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Correction

Correction: Liu, P., et al. Threshold Analysis and Stationary Distribution of a Stochastic Model with Relapse and Temporary Immunity. *Symmetry* 2020, *12*, 331

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Received: 7 April 2020; Accepted: 8 April 2020; Published: 15 April 2020

The authors wish to make the following corrections and explanations to this paper [1]:

(1) The authors in [2] used the integral Markov semigroup theory to prove the existence of a stationary distribution of a stochastic system, thus paper [1] should cite [2] on page 11, lines 5–6, although the article [2] has been previously cited in [1] as Reference 14. Consequently, the authors wish to correct "To handle this situation, we apply the integral Markov semigroup theory, presented in [24,25]." to "To handle this situation, we apply the integral Markov semigroup theory, presented in [14,24,25]."

(2) We have found two inadvertent errors on page 15, lines 2 and 5 in the paper [1]. We would like to make the correction: $LV_a(I,R)$ should be corrected to $LV_2(I,R)$, and $1-R_S^0$ should be corrected to $1-\frac{1}{R_S^0}$.

(3) We explain the differences between articles [1,2] which has been cited in [1] as Reference 14.

1. Different Models

The model of article [1] is:

$$\begin{cases} dS = \Big[\mu-\mu S-p\beta_1 SI-(1-p)\frac{\beta_2 SI}{1+mS+nI}+\gamma_2 R\Big]dt-p\sigma_1 SIdB_1(t)-(1-p)\frac{\sigma_2 SI}{1+mS+nI}dB_2(t), \\ dI = \Big[p\beta_1 SI+(1-p)\frac{\beta_2 SI}{1+mS+nI}-(\mu+\eta)I+\gamma_1 R\Big]dt+p\sigma_1 SIdB_1(t)+(1-p)\frac{\sigma_2 SI}{1+mS+nI}dB_2(t), \\ dR = [\eta I-(\mu+\gamma_1+\gamma_2)R]dt. \end{cases} \tag{1}$$

The model of article [2] is:

$$\begin{cases} dS = (\mu-\mu_1 S-\beta Sf(I)+\varepsilon I+\gamma R)dt-\sigma Sf(I)dB(t), \\ dI = [\beta Sf(I)-(\mu_2+\varepsilon+\lambda)I+\sigma R]dt+\sigma Sf(I)dB(t), \\ dR = (\lambda I-(\mu_3+\gamma+\sigma)R)dt. \end{cases} \tag{2}$$

The main difference between the two models is that (i) the model (1) mainly considers two different infectious forms, namely bilinear incidence rate and Beddington–DeAngelis (BD) saturated incidence rate, and the BD incidence rate here is different from the general incidence rate in [2], and $\beta Sf(I)$ the incidence rate considered in Reference 14 cannot include the BD saturated incidence rate in [1] $(1-p)\frac{\beta_2 SI}{1+mS+nI}$, because $1+mS+nI$, the denominator of BD saturated incidence rate contains S and I. (ii) we also consider two different kinds of white noise interference, namely, $p\sigma_1 SIdB_1(t)$ and $(1-p)\frac{\sigma_2 SI}{1+mS+nI}dB_2$, which are different from one white noise interference in [2], $\sigma Sf(I)dB(t)$.

2. The Different Purposes of Researches

The main purpose of our article [1] is to study the comprehensive effects of two different infection rate functions and two different kinds of white noise interference on stochastic dynamics. Therefore, it also brings some difficulties to the proof and analysis of our theorems, which is also the main purpose of our article.

3. The Different Specific Techniques for the Proof of Theorems

The specific research techniques of the two papers are different due to the different models. Especially in the proving process of theorems 1–7, the constructed function V and the differential of the V function dV are different. Moreover, in the mathematical derivation process, in order to deal with the nonlinear terms of BD saturated incidence rate and white noise interference, some differential inequality techniques are required.

4. The Different Conditions of the Theorems and Results

The conditions of the theorems in the two articles [1,2] are also different, especially the thresholds R_0 and R_S^0.

The thresholds R_0 and R_S^0 in article [2] are

$$R_0 = \frac{\beta\mu f'(0)}{\mu_1\left(\mu_2 + \varepsilon + \lambda - \frac{\sigma\lambda}{\mu_3+\gamma+\sigma}\right)} \tag{3}$$

$$R_S^0 = R_0 - \frac{\rho^2\left(\frac{\mu}{\mu_1} f'(0)\right)^2}{2\left(\mu_2 + \varepsilon + \lambda - \frac{\sigma\lambda}{\mu_3+\gamma+\sigma}\right)}. \tag{4}$$

The thresholds R_0 and R_S^0 in article [1] are

$$R_0 = \frac{\beta_1 p + \beta_2(1-p)}{\mu + \eta + \frac{1}{2}\sigma_1^2 p^2 + \frac{1}{2}\sigma_2^2(1-p)^2 - \frac{\gamma_1\eta}{\mu+\gamma_1+\gamma_2}}, \tag{5}$$

$$R_S^0 = \frac{p\beta_1 + \frac{(1-p)\beta_2}{1+m+n}}{\mu + \eta + \frac{1}{2}\sigma_1^2 p^2 + \frac{1}{2}\sigma_2^2(1-p)^2 - \frac{\gamma_1\eta}{\mu+\gamma_1+\gamma_2}}. \tag{6}$$

The thresholds R_0 and R_S^0 in article [1] imply the comprehensive effects of two different infection rate functions and two different kinds of white noise interference on stochastic dynamics.

Both articles [1,2] used Ito's formula and semigroup theory, then the lemma, some mathematical symbols, mathematical formulas and some steps may be similar and identical. Therefore, we gained some knowledge from [2] as Reference 14 of [1], and cited it. Here, we thank the authors of all References, especially [2].

The authors would like to apologize for any inconvenience caused to the readers by these changes.

References

1. Liu, P.; Meng, X.; Qi, H. Threshold Analysis and Stationary Distribution of a Stochastic Model with Relapse and Temporary Immunity. *Symmetry* **2020**, *12*, 331. [CrossRef]
2. Fatini, M.E.; Khalifi, M.E.; Gerlach, R.; Laaribi, A.; Taki, R. Stationary distribution and threshold dynamics of a stochastic SIRS model with a general incidence. *Phys. A* **2019**, *534*, 120696. [CrossRef]