symmetry

MDPI

Article

Testing Relativistic Time Dilation beyond the Weak-Field Post-Newtonian Approximation

Yaakov Friedman [1,*] and Esra Yudkin [2]

[1] Jerusalem College of Technology, Departments of Mathematics and Physics, Extended Relativity Research Center, P.O.B. 16031, Jerusalem 91160, Israel

[2] Jerusalem College of Technology, Departments of Physics, Extended Relativity Research Center, P.O.B. 16031, Jerusalem 91160, Israel; ekyudkin@gmail.com

* Correspondence: friedman@g.jct.ac.il

Received: 26 February 2020; Accepted: 24 March 2020; Published: 1 April 2020

check for updates

Abstract: In General Relativity, the gravitational field of a spherically symmetric non-rotating body is described by the Schwarzschild metric. This metric is invariant under time reversal, which implies that the power series expansion of the time dilation contains only even powers of v/c. The weak-field post-Newtonian approximation defines the relativistic time dilation of order ϵ (or of order $(v/c)^2$) of the small parameter. The next non-zero term of the time dilation is expected to be of order ϵ^2, which is impossible to measure with current technology. The new model presented here, called Relativistic Newtonian Dynamics, describes the field with respect to the coordinate system of a far-removed observer. The resulting metric preserves the symmetries of the problem and satisfies Einstein's field equations, but predicts an additional term of order $\epsilon^{3/2}$ for the time dilation. This term will cause an additional periodic time delay for clocks in eccentric orbits. The analysis of the gravitational redshift data from the Galileo satellites in eccentric orbits indicates that, by performing an improved satellite mission, it would be possible to test this additional time delay. This would reveal which of the coordinate systems and which of the above metrics are real. In addition to the increase of accuracy of the time dilation predictions, such an experiment could determine whether the metric of a spherically symmetric body is time reversible and whether the speed of light propagating toward the gravitating body is the same as the speed propagating away from it. More accurate time dilation and one-way speed of light formulas are important for astronomical research and for global positioning systems.

Keywords: astrophysical studies of gravity; relativistic time delay; gravitational redshift; alternative theories; breaking of time reversal

1. Introduction

Einstein's General Relativity (*GR*) has succeeded in explaining non-classical behavior in astrophysics. In *GR*, the gravitational force curves spacetime, and the curving is expressed by a metric [1]. For the gravitational field of a non-rotating, spherically symmetric body, Einstein's field equations lead to the Schwarzschild metric, usually expressed in Schwarzschild coordinates [2]. In these coordinates, the metric is invariant under time reversal. A central prediction of *GR* is time dilation experienced by clocks moving in a varying gravitational field.

The experimental verification of gravitational time dilation, based on the Schwarzschild metric, was obtained in 1960 by Pound and Rebka [3]. The gravitational time dilation was measured more accurately in 1976 by the Vessot–Levine rocket experiment, named Probe A [4]. The European Galileo satellite experiment 2014–2017 verified the *GR* prediction for the time shift with even higher accuracy [5,6]. This experiment measured the time delay of satellite clocks with respect to the clocks at rest on the Earth's surface. The relativistic time delay is caused by the gravitational redshift, which depends on the position of the clock in the gravitation field, and the relativistic Doppler effect, which depends on its velocity. The rotation of the Earth is neglected in gravitational redshift predictions.

Relativistic Newtonian Dynamics (RND) is an alternative relativistic theory of gravitation, developed recently by Yaakov Friedman and his collaborators (see [7,8]). As in GR, also here motion is by a geodesic with respect to a particular metric. However, RND uses the coordinate system of a far-removed observer at rest with respect to the source. Any event can be labeled effectively with respect to these coordinates by performing various measurements, while the traditional Schwarzschild coordinates, in which radial distance is defined by the area of a sphere, cannot be measured. For the gravitational field of a stationary, spherically symmetric body, the RND metric is similar to Whitehead's metric [9], which was shown by Eddington [10] to satisfy Einstein's field equations. Moreover, the RND metric coincides with the Schwarzschild metric in Eddington–Finkelstein coordinates [10,11]. While Eddington–Finkelstein coordinates are generally considered to be non-physical, we claim that our coordinate system is the one actually used to measure time dilation [5]. Note that, for an extended, spherically symmetric body, the RND metric differs from the corresponding metric [12] in Whitehead gravitation.

To justify the use of Eddington–Finkelstein coordinates for Schwarzschild geometry, in [13] (p. 248), the authors wrote "In general, if we wish to write down a solution of Einstein's field equation then we need to do so in some particular coordinate system. However, what, if any, is the significance of any such system? For example, suppose we take the Schwarzschild solution and apply some complicated coordinate transformation $x^\mu \to x'^\mu$. The resulting metric will still be a solution of the empty-space field equation, of course, but there is likely to be a little or no physical or geometric significance attached to the new coordinates x'^μ." Eddington–Finkelstein coordinates are derived from the traditional Schwarzschild ones by applying a non-linear transformation. In [13], the use of these coordinates is justified, and they are used to explain some known effects in astronomy.

RND predicts the same trajectories for planetary motion as GR and passes all classical tests of GR. For a gravitational field of an extended spherically symmetric body, the relativistic time dilation formula derived from the RND metric coincides with the GR one on the weak-field post-Newtonian approximation (of order ϵ of the small parameter). In contrast to GR, where the next non-zero term is of order ϵ^2, RND produces a non-zero term of order $\epsilon^{3/2}$. This additional term depends on both the position and velocity of the clock and results from breaking the time reversibility symmetry of the field's metric.

The data analysis [5,6] of the time shift of clocks on Galileo satellites reveal that reanalyzing these data may reveal this additional RND term. It also shows how to design an improved satellite experiment to determine whether the additional term, predicted by RND, could be observed. Such an experiment will test the GR time delay prediction, based on the Schwarzschild coordinates and metric, that terms of order $\epsilon^{3/2}$ must vanish for a spherically symmetric gravitational field. If the experiment produces a non-zero value for this term, this would show that the RND coordinates are more physical and its metric describes the field more precisely.

2. Relativistic Newtonian Dynamics

Relativistic Newtonian Dynamics is a geometric theory of gravitation. The dynamics of a moving object is described by its worldline in spacetime (which we call lab spacetime), as observed by an observer at rest far from the source of the field. We assume that the lab spacetime is the flat Minkowski spacetime of the far-away observer, whose frame we call K. To define the spacetime coordinates of any event in Earth's vicinity, we may use base stations positioned on the Earth's surface. These stations are equipped with synchronized atomic clocks and laser ranging equipment which will measure the time that the event was observed at the station and the distance from the station. Using the coordinates of the base station and the information about the gravitational field, it is possible to calculate the coordinates of the event in the spacetime of our observer. The gravitational field defines a metric on this spacetime, and the motion of any object is by a geodesic with respect to this metric. In [5], the time shift of Galileo clocks was calculated with respect to the clock in such a frame.

RND does not use Einstein's field equations to define the metric. Rather, the metric in *RND* is derived [7] entirely from the assumption that the field propagates with the speed of light, the 4D symmetry of the problem and the Newtonian limit. For the gravitational field of a spherically symmetric, non-rotating body of mass M, the line interval of the *RND* metric in spherical coordinates is

$$ds^2 = (1-\phi(r))c^2dt^2 - 2c\phi(r)dtdr - (1+\phi(r))dr^2 \\ - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1}$$

where

$$\phi(r) = \frac{r_s}{r} \tag{2}$$

is the dimensionless potential of the gravitational field, and

$$r_s = \frac{2GM}{c^2}, \tag{3}$$

is the *Schwarzschild radius*. This metric coincides with Whitehead's metric for such a field. From the derivation [7] of the metric in Equation (1), it follows that this is also the metric of a gravitation field generated by an extended, spherically symmetric body outside this body.

The proper time τ of an object (for example, the time of the satellite clock) in a gravitational field is defined by

$$cd\tau = ds, \tag{4}$$

and differentiation of any variable x by τ is denoted by $\dot{x}$. Dividing Equation (1) by $c^2d\tau^2$ and using the definition in Equation (4) of τ, we obtain

$$1 = \dot{t}^2\left(1 - \beta^2 - \phi(r)(1+\beta_r)^2\right), \tag{5}$$

where $\mathbf{v}$ is the velocity of the object, $\beta^2 = v^2/c^2$ and $\beta_r = \frac{1}{c}\frac{\dot{r}}{\dot{t}}$ is its unit-free radial component, as they are observed in K .

We can now explicitly define the connection between τ, the time of the satellite clock, and the lab frame time t, by introducing a γ factor satisfying

$$dt = \gamma d\tau\,. \tag{6}$$

From (5),

$$\gamma = \frac{1}{\sqrt{1-\beta^2-\phi(r)(1+\beta_r)^2}}. \tag{7}$$

If the field vanishes, we have $\phi(r) = 0$, and Equation (7) becomes the time dilation γ factor of special relativity. If the object is at rest, then $\beta = \beta_r = 0$, and Equation (7) is the gravitational time dilation. Thus, our γ factor properly incorporates both known time dilations.

Let us compare our metric and time dilation formulae to the corresponding formulae in *GR* based on the Schwarzschild metric. In Schwarzschild coordinates, the line interval of the Schwarzschild metric in spherical coordinates is

$$ds_s^2 = (1-\phi(r))c^2dt_s^2 - (1-\phi(r))^{-1}dr^2 - \\ r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{8}$$

with $\phi(r)$ defined by Equation (2). The Schwarzschild equivalent of Equation (5) is

$$1 = \dot{t}_s^2 \left(1 - \beta^2 - \phi(r)(1 + (1 - \phi(r))^{-1}\beta_r^2)\right), \tag{9}$$

which yields a time dilation factor

$$\gamma_s = \frac{1}{\sqrt{1 - \beta^2 - \phi(r)(1 + (1 - \phi(r))^{-1}\beta_r^2)}}. \tag{10}$$

To compare the *RND* and the Schwarzschild metrics, we use a standard assumption of post-Newtonian theory that $\beta^2 \sim \phi(r) \sim \epsilon$, where ϵ is a small parameter used for bookkeeping. Comparing the metrics in Equations (1) and (8), we see that they differ only in two coefficients: a difference in the dr^2 coefficient, of order ϵ^2, and of the $dtdr$ coefficient, of order ϵ, which is not present in Equation (8). Moreover, the Schwarzschild metric is invariant under time reversal, while the *RND* metric is not. We are not aware of a physical reason why the relativistic gravitational field of a source at rest should be invariant under time reversal.

The expansion to order less than ϵ^2 of the *GR* time dilation formula in Equation (10) is

$$\gamma_s \approx 1 + \frac{\beta^2}{2} + \frac{r_s}{2r}. \tag{11}$$

The corresponding *RND* formula (7) is

$$\gamma \approx 1 + \frac{\beta^2}{2} + \frac{r_s}{2r} + \beta_r \frac{r_s}{r}. \tag{12}$$

The difference between the two models' predictions is the $\beta_r \frac{r_s}{r}$ term of order $\epsilon^{3/2}$. Note that this term depends on both the clock's position in the field and its velocity.

3. The Time Delay Factor of Clocks on Satellites in Eccentric Orbits

The two Galileo 5 and 6 satellites carrying passive hydrogen masers moved in eccentric elliptic orbits around the Earth for about three years (2014–2017). The time delay of their clocks with respect to the clocks of the European Global Navigation Satellite System (EGNSS) was measured. The EGNSS clocks are synchronized to the base station clocks on the Earth's surface. The analysis of this time delay is based on the satellites' trajectories in the Geocentric Celestial Reference System. For simplicity, we rotate this system so that the motion of the satellites is in the plane $\theta = \pi/2$ and denote this reference system by K. At this point, we ignore the influences of the Sun and the Moon on our experiment. These influences will be considered later.

The expected time delay, with accuracy less than ϵ^2, of the clocks on Galileo satellites can be calculated from Equation (11) for *GR* and from Equation (12) for *RND*. To do this, we have to find the radial distance $r(\tau)$ of the satellite from the center of the Earth and its velocity $\mathbf{v}(\tau)$ in K.

In both models, since the metrics in Equations (1) and (8) are independent of φ, the momentum corresponding to this variable is conserved, implying

$$r^2\dot{\varphi} = J, \tag{13}$$

where J has the meaning of angular momentum per unit mass. The equations for $r(\tau)$ are also the same (see [7]):

$$\dot{r}^2 + (1 - \phi(r))\left(c^2 + \frac{J^2}{r^2}\right) = E. \tag{14}$$

The constants of motion J, E could be found from the initial conditions. Thus, the trajectory $r(\varphi)$ can be derived from these formulas and will be the same for both models.

The value of $\beta^2 = \mathbf{v}^2/c^2$ on the trajectory can be derived from the fact that $\mathbf{v}^2 = \dot{\mathbf{r}}^2/\dot{t}^2$ and that the square of the proper velocity is $\dot{\mathbf{r}}^2 = \dot{r}^2 + r^2\dot{\varphi}^2 = \dot{r}^2 + J^2/r^2$ is the same for both models. Thus, to estimate the difference in β^2 between the models, only the $\dot{t}$ formula is needed.

From [7], in *GR* based on the Schwarzschild metric, we have

$$\dot{t}_s = \frac{\sqrt{E}/c}{1-\phi(r)}, \tag{15}$$

while, in *RND*,

$$\dot{t} = \frac{(\sqrt{E} + \phi(r)\dot{r})/c}{1-\phi(r)}. \tag{16}$$

The expansion to order less than ϵ^2 of β^2 in *RND* is

$$\beta^2 = \frac{\dot{\mathbf{r}}^2}{c^2\dot{t}^2} = \frac{\dot{\mathbf{r}}^2(1-\phi(r))^2}{E}\left(1+\frac{\phi(r)\dot{r}}{\sqrt{E}}\right)^{-2}$$
$$\approx \beta_s^2(1-2\beta_r\phi(r)/\sqrt{E}) \approx \beta_s^2,$$

where β_s^2 is the value of β^2 in *GR*. This implies that, to ϵ^2 accuracy, there is no difference between β^2 in the two models. Thus, to this level of accuracy, the difference between the time dilation between the two models is

$$\gamma - \gamma_s \approx \beta_r \frac{r_s}{r}. \tag{17}$$

4. Time Shift between the Clocks on the Galileo Satellites and EGNSS Clocks

To define the shift between the clocks on the Galileo satellites and EGNSS base station clocks, we use the difference between these clocks and a clock measuring time in K. Denote by t_0 the time of initial clock synchronization.

Since the base station clock is at rest in a given gravitational potential, the time shift $\Delta t_b(t)$ between it and the lab clock readings is linear, and $\Delta t_b(t) = \gamma_b(t-t_0)$, where γ_b is the gravitational time dilation factor of the Earth's gravitation field on the Earth's surface.

The difference $\Delta t_m(t)$ between the maser clock on the satellite and the lab clock, according to *RND*, is

$$\Delta t_m(t) = \int_{t_0}^{t} \gamma(\tau)d\tau - (t-t_0),$$

and, using (12), it is

$$\Delta t_m(t) \approx \int_{t_0}^{t}\left(\frac{\beta^2}{2} + \frac{r_s}{2r} + \beta_r\frac{r_s}{r}\right)d\tau. \tag{18}$$

This difference according to *GR* is

$$\Delta t_s(t) = \int_{t_0}^{t} \gamma_s(\tau)d\tau - (t-t_0),$$

and, using Equation (11), it is

$$\Delta t_s(t) \approx \int_{t_0}^{t}\left(\frac{\beta^2}{2} + \frac{r_s}{2r}\right)d\tau. \tag{19}$$

Thus, the difference in time shifts $DT(t) = \Delta t_m(t) - \Delta t_s(t)$ predicted by the two models is given by

$$DT(t) = \int_{t_0}^{t} \beta_r(\tau)\frac{r_s}{r(\tau)}d\tau. \tag{20}$$

Since $\beta_r(\tau)$ is negative on the part of the orbit from the apogee to the perigee and positive on the other part, the time shift $DT(t)$ is periodic with the orbital period T.

The time shift $\Delta t_m(t)$ has a linear term $A(t-t_0)$. The coefficient of proportionality A could be defined by

$$A = \frac{1}{T}\int_T \gamma(\tau)d\tau.$$

The constant A is independent of the choice of revolution. The remaining time shift is periodic with period T.

The expected shift between the clocks on the satellites and the EGNSS clocks should be

$$\Delta t(t) = \Delta t_m(t) - \Delta t_b(t) = \Delta t_s(t) - \Delta t_b(t) + DT(t). \tag{21}$$

This shift has a linear term and a periodic one with period T. The periodic part keeps all the information on variation of the relativistic time dilation during one revolution and is free from slow varying parameters influencing the clock rate. Since $DT(t)$, defined by Equation (20), is periodic with period T, it is combined with the periodic part of the time shift.

We solved numerically $r(\tau)$ and $\beta_r(\tau)$ for the Galileo satellite trajectories based on the information from [5,6]. We calculated the periodic part of the time shift by use of Equation (18) and $DT(t)$ by use of Equation (20). Figure 1 presents these results.

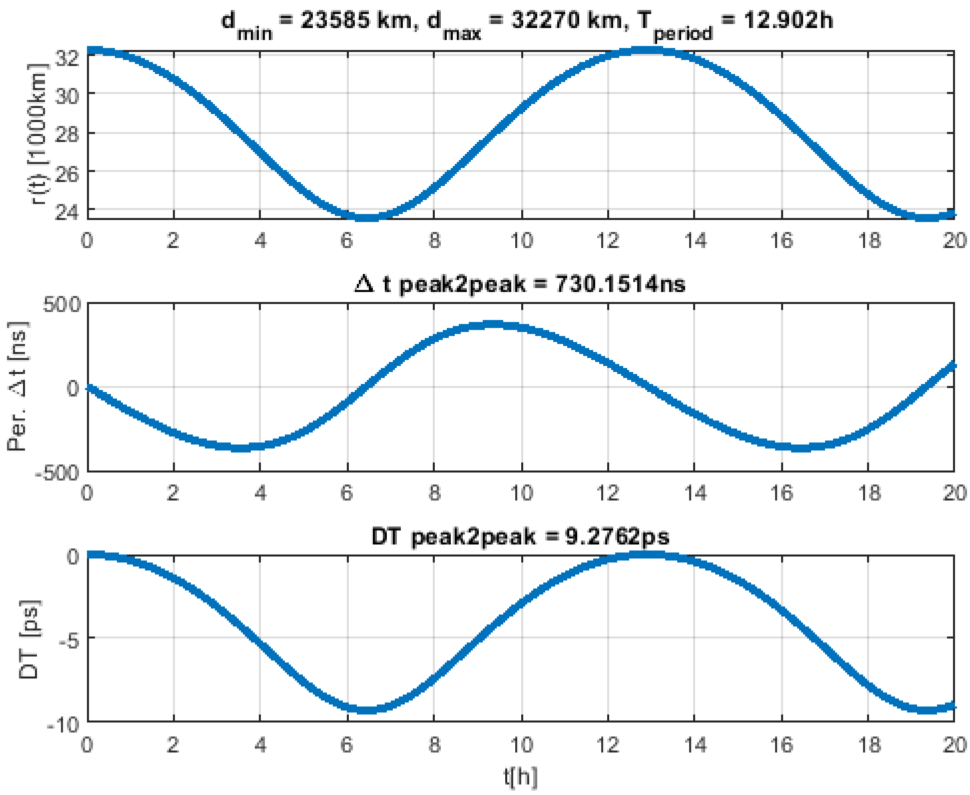

Figure 1. The radial distance $r(t)[10^3 km]$ (**top**), the periodic part of the time shift $\Delta t(t)[ns]$, as predicted by GR (**middle**) and the RND additional time shift $DT(t)[ps]$ (**bottom**) for the Galileo satellites.

By comparing $DT(t)$ with the observed residuals, presented in Figure 5 of [6], we see that the accuracy of the approximation of the the observed time delay at each period is only of one order larger than the additional term predicted by RND. This additional term might possibly be revealed by averaging the periodic part of the time shift over a large number of revolutions. Since the orbital period was 13 h, and the effect of the moon and the sun on the time dilation has a different period, such averaging will minimize their effect. Averaging reduces the random measurement error by a factor of $\sqrt{n}$, where n is the number of revolutions in the sample. Thus, if there are no non-random errors of period T, we should observe the $DT(t)$ term from the data.

5. Improved Experiment to Test Relativistic Time Dilation

To test the additional $DT(t)$ term in the time dilation more accurately, we have to perform an experiment in which this term will be more significant and will be measurable with higher confidence. To do this, we have to enlarge the eccentricity of the orbit and get the perigee as close to the Earth as possible. Having a shorter orbital period would help to gather statistically significant data in shorter time.

Below, in Figure 2, are the simulation results for an orbit with perigee distance 7200 km and apogee distance 20,000 km from the center of the Earth.

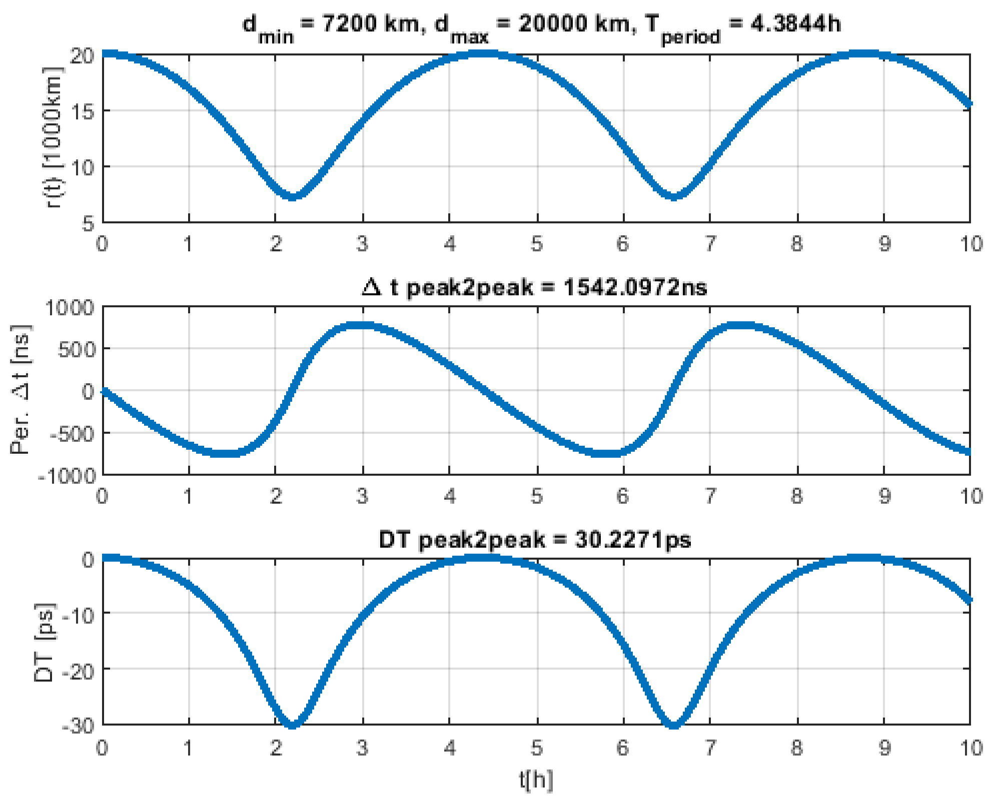

Figure 2. The radial distance $r(t)[10^3 km]$ (**top**), the periodic part of the time shift $\Delta t(t)[ns]$, as predicted by GR (**middle**) and the RND additional time shift $DT(t)[ps]$ (**bottom**) for the proposed satellite.

From the simulation, we observe that around the perigee, for almost 3 h, the $DT(t)$ will be in the $-$(10–30) ps range and disappear at the apogee. We expect that this could be measured. By decreasing the orbit perigee distance and increasing apogee distance, the amplitude of $DT(t)$ may increase threefold, but this would also lead to the increase of the orbital period.

The result of this experiment would show whether a term of order $\epsilon^{3/2}$ is present in the relativistic time dilation. This would show which of the coordinate systems is more physical and whether the metric of a spherically symmetric gravitational field is time reversible. As shown in [7], if the additional term is non-zero, this implies a correction to the one-way speed of light in a gravitation field. This correction may be needed for the next level of accuracy in satellite navigation systems and global positioning systems.

Generally, to test a term of order $\epsilon^{3/2}$ in time dilation, you need a strong gravitation field and velocities close to the speed of light. The Earth's gravitational field is not strong enough and the velocities of the satellite are much smaller than the speed of light, implying that it is impossible to measure this term directly. Nevertheless, the term $DT(t)$, which is obtained by integration of this term for several hours, could be measured.

6. Discussion

To date, relativistic effects of gravity have been tested at the level of the weak-field post-Newtonian approximation of GR. Gravitational time dilation is one of the main relativistic effects, which was tested with high accuracy in several experiments. We propose here to test relativistic time dilation in a gravitational field of a spherically symmetric body beyond the weak-field post-Newtonian approximation. Why does this become possible now?

1. The weak-field post-Newtonian approximation of GR considers corrections of order ϵ of the small parameter. The next non-zero term in GR, based on the Schwarzschild metric, is of order ϵ^2, which is impossible to measure with current technology. The time dilation formula based on the Relativistic Newtonian Dynamics model has a non-zero term of order $\epsilon^{3/2}$, which we propose to measure.
2. The additional time dilation term is significantly large to be measured directly only on stars with highly eccentric orbits near a black hole. What we propose is to test its effect on the time delay a clock placed on a satellite in an eccentric orbit around the Earth. Since this time delay is an integral of the time dilation for several hours, the contribution of the additional term can be observed.
3. The expected additional time delay is periodic with the orbital period of the satellite. By considering only the periodic part of the time delay data, we can remove unwanted effects influencing the time dilation. Averaging the periodic time delay over several rounds reduces the random error in the data.

This experiment would reveal whether the Schwarzschild metric in Schwarzschild coordinates or the RND metric in its coordinates is to be preferred. The experiment would increase the accuracy of the time dilation predictions, which is important for astronomical research, since it introduces a correction to the gravitational redshift. The experiment would also determine whether the metric of a spherically symmetric body is time reversible. If the experiment were to observe the additional term of the time delay, this would imply that the speed of light propagating toward the gravitating body remains c, while the speed of light propagating away from it is reduced by more than predicted by the Schwarzschild model. A more accurate one-way speed of light formula is important for synchronization of clocks and analysis of the data in global positioning systems.

Author Contributions: Conceptualization, Y.F.; methodology, Y.F.; software, E.Y.; validation, E.Y.; investigation, Y.F.; resources, Y.F., E.Y.; writing–original draft preparation, Y.F.; writing–review and editing, Y.F.; supervision, Y.F.; project administration, Y.F. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Acknowledgments: I want to thank Tzvi Scarr for editorial support and Eric David Kramer for constructive comments.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Misner, C.; Thorne, K.; Wheeler, J. *Gravitation*; Freeman: San Francisco, CA, USA, 1973.
2. Schwarzschild, K. Sitzungsber, Preuss Akad. *Wiss. Math.* **1921**, *1*, 966.

3. Pound, R.V.; Rebka, G.A., Jr. Gravitational red-shift in nuclear resonance. *Phys. Rev. Lett.* **1959**, *3*, 439. [CrossRef]
4. Vessot, R.F.; Levine, M.W.; Mattison, E.M.; Blomberg, E.L.; Hoffman, T.E.; Nystrom, G.U.; Watts, J.W. Test of Relativistic Gravitation with a Space-Borne Hydrogen Maser. *Phys. Rev. Lett.* **1980**, *45*, 2081–2084. [CrossRef]
5. Herrmann, S.; Finke, F.; Lülf, M.; Kichakova, O.; Puetzfeld, D.; Knickmann, D.; Dittus, H. Test of the Gravitation Redshif with Galileo Satellites in an Accentric Orbit. *Phys. Rev. Lett.* **2018**, *121*, 231102. [CrossRef] [PubMed]
6. Delva, P.; Puchades, N.; Schönemann, E.; Dilssner, F.; Courde, C.; Bertone, S.; Prieto-Cerdeira, R. A new test of gravitational redshift using Galileo satellites: The GREAT experiment. *Comptes Rendus Phys.* **2019**, *20*, 175–182. [CrossRef]
7. Friedman, Y. Relativistic gravitation based on Symmetry. *Symmetry* **2020**, *12*, 183. [CrossRef]
8. Friedman, Y.; Stav, S. New metrics of a spherically symmetric gravitational field passing classical tests of general relativity. *Europhys. Lett.* **2019**, *126*, 29001. [CrossRef]
9. Whitehead, A.N. *The Principle of Relativity*; Cambridge University Press: Cambridge, UK, 1922.
10. Eddington, A.S. A comparison of Whitehead's and Enstein's formulae. *Nature* **1924**, *113*, 192. [CrossRef]
11. Finkelstein, D. Past-Future Asymmetry of the Gravitational Field of a Point Particle. *Phys. Rev.* **1958**, *110*, 965. [CrossRef]
12. Synge, J.L. Orbits and Rays in the Gravitational Field of a Finite Sphere according to the Theory of A. N. Whitehead. *Proc. R. Soc.* **1952**, *211*, 303–319.
13. Hobson, M.P.; Efstathiou, G.; Lasenby, A.N. *General Relativity. An Introduction for Physicists*; Cambridge University Press: Cambridge, UK, 2007.