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Quantum-Gravity Screening Effect of the Cosmological Constant in the DeSitter Space–Time

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Abstract: Small-amplitude quantum-gravity periodic perturbations of the metric tensor, occurring in sequences of phase-shifted oscillations, are investigated for vacuum conditions and in the context of the manifestly-covariant theory of quantum gravity. The theoretical background is provided by the Hamiltonian representation of the quantum hydrodynamic equations yielding, in turn, quantum modifications of the Einstein field equations. It is shown that in the case of the DeSitter space–time sequences of small-size periodic perturbations with prescribed frequency are actually permitted, each one with its characteristic initial phase. The same perturbations give rise to non-linear modifications of the Einstein field equations in terms of a suitable stochastic-averaged and divergence-free quantum stress-energy tensor. As a result, a quantum-driven screening effect arises which is shown to affect the magnitude of the cosmological constant. Observable features on the DeSitter space–time solution and on the graviton mass estimate are pointed out.

Keywords: covariant quantum gravity; cosmological constant; DeSitter space–time; massive gravitons

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1. Introduction

A basic challenge in theoretical astrophysics and quantum gravity (QG) is about the search of possible quantum effects of space–time characterizing its microscopic QG-description, which are capable of influencing its large-scale behavior and global structure even in a cosmological scenario [1–6]. More precisely, the issue refers to the possible identification of quantum phenomena that might produce observable, namely macroscopic, modifications of space–time within the context of the Standard Formulation of General Relativity (SF-GR) based on the Einstein field equations (EFE). The goal of this paper refers more precisely to the identification of possible quantum screeening effects of the cosmological constant which arise in a deSitter space-time. The establishment of features that might affect the large-scale behavior of classical solutions of the same EFE expressed in terms of the structure of background space–time has been a subject of increasing interest in the scientific community (e.g., see Refs. [7–11]). In fact, over the past fifty years, a plethora of disparate approaches and theories have been devoted to quantum gravity. Concerning in particular the problems of the cosmological constant and the large-scale structure of the universe, a proper comparison among them can only be based on observations, i.e., the application of such theories to the description of observational data. The issue is important primarily for resolving existing questions in cosmology and astronomy, like for example the problems of dark matter and dark energy, for which conclusions and particular solutions applying to the DeSitter space–time might provide a relevant framework [12,13].

That such a problem, together with the goal anticipated above, makes sense at all is not even obvious and has remained a challenging open issue since the early days of QG which, following the original attempt by Dirac [14], led to the formulation of canonical Hamiltonian theory of GR exemplified by the famous 3+1 ADM representation of classical GR (Arnowitt, Deser and Misner 1962 [15]) and the subsequent identification of the quantum Wheeler–DeWitt equation [16]. The same equation, which prescribes the so-called "universe" quantum state function ψ, has become since then very popular in the specialized literature. In fact, it underlies most of quantization approaches to gravity formulated in the last fifty years, which include in particular canonical quantization theory [16] and loop quantum gravity [17,18].

Despite of this, the question of identifying the appropriate representation or set up for QG theory remains open. In fact, the Wheeler–DeWitt equation suffers from numerous shortcomings, which concern for example divergences due to functional derivatives and operator ordering ambiguities [19]. However, in the following we discuss three of them which are most pertinent in connection with the content of the theory reported in this paper. *Shortcoming #1* is the so-called problem of time, namely the fact that the Wheeler–DeWitt equation plays the role of a constraint relation on the wave function and is not represented as a first-order hyperbolic PDE with respect to a suitable dynamical time variable, as is the case of the Schroedinger equation in quantum mechanics [20]. Indeed, the absence of an external time parameter is not characteristic of the Wheeler–DeWitt equation itself but already appears at the classical level in the ADM constrained Hamiltonian formulation of GR (in the sense of Dirac). Although a time-like parameter can be recovered at the quantum level in various ways, its definition carries an additional conceptual problem. In fact, in the absence of a background space–time and of manifest covariance of the Wheeler–DeWitt theory, the same coordinate-time appears simultaneously as the dynamical parameter and a component of space–time which must be quantized by solving the same equation. In such a framework, the problem of time is included here as a shortcoming of the Wheeler–DeWitt equation which is distinguished from CQG-theory on this issue. As a consequence of its definition, the same equation holds therefore only for stationary solutions, in the sense of absence of dynamics with respect to an invariant time-parameter to be properly identified and distinguished by a coordinate-time of some sort. It is precisely this feature which makes the Wheeler–DeWitt equation unsuitable for investigating non-stationary quantum phenomena of the type considered in this paper and which arise due to non-equilibrium initial conditions of the quantum-wave equation, for which the problem of time is resolved through the definition of an explicit invariant evolution parameter. The same feature has apparently led Isham to give the following strong statement: "*...although it may be heretical to suggest it, the Wheeler–DeWitt equation—elegant though it be—may be completely the wrong way of formulating a quantum theory of gravity*" [21].

However, there are further serious shortcomings of the Wheeler–DeWitt equation raising, in turn, the question of its validity. Let us briefly point out and critically examine some of them. A second one is that, unlike what happens in standard non-relativistic Quantum Mechanics (QM [22]), for the same equation $|\psi|^2$ is not a probability density (*Shortcoming #2*). The implication is serious because, as a consequence, the issue of quantum unitarity, i.e., conservation of quantum probability, cannot be posted in such a context. The feature appears counter-intuitive and unphysical. Indeed, the property of unitarity, or respectively non-unitarity, in QM is usually attributed to the probability of existence or respectively decay of a quantum state.

Finally, there is a third aspect of the Wheeler–DeWitt equation which becomes crucial in the present context. This is related to the (missing) property of manifest covariance, in turn, based on the assumption that physical laws should be the same in the whole universe. In fact, any consistent physical theory (and indeed any physical law expressed in terms of an equation of some sort) should be the same in any part of the universe, and hence for this to be true the same theories should be intrinsically coordinate- (i.e., GR-) frame independent. In other words, the same theory/equation should be covariant with respect to the group of 4−dimensional local point transformations realized by diffeomeorphism of the type

$$r \Longleftrightarrow r' = r'(r), \tag{1}$$

which transform in each other two arbitrary coordinate systems (or GR-frames), while leaving invariant the differential-manifold structure of the background space–time [23]. It should be clear that here the focus is on the property of manifest covariance, which represents a more stringent requirement than that of general covariance. In fact, general covariance still underlies the Wheeler–DeWitt equation, in the same way it does at the classical level for ADM theory when a preferred reference system has been introduced upon invoking space–time foliation, consistent with the asynchronous Lagrangian formulation leading to EFE. In particular, the Hamiltonian constraint and momentum constraint of the theory respectively impose the time and space diffeomorphism invariance, even though separately, and the corresponding Hamiltonian equations of motion warrant the diffeomorphism invariance for the remaining six degrees of freedom. However, precisely because of its foundational derivation, and in contrast with the basic principle of the DeDonder–Weyl formalism, the Wheeler–DeWitt equation is intrinsically not manifestly covariant (Shortcoming #3), in the sense that it is not written in 4−tensor notation. The reason why this happens is that it relies on a preliminary 3+1 foliation of space–time. This means that the same foliation is preserved only for the subset of point transformations among GR-frames which do not mix time and space coordinates (a feature which therefore rules out by itself manifest covariance). Furthermore, by construction it is based on a non-tensor prescription of the classical and quantum canonical variables, according to which canonical variables do not preserve their tensor form under the group or local point transformations (1).

In our view, this aspect of the Wheeler–DeWitt equation should not be overlooked. The precise reasons why manifest covariance is crucial in the present context is two-fold:

- The first one concerns the representation of the quantum-wave equation, which should be cast in a manifestly-covariant form. A prerequisite for this to happen is, of course, that the quantum canonical variables (i.e., both the Lagrangian coordinates and the conjugate canonical momentum operators) should be expressed in 4−tensor form.
- The second one is that QG-theory should be such to permit a second-quantization theory for the gravitational field in the sense indicated above. More precisely, QG-theory should be capable of prescribing also the non-linear quantum modifications of the background space–time. Thus, besides prescribing the universe quantum state function ψ, QG-theory should predict also the related quantum-modified form of EFE which determines the background metric field tensor. As a consequence, the same equation—just as the original EFE - should be realized by means of a tensor (and therefore frame-independent) PDE, so that it must preserve its form with respect to the group of coordinate transformations (1) between two arbitrary GR-frames.

The implication is therefore that QG-theory should admit a frame (i.e., coordinate) independent character so to result intrinsically manifestly covariant in form. This should, therefore, be regarded as a mandatory property ultimately stemming from SF-GR itself.

Incidentally, all such requirements are not permitted or are violated in the case of the Wheeler-DeWitt equation. However, a new axiomatic (i.e., first-principle) approach has been recently achieved which altogether avoids the previous shortcomings and thus, in these respects, yields a self-consistent theory of QG. This is realized by the theory of manifestly-covariant quantum gravity (CQG-theory) proposed in Refs. [24,25], which is based on the discovery of classical and corresponding non-perturbative quantum Hamiltonian structures of General Relativity (GR) which are mutually related by means of standard covariant canonical quantization methods (see Ref. [26]). Although it is not within the scope of the present work to prove and justify the new manifestly-covariant Hamiltonian formalism for GR reported in Refs. [24,25], it is nevertheless instructive to clarify its scope. Indeed, such Hamiltonian formalism differs both in form (e.g., no superspace, no functional quantities, etc.) and content (different equation of motion) from the ADM one at the classical level and from the canonical one of DeWitt at the quantum level. However, remarkably, it remains fully consistent with the customary manifestly-covariant Lagrangian formulation of GR and EFE. At the classical level the fundamental departure between the two approaches lies primarily in the choice of variational principles on which they are built on. These are labeled respectively as asynchronous and synchronous ones (according to the nomenclature introduced in Ref. [27]), depending on the way in which the $4-$volume element $d\Omega = \sqrt{-g}d^4x$ (with g denoting the determinant of the metric tensor) is treated during variations.

Indeed, according to the original Einstein's approach, EFE is determined in terms of the Einstein-Hilbert asynchronous variational principle, namely in which the invariant volume element $d\Omega$ is considered variational. As a consequence, by construction the corresponding Lagrangian density is not a $4-$scalar, since it carries the quantity $\sqrt{-g}$. In such a framework the variational metric tensor exhibits the same geometrical and physical qualitative properties of the extremal one, so that in particular, it is allowed to raise and lower tensor indices (geometrical property) while it has an identically-vanishing covariant derivative (physical property). We stress that the canonical Hamiltonian theory of GR which leads to the ADM formulation (and determined by the same Einstein–Hilbert asynchronous variational principle) has analogies with non-relativistic classical mechanics where time is considered as a separate coordinate (absolute time). Indeed it demands a preliminary 3+1 foliation of space–time where again coordinate-time is singled out to parametrize dynamical evolution. This requires in fact that canonical variables, in particular conjugate momenta, are prescribed explicitly in terms of the same coordinate-time so that they actually realize non-tensor quantities. As a consequence, it is obvious that the resulting classical and quantum Hamiltonian structures of the theory which are determined in this way become intrinsically not manifestly-covariant.

However, it is well-known that for continuum-field theory an alternative Hamiltonian formulation exists, which is well-established in mathematical physics and dates back to the pioneering work by DeDonder (1930, [28]) and Weyl (1935, [29]). Such an approach is known in the literature as DeDonder–Weyl variational formulation, although it has also been subsequently denoted as "multi-symplectic" or "poly-symplectic field theory" (see for example Refs. [30–33]). Its basic requirement is the validity of the principle of manifest covariance of variational theory, whereby both Lagrangian density function and variational fields are represented by tensorial entities, a character also inherited by the corresponding Hamiltonian theory. The manifestly-covariant Hamiltonian theory of continuum field in DeDonder–Weyl formalism, therefore, arises as an intrinsic property of field theory expressing the existence of an underlying Hamiltonian structure of field dynamics, which is complementary to the non-manifestly-covariant one.

The implementation of the DeDonder–Weyl formalism for GR requires however the adoption of a variational principle warranting the validity of manifest covariance of the theory. In Ref. [27] this was found to be expressed by a synchronous Lagrangian variational principle of the type indicated above, i.e., in which by construction the invariant $4-$volume element $d\Omega$ and the factor $\sqrt{-g}$ in the Lagrangian density are considered constant in the evaluation of the functional derivative. A basic feature of the new variational approach is the use of superabundant variables. These are now identified with the components of the variational symmetric metric tensor $g_{\mu\nu}$, with properties to be distinguished from those of the background metric tensor (or extremal one) $\widehat{g}_{\mu\nu}$. In particular, this means that $g_{\mu\nu}$ does not raise or lower tensor indices while exhibiting non-vanishing covariant derivatives (i.e., which ultimately are associated with the conjugate canonical momenta). Nevertheless, the identity $g_{\mu\nu} = \widehat{g}_{\mu\nu}$ is warranted for the extremal EFE, proving the equivalence of the synchronous variational principle with the Einstein–Hilbert one. However, the corresponding manifestly-covariant Hamiltonian theory of GR built upon such Lagrangian principle represents a novel Hamiltonian formulation of GR, which is admitted on physical grounds and remains distinguished from the ADM one, expressing a different Hamiltonian character of gravitational theory. Besides the fulfillment of manifest-covariance property, it affords also a Hamilton-Jacobi representation of classical GR and its quantization by implementing canonical quantization methods well established in quantum mechanics and field theory, leading to the formulation of CQG-theory. The analytical progress reached so far in such a framework distinguishes CQG-theory from canonical approaches, both for physical and mathematical aspects.

In particular, CQG-theory exhibits both a suitable invariant proper-time parametrization and a well-definite behavior with respect of the Quantum Unitarity Principle. More precisely:

(a) In the absence of quantum sinks, $|\psi|^2$ identifies a quantum probability density [26] which satisfies a quantum continuity equation. In this case $|\psi|^2$ identifies the quantum probability density of the existence of massive graviton particles.

(b) On the contrary, in the presence of quantum sinks $|\psi|^2$ might not be locally conserved, as investigated in Ref. [34]. In such a case, in fact, $|\psi|^2$ satisfies a modified quantum continuity equation, accounting for the decay of $|\psi|^2$ produced by the effective loss of gravitons (see related discussion in Section 4).

As a consequence of manifest covariance, the same Hamiltonian representations are cast in $4-$tensor frame-independent forms while the quantum probability density $|\psi|^2$ acquires the character of a physical observable and therefore necessarily coincides with a $4-$scalar. The obvious implication is that in such a context the traditional setting in terms of a preliminary $3+1$ foliation of space–time [35] becomes superfluous and hence can be actually avoided. The CQG-theory implies, in particular, the validity of $4-$tensor quantum Hamilton equations, to be intended as quantum hydrodynamic equations associated with the quantum wave function ψ, and prescribed in terms of a Hamiltonian hydrodynamic state $x = \left(g^{\mu\nu}, \Pi_{\mu\nu}\right)$, where $g^{\mu\nu}$ and $\Pi_{\mu\nu}$ denote independent $4-$tensor canonical variables identified respectively with the generalized Lagrangian coordinates and conjugate canonical momenta. The latter canonical theory has permitted the explicit construction of non-perturbative quantum-modified EFE [7,36], identified with a suitable stationary form of the quantum Hamiltonian equations obtained by imposing suitable "equilibrium" initial conditions to the Hamiltonian hydrodynamic state, characterized by having vanishing canonical momentum. This made possible also a novel quantum prescription of the cosmological constant (CC), generated by the non-linear Bohm potential associated with the gravitational field quantum-vacuum self-interaction occurring among massive gravitons. The study of the quantum-modified EFE has shown that a cosmological DeSitter solution for the corresponding background metric tensor $\widehat{g} \equiv \widehat{g}_{\mu\nu}$ remains warranted in terms of the quantum CC.

The purpose of this paper is to extend the outcome of Ref. [7], searching for additional quantum-gravity effects determined in the framework of CQG-theory that can further contribute physically-observable quantum modifications of the classical EFE. The goal is twofold. First, the investigation concerns the treatment of quantum effects arising due to the presence of non-equilibrium initial perturbations of the canonical hydrodynamic state in the quantum Hamilton equations, whereby both the quantum metric tensor and its canonical momentum are allowed to be non-vanishing. It is shown that this can generate additional quantum corrections to the EFE solving for the background metric tensor $\widehat{g} \equiv \widehat{g}_{\mu\nu}$. Such an equation generalizes the one obtained in Ref. [7] with the inclusion of a suitably-defined stochastic-averaged stress-energy tensor, and in view of its derivation procedure it is referred to as momentum quantum-modified EFE. The second target consists in pointing out the existence of a screening effect on the quantum CC generated by quantum oscillatory perturbations of the background metric tensor, and the study of the observable features that this could produce on the physical properties of space–time and the cosmological graviton mass estimate. More precisely, in the following we intend to show that:

(1) An ensemble of N small-amplitude quantum periodic oscillatory perturbations $\delta g_{\mu\nu}^{(i)}$ of the background metric tensor $\widehat{g}_{\mu\nu}$ can exist, for $i = 1, N$, which are of the form

$$\delta g_{\mu\nu}^{(i)} = f\left(s + \varphi_i\right) \widehat{g}_{\mu\nu}, \tag{2}$$

where $f(s+\varphi_i)$ is a periodic 4−scalar function of s with period $\tau > 0$ and φ_i, for $i = 1, N$, is a constant initial phase. Hence by construction $f(s+\varphi_i)$ is actually defined for arbitrary $s \in \mathbb{R}$.

(2) The background metric tensor $\widehat{g}_{\mu\nu}$ is a stationary solution of the corresponding quantum-modified EFE. Therefore, we denote by $\langle\rangle_s \equiv \frac{1}{\tau}\int_0^\tau ds$ the average performed on the explicit dependence in terms of the proper-time s only (which is considered as a parameter), while keeping constant the 4−position $r^\mu(s)$ along a geodesic curve. This means that while performing the s−averaging the variable s is considered independent (and hence takes in principle arbitrary real values), so that by construction $\widehat{g}_{\mu\nu} = \langle\widehat{g}_{\mu\nu}\rangle_s$, while it is assumed that $\left\langle \delta g^{(i)}_{\mu\nu}\right\rangle_s = 0$ for all $i = 1, N$.

(3) The quantum perturbations $\delta g^{(i)}_{\mu\nu}$ affect the prescription of the CC, which is shown to take the form

$$\Lambda_S = K\Lambda_{CQG}(s_o), \tag{3}$$

where $K < 1$ is the quantum screening factor depending on the proper-time averaged effect carried by quantum gravitational perturbations, so that Λ_S identifies the quantum-screened CC.

(4) The structure of the corresponding quantum-modified EFE now includes an additional source term produced by proper-time averaged perturbations. Nevertheless, the customary vacuum-field representation for the background Ricci tensor is recovered, namely

$$\widehat{R}_{\mu\nu} = \Lambda_S\widehat{g}_{\mu\nu}. \tag{4}$$

(5) As a consequence, the background metric tensor $\widehat{g}_{\mu\nu}$ determined in this way recovers again a DeSitter space–time structure solution, in which however the radius of the event horizon is effectively increased by the factor $\frac{1}{K}$ due to the screening of the CC, while the Ricci 4−scalar is correspondingly reduced with respect to its value in the absence of the same screening effect.

(6) Based on the quantum screening mechanism, a new estimate for the quantum-graviton rest-mass predicted to arise in the DeSitter space–time is performed. Assuming that the screening effect is affecting the experimental CC-estimate, then it is shown that the graviton mass prediction can be effectively increased by a factor $\frac{1}{\sqrt{K}}$ with respect to the one in the absence of screening.

2. Quantum-Modified Einstein Field Equations

In this section, the theoretical framework underlying the present research is recalled. The starting point is represented by the manifestly-covariant 4−scalar quantum-gravity wave equation (CQG-wave equation) obtained in Refs. [25,26] and in turn based on manifestly-covariant canonical quantization. Let us consider here for definiteness the case in which no quantum sink is present [7]. Then, consistent with such a requirement, one can readily prove that Hamilton–Jacobi quantization [26] permits us to identify its representation and interpretation which are formally analogous to those laying at the foundation of Standard Quantum Mechanics, i.e., of the Schroedinger equation. Its form is provided by a linear evolution PDE

$$i\hbar\frac{d}{ds}\psi(s) = H^{(q)}_R\psi(s), \tag{5}$$

with $\frac{d}{ds} = \left.\frac{d}{ds}\right|_s + \frac{\partial}{\partial s}$ denoting the covariant s−derivative in Eulerian form and $H^{(q)}_R$ a suitable gauge-dependent self-adjoint quantum Hamiltonian operator earlier reported in Ref. [25]. Here the notation is given according to the same references, so that in particular for arbitrary s belonging to the time axis $I \equiv \mathbb{R}$ and corresponding 4−position $r = r(s)$ along a field geodetics, $\psi(s) \equiv \psi(g, \widehat{g}, r, s)$ denotes the 4−scalar quantum wave function associated with a graviton particle belonging to $r = r(s)$. Furthermore, $g = \{g_{\mu\nu}\}$ is the quantum generalized-coordinate field which spans the 10−dimensional real vector space $U_g \subseteq \mathbb{R}^{10}$ of the same wave-function, i.e., the set on which the associated quantum probability density function $\rho(s) = |\psi(s)|^2$ (quantum PDF) is prescribed.

It is important to recall here two crucial features of CQG-theory earlier pointed out [7,24]. The first one lies in the distinction between the quantum tensor $g_{\mu\nu}$, which identifies the continuum Lagrangian coordinates carrying the quantum physical properties of the gravitational field, and the background metric tensor $\widehat{g}_{\mu\nu}$ which instead describes the geometry of space–time. By definition, the tensor $g_{\mu\nu}$ is such that $g^{\mu\nu}g_{\mu\nu} \neq \delta_{\mu}^{\mu}$, while identically the normalization condition $\widehat{g}^{\mu\nu}\widehat{g}_{\mu\nu} = 4$ applies to the classical field. Accordingly, the quantum field $g_{\mu\nu}$ is allowed to exhibit a quantum dynamical behavior which deviates from $\widehat{g}_{\mu\nu}$ and to acquire a non-vanishing quantum momentum $\Pi_{\mu\nu}$, while thanks to the same second-quantization scheme proposed here (see also Ref. [7]) these quantum contributions can be included in the EFE and therefore they can in turn ultimately affect the solution of $\widehat{g}_{\mu\nu}$ itself.

The second feature is that the background metric tensor $\widehat{g}_{\mu\nu}$ is actually determined provided suitable boundary conditions are set. In other words, besides the emerging QG feature pointed out in Ref. [36], the quantum wave-equation which lies at the basic of CQG-theory prescribes also uniquely the PDEs, referred to here as quantum-modified Einstein field equations, which in turn determine $\widehat{g}_{\mu\nu}$. This means, therefore, that CQG-theory provides at the same time a self-consistent treatment of classical and quantum gravity, the first one containing however second-quantization corrections [7] carried by CQG-theory itself. In this section, we intend to further elaborate on this important aspect of the theory.

The CQG-wave Equation (5) can be represented in terms of an equivalent set of quantum hydrodynamic equations upon introducing the Madelung representation

$$\psi(g,\widehat{g},r,s) = \sqrt{\rho}\exp\left\{\frac{i}{\hbar}\mathcal{S}^{(q)}\right\}, \tag{6}$$

where the quantum fluid fields $\left\{\rho, S^{(q)}\right\} \equiv \left\{\rho(g,\widehat{g},r,s), \mathcal{S}^{(q)}(g,\widehat{g},r,s)\right\}$ identify respectively the 4−scalar quantum PDF and quantum phase-function. As a result, the same quantum fluid fields can be shown to satisfy the set of GR-quantum hydrodynamic equations (CQG-QHE) identified with the continuity and quantum Hamilton–Jacobi equations

$$\begin{aligned} \frac{d\rho}{ds} + \frac{\partial}{\partial g_{\mu\nu}}\left(\rho V_{\mu\nu}\right) &= 0, &(7)\\ \frac{d\mathcal{S}^{(q)}}{ds} + H^{(q)} &= 0, &(8)\end{aligned}$$

where $S^{(q)} \equiv S^{(q)}(g,\widehat{g},r,s)$ is the quantum phase-function. In addition, $V_{\mu\nu} \equiv \frac{1}{\alpha L}\frac{\partial \mathcal{S}^{(q)}}{\partial g^{\mu\nu}}$, where αL is a dimensional constant which is related to the graviton mass estimate given in Ref. [25], while $H^{(q)}$ denotes the effective quantum Hamiltonian density

$$H^{(q)} = \frac{1}{2\alpha L}\frac{\partial \mathcal{S}^{(q)}}{\partial g^{\mu\nu}}\frac{\partial \mathcal{S}^{(q)}}{\partial g_{\mu\nu}} + V_{QM} + V_o. \tag{9}$$

Here the case of vacuum conditions is considered, namely absence of classical sources, so that V_o and V_{QM} identify respectively the vacuum effective potential and quantum Bohm interaction potential [37] given by

$$V_o = \alpha L\left(2 - \frac{1}{4}g^{\mu\nu}g_{\mu\nu}\right)g^{\alpha\beta}\widehat{R}_{\alpha\beta}, \tag{10}$$

$$V_{QM} \equiv \frac{\hbar^2}{8\alpha L}\frac{\partial \ln\rho}{\partial g^{\mu\nu}}\frac{\partial \ln\rho}{\partial g_{\mu\nu}} - \frac{\hbar^2}{4\alpha L}\frac{\partial^2\rho}{\rho\partial g_{\mu\nu}\partial g^{\mu\nu}}. \tag{11}$$

An analytical treatment of the continuity Equation (7) has been carried out in Ref. [36] in terms of the generalized Lagrangian-path representation of the CQG-QHE, which realizes a stochastic trajectory-based formulation of the CQG-wave equation. This allows for the construction of an analytical solution for the PDF $\rho(s)$ to be expressed as a shifted Gaussian distribution of the type

$$\rho(s) = \rho_G \exp\left\{ -\int_{s_o}^{s} ds' \frac{\partial V_\nu^\mu(s')}{\partial g_\nu^\mu(s')} \right\}, \tag{12}$$

where $\rho_G = \rho(\Delta g - \widehat{g})$ is the shifted Gaussian PDF

$$\rho_G \equiv \frac{1}{\pi^5 r_{th}^{10}} \exp\left\{ -\frac{(\Delta g - \widehat{g})^2}{r_{th}^2} \right\}. \tag{13}$$

Here r_{th}^2 is the dimensionless invariant semi-amplitude width of the Gaussian quantum PDF, while in short-notation the exponent $(\Delta g - \widehat{g})^2$ stands for the 4−scalar defined as $(\Delta g - \widehat{g})^2 \equiv \left(\Delta g_{\mu\nu} - \widehat{g}_{\mu\nu}\right)(\Delta g^{\mu\nu} - \widehat{g}^{\mu\nu})$. More precisely, $\widehat{g} = \widehat{g}_{\mu\nu}$ is the background metric tensor and $\Delta g \equiv \Delta g_{\mu\nu}$ identifies a stochastic displacement symmetric tensor field associated with each stochastic quantum trajectory. As proved in Ref. [36], the generalized Lagrangian-path theory is constructed in such a way that the stochastic average over Δg of quantum observables (denoted with the symbol $<>$) coincides with their quantum expectation value, so that in particular the emergent gravity relationship $\langle \Delta g_{\mu\nu} \rangle = \widehat{g}_{\mu\nu}$ holds.

Let us now consider the quantum Hamilton–Jacobi Equation (8). It has been proved that a quantum Hamiltonian structure analogous to that holding for the classical GR-Hamilton equations can be established, which is represented by the set $\left\{x, H^{(q)}\right\}$, where the 4−tensor canonical state $x \equiv (g_{\mu\nu}, \Pi^{\mu\nu})$ is the Hamiltonian hydrodynamic state, with $\Pi^{\mu\nu} = \frac{\partial S^{(q)}}{\partial g_{\mu\nu}}$, and $H^{(q)}$ is the effective quantum Hamiltonian density defined above in Equation (9). This permits to represent Equation (8) equivalently as a set of manifestly-covariant quantum Hamilton equations, which take to form of evolution equations in terms of the proper-time invariant parameter s. In vacuum these equations are written as

$$\frac{d}{ds} g^{\mu\nu} = \frac{\Pi^{\mu\nu}}{\alpha L}, \tag{14}$$

$$\frac{d}{ds} \Pi_{\mu\nu} = -\frac{\partial}{\partial g^{\mu\nu}} (V_o + V_{QM}), \tag{15}$$

which are subject to generic initial conditions of the type $x(s_o) = x_o \equiv \left(g_o^{\mu\nu}, \Pi_{(o)\mu\nu}\right)$.

The quantum Hamilton Equations (14) and (15) generate corresponding quantum-modified EFE. In Ref. [7] this occurrence was established as follows:

(1) by imposing the "equilibrium" initial conditions

$$x(s_o) = \left(g_{(o)}^{\mu\nu} \equiv \widehat{g}^{\mu\nu}, \Pi_{(o)\mu\nu} \equiv 0\right), \tag{16}$$

namely requiring that the initial quantum tensor $g^{\mu\nu}$ coincides with the background one and its corresponding momentum (i.e., its covariant derivative) is identically vanishing;

(2) by taking the deterministic limit $\Delta g_{\mu\nu} \to 0$ in the contribution arising from the Bohm quantum potential for the quantum PDF (12), which amounts to require that the stochastic quantum trajectories driving the quantum wave function collapse on the single deterministic classical trajectory.

Under these assumptions and in vacuum conditions, the quantum Hamilton equations imply the *quantum-modified EFE*, which have been obtained in Ref. [7]. The latter take the form

$$\widehat{R}_{\mu\nu} - \frac{1}{2}\left[\widehat{R} - 2\Lambda_{CQG}(s)\right]\widehat{g}_{\mu\nu}(s) = 0, \tag{17}$$

where $\widehat{g}_{\mu\nu}(s) \equiv \widehat{g}_{\mu\nu}(r(s), s)$ denotes the classical deterministic background metric tensor, while $\widehat{R}_{\mu\nu}$ together with $\widehat{R}$ (respectively the Ricci tensor and Ricci $4-$scalar) are evaluated in terms of $\widehat{g}_{\mu\nu}(s)$, so that they are also actually treated as deterministic classical fields. Furthermore, $\Lambda_{CQG}(s)$ is the quantum CC reported in Ref. [7] and determined by CQG-theory, which arises due to the Bohm quantum vacuum interaction among massive gravitons. The tensorial term $\Lambda_{CQG}(s)\,\widehat{g}_{\mu\nu}(s)$ yields the quantum modification to the classical EFE, whereby in this framework the nature of the same $\Lambda_{CQG}(s)$ is purely quantum and is a characteristic contribution of CQG-theory. Notice that in this picture the background metric tensor $\widehat{g}_{\mu\nu}(s)$ can in principle depend both explicitly and implicitly on the proper-time s, to be identified here with the arc-length of a suitable family of time-like geodesics. The implicit dependence is carried by the $4-$position $r = r(s)$ which is evaluated along a non-null geodesics $\{r = r(s), s \in I_1\}$. Here, $I_1 \equiv [s_o, s_1]$ is a suitable proper-time interval, where the extrema s_o and s_1 identify the proper-times at which respectively the graviton geodesics begins and ends. However, for definiteness, in the following we shall restrict our analysis to the case in which the CC is stationary, namely is independent of proper time. As shown in Ref. [7] this corresponds to an admissible initial condition for the quantum wave equation, which implies that for arbitrary s the identity $\Lambda_{CQG}(s) \equiv \Lambda_{CQG}(s_o)$ holds, where

$$\Lambda_{CQG}(s_o) = \frac{\hbar^2}{(\alpha L)^2}\frac{1}{r_{th}^4} \tag{18}$$

and $\hbar$ is the reduced Planck constant. As a consequence of the stationarity condition indicated above, it follows that the background metric tensor solution of Equation (17) is stationary too, in the sense that it cannot depend explicitly on proper-time, so that symbolically in the following $\widehat{g}_{\mu\nu} \equiv \widehat{g}_{\mu\nu}(r(s))$.

A particular realization of $\widehat{g}_{\mu\nu}$, solution of Equation (17), can be shown to be the DeSitter space–time expressed in terms of the CC Λ_{CQG}. Thus, upon introducing the $4-$scalar function $B \equiv \left(1 - \frac{r^2}{A^2}\right)$, the background metric tensor in spherical coordinates $(ct, r, \vartheta, \varphi)$ can be written as $\widehat{g}_{\mu\nu} = diag\left\{B, B^{-1}, r^2, r^2 \sin^2\vartheta\right\}$, so that the corresponding Riemann distance takes the form $ds^2 = Bc^2dt^2 - B^{-1}dr^2 + r^2 d\Omega^2$. Here the parameter A is related to the "radius" of the DeSitter space–time, which in the framework of the quantum modified EFE depends on Λ_{CQG} by means of the prescription

$$A = \sqrt{\frac{3}{\Lambda_{CQG}}}. \tag{19}$$

We conclude by pointing out that, due to the choice of initial conditions $x(s_o)$ given above, in the quantum-modified EFE the only quantum contribution is carried by the quantum CC, and is therefore associated with the potential term V_{QM}. As a result, possible dynamical contributions associated with non-vanishing canonical momenta $\Pi^{\mu\nu}$ remain ruled out in such a case.

3. Momentum Quantum-Modified Einstein Field Equations

Based on the results established in Ref. [7], in this section we explore the possibility of obtaining a generalization of the quantum-modified EFE (17) that warrants at the same time the consistency with the quantum Hamilton Equations (14) and (15) expressed in evolution form in terms of the invariant proper-time s. In order to reach the goal, initial conditions different from those adopted previously and reported in Equation (16) are implemented. More precisely, for the present investigation, more general "non-equilibrium" initial conditions of the form

$$x(s_o) = \left(g^{\mu\nu}_{(o)} \equiv \widehat{g}^{\mu\nu}(s_o) + \delta g^{\mu\nu}(s_o), \Pi_{(o)\mu\nu} \equiv \delta\pi_{\mu\nu}(s_o)\right) \tag{20}$$

are considered. This means that the initial tensor of the quantum gravitational field $g^{\mu\nu}_{(o)}$ is allowed to deviate from the deterministic background metric tensor by the quantity $\delta g^{\mu\nu}(s_o)$, while its conjugate momentum is correspondingly non-vanishing in terms of $\delta\pi_{\mu\nu}(s_o)$. It follows that, according to this new prescription, initial conditions for both canonical variables are introduced here, in difference with Equation (16). Nevertheless, in this regard we notice that, for a single perturbation, the only possible meaningful initial condition is the one in which $\delta g^{\mu\nu}(s_o) = 0$ and $\delta\pi_{\mu\nu}(s_o) \neq 0$. In fact, if the converse case of $\delta g^{\mu\nu}(s_o) \neq 0$ but $\delta\pi_{\mu\nu}(s_o) = 0$ is considered, one can always absorb $\delta g^{\mu\nu}(s_o)$ into the prescription of $\widehat{g}^{\mu\nu}(s_o)$, so that the customary case of the quantum-modified EFE is recovered. However, in the more general case of superposition of different perturbations of the type (2) this is not possible. Therefore, under such a prescription and given validity of Equation (20), for the i-th perturbation, identifying $\delta g_{\mu\nu} \equiv \delta g^{(i)}_{\mu\nu}$, one must introduce accordingly the decomposition

$$g^{\mu\nu}(s) = \widehat{g}^{\mu\nu} + \delta g^{\mu\nu}(s), \tag{21}$$

$$\Pi_{\mu\nu}(s) = \delta\pi_{\mu\nu}(s), \tag{22}$$

holding at the generic proper-time s. Here, both $\delta\pi_{\mu\nu}(s)$ and $\delta g^{\mu\nu}(s)$ are treated as stochastic fields with respect to the background fields, while proper-time averages on arbitrary functions of $\delta\pi_{\mu\nu}(s)$ and $\delta g^{\mu\nu}(s)$ are treated as classical observables. Correspondingly, the quantum Hamilton Equations (14) and (15) imply therefore

$$\frac{d}{ds}\delta g^{\mu\nu} = \frac{\delta\pi^{\mu\nu}}{\alpha L}, \tag{23}$$

$$\frac{d}{ds}\delta\pi_{\mu\nu} = -\frac{\partial}{\partial g^{\mu\nu}}(V_o + V_{QM}), \tag{24}$$

where in the second equation the rhs is a function of $\delta g^{\mu\nu}(s)$. In particular, because of the definition of the potential V_o (see Equation (10)), which is a quadratic function of $g^{\mu\nu}(s)$, Equation (24) carries necessarily non-linear dependences on the displacement tensor $\delta g^{\mu\nu}(s)$. Furthermore, the momentum equation yields a second-order proper-time derivative of the same tensor $\delta g^{\mu\nu}$ when replaced in Equation (24).

Direct evaluation of the rhs of Equation (24), upon also letting the deterministic limit $\Delta g_{\mu\nu} \to 0$ in the quantum PDF, yields two independent equations, respectively for $\delta g_{\mu\nu}$ and $\widehat{g}_{\mu\nu}$. The first one, written in the linear approximation for the ith-perturbation, to be considered suitably small with respect to the background metric tensor $\widehat{g}_{\mu\nu}$, becomes

$$\frac{d^2}{ds^2}\delta g^{(i)}_{\mu\nu} = -\frac{1}{2}\left[\widehat{g}_{\mu\nu}\widehat{R}_{\alpha\beta} + \widehat{R}_{\mu\nu}\widehat{g}_{\alpha\beta}\right]\delta g^{(i)\alpha\beta} - \frac{1}{2}\widehat{R}\delta g^{(i)}_{\mu\nu}, \tag{25}$$

which represents a harmonic dynamical equation for each $\delta g^{(i)}_{\mu\nu}$ driven by the background curvature tensors of space–time. Instead, Equation (24) yields now for $\widehat{g}_{\mu\nu}$ the equation

$$\widehat{R}_{\mu\nu} - \frac{1}{2}\left[\widehat{R} - 2\Lambda_{CQG}\right]\widehat{g}_{\mu\nu} = \left\langle T_{\mu\nu}\right\rangle_s, \tag{26}$$

to be referred to as momentum quantum-modified EFE. Its solution, represented by the metric tensor $\widehat{g}_{\mu\nu}$ again consistently prescribes the tensor properties of the theory. The difference between Equation (17) and Equation (26) lies in the tensor term $\left\langle T_{\mu\nu}\right\rangle_s$ which is characteristic of non-vanishing momentum contributions retained in the quantum Hamilton equations, and which were previously excluded in the treatment of Ref. [7]. More precisely, here $\left\langle T_{\mu\nu}\right\rangle_s$ identifies, up to a dimensional constant, a stress–energy tensor which carries proper-time averaged quantum contributions of all the ensemble of perturbations. This is found to be defined as

$$\left\langle T_{\mu\nu}\right\rangle_s \equiv \sum_{i=1}^{N}\left[\frac{1}{2}\widehat{R}_{\alpha\beta}\left\langle \delta g^{(i)\alpha\beta}\delta g^{(i)}_{\mu\nu}\right\rangle_s + \frac{1}{4}\widehat{R}_{\mu\nu}\left\langle \delta g^{(i)\alpha\beta}\delta g^{(i)}_{\alpha\beta}\right\rangle_s\right]. \tag{27}$$

Hence, $\left\langle T_{\mu\nu}\right\rangle_s$ contains the non-linear (i.e., quadratic) corrections in $\delta g_{\mu\nu}$, while the proper-time averages extrapolate its stochastic quantum character and warrant it is an observable in Equation (26).

We stress that, although each term $\delta g_{\mu\nu}^{(i)}$ of the perturbation sequence is assumed small, the non-linear contribution carried by $\left\langle T_{\mu\nu}\right\rangle_s$ gives actually a non-perturbative character to Equation (26). The form of Equation (26) suggests to seek a solution for the Ricci tensor of the form $\widehat{R}_{\mu\nu} = K\Lambda_{CQG}\widehat{g}_{\mu\nu}$, and consequently $\widehat{R} = 4K\Lambda_{CQG}$. It can be easily shown that such a solution exhibits the remarkable feature of warranting also the divergence-free condition of $\left\langle T_{\mu\nu}\right\rangle_s$ and of the whole Equation (26). In particular, substituting in Equation (26) then it follows that the 4−scalar K is given by

$$K = \frac{1}{1 + \frac{1}{4}\sum_{i=1}^{N}\left[\left\langle \delta g^{(i)\alpha\beta}\delta g_{\alpha\beta}^{(i)}\right\rangle_s + \frac{1}{2}\left\langle \delta g^{(i)2}\right\rangle_s\right]}, \tag{28}$$

where $\delta g^{(i)} \equiv \delta g_{\mu\nu}^{(i)}\widehat{g}^{\mu\nu}$ is the trace and consequently $\left\langle \delta g^{(i)2}\right\rangle_s > 0$, while by construction $\left\langle \delta g^{(i)\alpha\beta}\delta g_{\alpha\beta}^{(i)}\right\rangle_s > 0$, so that necessarily $K < 1$, which therefore identifies a quantum screening factor. This proves the validity of the representation (3) and the fact that the same CC Λ_S is proper-time independent.

The same result permits us to cast Equation (25) for the ith-perturbation in the form

$$\frac{d^2}{ds^2}\delta g_{\mu\nu}^{(i)} = -\Lambda_S\left(\widehat{g}_{\mu\nu}\delta g^{(i)} + 2\delta g_{\mu\nu}^{(i)}\right), \tag{29}$$

which implies

$$\frac{d^2}{ds^2}\delta g^{(i)} = -6\Lambda_S\delta g^{(i)}. \tag{30}$$

This is a harmonic oscillator equation with a generic solution for the trace of the form $\delta g^{(i)} = \Phi\sin\left(\sqrt{6\Lambda_S}\,(s+\varphi_i)\right)$, namely

$$\delta g_{\mu\nu}^{(i)} = \frac{1}{4}\Phi\sin\left(\sqrt{6\Lambda_S}\,(s+\varphi_i)\right)\widehat{g}_{\mu\nu}, \tag{31}$$

where φ_i for $i = 1, N$ are assumed to be uniformly spread in the interval $[0,\tau]$, while Φ is a constant amplitude. The conclusion is therefore that the function $f(s+\varphi_i)$ introduced in Equation (2) is found to be $f(s+\varphi_i) \equiv \frac{1}{4}\Phi\sin\left(\sqrt{6\Lambda_S}\,(s+\varphi_i)\right)$, which indeed is periodic in s with period $\tau = \frac{2\pi}{\sqrt{6\Lambda_S}}$. As a consequence, proper-time averages can be computed analytically, so that the screening coefficient K is given by

$$K = \frac{1}{1 + \frac{3N}{32}\Phi^2}. \tag{32}$$

The solution for the i-th perturbation $\delta g_{\mu\nu}^{(i)}$ implies that at the proper-time $s_o = 0$ the initial perturbation of metric tensor is

$$\delta g_{\mu\nu}^{(i)}(s_o) = \frac{1}{4}\Phi\sin\left(\sqrt{6\Lambda_S}\varphi_i\right)\widehat{g}_{\mu\nu}, \tag{33}$$

while the corresponding initial condition for $\delta\pi_{\mu\nu}^{(i)}(s_o)$ is found to be

$$\delta\pi_{\mu\nu}^{(i)}(s_o) = \frac{1}{4}\Phi\alpha L\sqrt{6\Lambda_S}\cos\left(\sqrt{6\Lambda_S}\varphi_i\right)\widehat{g}_{\mu\nu}(s_o), \tag{34}$$

consistent with the previous considerations. We further notice that the solutions for $\delta g_{\mu\nu}^{(i)}$ and $\delta\pi_{\mu\nu}^{(i)}$ identify quantum periodic oscillatory perturbations. Regarding the physical interpretation, the quantum perturbations considered here do not have the character of wave perturbations, namely as travelling waves propagating in space–time. We remark that, although the amplitude Φ of the perturbations can be in principle assumed to be such that the corresponding perturbation $\delta g_{\mu\nu}^{(i)}$ remains much smaller than $\widehat{g}_{\mu\nu}$, e.g., below the current accuracy level of experimental observations aimed at measuring the same $\widehat{g}_{\mu\nu}$, the overall contribution to the screening factor K can be nevertheless of $O(1)$ and therefore significant. This may occur provided the quantity $\frac{3N}{32}\Phi^2$ is at least of $O(1)$ or larger. Thus, for example, letting $\Phi \sim 5 \times 10^{-4}$ and $N \sim 10^8$ (which justifies the statistical average hypothesis) yields $K \sim 0.3$. Under these conditions, the metric tensor itself is affected.

4. Case of Quantum Non-Unitarity

Let us consider here for completeness also the case of quantum sinks treated in Ref. [34]. Correspondingly, the CQG-wave Equation (5) is replaced with the modified equation

$$i\hbar \frac{d}{ds}\psi(g,s) = \left\{ H_R^{(q)} + \frac{i\hbar}{2} Q_L^{(q)} \right\} \psi(g,s), \tag{35}$$

in which $i\hbar Q_L^{(q)}$ and $Q_L^{(q)}$ denote respectively an appropriate $4-$scalar it (or capture) quantum operator and a real function, denoted as graviton capture term. In particular, $Q_L^{(q)}$, which is assumed to hold for the deSitter event horizon, following Ref. [34] is taken of the form

$$Q_L^{(q)} = f(r,s)\Theta(\varepsilon - \Delta). \tag{36}$$

Here, $f(r,s)$ is an arbitrary smooth and bounded real function. Furthermore, the theta function $\Theta(\varepsilon - \Delta)$(with $\varepsilon > 0$ to be assumed $\ll 1$ and $\Delta = \Delta(r)$ being a monotonic function of the radial displacement from an event horizon) takes into account the spatial localization of the quantum sink, which is assumed to be located suitably close to the DeSitter event horizon [34]. In particular, in the case in which $f(r,s) \leq 0$ the operator $\frac{i\hbar}{2} Q_L^{(q)}$ identifies a localized quantum *pit* or *capture* operator. As a consequence, $Q_L^{(q)}$ is non-zero only in a suitable domain close to the event horizon domain defined above. Introducing once again the Madelung representation (6) one notices that now $\rho \equiv \rho(g,\widehat{g},r,s)$ generally is no more a probability density since it follows that

$$\int_{U_g} d(g)\rho(g,\widehat{g},r,s) \leq 1, \tag{37}$$

with $U_g \subseteq \mathbb{R}^{10}$ denoting the $10-$dimensional configuration space spanned by the symmetric coordinate field $g \equiv \{g_{\mu\nu}\}$. A corresponding set of quantum hydrodynamic equations analogous to Equations (7) and (8) is therefore recovered. In particular, one finds that the first one is now replaced with the PDE in Eulerian form

$$\frac{d\rho}{ds} + \frac{\partial}{\partial g_{\mu\nu}}\left(\rho V_{\mu\nu}\right) = Q_L^{(q)}, \tag{38}$$

which realizes the non-unitary generalization of the quantum continuity equation. In analogy to Equation (7), the modified Equation (38) can be solved explicitly to give the analytical solution for the PDF $\rho \equiv \rho_M(s)$, where

$$\rho_M(s) = \rho(s)\,\eta(s), \tag{39}$$

with

$$\eta(s) \equiv \exp\left\{ \int_{s_o}^{s} ds' f(r(s'),s')\Theta(\varepsilon - \Delta) \right\}, \tag{40}$$

carrying the new non-stationary modification produced by the graviton capture term $f(r(s'),s')\Theta(\varepsilon - \Delta)$ which appears in the previous modified quantum continuity equation. We stress furthermore that the quantum Hamilton-Jacobi Equation (8) remains unchanged. As a consequence, the subsequent theory developed in Section 3 still applies.

5. Physical Properties and Applications: DeSitter Solution and Graviton Mass

We now proceed to elucidate the physical implications of the quantum screening mechanism. First we stress that validity of Equation (4) implies that the space–time metric tensor still admits a DeSitter type solution expressed in terms of Λ_S. More precisely, in analogy with Section 2, we introduce the 4−scalar function $B_S \equiv \left(1 - \frac{r^2}{A_S^2}\right)$, with A_S identifying the "radius" of the DeSitter space–time in the presence of screening effect. Then, in spherical coordinates $(ct, r, \vartheta, \varphi)$ the background metric tensor can be written as $\widehat{g}_{\mu\nu} = diag\left\{B_S, B_S^{-1}, r^2, r^2 \sin^2\vartheta\right\}$, so that the corresponding Riemann distance takes the form $ds^2 = B_S c^2 dt^2 - B_S^{-1} dr^2 + r^2 d\Omega^2$. Here, the parameter A_S is related to Λ_S by means of the prescription

$$A_S = \sqrt{\frac{3}{\Lambda_S}}. \tag{41}$$

Comparison between the DeSitter radius solutions A and A_S, corresponding respectively to the two cases of quantum-modified and momentum quantum-modified EFE, permits to establish the inequality

$$A_S > A \Longrightarrow \sqrt{\frac{3}{\Lambda_S}} > \sqrt{\frac{3}{\Lambda_{CQG}}}, \tag{42}$$

so that effectively the radius of the DeSitter event horizon is increased if the screening mechanism of the quantum CC is considered.

Furthermore, denoting here for convenience by $\widehat{R}_{\Lambda_S}$ and $\widehat{R}_{\Lambda_{CQG}}$ the Ricci 4−scalars associated with the quantum-modified and the momentum quantum-modified EFE, the following inequality is similarly established on this basis:

$$0 < \widehat{R}_{\Lambda_S} < \widehat{R}_{\Lambda_{CQG}}. \tag{43}$$

This relationship states that, in vacuum configurations considered here:

(1) The classical Ricci 4−scalar obtained by taking the semi-classical limit of the quantum-gravity contributions is identically vanishing, namely the classical space–time is flat in a vacuum.

(2) In the framework of CQG-theory, quantum contributions retained in the EFE always act so to produce a non-vanishing and positive curvature of space–time.

(3) The Ricci 4−scalar obtained in the framework of the quantum-modified EFE due to the Bohm potential interaction is greater than the corresponding one established by the momentum quantum-modified EFE which includes also corrections due to the stress-energy tensor $\left\langle T_{\mu\nu}\right\rangle_S$.

Validity of Equations (42) and (43) then proves that a physical observable feature distinguishes the DeSitter solutions considered here, depending on whether the screening effect is set in or not. In particular, the momentum quantum-modified EFE predicts a different geometry of space–time in which the accessible physical domain, related to the magnitude of A_S is increased, while the corresponding invariant curvature measured by $\widehat{R}_{\Lambda_S}$ is decreased. Hence, momentum corrections to the EFE contribute to making the space–time flatter and to set the ideal surface of the DeSitter event horizon further away.

The theoretical foundation for the screening effect of the CC pointed out here is unique in its formulation. In fact, although the possible occurrence of a physical mechanism of this type has been pointed out recently in Ref. [38], the framework adopted here remains unprecedented for the following distinguishing features: (1) The CC is not an ad hoc classical term that is included a priori in the field equations, but its prescription has a well-defined quantum origin, since it is generated by the quantum-gravity Bohm interaction occurring among massive gravitons. (2) The classical EFE are modified by explicit self-consistent quantum contributions, represented by a stochastic-averaged and divergence-free quantum stress-energy tensor, rather than by *ad hoc* phenomenological extra-terms included in the classical Einstein–Hilbert action, as it is the case of the Energy–Momentum Log Gravity theory considered in Ref. [38]. (3) Unlike the solution reported in Ref. [38], the present derivation exhibits the remarkable feature of warranting also the divergence-free condition of $\langle T_{\mu\nu} \rangle_s$ and of the whole Equation (26). (4) The case of vacuum quantum-modified EFE is considered here, thus excluding the effect of classical field sources. Hence, the screening effect depicted here is purely quantum in character, arising specifically due to quantum-gravity non-equilibrium initial perturbations.

Finally, a crucial issue concerns the physical implications of the screening effect of the CC on the mass estimate for massive gravitons predicted by CQG-theory in a cosmological scenario like the DeSitter space–time. The invariant graviton rest-mass m_o was estimated in Ref. [25] in terms of the ground-state eigenvalue energy occurring in the presence of discrete energy levels generated by the potential associated with the same CC. In such a case the upper bound estimate is provided by

$$m_o \lesssim 0.326 \frac{\hbar \sqrt{|\Lambda_{CQG}|}}{c}, \tag{44}$$

so that the theory prescribes m_o as a function of Λ_{CQG}, i.e., $m_o = m_o(\Lambda_{CQG})$. The numerical value for m_o was obtained in Ref. [25] by assuming that the observationally-measured value of the CC, to be denoted Λ_{obs}, coincides with Λ_{CQG}, so that there enter no additional (quantum and/or classical) contributions to the CC other than the one provided by CQG-theory. Hence, letting $\Lambda_{CQG} = \Lambda_{obs}$ in Equation (44) and adopting for Λ_{obs} the current astrophysical estimated value $\Lambda \cong 1.2 \times 10^{-52} m^{-2}$ [39], one finds that $m_o(\Lambda_{obs}) \cong 1.26 \times 10^{-69} kg \sim 7 \times 10^{-34} eV/c^2$, so that the resulting graviton-to-electron mass ratio is $\frac{m_o}{m_e} \cong 1.38 \times 10^{-39}$, with m_e denoting the electron rest-mass. However, if we instead assume that the screening mechanism pointed out in this Letter is effectively operating, then necessarily the previous numerical estimate requires that $\Lambda_{obs} = \Lambda_S = K\Lambda_{CQG}$, so that $m_o(\Lambda_{obs}) = m_o(\Lambda_S)$. This means that the graviton rest-mass estimated in this way might be affected itself by the screening factor K. Thus, replacing in Equation (44) Λ_{CQG} with Λ_S, the following mass-enhancement relationship applies:

$$m_o(\Lambda_{CQG}) = \frac{1}{\sqrt{K}} m_o(\Lambda_S). \tag{45}$$

As a consequence, since $K < 1$, the numerical value for the graviton rest-mass obtained under the assumption $m_o(\Lambda_{obs}) = m_o(\Lambda_S)$ is effectively increased by a factor $\frac{1}{\sqrt{K}}$. From the physical point of view, this implies that, if the screening mechanism is in place, the real graviton rest-mass can be higher than the one inferred by adopting the observationally-deduced value of the CC. On similar grounds, it is immediate to determine the corresponding correction for the Compton wave-length $\lambda_C \equiv \frac{\hbar}{m_o c}$ associated with the massive graviton. If the physical mass increases, then λ_C must decreases accordingly. In fact, one obtains that $\lambda_C(\Lambda_{CQG}) = \sqrt{K} \lambda_C(\Lambda_S)$.

6. Conclusions

From the previous analysis, it follows that non-linear quantum-gravity perturbations of the DeSitter space–time arising in vacuum are actually permitted in the framework of CQG-theory giving rise, in turn, to non-perturbative momentum quantum-modified Einstein field equations. This occurs provided they are determined in terms of sequences of phase-shifted periodic perturbations of equal amplitude and prescribed frequency. The theoretical background is provided by the Hamiltonian representation of the quantum hydrodynamic equations in the context of the manifestly-covariant theory of quantum gravity. As a result, new quantum modifications of the EFE have been obtained, which involve also a new analytical estimate for the cosmological constant (CC), denoted here as Λ_S. The latter has been shown to arise by a quantum-driven screening effect generated by the same perturbations. We stress that the physical origin of the non-equilibrium periodic oscillations is intrinsically quantum. The underlying physical motivations are as follows. First, the effective kinetic energy carried by the perturbations is drawn from the effective potential energy carried by the quantum CC. As a consequence, the CC Λ_{CQG} necessarily decreases when such perturbations are set in. Second, the physical origin of the CC Λ_{CQG} is purely quantum. In fact, according to Ref. [7], it is ascribed exclusively to the Bohm quantum vacuum interaction occurring among gravitons. Third, such occurrences can only arise provided Λ_{CQG} is non-vanishing. Indeed, if we drop beforehand the contribution of the quantum CC Λ_{CQG} entering the quantum-modified Einstein Equations (17), then the same oscillatory perturbations simply cannot exist. Fourth, physical implications concerning the graviton rest-mass estimate in a cosmological deSitter space–time have been addressed, proving that because of the screening effect of the CC, the same graviton rest mass might be higher than the one previously obtained in Ref. [25] adopting the observationally-measured value of the CC. Finally, a further interesting conclusion concerns the non-unitary generalization of CQG-theory reported in Section 4. In fact, it shows that a quantum sink which is assumed spatially localized near the DeSitter event horizon, does not change the quantum screening effect pointed out here. As a consequence, the CC and graviton rest mass estimates determined in this way are unaffected, while the entropic theorems pointed out in Ref. [34] remain equally unchanged.

Besides the foundations of QG and the axiomatic formulation of CQG-theory, the conclusions reached here are meaningful in the context of QG, theoretical astrophysics and cosmology for their potential physical relevance. The same conclusions corroborate and stress the importance of the adoption of a first-principle QG approach like CQG-theory. In particular, the conceptual shortcomings of the Wheeler–DeWitt equation, and implicitly of the related quantization theories based on that, have been shown to be overcome in the context of CQG-theory. Finally, the first-principle formulation of the momentum quantum-modified Einstein field equations based on a second-quantization approach has been achieved, together with the discovery of the CC screening and mass-enhancement effects which hold even in the case of quantum sink effects close to the DeSitter event horizon.

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