algorithm 3 for $p=1, \lambda=0.6, p=3, \lambda=0.9-1.0, p=2, \lambda=0.6-1.0, p=3, \lambda=0.7-0.9$ and $p=10, \lambda=0.5-1.0$ (Table 9), the order is same is obtained.

From Table 11, we have seen that the ranking coincides with the already proposed methods for higher values of the parameter $\lambda$. This shows that the already proposed methods support the voting by majority or maximum group utility.

From Table 11, we have seen that the proposed method is more general. The previously proposed methods aggregate the information and provide the best alternative only with the voting by majority or maximum group utility. While the proposed method covers all aspects of the decision making, i.e., the voting by a majority, consensus, and veto by choosing different values of the parameter $\lambda$.

Table 11. Coincide Ranking for the Parameters $p$ and $\lambda$.

| Method | Algorithm | Coincide Ranking for the Parameters $p$ and $\lambda$ |
| :---: | :---: | :---: |
| Feng et al. [14] Xu and Yager [51] Yager [58] | Algorithm 2 | $\begin{aligned} & p=1, \lambda=0.7-1.0 p=2, \lambda=0.8-1.0 \\ & p=3, \lambda=0.9-1.0 p=5, \lambda=0.9-1.0 \\ & p=10, \lambda=0.9-1.0 \end{aligned}$ |
|  | Algorithm 4 | $\begin{aligned} & p=2, \lambda=0.8-1.0 p=3, \lambda=0.9-1.0 \\ & p=5, \lambda=0.8-1.0 p=10, \lambda=0.8-1.0 \end{aligned}$ |
| Khan et al. [46] Xu [50] Wang and Liu [55] Zhao et al. [56] Garg [57] Yager [58] | Algorithm 1 | $\begin{aligned} & p=1, \lambda=0.6-0.8 p=2, \lambda=0.8-1.0 \\ & p=3, \lambda=0.6 \end{aligned}$ |
|  | Algorithm 3 | $\begin{aligned} & p=1, \lambda=0.6-0.6 p=3, \lambda=0.9-1.0 \\ & p=2, \lambda=0.6-1.0 p=3, \lambda=0.7-0.9 \\ & p=10, \lambda=0.5-1.0 \end{aligned}$ |

## 7. Conclusions

The concept of GIFSS has redefined and clarified by F. Feng. In this paper, we have introduced the Euclidean, Hamming, and generalized distance measures for GIFSSs and discussed their properties. The numerical examples of decision making and pattern recognition have discussed based on the proposed distance measures. We also developed a remoteness based VIKOR method for GIFSSs. The displaced and fixed ideal IFVs have defined. The displaced and fixed remoteness indexes have defined for IFVs. The four new ranking indexes based on the displaced and fixed ideals displaced and fixed remoteness indexes, and precise and IF importance weights have defined. Different procedures of obtaining precise and IF importance weights have discussed. Four algorithms based on the new ranking indexes, displaced and fixed ideals, displaced and fixed remoteness indexes, and precise and IF importance weights have been proposed. The four algorithms representing the four remoteness based VIKOR methods. In the end, the selection of renewable energy sources problem is solved by using the above proposed four algorithms representing the four remoteness-based VIKOR methods.

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