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# Similarity Measures of Quadripartitioned Single Valued Bipolar Neutrosophic Sets and Its Application in Multi-Criteria Decision Making Problems

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Received: 9 May 2020; Accepted: 11 June 2020; Published: 16 June 2020



**Abstract:** In this paper, a definition of quadripartitioned single valued bipolar neutrosophic set (QSVBNS) is introduced as a generalization of both quadripartitioned single valued neutrosophic sets (QSVNS) and bipolar neutrosophic sets (BNS). There is an inherent symmetry in the definition of QSVBNS. Some operations on them are defined and a set theoretic study is accomplished. Various similarity measures and distance measures are defined on QSVBNS. An algorithm relating to multi-criteria decision making problem is presented based on quadripartitioned bipolar weighted similarity measure. Finally, an example is shown to verify the flexibility of the given method and the advantage of considering QSVBNS in place of fuzzy sets and bipolar fuzzy sets.

**Keywords:** neutrosophic sets; quadripartitioned bipolar neutrosophic sets; similarity measure; decision making

## 1. Introduction

Multiple-criteria decision making (MCDM) is a branch of decision making theory where the aim of an individual is to select the most acceptable alternatives among the feasible ones under some criteria. This criteria dependence in decision can be found in real life on a regular basis. While handling some real life decision making problems, a decision maker often faces trouble due to the presence of several kinds of uncertainties in the data, which is very natural. An attempt was made for the first time by Zadeh [1] by introducing a novel concept of fuzzy set theory. Immediately after that, several improvisations of fuzzy sets were made and implemented in the decision making process. For instance, rough sets by Pawlak [2], intuitionistic fuzzy sets [3], interval valued intuitionistic fuzzy sets [4] by Atanassov, soft sets by Molodtsov [5], etc. Unlike the classical logic, a fuzzy set associates a degree of membership value to every element of the universe of discourse, which can range from 0 to 1, whereas an intuitionistic fuzzy set associate a degree of membership  $\mu \in [0, 1]$  and a degree of non-membership  $\nu \in [0, 1]$ , where  $0 \leq \mu + \nu \leq 1$ . The margin of indeterminacy or hesitation  $\pi$  is defined as  $\pi = 1 - \mu - \nu$ . Smarandache in [6,7] proposed neutrosophic sets. In neutrosophic sets, the indeterminacy membership function walks along independently of the truth membership or of the falsity membership. Neutrosophic theory has been widely explored by researchers (see [8–13]) for application purpose in handling real life situations involving uncertainty. Although the hesitation margin of neutrosophic theory is independent of the truth or falsity membership, looks more general than intuitionistic fuzzy sets yet. Recently, in [14] Atanassov et al. studied the relations between inconsistent intuitionistic fuzzy sets [15], picture fuzzy sets [16], neutrosophic sets [7] and intuitionistic

fuzzy sets [3]; however, it remains in doubt that whether the indeterminacy associated to a particular element occurs due to the belongingness of the element or the non-belongingness. This has been pointed out by Chatterjee et al. [17] while introducing a more general structure of neutrosophic set viz. quadripartitioned single valued neutrosophic set (QSVNS). The idea of QSVNS is actually stretched from Smarandache's four numerical-valued neutrosophic logic [18] and Belnap's four valued logic [19], where the indeterminacy is divided into two parts, namely, "unknown" i.e., neither true nor false and "contradiction" i.e., both true and false. In the context of neutrosophic study however, the QSVNS looks quite logical. Also in their study, Chatterjee et al. [17] analyzed a real life example for a better understanding of a QSVNS environment and showed that such situations occur very naturally. They have also solved a decision making problem pertaining to pattern recognition showing the application capability of QSVNS.

Bipolarity often reflects the tendency of the human mind in reasoning to make a decision on the basis of +ve and -ve information. Lee [20,21] introduced bipolar fuzzy set as an extension of fuzzy set. Bipolar fuzzy model with some hybrid structures were studied in [22–25] shows the flexibility of bipolar fuzzy sets for solving decision making problems. Bipolar fuzzy set and neutrosophic set have been put together for a more general framework viz. bipolar neutrosophic sets (BNS) by Deli et al. [26]. Sahin et al. [27,28] introduced the Jaccard vector similarity measure, hybrid vector similarity measure, and dice similarity measure with applications to decision making problems. Jamil et al. [29] applied bipolar neutrosophic Hamacher averaging operator in group decision making problems. Bipolar neutrosophic sets help in handling uncertain information in a reliable way. It is an extension of the bipolar fuzzy set and neutrosophic set, which can deal with real life problems involving positive and negative information. Looking at the work of Chatterjee et al. [17], unlike neutrosophic set, it is in doubt whether the negative indeterminacy associated to some elements of the universe of discourse is due to the occurrence of the counter-property or the non-occurrence of the counter-property.

To overcome the aforesaid situation we merged the bipolar neutrosophic set and QSVNS to introduce a more general structure, namely, quadripartitioned single valued bipolar neutrosophic sets (QSVBNS). The word "quadripartitioned" refers to four values i.e., in case of QSVBNS the negative indeterminacy of the BNS is divided into two parts alongside truth and falsity membership alike QSVNS.

First, we develop some set theoretic results on QSVBNS and then formulas for similarity measures were framed, and finally a real life problem was dealt with using the MCDM method in this setting. A comparison was made in application of a real life problem, where it is seen that the use of QSVBN system gives a better result compared to fuzzy sets and bipolar fuzzy sets. The paper is organized as follows: Section 2 recalls some preliminaries results. In Section 3, QSVBNS is introduced and some basic operations on QSVBNS are dealt with; an example of QSVBNS is also presented. Several similarity measures between QSVBNS are defined and their properties are studied in Section 4. In Section 5 we give an algorithm based on quadripartitioned bipolar weighted similarity measure to deal with the multi-criteria decision making problem in a QSVBN environment. Based on the given algorithm, a real life problem in decision making is solved in Section 6. A detailed discussion about the obtained result is analyzed in Section 7. Finally, Section 8 concludes the paper.

## 2. Preliminaries

**Definition 1** ([7]). *Let  $X$  be a universal set. A single valued neutrosophic set  $A$  over  $X$  is defined as,  $A = \{ \langle x, \langle T_A(x), I_A(x), F_A(x) \rangle \rangle : x \in X \}$ , where,  $T_A(x), I_A(x), F_A(x)$  are respectively called truth-membership function, indeterminacy-membership function and falsity-membership function. These are defined by  $T_A : X \rightarrow [0, 1]$ ,  $I_A : X \rightarrow [0, 1]$ ,  $F_A : X \rightarrow [0, 1]$  respectively with the property that  $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ .*

**Definition 2** ([17]). *Let  $X$  be a universal set. A quadripartitioned neutrosophic set (QSVNS)  $A$ , over  $X$  is defined as,  $A = \{ \langle x, \langle T_A(x), C_A(x), U_A(x), F_A(x) \rangle \rangle : x \in X \}$ , where  $T_A(x), C_A(x), U_A(x), F_A(x)$  are respectively called truth-membership function, contradiction-membership function, ignorance-membership*

function, and falsity-membership function. These are defined by  $T_A : X \rightarrow [0, 1]$ ,  $C_A : X \rightarrow [0, 1]$ ,  $U_A : X \rightarrow [0, 1]$ ,  $F_A : X \rightarrow [0, 1]$  respectively with the property that  $0 \leq T_A(x) + C_A(x) + U_A(x) + F_A(x) \leq 4$ .

When  $X$  is discrete,  $A$  is represented as,  $A = \sum_{k=i}^n \langle T_A(x_i), C_A(x_i), U_A(x_i), F_A(x_i) \rangle / x_i$ ,  $x_i \in X$ . When  $X$  is continuous,  $A$  is represented as,  $\int_X \langle T_A(x), C_A(x), U_A(x), F_A(x) \rangle / x$ ,  $x \in X$ .

**Definition 3** ([26]). A bipolar neutrosophic set (BNS)  $A$  in  $X$  is defined to be an object of the form  $A = \{ \langle x, T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x) \rangle : x \in X \}$ , where,  $T^+, I^+, F^+$  are functions from  $X$  to  $[0, 1]$  and  $T^-, I^-, F^-$  are functions from  $X$  to  $[-1, 0]$ . The +ve membership degrees  $T^+(x), I^+(x), F^+(x)$  denote respectively the truth-membership, indeterminate-membership, and falsity-membership of  $x \in X$  corresponding to a bipolar neutrosophic set  $A$  and the -ve membership degrees  $T^-(x), I^-(x), F^-(x)$  denote respectively the truth-membership, indeterminate-membership, and falsity-membership  $x \in X$  to some implicit counter-property corresponding to a bipolar neutrosophic set  $A$ .

**Definition 4** ([17]). Let  $A$  and  $B$  be two QSVNS. Then

- (1)  $A \subseteq B$  if  $T_A(x) \leq T_B(x), C_A(x) \leq C_B(x), U_A(x) \geq U_B(x), F_A(x) \geq F_B(x)$  for all  $x \in X$ .
- (2) The complement of  $A$  is denoted by  $A^c$  and is defined as  $A^c = \sum_{i=1}^n \langle F_A(x_i), U_A(x_i), C_A(x_i), T_A(x_i) \rangle / x_i$ ,  $x_i \in X$ , where  $T_{A^c}(x_i) = F_A(x_i)$ ,  $C_{A^c}(x_i) = U_A(x_i)$  and  $U_{A^c}(x_i) = C_A(x_i)$ ,  $F_{A^c}(x_i) = T_A(x_i)$ .
- (3)  $A \cup B$  is defined as  $A \cup B = \sum_{i=1}^n \langle T_A(x_i) \vee T_B(x_i), C_A(x_i) \vee C_B(x_i), U_A(x_i) \wedge U_B(x_i), F_A(x_i) \wedge F_B(x_i) \rangle / X$ .
- (4)  $A \cap B$  is defined as  $A \cap B = \sum_{i=1}^n \langle T_A(x_i) \wedge T_B(x_i), C_A(x_i) \wedge C_B(x_i), U_A(x_i) \vee U_B(x_i), F_A(x_i) \vee F_B(x_i) \rangle / X$ .

**Definition 5** ([26]). Let  $A_1 = \langle x, T_1^+(x), I_1^+(x), F_1^+(x), T_1^-(x), I_1^-(x), F_1^-(x) \rangle$ ,  $A_2 = \langle x, T_2^+(x), I_2^+(x), F_2^+(x), T_2^-(x), I_2^-(x), F_2^-(x) \rangle$  be two bipolar neutrosophic sets (BNS) over the universe of discourse  $X$ . Then,

- (1)  $A_1 \subseteq A_2$  if  $T_1^+(x) \leq T_2^+(x), I_1^+(x) \leq I_2^+(x), F_1^+(x) \geq F_2^+(x)$  and  $T_1^-(x) \geq T_2^-(x), I_1^-(x) \geq I_2^-(x), F_1^-(x) \leq F_2^-(x)$ .
- (2)  $A^c = \langle x, 1 - T_A^+(x), 1 - I_A^+(x), 1 - F_A^+(x), -1 - T_A^-(x), -1 - I_A^-(x), -1 - F_A^-(x) \rangle$ .
- (3)  $A_1 \cup A_2 = \langle x, \max(T_1^+(x), T_2^+(x)), \frac{I_1^+(x) + I_2^+(x)}{2}, \min(F_1^+(x), F_2^+(x)), \min(T_1^-(x), T_2^-(x)), \frac{I_1^-(x) + I_2^-(x)}{2}, \max(F_1^-(x), F_2^-(x)) \rangle$  for all  $x \in X$ .
- (4)  $A_1 \cap A_2 = \langle x, \min(T_1^+(x), T_2^+(x)), \frac{I_1^+(x) + I_2^+(x)}{2}, \max(F_1^+(x), F_2^+(x)), \max(T_1^-(x), T_2^-(x)), \frac{I_1^-(x) + I_2^-(x)}{2}, \min(F_1^-(x), F_2^-(x)) \rangle$  for all  $x \in X$ .

### 3. Quadripartitioned Single Valued Bipolar Neutrosophic Sets

In this section, we introduce the concept of quadripartitioned single valued bipolar neutrosophic sets (QSVBNS).

**Definition 6.** A quadripartitioned single valued bipolar neutrosophic set (QSVBNS)  $A$  in  $X$  defined as an object of the form  $A = \langle x, T_A^P(x), C_A^P(x), U_A^P(x), F_A^P(x), T_A^N(x), C_A^N(x), U_A^N(x), F_A^N(x) \rangle : x \in X$ , where,  $T_A^P, C_A^P, U_A^P, F_A^P : X \rightarrow [0, 1]$  and  $T_A^N, C_A^N, U_A^N, F_A^N : X \rightarrow [-1, 0]$ . The positive membership degrees  $T_A^P(x), C_A^P(x), U_A^P(x), F_A^P(x)$  denote respectively the truth-membership, a contradiction-membership, an ignorance membership, and falsity membership of  $x \in X$  corresponding to a QSVBNS  $A$ . The negative membership degrees  $T_A^N(x), C_A^N(x), U_A^N(x), F_A^N(x)$  denote respectively the truth-membership, a contradiction-membership, an ignorance membership, and falsity membership of  $x \in X$  to some explicit counter-property corresponding to a QSVBNS  $A$ .

With respect to  $(T_A^P(x), F_A^P(x))$  and  $(C_A^P(x), U_A^P(x))$  and +ve and -ve membership grade, there is a sense of symmetry in the structure of QSVBNS.

**Example 1.** Suppose an environment organization desires to know peoples opinion on the following statement: “The fashion industry has helped economic growth but it also has a bad impact on the environment due to a large amount of carbon emissions”. To help the cause, a group of four experts, say  $X = \{x_1, x_2, x_3, x_4\}$ , has been asked to give their opinion. The statement can be divided into two parts as:

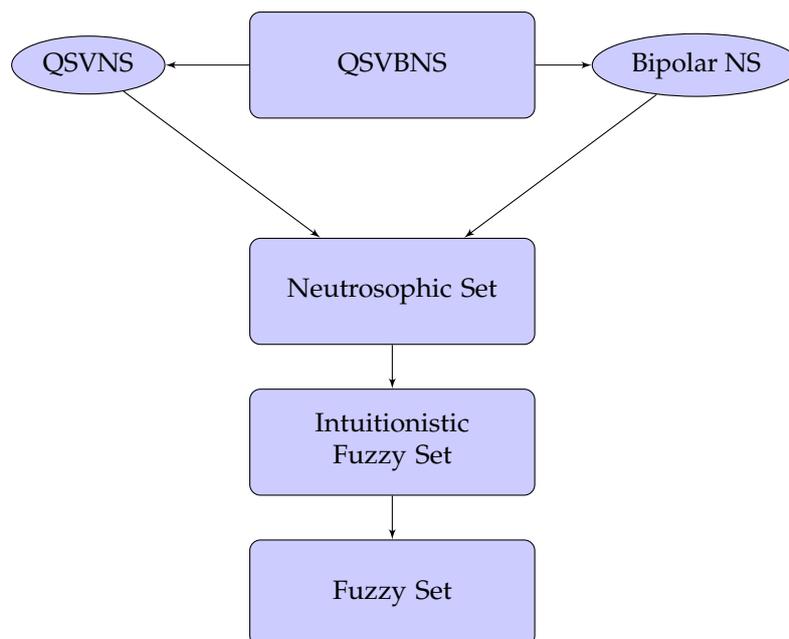
- (a) The fashion industry has helped economic growth and to a counter property of that:
- (b) The fashion industry has a bad impact on the environment due to a large amount of carbon emissions.

It may so happen that the opinion has the following outcomes: “a degree of agreement with statement (a) and disagreement with statement (b)”, “a degree of agreement and disagreement with both the statements (a) and (b)”, “a degree of neither agreement nor disagreement regarding both the statements”, and “a degree of disagreement with statement (a) and agreement with statement (b)”. According to the views of the four experts, the outcome represented in terms of QSVBNS as follows:

$$\langle 0.9, 0.4, 0.3, 0.1, -0.1, -0.4, -0.3, -0.9 \rangle / x_1 + \langle 0.3, 0.8, 0.2, 0.3, -0.4, -0.4, -0.9, -0.5 \rangle / x_2 + \langle 0.2, 0.5, 0.8, 0.4, -0.2, -0.1, -0.8, -0.3 \rangle / x_3 + \langle 0.2, 0.5, 0.5, 0.9, -0.8, -0.1, -0.4, -0.2 \rangle / x_4$$

the above QSVBNS reflects that the expert  $x_1$  agrees to the fact that the fashion industry has helped economic growth, whereas the expert  $x_2$  believes that fashion industry might have helped economic growth but it also has affected the environment a bit. On the other side the expert  $x_3$  is ignorant regarding the truth of both the statements and the expert  $x_4$  opines that fashion industry does not have much impact on the world economy but he believes that it causes damage to the environment.

**Remark 1.** The relationship between QSVBNS and other extensions of fuzzy sets are diagrammatically depicted in the following figure (Figure 1).



**Figure 1.** Relationship of quadripartioned single valued bipolar neutrosophic sets (QSVBNS) with other extensions of fuzzy sets.

**Definition 7.** A QSVBNS,  $A$  over a universe  $X$  is said to be an absolute QSVBNS, denoted by  $\mathbf{X}$ , if for each  $x \in X$ ,  $T_A^P(x) = 1$ ,  $C_A^P(x) = 1$ ,  $U_A^P(x) = 0$ ,  $F_A^P(x) = 0$ ,  $T_A^N(x) = -1$ ,  $C_A^N(x) = -1$ ,  $U_A^N(x) = 0$ ,  $F_A^N(x) = 0$ .

**Definition 8.** A QSVBNS,  $A$  over a universe  $X$  is said to be a null QSVBNS, denoted by  $\mathbf{\Theta}$ , if for each  $x \in X$  the membership values are respectively  $T_A^P(x) = 0$ ,  $C_A^P(x) = 0$ ,  $U_A^P(x) = 1$ ,  $F_A^P(x) = 1$ ,  $T_A^N(x) = 0$ ,  $C_A^N(x) = 0$ ,  $U_A^N(x) = -1$ ,  $F_A^N(x) = -1$ .

Alongside this, we expound some set theoretic operations on quadripartitioned single valued bipolar neutrosophic sets over a universe  $X$  and analyze some of their properties.

**Definition 9.** Let  $A$  and  $B$  be two QSVBNS over  $X$ . Then  $A$  is said to be included in  $B$ , denoted by  $A \subseteq B$ , if for each  $x \in X$ ,  $T_A^P(x) \leq T_B^P(x)$ ,  $C_A^P(x) \leq C_B^P(x)$ ,  $U_A^P(x) \geq U_B^P(x)$ ,  $F_A^P(x) \geq F_B^P(x)$  and  $T_A^N(x) \geq T_B^N(x)$ ,  $C_A^N(x) \geq C_B^N(x)$ ,  $U_A^N(x) \leq U_B^N(x)$ ,  $F_A^N(x) \leq F_B^N(x)$ .

**Definition 10.** Two QSVBNSs  $A$  and  $B$  are said to be equal if for each  $x \in X$ ,  $T_A^P(x) = T_B^P(x)$ ,  $C_A^P(x) = C_B^P(x)$ ,  $U_A^P(x) = U_B^P(x)$ ,  $F_A^P(x) = F_B^P(x)$  and  $T_A^N(x) = T_B^N(x)$ ,  $C_A^N(x) = C_B^N(x)$ ,  $U_A^N(x) = U_B^N(x)$ ,  $F_A^N(x) = F_B^N(x)$ .

**Definition 11.** The complement of a QSVBNS  $A$ , denoted by  $A^c$ , is defined as,  $A^c = \langle x, F_A^P(x), U_A^P(x), C_A^P(x), T_A^P(x), F_A^N(x), U_A^N(x), C_A^N(x), T_A^N(x) \rangle : x \in X$ , where,  $T_{A^c}^P(x) = F_A^P(x)$ ,  $C_{A^c}^P(x) = U_A^P(x)$ ,  $U_{A^c}^P(x) = C_A^P(x)$ ,  $F_{A^c}^P(x) = T_A^P(x)$  and  $T_{A^c}^N(x) = F_A^N(x)$ ,  $C_{A^c}^N(x) = U_A^N(x)$ ,  $U_{A^c}^N(x) = C_A^N(x)$ ,  $F_{A^c}^N(x) = T_A^N(x)$ ,  $x \in X$ .

**Definition 12.** The union of two QSVBNS  $A$  and  $B$ , denoted by  $A \cup B$  is defined as,  $A \cup B = \langle x, T_A^P(x) \vee T_B^P(x), C_A^P(x) \vee C_B^P(x), U_A^P(x) \wedge U_B^P(x), F_A^P(x) \wedge F_B^P(x), T_A^N(x) \wedge T_B^N(x), C_A^N(x) \wedge C_B^N(x), U_A^N(x) \vee U_B^N(x), F_A^N(x) \vee F_B^N(x) \rangle : x \in X$ .

**Definition 13.** The intersection of two QSVBNS,  $A$  and  $B$ , denoted by  $A \cap B$  is defined as,  $A \cap B = \langle x, T_A^P(x) \wedge T_B^P(x), C_A^P(x) \wedge C_B^P(x), U_A^P(x) \vee U_B^P(x), F_A^P(x) \vee F_B^P(x), T_A^N(x) \vee T_B^N(x), C_A^N(x) \vee C_B^N(x), U_A^N(x) \wedge U_B^N(x), F_A^N(x) \wedge F_B^N(x) \rangle : x \in X$ .

**Example 2.** For two QSVBNS  $A$  and  $B$  over  $X$  given by  $A = \langle 0.8, 0.6, 0.4, 0.1, -0.2, -0.3, -0.5, -0.7 \rangle / x_1 + \langle 0.6, 0.5, 0.2, 0.3, -0.5, -0.4, -0.7, -0.8 \rangle / x_2 + \langle 0.2, 0.5, 0.6, 0.7, -0.6, -0.1, -0.5, -0.7 \rangle / x_3$  and  $B = \langle 0.6, 0.5, 0.4, 0.3, -0.4, -0.7, -0.5, -0.6 \rangle / x_1 + \langle 0.4, 0.5, 0.7, 0.5, -0.6, -0.4, -0.3, -0.4 \rangle / x_2 + \langle 0.3, 0.7, 0.4, 0.2, -0.2, -0.1, -0.4, -0.8 \rangle / x_3$

- (1)  $A^c = \langle 0.1, 0.4, 0.6, 0.8, -0.7, -0.5, -0.3, -0.2 \rangle / x_1 + \langle 0.3, 0.2, 0.5, 0.6, -0.8, -0.7, -0.4, -0.5 \rangle / x_2 + \langle 0.7, 0.6, 0.5, 0.2, -0.7, -0.5, -0.1, -0.6 \rangle / x_3$ .
- (2)  $A \cup B = \langle 0.8, 0.6, 0.4, 0.1, -0.4, -0.7, -0.5, -0.6 \rangle / x_1 + \langle 0.6, 0.5, 0.2, 0.3, -0.6, -0.4, -0.3, -0.4 \rangle / x_2 + \langle 0.3, 0.7, 0.4, 0.2, -0.6, -0.1, -0.4, -0.7 \rangle / x_3$ .
- (3)  $A \cap B = \langle 0.6, 0.5, 0.4, 0.3, -0.2, -0.3, -0.5, -0.6 \rangle / x_1 + \langle 0.4, 0.5, 0.7, 0.5, -0.5, -0.4, -0.7, -0.8 \rangle / x_2 + \langle 0.2, 0.5, 0.6, 0.7, -0.6, -0.2, -0.5, -0.8 \rangle / x_3$ .

**Theorem 1.** Under the aforesaid set-theoretic operation, the quadripartitioned single valued bipolar neutrosophic sets satisfy the following properties:

(1) Identity law:

- $A \cup \mathbf{\Theta} = A$  and  $A \cap \mathbf{X} = A$ .

(2) Commutative law:

- $A \cup B = B \cup A$  and  $A \cap B = B \cap A$ .

(3) Associative law:

- $(A \cup B) \cup C = A \cup (B \cup C)$  and  $(A \cap B) \cap C = A \cap (B \cap C)$ .

(4) Distributive law:

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$  and  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ .

(5) De-Morgan's law:

- $(A \cup B)^c = A^c \cap B^c$  and  $(A \cap B)^c = A^c \cup B^c$ .

**Proof.** Proofs are plain-dealing.  $\square$

#### 4. Similarity Measure of Quadripartitioned Bipolar Neutrosophic Sets

Here we provide the definition of similarity measure between two QSVBNS  $A, B$  over a universe  $X$ .

**Definition 14.** Let  $QSVBNS(X)$  indicate the set of all QSVBNS over the universe  $X$ . Let  $S : QSVBNS(X) \times QSVBNS(X) \rightarrow [0, 1]$  be a function satisfying the following properties for all  $A, B \in QSVBNS(X)$ :

(S1)  $0 \leq S(A, B) < 1$  and  $S(A, B) = 1$  iff  $A = B$ ,

(S2)  $S(A, B) = S(B, A)$ ,

(S3) for any  $A, B, C \in QSVBNS(X)$  with  $A \subseteq B \subseteq C$ ,  $S(A, C) \leq S(A, B) \wedge S(B, C)$ .

then  $S$  is said to be a similarity measure.

Based on the membership functions of two QSVBNS, we prescribe some functions which measures the differences between the membership values of two QSVBNS. Let  $A, B \in QSVBNS$ . For each  $x_k \in X$ ,  $k = 1, 2, \dots, n$  and for each  $i = 1, 2, 3, 4$  and  $j = 1, 2$  define the functions  $\delta_i^{A,B}, \lambda_j^{A,B} : QSVBNS(X) \times QSVBNS(X) \rightarrow [0, 1]$  respectively,

- $\delta_1^{A,B}(x_k) = \frac{1}{2}(|T_A^P(x_k) - T_B^P(x_k)| + |T_A^N(x_k) - T_B^N(x_k)|)$ ,
- $\delta_2^{A,B}(x_k) = \frac{1}{2}(|F_A^P(x_k) - F_B^P(x_k)| + |F_A^N(x_k) - F_B^N(x_k)|)$ ,
- $\delta_3^{A,B}(x_k) = \frac{1}{4}(\delta_1^{A,B}(x_k) + \delta_2^{A,B}(x_k) + |C_A^P(x_k) - C_B^P(x_k)| + |C_A^N(x_k) - C_B^N(x_k)|)$ ,
- $\delta_4^{A,B}(x_k) = \frac{1}{2}(|U_A^P(x_k) - U_B^P(x_k)| + |U_A^N(x_k) - U_B^N(x_k)|)$ ,
- $\lambda_1^{A,B}(x_k) = \frac{1}{4}(|T_A^P(x_k)C_A^P(x_k) - T_B^P(x_k)C_B^P(x_k)| + |T_A^N(x_k)C_A^N(x_k) - T_B^N(x_k)C_B^N(x_k)| + |C_A^P(x_k) - C_B^P(x_k)| + |C_A^N(x_k) - C_B^N(x_k)|)$ ,
- $\lambda_2^{A,B}(x_k) = \frac{1}{4}(\delta_1^{A,B}(x_k) + \delta_2^{A,B}(x_k) + |U_A^P(x_k) - U_B^P(x_k)| + |U_A^N(x_k) - U_B^N(x_k)|)$ .

**Remark 2.** The function  $\mathcal{T}_1(A, B)$  defined by  $\mathcal{T}_1(A, B) = 1 - \frac{1}{4n} \sum_{k=1}^n \sum_{i=1}^4 \delta_i^{A,B}(x_k)$ ,  $A, B \in QSVBNS(X)$  is shown to be a similarity measure in the following theorem.

**Theorem 2.**  $\mathcal{T}_1(A, B)$  is a similarity measure between two quadripartitioned bipolar neutrosophic sets  $A$  and  $B$  over  $X$ .

**Proof.** Since  $T_A^P(x_k), C_A^P(x_k), U_A^P(x_k), F_A^P(x_k) \in [0, 1]$  and  $T_A^N(x_k), C_A^N(x_k), U_A^N(x_k), F_A^N(x_k) \in [-1, 0]$  for all  $x_k \in X$  it follows that  $\delta_1^{A,B}(x_k)$  attains its maximum value 1 whenever one of the positive truth membership values corresponding to  $A$  and  $B$  is 1 and the other is 0 and one of the negative truth

membership values corresponding to  $A$  and  $B$  is  $-1$  and the other is  $0$ . Similarly for each  $x_k \in X$ ,  $\delta_1^{A,B}(x_k)$  attains its minimum value  $0$  whenever  $T_A^P(x_k) = T_B^P(x_k)$  and  $T_A^N(x_k) = T_B^N(x_k)$ . Thus for all  $x_k \in X$ ,  $0 \leq \delta_1^{A,B}(x_k) \leq 1, \forall x_k \in X$ . Likewise it can be shown that  $0 \leq \delta_2^{A,B}(x_k), \delta_3^{A,B}(x_k), \delta_4^{A,B}(x_k) \leq 1$ .

Therefore for all  $x_k \in X$  we have,

$$\begin{aligned} & 0 \leq \delta_1^{A,B}(x_k) + \delta_2^{A,B}(x_k) + \delta_3^{A,B}(x_k) + \delta_4^{A,B}(x_k) \leq 4 \\ \implies & 0 \leq \frac{1}{4}[\delta_1^{A,B}(x_k) + \delta_2^{A,B}(x_k) + \delta_3^{A,B}(x_k) + \delta_4^{A,B}(x_k)] \leq 1 \\ \implies & 0 \leq \frac{1}{4n} \sum_{k=1}^n \sum_{i=1}^4 \delta_i^{A,B}(x_k) \leq 1 \\ \implies & -1 \leq -\frac{1}{4n} \sum_{k=1}^n \sum_{i=1}^4 \delta_i^{A,B}(x_k) \leq 0 \\ \implies & 0 \leq 1 - \frac{1}{4n} \sum_{k=1}^n \sum_{i=1}^4 \delta_i^{A,B}(x_k) \leq 1 \\ \implies & 0 \leq \mathcal{T}_1(A, B) \leq 1 \end{aligned}$$

Again,  $\mathcal{T}_1(A, B) = 1 \iff \sum_{k=1}^n \sum_{i=1}^4 \delta_i^{A,B}(x_k) = 0 \iff \delta_1^{A,B}(x_k) = \delta_2^{A,B}(x_k) = \delta_3^{A,B}(x_k) = \delta_4^{A,B}(x_k) = 0 \iff T_A^P(x_k) = T_B^P(x_k), C_A^P(x_k) = C_B^P(x_k), U_A^P(x_k) = U_B^P(x_k), F_A^P(x_k) = F_B^P(x_k)$  and  $T_A^N(x_k) = T_B^N(x_k), C_A^N(x_k) = C_B^N(x_k), U_A^N(x_k) = U_B^N(x_k), F_A^N(x_k) = F_B^N(x_k) \iff A = B$ .

It is easy to prove that  $\mathcal{T}_1(A, B) = \mathcal{T}_1(B, A)$ .

Finally let  $A \subset B \subset C$ . Then for all  $x_k \in X$ , we have

$$T_A^P(x_k) \leq T_B^P(x_k) \leq T_C^P(x_k), T_A^N(x_k) \geq T_B^N(x_k) \geq T_C^N(x_k), C_A^P(x_k) \leq C_B^P(x_k) \leq C_C^P(x_k), C_A^N(x_k) \geq C_B^N(x_k) \geq C_C^N(x_k), U_A^P(x_k) \geq U_B^P(x_k) \geq U_C^P(x_k), U_A^N(x_k) \leq U_B^N(x_k) \leq U_C^N(x_k), F_A^P(x_k) \geq F_B^P(x_k) \geq F_C^P(x_k), F_A^N(x_k) \leq F_B^N(x_k) \leq F_C^N(x_k).$$

Then,  $|T_A^P(x_k) - T_B^P(x_k)| \leq |T_A^P(x_k) - T_C^P(x_k)|, |T_A^N(x_k) - T_B^N(x_k)| \leq |T_A^N(x_k) - T_C^N(x_k)|$ .

Therefore,  $\frac{1}{2}(|T_A^P(x_k) - T_B^P(x_k)| + |T_A^N(x_k) - T_B^N(x_k)|) \leq \frac{1}{2}(|T_A^P(x_k) - T_C^P(x_k)| + |T_A^N(x_k) - T_C^N(x_k)|) \implies \delta_1^{A,B}(x_k) \leq \delta_1^{A,C}(x_k), \forall x_k \in X \dots\dots\dots(*)$

Considering a similar process, it can be shown that  $\delta_2^{A,B}(x_k) \leq \delta_2^{A,C}(x_k), \delta_3^{A,B}(x_k) \leq \delta_3^{A,C}(x_k)$  and  $\delta_4^{A,B}(x_k) \leq \delta_4^{A,C}(x_k)$ .

Therefore,

$$\begin{aligned} & \sum_{i=1}^4 \delta_i^{A,C}(x_k) \geq \sum_{i=1}^4 \delta_i^{A,B}(x_k) \\ \implies & \sum_{k=1}^n \sum_{i=1}^4 \delta_i^{A,C}(x_k) \geq \sum_{k=1}^n \sum_{i=1}^4 \delta_i^{A,B}(x_k) \\ \implies & 1 - \frac{1}{4n} \sum_{k=1}^n \sum_{i=1}^4 \delta_i^{A,B}(x_k) \geq 1 - \frac{1}{4n} \sum_{k=1}^n \sum_{i=1}^4 \delta_i^{A,C}(x_k) \\ \implies & \mathcal{T}_1(A, B) \geq \mathcal{T}_1(A, C). \end{aligned}$$

Similarly  $\mathcal{T}_1(B, C) \geq \mathcal{T}_1(A, C)$ . Therefore,  $\mathcal{T}_1(A, C) \leq \mathcal{T}_1(A, B) \wedge \mathcal{T}_1(B, C)$ . Hence the proof.  $\square$

**Remark 3.** Define  $\mathcal{T}_2(A, B) = 1 - \left[ \frac{1}{n} \sum_{k=1}^n \left( \frac{1}{4} (\delta_1^{A,B}(x_k) + \delta_2^{A,B}(x_k) + \lambda_1^{A,B}(x_k) + \lambda_2^{A,B}(x_k)) \right)^p \right]^{\frac{1}{p}}$ , where  $p$ , a positive integer, is defined to be the order of the similarity.

**Theorem 3.**  $\mathcal{T}_2(A, B)$  is a similarity measure between two quadripartioned bipolar neutrosophic sets  $A$  and  $B$  over  $X$ .

**Proof.**  $\mathcal{T}_2(A, B) = \mathcal{T}_2(B, A)$  is quite obvious.

Under any of the the following condition  $\lambda_1^{A,B}(x_k)$  attains its maximum value 1,

- (1)  $T_A^P(x_k) = C_A^P(x_k) = 1$  and at least one of  $T_B^P(x_k)$  or  $C_B^P(x_k)$  is 0 or  $T_B^P(x_k) = C_B^P(x_k) = 1$  and at least one of  $T_A^P(x_k)$  or  $C_A^P(x_k)$  is 0,
- (2)  $T_A^N(x_k) = C_A^N(x_k) = -1$  and at least one of  $T_B^N(x_k)$  or  $C_B^N(x_k)$  is 0 or  $T_B^N(x_k) = C_B^N(x_k) = -1$  and at least one of  $T_A^N(x_k)$  or  $C_A^N(x_k)$  is 0,
- (3)  $C_A^P(x_k) = 1$  and  $C_B^P(x_k) = 0$  or  $C_A^P(x_k) = 0$  and  $C_B^P(x_k) = 1$ ,
- (4)  $C_A^N(x_k) = -1$  and  $C_B^N(x_k) = 0$  or  $C_A^N(x_k) = 0$  and  $C_B^N(x_k) = -1$ .

Also the minimum value of  $\lambda_1^{A,B}(x_k)$  is 0. Thus  $0 \leq \lambda_1^{A,B}(x_k) \leq 1$  for all  $x_k \in X$ . Also  $0 \leq \lambda_2^{A,B}(x_k) \leq 1$  follows from similar arguments. Therefore,

$$\begin{aligned} & 0 \leq \delta_1^{A,B}(x_k) + \delta_2^{A,B}(x_k) + \lambda_1^{A,B}(x_k) + \lambda_2^{A,B}(x_k) \leq 4 \\ \Rightarrow & 0 \leq \left(\frac{1}{4}(\delta_1^{A,B}(x_k) + \delta_2^{A,B}(x_k) + \lambda_1^{A,B}(x_k) + \lambda_2^{A,B}(x_k))\right)^p \leq 1, \forall p \geq 1 \\ \Rightarrow & 0 \leq \sum_{k=1}^n \left(\frac{1}{4}(\delta_1^{A,B}(x_k) + \delta_2^{A,B}(x_k) + \lambda_1^{A,B}(x_k) + \lambda_2^{A,B}(x_k))\right)^p \leq n \\ \Rightarrow & 0 \leq \frac{1}{n} \sum_{k=1}^n \left(\frac{1}{4}(\delta_1^{A,B}(x_k) + \delta_2^{A,B}(x_k) + \lambda_1^{A,B}(x_k) + \lambda_2^{A,B}(x_k))\right)^p \leq 1 \\ \Rightarrow & 0 \leq \left[\frac{1}{n} \sum_{k=1}^n \left(\frac{1}{4}(\delta_1^{A,B}(x_k) + \delta_2^{A,B}(x_k) + \lambda_1^{A,B}(x_k) + \lambda_2^{A,B}(x_k))\right)^p\right]^{\frac{1}{p}} \leq 1 \\ \Rightarrow & 0 \leq 1 - \left[\frac{1}{n} \sum_{k=1}^n \left(\frac{1}{4}(\delta_1^{A,B}(x_k) + \delta_2^{A,B}(x_k) + \lambda_1^{A,B}(x_k) + \lambda_2^{A,B}(x_k))\right)^p\right]^{\frac{1}{p}} \leq 1 \\ \Rightarrow & 0 \leq \mathcal{T}_2(A, B) \leq 1 \end{aligned}$$

To show the triangular inequality suppose  $P \subset Q \subset R$ . Then for all  $x_k \in X$ , we have,  $T_P^P(x_k) \leq T_Q^P(x_k) \leq T_R^P(x_k)$ ,  $T_P^N(x_k) \geq T_Q^N(x_k) \geq T_R^N(x_k)$ ,  $C_P^P(x_k) \leq C_Q^P(x_k) \leq C_R^P(x_k)$ ,  $C_P^N(x_k) \geq C_Q^N(x_k) \geq C_R^N(x_k)$ ,  $U_P^P(x_k) \geq U_Q^P(x_k) \geq U_R^P(x_k)$ ,  $U_P^N(x_k) \leq U_Q^N(x_k) \leq U_R^N(x_k)$ ,  $F_P^P(x_k) \geq F_Q^P(x_k) \geq F_R^P(x_k)$ ,  $F_P^N(x_k) \leq F_Q^N(x_k) \leq F_R^N(x_k)$ .

Then  $\delta_1^{P,R}(x_k) \geq \delta_1^{P,Q}(x_k)$  and  $\delta_2^{P,R}(x_k) \geq \delta_2^{P,Q}(x_k)$  follows from (\*) of Theorem 2. Next consider  $\lambda_1^{P,Q}(x_k)$  and  $\lambda_1^{P,R}(x_k)$ . From above inequalities we have,  $T_P^P(x_k)C_P^P(x_k) \leq T_Q^P(x_k)C_Q^P(x_k) \leq T_R^P(x_k)C_R^P(x_k) \Rightarrow |T_P^P(x_k)C_P^P(x_k) - T_Q^P(x_k)C_Q^P(x_k)| \leq |T_P^P(x_k)C_P^P(x_k) - T_R^P(x_k)C_R^P(x_k)|$ .

Similarly,  $|T_P^N(x_k)C_P^N(x_k) - T_Q^N(x_k)C_Q^N(x_k)| \leq |T_P^N(x_k)C_P^N(x_k) - T_R^N(x_k)C_R^N(x_k)|$  and  $|C_P^P(x_k) - C_Q^P(x_k)| \leq |C_P^P(x_k) - C_Q^P(x_k)|$ ,  $|C_P^N(x_k) - C_Q^N(x_k)| \leq |C_P^N(x_k) - C_Q^N(x_k)|$ .

Then,  $\frac{1}{4}(|T_P^P(x_k)C_P^P(x_k) - T_R^P(x_k)C_R^P(x_k)| + |T_P^N(x_k)C_P^N(x_k) - T_R^N(x_k)C_R^N(x_k)| + |C_P^P(x_k) - C_R^P(x_k)| + |C_P^N(x_k) - C_R^N(x_k)|) \geq \frac{1}{4}(|T_P^P(x_k)C_P^P(x_k) - T_Q^P(x_k)C_Q^P(x_k)| + |T_P^N(x_k)C_P^N(x_k) - T_Q^N(x_k)C_Q^N(x_k)| + |C_P^P(x_k) - C_Q^P(x_k)| + |C_P^N(x_k) - C_Q^N(x_k)|)$   
 $\Rightarrow \lambda_1^{P,R}(x_k) \geq \lambda_1^{P,Q}(x_k)$

Similarly it can shown that  $\lambda_2^{P,R}(x_k) \geq \lambda_2^{P,Q}(x_k)$ .

$$\begin{aligned} & \frac{1}{4}(\delta_1^{P,R}(x_k) + \delta_2^{P,R}(x_k) + \lambda_1^{P,R}(x_k) + \lambda_2^{P,R}(x_k)) \geq \frac{1}{4}(\delta_1^{P,Q}(x_k) + \delta_2^{P,Q}(x_k) + \lambda_1^{P,Q}(x_k) + \lambda_2^{P,Q}(x_k)) \\ \Rightarrow & \sum_{k=1}^n \left(\frac{1}{4}(\delta_1^{P,R}(x_k) + \delta_2^{P,R}(x_k) + \lambda_1^{P,R}(x_k) + \lambda_2^{P,R}(x_k))\right)^p \geq \sum_{k=1}^n \left(\frac{1}{4}(\delta_1^{P,Q}(x_k) + \delta_2^{P,Q}(x_k) + \lambda_1^{P,Q}(x_k) + \lambda_2^{P,Q}(x_k))\right)^p \\ & \lambda_1^{P,Q}(x_k) + \lambda_2^{P,Q}(x_k)) \Rightarrow 1 - \left[\frac{1}{n} \sum_{k=1}^n \left(\frac{1}{4}(\delta_1^{P,R}(x_k) + \delta_2^{P,R}(x_k) + \lambda_1^{P,R}(x_k) + \lambda_2^{P,R}(x_k))\right)^p\right]^{\frac{1}{p}} \leq \\ & 1 - \left[\frac{1}{n} \sum_{k=1}^n \left(\frac{1}{4}(\delta_1^{P,Q}(x_k) + \delta_2^{P,Q}(x_k) + \lambda_1^{P,Q}(x_k) + \lambda_2^{P,Q}(x_k))\right)^p\right]^{\frac{1}{p}} \\ \Rightarrow & \mathcal{T}_2(P, R) \leq \mathcal{T}_2(P, Q) \end{aligned}$$

It is identical to show  $\mathcal{T}_2(P, R) \leq \mathcal{T}_2(Q, R)$ . Therefore,  $\mathcal{T}_2(P, R) \leq \mathcal{T}_2(P, Q) \wedge \mathcal{T}_2(Q, R)$ . This completes the proof.  $\square$

We define below the weighted similarity measure between two QSVBNS  $A, B$  over a universe  $X$ .

**Definition 15.** The weighted similarity measure of two QSVBNS  $A, B$  is defined as,  $\mathcal{T}_2^w(A, B) = 1 - \left[ \frac{1}{n} \sum_{k=1}^n w_k \left( \frac{1}{4} (\delta_1^{A,B}(x_k) + \delta_2^{A,B}(x_k) + \lambda_1^{A,B}(x_k) + \lambda_2^{A,B}(x_k)) \right)^p \right]^{\frac{1}{p}}$ , where,  $(w_1, w_2, \dots, w_n)^T$  is the weight vector assigned to the element  $x_1, x_2, \dots, x_n$  of the universe  $X$ , such that  $0 \leq w_k \leq 1, k = 1, 2, \dots, n$  and  $\sum_{k=1}^n w_k = 1$ . It is effortless to find out that  $\mathcal{T}_2^w(A, B)$  satisfies the conditions of similarity measure.

**Remark 4.** We define the following functions based on one particular membership function for two QSVBNSs  $A$  and  $B$  over the universe of discourse  $X$ , provided the denominators never vanish

$$\begin{aligned} \mathcal{T}_T^P(A, B) &= \frac{\sum_{k=1}^n \min(T_A^P(x_k), T_B^P(x_k))}{\sum_{k=1}^n \max(T_A^P(x_k), T_B^P(x_k))}, & \mathcal{T}_C^P(A, B) &= \frac{\sum_{k=1}^n \min(C_A^P(x_k), C_B^P(x_k))}{\sum_{k=1}^n \max(C_A^P(x_k), C_B^P(x_k))}, \\ \mathcal{T}_U^P(A, B) &= \frac{\sum_{k=1}^n \min(U_A^P(x_k), U_B^P(x_k))}{\sum_{k=1}^n \max(U_A^P(x_k), U_B^P(x_k))}, & \mathcal{T}_F^P(A, B) &= \frac{\sum_{k=1}^n \min(F_A^P(x_k), F_B^P(x_k))}{\sum_{k=1}^n \max(F_A^P(x_k), F_B^P(x_k))}, \\ \mathcal{T}_T^N(A, B) &= \frac{\sum_{k=1}^n \max(T_A^N(x_k), T_B^N(x_k))}{\sum_{k=1}^n \min(T_A^N(x_k), T_B^N(x_k))}, & \mathcal{T}_C^N(A, B) &= \frac{\sum_{k=1}^n \max(C_A^N(x_k), C_B^N(x_k))}{\sum_{k=1}^n \min(C_A^N(x_k), C_B^N(x_k))}, \\ \mathcal{T}_U^N(A, B) &= \frac{\sum_{k=1}^n \max(U_A^N(x_k), U_B^N(x_k))}{\sum_{k=1}^n \min(U_A^N(x_k), U_B^N(x_k))}, & \mathcal{T}_F^N(A, B) &= \frac{\sum_{k=1}^n \max(F_A^N(x_k), F_B^N(x_k))}{\sum_{k=1}^n \min(F_A^N(x_k), F_B^N(x_k))}. \end{aligned}$$

The following is a definition of generalized similarity measure between two QSVBNS  $A, B$  whose value set is the set of all  $2 \times 4$  matrices over  $\mathbb{R}$ .

**Definition 16.** Let  $X = \{x_1, x_2, \dots, x_n\}$  be a finite universe of discourse. For two QSVBNSs  $A, B$  define a mapping  $\mathcal{L} : \text{QSVBNS}(X) \times \text{QSVBNS}(X) \rightarrow \mathbb{M}_{2 \times 4}(\mathbb{R})$  by

$$\mathcal{L}(A, B) = \begin{pmatrix} \mathcal{T}_T^P(A, B) & \mathcal{T}_C^P(A, B) & \mathcal{T}_U^P(A, B) & \mathcal{T}_F^P(A, B) \\ \mathcal{T}_T^N(A, B) & \mathcal{T}_C^N(A, B) & \mathcal{T}_U^N(A, B) & \mathcal{T}_F^N(A, B) \end{pmatrix},$$

and a partial order relation " $\preceq$ " on  $\mathbb{M}_{2 \times 4}(\mathbb{R})$  as:

$$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{pmatrix} \preceq \begin{pmatrix} b_1 & b_2 & b_3 & b_4 \\ b_5 & b_6 & b_7 & b_8 \end{pmatrix}, \text{ if } a_i \leq b_i \forall i.$$

Also define

$$\tilde{0} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{ and } \tilde{1} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

**Remark 5.** Then for  $A, B \in \text{QSVBNS}(X)$

- (1)  $\tilde{0} \preceq \mathcal{L}(A, B) \preceq \tilde{1}$ ,
- (2)  $\mathcal{L}(A, B) = \mathcal{L}(B, A)$ ,
- (3) for  $A \subset B \subset C$ ,  $\mathcal{L}(A, C) \preceq \mathcal{L}(A, B) \wedge \mathcal{L}(B, C)$ .

We give an outline of the proof as:

Let  $A \subset B \subset C$ . Then for all  $x_k \in X$ ,  $T_A^N(x_k) \geq T_B^N(x_k) \geq T_C^N(x_k)$ . Consequently,  $\min(T_A^N(x_k), T_B^N(x_k)) = T_B^N(x_k)$ ,  $\max(T_A^N(x_k), T_B^N(x_k)) = T_A^N(x_k)$ ,  $\min(T_A^N(x_k), T_C^N(x_k)) = T_C^N(x_k)$ ,  $\max(T_A^N(x_k), T_C^N(x_k)) = T_A^N(x_k)$ .

$$\text{Then we have, } \sum_{k=1}^n T_B^N(x_k) \geq \sum_{k=1}^n T_C^N(x_k) \Rightarrow \frac{1}{\sum_{k=1}^n T_B^N(x_k)} \leq \frac{1}{\sum_{k=1}^n T_C^N(x_k)} \Rightarrow \frac{\sum_{k=1}^n T_A^N(x_k)}{\sum_{k=1}^n T_B^N(x_k)} \geq \frac{\sum_{k=1}^n T_A^N(x_k)}{\sum_{k=1}^n T_C^N(x_k)}$$

$$\Rightarrow \mathcal{T}_T^N(A, B) \geq \mathcal{T}_T^N(A, C).$$

A similar process follows for  $\mathcal{T}_T^N(B, C) \geq \mathcal{T}_T^N(A, C)$ . Hence  $\mathcal{T}_T^N(A, C) \leq \mathcal{T}_T^N(A, B) \wedge \mathcal{T}_T^N(B, C)$ . Hence  $\mathcal{L}$  is a generalized similarity measure.

**Definition 17.** Let  $d : \text{QSVBNS}(X) \times \text{QSVBNS}(X) \rightarrow \mathbb{R}^+ \cup \{0\}$  be a mapping satisfying the following conditions:

- (d1)  $d(A, B) \geq 0$  and  $d(A, B) = 0$  iff  $A = B$ ,
- (d2)  $d(A, B) = d(B, A)$ ,
- (d3)  $d(A, C) \leq d(A, B) + d(B, C)$ .

Then  $d$  is said to be a distance based measure between two QSVBNS  $A$  and  $B$ .

We now define the Hamming distance, normalized Hamming distance, Euclidean distance, and normalized Euclidean distance between two QSVBNSs  $A, B \in \text{QSVBNS}(X)$ ,

(1) The Hamming distance:

$$\bullet d_H(A, B) = \sum_{k=1}^n (|T_A^P(x_k) - T_B^P(x_k)| + |C_A^P(x_k) - C_B^P(x_k)| + |U_A^P(x_k) - U_B^P(x_k)| + |F_A^P(x_k) - F_B^P(x_k)| + |T_A^N(x_k) - T_B^N(x_k)| + |C_A^N(x_k) - C_B^N(x_k)| + |U_A^N(x_k) - U_B^N(x_k)| + |F_A^N(x_k) - F_B^N(x_k)|),$$

(2) Normalized Hamming distance:

$$\bullet d_{NH}(A, B) = \frac{1}{8n} d_H(A, B),$$

(3) The Euclidean distance:

$$\bullet d_E(A, B) = \left( \sum_{k=1}^n (|T_A^P(x_k) - T_B^P(x_k)|^2 + |C_A^P(x_k) - C_B^P(x_k)|^2 + |U_A^P(x_k) - U_B^P(x_k)|^2 + |F_A^P(x_k) - F_B^P(x_k)|^2 + |T_A^N(x_k) - T_B^N(x_k)|^2 + |C_A^N(x_k) - C_B^N(x_k)|^2 + |U_A^N(x_k) - U_B^N(x_k)|^2 + |F_A^N(x_k) - F_B^N(x_k)|^2) \right)^{\frac{1}{2}},$$

(4) Normalized Euclidean distance:

$$\bullet d_{NE}(A, B) = \frac{d_E(A, B)}{2n\sqrt{2}}.$$

### 5. QBN-Multi-Criteria Decision Making Method

**Definition 18.** Let  $U = (u_1, u_2, \dots, u_m)$  be a set of alternatives,  $A = (a_1, a_2, \dots, a_n)$  be the set of attributes,  $w = (w_1, w_2, \dots, w_n)^T$  be the weight vector assigned to  $a_j$  ( $j = 1, 2, \dots, n$ ) such that  $w_k \geq 0$  and  $\sum_{k=1}^n w_k = 1$ .

Let  $[b_{ij}]_{m \times n} = \langle T_{ij}^P, C_{ij}^P, U_{ij}^P, F_{ij}^P, T_{ij}^N, C_{ij}^N, U_{ij}^N, F_{ij}^N \rangle$  represents the rating values of the alternatives in term of QSVBNS.

Then, 
$$\begin{pmatrix} & a_1 & a_2 & \cdots & a_n \\ u_1 & b_{1,1} & b_{1,2} & \cdots & b_{1,n} \\ u_2 & b_{2,1} & b_{2,2} & \cdots & b_{2,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_m & b_{m,1} & b_{m,2} & \cdots & b_{m,n} \end{pmatrix}$$
 is called the QBN-multi-attribute decision making matrix.

The positive ideal QBN solution of the decision matrix  $[b_{ij}]_{m \times n}$  is defined as:  $\bar{b}_j^* = \langle \max_i \{T_{ij}^P\}, \max_i \{C_{ij}^P\}, \min_i \{U_{ij}^P\}, \min_i \{F_{ij}^P\}, \min_i \{T_{ij}^N\}, \min_i \{C_{ij}^N\}, \max_i \{U_{ij}^N\}, \max_i \{F_{ij}^N\} \rangle$ . The negative ideal QBN solution of the decision matrix  $[b_{ij}]_{m \times n}$  is defined as:

$$\underline{b}_j^* = \langle \min_i \{T_{ij}^P\}, \min_i \{C_{ij}^P\}, \max_i \{U_{ij}^P\}, \max_i \{F_{ij}^P\}, \max_i \{T_{ij}^N\}, \max_i \{C_{ij}^N\}, \min_i \{U_{ij}^N\}, \min_i \{F_{ij}^N\} \rangle.$$

We now propose an algorithm based on the quadripartitioned weighted similarity measure to select the best alternative for multi-attribute decision making problem in quadripartitioned bipolar neutrosophic environment which is given in Algorithm 1 :

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**Algorithm 1: Algorithm based on The quadripartitioned weighted similarty measure**

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- Step 1. Give the QBN-multi-attribute decision making matrix  $[b_{ij}]_{m \times n}$  to the decision maker.
- Step 2. Compute the positive ideal QBN solution  $\bar{b}_j^*$  and negative ideal QBN solution  $\underline{b}_j^*$  for the decision matrix  $[b_{ij}]_{m \times n}$ .
- Step 3. Determine  $\mathcal{T}_2^w(\bar{b}_j^*, b_i)$  for  $j = 1, 2, \dots, m$ , the weighted quadripartitioned similarity measure between positive ideal solution  $\bar{b}_j^*$  and  $b_i = [b_{ij}]_{1 \times n}$  for  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$  and  $\mathcal{T}_2^w(\underline{b}_j^*, b_i)$  for  $j = 1, 2, \dots, m$ , the weighted quadripartitioned similarity measure between negative ideal solution  $\underline{b}_j^*$  and  $b_i = [b_{ij}]_{1 \times n}$  for  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$  as:

- $\mathcal{T}_2^w(\bar{b}_j^*, b_i) = 1 - \left[ \frac{1}{n} \sum_{k=1}^n w_k \left( \frac{1}{4} (\delta_1^{\bar{b}_j^*, b_i}(x_k) + \delta_2^{\bar{b}_j^*, b_i}(x_k) + \lambda_1^{\bar{b}_j^*, b_i}(x_k) + \lambda_2^{\bar{b}_j^*, b_i}(x_k)) \right)^p \right]^{\frac{1}{p}}$ ,
- $\mathcal{T}_2^w(\underline{b}_j^*, b_i) = 1 - \left[ \frac{1}{n} \sum_{k=1}^n w_k \left( \frac{1}{4} (\delta_1^{\underline{b}_j^*, b_i}(x_k) + \delta_2^{\underline{b}_j^*, b_i}(x_k) + \lambda_1^{\underline{b}_j^*, b_i}(x_k) + \lambda_2^{\underline{b}_j^*, b_i}(x_k)) \right)^p \right]^{\frac{1}{p}}$ ,

- Step 4. Figure out the non-increasing order of the average ideal solution,  $\frac{\mathcal{T}_2^w(\bar{b}_j^*, b_i) + \mathcal{T}_2^w(\underline{b}_j^*, b_i)}{2}$  for  $j = 1, 2, \dots, m$  and select the best alternatives.
- 

### 6. Illustrative Example

The following group decision making problem, which has been studied by Wu et al. [30], is taken into consideration in a quadripartitioned bipolar neutrosophic environment. Climate change in a global environment is a worrying sign. Industries have shifted their focus toward green production. A car company is eager to choose the most suitable green supplier for one of the key elements in its manufacturing process. After the pre-evaluation, four suppliers  $A_i$ , ( $i = 1, 2, 3, 4$ ), have been short-listed for evaluation on the basis of the concerned criteria:  $a_1$ : is the product quality,  $a_2$ : is technological capability, and  $a_3$ : is pollution control. The weight vector of the concerned criteria are  $\{w_1, w_2, w_3\}^T = \{0.3, 0.3, 0.4\}^T$ . To determine the decision information an expert is appointed to gather the criteria values for the four possible alternatives in a QBN environment which is given in Algorithm 2.

**Algorithm 2: Decision making algorithm for the four possible alternatives in a QBN environment**

Step 1. The decision matrix  $[b_{ij}]_{4 \times 3}$  given by the expert shown in Table 1.

Step 2. The positive ideal quadripartitioned bipolar neutrosophic solution and negative ideal quadripartitioned bipolar neutrosophic solutions are calculated as:  $\bar{\mathcal{A}}^* = [\langle 0.8, 0.6, 0.2, 0.2, -0.6, -0.6, -0.4, -0.4 \rangle, \langle 0.9, 0.4, 0.2, 0.2, -0.5, -0.3, -0.2, -0.4 \rangle, \langle 0.9, 0.6, 0.4, 0.3, -0.5, -0.4, -0.1, -0.3 \rangle]$  and  $\underline{\mathcal{A}}^* = [\langle 0.4, 0.2, 0.5, 0.5, -0.2, -0.3, -0.7, -0.8 \rangle, \langle 0.6, 0.1, 0.6, 0.5, -0.1, -0.2, -0.5, -0.6 \rangle, \langle 0.6, 0.1, 0.5, 0.7, -0.1, -0.2, -0.6, -0.8 \rangle]$

Step 3. The weighted quadripartitioned similarity measure  $\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_i)$ ,  $i = 1, 2, 3, 4$ , and  $\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_i)$ ,  $i = 1, 2, 3, 4$  (shown in Table 2) are computed.

**Table 1.** Decision making matrix.

	$a_1$	$a_2$	$a_3$
$\mathcal{A}_1$	$\langle 0.4, 0.5, 0.3, 0.2, -0.6, -0.4, -0.5, -0.7 \rangle$	$\langle 0.6, 0.1, 0.2, 0.3, -0.4, -0.3, -0.2, -0.5 \rangle$	$\langle 0.8, 0.6, 0.5, 0.7, -0.3, -0.2, -0.1, -0.4 \rangle$
$\mathcal{A}_2$	$\langle 0.6, 0.4, 0.2, 0.5, -0.4, -0.5, -0.7, -0.8 \rangle$	$\langle 0.6, 0.2, 0.3, 0.4, -0.5, -0.2, -0.3, -0.4 \rangle$	$\langle 0.7, 0.4, 0.5, 0.6, -0.1, -0.3, -0.4, -0.5 \rangle$
$\mathcal{A}_3$	$\langle 0.7, 0.2, 0.4, 0.3, -0.2, -0.6, -0.4, -0.5 \rangle$	$\langle 0.9, 0.3, 0.6, 0.5, -0.2, -0.2, -0.5, -0.4 \rangle$	$\langle 0.6, 0.1, 0.5, 0.4, -0.2, -0.4, -0.6, -0.3 \rangle$
$\mathcal{A}_4$	$\langle 0.8, 0.6, 0.5, 0.3, -0.5, -0.3, -0.6, -0.4 \rangle$	$\langle 0.6, 0.4, 0.3, 0.2, -0.1, -0.3, -0.4, -0.6 \rangle$	$\langle 0.9, 0.6, 0.4, 0.3, -0.5, -0.3, -0.6, -0.8 \rangle$

**7. Discussion**

In this section, a detailed analysis about the above introduced algorithm and the obtained result based on the algorithm is carried through. The weighted quadripartitioned similarity measure between the alternatives ( $\mathcal{A}_j$ ,  $j = 1, 2, 3, 4$ ) and the positive ideal quadripartitioned bipolar neutrosophic solution is shown in Table 2 and the same with the negative ideal quadripartitioned bipolar neutrosophic solution is given in Table 2. The main objective of the proposed algorithm i.e., the average ideal solution is obtained. The ranking results based on these similarity measures is highlighted in the final Table (Table 3).

Deli et al. [28] and Sahin et al. [27], in their study on bipolar neutrosophic sets, obtained the ranking result without considering the negative ideal solution. From their algorithm, it can be seen that if the decision maker is asked to choose an alternative emphasizing more on the satisfaction degree of the given criteria than the satisfaction degree of the counter-property of the criteria, the similarity measure with the positive ideal solution can be used to get the most suitable alternatives. In the reverse case, the similarity measure with negative ideal solution will be more fruitful.

In the case of QSVBNS, our algorithm follows the same footsteps as Deli et al. [28]; however, the major difference is that we have used the average of the measure values of positive ideal solution and negative ideal solution. From the proposed algorithm, it seems that the ranking results to choose the best alternatives using a positive ideal solution and a negative ideal solution will give exactly the opposite ranking, but this is not the case, as can be seen in Table 2. The similarity index  $p$  in the weighted quadripartitioned similarity measure also has a big role to play. From the study, we have found that as the similarity index starts to take more higher values, the formula predicts more accurately, as the difference between the first two choices of the alternatives starts increasing for higher values of  $p$ . From Figure 2, it is observed that for  $p = 1, 2$ , the alternative values are quite close and  $\mathcal{A}_3$  comes out as the best alternative with a very small margin from  $\mathcal{A}_1$ . For  $p = 3$ ,  $\mathcal{A}_1$  overtakes  $\mathcal{A}_3$  as the best alternative and as  $p$  starts increasing,  $\mathcal{A}_1$  remains the best among the four alternatives and the margin of the second best choice  $\mathcal{A}_3$  starts increasing. The differences between the alternative values is shown in Figure 2 for  $p = 1, 2, 3$ . In Figure 3, it can be observed that for  $p = 4, 5, 6$ ,  $\mathcal{A}_1$  enlarge by a

bit more margin than  $\mathcal{A}_3$ . In Figure 4, the comparison between the best two alternatives  $\mathcal{A}_1$  and  $\mathcal{A}_3$  is shown. It can be seen that for a very large value of  $p$ ,  $\mathcal{A}_1 - \mathcal{A}_3 \approx 0.1$ .

Table 2. Similarity measures for different values of  $p$ .

Similarity Index	Positive Ideal Solution	Negative Ideal Solution	Average Ideal Solution
$p = 1$	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_1) = 0.952416$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_1) = 0.94$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_1) = 0.946208$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_2) = 0.929125$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_2) = 0.954604$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_2) = 0.941864$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_3) = 0.936681$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_3) = 0.960833$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_3) = 0.948757$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_4) = 0.957291$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_4) = 0.928791$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_4) = 0.943041$
$p = 2$	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_1) = 0.917337$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_1) = 0.895453$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_1) = 0.906395$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_2) = 0.874914$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_2) = 0.921020$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_2) = 0.897967$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_3) = 0.889566$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_3) = 0.927222$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_3) = 0.908394$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_4) = 0.923766$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_4) = 0.870422$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_4) = 0.897094$
$p = 3$	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_1) = 0.900428$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_1) = 0.873755$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_1) = 0.887092$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_2) = 0.847426$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_2) = 0.904713$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_2) = 0.876069$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_3) = 0.866476$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_3) = 0.907610$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_3) = 0.887043$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_4) = 0.905738$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_4) = 0.839067$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_4) = 0.872402$
$p = 4$	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_1) = 0.890553$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_1) = 0.860959$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_1) = 0.875756$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_2) = 0.830665$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_2) = 0.895086$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_2) = 0.862875$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_3) = 0.852716$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_3) = 0.894353$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_3) = 0.873535$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_4) = 0.893862$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_4) = 0.819565$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_4) = 0.856714$
$p = 5$	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_1) = 0.884025$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_1) = 0.852444$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_1) = 0.868235$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_2) = 0.819257$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_2) = 0.888633$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_2) = 0.853945$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_3) = 0.843424$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_3) = 0.884681$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_3) = 0.864052$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_4) = 0.885108$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_4) = 0.806328$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_4) = 0.845718$
$p = 6$	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_1) = 0.879341$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_1) = 0.846318$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_1) = 0.862830$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_2) = 0.810950$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_2) = 0.883926$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_2) = 0.847438$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_3) = 0.836617$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_3) = 0.877287$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_3) = 0.856952$
	$\mathcal{T}_2^w(\bar{\mathcal{A}}^*, \mathcal{A}_4) = 0.878257$	$\mathcal{T}_2^w(\underline{\mathcal{A}}^*, \mathcal{A}_4) = 0.796801$	$\mathcal{T}_2^w(\mathcal{A}^*, \mathcal{A}_4) = 0.837529$

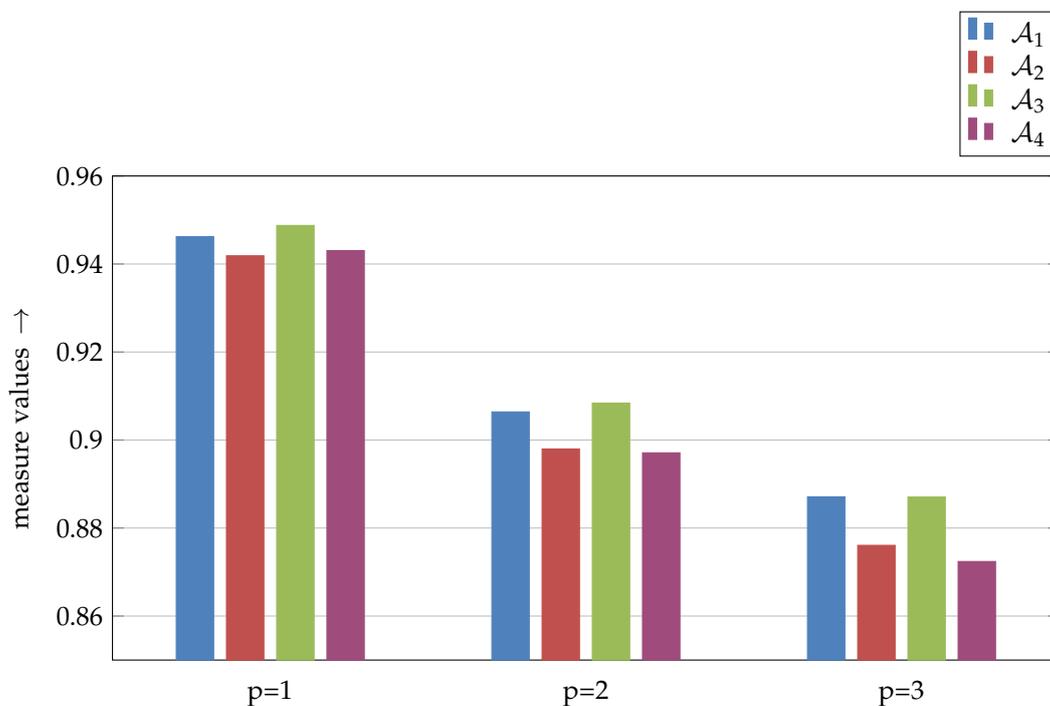


Figure 2. Comparison between the alternatives for different similarity indexes.

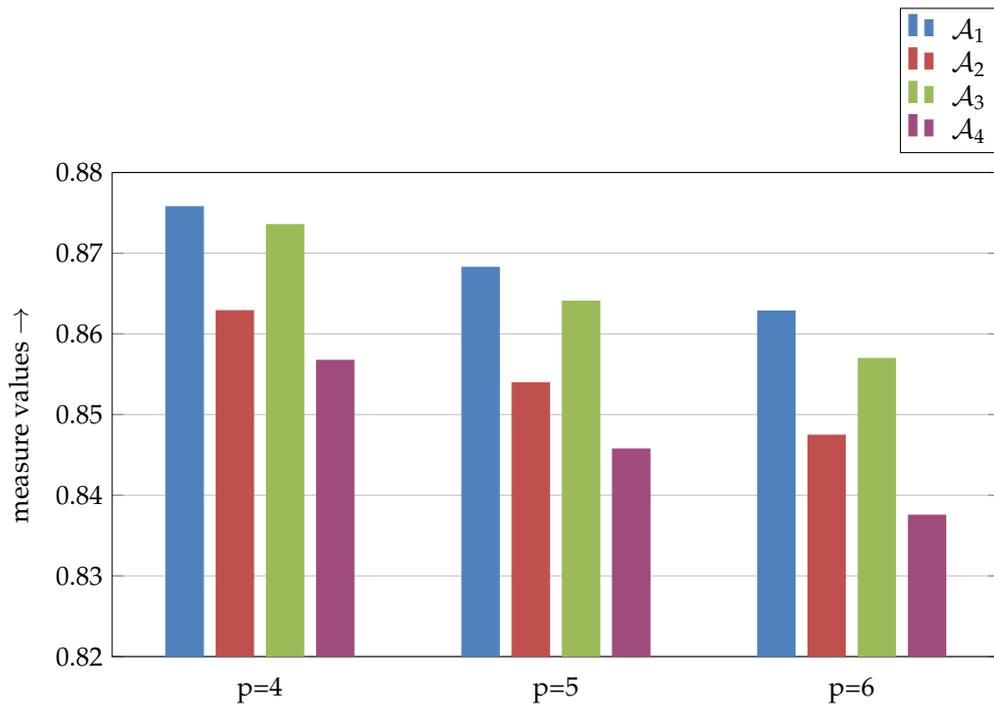


Figure 3. Comparison between the alternatives for different similarity indexes.

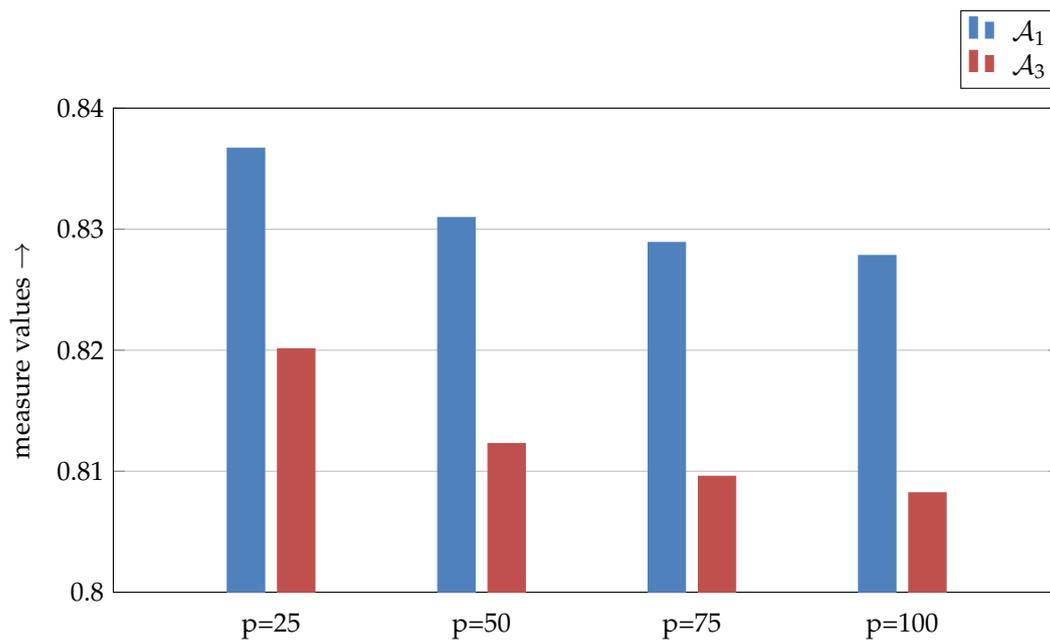


Figure 4. Comparison between the best two alternatives  $A_1$  and  $A_3$  for some higher similarity index.

From the above analysis it is clear that  $A_1$  is the most suitable alternative to the decision maker. The proposed method can get the better of the decision making problem with quadripartioned bipolar neutrosophic information. Also, the higher similarity index produces a more accurate method by indicating clear differences between the alternatives. A comparison of the above discussed MCDM problem is made in a fuzzy system and a bipolar fuzzy system, where it is observed that in a fuzzy system, the same conclusion is reached at  $p = 5$  and in bipolar fuzzy system the result fluctuate between  $A_1$  and  $A_3$ . So, we think that the proposed method can serve immensely in decision making purpose.

Table 3. Ranking results.

Similarity Index	Based on Positive Ideal Solution	Based on Negative Ideal Solution	Based on Average Value
$p = 1$	$\mathcal{A}_4 \succ \mathcal{A}_1 \succ \mathcal{A}_3 \succ \mathcal{A}_2$	$\mathcal{A}_3 \succ \mathcal{A}_2 \succ \mathcal{A}_1 \succ \mathcal{A}_4$	$\mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_4 \succ \mathcal{A}_2$
$p = 2$	$\mathcal{A}_4 \succ \mathcal{A}_1 \succ \mathcal{A}_3 \succ \mathcal{A}_2$	$\mathcal{A}_3 \succ \mathcal{A}_2 \succ \mathcal{A}_1 \succ \mathcal{A}_4$	$\mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_2 \succ \mathcal{A}_4$
$p = 3$	$\mathcal{A}_4 \succ \mathcal{A}_1 \succ \mathcal{A}_3 \succ \mathcal{A}_2$	$\mathcal{A}_3 \succ \mathcal{A}_2 \succ \mathcal{A}_1 \succ \mathcal{A}_4$	$\mathcal{A}_1 \succ \mathcal{A}_3 \succ \mathcal{A}_2 \succ \mathcal{A}_4$
$p = 4$	$\mathcal{A}_4 \succ \mathcal{A}_1 \succ \mathcal{A}_3 \succ \mathcal{A}_2$	$\mathcal{A}_2 \succ \mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_4$	$\mathcal{A}_1 \succ \mathcal{A}_3 \succ \mathcal{A}_2 \succ \mathcal{A}_4$
$p = 5$	$\mathcal{A}_4 \succ \mathcal{A}_1 \succ \mathcal{A}_3 \succ \mathcal{A}_2$	$\mathcal{A}_2 \succ \mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_4$	$\mathcal{A}_1 \succ \mathcal{A}_3 \succ \mathcal{A}_2 \succ \mathcal{A}_4$
$p = 6$	$\mathcal{A}_1 \succ \mathcal{A}_4 \succ \mathcal{A}_3 \succ \mathcal{A}_2$	$\mathcal{A}_2 \succ \mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_4$	$\mathcal{A}_1 \succ \mathcal{A}_3 \succ \mathcal{A}_2 \succ \mathcal{A}_4$

## 8. Conclusions

This paper introduces a novel concept of QSVBNS. Some set theoretic operations and several similarity measures have been stated. Also, a real life example of QSVBNS is presented for brief understanding. Decision making problem has been considered in a QSVBN environment and dealt with successfully.

BNS has been a successful tool for decision maker as bipolarity often occurs as a common phenomenon in human thinking. QSVBNS comes handy in situations where a person is unsure about the truth, false, both true and false or neither of them while handling a bipolar information system. The introduction of QSVBNS will help the cause of decision making problems and open new branch of neutrosophy. Future work may involve study of aggregation operators involving QSVBNS while dealing with decision making problems.

**Author Contributions:** Created and conceptualized ideas, S.R. and J.-G.L.; writing—original draft preparation, S.R.; writing—review and editing, A.P. and S.K.S.; funding acquisition, J.-G.L. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was partially supported by a Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2018R1D1A1B07049321).

**Conflicts of Interest:** The authors declare no conflict of interest.

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