



## Curves in 4-dimensions

**>** *with(DifferentialGeometry) : with(Tensor) : with(JetCalculus) : with(Tools) :*

**>** *DGsetup([t], [x, y, z], M, 4) :*

**M >** *Preferences("JetNotation", "JetNotation1") :*

**M >**  $\omega \triangleq \text{evalDG}(dt \&w dy[] \&w dx[] \&w dz[])$

$$\omega \triangleq dt \wedge dy[] \wedge dx[] \wedge dz[] \quad (1.1)$$

**Invariant vector v0**

**M >**  $v0 \triangleq \text{evalDG}(t D_t Cx[] D_x[] Cy[] D_y[] Cz[] D_z[])$

$$v0 \triangleq t D_t Cx[] D_x[] Cy[] D_y[] Cz[] D_z[] \quad (1.2)$$

**w1 corresponds the derivative of v0**

**M >**  $w1 \triangleq \text{evalDG}(D_t Cx[1] D_x[] Cy[1] D_y[] Cz[1] D_z[])$

$$w1 \triangleq D_t Cx_1 D_x[] Cy_1 D_y[] Cz_1 D_z[] \quad (1.3)$$

**here k1 corresponds to 1/β**

**M >**  $v1 \triangleq \text{evalDG}\left(\frac{w1}{k1}\right)$

$$v1 \triangleq \frac{D_t}{k1} \wedge \frac{x_1 D_x[]}{k1} \wedge \frac{y_1 D_y[]}{k1} \wedge \frac{z_1 D_z[]}{k1} \quad (1.4)$$

**solve the normalization criterion to determine k1.**

**M >**  $tmp1 \triangleq \text{solve}(\text{Hook}([v0, v1], \omega) \ll 1, k1) :$

**substitute back into v1, thus giving v1 uniquely**

**M >**  $v1 \triangleq \text{subs}(k1 = tmp1, v1)$

$$v1 \triangleq \frac{D_t}{t y_1 Cx[] z_1 K z[] x_1 K y[]} \wedge \frac{x_1 D_x[]}{t y_1 Cx[] z_1 K z[] x_1 K y[]} \quad (1.5)$$

$$\wedge \frac{y_1 D_y[]}{t y_1 Cx[] z_1 K z[] x_1 K y[]} \wedge \frac{z_1 D_z[]}{t y_1 Cx[] z_1 K z[] x_1 K y[]}$$

**input the derivative of the vector v0**

**M >**  $w2 \triangleq \text{evalDG}(x[1, 1] D_x[] Cy[1, 1] D_y[] Cz[1, 1] D_z[])$

**input the chain rule relation with v2**

**M >**  $v2 \triangleq \text{evalDG}\left(\frac{w2 \ll k2 v1}{tmp1^2}\right)$

$$v2 \triangleq k \frac{k2 D_t}{(t y_1 Cx[] z_1 K x_1 z[] K y[])^3} \quad (1.6)$$

$$\ll \frac{(k t x_{1,1} y_1 K x[] x_{1,1} z_1 C x_1 x_{1,1} z[] \wedge k2 x_1 C x_{1,1} y[]) D_x[]}{(t y_1 Cx[] z_1 K x_1 z[] K y[])^3}$$

$$\ll \frac{(k t y_{1,1} y_1 K x[] y_{1,1} z_1 C x_1 y_{1,1} z[] \wedge k2 y_1 C y[] y_{1,1}) D_y[]}{(t y_1 Cx[] z_1 K x_1 z[] K y[])^3}$$

$$\ll \frac{(k t y_1 z_{1,1} K x[] z_{1,1} z_1 C x_1 z[] z_{1,1} \wedge k2 z_1 C y[] z_{1,1}) D_z[]}{(t y_1 Cx[] z_1 K x_1 z[] K y[])^3}$$

**solve to obtain k2 and input it to get v2 uniquely**

**M >**  $tmp2 \triangleq \text{solve}(\text{Hook}([v0, v2], \omega), k2) :$

**M** > v2 d subs(k2=tmp2, v2) :

we can then calculate the first invariant by computing  $\omega(v1, v2)$

**M** > I<sub>2</sub> d Hook([v1, v2],  $\omega$ )

$$I_2 d \frac{x_1 z_{1,1} K x_{1,1} z_1 C y_{1,1}}{(t y_1 C x[ ] z_1 K x_1 z[ ] K y[ ])^3} \quad (1.7)$$

same story, take the next derivative of v0, solve the chain rule relation and normalize to find k3 and substitute back into v3 to obtain v3 uniquely

**M** > w3 d evalDG(x[1, 1, 1] D\_x[ ] C y[1, 1, 1] D\_y[ ] C z[1, 1, 1] D\_z[ ] ) :

**M** > v3 d evalDG $\left( \frac{(w3 K 3 tmp1 tmp2 v2 K k3 v1)}{tmp1^3} \right) :$

**M** > tmp3 d solve(Hook([v0, v3],  $\omega$ ), k3) :

then v3 is uniquely determined

**M** > v3 d simplify(subs(k3=tmp3, v3)) :

can now calculate more invariants, in fact two more

**M** > I<sub>3a</sub> d Hook([v1, v3],  $\omega$ ); I<sub>3b</sub> d Hook([v2, v3],  $\omega$ )

$$I_{3a} d \frac{1}{(t y_1 C x[ ] z_1 K x_1 z[ ] K y[ ])^5} (t x_1 y_1 z_{1,1,1} K 3 t x_1 y_{1,1} z_{1,1} C 3 t x_{1,1} y_{1,1} z_1$$

$$K t x_{1,1,1} y_1 z_1 C x[ ] x_1 z_{1,1,1} K 3 x[ ] x_1 z_{1,1}^2 C 3 x[ ] x_{1,1} z_1 z_{1,1} K x[ ] x_{1,1,1} z_1^2$$

$$K x_1^2 z[ ] z_{1,1,1} C 3 x_1 x_{1,1} z[ ] z_{1,1} C x_1 x_{1,1,1} z[ ] z_1 K 3 x_{1,1}^2 z[ ] z_1 C t y_1 y_{1,1,1}$$

$$K 3 t y_{1,1}^2 K 3 x[ ] y_{1,1} z_{1,1} C y_{1,1,1} x[ ] z_1 K x_1 y[ ] z_{1,1,1} K y_{1,1,1} z[ ] x_1$$

$$C 3 x_{1,1} y_{1,1} z[ ] C x_{1,1,1} y[ ] z_1 K y[ ] y_{1,1,1})$$

$$I_{3b} d k \frac{1}{(t y_1 C x[ ] z_1 K x_1 z[ ] K y[ ])^6} (t x_1 y_{1,1} z_{1,1,1} K t x_1 y_{1,1,1} z_{1,1} K t x_{1,1} y_1 z_{1,1,1}$$

$$C t x_{1,1} y_{1,1,1} z_1 C t x_{1,1,1} y_1 z_{1,1} K t x_{1,1,1} y_{1,1} z_1 K x[ ] y_{1,1} z_{1,1,1} C x[ ] y_{1,1,1} z_{1,1}$$

$$C x_{1,1} y[ ] z_{1,1,1} K x_{1,1} y_{1,1,1} z[ ] K x_{1,1,1} y[ ] z_{1,1} C x_{1,1,1} y_{1,1} z[ ] ) \quad (1.8)$$

continue the same procedure again

**M** > w4 d evalDG(x[1, 1, 1, 1] D\_x[ ] C y[1, 1, 1, 1] D\_y[ ] C z[1, 1, 1, 1] D\_z[ ] ) :

**M** > v4 d evalDG $\left( \frac{w4 K 6 tmp1^2 tmp2 v3 K (4 tmp1 tmp3 C 3 tmp2^2) v2 K k4 v1}{tmp1^4} \right) :$

**M** > tmp4 d solve(Hook([v0, v4],  $\omega$ ), k4) :

this gives ut v4 uniquely

**M** > v4 d subs(k4=tmp4, v4) :

we can then compute the final invariants

**M** > I<sub>4a</sub> d Hook([v1, v4],  $\omega$ )

$$I_{4a} d \frac{1}{(t y_1 C x[ ] z_1 K x_1 z[ ] K y[ ])^7} (t^2 x_1 y_1^2 z_{1,1,1,1} K 6 t^2 x_1 y_1 y_{1,1} z_{1,1,1}$$

$$K 4 t^2 x_1 y_1 y_{1,1,1} z_{1,1} C 15 t^2 x_1 y_1^2 z_{1,1} C 4 t^2 x_{1,1} y_1 y_{1,1,1} z_1 K 15 t^2 x_{1,1} y_{1,1}^2 z_1$$

$$C 6 t^2 x_{1,1,1} y_1 y_{1,1} z_1 K t^2 x_{1,1,1,1} y_1^2 z_1 C 2 t x[ ] x_1 y_1 z_{1,1,1,1}$$

$$K 10 t x[ ] x_1 y_1 z_{1,1} z_{1,1,1} K 6 t x[ ] x_1 y_{1,1} z_{1,1,1} C 30 t x[ ] x_1 y_{1,1} z_{1,1}^2$$

$$\begin{aligned}
& K 4 t x[ ] x_1 y_{1,1,1} z_1 z_{1,1} C 4 t x[ ] x_{1,1} y_1 z_1 z_{1,1} K 30 t x[ ] x_{1,1} y_{1,1} z_1 z_{1,1} \\
& C 4 t x[ ] x_{1,1} y_{1,1} z_1^2 C 6 t x[ ] x_{1,1,1} y_1 z_1 z_{1,1} C 6 t x[ ] x_{1,1,1} y_{1,1} z_1^2 \\
& K 2 t x[ ] x_{1,1,1,1} y_1 z_1^2 K 2 t x_1^2 y_1 z[ ] z_{1,1,1,1} C 6 t x_1^2 y_{1,1} z[ ] z_{1,1,1} C 4 t \\
& x_1^2 y_{1,1,1} z[ ] z_{1,1} C 6 t x_1 x_{1,1} y_1 z[ ] z_{1,1,1} K 30 t x_1 x_{1,1} y_{1,1} z[ ] z_{1,1} \\
& K 4 t x_1 x_{1,1} y_{1,1} z[ ] z_1 C 4 t x_1 x_{1,1,1} y_1 z[ ] z_{1,1} K 6 t x_1 x_{1,1,1} y_{1,1} z[ ] z_1 \\
& C 2 t x_1 x_{1,1,1,1} y_1 z[ ] z_1 C 30 t x_{1,1}^2 y_{1,1} z[ ] z_1 K 10 t x_{1,1} x_{1,1,1} y_1 z[ ] z_1 C x[ ]^2 x_1 \\
& z_1^2 z_{1,1,1,1} K 10 x[ ]^2 x_1 z_1 z_{1,1} z_{1,1,1} C 15 x[ ]^2 x_1 z_{1,1}^3 C 4 x[ ]^2 x_{1,1} z_{1,1}^2 z_{1,1,1} \\
& K 15 x[ ]^2 x_{1,1} z_1 z_{1,1}^2 C 6 x[ ]^2 x_{1,1,1} z_{1,1}^2 z_1 K x[ ]^2 x_{1,1,1,1} z_{1,1}^3 K 2 x[ ] \\
& x_1^2 z[ ] z_1 z_{1,1,1,1} C 10 x[ ] x_1^2 z[ ] z_{1,1} z_{1,1,1} C 2 x[ ] x_1 x_{1,1} z[ ] z_1 z_{1,1,1} \\
& K 30 x[ ] x_1 x_{1,1} z[ ] z_{1,1}^2 K 2 x[ ] x_1 x_{1,1,1} z[ ] z_1 z_{1,1} C 2 x[ ] x_1 x_{1,1,1,1} z[ ] z_1^2 \\
& C 30 x[ ] x_{1,1}^2 z[ ] z_1 z_{1,1} K 10 x[ ] x_{1,1} x_{1,1,1} z[ ] z_1^2 C x_1^3 z[ ]^2 z_{1,1,1,1} K 6 \\
& x_1^2 x_{1,1} z[ ]^2 z_{1,1,1} K 4 x_1^2 x_{1,1,1} z[ ]^2 z_{1,1} K x_1^2 x_{1,1,1,1} z[ ]^2 z_1 C 15 x_1 x_{1,1}^2 z[ ]^2 z_{1,1} \\
& C 10 x_1 x_{1,1} x_{1,1,1} z[ ]^2 z_1 K 15 x_{1,1}^3 z[ ]^2 z_1 C t^2 y_1^2 y_{1,1,1,1} K 10 t^2 y_1 y_{1,1} y_{1,1,1} \\
& C 15 t^2 y_{1,1}^3 K 4 t x[ ] y_1 y_{1,1} z_{1,1,1} K 6 t x[ ] y_1 y_{1,1,1} z_{1,1} C 2 t x[ ] y_1 y_{1,1,1,1} z_1 \\
& C 30 t x[ ] y_{1,1}^2 z_{1,1} K 10 t x[ ] y_{1,1} y_{1,1,1} z_1 K 2 t x_1 y[ ] y_1 z_{1,1,1,1} \\
& C 6 t x_1 y[ ] y_{1,1} z_{1,1,1} C 4 t x_1 y[ ] y_{1,1,1} z_{1,1} K 2 t x_1 y_1 y_{1,1,1,1} z[ ] \\
& C 10 t x_1 y_{1,1} y_{1,1,1} z[ ] K 4 t x_{1,1} y[ ] y_{1,1,1} z_1 C 6 t x_{1,1} y_1 y_{1,1,1} z[ ] K 30 t x_{1,1} \\
& y_{1,1}^2 z[ ] K 6 t x_{1,1,1} y[ ] y_{1,1} z_1 C 4 t x_{1,1,1} y_1 y_{1,1} z[ ] C 2 t x_{1,1,1,1} y[ ] y_1 z_1 \\
& K 4 x[ ]^2 y_{1,1} z_1 z_{1,1,1} C 15 x[ ]^2 y_{1,1} z_{1,1}^2 K 6 x[ ]^2 y_{1,1,1} z_1 z_{1,1} C x[ ]^2 y_{1,1,1,1} z_{1,1}^2 \\
& K 2 x[ ] x_1 y[ ] z_1 z_{1,1,1,1} C 10 x[ ] x_1 y[ ] z_{1,1} z_{1,1,1} C 4 x[ ] x_1 y_{1,1} z[ ] z_{1,1,1} \\
& C 6 x[ ] x_1 y_{1,1,1} z[ ] z_{1,1} K 2 x[ ] x_1 y_{1,1,1,1} z[ ] z_1 K 4 x[ ] x_{1,1} y[ ] z_1 z_{1,1,1} \\
& K 30 x[ ] x_{1,1} y_{1,1} z[ ] z_{1,1} C 6 x[ ] x_{1,1} y_{1,1,1} z[ ] z_1 K 6 x[ ] x_{1,1,1} y[ ] z_1 z_{1,1} \\
& C 4 x[ ] x_{1,1,1} y_{1,1} z[ ] z_1 C 2 x[ ] x_{1,1,1,1} y[ ] z_1^2 C 2 x_1^2 y[ ] z[ ] z_{1,1,1,1} C \\
& x_1^2 y_{1,1,1,1} z[ ]^2 K 6 x_1 x_{1,1} y[ ] z[ ] z_{1,1,1} K 6 x_1 x_{1,1} y_{1,1,1} z[ ]^2 \\
& K 4 x_1 x_{1,1,1} y[ ] z[ ] z_{1,1} K 4 x_1 x_{1,1,1} y_{1,1} z[ ]^2 K 2 x_1 x_{1,1,1,1} y[ ] z[ ] z_1 C 15 \\
& x_{1,1}^2 y_{1,1} z[ ]^2 C 10 x_{1,1} x_{1,1,1} y[ ] z[ ] z_1 K 2 t y[ ] y_1 y_{1,1,1,1} C 10 t y[ ] y_{1,1} y_{1,1,1} \\
& C 4 x[ ] y[ ] y_{1,1} z_{1,1,1} C 6 x[ ] y[ ] y_{1,1,1} z_{1,1} K 2 x[ ] y[ ] y_{1,1,1,1} z_1 \\
& C x_1 y[ ]^2 z_{1,1,1,1} C 2 x_1 y[ ] y_{1,1,1,1} z[ ] K 6 x_{1,1} y[ ] y_{1,1,1} z[ ] \\
& K 4 x_{1,1,1} y[ ] y_{1,1} z[ ] K x_{1,1,1,1} y[ ]^2 z_1 C y[ ]^2 y_{1,1,1,1} )
\end{aligned}$$

$$\mathbf{M} > I_{4b} \text{d} \text{ Hook}([v2, v4], \omega)$$

$$I_{4b} \text{d} \text{ k} \frac{1}{(t y_1 C x[ ] z_1 K x_1 z[ ] K y[ ] )^8} (t^2 x_1 y_1 y_{1,1} z_{1,1,1,1} K t^2 x_1 y_1 y_{1,1,1,1} z_{1,1} \quad (1.10)$$

$$\begin{aligned}
& K 6 t^2 x_1 y_{1,1}^2 z_{1,1,1} C 6 t^2 x_1 y_{1,1} y_{1,1,1} z_{1,1} K t^2 x_{1,1} y_1^2 z_{1,1,1,1} \\
& C 6 t^2 x_{1,1} y_1 y_{1,1} z_{1,1,1} C t^2 x_{1,1} y_1 y_{1,1,1} z_1 K 6 t^2 x_{1,1} y_{1,1} y_{1,1,1} z_1 \\
& K 6 t^2 x_{1,1,1} y_1 y_{1,1} z_{1,1} C 6 t^2 x_{1,1,1} y_1^2 z_1 C t^2 x_{1,1,1} y_1^2 z_{1,1} K t^2 x_{1,1,1} y_1 y_{1,1} z_1 \\
& C t x[ ] x_1 y_{1,1} z_1 z_{1,1,1,1} K 6 t x[ ] x_1 y_{1,1} z_{1,1} z_{1,1,1} C 6 t x[ ] x_1 y_{1,1,1} z_1^2 \\
& K t x[ ] x_1 y_{1,1,1} z_1 z_{1,1} K t x[ ] x_{1,1} y_1 z_1 z_{1,1,1,1} C 6 t x[ ] x_{1,1} y_1 z_{1,1} z_{1,1,1} \\
& K 6 t x[ ] x_{1,1} y_{1,1} z_1 z_{1,1} C t x[ ] x_{1,1} y_{1,1,1} z_1^2 K 6 t x[ ] x_{1,1,1} y_1 z_1^2 \\
& C 6 t x[ ] x_{1,1,1} y_{1,1} z_1 z_{1,1} C t x[ ] x_{1,1,1} y_1 z_1 z_{1,1} K t x[ ] x_{1,1,1} y_{1,1} z_1^2 K t \\
& x_1^2 y_{1,1} z[ ] z_{1,1,1,1} C t x_1^2 y_{1,1,1,1} z[ ] z_{1,1} C t x_1 x_{1,1} y_1 z[ ] z_{1,1,1,1} \\
& C 6 t x_1 x_{1,1} y_{1,1} z[ ] z_{1,1,1} K 6 t x_1 x_{1,1} y_{1,1} z[ ] z_{1,1} K t x_1 x_{1,1} y_{1,1,1,1} z[ ] z_1 \\
& K t x_1 x_{1,1,1,1} y_1 z[ ] z_{1,1} C t x_1 x_{1,1,1,1} y_{1,1} z[ ] z_1 K 6 t x_1^2 y_1 z[ ] z_{1,1,1} C 6 t \\
& x_1^2 y_{1,1,1,1} z[ ] z_1 C 6 t x_{1,1} x_{1,1,1} y_1 z[ ] z_{1,1} K 6 t x_{1,1} x_{1,1,1} y_{1,1} z[ ] z_1 \\
& K t x[ ] y_1 y_{1,1} z_{1,1,1,1} C t x[ ] y_1 y_{1,1,1,1} z_{1,1} C 6 t x[ ] y_1^2 z_{1,1,1,1} \\
& K 6 t x[ ] y_{1,1} y_{1,1,1} z_{1,1} K t x_1 y[ ] y_{1,1} z_{1,1,1,1} C t x_1 y[ ] y_{1,1,1,1} z_{1,1} \\
& C 2 t x_{1,1} y[ ] y_1 z_{1,1,1,1} K 6 t x_{1,1} y[ ] y_{1,1} z_{1,1,1} K t x_{1,1} y[ ] y_{1,1,1,1} z_1 \\
& K t x_{1,1} y_1 y_{1,1,1,1} z[ ] C 6 t x_{1,1} y_{1,1} y_{1,1,1} z[ ] C 6 t x_{1,1,1} y[ ] y_{1,1} z_{1,1} K 6 t x_{1,1,1} \\
& y_1^2 z[ ] K 2 t x_{1,1,1,1} y[ ] y_1 z_{1,1} C t x_{1,1,1,1} y[ ] y_{1,1} z_1 C t x_{1,1,1,1} y_1 y_{1,1} z[ ] \\
& K x[ ]^2 y_{1,1} z_1 z_{1,1,1,1} C 6 x[ ]^2 y_{1,1} z_{1,1} z_{1,1,1,1} K 6 x[ ]^2 y_{1,1,1,1} z_{1,1}^2 \\
& C x[ ]^2 y_{1,1,1,1} z_1 z_{1,1} C x[ ] x_1 y_{1,1} z[ ] z_{1,1,1,1} K x[ ] x_1 y_{1,1,1,1} z[ ] z_{1,1} \\
& C x[ ] x_{1,1} y[ ] z_1 z_{1,1,1,1} K 6 x[ ] x_{1,1} y[ ] z_{1,1} z_{1,1,1} K 6 x[ ] x_{1,1} y_{1,1} z[ ] z_{1,1,1} \\
& C 12 x[ ] x_{1,1} y_{1,1,1} z[ ] z_{1,1} K x[ ] x_{1,1} y_{1,1,1,1} z[ ] z_1 C 6 x[ ] x_{1,1,1} y[ ] z_{1,1}^2 \\
& K 6 x[ ] x_{1,1,1} y_{1,1} z[ ] z_{1,1} K x[ ] x_{1,1,1,1} y[ ] z_1 z_{1,1} C x[ ] x_{1,1,1,1} y_{1,1} z[ ] z_1 \\
& K x_1 x_{1,1} y[ ] z[ ] z_{1,1,1,1} C x_1 x_{1,1} y_{1,1,1,1} z[ ]^2 C x_1 x_{1,1,1,1} y[ ] z[ ] z_{1,1} \\
& K x_1 x_{1,1,1,1} y_{1,1} z[ ]^2 C 6 x_{1,1}^2 y[ ] z[ ] z_{1,1,1} K 6 x_{1,1}^2 y_{1,1,1,1} z[ ]^2 \\
& K 6 x_{1,1} x_{1,1,1,1} y[ ] z[ ] z_{1,1} C 6 x_{1,1} x_{1,1,1,1} y_{1,1} z[ ]^2 C x[ ] y[ ] y_{1,1} z_{1,1,1,1} \\
& K x[ ] y[ ] y_{1,1,1,1} z_{1,1} K x_{1,1} y[ ]^2 z_{1,1,1,1} C x_{1,1} y[ ] y_{1,1,1,1} z[ ] \\
& C x_{1,1,1,1} y[ ]^2 z_{1,1} K x_{1,1,1,1} y[ ] y_{1,1} z[ ] )
\end{aligned}$$

$$\mathbf{M} > I_{4c} \triangleleft Hook([v3, v4], \omega)$$

$$I_{4c} \triangleleft K \frac{1}{(ty_1 Cx[ ] z_1 K x_1 z[ ] K y[ ])^{10}} \left( t^3 x_1 y_1^2 y_{1,1,1} z_{1,1,1,1} K t^3 x_1 y_1^2 y_{1,1,1,1} z_{1,1,1,1} \right) \quad (1.11)$$

$$\begin{aligned}
& K 3 t^3 x_1 y_1 y_{1,1}^2 z_{1,1,1,1} C 4 t^3 x_1 y_1 y_{1,1} y_{1,1,1} z_{1,1,1} C 3 t^3 x_1 y_1 y_{1,1} y_{1,1,1,1} z_{1,1} \\
& K 4 t^3 x_1 y_1 y_{1,1,1}^2 z_{1,1} C 3 t^3 x_1 y_{1,1}^3 z_{1,1,1} K 3 t^3 x_1 y_{1,1}^2 y_{1,1,1} z_{1,1} C 3 t^3 x_{1,1} \\
& y_1^2 y_{1,1} z_{1,1,1,1} K 4 t^3 x_{1,1} y_1^2 y_{1,1} z_{1,1,1} K 3 t^3 x_{1,1} y_1 y_{1,1}^2 z_{1,1,1}
\end{aligned}$$

$$\begin{aligned}
& K 3 \ell^3 x_{1,1} y_1 y_{1,1} y_{1,1,1,1} z_1 C 4 \ell^3 x_{1,1} y_1 y_{1,1,1,1}^2 z_1 C 3 \ell^3 x_{1,1} y_{1,1}^2 y_{1,1,1,1} z_1 K \ell^3 x_{1,1,1,1} \\
& y_1^3 z_{1,1,1,1} C 4 \ell^3 x_{1,1,1,1} y_1^2 y_{1,1,1,1} z_1 C \ell^3 x_{1,1,1,1} y_1^2 y_{1,1,1,1} z_1 C 3 \ell^3 x_{1,1,1,1} y_1 y_{1,1}^2 z_{1,1} \\
& K 4 \ell^3 x_{1,1,1,1} y_1 y_{1,1} y_{1,1,1,1} z_1 K 3 \ell^3 x_{1,1,1,1} y_{1,1}^3 z_1 C \ell^3 x_{1,1,1,1} y_1^3 z_{1,1,1} K 3 \ell^3 x_{1,1,1,1} \\
& y_1^2 y_{1,1} z_{1,1} K \ell^3 x_{1,1,1,1} y_1^2 y_{1,1,1,1} z_1 C 3 \ell^3 x_{1,1,1,1} y_1 y_{1,1}^2 z_1 \\
& K 3 \ell^2 x[ ] x_1 y_1 y_{1,1} z_{1,1} z_{1,1,1,1} C 4 \ell^2 x[ ] x_1 y_1 y_{1,1} z_{1,1,1,1}^2 \\
& C 2 \ell^2 x[ ] x_1 y_1 y_{1,1,1,1} z_1 z_{1,1,1,1} K 4 \ell^2 x[ ] x_1 y_1 y_{1,1,1,1} z_{1,1} z_{1,1,1,1} \\
& K 2 \ell^2 x[ ] x_1 y_1 y_{1,1,1,1} z_1 z_{1,1,1,1} C 3 \ell^2 x[ ] x_1 y_1 y_{1,1,1,1} z_{1,1}^2 K 3 \ell^2 x[ ] x_1 \\
& y_{1,1}^2 z_1 z_{1,1,1,1} C 6 \ell^2 x[ ] x_1 y_{1,1}^2 z_{1,1} z_{1,1,1,1} C 4 \ell^2 x[ ] x_1 y_{1,1} y_{1,1,1,1} z_1 z_{1,1,1,1} \\
& K 6 \ell^2 x[ ] x_1 y_{1,1} y_{1,1,1,1} z_{1,1}^2 C 3 \ell^2 x[ ] x_1 y_{1,1} y_{1,1,1,1} z_1 z_{1,1} K 4 \ell^2 x[ ] x_1 \\
& y_{1,1,1,1}^2 z_1 z_{1,1} C 3 \ell^2 x[ ] x_{1,1} y_{1,1}^2 z_{1,1} z_{1,1,1,1} K 4 \ell^2 x[ ] x_{1,1} y_{1,1}^2 z_{1,1,1,1}^2 \\
& C 3 \ell^2 x[ ] x_{1,1} y_1 y_{1,1} z_1 z_{1,1,1,1} K 6 \ell^2 x[ ] x_{1,1} y_1 y_{1,1} z_{1,1} z_{1,1,1,1} \\
& K 3 \ell^2 x[ ] x_{1,1} y_1 y_{1,1,1,1} z_1 z_{1,1} C 6 \ell^2 x[ ] x_{1,1} y_{1,1} y_{1,1,1,1} z_1 z_{1,1} \\
& K 3 \ell^2 x[ ] x_{1,1} y_{1,1} y_{1,1,1,1} z_{1,1}^2 C 4 \ell^2 x[ ] x_{1,1} y_{1,1}^2 z_{1,1}^2 K 2 \ell^2 x[ ] x_{1,1,1,1} y_{1,1}^2 z_1 z_{1,1,1,1} \\
& C 4 \ell^2 x[ ] x_{1,1,1,1} y_{1,1}^2 z_{1,1} z_{1,1,1,1} K 4 \ell^2 x[ ] x_{1,1,1,1} y_1 y_{1,1} z_1 z_{1,1,1,1} \\
& C 6 \ell^2 x[ ] x_{1,1,1,1} y_1 y_{1,1} z_{1,1}^2 C 4 \ell^2 x[ ] x_{1,1,1,1} y_1 y_{1,1,1,1} z_1 z_{1,1} \\
& C 2 \ell^2 x[ ] x_{1,1,1,1} y_1 y_{1,1,1,1} z_{1,1}^2 K 6 \ell^2 x[ ] x_{1,1,1,1} y_{1,1}^2 z_1 z_{1,1} \\
& K 4 \ell^2 x[ ] x_{1,1,1,1} y_{1,1} y_{1,1,1,1} z_{1,1}^2 C 2 \ell^2 x[ ] x_{1,1,1,1} y_{1,1}^2 z_1 z_{1,1,1,1} K 3 \ell^2 x[ ] x_{1,1,1,1} y_{1,1}^2 z_{1,1}^2 \\
& K 2 \ell^2 x[ ] x_{1,1,1,1} y_1 y_{1,1,1,1} z_{1,1}^2 C 3 \ell^2 x[ ] x_{1,1,1,1} y_{1,1}^2 z_{1,1}^2 K 2 \ell^2 x_{1,1}^2 y_1 y_{1,1,1,1} z[ ] z_{1,1,1,1} \\
& C 2 \ell^2 x_{1,1}^2 y_1 y_{1,1,1,1} z[ ] z_{1,1,1,1} C 3 \ell^2 x_{1,1}^2 y_{1,1}^2 z[ ] z_{1,1,1,1} K 4 \ell^2 x_{1,1}^2 y_{1,1} y_{1,1,1,1} z[ ] z_{1,1,1,1} \\
& K 3 \ell^2 x_{1,1}^2 y_{1,1} y_{1,1,1,1} z[ ] z_{1,1,1,1} C 4 \ell^2 x_{1,1}^2 y_{1,1,1,1}^2 z[ ] z_{1,1,1,1} C 4 \ell^2 x_1 x_{1,1} y_1 y_{1,1,1,1} z[ ] z_{1,1,1,1} \\
& K 3 \ell^2 x_1 x_{1,1} y_1 y_{1,1,1,1} z[ ] z_{1,1,1,1} K 6 \ell^2 x_1 x_{1,1} y_{1,1}^2 z[ ] z_{1,1,1,1} \\
& C 6 \ell^2 x_1 x_{1,1} y_{1,1} y_{1,1,1,1} z[ ] z_{1,1,1,1} C 3 \ell^2 x_1 x_{1,1} y_{1,1} y_{1,1,1,1} z[ ] z_{1,1} K 4 \ell^2 x_1 x_{1,1} \\
& y_{1,1,1,1}^2 z[ ] z_{1,1} C 2 \ell^2 x_1 x_{1,1,1,1} y_{1,1}^2 z[ ] z_{1,1,1,1} K 4 \ell^2 x_1 x_{1,1,1,1} y_1 y_{1,1} z[ ] z_{1,1,1,1} \\
& K 2 \ell^2 x_1 x_{1,1,1,1} y_1 y_{1,1,1,1} z[ ] z_{1,1} C 4 \ell^2 x_1 x_{1,1,1,1} y_{1,1} y_{1,1,1,1} z[ ] z_{1,1} K 2 \ell^2 x_1 x_{1,1,1,1} \\
& y_{1,1}^2 z[ ] z_{1,1,1,1} C 3 \ell^2 x_1 x_{1,1,1,1} y_1 y_{1,1} z[ ] z_{1,1} C 2 \ell^2 x_1 x_{1,1,1,1} y_1 y_{1,1,1,1} z[ ] z_{1,1} \\
& K 3 \ell^2 x_1 x_{1,1,1,1} y_{1,1}^2 z[ ] z_{1,1} K 3 \ell^2 x_{1,1}^2 y_{1,1}^2 z[ ] z_{1,1,1,1} C 6 \ell^2 x_{1,1}^2 y_1 y_{1,1} z[ ] z_{1,1,1,1} \\
& C 3 \ell^2 x_{1,1}^2 y_1 y_{1,1,1,1} z[ ] z_{1,1} K 6 \ell^2 x_{1,1}^2 y_{1,1} y_{1,1,1,1} z[ ] z_{1,1} C 4 \ell^2 x_{1,1} x_{1,1,1,1} y_{1,1}^2 z[ ] z_{1,1,1,1} \\
& K 6 \ell^2 x_{1,1} x_{1,1,1,1} y_1 y_{1,1} z[ ] z_{1,1} K 4 \ell^2 x_{1,1} x_{1,1,1,1} y_1 y_{1,1,1,1} z[ ] z_{1,1} C 6 \ell^2 x_{1,1} x_{1,1,1,1} \\
& y_{1,1}^2 z[ ] z_{1,1} C 3 \ell^2 x_{1,1} x_{1,1,1,1} y_{1,1}^2 z[ ] z_{1,1} K 3 \ell^2 x_{1,1} x_{1,1,1,1} y_1 y_{1,1} z[ ] z_{1,1} K 4 \ell^2 x_{1,1,1,1}^2 \\
& y_{1,1}^2 z[ ] z_{1,1} C 4 \ell^2 x_{1,1,1,1}^2 y_1 y_{1,1} z[ ] z_{1,1} K 3 t x[ ]^2 x_1 y_{1,1} z_1 z_{1,1} z_{1,1,1,1} \\
& C 4 t x[ ]^2 x_1 y_{1,1} z_{1,1}^2 C 3 t x[ ]^2 x_1 y_{1,1} z_{1,1}^2 z_{1,1,1,1} C t x[ ]^2 x_1 y_{1,1,1,1} z_{1,1}^2 z_{1,1,1,1} \\
& K 4 t x[ ]^2 x_1 y_{1,1,1,1} z_1 z_{1,1} z_{1,1,1,1} K 3 t x[ ]^2 x_1 y_{1,1,1,1} z_{1,1}^3 K t x[ ]^2 x_1 y_{1,1,1,1} z_{1,1}^2 z_{1,1,1,1}
\end{aligned}$$

$$\begin{aligned}
& C3tx[ ]^2x_1y_{1,1,1,1}z_1z_{1,1}^2C3tx[ ]^2x_{1,1}y_1z_1z_{1,1}z_{1,1,1,1}K4tx[ ]^2x_{1,1}y_1z_1z_{1,1,1}^2 \\
& K3tx[ ]^2x_{1,1}y_1z_{1,1}^2z_{1,1,1}C4tx[ ]^2x_{1,1}y_{1,1}z_1^2z_{1,1,1}C3tx[ ]^2x_{1,1}y_{1,1}z_1z_{1,1}^2 \\
& K3tx[ ]^2x_{1,1}y_{1,1,1,1}z_1^2z_{1,1}Ktx[ ]^2x_{1,1,1}y_1z_1^2z_{1,1,1,1} \\
& C4tx[ ]^2x_{1,1,1}y_1z_1z_{1,1}z_{1,1,1}C3tx[ ]^2x_{1,1,1}y_1z_{1,1}^3K4tx[ ]^2x_{1,1,1}y_{1,1}z_1^2z_{1,1,1} \\
& K3tx[ ]^2x_{1,1,1}y_{1,1}z_1z_{1,1}^2Ctx[ ]^2x_{1,1,1}y_{1,1,1}z_1^3Ctx[ ]^2x_{1,1,1,1}y_1z_1^2z_{1,1,1} \\
& K3tx[ ]^2x_{1,1,1,1}y_1z_1z_{1,1}^2C3tx[ ]^2x_{1,1,1,1}y_{1,1}z_1^2z_{1,1}Ktx[ ]^2x_{1,1,1,1}y_{1,1}z_1^3 \\
& C3tx[ ]x_1^2y_{1,1}z[ ]z_{1,1}z_{1,1,1,1}K4tx[ ]x_1^2y_{1,1}z[ ]z_{1,1,1}^2K2tx[ ] \\
& x_1^2y_{1,1,1}z[ ]z_1z_{1,1,1,1}C4tx[ ]x_1^2y_{1,1,1}z[ ]z_{1,1}z_{1,1,1}C2tx[ ] \\
& x_1^2y_{1,1,1,1}z[ ]z_1z_{1,1,1}K3tx[ ]x_1^2y_{1,1,1,1}z[ ]z_{1,1}^2 \\
& K3tx[ ]x_1x_{1,1}y_1z[ ]z_{1,1}z_{1,1,1,1}C4tx[ ]x_1x_{1,1}y_1z[ ]z_{1,1,1}^2 \\
& C3tx[ ]x_1x_{1,1}y_{1,1}z[ ]z_1z_{1,1,1,1}K6tx[ ]x_1x_{1,1}y_{1,1}z[ ]z_{1,1}z_{1,1,1} \\
& K4tx[ ]x_1x_{1,1}y_{1,1,1}z[ ]z_1z_{1,1,1}C6tx[ ]x_1x_{1,1}y_{1,1,1}z[ ]z_{1,1}^2 \\
& C2tx[ ]x_1x_{1,1,1}y_1z[ ]z_1z_{1,1,1,1}K4tx[ ]x_1x_{1,1,1}y_1z[ ]z_{1,1}z_{1,1,1} \\
& C4tx[ ]x_1x_{1,1,1}y_{1,1}z[ ]z_1z_{1,1}K2tx[ ]x_1x_{1,1,1}y_{1,1,1}z[ ]z_{1,1}^2 \\
& K2tx[ ]x_1x_{1,1,1,1}y_1z[ ]z_1z_{1,1,1}C3tx[ ]x_1x_{1,1,1,1}y_1z[ ]z_{1,1}^2 \\
& K3tx[ ]x_1x_{1,1,1,1}y_{1,1}z[ ]z_1z_{1,1}C2tx[ ]x_1x_{1,1,1,1}y_{1,1,1}z[ ]z_{1,1}^2K3tx[ ] \\
& x_{1,1}^2y_1z[ ]z_1z_{1,1,1,1}C6tx[ ]x_{1,1}^2y_1z[ ]z_{1,1}z_{1,1,1}K6tx[ ]x_{1,1}^2y_{1,1,1}z[ ]z_1z_{1,1} \\
& C3tx[ ]x_{1,1}^2y_{1,1,1,1}z[ ]z_{1,1}^2C4tx[ ]x_{1,1}x_{1,1,1}y_1z[ ]z_1z_{1,1,1} \\
& K6tx[ ]x_{1,1}x_{1,1,1}y_1z[ ]z_{1,1}^2C6tx[ ]x_{1,1}x_{1,1,1}y_{1,1}z[ ]z_1z_{1,1} \\
& K4tx[ ]x_{1,1}x_{1,1,1,1}y_{1,1}z[ ]z_{1,1}^2C3tx[ ]x_{1,1}x_{1,1,1,1}y_1z[ ]z_1z_{1,1} \\
& K3tx[ ]x_{1,1}x_{1,1,1,1}y_{1,1}z[ ]z_{1,1}^2K4tx[ ]x_{1,1,1}^2y_1z[ ]z_1z_{1,1}C4tx[ ] \\
& x_{1,1,1}^2y_{1,1}z[ ]z_{1,1}^2Ctx_{1,1}^3y_{1,1,1}z[ ]^2z_{1,1,1,1}Ktx_{1,1,1,1}^3y_{1,1,1}z[ ]^2z_{1,1,1}K3t \\
& x_{1,1}^2x_{1,1}y_{1,1}z[ ]^2z_{1,1,1,1}C3tx_{1,1}^2x_{1,1,1,1,1}z[ ]^2z_{1,1}Ktx_{1,1,1,1}^2x_{1,1}y_1z[ ]^2z_{1,1,1,1} \\
& C4tx_{1,1}^2x_{1,1,1}y_{1,1}z[ ]^2z_{1,1,1}K4tx_{1,1}^2x_{1,1,1}y_{1,1,1}z[ ]^2z_{1,1}Ct \\
& x_{1,1}^2x_{1,1,1}y_{1,1,1,1}z[ ]^2z_1Ctx_{1,1}^2x_{1,1,1,1}y_1z[ ]^2z_{1,1,1}Ktx_{1,1,1,1}^2x_{1,1,1,1}y_{1,1,1}z[ ]^2z_1 \\
& C3tx_1x_{1,1}^2y_1z[ ]^2z_{1,1,1,1}C3tx_1x_{1,1}^2y_{1,1}z[ ]^2z_{1,1,1}K3tx_1x_{1,1}^2y_{1,1,1}z[ ]^2z_{1,1} \\
& K3tx_1x_{1,1}^2y_{1,1,1,1}z[ ]^2z_1K4tx_1x_{1,1}x_{1,1,1}y_1z[ ]^2z_{1,1,1} \\
& C4tx_1x_{1,1}x_{1,1,1}y_{1,1,1}z[ ]^2z_1K3tx_1x_{1,1}x_{1,1,1,1}y_1z[ ]^2z_{1,1} \\
& C3tx_1x_{1,1}x_{1,1,1,1}y_{1,1}z[ ]^2z_1C4tx_1x_{1,1,1}^2y_1z[ ]^2z_{1,1}K4tx_1x_{1,1,1}^2y_{1,1}z[ ]^2z_1 \\
& K3tx_{1,1}^3y_1z[ ]^2z_{1,1,1}C3tx_{1,1}^3y_{1,1,1}z[ ]^2z_1C3tx_{1,1}^2x_{1,1,1}y_1z[ ]^2z_{1,1}K3t \\
& x_{1,1}^2x_{1,1,1}y_{1,1}z[ ]^2z_1Kf^2x[ ]y_1^2y_{1,1,1}z_{1,1,1,1}Cf^2x[ ]y_1^2y_{1,1,1,1}z_{1,1,1} \\
& C3f^2x[ ]y_1y_{1,1}^2z_{1,1,1,1}K4f^2x[ ]y_1y_{1,1}y_{1,1,1}z_{1,1,1}K3f^2x[ ]y_1y_{1,1}y_{1,1,1,1}z_{1,1}
\end{aligned}$$

$$\begin{aligned}
& C4 \ell^2 x[ ] y_1 y_{1,1,1}^2 z_{1,1} K3 \ell^2 x[ ] y_{1,1}^3 z_{1,1,1} C3 \ell^2 x[ ] y_{1,1}^2 y_{1,1,1} z_{1,1} \\
& K2 \ell^2 x_1 y[ ] y_1 y_{1,1,1,1} z_{1,1,1,1} C2 \ell^2 x_1 y[ ] y_1 y_{1,1,1,1} z_{1,1,1,1} C3 \ell^2 x_1 y[ ] y_{1,1}^2 z_{1,1,1,1} \\
& K4 \ell^2 x_1 y[ ] y_{1,1} y_{1,1,1} z_{1,1,1} K3 \ell^2 x_1 y[ ] y_{1,1} y_{1,1,1,1} z_{1,1} C4 \ell^2 x_1 y[ ] y_{1,1,1}^2 z_{1,1} \\
& K6 \ell^2 x_{1,1} y[ ] y_1 y_{1,1} z_{1,1,1,1} C8 \ell^2 x_{1,1} y[ ] y_1 y_{1,1,1} z_{1,1,1} C3 \ell^2 x_{1,1} y[ ] y_{1,1}^2 z_{1,1,1} \\
& C3 \ell^2 x_{1,1} y[ ] y_{1,1} y_{1,1,1,1} z_1 K4 \ell^2 x_{1,1} y[ ] y_{1,1}^2 z_1 C3 \ell^2 x_{1,1} y_1 y_{1,1} y_{1,1,1,1} z[ ] \\
& K4 \ell^2 x_{1,1} y_1 y_{1,1,1}^2 z[ ] K3 \ell^2 x_{1,1} y_{1,1}^2 y_{1,1,1} z[ ] C3 \ell^2 x_{1,1,1} y[ ] y_{1,1}^2 z_{1,1,1,1} \\
& K8 \ell^2 x_{1,1,1} y[ ] y_1 y_{1,1,1} z_{1,1} K2 \ell^2 x_{1,1,1} y[ ] y_1 y_{1,1,1,1} z_1 K3 \ell^2 x_{1,1,1} y[ ] y_{1,1}^2 z_{1,1} \\
& C4 \ell^2 x_{1,1,1} y[ ] y_{1,1} y_{1,1,1} z_1 K \ell^2 x_{1,1,1} y_{1,1}^2 y_{1,1,1,1} z[ ] C4 \ell^2 x_{1,1,1} y_1 y_{1,1} y_{1,1,1} z[ ] \\
& C3 \ell^2 x_{1,1,1} y_{1,1}^3 z[ ] K3 \ell^2 x_{1,1,1,1} y[ ] y_{1,1}^2 z_{1,1,1} C6 \ell^2 x_{1,1,1,1} y[ ] y_1 y_{1,1} z_{1,1} \\
& C2 \ell^2 x_{1,1,1,1} y[ ] y_1 y_{1,1,1} z_1 K3 \ell^2 x_{1,1,1,1} y[ ] y_{1,1}^2 z_1 C \ell^2 x_{1,1,1,1} y_{1,1}^2 y_{1,1,1} z[ ] \\
& K3 \ell^2 x_{1,1,1,1} y_1 y_{1,1}^2 z[ ] C3 tx[ ]^2 y_1 y_{1,1} z_{1,1} z_{1,1,1,1} K4 tx[ ]^2 y_1 y_{1,1} z_{1,1,1}^2 \\
& K2 tx[ ]^2 y_1 y_{1,1,1} z_1 z_{1,1,1,1} C4 tx[ ]^2 y_1 y_{1,1,1} z_{1,1} z_{1,1,1} \\
& C2 tx[ ]^2 y_1 y_{1,1,1,1} z_1 z_{1,1,1} K3 tx[ ]^2 y_1 y_{1,1,1,1} z_{1,1}^2 C3 tx[ ]^2 y_{1,1}^2 z_1 z_{1,1,1,1} \\
& K6 tx[ ]^2 y_{1,1}^2 z_{1,1} z_{1,1,1} K4 tx[ ]^2 y_{1,1} y_{1,1,1} z_1 z_{1,1,1} C6 tx[ ]^2 y_{1,1} y_{1,1,1} z_{1,1}^2 \\
& K3 tx[ ]^2 y_{1,1} y_{1,1,1,1} z_1 z_{1,1} C4 tx[ ]^2 y_{1,1}^2 z_{1,1} z_{1,1} \\
& C3 tx[ ] x_1 y[ ] y_{1,1} z_{1,1} z_{1,1,1,1} K4 tx[ ] x_1 y[ ] y_{1,1} z_{1,1,1}^2 \\
& K2 tx[ ] x_1 y[ ] y_{1,1,1} z_1 z_{1,1,1,1} C4 tx[ ] x_1 y[ ] y_{1,1,1} z_{1,1} z_{1,1,1} \\
& C2 tx[ ] x_1 y[ ] y_{1,1,1,1} z_1 z_{1,1,1} K3 tx[ ] x_1 y[ ] y_{1,1,1,1} z_{1,1}^2 \\
& C2 tx[ ] x_1 y_1 y_{1,1,1} z[ ] z_{1,1,1,1} K2 tx[ ] x_1 y_1 y_{1,1,1,1} z[ ] z_{1,1,1} K3 tx[ ] x_1 \\
& y_{1,1}^2 z[ ] z_{1,1,1,1} C4 tx[ ] x_1 y_{1,1} y_{1,1,1} z[ ] z_{1,1,1} C3 tx[ ] x_1 y_{1,1} y_{1,1,1,1} z[ ] z_{1,1} \\
& K4 tx[ ] x_1 y_{1,1}^2 z[ ] z_{1,1} K6 tx[ ] x_{1,1} y[ ] y_1 z_{1,1} z_{1,1,1,1} C8 tx[ ] x_{1,1} y[ ] y_1 \\
& z_{1,1,1}^2 K3 tx[ ] x_{1,1} y[ ] y_{1,1} z_1 z_{1,1,1,1} C6 tx[ ] x_{1,1} y[ ] y_{1,1} z_{1,1} z_{1,1,1} \\
& C3 tx[ ] x_{1,1} y[ ] y_{1,1,1,1} z_1 z_{1,1} K3 tx[ ] x_{1,1} y_1 y_{1,1} z[ ] z_{1,1,1,1} \\
& K4 tx[ ] x_{1,1} y_1 y_{1,1,1} z[ ] z_{1,1,1} C6 tx[ ] x_{1,1} y_1 y_{1,1,1,1} z[ ] z_{1,1} C6 tx[ ] x_{1,1} \\
& y_{1,1}^2 z[ ] z_{1,1,1} K12 tx[ ] x_{1,1} y_{1,1} y_{1,1,1} z[ ] z_{1,1} C3 tx[ ] x_{1,1} y_{1,1} y_{1,1,1,1} z[ ] z_1 \\
& K4 tx[ ] x_{1,1} y_{1,1}^2 z[ ] z_1 C4 tx[ ] x_{1,1,1} y[ ] y_1 z_1 z_{1,1,1,1} \\
& K8 tx[ ] x_{1,1,1} y[ ] y_1 z_{1,1} z_{1,1,1} C4 tx[ ] x_{1,1,1} y[ ] y_{1,1} z_1 z_{1,1,1} \\
& K6 tx[ ] x_{1,1,1} y[ ] y_{1,1} z_{1,1}^2 K4 tx[ ] x_{1,1,1} y[ ] y_{1,1,1} z_1 z_{1,1} \\
& K2 tx[ ] x_{1,1,1} y[ ] y_{1,1,1,1} z_{1,1}^2 C8 tx[ ] x_{1,1,1} y_1 y_{1,1} z[ ] z_{1,1,1} \\
& K4 tx[ ] x_{1,1,1} y_1 y_{1,1,1} z[ ] z_{1,1} K2 tx[ ] x_{1,1,1} y_1 y_{1,1,1,1} z[ ] z_1 C6 tx[ ] x_{1,1,1} \\
& y_{1,1}^2 z[ ] z_{1,1} C4 tx[ ] x_{1,1,1} y_{1,1} y_{1,1,1} z[ ] z_1 K4 tx[ ] x_{1,1,1,1} y[ ] y_1 z_1 z_{1,1,1} \\
& C6 tx[ ] x_{1,1,1,1} y[ ] y_1 z_{1,1}^2 C2 tx[ ] x_{1,1,1,1} y[ ] y_{1,1} z_1^2
\end{aligned}$$

$$\begin{aligned}
& K 3 t x[ ] x_{1,1,1,1} y_1 y_{1,1} z[ ] z_{1,1} C 2 t x[ ] x_{1,1,1,1} y_1 y_{1,1} z[ ] z_1 K 3 t x[ ] x_{1,1,1,1} \\
& y_{1,1}^2 z[ ] z_1 C 2 t x_1^2 y[ ] y_{1,1,1} z[ ] z_{1,1,1,1} K 2 t x_1^2 y[ ] y_{1,1,1,1} z[ ] z_{1,1,1} \\
& K 4 t x_1 x_{1,1} y[ ] y_{1,1,1} z[ ] z_{1,1,1} C 3 t x_1 x_{1,1} y[ ] y_{1,1,1,1} z[ ] z_{1,1} \\
& K 3 t x_1 x_{1,1} y_{1,1} y_{1,1,1,1} z[ ]^2 C 4 t x_1 x_{1,1} y_{1,1}^2 z[ ]^2 \\
& K 4 t x_1 x_{1,1,1} y[ ] y_1 z[ ] z_{1,1,1,1} C 4 t x_1 x_{1,1,1} y[ ] y_{1,1} z[ ] z_{1,1,1} \\
& C 2 t x_1 x_{1,1,1} y[ ] y_{1,1,1,1} z[ ] z_1 C 2 t x_1 x_{1,1,1} y_1 y_{1,1,1,1} z[ ]^2 \\
& K 4 t x_1 x_{1,1,1} y_{1,1} y_{1,1,1} z[ ]^2 C 4 t x_1 x_{1,1,1} y[ ] y_1 z[ ] z_{1,1,1} \\
& K 3 t x_1 x_{1,1,1,1} y[ ] y_{1,1} z[ ] z_{1,1} K 2 t x_1 x_{1,1,1,1} y[ ] y_{1,1} z[ ] z_1 \\
& K 2 t x_1 x_{1,1,1,1} y_1 y_{1,1,1} z[ ]^2 C 3 t x_1 x_{1,1,1,1} y_{1,1}^2 z[ ]^2 C 6 t x_{1,1}^2 y[ ] y_1 z[ ] z_{1,1,1,1} \\
& K 6 t x_{1,1}^2 y[ ] y_{1,1} z[ ] z_{1,1,1} K 3 t x_{1,1}^2 y[ ] y_{1,1,1,1} z[ ] z_1 K 3 t x_{1,1}^2 y_1 y_{1,1,1,1} z[ ]^2 \\
& C 6 t x_{1,1}^2 y_{1,1} y_{1,1,1} z[ ]^2 K 8 t x_{1,1} x_{1,1,1} y[ ] y_1 z[ ] z_{1,1,1} \\
& C 6 t x_{1,1} x_{1,1,1} y[ ] y_{1,1} z[ ] z_{1,1} C 4 t x_{1,1} x_{1,1,1} y[ ] y_{1,1,1} z[ ] z_1 \\
& C 4 t x_{1,1} x_{1,1,1} y_1 y_{1,1,1} z[ ]^2 K 6 t x_{1,1} x_{1,1,1} y_{1,1}^2 z[ ]^2 \\
& K 6 t x_{1,1} x_{1,1,1,1} y[ ] y_1 z[ ] z_{1,1} C 3 t x_{1,1} x_{1,1,1,1} y[ ] y_{1,1} z[ ] z_1 \\
& C 3 t x_{1,1} x_{1,1,1,1} y_1 y_{1,1} z[ ]^2 C 8 t x_{1,1,1}^2 y[ ] y_1 z[ ] z_{1,1} K 4 t x_{1,1,1}^2 y[ ] y_{1,1} z[ ] z_1 \\
& K 4 t x_{1,1,1}^2 y_1 y_{1,1} z[ ]^2 C 3 x[ ]^3 y_{1,1} z_1 z_{1,1} z_{1,1,1,1} K 4 x[ ]^3 y_{1,1} z_1 z_{1,1,1}^2 \\
& K 3 x[ ]^3 y_{1,1} z_{1,1}^2 z_{1,1,1} K x[ ]^3 y_{1,1,1} z_{1,1,1,1}^2 C 4 x[ ]^3 y_{1,1,1} z_1 z_{1,1} z_{1,1,1} \\
& C 3 x[ ]^3 y_{1,1,1} z_{1,1}^3 C x[ ]^3 y_{1,1,1,1} z_{1,1,1}^2 K 3 x[ ]^3 y_{1,1,1,1} z_1 z_{1,1}^2 \\
& K 3 x[ ]^2 x_1 y_{1,1} z[ ] z_{1,1} z_{1,1,1,1} C 4 x[ ]^2 x_1 y_{1,1} z[ ] z_{1,1,1}^2 \\
& C 2 x[ ]^2 x_1 y_{1,1,1} z[ ] z_1 z_{1,1,1,1} K 4 x[ ]^2 x_1 y_{1,1,1} z[ ] z_{1,1} z_{1,1,1} \\
& K 2 x[ ]^2 x_1 y_{1,1,1,1} z[ ] z_1 z_{1,1,1} C 3 x[ ]^2 x_1 y_{1,1,1,1} z[ ] z_{1,1}^2 \\
& K 3 x[ ]^2 x_{1,1} y[ ] z_1 z_{1,1} z_{1,1,1,1} C 4 x[ ]^2 x_{1,1} y[ ] z_1 z_{1,1,1}^2 C 3 x[ ]^2 x_{1,1} y[ ] \\
& z_{1,1}^2 z_{1,1,1} K 3 x[ ]^2 x_{1,1} y_{1,1} z[ ] z_1 z_{1,1,1,1} C 6 x[ ]^2 x_{1,1} y_{1,1} z[ ] z_{1,1} z_{1,1,1} \\
& K 4 x[ ]^2 x_{1,1} y_{1,1,1} z[ ] z_1 z_{1,1,1} K 9 x[ ]^2 x_{1,1} y_{1,1,1} z[ ] z_{1,1}^2 \\
& C 6 x[ ]^2 x_{1,1} y_{1,1,1,1} z[ ] z_1 z_{1,1} C x[ ]^2 x_{1,1,1} y[ ] z_{1,1}^2 z_{1,1,1,1} \\
& K 4 x[ ]^2 x_{1,1,1} y[ ] z_1 z_{1,1} z_{1,1,1} K 3 x[ ]^2 x_{1,1,1} y[ ] z_{1,1}^3 \\
& C 8 x[ ]^2 x_{1,1,1} y_{1,1} z[ ] z_1 z_{1,1,1} C 3 x[ ]^2 x_{1,1,1} y_{1,1} z[ ] z_{1,1}^2 \\
& K 4 x[ ]^2 x_{1,1,1} y_{1,1,1} z[ ] z_1 z_{1,1} K x[ ]^2 x_{1,1,1} y_{1,1,1,1} z[ ] z_{1,1}^2 K x[ ]^2 x_{1,1,1,1} y[ ] \\
& z_{1,1}^2 z_{1,1,1} C 3 x[ ]^2 x_{1,1,1,1} y[ ] z_1 z_{1,1}^2 K 3 x[ ]^2 x_{1,1,1,1} y_{1,1} z[ ] z_1 z_{1,1} \\
& C x[ ]^2 x_{1,1,1,1} y_{1,1,1} z[ ] z_{1,1}^2 K x[ ]^2 x_{1,1,1,1} y_{1,1} z[ ]^2 z_{1,1,1,1} C x[ ]^2 x_{1,1,1,1} y_{1,1,1} z[ ]^2 z_{1,1,1,1} \\
& C 3 x[ ] x_1 x_{1,1} y[ ] z[ ] z_{1,1} z_{1,1,1,1} K 4 x[ ] x_1 x_{1,1} y[ ] z[ ] z_{1,1,1}^2 \\
& C 3 x[ ] x_1 x_{1,1} y_{1,1} z[ ]^2 z_{1,1,1,1} C 4 x[ ] x_1 x_{1,1} y_{1,1,1} z[ ]^2 z_{1,1,1}
\end{aligned}$$

$$\begin{aligned}
& K 6 x[ ] x_1 x_{1,1} y_{1,1,1} z[ ]^2 z_{1,1} K 2 x[ ] x_1 x_{1,1} y[ ] z[ ] z_1 z_{1,1,1,1} \\
& C 4 x[ ] x_1 x_{1,1,1} y[ ] z[ ] z_{1,1} z_{1,1,1} K 8 x[ ] x_1 x_{1,1,1} y_{1,1} z[ ]^2 z_{1,1,1} \\
& C 4 x[ ] x_1 x_{1,1,1} y_{1,1,1} z[ ]^2 z_{1,1} C 2 x[ ] x_1 x_{1,1,1} y_{1,1,1} z[ ]^2 z_1 \\
& C 2 x[ ] x_1 x_{1,1,1,1} y[ ] z[ ] z_1 z_{1,1,1} K 3 x[ ] x_1 x_{1,1,1,1} y[ ] z[ ] z_{1,1}^2 \\
& C 3 x[ ] x_1 x_{1,1,1,1} y_{1,1} z[ ]^2 z_{1,1} K 2 x[ ] x_1 x_{1,1,1,1} y_{1,1,1} z[ ]^2 z_1 C 3 x[ ] \\
& x_{1,1}^2 y[ ] z[ ] z_1 z_{1,1,1,1} K 6 x[ ] x_{1,1}^2 y[ ] z[ ] z_{1,1} z_{1,1,1} K 3 x[ ] x_{1,1}^2 y_{1,1} z[ ]^2 z_{1,1,1} \\
& C 9 x[ ] x_{1,1}^2 y_{1,1,1} z[ ]^2 z_{1,1} K 3 x[ ] x_{1,1}^2 y_{1,1,1,1} z[ ]^2 z_1 \\
& K 4 x[ ] x_{1,1} x_{1,1,1} y[ ] z[ ] z_1 z_{1,1,1} C 6 x[ ] x_{1,1} x_{1,1,1} y[ ] z[ ] z_{1,1}^2 \\
& K 6 x[ ] x_{1,1} x_{1,1,1} y_{1,1} z[ ]^2 z_{1,1} C 4 x[ ] x_{1,1} x_{1,1,1} y_{1,1} z[ ]^2 z_1 \\
& K 3 x[ ] x_{1,1} x_{1,1,1,1} y[ ] z[ ] z_1 z_{1,1} C 3 x[ ] x_{1,1} x_{1,1,1,1} y_{1,1} z[ ]^2 z_1 C 4 x[ ] \\
& x_{1,1,1}^2 y[ ] z[ ] z_1 z_{1,1} K 4 x[ ] x_{1,1,1}^2 y_{1,1} z[ ]^2 z_1 C x_{1,1,1}^2 y[ ] z[ ]^2 z_{1,1,1,1} K \\
& x_{1,1,1}^2 x_{1,1,1} y_{1,1,1,1} z[ ]^3 K x_{1,1,1,1}^2 y[ ] z[ ]^2 z_{1,1,1} C x_{1,1,1,1}^2 y_{1,1,1} z[ ]^3 K 3 x_1 \\
& x_{1,1}^2 y[ ] z[ ]^2 z_{1,1,1,1} C 3 x_1 x_{1,1}^2 y_{1,1,1,1} z[ ]^3 C 4 x_1 x_{1,1} x_{1,1,1} y[ ] z[ ]^2 z_{1,1,1} \\
& K 4 x_1 x_{1,1} x_{1,1,1} y_{1,1,1} z[ ]^3 C 3 x_1 x_{1,1} x_{1,1,1} y[ ] z[ ]^2 z_{1,1} \\
& K 3 x_1 x_{1,1} x_{1,1,1,1} y_{1,1} z[ ]^3 K 4 x_1 x_{1,1,1}^2 y[ ] z[ ]^2 z_{1,1} C 4 x_1 x_{1,1,1}^2 y_{1,1} z[ ]^3 C 3 \\
& x_{1,1}^3 y[ ] z[ ]^2 z_{1,1,1} K 3 x_{1,1}^3 y_{1,1,1} z[ ]^3 K 3 x_{1,1}^2 x_{1,1,1} y[ ] z[ ]^2 z_{1,1} C 3 \\
& x_{1,1}^2 x_{1,1,1} y_{1,1} z[ ]^3 C 2 t x[ ] y[ ] y_1 y_{1,1,1} z_{1,1,1,1} K 2 t x[ ] y[ ] y_1 y_{1,1,1,1} z_{1,1,1} \\
& K 3 t x[ ] y[ ] y_{1,1}^2 z_{1,1,1,1} C 4 t x[ ] y[ ] y_{1,1} y_{1,1,1} z_{1,1,1} \\
& C 3 t x[ ] y[ ] y_{1,1} y_{1,1,1,1} z_{1,1} K 4 t x[ ] y[ ] y_{1,1,1}^2 z_{1,1} C t x_1 y[ ]^2 y_{1,1,1} z_{1,1,1,1} \\
& K t x_1 y[ ]^2 y_{1,1,1,1} z_{1,1,1} C 3 t x_{1,1} y[ ]^2 y_{1,1} z_{1,1,1,1} K 4 t x_{1,1} y[ ]^2 y_{1,1,1} z_{1,1,1} \\
& K 3 t x_{1,1} y[ ] y_{1,1} y_{1,1,1,1} z[ ] C 4 t x_{1,1} y[ ] y_{1,1,1}^2 z[ ] K 3 t x_{1,1,1} y[ ]^2 y_1 z_{1,1,1,1} \\
& C 4 t x_{1,1,1} y[ ]^2 y_{1,1,1} z_{1,1} C t x_{1,1,1} y[ ]^2 y_{1,1,1,1} z_1 C 2 t x_{1,1,1} y[ ] y_1 y_{1,1,1,1} z[ ] \\
& K 4 t x_{1,1,1} y[ ] y_{1,1} y_{1,1,1} z[ ] C 3 t x_{1,1,1,1} y[ ]^2 y_1 z_{1,1,1} K 3 t x_{1,1,1,1} y[ ]^2 y_{1,1} z_{1,1} \\
& K t x_{1,1,1,1} y[ ]^2 y_{1,1,1} z_1 K 2 t x_{1,1,1,1} y[ ] y_1 y_{1,1,1} z[ ] C 3 t x_{1,1,1,1} y[ ] y_{1,1}^2 z[ ] \\
& K 3 x[ ]^2 y[ ] y_{1,1} z_{1,1} z_{1,1,1,1} C 4 x[ ]^2 y[ ] y_{1,1} z_{1,1,1}^2 C 2 x[ ]^2 y[ ] y_{1,1,1} z_1 z_{1,1,1,1} \\
& K 4 x[ ]^2 y[ ] y_{1,1,1} z_{1,1} z_{1,1,1} K 2 x[ ]^2 y[ ] y_{1,1,1,1} z_1 z_{1,1,1} C 3 x[ ]^2 y[ ] y_{1,1,1,1} \\
& z_{1,1}^2 K 2 x[ ] x_1 y[ ] y_{1,1,1} z[ ] z_{1,1,1,1} C 2 x[ ] x_1 y[ ] y_{1,1,1,1} z[ ] z_{1,1,1} \\
& C 3 x[ ] x_{1,1} y[ ]^2 z_{1,1} z_{1,1,1,1} K 4 x[ ] x_{1,1} y[ ]^2 z_{1,1,1}^2 \\
& C 3 x[ ] x_{1,1} y[ ] y_{1,1} z[ ] z_{1,1,1,1} C 4 x[ ] x_{1,1} y[ ] y_{1,1,1} z[ ] z_{1,1,1} \\
& K 6 x[ ] x_{1,1} y[ ] y_{1,1,1,1} z[ ] z_{1,1} K 2 x[ ] x_{1,1,1} y[ ]^2 z_1 z_{1,1,1,1} \\
& C 4 x[ ] x_{1,1,1} y[ ]^2 z_{1,1} z_{1,1,1} K 8 x[ ] x_{1,1,1} y[ ] y_{1,1} z[ ] z_{1,1,1} \\
& C 4 x[ ] x_{1,1,1} y[ ] y_{1,1,1} z[ ] z_{1,1} C 2 x[ ] x_{1,1,1} y[ ] y_{1,1,1,1} z[ ] z_1
\end{aligned}$$

$$\begin{aligned}
& C^2 x[ ] x_{1,1,1,1} y[ ]^2 z_{1,1,1} K^3 x[ ] x_{1,1,1,1} y[ ]^2 z_{1,1}^2 \\
& C^3 x[ ] x_{1,1,1,1} y[ ] y_{1,1} z[ ] z_{1,1} K^2 x[ ] x_{1,1,1,1} y[ ] y_{1,1} z[ ] z_{1,1} \\
& C^2 x_1 x_{1,1,1,1} y[ ]^2 z[ ] z_{1,1,1,1} K^2 x_1 x_{1,1,1,1} y[ ] y_{1,1,1,1} z[ ]^2 \\
& K^2 x_1 x_{1,1,1,1} y[ ]^2 z[ ] z_{1,1,1} C^2 x_1 x_{1,1,1,1} y[ ] y_{1,1,1} z[ ]^2 K^3 \\
& x_{1,1}^2 y[ ]^2 z[ ] z_{1,1,1,1} C^3 x_{1,1}^2 y[ ] y_{1,1,1,1} z[ ]^2 C^4 x_{1,1} x_{1,1,1,1} y[ ]^2 z[ ] z_{1,1,1} \\
& K^4 x_{1,1} x_{1,1,1,1} y[ ] y_{1,1,1} z[ ]^2 C^3 x_{1,1} x_{1,1,1,1} y[ ]^2 z[ ] z_{1,1} \\
& K^3 x_{1,1} x_{1,1,1,1} y[ ] y_{1,1} z[ ]^2 K^4 x_{1,1}^2 y[ ]^2 z[ ] z_{1,1} C^4 x_{1,1}^2 y[ ] y_{1,1} z[ ]^2 \\
& K x[ ] y[ ]^2 y_{1,1,1,1} z_{1,1,1,1} C x[ ] y[ ]^2 y_{1,1,1,1} z_{1,1,1} C x_{1,1,1} y[ ]^3 z_{1,1,1,1} \\
& K x_{1,1,1} y[ ]^2 y_{1,1,1,1} z[ ] K x_{1,1,1,1} y[ ]^3 z_{1,1,1} C x_{1,1,1,1} y[ ]^2 y_{1,1,1} z[ ] )
\end{aligned}$$

**M** >

## Hypersurfaces in 4-dimensions

**M** > restart with(DifferentialGeometry) : with(Tensor) : with(JetCalculus) :  
with(Tools) :

**M** > DGsetup([x, y, z], [u], M, 3) :

**M** > Preferences("JetNotation", "JetNotation1") :

**M** >  $\omega \triangleq \text{evalDG}(dx \&w dz \&w dy \&w du[ ])$

$$\omega \triangleq dx \wedge dz \wedge dy \wedge du[ ]$$

(2.1)

This time too, we have the invariant vector  $v_0$

**M** >  $v_0 \triangleq \text{evalDG}(x D_x C y D_y C z D_z C u[ ] D_u[ ])$

$$v_0 \triangleq x D_x C y D_y C z D_z C u[ ] D_u[ ]$$

(2.2)

Input a non-canonical basis for  $T_p M$  and use this to compute a non-canonical basis for the orthogonal complement with respect to  $\omega$

**M** >  $X \triangleq \text{evalDG}(D_x C u[1] D_u[ ])$  :  $Y \triangleq \text{evalDG}(D_y C u[2] D_u[ ])$  :  $Z \triangleq \text{evalDG}(D_z C u[3] D_u[ ])$  :

**M** > solve({Hook([X, evalDG(a D\_x C b D\_y C c D\_z C d D\_u[ ])],  $\omega$ ), Hook([Y, evalDG(a D\_x C b D\_y C c D\_z C d D\_u[ ])],  $\omega$ ), Hook([Z, evalDG(a D\_x C b D\_y C c D\_z C d D\_u[ ])],  $\omega$ )}, {a, b, c, d})

$$\{a = k u_3 b, b = b, c = u_1 b, d = u_2 b\}$$

(2.3)

Take  $b = 1$ , this gives the vector

**M** >  $w_1 \triangleq \text{evalDG}(k u[3] D_x C D_y C u[1] D_z C u[2] D_u[ ])$  :

which can be used to do normalization, obtaining the first invariant horizontal vector

**M** >  $tmp1 \triangleq \text{solve}(\text{Hook}([v_0, k_1 w_1], \omega) K 1, k_1)$  :

**M** >  $v_1 \triangleq \text{evalDG}(tmp1 w_1)$

$$v_1 \triangleq k \frac{u_3 D_x}{x u_1 C y u_2 C z u_3 K u[ ]} C \frac{D_y}{x u_1 C y u_2 C z u_3 K u[ ]}$$

(2.4)

$$C \frac{u_1 D_z}{x u_1 C y u_2 C z u_3 K u[ ]} C \frac{u_2 D_u[ ]}{x u_1 C y u_2 C z u_3 K u[ ]}$$

Next, take a representative  $q = -u + u(x, y, z)$  for the hypersurface and compute its differential

**M** >  $dq \triangleq \text{evalDG}(k du[ ] C u[1] dx C u[2] dy C u[3] dz)$  :

The symmetric differential and the differentiaial shares the same scaling factor, which we know compute

**M** > tmp2 d solve(Hook(k2 v0, dq) K 1, k2)

$$\text{tmp2} \triangleq \frac{1}{x u_1 C y u_2 C z u_3 K u[ ]} \quad (2.5)$$

hence the symmetric differential is

**M** > Q d evalDG(tmp2 ( u[1, 1] dx &s dx C 2 u[1, 2] dx &s dy C 2 u[1, 3] dx &s dz  
C u[2, 2] dy &s dy C 2 u[2, 3] dy &s dz C u[3, 3] dz &s dz ))

$$\begin{aligned} Q \triangleq & \frac{u_{1,1} dx}{x u_1 C y u_2 C z u_3 K u[ ]} dx C \frac{u_{1,2} dx}{x u_1 C y u_2 C z u_3 K u[ ]} dy \\ & C \frac{u_{1,3} dx}{x u_1 C y u_2 C z u_3 K u[ ]} dz C \frac{u_{1,2} dy}{x u_1 C y u_2 C z u_3 K u[ ]} dx \\ & C \frac{u_{2,2} dy}{x u_1 C y u_2 C z u_3 K u[ ]} dy C \frac{u_{2,3} dy}{x u_1 C y u_2 C z u_3 K u[ ]} dz \\ & C \frac{u_{1,3} dz}{x u_1 C y u_2 C z u_3 K u[ ]} dx C \frac{u_{2,3} dz}{x u_1 C y u_2 C z u_3 K u[ ]} dy \\ & C \frac{u_{3,3} dz}{x u_1 C y u_2 C z u_3 K u[ ]} dz \end{aligned} \quad (2.6)$$

which gives the first invariant as

**M** > I<sub>2a</sub> d Hook([vI, vI], Q)

$$I_{2a} \triangleq \frac{u_{1,3}^2 u_{3,3} K^2 u_1 u_3 u_{1,3} C u_3^2 u_{1,1} C^2 u_1 u_{2,3} K^2 u_3 u_{1,2} C u_{2,2}}{(x u_1 C y u_2 C z u_3 K u[ ])^3} \quad (2.7)$$

Input more information we have gathered

**M** > α d Hook(v0, ω)

$$\alpha \triangleq k z dx K u[ ] dy C x dz C y du[ ] \quad (2.8)$$

**M** > β d convert(ContractIndices(Q &t vI, [[1, 3]]), DGform)

$$\begin{aligned} \beta \triangleq & \frac{(u_1 u_{1,3} K u_3 u_{1,1} C u_{1,2}) dx}{(x u_1 C y u_2 C z u_3 K u[ ])^2} C \frac{(u_1 u_{2,3} K u_3 u_{1,2} C u_{2,2}) dy}{(x u_1 C y u_2 C z u_3 K u[ ])^2} \\ & C \frac{(u_1 u_{3,3} K u_3 u_{1,3} C u_{2,3}) dz}{(x u_1 C y u_2 C z u_3 K u[ ])^2} \end{aligned} \quad (2.9)$$

using α and β we can find the missing invariant horizontal vectors needed. Pick a basis for T\_{p}M and use it to construct a basis for ker α

**M** > W d evalDG(a X C b Y C c Z) :

**M** > tmp3 d solve(Hook(W, α), {a, b, c})

$$\text{tmp3} \triangleq \left\{ a = k \frac{b y u_2 C c y u_3 K u[ ] b C x c}{y u_1 K z}, b = b, c = c \right\} \quad (2.10)$$

**M** > W1 d evalDG(K(y u[1] K z) DGsimplify(eval(eval(W, tmp3), {b = 1, c = 0}))) :

**M** > W2 d evalDG(K(y u[1] K z) DGsimplify(eval(eval(W, tmp3), {b = 0, c = 1}))) :

using this basis we can take a general vector in ker α

**M** > w2 d evalDG(a W1 C b W2) : w3 d evalDG(c W1 C d W2) :

then we solve and fix a,b,c,d by the conditions we have described in the paper

**M** > tmp4 d solve( {Hook([v1, w3], Q), Hook([w2, w3], Q), Hook([v1, w2], Q) K 1,  
Hook([w2, w3], w) K 1}, {a, b, c, d}) :

this gives the final vectors

**M** > v2 d subs(tmp4, w2) : v3 d subs(tmp4, w3) :

which can now be used to compute our remaining invariants

**M** > I<sub>2b</sub> d Hook([v2, v2], Q); I<sub>2c</sub> d Hook([v3, v3], Q)

$$I_{2b} d \left( (x u_1 C y u_2 C z u_3 K u[ ])^3 (y^2 u_1^2 u_{2,2} u_{3,3} K y^2 u_1^2 u_{2,3}^2 K 2 y^2 u_1 u_2 u_{1,2} u_{3,3} \right. \\ C 2 y^2 u_1 u_2 u_{1,3} u_{2,3} C 2 y^2 u_1 u_3 u_{1,2} u_{2,3} K 2 y^2 u_1 u_3 u_{1,3} u_{2,2} C y^2 u_2^2 u_{1,1} u_{3,3} \\ K y^2 u_2^2 u_{1,3}^2 K 2 y^2 u_2 u_3 u_{1,1} u_{2,3} C 2 y^2 u_2 u_3 u_{1,2} u_{1,3} C y^2 u_3^2 u_{1,1} u_{2,2} K y^2 u_3^2 u_{1,2}^2 \\ C 2 x y u_1 u_{1,2} u_{2,3} K 2 x y u_1 u_{1,3} u_{2,2} K 2 x y u_2 u_{1,1} u_{2,3} C 2 x y u_2 u_{1,2} u_{1,3} \\ C 2 x y u_3 u_{1,1} u_{2,2} K 2 x y u_3 u_{1,2}^2 K 2 y z u_1 u_{2,2} u_{3,3} C 2 y z u_1 u_{2,3}^2 \\ C 2 y z u_2 u_{1,2} u_{3,3} K 2 y z u_2 u_{1,3} u_{2,3} K 2 y z u_3 u_{1,2} u_{2,3} C 2 y z u_3 u_{1,3} u_{2,2} \\ C 2 y u[ ] u_1 u_{1,2} u_{3,3} K 2 y u[ ] u_1 u_{1,3} u_{2,3} K 2 y u[ ] u_2 u_{1,1} u_{3,3} C 2 y u[ ] u_2 u_{1,3}^2 \\ C 2 y u[ ] u_3 u_{1,1} u_{2,3} K 2 y u[ ] u_3 u_{1,2} u_{1,3} C x^2 u_{1,1} u_{2,2} K x^2 u_{1,2}^2 K 2 x z u_{1,2} u_{2,3} \\ C 2 x z u_{1,3} u_{2,2} C 2 x u[ ] u_{1,1} u_{2,3} K 2 x u[ ] u_{1,2} u_{1,3} C z^2 u_{2,2} u_{3,3} K z^2 u_{2,3}^2 \\ K 2 z u[ ] u_{1,2} u_{3,3} C 2 z u[ ] u_{1,3} u_{2,3} C u[ ]^2 u_{1,1} u_{3,3} K u[ ]^2 u_{1,3}^2) / (y^2 u_1^4 u_{2,2} \\ u_{3,3}^2 K y^2 u_1^4 u_{2,3}^2 u_{3,3} K 2 y^2 u_1^3 u_2 u_{1,2} u_{2,3}^2 C 2 y^2 u_1^3 u_2 u_{1,3} u_{2,3} u_{3,3} C 2 y^2 \\ u_1^3 u_3 u_{1,2} u_{2,3} u_{3,3} K 4 y^2 u_1^3 u_3 u_{1,3} u_{2,2} u_{3,3} C 2 y^2 u_1^3 u_3 u_{1,3} u_{2,3}^2 C y^2 u_1^2 u_2^2 u_{1,1} u_{3,3}^2 \\ K y^2 u_1^2 u_2^2 u_{1,3}^2 u_{3,3} K 2 y^2 u_1^2 u_2 u_3 u_{1,1} u_{2,3} u_{3,3} C 6 y^2 u_1^2 u_2 u_3 u_{1,2} u_{1,3} u_{3,3} K 4 y^2 \\ u_1^2 u_2 u_3 u_{1,3}^2 u_{2,3} C 2 y^2 u_1^2 u_3^2 u_{1,1} u_{2,2} u_{3,3} K y^2 u_1^2 u_3^2 u_{1,1} u_{2,3}^2 K y^2 u_1^2 u_3^2 u_{1,2}^2 u_{3,3} \\ K 4 y^2 u_1^2 u_3^2 u_{1,2} u_{1,3} u_{2,3} C 4 y^2 u_1^2 u_3^2 u_{1,3}^2 u_{2,2} K 2 y^2 u_1 u_2^2 u_3 u_{1,1} u_{1,3} u_{3,3} \\ C 2 y^2 u_1 u_2^2 u_3 u_{1,3}^2 K 2 y^2 u_1 u_2 u_3^2 u_{1,1} u_{1,2} u_{3,3} C 6 y^2 u_1 u_2 u_3^2 u_{1,1} u_{1,3} u_{2,3} \\ K 4 y^2 u_1 u_2 u_3^2 u_{1,2} u_{1,3}^2 C 2 y^2 u_1 u_3^3 u_{1,1} u_{1,2} u_{2,3} K 4 y^2 u_1 u_3^3 u_{1,1} u_{1,3} u_{2,2} \\ C 2 y^2 u_1 u_3^2 u_{1,2}^2 u_{1,3} C y^2 u_2^2 u_3^2 u_{1,1}^2 u_{3,3} K y^2 u_2^2 u_3^2 u_{1,1} u_{1,3}^2 K 2 y^2 u_2 u_3^3 u_{1,1}^2 u_{2,3} \\ C 2 y^2 u_2 u_3^3 u_{1,1} u_{1,2} u_{1,3} C y^2 u_3^4 u_{1,1}^2 u_{2,2} K y^2 u_3^4 u_{1,1} u_{1,2}^2 C 2 x y u_1^3 u_{1,2} u_{2,3} u_{3,3} \\ K 2 x y u_1^3 u_{1,3} u_{2,2} u_{3,3} K 2 x y u_1^2 u_2 u_{1,1} u_{2,3} u_{3,3} C 2 x y u_1^2 u_2 u_{1,2} u_{1,3} u_{3,3} \\ C 2 x y u_1^2 u_3 u_{1,1} u_{2,2} u_{3,3} K 2 x y u_1^2 u_3 u_{1,2}^2 u_{3,3} K 4 x y u_1^2 u_3 u_{1,2} u_{1,3} u_{2,3} C 4 x y \\ u_1^2 u_3 u_{1,3}^2 u_{2,2} C 4 x y u_1 u_2 u_3 u_{1,1} u_{1,3} u_{2,3} K 4 x y u_1 u_2 u_3 u_{1,2} u_{1,3}^2 C 2 x y u_1 \\ u_3^2 u_{1,1} u_{1,2} u_{2,3} K 6 x y u_1 u_3^2 u_{1,1} u_{1,3} u_{2,2} C 4 x y u_1 u_3^2 u_{1,2}^2 u_{1,3} K 2 x y u_2 u_3^2 \\ u_{1,1}^2 u_{2,3} C 2 x y u_2 u_3^2 u_{1,1} u_{1,2} u_{1,3} C 2 x y u_3^3 u_{1,1}^2 u_{2,2} K 2 x y u_3^3 u_{1,1} u_{1,2}^2 C 2 y^2 \\ u_1^3 u_{2,2} u_{2,3} u_{3,3} K 2 y^2 u_1^3 u_{2,3}^2 K 4 y^2 u_1^2 u_2 u_{1,2} u_{2,3} u_{3,3} C 4 y^2 u_1^2 u_2 u_{1,3} u_{2,3}^2 K 2 y^2 \\ u_1^2 u_3 u_{1,2} u_{2,2} u_{3,3} C 6 y^2 u_1^2 u_3 u_{1,2} u_{2,3}^2 K 4 y^2 u_1^2 u_3 u_{1,3} u_{2,2} u_{2,3} C 2 y^2 u_1 \\ u_2^2 u_{1,1} u_{2,3} u_{3,3} K 2 y^2 u_1 u_2^2 u_{1,3}^2 u_{2,3} K 4 y^2 u_1 u_2 u_3 u_{1,1} u_{2,3}^2 C 4 y^2 u_1 u_2 u_3 u_{1,2}^2 u_{3,3} \\$$

$$\begin{aligned}
& C 2 y^2 u_1 u_3^2 u_{1,1} u_{2,2} u_{2,3} K 6 y^2 u_1 u_3^2 u_{1,2}^2 u_{2,3} C 4 y^2 u_1 u_3^2 u_{1,2} u_{1,3} u_{2,2} K 2 y^2 \\
& u_2^2 u_3 u_{1,1} u_{1,2} u_{3,3} C 2 y^2 u_2^2 u_3 u_{1,2} u_{1,3}^2 C 4 y^2 u_2 u_3^2 u_{1,1} u_{1,2} u_{2,3} K 4 y^2 u_2 u_3^2 \\
& u_{1,2}^2 u_{1,3} K 2 y^2 u_3^3 u_{1,1} u_{1,2} u_{2,2} C 2 y^2 u_3^3 u_{1,2}^2 K 2 y z u_1^3 u_{2,2} u_{2,3}^2 C 2 y z u_1^3 u_{2,3}^2 u_{3,3} \\
& C 2 y z u_1^2 u_2 u_{1,2} u_{2,3}^2 K 2 y z u_1^2 u_2 u_{1,3} u_{2,3} u_{3,3} K 2 y z u_1^2 u_3 u_{1,2} u_{2,3} u_{3,3} C 6 y z \\
& u_1^2 u_3 u_{1,3} u_{2,2} u_{3,3} K 4 y z u_1^2 u_3 u_{1,3} u_{2,3}^2 K 4 y z u_1 u_2 u_3 u_{1,2} u_{1,3} u_{3,3} \\
& C 4 y z u_1 u_2 u_3 u_{1,3}^2 u_{2,3} K 2 y z u_1 u_3^2 u_{1,1} u_{2,2} u_{3,3} C 2 y z u_1 u_3^2 u_{1,1} u_{2,3}^2 C 4 y z u_1 \\
& u_3^2 u_{1,2} u_{1,3} u_{2,3} K 4 y z u_1 u_3^2 u_{1,3}^2 u_{2,2} C 2 y z u_2 u_3^2 u_{1,1} u_{1,2} u_{3,3} K 2 y z u_2 \\
& u_3^2 u_{1,1} u_{1,3} u_{2,3} K 2 y z u_3^3 u_{1,1} u_{1,2} u_{2,3} C 2 y z u_3^3 u_{1,1} u_{1,3} u_{2,2} C 2 y u[ ] u_1^3 u_{1,2} \\
& u_{3,3}^2 K 2 y u[ ] u_1^3 u_{1,3} u_{2,3} u_{3,3} K 2 y u[ ] u_1^2 u_2 u_{1,1} u_{3,3}^2 C 2 y u[ ] u_1^2 u_2 u_{1,3}^2 u_{3,3} \\
& C 2 y u[ ] u_1^2 u_3 u_{1,1} u_{2,3} u_{3,3} K 6 y u[ ] u_1^2 u_3 u_{1,2} u_{1,3} u_{3,3} C 4 y u[ ] u_1^2 u_3 u_{1,3}^2 u_{2,3} \\
& C 4 y u[ ] u_1 u_2 u_3 u_{1,1} u_{1,3} u_{3,3} K 4 y u[ ] u_1 u_2 u_3 u_{1,3}^3 C 2 y u[ ] u_1 u_3^2 u_{1,1} u_{1,2} u_{3,3} \\
& K 6 y u[ ] u_1 u_3^2 u_{1,1} u_{1,3} u_{2,3} C 4 y u[ ] u_1 u_3^2 u_{1,2} u_{1,3}^2 K 2 y u[ ] u_2 u_3^2 u_{1,1}^2 u_{3,3} \\
& C 2 y u[ ] u_2 u_3^2 u_{1,1} u_{1,3}^2 C 2 y u[ ] u_3^3 u_{1,1}^2 u_{2,3} K 2 y u[ ] u_3^3 u_{1,1} u_{1,2} u_{1,3} C x^2 \\
& u_1^2 u_{1,1} u_{2,3}^2 K 2 x^2 u_1^2 u_{1,2} u_{1,3} u_{2,3} C x^2 u_1^2 u_{1,3}^2 u_{2,2} K 2 x^2 u_1 u_3 u_{1,1} u_{1,3} u_{2,2} \\
& C 2 x^2 u_1 u_3 u_{1,2}^2 u_{1,3} C u_{2,2} u_{1,1}^2 u_3^2 x^2 K u_{1,2}^2 u_{1,1} u_3^2 x^2 C 4 x y u_1^2 u_{1,2} u_{2,3}^2 K 4 x y \\
& u_1^2 u_{1,3} u_{2,2} u_{2,3} K 2 x y u_1 u_2 u_{1,1} u_{2,2} u_{3,3} K 2 x y u_1 u_2 u_{1,1} u_{2,3}^2 C 2 x y u_1 u_2 \\
& u_{1,2}^2 u_{3,3} C 2 x y u_1 u_2 u_{1,3}^2 u_{2,2} C 4 x y u_1 u_3 u_{1,1} u_{2,2} u_{2,3} K 8 x y u_1 u_3 u_{1,2}^2 u_{2,3} \\
& C 4 x y u_1 u_3 u_{1,2} u_{1,3} u_{2,2} C 4 x y u_2 u_3 u_{1,1} u_{1,2} u_{2,3} K 4 x y u_2 u_3 u_{1,2}^2 u_{1,3} K 4 x y \\
& u_3^2 u_{1,1} u_{1,2} u_{2,2} C 4 x y u_3^2 u_{1,2}^2 K 2 x z u_1^2 u_{1,2} u_{2,3} u_{3,3} C 2 x z u_1^2 u_{1,3} u_{2,2} u_{3,3} \\
& K 2 x z u_1 u_3 u_{1,1} u_{2,2} u_{3,3} C 2 x z u_1 u_3 u_{1,1} u_{2,3}^2 C 2 x z u_1 u_3 u_{1,2}^2 u_{3,3} K 2 x z u_1 u_3 \\
& u_{1,3}^2 u_{2,2} K 2 x z u_3^2 u_{1,1} u_{1,2} u_{2,3} C 2 x z u_3^2 u_{1,1} u_{1,3} u_{2,2} C 2 x u[ ] u_1^2 u_{1,1} u_{2,3} u_{3,3} \\
& K 2 x u[ ] u_1^2 u_{1,2} u_{1,3} u_{3,3} K 4 x u[ ] u_1 u_3 u_{1,1} u_{1,3} u_{2,3} C 4 x u[ ] u_1 u_3 u_{1,2} u_{1,3}^2 \\
& C 2 u_{2,3} u_{1,1}^2 u_3^2 u[ ] x K 2 u_{1,3} u_{1,2} u_{1,1} u_3^2 u[ ] x C y^2 u_1^2 u_{2,2}^2 u_{3,3} K y^2 u_1^2 u_{2,2}^2 u_{2,3}^2 \\
& K 2 y^2 u_1 u_2 u_{1,2} u_{2,2} u_{3,3} C 2 y^2 u_1 u_2 u_{1,3} u_{2,2} u_{2,3} C 2 y^2 u_1 u_3 u_{1,2} u_{2,2} u_{2,3} \\
& K 2 y^2 u_1 u_3 u_{1,3} u_{2,2}^2 C y^2 u_2^2 u_{1,1} u_{2,3}^2 C y^2 u_2^2 u_{1,2}^2 u_{3,3} K 2 y^2 u_2^2 u_{1,2} u_{1,3} u_{2,3} \\
& K 2 y^2 u_2 u_3 u_{1,1} u_{2,2} u_{2,3} C 2 y^2 u_2 u_3 u_{1,2} u_{1,3} u_{2,2} C y^2 u_3^2 u_{1,1} u_{2,2}^2 K y^2 u_3^2 u_{1,2}^2 u_{2,2}^2 \\
& K 4 y z u_1^2 u_{2,2} u_{2,3} u_{3,3} C 4 y z u_1^2 u_{2,3}^3 C 4 y z u_1 u_2 u_{1,2} u_{2,3} u_{3,3} K 4 y z u_1 u_2 u_{1,3} \\
& u_{2,3}^2 C 4 y z u_1 u_3 u_{1,2} u_{2,2} u_{3,3} K 8 y z u_1 u_3 u_{1,2} u_{2,3}^2 C 4 y z u_1 u_3 u_{1,3} u_{2,2} u_{2,3} \\
& K 2 y z u_2 u_3 u_{1,1} u_{2,2} u_{3,3} C 2 y z u_2 u_3 u_{1,1} u_{2,3}^2 K 2 y z u_2 u_3 u_{1,2}^2 u_{3,3} C 2 y z u_2 u_3 \\
& u_{1,3}^2 u_{2,2} C 4 y z u_3^2 u_{1,2}^2 u_{2,3} K 4 y z u_3^2 u_{1,2} u_{1,3} u_{2,2} C 4 y u[ ] u_1^2 u_{1,2} u_{2,3} u_{3,3} \\
& K 4 y u[ ] u_1^2 u_{1,3} u_{2,3}^2 K 4 y u[ ] u_1 u_2 u_{1,1} u_{2,3} u_{3,3} C 4 y u[ ] u_1 u_2 u_{1,3}^2 u_{2,3} \\
& C 4 y u[ ] u_1 u_3 u_{1,1} u_{2,3}^2 K 4 y u[ ] u_1 u_3 u_{1,2}^2 u_{3,3} C 4 y u[ ] u_2 u_3 u_{1,1} u_{1,2} u_{3,3}
\end{aligned}$$

$$\begin{aligned}
& K 4 y u[ ] u_2 u_3 u_{1,2} u_{1,3}^2 K 4 y u[ ] u_3^2 u_{1,1} u_{1,2} u_{2,3} C 4 y u[ ] u_3^2 u_{1,2}^2 u_{1,3} C z^2 \\
& u_1^2 u_{2,2} u_{3,3}^2 K z^2 u_1^2 u_{2,3}^2 u_{3,3} K 2 z^2 u_1 u_3 u_{1,3} u_{2,2} u_{3,3} C 2 z^2 u_1 u_3 u_{1,3} u_{2,3}^2 C z^2 u_3^2 \\
& u_{1,2}^2 u_{3,3} K 2 z^2 u_3^2 u_{1,2} u_{1,3} u_{2,3} C z^2 u_3^2 u_{1,3}^2 u_{2,2} K 2 z u[ ] u_1^2 u_{1,2} u_{3,3}^2 C 2 z u[ ] \\
& u_1^2 u_{1,3} u_{2,3} u_{3,3} C 4 z u[ ] u_1 u_3 u_{1,2} u_{1,3} u_{3,3} K 4 z u[ ] u_1 u_3 u_{1,3}^2 u_{2,3} K 2 z u[ ] \\
& u_3^2 u_{1,1} u_{1,2} u_{3,3} C 2 z u[ ] u_3^2 u_{1,1} u_{1,3} u_{2,3} C u[ ]^2 u_1^2 u_{1,1} u_{3,3}^2 K u[ ]^2 u_1^2 u_{1,3}^2 u_{3,3} \\
& K 2 u[ ]^2 u_1 u_3 u_{1,1} u_{1,3} u_{3,3} C 2 u[ ]^2 u_1 u_3 u_{1,3}^3 C u_{3,3} u_{1,1}^2 u_{3,3}^2 u[ ]^2 K u_{1,3}^2 u_{1,1} \\
& u_3^2 u[ ]^2 C 2 x^2 u_1 u_{1,1} u_{2,2} u_{2,3} K 2 x^2 u_1 u_{1,2}^2 u_{2,3} K 2 u_{2,2} u_{1,2} u_{1,1} u_3 x^2 C 2 \\
& u_{1,2}^3 u_3 x^2 C 2 x y u_1 u_{1,2} u_{2,2} u_{2,3} K 2 x y u_1 u_{1,3} u_{2,2}^2 K 2 x y u_2 u_{1,1} u_{2,2} u_{2,3} \\
& C 2 x y u_2 u_{1,2} u_{1,3} u_{2,2} C 2 x y u_3 u_{1,1} u_{2,2}^2 K 2 x y u_3 u_{1,2}^2 u_{2,2} K 4 x z u_1 u_{1,2} u_{2,3}^2 \\
& C 4 x z u_1 u_{1,3} u_{2,2} u_{2,3} C 4 x z u_3 u_{1,2}^2 u_{2,3} K 4 x z u_3 u_{1,2} u_{1,3} u_{2,2} \\
& C 2 x u[ ] u_1 u_{1,1} u_{2,2} u_{3,3} C 2 x u[ ] u_1 u_{1,1} u_{2,3}^2 K 2 x u[ ] u_1 u_{1,2}^2 u_{3,3} K 2 x u[ ] u_1 \\
& u_{1,3}^2 u_{2,2} K 4 u_{2,3} u_{1,2} u_{1,1} u_3 u[ ] x C 4 u_{1,3} u_{1,2}^2 u_3 u[ ] x K 2 y z u_1 u_{2,2}^2 u_{3,3} \\
& C 2 y z u_1 u_{2,2} u_{2,3}^2 C 2 y z u_2 u_{1,2} u_{2,2} u_{3,3} K 2 y z u_2 u_{1,3} u_{2,2} u_{2,3} \\
& K 2 y z u_3 u_{1,2} u_{2,2} u_{2,3} C 2 y z u_3 u_{1,3} u_{2,2}^2 C 2 y u[ ] u_1 u_{1,2} u_{2,2} u_{3,3} \\
& K 2 y u[ ] u_1 u_{1,3} u_{2,2} u_{2,3} K 2 y u[ ] u_2 u_{1,1} u_{2,3}^2 K 2 y u[ ] u_2 u_{1,2}^2 u_{3,3} \\
& C 4 y u[ ] u_2 u_{1,2} u_{1,3} u_{2,3} C 2 y u[ ] u_3 u_{1,1} u_{2,2} u_{2,3} K 2 y u[ ] u_3 u_{1,2} u_{1,3} u_{2,2} \\
& C 2 z^2 u_1 u_{2,2} u_{2,3} u_{3,3} K 2 z^2 u_1 u_{2,3}^3 K 2 z^2 u_3 u_{1,2} u_{2,2} u_{3,3} C 2 z^2 u_3 u_{1,2} u_{2,3}^2 \\
& K 4 z u[ ] u_1 u_{1,2} u_{2,3} u_{3,3} C 4 z u[ ] u_1 u_{1,3} u_{2,3}^2 C 2 z u[ ] u_3 u_{1,1} u_{2,2} u_{3,3} \\
& K 2 z u[ ] u_3 u_{1,1} u_{2,3}^2 C 2 z u[ ] u_3 u_{1,2}^2 u_{3,3} K 2 z u[ ] u_3 u_{1,3}^2 u_{2,2} \\
& C 2 u[ ]^2 u_1 u_{1,1} u_{2,3} u_{3,3} K 2 u[ ]^2 u_1 u_{1,3}^2 u_{2,3} K 2 u_{3,3} u_{1,2} u_{1,1} u_3 u[ ]^2 C 2 \\
& u_{1,3}^2 u_{1,2} u_3 u[ ]^2 C u_{2,2}^2 u_{1,1} x^2 K u_{2,2} u_{1,2}^2 x^2 K 2 x z u_{1,2} u_{2,2} u_{2,3} C 2 x z u_{1,3} u_{2,2}^2 \\
& C 2 u_{2,3} u_{2,2} u_{1,1} u[ ] x K 2 u_{2,2} u_{1,3} u_{1,2} u[ ] x C z^2 u_{2,2}^2 u_{3,3} K z^2 u_{2,2} u_{2,3}^2 \\
& K 2 z u[ ] u_{1,2} u_{2,2} u_{3,3} C 2 z u[ ] u_{1,3} u_{2,2} u_{2,3} C u_{2,3}^2 u_{1,1} u[ ]^2 C u_{3,3} u_{1,2}^2 u[ ]^2 \\
& K 2 u_{2,3} u_{1,3} u_{1,2} u[ ]^2)
\end{aligned}$$

$$\begin{aligned}
I_{2c} \text{ d } & (y^2 u_1^4 u_{2,2} u_{3,3}^2 K y^2 u_1^4 u_{2,3}^2 u_{3,3} K 2 y^2 u_1^3 u_2 u_{1,2} u_{3,3}^2 C 2 y^2 u_1^3 u_2 u_{1,3} u_{2,3} u_{3,3} \\
& C 2 y^2 u_1^3 u_3 u_{1,2} u_{2,3} u_{3,3} K 4 y^2 u_1^3 u_3 u_{1,3} u_{2,2} u_{3,3} C 2 y^2 u_1^3 u_3 u_{1,3} u_{2,3}^2 C y^2 u_1^2 \\
& u_{2,1}^2 u_{1,1} u_{3,3}^2 K y^2 u_1^2 u_{2,2}^2 u_{3,3}^2 K 2 y^2 u_1^2 u_2 u_3 u_{1,1} u_{2,3} u_{3,3} C 6 y^2 \\
& u_1^2 u_2 u_3 u_{1,2} u_{1,3} u_{3,3} K 4 y^2 u_1^2 u_2 u_3 u_{1,3}^2 u_{2,3} C 2 y^2 u_1^2 u_3^2 u_{1,1} u_{2,2} u_{3,3} K y^2 u_1^2 \\
& u_3^2 u_{1,1} u_{2,3}^2 K y^2 u_1^2 u_3^2 u_{1,2}^2 u_{3,3} K 4 y^2 u_1^2 u_3^2 u_{1,2} u_{1,3} u_{2,3} C 4 y^2 u_1^2 u_3^2 u_{1,3}^2 u_{2,2} \\
& K 2 y^2 u_1 u_2^2 u_3 u_{1,1} u_{1,3} u_{3,3} C 2 y^2 u_1 u_2^2 u_3 u_{1,3}^3 K 2 y^2 u_1 u_2 u_3^2 u_{1,1} u_{1,2} u_{3,3} \\
& C 6 y^2 u_1 u_2 u_3^2 u_{1,1} u_{1,3} u_{2,3} K 4 y^2 u_1 u_2 u_3^2 u_{1,2} u_{1,3}^2 C 2 y^2 u_1 u_3^3 u_{1,1} u_{1,2} u_{2,3} \\
& K 4 y^2 u_1 u_3^3 u_{1,1} u_{1,3} u_{2,2} C 2 y^2 u_1 u_3^3 u_{1,2}^2 u_{1,3} C y^2 u_2^2 u_3^2 u_{1,1}^2 u_{3,3} K y^2 u_2^2 u_3^2 u_{1,1}
\end{aligned} \tag{2.11}$$

$$\begin{aligned}
& u_{1,3}^2 K 2 y^2 u_2 u_3^3 u_{1,1}^2 u_{2,3} C 2 y^2 u_2 u_3^3 u_{1,1} u_{1,2} u_{1,3} C y^2 u_3^4 u_{1,1}^2 u_{2,2} K y^2 u_3^4 u_{1,1} u_{1,2}^2 \\
& C 2 x y u_1^3 u_{1,2} u_{2,3} u_{3,3} K 2 x y u_1^3 u_{1,3} u_{2,2} u_{3,3} K 2 x y u_1^2 u_2 u_{1,1} u_{2,3} u_{3,3} C 2 x y \\
& u_1^2 u_2 u_{1,2} u_{1,3} u_{3,3} C 2 x y u_1^2 u_3 u_{1,1} u_{2,2} u_{3,3} K 2 x y u_1^2 u_3 u_{1,2}^2 u_{3,3} K 4 x y \\
& u_1^2 u_3 u_{1,2} u_{1,3} u_{2,3} C 4 x y u_1^2 u_3 u_{1,3}^2 u_{2,2} C 4 x y u_1 u_2 u_3 u_{1,1} u_{1,3} u_{2,3} \\
& K 4 x y u_1 u_2 u_3 u_{1,2} u_{1,3}^2 C 2 x y u_1 u_3^2 u_{1,1} u_{1,2} u_{2,3} K 6 x y u_1 u_3^2 u_{1,1} u_{1,3} u_{2,2} \\
& C 4 x y u_1 u_3^2 u_{1,2}^2 u_{1,3} K 2 x y u_2 u_3^2 u_{1,1}^2 u_{2,3} C 2 x y u_2 u_3^2 u_{1,1} u_{1,2} u_{1,3} C 2 x y u_3^3 \\
& u_{1,1}^2 u_{2,2} K 2 x y u_3^3 u_{1,1} u_{1,2}^2 C 2 y^2 u_1^3 u_{2,2} u_{2,3} u_{3,3} K 2 y^2 u_1^3 u_{2,3}^3 K 4 y^2 \\
& u_1^2 u_2 u_{1,2} u_{2,3} u_{3,3} C 4 y^2 u_1^2 u_2 u_{1,3} u_{2,3}^2 K 2 y^2 u_1^2 u_3 u_{1,2} u_{2,2} u_{3,3} C 6 y^2 u_1^2 u_3 u_{1,2} \\
& u_{2,3}^2 K 4 y^2 u_1^2 u_3 u_{1,3} u_{2,2} u_{2,3} C 2 y^2 u_1 u_2^2 u_{1,1} u_{2,3} u_{3,3} K 2 y^2 u_1 u_2^2 u_{1,3} u_{2,3} \\
& K 4 y^2 u_1 u_2 u_3 u_{1,1} u_{2,3}^2 C 4 y^2 u_1 u_2 u_3 u_{1,2}^2 u_{3,3} C 2 y^2 u_1 u_3^2 u_{1,1} u_{2,2} u_{2,3} K 6 y^2 u_1 \\
& u_3^2 u_{1,2}^2 u_{2,3} C 4 y^2 u_1 u_3^2 u_{1,2} u_{1,3} u_{2,2} K 2 y^2 u_2^2 u_3 u_{1,1} u_{1,2} u_{3,3} C 2 y^2 u_2^2 u_3 u_{1,2} u_{1,3}^2 \\
& C 4 y^2 u_2 u_3^2 u_{1,1} u_{1,2} u_{2,3} K 4 y^2 u_2 u_3^2 u_{1,2}^2 u_{1,3} K 2 y^2 u_3^3 u_{1,1} u_{1,2} u_{2,2} C 2 y^2 u_3^3 u_{1,2}^3 \\
& K 2 y z u_1^3 u_{2,2} u_{2,3}^2 C 2 y z u_1^3 u_{2,3}^2 u_{3,3} C 2 y z u_1^2 u_2 u_{1,2} u_{2,3}^2 K 2 y z \\
& u_1^2 u_2 u_{1,3} u_{2,3} u_{3,3} K 2 y z u_1^2 u_3 u_{1,2} u_{2,3} u_{3,3} C 6 y z u_1^2 u_3 u_{1,3} u_{2,2} u_{3,3} K 4 y z \\
& u_1^2 u_3 u_{1,3} u_{2,3}^2 K 4 y z u_1 u_2 u_3 u_{1,2} u_{1,3} u_{3,3} C 4 y z u_1 u_2 u_3 u_{1,3}^2 u_{2,3} K 2 y z u_1 \\
& u_3^2 u_{1,1} u_{2,2} u_{3,3} C 2 y z u_1 u_3^2 u_{1,1} u_{2,3}^2 C 4 y z u_1 u_3^2 u_{1,2} u_{1,3} u_{2,3} K 4 y z u_1 u_3^2 \\
& u_{1,3}^2 u_{2,2} C 2 y z u_2 u_3^2 u_{1,1} u_{1,2} u_{3,3} K 2 y z u_2 u_3^2 u_{1,1} u_{1,3} u_{2,3} K 2 y z u_3^3 u_{1,1} u_{1,2} u_{2,3} \\
& C 2 y z u_3^3 u_{1,1} u_{1,3} u_{2,2} C 2 y u[ ] u_1^3 u_{1,2} u_{2,3}^2 K 2 y u[ ] u_1^3 u_{1,3} u_{2,3} u_{3,3} K 2 y u[ ] \\
& u_1^2 u_2 u_{1,1} u_{2,3}^2 C 2 y u[ ] u_1^2 u_2 u_{1,3}^2 u_{3,3} C 2 y u[ ] u_1^2 u_3 u_{1,1} u_{2,3} u_{3,3} K 6 y u[ ] \\
& u_1^2 u_3 u_{1,2} u_{1,3} u_{3,3} C 4 y u[ ] u_1^2 u_3 u_{1,3}^2 u_{2,3} C 4 y u[ ] u_1 u_2 u_3 u_{1,1} u_{1,3} u_{3,3} \\
& K 4 y u[ ] u_1 u_2 u_3 u_{1,3}^3 C 2 y u[ ] u_1 u_3^2 u_{1,1} u_{1,2} u_{3,3} K 6 y u[ ] u_1 u_3^2 u_{1,1} u_{1,3} u_{2,3} \\
& C 4 y u[ ] u_1 u_3^2 u_{1,2} u_{1,3}^2 K 2 y u[ ] u_2 u_3^2 u_{1,1}^2 u_{3,3} C 2 y u[ ] u_2 u_3^2 u_{1,1} u_{1,2}^2 \\
& C 2 y u[ ] u_3^3 u_{1,1}^2 u_{2,3} K 2 y u[ ] u_3^3 u_{1,1} u_{1,2} u_{1,3} C x^2 u_1^2 u_{1,1} u_{2,3}^2 K 2 x^2 \\
& u_1^2 u_{1,2} u_{1,3} u_{2,3} C x^2 u_1^2 u_{1,3}^2 u_{2,2} K 2 x^2 u_1 u_3 u_{1,1} u_{1,3} u_{2,2} C 2 x^2 u_1 u_3 u_{1,2}^2 u_{1,3} \\
& C u_{2,2} u_{1,1}^2 u_3^2 x^2 K u_{1,2}^2 u_{1,1} u_3^2 x^2 C 4 x y u_1^2 u_{1,2} u_{2,3}^2 K 4 x y u_1^2 u_{1,3} u_{2,2} u_{2,3} \\
& K 2 x y u_1 u_2 u_{1,1} u_{2,2} u_{3,3} K 2 x y u_1 u_2 u_{1,1} u_{2,3}^2 C 2 x y u_1 u_2 u_{1,2}^2 u_{3,3} C 2 x y u_1 u_2 \\
& u_{1,3}^2 u_{2,2} C 4 x y u_1 u_3 u_{1,1} u_{2,2} u_{2,3} K 8 x y u_1 u_3 u_{1,2}^2 u_{2,3} C 4 x y u_1 u_3 u_{1,2} u_{1,3} u_{2,2} \\
& C 4 x y u_2 u_3 u_{1,1} u_{1,2} u_{2,3} K 4 x y u_2 u_3 u_{1,2}^2 u_{1,3} K 4 x y u_3^2 u_{1,1} u_{1,2} u_{2,2} C 4 x y u_3^2 \\
& u_{1,2}^3 K 2 x z u_1^2 u_{1,2} u_{2,3} u_{3,3} C 2 x z u_1^2 u_{1,3} u_{2,2} u_{3,3} K 2 x z u_1 u_3 u_{1,1} u_{2,2} u_{3,3} \\
& C 2 x z u_1 u_3 u_{1,1} u_{2,3}^2 C 2 x z u_1 u_3 u_{1,2}^2 u_{3,3} K 2 x z u_1 u_3 u_{1,3}^2 u_{2,2} K 2 x z \\
& u_3^2 u_{1,1} u_{1,2} u_{2,3} C 2 x z u_3^2 u_{1,1} u_{1,3} u_{2,2} C 2 x u[ ] u_1^2 u_{1,1} u_{2,3} u_{3,3} K 2 x u[ ] \\
& u_1^2 u_{1,2} u_{1,3} u_{3,3} K 4 x u[ ] u_1 u_3 u_{1,1} u_{1,3} u_{2,3} C 4 x u[ ] u_1 u_3 u_{1,2} u_{1,3}^2 C 2 u_{2,3} u_{1,1}^2
\end{aligned}$$

$$\begin{aligned}
& u_3^2 u[ ] x K 2 u_{1,3} u_{1,2} u_{1,1} u_3^2 u[ ] x C y^2 u_1^2 u_{2,2}^2 u_{3,3} K y^2 u_1^2 u_{2,2}^2 u_{2,3}^2 \\
& K 2 y^2 u_1 u_2 u_{1,2} u_{2,2} u_{3,3} C 2 y^2 u_1 u_2 u_{1,3} u_{2,2} u_{2,3} C 2 y^2 u_1 u_3 u_{1,2} u_{2,2} u_{2,3} \\
& K 2 y^2 u_1 u_3 u_{1,3} u_{2,2}^2 C y^2 u_2^2 u_{1,1} u_{2,3}^2 C y^2 u_2^2 u_{1,2}^2 u_{3,3} K 2 y^2 u_2^2 u_{1,2} u_{1,3} u_{2,3} \\
& K 2 y^2 u_2 u_3 u_{1,1} u_{2,2} u_{2,3} C 2 y^2 u_2 u_3 u_{1,2} u_{1,3} u_{2,2} C y^2 u_3^2 u_{1,1} u_{2,2}^2 K y^2 u_3^2 u_{1,2}^2 u_{2,2} \\
& K 4 y z u_1^2 u_{2,2} u_{2,3} u_{3,3} C 4 y z u_1^2 u_{2,3}^3 C 4 y z u_1 u_2 u_{1,2} u_{2,3} u_{3,3} K 4 y z u_1 u_2 u_{1,3} \\
& u_{2,3}^2 C 4 y z u_1 u_3 u_{1,2} u_{2,2} u_{3,3} K 8 y z u_1 u_3 u_{1,2} u_{2,3}^2 C 4 y z u_1 u_3 u_{1,3} u_{2,2} u_{2,3} \\
& K 2 y z u_2 u_3 u_{1,1} u_{2,2} u_{3,3} C 2 y z u_2 u_3 u_{1,1} u_{2,3}^2 K 2 y z u_2 u_3 u_{1,2}^2 u_{3,3} C 2 y z u_2 u_3 \\
& u_{1,3}^2 u_{2,2} C 4 y z u_3^2 u_{1,2}^2 u_{2,3} K 4 y z u_3^2 u_{1,2} u_{1,3} u_{2,2} C 4 y u[ ] u_1^2 u_{1,2} u_{2,3} u_{3,3} \\
& K 4 y u[ ] u_1^2 u_{1,3} u_{2,3}^2 K 4 y u[ ] u_1 u_2 u_{1,1} u_{2,3} u_{3,3} C 4 y u[ ] u_1 u_2 u_{1,3}^2 u_{2,3} \\
& C 4 y u[ ] u_1 u_3 u_{1,1} u_{2,3}^2 K 4 y u[ ] u_1 u_3 u_{1,2}^2 u_{3,3} C 4 y u[ ] u_2 u_3 u_{1,1} u_{1,2} u_{3,3} \\
& K 4 y u[ ] u_2 u_3 u_{1,2} u_{1,3}^2 K 4 y u[ ] u_3^2 u_{1,1} u_{1,2} u_{2,3} C 4 y u[ ] u_3^2 u_{1,2}^2 u_{1,3} C z^2 \\
& u_1^2 u_{2,2} u_{3,3}^2 K z^2 u_1^2 u_{2,3}^2 u_{3,3} K 2 z^2 u_1 u_3 u_{1,3} u_{2,2} u_{3,3} C 2 z^2 u_1 u_3 u_{1,3} u_{2,3}^2 C z^2 u_3^2 \\
& u_{1,2}^2 u_{3,3} K 2 z^2 u_3^2 u_{1,2} u_{1,3} u_{2,3} C z^2 u_3^2 u_{1,3}^2 u_{2,2} K 2 z u[ ] u_1^2 u_{1,2} u_{2,3}^2 C 2 z u[ ] \\
& u_1^2 u_{1,3} u_{2,3} u_{3,3} C 4 z u[ ] u_1 u_3 u_{1,2} u_{1,3} u_{3,3} K 4 z u[ ] u_1 u_3 u_{1,3}^2 u_{2,3} K 2 z u[ ] \\
& u_3^2 u_{1,1} u_{1,2} u_{3,3} C 2 z u[ ] u_3^2 u_{1,1} u_{1,3} u_{2,3} C u[ ]^2 u_1^2 u_{1,1} u_{2,3}^2 K u[ ]^2 u_1^2 u_{1,3}^2 u_{3,3} \\
& K 2 u[ ]^2 u_1 u_3 u_{1,1} u_{1,3} u_{3,3} C 2 u[ ]^2 u_1 u_3 u_{1,3}^3 C u_{3,3} u_{1,1}^2 u_3^2 u[ ]^2 K u_{1,3}^2 u_{1,1} \\
& u_3^2 u[ ]^2 C 2 x^2 u_1 u_{1,1} u_{2,2} u_{2,3} K 2 x^2 u_1 u_{1,2}^2 u_{2,3} K 2 u_{2,2} u_{1,2} u_{1,1} u_3 x^2 C 2 \\
& u_{1,2}^3 u_3 x^2 C 2 x y u_1 u_{1,2} u_{2,2} u_{2,3} K 2 x y u_1 u_{1,3} u_{2,2}^2 K 2 x y u_2 u_{1,1} u_{2,2} u_{2,3} \\
& C 2 x y u_2 u_{1,2} u_{1,3} u_{2,2} C 2 x y u_3 u_{1,1} u_{2,2}^2 K 2 x y u_3 u_{1,2}^2 u_{2,2} K 4 x z u_1 u_{1,2} u_{2,3}^2 \\
& C 4 x z u_1 u_{1,3} u_{2,2} u_{2,3} C 4 x z u_3 u_{1,2}^2 u_{2,3} K 4 x z u_3 u_{1,2} u_{1,3} u_{2,2} \\
& C 2 x u[ ] u_1 u_{1,1} u_{2,2} u_{3,3} C 2 x u[ ] u_1 u_{1,1} u_{2,3}^2 K 2 x u[ ] u_1 u_{1,2}^2 u_{3,3} K 2 x u[ ] u_1 \\
& u_{1,3}^2 u_{2,2} K 4 u_{2,3} u_{1,2} u_{1,1} u_3 u[ ] x C 4 u_{1,3} u_{1,2}^2 u_3 u[ ] x K 2 y z u_1 u_{2,2}^2 u_{3,3} \\
& C 2 y z u_1 u_{2,2} u_{2,3}^2 C 2 y z u_2 u_{1,2} u_{2,2} u_{3,3} K 2 y z u_2 u_{1,3} u_{2,2} u_{2,3} \\
& K 2 y z u_3 u_{1,2} u_{2,2} u_{2,3} C 2 y z u_3 u_{1,3} u_{2,2}^2 C 2 y u[ ] u_1 u_{1,2} u_{2,2} u_{3,3} \\
& K 2 y u[ ] u_1 u_{1,3} u_{2,2} u_{2,3} K 2 y u[ ] u_2 u_{1,1} u_{2,3}^2 K 2 y u[ ] u_2 u_{1,2}^2 u_{3,3} \\
& C 4 y u[ ] u_2 u_{1,2} u_{1,3} u_{2,3} C 2 y u[ ] u_3 u_{1,1} u_{2,2} u_{2,3} K 2 y u[ ] u_3 u_{1,2} u_{1,3} u_{2,2} \\
& C 2 z^2 u_1 u_{2,2} u_{2,3} u_{3,3} K 2 z^2 u_1 u_{2,3}^3 K 2 z^2 u_3 u_{1,2} u_{2,2} u_{3,3} C 2 z^2 u_3 u_{1,2} u_{2,3}^2 \\
& K 4 z u[ ] u_1 u_{1,2} u_{2,3} u_{3,3} C 4 z u[ ] u_1 u_{1,3} u_{2,3}^2 C 2 z u[ ] u_3 u_{1,1} u_{2,2} u_{3,3} \\
& K 2 z u[ ] u_3 u_{1,1} u_{2,3}^2 C 2 z u[ ] u_3 u_{1,2}^2 u_{3,3} K 2 z u[ ] u_3 u_{1,3}^2 u_{2,2} \\
& C 2 u[ ]^2 u_1 u_{1,1} u_{2,3} u_{3,3} K 2 u[ ]^2 u_1 u_{1,3}^2 u_{2,3} K 2 u_{3,3} u_{1,2} u_{1,1} u_3 u[ ]^2 C 2 \\
& u_{1,3}^2 u_{1,2} u_3 u[ ]^2 C u_{2,2}^2 u_{1,1} x^2 K u_{2,2} u_{1,2}^2 x^2 K 2 x z u_{1,2} u_{2,2} u_{2,3} C 2 x z u_{1,3} u_{2,2}^2 \\
& C 2 u_{2,3} u_{2,2} u_{1,1} u[ ] x K 2 u_{2,2} u_{1,3} u_{1,2} u[ ] x C z^2 u_{2,2}^2 u_{3,3} K z^2 u_{2,2} u_{2,3}^2
\end{aligned}$$

$$\frac{\begin{aligned} &K^2 z u[ ] u_{1,2} u_{2,2} u_{3,3} C^2 z u[ ] u_{1,3} u_{2,2} u_{2,3} C u_{2,3}^2 u_{1,1} u[ ]^2 C u_{3,3} u_{1,2}^2 u[ ]^2 \\ &K^2 u_{2,3} u_{1,3} u_{1,2} u[ ]^2 \end{aligned}}{\left( (x^3 u_1^3 C^3 x^2 y u_1^2 u_2 C^3 x^2 z u_1^2 u_3 C^3 x y^2 u_1 u_2^2 \right.} \\ \left. C^6 x y z u_1 u_2 u_3 C^3 x z^2 u_1 u_3^2 C y^3 u_2^3 C^3 y^2 z u_2^2 u_3 C^3 y z^2 u_2 u_3^2 C z^3 u_3^3 K^3 x^2 u[ ] \right.} \\ \left. u_1^2 K^6 x y u[ ] u_1 u_2 K^6 x z u[ ] u_1 u_3 K^3 y^2 u[ ] u_2^2 K^6 y z u[ ] u_2 u_3 K^3 z^2 u[ ] u_3^2 \right.} \\ \left. C^3 x u[ ]^2 u_1 C^3 y u[ ]^2 u_2 C^3 z u[ ]^2 u_3 K u[ ]^3 \right)^2 (x u_1 C y u_2 C z u_3 K u[ ] ) \Bigg) \\ \mathbf{M} >$$

## Surfaces in 4-dimensions

$\mathbf{M} >$  *restart with(DifferentialGeometry) : with(Tensor) : with(JetCalculus) : with(Tools) :*

$\mathbf{M} >$  *DGsetup([t, s], [x, y], M, 4) :*

$\mathbf{M} >$  *Preferences("JetNotation", "JetNotation1") :*

$\mathbf{M} >$   $\omega \triangleq \text{evalDG}(dt \&w dx[ ] C ds \&w dy[ ])$

$$\omega \triangleq dt \wedge dx[ ] C ds \wedge dy[ ] \quad (3.1)$$

$\mathbf{M} >$   $v0 \triangleq \text{evalDG}(t D_t C s D_s C x[ ] D_x[ ] C y[ ] D_y[ ])$

$$v0 \triangleq t D_t C s D_s C x[ ] D_x[ ] C y[ ] D_y[ ] \quad (3.2)$$

*Insert a non-canonical basis on the tangent space*

$\mathbf{M} >$   $X \triangleq \text{evalDG}(D_t C x[1] D_x[ ] C y[1] D_y[ ])$ ;  $Y \triangleq \text{evalDG}(D_s C x[2] D_x[ ] C y[2] D_y[ ])$

$$X \triangleq D_t C x_1 D_x[ ] C y_1 D_y[ ]$$

$$Y \triangleq D_s C x_2 D_x[ ] C y_2 D_y[ ] \quad (3.3)$$

*Take an arbitrary vector*

$\mathbf{M} >$   $W^t \triangleq \text{evalDG}(a D_t C b D_s C c D_x[ ] C d D_y[ ])$

$$W^t \triangleq a D_t C b D_s C c D_x[ ] C d D_y[ ] \quad (3.4)$$

*use it to compute a basis for the orthogonal complement of  $T_{\{p\}}M$  with respect to  $\omega$*

$\mathbf{M} >$   $tmp1 \triangleq \text{solve}(\{Hook([X, W^t], \omega), Hook([Y, W^t], \omega)\}, \{a, b, c, d\}) :$

$\mathbf{M} >$   $Wtmp \triangleq \text{subs}(tmp1, W^t) :$

$\mathbf{M} >$   $X^t \triangleq \text{DGSimplify}(\text{eval}(Wtmp, \{a=1, b=0\})) : Y^t \triangleq \text{DGSimplify}(\text{eval}(Wtmp, \{a=0, b=1\})) :$

*using these we can compute a canonical basis for  $T_{\{p\}}M$*

$\mathbf{M} >$   $v1 \triangleq \text{evalDG}\left(\frac{y[2] s C x[2] t K y[ ]}{x[2] k y[1]} D_t C \left(\frac{k y[1] s K x[1] t C x[ ]}{x[2] k y[1]}\right) D_s \right.$

$$C \left( \frac{(k y[1] s C x[ ] ) x[2] C x[1] (y[2] s K y[ ] )}{x[2] k y[1]} \right) D_x[ ]$$

$$C \left( \frac{(x[2] t K y[ ] ) y[1] K y[2] (x[1] t K x[ ] )}{x[2] k y[1]} \right) D_y[ ] \Bigg)$$

$$v1 \triangleq \frac{(y_2 s C x_2 t K y[ ] ) D_t}{x_2 K y_1} K \frac{(y_1 s C x_1 t K x[ ] ) D_s}{x_2 K y_1}$$

$$C \frac{(s x_1 y_2 K s x_2 y_1 C x[ ] x_2 K x_1 y[ ] ) D_x[ ]}{x_2 K y_1}$$

(3.5)

$$\begin{aligned}
& \mathbf{K} \frac{(t x_1 y_2 \mathbf{K} t x_2 y_1 \mathbf{K} x[ ] y_2 \mathbf{C} y[ ] y_1) D_{-y}[ ]}{x_2 \mathbf{K} y_1} \\
\mathbf{M} > v2 \mathbf{d} \text{ evalDG} \left( \left( \frac{k y[2] s \mathbf{K} y[1] t \mathbf{C} y[ ]}{x[2] \mathbf{K} y[1]} \right) D_{-t} \mathbf{C} \left( \frac{x[2] s \mathbf{C} x[1] t \mathbf{K} x[ ]}{x[2] \mathbf{K} y[1]} \right) D_{-s} \right. \\
& \quad \mathbf{C} \left( \frac{(x[2] s \mathbf{K} x[ ] ) y[1] \mathbf{K} x[1] (y[2] s \mathbf{K} y[ ] )}{x[2] \mathbf{K} y[1]} \right) D_{-x}[ ] \\
& \quad \left. \mathbf{C} \left( \frac{(k y[1] t \mathbf{C} y[ ] ) x[2] \mathbf{C} y[2] \$ (x[1] t \mathbf{K} x[ ] )}{x[2] \mathbf{K} y[1]} \right) D_{-y}[ ] \right) \\
v2 \mathbf{d} \mathbf{K} \frac{(y_2 s \mathbf{C} t y_1 \mathbf{K} y[ ] ) D_{-t}}{x_2 \mathbf{K} y_1} \mathbf{C} \frac{(x_2 s \mathbf{C} x_1 t \mathbf{K} x[ ] ) D_{-s}}{x_2 \mathbf{K} y_1} \\
& \mathbf{K} \frac{(s x_1 y_2 \mathbf{K} s x_2 y_1 \mathbf{C} x[ ] y_1 \mathbf{K} x_1 y[ ] ) D_{-x}[ ]}{x_2 \mathbf{K} y_1} \\
& \mathbf{C} \frac{(t x_1 y_2 \mathbf{K} t x_2 y_1 \mathbf{K} x[ ] y_2 \mathbf{C} x_2 y[ ] ) D_{-y}[ ]}{x_2 \mathbf{K} y_1}
\end{aligned} \tag{3.6}$$

Arbitrary vectors in  $T_{\{p\}\Sigma}$  and  $T_{\{p\}\Sigma}^{\dagger}$  to be determined a quadratic form expression in the basis  $d^2 f$  and  $d^2 g$

$$\begin{aligned}
\mathbf{M} > w1tmp \mathbf{d} \text{ evalDG}(k1 X \mathbf{C} k2 Y) : w2tmp \mathbf{d} \text{ evalDG}(l1 X^{\dagger} \mathbf{C} l2 Y^{\dagger}) : \\
\mathbf{M} > H1 \mathbf{d} \text{ evalDG}(x[1, 1] dt \& s dt \mathbf{C} 2 x[1, 2] dt \& s ds \mathbf{C} x[2, 2] ds \& s ds) : H2 \\
& \quad \mathbf{d} \text{ evalDG}(y[1, 1] dt \& s dt \mathbf{C} 2 y[1, 2] dt \& s ds \mathbf{C} y[2, 2] ds \& s ds) : \\
\mathbf{M} > Q1 \mathbf{d} \text{ evalDG}(a H1 \mathbf{C} b H2) \\
Q1 \mathbf{d} (a x_{1,1} \mathbf{C} b y_{1,1}) dt dt \mathbf{C} (a x_{1,2} \mathbf{C} b y_{1,2}) dt ds \mathbf{C} (a x_{1,2} \mathbf{C} b y_{1,2}) ds dt \\
& \quad \mathbf{C} (a x_{2,2} \mathbf{C} b y_{2,2}) ds ds
\end{aligned} \tag{3.7}$$

Fix Q1 up to scale by  $Q(v1, v1) = 0$ .

$$\begin{aligned}
\mathbf{M} > tmp1 \mathbf{d} \text{ solve}(\text{Hook}([v1, v1], Q1), \{a\}) : \\
\mathbf{M} > Q1 \mathbf{d} \text{ subs}(tmp1, Q1) :
\end{aligned}$$

We can now determine w1tmp uniquely as the second null-like vector to Q1

$$\begin{aligned}
\mathbf{M} > tmp2 \mathbf{d} \text{ solve}(\{\text{Hook}([v1, w1tmp], \omega) \mathbf{K} 1, \text{Hook}([w1tmp, w1tmp], Q1)\}, \{k1, \\
& \quad k2\}) : \\
\mathbf{M} > w1 \mathbf{d} \text{ subs}(tmp2, w1tmp) :
\end{aligned}$$

Having obtained w1 uniquely we can fix Q1

$$\begin{aligned}
\mathbf{M} > tmp3 \mathbf{d} \text{ solve}(\{\text{Hook}([v1, w1], Q1) \mathbf{K} 1\}, \{b\}) : \\
\mathbf{M} > Q1 \mathbf{d} \text{ subs}(tmp3, Q1) :
\end{aligned}$$

Next, we find the corresponding 1-form

$$\begin{aligned}
\mathbf{M} > df \mathbf{d} \text{ evalDG}(dx[ ] \mathbf{K} x[1] dt \mathbf{K} x[2] ds); dg \mathbf{d} \text{ evalDG}(dy[ ] \mathbf{K} y[1] dt \mathbf{K} y[2] ds) \\
& \quad df \mathbf{d} \mathbf{K} x_1 dt \mathbf{K} x_2 ds \mathbf{C} dx[ ] \\
& \quad dg \mathbf{d} \mathbf{K} y_1 dt \mathbf{K} y_2 ds \mathbf{C} dy[ ]
\end{aligned} \tag{3.8}$$

$$\begin{aligned}
\mathbf{M} > \sigma1 \mathbf{d} \text{ evalDG}(a df \mathbf{C} b dg) \\
& \quad \sigma1 \mathbf{d} \mathbf{K} (a x_1 \mathbf{C} b y_1) dt \mathbf{K} (a x_2 \mathbf{C} b y_2) ds \mathbf{C} a dx[ ] \mathbf{C} b dy[ ]
\end{aligned} \tag{3.9}$$

Recall that the 1-form corresponding to the quadratic form has the same coefficients, hence we substitute these into  $\sigma1$ , determining it uniquely.

$$\mathbf{M} > \sigma1 \mathbf{d} \text{ subs}(tmp3, \text{subs}(tmp1, \sigma1)) :$$

Now we can compute our first invariant

$$\mathbf{M} > I_{2a} \mathbf{d} \text{ Hook}(v2, \sigma1) :$$

To find the remaining invariants we start with the 1-form and determine it's quadratic form. This again boils down to just finding the coefficients

$$\begin{aligned} \mathbf{M} > \sigma 2 \sqsubset \text{evalDG}(c \, df \sqsubset d \, dg) \\ & \sigma 2 \sqsubset \mathbb{K}(c \, x_1 \sqsubset d \, y_1) \, dt \mathbb{K}(c \, x_2 \sqsubset d \, y_2) \, ds \sqsubset c \, dx[ ] \sqsubset d \, dy[ ] \end{aligned} \quad (3.10)$$

The vector w2tmp is not determined, so the first thing to do is find it

$$\mathbf{M} > \text{tmp4} \sqsubset \text{solve}(\{ \text{Hook}([v2, w2tmp], \omega) \mathbb{K} 1, \text{Hook}(w2tmp, \sigma 1) \}, \{l1, l2\}) :$$

$$\mathbf{M} > w2 \sqsubset \text{subs}(\text{tmp4}, w2tmp) :$$

Having found w2 we can move on to obtaining  $\sigma 2$

$$\mathbf{M} > \text{tmp5} \sqsubset \text{solve}(\{ \text{Hook}(w2, \sigma 2) \mathbb{K} 1, \text{Hook}(v2, \sigma 2) \}, \{c, d\}) :$$

These coefficients will give the quadratic form Q2 uniquely

$$\begin{aligned} \mathbf{M} > Q2tmp \sqsubset \text{evalDG}(c \, H1 \sqsubset d \, H2) \\ Q2tmp \sqsubset (c \, x_{1,1} \sqsubset d \, y_{1,1}) \, dt \, dt \sqsubset (c \, x_{1,2} \sqsubset d \, y_{1,2}) \, dt \, ds \sqsubset (c \, x_{1,2} \sqsubset d \, y_{1,2}) \, ds \, dt \\ & \sqsubset (c \, x_{2,2} \sqsubset d \, y_{2,2}) \, ds \, ds \end{aligned} \quad (3.11)$$

$$\mathbf{M} > Q2 \sqsubset \text{subs}(\text{tmp5}, Q2tmp) :$$

the remaining invariants are then computed as

$$\mathbf{M} > I_{2b} \sqsubset \text{Hook}([v1, v1], Q2) \\ I_{2b} \sqsubset \frac{1}{(x_2 \mathbb{K} y_1)^3} (s^3 x_2 y_1^2 y_{2,2} \mathbb{K} 2 s^3 x_2 y_1 y_2 y_{1,2} \sqsubset s^3 x_2 y_2^2 y_{1,1} \mathbb{K} s^3 x_{1,1} y_2^3 \sqsubset 2 s^3 x_{1,2} y_1 \quad (3.12)$$

$$\begin{aligned} & y_2^2 \mathbb{K} s^3 x_{2,2} y_1^2 y_2 \sqsubset 2 s^2 t x_1 x_2 y_1 y_{2,2} \mathbb{K} 2 s^2 t x_1 x_2 y_2 y_{1,2} \sqsubset 2 s^2 t x_1 x_{1,2} y_2^2 \\ & \mathbb{K} 2 s^2 t x_1 x_{2,2} y_1 y_2 \sqsubset s^2 t x_1 y_1^2 y_{2,2} \mathbb{K} 2 s^2 t x_1 y_1 y_2 y_{1,2} \sqsubset s^2 t x_1 y_2^2 y_{1,1} \mathbb{K} 2 s^2 t \\ & x_2^2 y_1 y_{1,2} \sqsubset 2 s^2 t x_2^2 y_2 y_{1,1} \mathbb{K} 2 s^2 t x_2 x_{1,1} y_2^2 \sqsubset 2 s^2 t x_2 x_{1,2} y_1 y_2 \mathbb{K} s^2 t x_{1,1} y_1 y_2^2 \\ & \sqsubset 2 s^2 t x_{1,2} y_1^2 y_2 \mathbb{K} s^2 t x_{2,2} y_1^3 \sqsubset s^2 t x_1^2 x_2 y_{2,2} \mathbb{K} s^2 t x_1^2 x_{2,2} y_2 \sqsubset 2 s^2 t x_1^2 y_1 y_{2,2} \\ & \mathbb{K} 2 s^2 t x_1^2 y_2 y_{1,2} \mathbb{K} 2 s^2 t x_1 x_2^2 y_{1,2} \sqsubset 2 s^2 t x_1 x_2 x_{1,2} y_2 \mathbb{K} 2 s^2 t x_1 x_2 y_1 y_{1,2} \\ & \sqsubset 2 s^2 t x_1 x_2 y_2 y_{1,1} \sqsubset 2 s^2 t x_1 x_{1,2} y_1 y_2 \mathbb{K} 2 s^2 t x_1 x_{2,2} y_1^2 \sqsubset s^2 t x_2^3 y_{1,1} \mathbb{K} s^2 t x_2^2 x_{1,1} y_2 \\ & \mathbb{K} 2 s^2 t x_2 x_{1,1} y_1 y_2 \sqsubset 2 s^2 t x_2 x_{1,2} y_1^2 \sqsubset t^3 x_1^3 y_{2,2} \mathbb{K} 2 t^3 x_1^2 x_2 y_{1,2} \mathbb{K} t^3 x_1^2 x_{2,2} y_1 \sqsubset t^3 x_1 \\ & x_2^2 y_{1,1} \sqsubset 2 t^3 x_1 x_2 x_{1,2} y_1 \mathbb{K} t^3 x_2^2 x_{1,1} y_1 \mathbb{K} 2 s^2 x[ ] x_2 y_1 y_{2,2} \sqsubset 2 s^2 x[ ] x_2 y_2 y_{1,2} \\ & \mathbb{K} 2 s^2 x[ ] x_{1,2} y_2^2 \sqsubset 2 s^2 x[ ] x_{2,2} y_1 y_2 \mathbb{K} s^2 x[ ] y_1^2 y_{2,2} \sqsubset 2 s^2 x[ ] y_1 y_2 y_{1,2} \mathbb{K} s^2 x[ ] \\ & y_2^2 y_{1,1} \sqsubset 2 s^2 x_2 y[ ] y_1 y_{1,2} \mathbb{K} 2 s^2 x_2 y[ ] y_2 y_{1,1} \sqsubset 3 s^2 x_{1,1} y[ ] y_2^2 \mathbb{K} 4 s^2 x_{1,2} y[ ] y_1 y_2 \\ & \sqsubset s^2 x_{2,2} y[ ] y_1^2 \mathbb{K} 2 s t x[ ] x_1 x_2 y_{2,2} \sqsubset 2 s t x[ ] x_1 x_{2,2} y_2 \mathbb{K} 4 s t x[ ] x_1 y_1 y_{2,2} \\ & \sqsubset 4 s t x[ ] x_1 y_2 y_{1,2} \sqsubset 2 s t x[ ] x_2^2 y_{1,2} \mathbb{K} 2 s t x[ ] x_2 x_{1,2} y_2 \sqsubset 2 s t x[ ] x_2 y_1 y_{1,2} \\ & \mathbb{K} 2 s t x[ ] x_2 y_2 y_{1,1} \mathbb{K} 2 s t x[ ] x_{1,2} y_1 y_2 \sqsubset 2 s t x[ ] x_{2,2} y_1^2 \sqsubset 2 s t x_1 x_2 y[ ] y_{1,2} \\ & \mathbb{K} 4 s t x_1 x_{1,2} y[ ] y_2 \sqsubset 2 s t x_1 x_{2,2} y[ ] y_1 \sqsubset 2 s t x_1 y[ ] y_1 y_{1,2} \mathbb{K} 2 s t x_1 y[ ] y_2 y_{1,1} \\ & \mathbb{K} 2 s t x_2^2 y[ ] y_{1,1} \sqsubset 4 s t x_2 x_{1,1} y[ ] y_2 \mathbb{K} 2 s t x_2 x_{1,2} y[ ] y_1 \sqsubset 2 s t x_{1,1} y[ ] y_1 y_2 \\ & \mathbb{K} 2 s t x_{1,2} y[ ] y_1^2 \mathbb{K} 3 t^2 x[ ] x_1^2 y_{2,2} \sqsubset 4 t^2 x[ ] x_1 x_2 y_{1,2} \sqsubset 2 t^2 x[ ] x_1 x_{2,2} y_1 \mathbb{K} t^2 x[ ] \\ & x_2^2 y_{1,1} \mathbb{K} 2 t^2 x[ ] x_2 x_{1,2} y_1 \sqsubset t^2 x_1^2 x_{2,2} y[ ] \sqsubset 2 t^2 x_1^2 y[ ] y_{1,2} \mathbb{K} 2 t^2 x_1 x_2 x_{1,2} y[ ] \\ & \mathbb{K} 2 t^2 x_1 x_2 y[ ] y_{1,1} \mathbb{K} 2 t^2 x_1 x_{1,2} y[ ] y_1 \sqsubset t^2 x_2^2 x_{1,1} y[ ] \sqsubset 2 t^2 x_2 x_{1,1} y[ ] y_1 \end{aligned}$$

$$\begin{aligned}
& C s x[ ]^2 x_2 y_{2,2} K s x[ ]^2 x_2 y_{2,2} C 2 s x[ ]^2 y_1 y_{2,2} K 2 s x[ ]^2 y_2 y_{1,2} \\
& K 2 s x[ ] x_2 y[ ] y_{1,2} C 4 s x[ ] x_{1,2} y[ ] y_2 K 2 s x[ ] x_{2,2} y[ ] y_1 K 2 s x[ ] y[ ] y_1 y_{1,2} \\
& C 2 s x[ ] y[ ] y_2 y_{1,1} C s x_2 y[ ]^2 y_{1,1} K 3 s x_{1,1} y[ ]^2 y_2 C 2 s x_{1,2} y[ ]^2 y_1 \\
& C 3 t x[ ]^2 x_1 y_{2,2} K 2 t x[ ]^2 x_2 y_{1,2} K t x[ ]^2 x_{2,2} y_1 K 2 t x[ ] x_1 x_{2,2} y[ ] \\
& K 4 t x[ ] x_1 y[ ] y_{1,2} C 2 t x[ ] x_2 x_{1,2} y[ ] C 2 t x[ ] x_2 y[ ] y_{1,1} C 2 t x[ ] x_{1,2} y[ ] y_1 \\
& C 2 t x_1 x_{1,2} y[ ]^2 C t x_1 y[ ]^2 y_{1,1} K 2 t x_2 x_{1,1} y[ ]^2 K t x_{1,1} y[ ]^2 y_1 K x[ ]^3 y_{2,2} \\
& C x[ ]^2 x_{2,2} y[ ] C 2 x[ ]^2 y[ ] y_{1,2} K 2 x[ ] x_{1,2} y[ ]^2 K x[ ] y[ ]^2 y_{1,1} C x_{1,1} y[ ]^3
\end{aligned}$$

**M** >  $I_{2c} \sqsubset \text{Hook}([vI, wI], Q2)$

$$\begin{aligned}
I_{2c} \sqsubset K & \left( s^3 x_2 x_{1,1} y_1^2 y_{2,2}^2 K 2 s^3 x_2 x_{1,1} y_1 y_2 y_{1,2} y_{2,2} K s^3 x_2 x_{1,1} y_2^2 y_{1,1} y_{2,2} C 2 s^3 x_2 x_{1,1} \right. \\
& y_2^2 y_{1,2}^2 K 2 s^3 x_2 x_{1,2} y_1^2 y_{1,2} y_{2,2} C 4 s^3 x_2 x_{1,2} y_1 y_2 y_{1,1} y_{2,2} K 2 s^3 x_2 x_{1,2} y_2^2 y_{1,1} y_{1,2} \\
& K s^3 x_2 x_{2,2} y_1^2 y_{1,1} y_{2,2} C 2 s^3 x_2 x_{2,2} y_1^2 y_{1,2}^2 K 2 s^3 x_2 x_{2,2} y_1 y_2 y_{1,1} y_{1,2} C s^3 x_2 x_{2,2} y_2^2 \\
& y_{1,1}^2 C s^3 x_{1,1}^2 y_2^3 y_{2,2} K 2 s^3 x_{1,1} x_{1,2} y_1 y_2^2 y_{2,2} K 2 s^3 x_{1,1} x_{1,2} y_2^3 y_{1,2} K s^3 x_{1,1} x_{2,2} \\
& y_1^2 y_2 y_{2,2} C 4 s^3 x_{1,1} x_{2,2} y_1 y_2^2 y_{1,2} K s^3 x_{1,1} x_{2,2} y_2^3 y_{1,1} C 2 s^3 x_{1,2}^2 y_1^2 y_2 y_{2,2} C 2 s^3 x_{1,2}^2 \\
& y_2^3 y_{1,1} K 2 s^3 x_{1,2} x_{2,2} y_1^2 y_2 y_{1,2} K 2 s^3 x_{1,2} x_{2,2} y_1 y_2^2 y_{1,1} C s^3 x_{2,2}^2 y_1^2 y_2 y_{1,1} \\
& C 2 s^2 t x_1 x_2 x_{1,1} y_1 y_{2,2}^2 K 2 s^2 t x_1 x_2 x_{1,1} y_2 y_{1,2} y_{2,2} K 4 s^2 t x_1 x_2 x_{1,2} y_1 y_{1,2} y_{2,2} \\
& C 4 s^2 t x_1 x_2 x_{1,2} y_2 y_{1,1} y_{2,2} K 2 s^2 t x_1 x_2 x_{2,2} y_1 y_{1,1} y_{2,2} C 4 s^2 t x_1 x_2 x_{2,2} y_1 y_{1,2}^2 \\
& K 2 s^2 t x_1 x_2 x_{2,2} y_2 y_{1,1} y_{1,2} K 2 s^2 t x_1 x_{1,1} x_{1,2} y_2^2 y_{2,2} K 2 s^2 t x_1 x_{1,1} x_{2,2} y_1 y_2 y_{2,2} \\
& C 4 s^2 t x_1 x_{1,1} x_{2,2} y_2^2 y_{1,2} C s^2 t x_1 x_{1,1} y_1^2 y_{2,2}^2 K 2 s^2 t x_1 x_{1,1} y_1 y_2 y_{1,2} y_{2,2} \\
& K s^2 t x_1 x_{1,1} y_2^2 y_{1,1} y_{2,2} C 2 s^2 t x_1 x_{1,1} y_2^2 y_{1,2}^2 C 4 s^2 t x_1 x_{1,2}^2 y_1 y_2 y_{2,2} \\
& K 4 s^2 t x_1 x_{1,2} x_{2,2} y_1 y_2 y_{1,2} K 2 s^2 t x_1 x_{1,2} x_{2,2} y_2^2 y_{1,1} K 2 s^2 t x_1 x_{1,2} y_1^2 y_{1,2} y_{2,2} \\
& C 4 s^2 t x_1 x_{1,2} y_1 y_2 y_{1,1} y_{2,2} K 2 s^2 t x_1 x_{1,2} y_2^2 y_{1,1} y_{1,2} C 2 s^2 t x_1 x_{2,2}^2 y_1 y_2 y_{1,1} \\
& K s^2 t x_1 x_{2,2} y_1^2 y_{1,1} y_{2,2} C 2 s^2 t x_1 x_{2,2} y_1^2 y_{1,2}^2 K 2 s^2 t x_1 x_{2,2} y_1 y_2 y_{1,1} y_{1,2} \\
& C s^2 t x_1 x_{2,2} y_2^2 y_{1,1}^2 K 2 s^2 t x_2^2 x_{1,1} y_1 y_{1,2} y_{2,2} K 2 s^2 t x_2^2 x_{1,1} y_2 y_{1,1} y_{2,2} C 4 s^2 t \\
& x_2^2 x_{1,1} y_2 y_{1,2}^2 C 4 s^2 t x_2^2 x_{1,2} y_1 y_{1,1} y_{2,2} K 4 s^2 t x_2^2 x_{1,2} y_2 y_{1,1} y_{1,2} K 2 s^2 t \\
& x_2^2 x_{2,2} y_1 y_{1,1} y_{1,2} C 2 s^2 t x_2^2 x_{2,2} y_2 y_{1,1}^2 C 2 s^2 t x_2 x_{1,1}^2 y_2^2 y_{2,2} \\
& K 2 s^2 t x_2 x_{1,1} x_{1,2} y_1 y_2 y_{2,2} K 4 s^2 t x_2 x_{1,1} x_{1,2} y_2^2 y_{1,2} C 4 s^2 t x_2 x_{1,1} x_{2,2} y_1 y_2 y_{1,2} \\
& K 2 s^2 t x_2 x_{1,1} x_{2,2} y_2^2 y_{1,1} C 4 s^2 t x_2 x_{1,2}^2 y_2^2 y_{1,1} K 2 s^2 t x_2 x_{1,2} x_{2,2} y_1 y_2 y_{1,1} C s^2 t \\
& x_{1,1}^2 y_1 y_2^2 y_{2,2} K 2 s^2 t x_{1,1} x_{1,2} y_1^2 y_2 y_{2,2} K 2 s^2 t x_{1,1} x_{1,2} y_1 y_2^2 y_{1,2} K s^2 t x_{1,1} x_{2,2} \\
& y_1^3 y_{2,2} C 4 s^2 t x_{1,1} x_{2,2} y_1^2 y_2 y_{1,2} K s^2 t x_{1,1} x_{2,2} y_1 y_2^2 y_{1,1} C 2 s^2 t x_{1,2}^2 y_1^3 y_{2,2} C 2 s^2 t \\
& x_{1,2}^2 y_1 y_2^2 y_{1,1} K 2 s^2 t x_{1,2} x_{2,2} y_1^3 y_{1,2} K 2 s^2 t x_{1,2} x_{2,2} y_1^2 y_2 y_{1,1} C s^2 t x_{2,2}^2 y_1^3 y_{1,1} \\
& C s^2 t x_{1,1}^2 x_2 x_{1,1} y_{2,2}^2 K 2 s^2 t x_{1,1}^2 x_2 x_{1,2} y_{1,2} y_{2,2} K s^2 t x_{1,1}^2 x_2 x_{2,2} y_{1,1} y_{2,2} C 2 s^2 t \\
& x_{1,1}^2 x_2 x_{2,2} y_{1,2}^2 K s^2 t x_{1,1}^2 x_{1,2} x_{2,2} y_2 y_{2,2} C 2 s^2 t x_{1,1}^2 x_{1,1} y_1 y_{2,2}^2 K 2 s^2 t x_{1,1}^2 x_{1,1} y_2 y_{1,2} y_{2,2}
\end{aligned} \tag{3.13}$$

$$\begin{aligned}
& C 2 s t^2 x_1^2 x_{1,2}^2 y_2 y_{2,2} K 2 s t^2 x_1^2 x_{1,2} x_{2,2} y_2 y_{1,2} K 4 s t^2 x_1^2 x_{1,2} y_1 y_{1,2} y_{2,2} C 4 s t^2 \\
& x_1^2 x_{1,2} y_2 y_{1,1} y_{2,2} C s t^2 x_1^2 x_{2,2}^2 y_2 y_{1,1} K 2 s t^2 x_1^2 x_{2,2} y_1 y_{1,1} y_{2,2} C 4 s t^2 x_1^2 x_{2,2} y_1 y_{1,2}^2 \\
& K 2 s t^2 x_1^2 x_{2,2} y_2 y_{1,1} y_{1,2} K 2 s t^2 x_1 x_{2,2}^2 x_{1,1} y_{1,2} y_{2,2} C 4 s t^2 x_1 x_{2,2}^2 x_{1,2} y_{1,1} y_{2,2} \\
& K 2 s t^2 x_1 x_{2,2}^2 x_{2,2} y_{1,1} y_{1,2} K 2 s t^2 x_1 x_2 x_{1,1} x_{1,2} y_2 y_{2,2} C 4 s t^2 x_1 x_2 x_{1,1} x_{2,2} y_2 y_{1,2} \\
& K 2 s t^2 x_1 x_2 x_{1,1} y_1 y_{1,2} y_{2,2} K 2 s t^2 x_1 x_2 x_{1,1} y_2 y_{1,1} y_{2,2} C 4 s t^2 x_1 x_2 x_{1,1} y_2 y_{1,2}^2 \\
& K 2 s t^2 x_1 x_2 x_{1,2} x_{2,2} y_2 y_{1,1} C 4 s t^2 x_1 x_2 x_{1,2} y_1 y_{1,1} y_{2,2} K 4 s t^2 x_1 x_2 x_{1,2} y_2 y_{1,1} y_{1,2} \\
& K 2 s t^2 x_1 x_2 x_{2,2} y_1 y_{1,1} y_{1,2} C 2 s t^2 x_1 x_2 x_{2,2} y_2 y_{1,1}^2 K 2 s t^2 x_1 x_{1,1} x_{1,2} y_1 y_2 y_{2,2} \\
& K 2 s t^2 x_1 x_{1,1} x_{2,2} y_1^2 y_{2,2} C 4 s t^2 x_1 x_{1,1} x_{2,2} y_1 y_2 y_{1,2} C 4 s t^2 x_1 x_{1,2}^2 y_1^2 y_{2,2} \\
& K 4 s t^2 x_1 x_{1,2} x_{2,2} y_1^2 y_{1,2} K 2 s t^2 x_1 x_{1,2} x_{2,2} y_1 y_2 y_{1,1} C 2 s t^2 x_1 x_{2,2}^2 y_1^2 y_{1,1} K s t^2 \\
& x_2^3 x_{1,1} y_{1,1} y_{2,2} C 2 s t^2 x_2^3 x_{1,1} y_{1,2}^2 K 2 s t^2 x_2^3 x_{1,2} y_{1,1} y_{1,2} C s t^2 x_2^3 x_{2,2} y_{1,1}^2 C s t^2 x_2^2 \\
& x_{1,1}^2 y_2 y_{2,2} K 2 s t^2 x_2^2 x_{1,1} x_{1,2} y_2 y_{1,2} K s t^2 x_2^2 x_{1,1} x_{2,2} y_2 y_{1,1} C 2 s t^2 x_2^2 x_{1,2}^2 y_2 y_{1,1} \\
& C 2 s t^2 x_2 x_{1,1}^2 y_1 y_2 y_{2,2} K 2 s t^2 x_2 x_{1,1} x_{1,2} y_1^2 y_{2,2} K 4 s t^2 x_2 x_{1,1} x_{1,2} y_1 y_2 y_{1,2} \\
& C 4 s t^2 x_2 x_{1,1} x_{2,2} y_1^2 y_{1,2} K 2 s t^2 x_2 x_{1,1} x_{2,2} y_1 y_2 y_{1,1} C 4 s t^2 x_2 x_{1,2}^2 y_1 y_2 y_{1,1} \\
& K 2 s t^2 x_2 x_{1,2} x_{2,2} y_1^2 y_{1,1} C t^3 x_1^3 x_{1,1} y_{2,2}^2 K 2 t^3 x_1^3 x_{1,2} y_{1,2} y_{2,2} K t^3 x_1^3 x_{2,2} y_{1,1} y_{2,2} \\
& C 2 t^3 x_1^3 x_{2,2} y_{1,2}^2 K 2 t^3 x_1^2 x_2 x_{1,1} y_{1,2} y_{2,2} C 4 t^3 x_1^2 x_2 x_{1,2} y_{1,1} y_{2,2} K 2 t^3 \\
& x_1^2 x_2 x_{2,2} y_{1,1} y_{1,2} K t^3 x_1^2 x_{1,1} x_{2,2} y_1 y_{2,2} C 2 t^3 x_1^2 x_{1,2}^2 y_1 y_{2,2} K 2 t^3 x_1^2 x_{1,2} x_{2,2} y_1 y_{1,2} \\
& C t^3 x_1^2 x_{2,2}^2 y_1 y_{1,1} K t^3 x_1 x_2^2 x_{1,1} y_{1,1} y_{2,2} C 2 t^3 x_1 x_2^2 x_{1,1} y_{1,2}^2 K 2 t^3 x_1 x_2^2 x_{1,2} y_{1,1} y_{1,2} \\
& C t^3 x_1 x_2^2 x_{2,2} y_{1,1}^2 K 2 t^3 x_1 x_2 x_{1,1} x_{1,2} y_1 y_{2,2} C 4 t^3 x_1 x_2 x_{1,1} x_{2,2} y_1 y_{1,2} \\
& K 2 t^3 x_1 x_2 x_{1,2} x_{2,2} y_1 y_{1,1} C t^3 x_2^2 x_{1,1}^2 y_1 y_{2,2} K 2 t^3 x_2^2 x_{1,1} x_{1,2} y_1 y_{1,2} K t^3 \\
& x_2^2 x_{1,1} x_{2,2} y_1 y_{1,1} C 2 t^3 x_2^2 x_{1,2}^2 y_1 y_{1,1} K 2 s^2 x[ ] x_2 x_{1,1} y_1 y_{2,2}^2 \\
& C 2 s^2 x[ ] x_2 x_{1,1} y_2 y_{1,2} y_{2,2} C 4 s^2 x[ ] x_2 x_{1,2} y_1 y_{1,2} y_{2,2} \\
& K 4 s^2 x[ ] x_2 x_{1,2} y_2 y_{1,1} y_{2,2} C 2 s^2 x[ ] x_2 x_{2,2} y_1 y_{1,1} y_{2,2} K 4 s^2 x[ ] x_2 x_{2,2} y_1 y_{1,2}^2 \\
& C 2 s^2 x[ ] x_2 x_{2,2} y_2 y_{1,1} y_{1,2} C 2 s^2 x[ ] x_{1,1} x_{1,2} y_2^2 y_{2,2} C 2 s^2 x[ ] x_{1,1} x_{2,2} y_1 y_2 y_{2,2} \\
& K 4 s^2 x[ ] x_{1,1} x_{2,2} y_2^2 y_{1,2} K s^2 x[ ] x_{1,1} y_1^2 y_{2,2}^2 C 2 s^2 x[ ] x_{1,1} y_1 y_2 y_{1,2} y_{2,2} \\
& C s^2 x[ ] x_{1,1} y_2^2 y_{1,1} y_{2,2} K 2 s^2 x[ ] x_{1,1} y_2^2 y_{1,2}^2 K 4 s^2 x[ ] x_{1,2}^2 y_1 y_2 y_{2,2} \\
& C 4 s^2 x[ ] x_{1,2} x_{2,2} y_1 y_2 y_{1,2} C 2 s^2 x[ ] x_{1,2} x_{2,2} y_2^2 y_{1,1} C 2 s^2 x[ ] x_{1,2} y_1^2 y_{1,2} y_{2,2} \\
& K 4 s^2 x[ ] x_{1,2} y_1 y_2 y_{1,1} y_{2,2} C 2 s^2 x[ ] x_{1,2} y_2^2 y_{1,1} y_{1,2} K 2 s^2 x[ ] x_{2,2}^2 y_1 y_2 y_{1,1} \\
& C s^2 x[ ] x_{2,2} y_1^2 y_{1,1} y_{2,2} K 2 s^2 x[ ] x_{2,2} y_1^2 y_{1,2}^2 C 2 s^2 x[ ] x_{2,2} y_1 y_2 y_{1,1} y_{1,2} \\
& K s^2 x[ ] x_{2,2} y_2^2 y_{1,1}^2 C 2 s^2 x_2 x_{1,1} y[ ] y_1 y_{1,2} y_{2,2} C 2 s^2 x_2 x_{1,1} y[ ] y_2 y_{1,1} y_{2,2} \\
& K 4 s^2 x_2 x_{1,1} y[ ] y_2 y_{1,2}^2 K 4 s^2 x_2 x_{1,2} y[ ] y_1 y_{1,1} y_{2,2} C 4 s^2 x_2 x_{1,2} y[ ] y_2 y_{1,1} y_{1,2} \\
& C 2 s^2 x_2 x_{2,2} y[ ] y_1 y_{1,1} y_{1,2} K 2 s^2 x_2 x_{2,2} y[ ] y_2 y_{1,1}^2 K 3 s^2 x_{1,1}^2 y[ ] y_2^2 y_{2,2} \\
& C 4 s^2 x_{1,1} x_{1,2} y[ ] y_1 y_2 y_{2,2} C 6 s^2 x_{1,1} x_{1,2} y[ ] y_2^2 y_{1,2} C s^2 x_{1,1} x_{2,2} y[ ] y_1^2 y_{2,2}
\end{aligned}$$

$$\begin{aligned}
& K 8 s^2 x_{1,1} x_{2,2} y [ ] y_1 y_2 y_{1,2} C 3 s^2 x_{1,1} x_{2,2} y [ ] y_2^2 y_{1,1} K 2 s^2 x_{1,2}^2 y [ ] y_1^2 y_{2,2} K 6 s^2 \\
& x_{1,2}^2 y [ ] y_2^2 y_{1,1} C 2 s^2 x_{1,2} x_{2,2} y [ ] y_1^2 y_{1,2} C 4 s^2 x_{1,2} x_{2,2} y [ ] y_1 y_2 y_{1,1} K s^2 x_{2,2}^2 y [ ] \\
& y_1^2 y_{1,1} K 2 s t x [ ] x_1 x_2 x_{1,1} y_{2,2}^2 C 4 s t x [ ] x_1 x_2 x_{1,2} y_{1,2} y_{2,2} \\
& C 2 s t x [ ] x_1 x_2 x_{2,2} y_{1,1} y_{2,2} K 4 s t x [ ] x_1 x_2 x_{2,2} y_{1,2}^2 C 2 s t x [ ] x_1 x_{1,1} x_{2,2} y_2 y_{2,2} \\
& K 4 s t x [ ] x_1 x_{1,1} y_1 y_{2,2}^2 C 4 s t x [ ] x_1 x_{1,1} y_2 y_{1,2} y_{2,2} K 4 s t x [ ] x_1 x_{1,2}^2 y_2 y_{2,2} \\
& C 4 s t x [ ] x_1 x_{1,2} x_{2,2} y_2 y_{1,2} C 8 s t x [ ] x_1 x_{1,2} y_1 y_{1,2} y_{2,2} \\
& K 8 s t x [ ] x_1 x_{1,2} y_2 y_{1,1} y_{2,2} K 2 s t x [ ] x_1 x_{2,2}^2 y_2 y_{1,1} C 4 s t x [ ] x_1 x_{2,2} y_1 y_{1,1} y_{2,2} \\
& K 8 s t x [ ] x_1 x_{2,2} y_1 y_{1,2}^2 C 4 s t x [ ] x_1 x_{2,2} y_2 y_{1,1} y_{1,2} C 2 s t x [ ] x_2^2 x_{1,1} y_{1,2} y_{2,2} \\
& K 4 s t x [ ] x_2^2 x_{1,2} y_{1,1} y_{2,2} C 2 s t x [ ] x_2^2 x_{2,2} y_{1,1} y_{1,2} C 2 s t x [ ] x_2 x_{1,1} x_{1,2} y_2 y_{2,2} \\
& K 4 s t x [ ] x_2 x_{1,1} x_{2,2} y_2 y_{1,2} C 2 s t x [ ] x_2 x_{1,1} y_1 y_{1,2} y_{2,2} \\
& C 2 s t x [ ] x_2 x_{1,1} y_2 y_{1,1} y_{2,2} K 4 s t x [ ] x_2 x_{1,1} y_2 y_{1,2}^2 C 2 s t x [ ] x_2 x_{1,2} x_{2,2} y_2 y_{1,1} \\
& K 4 s t x [ ] x_2 x_{1,2} y_1 y_{1,1} y_{2,2} C 4 s t x [ ] x_2 x_{1,2} y_2 y_{1,1} y_{1,2} \\
& C 2 s t x [ ] x_2 x_{2,2} y_1 y_{1,1} y_{1,2} K 2 s t x [ ] x_2 x_{2,2} y_2 y_{1,1}^2 C 2 s t x [ ] x_{1,1} x_{1,2} y_1 y_2 y_{2,2} \\
& C 2 s t x [ ] x_{1,1} x_{2,2} y_1^2 y_{2,2} K 4 s t x [ ] x_{1,1} x_{2,2} y_1 y_2 y_{1,2} K 4 s t x [ ] x_{1,2}^2 y_1^2 y_{2,2} \\
& C 4 s t x [ ] x_{1,2} x_{2,2} y_1^2 y_{1,2} C 2 s t x [ ] x_{1,2} x_{2,2} y_1 y_2 y_{1,1} K 2 s t x [ ] x_{2,2}^2 y_1^2 y_{1,1} \\
& C 2 s t x_1 x_2 x_{1,1} y [ ] y_{1,2} y_{2,2} K 4 s t x_1 x_2 x_{1,2} y [ ] y_{1,1} y_{2,2} \\
& C 2 s t x_1 x_2 x_{2,2} y [ ] y_{1,1} y_{1,2} C 4 s t x_1 x_{1,1} x_{1,2} y [ ] y_2 y_{2,2} \\
& C 2 s t x_1 x_{1,1} x_{2,2} y [ ] y_1 y_{2,2} K 8 s t x_1 x_{1,1} x_{2,2} y [ ] y_2 y_{1,2} \\
& C 2 s t x_1 x_{1,1} y [ ] y_1 y_{1,2} y_{2,2} C 2 s t x_1 x_{1,1} y [ ] y_2 y_{1,1} y_{2,2} K 4 s t x_1 x_{1,1} y [ ] y_2 y_{1,2}^2 \\
& K 4 s t x_1 x_{1,2}^2 y [ ] y_1 y_{2,2} C 4 s t x_1 x_{1,2} x_{2,2} y [ ] y_1 y_{1,2} C 4 s t x_1 x_{1,2} x_{2,2} y [ ] y_2 y_{1,1} \\
& K 4 s t x_1 x_{1,2} y [ ] y_1 y_{1,1} y_{2,2} C 4 s t x_1 x_{1,2} y [ ] y_2 y_{1,1} y_{1,2} K 2 s t x_1 x_{2,2}^2 y [ ] y_1 y_{1,1} \\
& C 2 s t x_1 x_{2,2} y [ ] y_1 y_{1,1} y_{1,2} K 2 s t x_1 x_{2,2} y [ ] y_2 y_{1,1}^2 C 2 s t x_2^2 x_{1,1} y [ ] y_{1,1} y_{2,2} \\
& K 4 s t x_2^2 x_{1,1} y [ ] y_{1,2}^2 C 4 s t x_2^2 x_{1,2} y [ ] y_{1,1} y_{1,2} K 2 s t x_2^2 x_{2,2} y [ ] y_{1,1}^2 K 4 s t x_2 \\
& x_{1,1}^2 y [ ] y_2 y_{2,2} C 2 s t x_2 x_{1,1} x_{1,2} y [ ] y_1 y_{2,2} C 8 s t x_2 x_{1,1} x_{1,2} y [ ] y_2 y_{1,2} \\
& K 4 s t x_2 x_{1,1} x_{2,2} y [ ] y_1 y_{1,2} C 4 s t x_2 x_{1,1} x_{2,2} y [ ] y_2 y_{1,1} K 8 s t x_2 x_{1,2}^2 y [ ] y_2 y_{1,1} \\
& C 2 s t x_2 x_{1,2} x_{2,2} y [ ] y_1 y_{1,1} K 2 s t x_{1,1}^2 y [ ] y_1 y_2 y_{2,2} C 2 s t x_{1,1} x_{1,2} y [ ] y_1^2 y_{2,2} \\
& C 4 s t x_{1,1} x_{1,2} y [ ] y_1 y_2 y_{1,2} K 4 s t x_{1,1} x_{2,2} y [ ] y_1^2 y_{1,2} \\
& C 2 s t x_{1,1} x_{2,2} y [ ] y_1 y_2 y_{1,1} K 4 s t x_{1,2}^2 y [ ] y_1 y_2 y_{1,1} C 2 s t x_{1,2} x_{2,2} y [ ] y_1^2 y_{1,1} \\
& K 3 t^2 x [ ] x_1^2 x_{1,1} y_{2,2}^2 C 6 t^2 x [ ] x_1^2 x_{1,2} y_{1,2} y_{2,2} C 3 t^2 x [ ] x_1^2 x_{2,2} y_{1,1} y_{2,2} \\
& K 6 t^2 x [ ] x_1^2 x_{2,2} y_{1,2}^2 C 4 t^2 x [ ] x_1 x_2 x_{1,1} y_{1,2} y_{2,2} K 8 t^2 x [ ] x_1 x_2 x_{1,2} y_{1,1} y_{2,2} \\
& C 4 t^2 x [ ] x_1 x_2 x_{2,2} y_{1,1} y_{1,2} C 2 t^2 x [ ] x_1 x_{1,1} x_{2,2} y_1 y_{2,2} K 4 t^2 x [ ] x_1 x_{1,2}^2 y_1 y_{2,2} \\
& C 4 t^2 x [ ] x_1 x_{1,2} x_{2,2} y_1 y_{1,2} K 2 t^2 x [ ] x_1 x_{2,2}^2 y_1 y_{1,1} C t^2 x [ ] x_2^2 x_{1,1} y_{1,1} y_{2,2}
\end{aligned}$$

$$\begin{aligned}
& K 2 t^2 x[ ] x_{2,1}^2 x_{1,1} y_{1,2}^2 C 2 t^2 x[ ] x_{2,2}^2 x_{1,2} y_{1,1} y_{1,2} K t^2 x[ ] x_{2,2}^2 x_{2,2} y_{1,1}^2 \\
& C 2 t^2 x[ ] x_{2,1} x_{1,1} x_{1,2} y_1 y_{2,2} K 4 t^2 x[ ] x_{2,1} x_{1,1} x_{2,2} y_1 y_{1,2} \\
& C 2 t^2 x[ ] x_{2,1} x_{1,2} x_{2,2} y_1 y_{1,1} C t^2 x_{1,1}^2 x_{1,2} y[ ] y_{2,2} C 2 t^2 x_{1,1}^2 x_{1,1} y[ ] y_{1,2} y_{2,2} \\
& K 2 t^2 x_{1,1}^2 x_{1,2} y[ ] y_{2,2} C 2 t^2 x_{1,1}^2 x_{1,2} x_{2,2} y[ ] y_{1,2} K 4 t^2 x_{1,1}^2 x_{1,2} y[ ] y_{1,1} y_{2,2} K t^2 x_{1,1}^2 \\
& x_{2,2} y[ ] y_{1,1} C 2 t^2 x_{1,1}^2 x_{2,2} y[ ] y_{1,1} y_{1,2} C 2 t^2 x_1 x_2 x_{1,1} x_{1,2} y[ ] y_{2,2} \\
& K 4 t^2 x_1 x_2 x_{1,1} x_{2,2} y[ ] y_{1,2} C 2 t^2 x_1 x_2 x_{1,1} y[ ] y_{1,1} y_{2,2} K 4 t^2 x_1 x_2 x_{1,1} y[ ] y_{1,2}^2 \\
& C 2 t^2 x_1 x_2 x_{1,2} x_{2,2} y[ ] y_{1,1} C 4 t^2 x_1 x_2 x_{1,2} y[ ] y_{1,1} y_{1,2} K 2 t^2 x_1 x_2 x_{2,2} y[ ] y_{1,1}^2 \\
& C 2 t^2 x_1 x_{1,1} x_{1,2} y[ ] y_1 y_{2,2} K 4 t^2 x_1 x_{1,1} x_{2,2} y[ ] y_1 y_{1,2} \\
& C 2 t^2 x_1 x_{1,2} x_{2,2} y[ ] y_1 y_{1,1} K t^2 x_{2,1}^2 x_{1,1} y[ ] y_{2,2} C 2 t^2 x_{2,1}^2 x_{1,1} x_{1,2} y[ ] y_{1,2} C t^2 \\
& x_{2,1}^2 x_{1,1} x_{2,2} y[ ] y_{1,1} K 2 t^2 x_{2,1}^2 x_{1,2} y[ ] y_{1,1} K 2 t^2 x_2 x_{1,1}^2 y[ ] y_1 y_{2,2} \\
& C 4 t^2 x_2 x_{1,1} x_{1,2} y[ ] y_1 y_{1,2} C 2 t^2 x_2 x_{1,1} x_{2,2} y[ ] y_1 y_{1,1} K 4 t^2 x_2 x_{1,2}^2 y[ ] y_1 y_{1,1} \\
& C s x[ ]^2 x_{2,1} x_{1,1} y_{2,2}^2 K 2 s x[ ]^2 x_{2,1} x_{1,2} y_{1,2} y_{2,2} K s x[ ]^2 x_{2,2} x_{2,2} y_{1,1} y_{2,2} \\
& C 2 s x[ ]^2 x_{2,2} x_{2,2} y_{1,2}^2 K s x[ ]^2 x_{1,1} x_{2,2} y_2 y_{2,2} C 2 s x[ ]^2 x_{1,1} y_1 y_{2,2}^2 \\
& K 2 s x[ ]^2 x_{1,1} y_2 y_{1,2} y_{2,2} C 2 s x[ ]^2 x_{1,2}^2 y_2 y_{2,2} K 2 s x[ ]^2 x_{1,2} x_{2,2} y_2 y_{1,2} \\
& K 4 s x[ ]^2 x_{1,2} y_1 y_{1,2} y_{2,2} C 4 s x[ ]^2 x_{1,2} y_2 y_{1,1} y_{2,2} C s x[ ]^2 x_{2,2}^2 y_2 y_{1,1} \\
& K 2 s x[ ]^2 x_{2,2} y_1 y_{1,1} y_{2,2} C 4 s x[ ]^2 x_{2,2} y_1 y_{1,2}^2 K 2 s x[ ]^2 x_{2,2} y_2 y_{1,1} y_{1,2} \\
& K 2 s x[ ] x_{2,1} x_{1,1} y[ ] y_{1,2} y_{2,2} C 4 s x[ ] x_{2,1} x_{1,2} y[ ] y_{1,1} y_{2,2} \\
& K 2 s x[ ] x_{2,2} x_{2,2} y[ ] y_{1,1} y_{1,2} K 4 s x[ ] x_{1,1} x_{1,2} y[ ] y_2 y_{2,2} \\
& K 2 s x[ ] x_{1,1} x_{2,2} y[ ] y_1 y_{2,2} C 8 s x[ ] x_{1,1} x_{2,2} y[ ] y_2 y_{1,2} \\
& K 2 s x[ ] x_{1,1} y[ ] y_1 y_{1,2} y_{2,2} K 2 s x[ ] x_{1,1} y[ ] y_2 y_{1,1} y_{2,2} C 4 s x[ ] x_{1,1} y[ ] y_2 \\
& y_{1,2}^2 C 4 s x[ ] x_{1,2}^2 y[ ] y_1 y_{2,2} K 4 s x[ ] x_{1,2} x_{2,2} y[ ] y_1 y_{1,2} \\
& K 4 s x[ ] x_{1,2} x_{2,2} y[ ] y_2 y_{1,1} C 4 s x[ ] x_{1,2} y[ ] y_1 y_{1,1} y_{2,2} \\
& K 4 s x[ ] x_{1,2} y[ ] y_2 y_{1,1} y_{1,2} C 2 s x[ ] x_{2,2}^2 y[ ] y_1 y_{1,1} \\
& K 2 s x[ ] x_{2,2} y[ ] y_1 y_{1,1} y_{1,2} C 2 s x[ ] x_{2,2} y[ ] y_2 y_{1,1}^2 K s x_{2,1} x_{1,1} y[ ]^2 y_{1,1} y_{2,2} \\
& C 2 s x_{2,1} x_{1,1} y[ ]^2 y_{1,2}^2 K 2 s x_{2,1} x_{1,2} y[ ]^2 y_{1,1} y_{1,2} C s x_{2,2} x_{2,2} y[ ]^2 y_{1,1}^2 C 3 s \\
& x_{1,1}^2 y[ ]^2 y_2 y_{2,2} K 2 s x_{1,1} x_{1,2} y[ ]^2 y_1 y_{2,2} K 6 s x_{1,1} x_{1,2} y[ ]^2 y_2 y_{1,2} \\
& C 4 s x_{1,1} x_{2,2} y[ ]^2 y_1 y_{1,2} K 3 s x_{1,1} x_{2,2} y[ ]^2 y_2 y_{1,1} C 6 s x_{1,2}^2 y[ ]^2 y_2 y_{1,1} \\
& K 2 s x_{1,2} x_{2,2} y[ ]^2 y_1 y_{1,1} C 3 t x[ ]^2 x_1 x_{1,1} y_{2,2}^2 K 6 t x[ ]^2 x_1 x_{1,2} y_{1,2} y_{2,2} \\
& K 3 t x[ ]^2 x_1 x_{2,2} y_{1,1} y_{2,2} C 6 t x[ ]^2 x_1 x_{2,2} y_{1,2}^2 K 2 t x[ ]^2 x_2 x_{1,1} y_{1,2} y_{2,2} \\
& C 4 t x[ ]^2 x_2 x_{1,2} y_{1,1} y_{2,2} K 2 t x[ ]^2 x_2 x_{2,2} y_{1,1} y_{1,2} K t x[ ]^2 x_{1,1} x_{2,2} y_1 y_{2,2} \\
& C 2 t x[ ]^2 x_{1,2}^2 y_1 y_{2,2} K 2 t x[ ]^2 x_{1,2} x_{2,2} y_1 y_{1,2} C t x[ ]^2 x_{2,2}^2 y_1 y_{1,1} \\
& K 2 t x[ ] x_1 x_{1,1} x_{2,2} y[ ] y_{2,2} K 4 t x[ ] x_1 x_{1,1} y[ ] y_{1,2} y_{2,2} C 4 t x[ ] x_1 x_{1,2}^2 y[ ] y_{2,2}
\end{aligned}$$

$$\begin{aligned}
& K 4 t x[ ] x_1 x_{1,2} x_{2,2} y[ ] y_{1,2} C 8 t x[ ] x_1 x_{1,2} y[ ] y_{1,1} y_{2,2} C 2 t x[ ] x_1 x_{2,2}^2 y[ ] y_{1,1} \\
& K 4 t x[ ] x_1 x_{2,2} y[ ] y_{1,1} y_{1,2} K 2 t x[ ] x_2 x_{1,1} x_{1,2} y[ ] y_{2,2} \\
& C 4 t x[ ] x_2 x_{1,1} x_{2,2} y[ ] y_{1,2} K 2 t x[ ] x_2 x_{1,1} y[ ] y_{1,1} y_{2,2} C 4 t x[ ] x_2 x_{1,1} y[ ] y_{1,2}^2 \\
& K 2 t x[ ] x_2 x_{1,2} x_{2,2} y[ ] y_{1,1} K 4 t x[ ] x_2 x_{1,2} y[ ] y_{1,1} y_{1,2} C 2 t x[ ] x_2 x_{2,2} y[ ] y_{1,1}^2 \\
& K 2 t x[ ] x_{1,1} x_{1,2} y[ ] y_{1,1} y_{2,2} C 4 t x[ ] x_{1,1} x_{2,2} y[ ] y_{1,1} y_{1,2} \\
& K 2 t x[ ] x_{1,2} x_{2,2} y[ ] y_{1,1} y_{1,1} K 2 t x_1 x_{1,1} x_{1,2} y[ ] y_{2,2}^2 C 4 t x_1 x_{1,1} x_{2,2} y[ ] y_{1,2}^2 \\
& K t x_1 x_{1,1} y[ ] y_{1,1} y_{2,2}^2 C 2 t x_1 x_{1,1} y[ ] y_{1,2}^2 K 2 t x_1 x_{1,2} x_{2,2} y[ ] y_{1,1}^2 \\
& K 2 t x_1 x_{1,2} y[ ] y_{1,1} y_{1,2} C t x_1 x_{2,2} y[ ] y_{1,1}^2 C 2 t x_2 x_{1,1}^2 y[ ] y_{2,2}^2 \\
& K 4 t x_2 x_{1,1} x_{1,2} y[ ] y_{1,2} K 2 t x_2 x_{1,1} x_{2,2} y[ ] y_{1,1} C 4 t x_2 x_{1,2}^2 y[ ] y_{1,1} C t \\
& x_{1,1}^2 y[ ] y_{1,1} y_{2,2} K 2 t x_{1,1} x_{1,2} y[ ] y_{1,1} y_{1,2} K t x_{1,1} x_{2,2} y[ ] y_{1,1} y_{1,1} C 2 t \\
& x_{1,2}^2 y[ ] y_{1,1} y_{1,1} K x[ ] x_{1,1}^3 y_{2,2}^2 C 2 x[ ] x_{1,2}^3 y_{1,2} y_{2,2} C x[ ] x_{2,2}^3 y_{1,1} y_{2,2} \\
& K 2 x[ ] x_{2,2}^3 y_{1,2}^2 C x[ ] x_{1,1}^2 x_{2,2} y[ ] y_{2,2} C 2 x[ ] x_{1,1}^2 y[ ] y_{1,2} y_{2,2} K 2 x[ ]^2 \\
& x_{1,2}^2 y[ ] y_{2,2} C 2 x[ ] x_{1,2}^2 x_{2,2} y[ ] y_{1,2} K 4 x[ ] x_{1,2}^2 y[ ] y_{1,1} y_{2,2} K x[ ]^2 \\
& x_{2,2}^2 y[ ] y_{1,1} C 2 x[ ] x_{2,2}^2 y[ ] y_{1,1} y_{1,2} C 2 x[ ] x_{1,1} x_{1,2} y[ ] y_{2,2}^2 \\
& K 4 x[ ] x_{1,1} x_{2,2} y[ ] y_{1,2} C x[ ] x_{1,1} y[ ] y_{1,1} y_{2,2} K 2 x[ ] x_{1,1} y[ ] y_{1,2}^2 \\
& C 2 x[ ] x_{1,2} x_{2,2} y[ ] y_{1,1} C 2 x[ ] x_{1,2} y[ ] y_{1,1} y_{1,2} K x[ ] x_{2,2} y[ ] y_{1,1}^2 K \\
& x_{1,1}^2 y[ ] y_{2,2}^3 C 2 x_{1,1} x_{1,2} y[ ] y_{1,2}^3 C x_{1,1} x_{2,2} y[ ] y_{1,1}^3 K 2 x_{1,2}^2 y[ ] y_{1,1}^3) / \\
& (2 (s^2 x_{1,1} y_1 y_2 y_{2,2} K s^2 x_{1,1} y_{2,2}^2 y_{1,2} K s^2 x_{1,2} y_{1,2}^2 y_{2,2} C s^2 x_{1,2} y_{2,2}^2 y_{1,1} C s^2 x_{2,2} y_{1,1}^2 y_{1,2} \\
& K s^2 x_{2,2} y_1 y_2 y_{1,1} C s t x_1 x_{1,1} y_2 y_{2,2} K 2 s t x_1 x_{1,2} y_1 y_{2,2} C 2 s t x_1 x_{2,2} y_1 y_{1,2} \\
& K s t x_1 x_{2,2} y_2 y_{1,1} C s t x_2 x_{1,1} y_1 y_{2,2} K 2 s t x_2 x_{1,1} y_2 y_{1,2} C 2 s t x_2 x_{1,2} y_2 y_{1,1} \\
& K s t x_2 x_{2,2} y_1 y_{1,1} K t^2 x_1^2 x_{1,2} y_{2,2} C t^2 x_1^2 x_{2,2} y_{1,2} C t^2 x_1 x_2 x_{1,1} y_{2,2} \\
& K t^2 x_1 x_2 x_{2,2} y_{1,1} K t^2 x_2^2 x_{1,1} y_{1,2} C t^2 x_2^2 x_{1,2} y_{1,1} K s x[ ] x_{1,1} y_2 y_{2,2} \\
& C 2 s x[ ] x_{1,2} y_1 y_{2,2} K 2 s x[ ] x_{2,2} y_1 y_{1,2} C s x[ ] x_{2,2} y_2 y_{1,1} K s x_{1,1} y[ ] y_1 y_{2,2} \\
& C 2 s x_{1,1} y[ ] y_2 y_{1,2} K 2 s x_{1,2} y[ ] y_2 y_{1,1} C s x_{2,2} y[ ] y_1 y_{1,1} C 2 t x[ ] x_1 x_{1,2} y_{2,2} \\
& K 2 t x[ ] x_1 x_{2,2} y_{1,2} K t x[ ] x_2 x_{1,1} y_{2,2} C t x[ ] x_2 x_{2,2} y_{1,1} K t x_1 x_{1,1} y[ ] y_{2,2} \\
& C t x_1 x_{2,2} y[ ] y_{1,1} C 2 t x_2 x_{1,1} y[ ] y_{1,2} K 2 t x_2 x_{1,2} y[ ] y_{1,1} K x[ ]^2 x_{1,2} y_{2,2} \\
& C x[ ]^2 x_{2,2} y_{1,2} C x[ ] x_{1,1} y[ ] y_{2,2} K x[ ] x_{2,2} y[ ] y_{1,1} K x_{1,1} y[ ] y_{1,2}^2 \\
& C x_{1,2} y[ ] y_{1,1}^2) (x_2 K y_1)^2)
\end{aligned}$$

**M** >  $I_{2d} \triangleleft \text{Hook}([w1, w1], Q2)$

$$\begin{aligned}
I_{2d} \triangleleft & (s^3 x_2 x_{1,1}^2 y_1^2 y_{2,2}^3 K 2 s^3 x_2 x_{1,1}^2 y_1 y_2 y_{1,2} y_{2,2}^2 C s^3 x_2 x_{1,1}^2 y_{2,2}^2 y_{1,1} y_{2,2}^2 \\
& K 4 s^3 x_2 x_{1,1} x_{1,2} y_{1,2}^2 y_{1,2} y_{2,2}^2 C 8 s^3 x_2 x_{1,1} x_{1,2} y_1 y_2 y_{1,2}^2 y_{2,2} K 4 s^3 x_2 x_{1,1} x_{1,2} \\
& y_{2,2}^2 y_{1,1} y_{1,2} y_{2,2} K 2 s^3 x_2 x_{1,1} x_{2,2} y_{1,1}^2 y_{1,1} y_{2,2}^2 C 4 s^3 x_2 x_{1,1} x_{2,2} y_{1,1}^2 y_{1,2} y_{2,2}^2
\end{aligned}$$

(3.14)

$$\begin{aligned}
& \mathbb{C} 4 s^3 x_{2,1} x_{2,2} y_1 y_2 y_{1,1} y_{1,2} y_{2,2} \mathbb{K} 8 s^3 x_{2,1} x_{2,2} y_1 y_2 y_{1,2}^3 \mathbb{K} 2 s^3 x_{2,1} x_{2,2} y_2^2 \\
& y_{1,1}^2 y_{2,2} \mathbb{C} 4 s^3 x_{2,1} x_{2,2} y_2^2 y_{1,1} y_{1,2}^2 \mathbb{C} 4 s^3 x_{2,1}^2 y_{1,2}^2 y_{1,1} y_{2,2}^2 \mathbb{K} 8 s^3 x_{2,1}^2 y_{1,2}^2 y_1 y_2 y_{1,1} y_{1,2} y_{2,2} \mathbb{C} 4 s^3 x_{2,1}^2 y_{2,2}^2 y_{1,1}^2 y_{2,2} \mathbb{K} 4 s^3 x_{2,1} x_{2,2} y_1^2 y_{1,1} y_{1,2} y_{2,2} \\
& \mathbb{C} 8 s^3 x_{2,1} x_{2,2} y_1 y_2 y_{1,1} y_{1,2}^2 \mathbb{K} 4 s^3 x_{2,1} x_{2,2} y_2^2 y_{1,1}^2 y_{1,2} \mathbb{C} s^3 x_{2,1}^2 y_{1,1}^2 y_{1,2}^2 y_{2,2} \mathbb{K} 2 s^3 x_{2,1}^2 y_{1,1}^2 y_{2,2}^2 y_{1,1} y_{1,2} \mathbb{C} s^3 x_{2,1}^2 y_{2,2}^2 y_{1,1}^3 \mathbb{K} s^3 x_{1,1}^3 y_{2,2}^2 \mathbb{C} 2 s^3 x_{1,1}^2 x_{1,2} y_1 y_2^2 \\
& y_{2,2}^2 \mathbb{C} 4 s^3 x_{1,1}^2 x_{1,2} y_2^3 y_{1,2} y_{2,2} \mathbb{K} s^3 x_{1,1}^2 x_{2,2} y_1^2 y_2 y_{2,2}^2 \mathbb{C} 2 s^3 x_{1,1}^2 x_{2,2} y_2^3 y_{1,1} y_{2,2} \mathbb{K} 4 s^3 x_{1,1}^2 x_{2,2} y_2^3 y_{1,2}^2 \mathbb{K} 8 s^3 x_{1,1} x_{1,2}^2 y_1 y_2^2 y_{1,2} y_{2,2} \mathbb{K} 4 s^3 x_{1,1} x_{1,2}^2 y_2^3 y_{1,1} y_{2,2} \\
& \mathbb{C} 4 s^3 x_{1,1} x_{1,2} x_{2,2} y_1^2 y_2 y_{1,2} y_{2,2} \mathbb{K} 4 s^3 x_{1,1} x_{1,2} x_{2,2} y_1 y_2^2 y_{1,1} y_{2,2} \mathbb{C} 8 s^3 x_{1,1} x_{1,2} x_{2,2} y_1 y_2^2 y_{1,2}^2 \mathbb{C} 4 s^3 x_{1,1} x_{1,2} x_{2,2} y_2^3 y_{1,1} y_{1,2} \mathbb{C} 2 s^3 x_{1,1} x_{2,2}^2 \\
& y_1^2 y_2 y_{1,1} y_{2,2} \mathbb{K} 4 s^3 x_{1,1} x_{2,2}^2 y_1^2 y_2 y_{1,2}^2 \mathbb{K} s^3 x_{1,1} x_{2,2}^2 y_2^3 y_{1,1}^2 \mathbb{C} 8 s^3 x_{1,1}^3 x_{2,2} y_1 y_2^2 y_{1,1} y_{2,2} \mathbb{K} 4 s^3 x_{1,1}^2 x_{2,2} y_1^2 y_2 y_{1,1} y_{2,2} \mathbb{K} 8 s^3 x_{1,1}^2 x_{2,2} y_1 y_2^2 y_{1,1} y_{1,2} \mathbb{C} 4 s^3 x_{1,1} x_{2,2}^2 \\
& y_1^2 y_2 y_{1,1} y_{1,2} \mathbb{C} 2 s^3 x_{1,1} x_{2,2}^2 y_1 y_2^2 y_{1,1}^2 \mathbb{K} s^3 x_{2,2}^3 y_1^2 y_2 y_{1,1}^2 \mathbb{C} 2 s^2 t x_1 x_2 x_{1,1}^2 y_1 y_{2,2}^3 \\
& \mathbb{K} 2 s^2 t x_1 x_2 x_{1,1}^2 y_2 y_{1,2} y_{2,2}^2 \mathbb{K} 8 s^2 t x_1 x_2 x_{1,1} x_{1,2} y_1 y_{1,2} y_{2,2}^2 \\
& \mathbb{C} 8 s^2 t x_1 x_2 x_{1,1} x_{1,2} y_2 y_{1,2}^2 y_{2,2} \mathbb{K} 4 s^2 t x_1 x_2 x_{1,1} x_{2,2} y_1 y_{1,1} y_{2,2}^2 \\
& \mathbb{C} 8 s^2 t x_1 x_2 x_{1,1} x_{2,2} y_1 y_{1,2}^2 y_{2,2} \mathbb{C} 4 s^2 t x_1 x_2 x_{1,1} x_{2,2} y_2 y_{1,1} y_{1,2} y_{2,2} \\
& \mathbb{K} 8 s^2 t x_1 x_2 x_{1,1} x_{2,2} y_2 y_{1,2}^3 \mathbb{C} 8 s^2 t x_1 x_2 x_{1,2}^2 y_1 y_{1,1} y_{2,2}^2 \mathbb{K} 8 s^2 t x_1 x_2 \\
& x_{1,2}^2 y_2 y_{1,1} y_{1,2} y_{2,2} \mathbb{K} 8 s^2 t x_1 x_2 x_{1,2} x_{2,2} y_1 y_{1,1} y_{1,2} y_{2,2} \mathbb{C} 8 s^2 t x_1 x_2 x_{1,2} x_{2,2} y_2 y_{1,1} \\
& y_{1,2}^2 \mathbb{C} 2 s^2 t x_1 x_2 x_{2,2}^2 y_1 y_{1,1}^2 y_{2,2} \mathbb{K} 2 s^2 t x_1 x_2 x_{2,2}^2 y_2 y_{1,1}^2 y_{1,2} \mathbb{C} 2 s^2 t x_1 x_{1,1}^2 x_{1,2} y_2^2 \\
& y_{2,2}^2 \mathbb{K} 2 s^2 t x_1 x_{1,1}^2 x_{2,2} y_1 y_2 y_{2,2}^2 \mathbb{C} s^2 t x_1 x_{1,1}^2 y_1^3 y_{2,2}^2 \mathbb{K} 2 s^2 t x_1 x_{1,1}^2 y_1 y_2 y_{1,2} y_{2,2}^2 \\
& \mathbb{C} s^2 t x_1 x_{1,1}^2 y_2 y_{1,1} y_{2,2}^2 \mathbb{K} 8 s^2 t x_1 x_{1,1} x_{1,2}^2 y_2 y_{1,2} y_{2,2} \\
& \mathbb{C} 8 s^2 t x_1 x_{1,1} x_{1,2} x_{2,2} y_1 y_2 y_{1,2} y_{2,2} \mathbb{K} 4 s^2 t x_1 x_{1,1} x_{1,2} x_{2,2} y_2^2 y_{1,1} y_{2,2} \\
& \mathbb{C} 8 s^2 t x_1 x_{1,1} x_{1,2} x_{2,2} y_2^2 y_{1,2}^2 \mathbb{K} 4 s^2 t x_1 x_{1,1} x_{1,2} y_1^2 y_{1,2} y_{2,2}^2 \mathbb{C} 8 s^2 t x_1 x_{1,1} x_{1,2} y_1 y_2 \\
& y_{1,2}^2 y_{2,2} \mathbb{K} 4 s^2 t x_1 x_{1,1} x_{1,2} y_2^2 y_{1,1} y_{1,2} y_{2,2} \mathbb{C} 4 s^2 t x_1 x_{1,1} x_{2,2}^2 y_1 y_2 y_{1,1} y_{2,2} \\
& \mathbb{K} 8 s^2 t x_1 x_{1,1} x_{2,2}^2 y_1 y_2 y_{1,2}^2 \mathbb{K} 2 s^2 t x_1 x_{1,1} x_{2,2} y_1^2 y_{1,1} y_{2,2}^2 \mathbb{C} 4 s^2 t x_1 x_{1,1} x_{2,2} y_1^2 \\
& y_{1,2}^2 y_{2,2} \mathbb{C} 4 s^2 t x_1 x_{1,1} x_{2,2} y_1 y_2 y_{1,1} y_{1,2} y_{2,2} \mathbb{K} 8 s^2 t x_1 x_{1,1} x_{2,2} y_1 y_2 y_{1,2}^3 \\
& \mathbb{K} 2 s^2 t x_1 x_{1,1} x_{2,2} y_2^2 y_{1,1}^2 y_{2,2} \mathbb{C} 4 s^2 t x_1 x_{1,1} x_{2,2} y_2^2 y_{1,1} y_{1,2}^2 \mathbb{C} 8 s^2 t x_1 x_{1,1}^3 y_{2,2} \\
& y_{2,2}^2 y_{1,1} y_{2,2} \mathbb{K} 8 s^2 t x_1 x_{1,2}^2 x_{2,2} y_1 y_2 y_{1,1} y_{2,2} \mathbb{K} 8 s^2 t x_1 x_{1,2}^2 x_{2,2} y_2^2 y_{1,1} y_{1,2} \mathbb{C} 4 s^2 t x_1 \\
& x_{1,2}^2 y_1^2 y_{1,1} y_{2,2}^2 \mathbb{K} 8 s^2 t x_1 x_{1,2}^2 y_1 y_2 y_{1,1} y_{1,2} y_{2,2} \mathbb{C} 4 s^2 t x_1 x_{1,2}^2 y_2^2 y_{1,1}^2 y_{2,2} \\
& \mathbb{C} 8 s^2 t x_1 x_{1,2} x_{2,2}^2 y_1 y_2 y_{1,1} y_{1,2} \mathbb{C} 2 s^2 t x_1 x_{1,2} x_{2,2}^2 y_2^2 y_{1,1}^2 \mathbb{K} 4 s^2 t x_1 x_{1,2} x_{2,2} \\
& y_1^2 y_{1,1} y_{1,2} y_{2,2} \mathbb{C} 8 s^2 t x_1 x_{1,2} x_{2,2} y_1 y_2 y_{1,1} y_{1,2}^2 \mathbb{K} 4 s^2 t x_1 x_{1,2} x_{2,2} y_2^2 y_{1,1}^2 y_{1,2} \\
& \mathbb{K} 2 s^2 t x_1 x_{2,2}^3 y_1 y_2 y_{1,1}^2 \mathbb{C} s^2 t x_1 x_{2,2}^2 y_1^2 y_{1,1}^2 y_{2,2} \mathbb{K} 2 s^2 t x_1 x_{2,2}^2 y_1 y_2 y_{1,1}^2 y_{1,2} \mathbb{C} s^2 t x_1 \\
& x_{2,2}^2 y_2^3 y_{1,1} \mathbb{K} 2 s^2 t x_2^2 x_{1,1}^2 y_1 y_{1,2} y_{2,2}^2 \mathbb{C} 2 s^2 t x_2^2 x_{1,1}^2 y_2 y_{1,1} y_{2,2}^2 \mathbb{C} 8 s^2 t x_2^2 x_{1,1} x_{1,2} y_1
\end{aligned}$$

$$\begin{aligned}
& y_{1,2}^2 y_{2,2} K 8 s^2 t x_{2,1}^2 x_{1,1} x_{1,2} y_{2,1} y_{1,1} y_{1,2} y_{2,2} C 4 s^2 t x_{2,1}^2 x_{1,1} x_{2,2} y_1 y_{1,1} y_{1,2} y_{2,2} \\
& K 8 s^2 t x_{2,1}^2 x_{1,1} x_{2,2} y_1 y_{1,2}^3 K 4 s^2 t x_{2,1}^2 x_{1,1} x_{2,2} y_2 y_{1,1}^2 y_{2,2} C 8 s^2 t x_{2,1}^2 x_{1,1} x_{2,2} y_2 y_{1,1} \\
& y_{1,2}^2 K 8 s^2 t x_{2,1}^2 x_{1,2}^2 y_1 y_{1,1} y_{1,2} y_{2,2} C 8 s^2 t x_{2,1}^2 x_{1,2}^2 y_2 y_{1,1}^2 y_{2,2} C 8 s^2 t \\
& x_{2,1}^2 x_{1,2} x_{2,2} y_1 y_{1,1} y_{1,2}^2 K 8 s^2 t x_{2,1}^2 x_{1,2} x_{2,2} y_2 y_{1,1}^2 y_{1,2} K 2 s^2 t x_{2,1}^2 x_{2,2}^2 y_1 y_{1,1}^2 y_{1,2} \\
& C 2 s^2 t x_{2,1}^2 x_{2,2}^2 y_2 y_{1,1}^3 K 2 s^2 t x_2 x_{1,1}^3 y_2^2 y_{2,2}^2 C 2 s^2 t x_2 x_{1,1}^2 x_{1,2} y_1 y_2 y_{2,2}^2 C 8 s^2 t x_2 \\
& x_{1,1}^2 x_{1,2} y_2^2 y_{1,2} y_{2,2} C 4 s^2 t x_2 x_{1,1}^2 x_{2,2} y_2^2 y_{1,1} y_{2,2} K 8 s^2 t x_2 x_{1,1}^2 x_{2,2} y_2^2 y_{1,2}^2 \\
& K 8 s^2 t x_2 x_{1,1} x_{1,2}^2 y_1 y_2 y_{1,2} y_{2,2} K 8 s^2 t x_2 x_{1,1} x_{1,2}^2 y_2^2 y_{1,1} y_{2,2} \\
& K 4 s^2 t x_2 x_{1,1} x_{1,2} x_{2,2} y_1 y_2 y_{1,1} y_{2,2} C 8 s^2 t x_2 x_{1,1} x_{1,2} x_{2,2} y_1 y_2 y_{1,2}^2 \\
& C 8 s^2 t x_2 x_{1,1} x_{1,2} x_{2,2} y_2^2 y_{1,1} y_{1,2} K 2 s^2 t x_2 x_{1,1} x_{2,2}^2 y_2^2 y_{1,1}^2 C 8 s^2 t x_2 \\
& x_{1,2}^3 y_1 y_2 y_{1,1} y_{2,2} K 8 s^2 t x_2 x_{1,2}^2 x_{2,2} y_1 y_2 y_{1,1} y_{1,2} C 2 s^2 t x_2 x_{1,2} x_{2,2}^2 y_1 y_2 y_{1,1}^2 K s^2 t \\
& x_{1,1}^3 y_1 y_2^2 y_{2,2}^2 C 2 s^2 t x_{1,1}^2 x_{1,2} y_1^2 y_2 y_{2,2}^2 C 4 s^2 t x_{1,1}^2 x_{1,2} y_1 y_2^2 y_{1,2} y_{2,2} K s^2 t x_{1,1}^2 x_{2,2} \\
& y_1^2 y_{2,2}^2 C 2 s^2 t x_{1,1}^2 x_{2,2} y_1 y_2^2 y_{1,1} y_{2,2} K 4 s^2 t x_{1,1}^2 x_{2,2} y_1 y_2^2 y_{1,2}^2 K 8 s^2 t x_{1,1} x_{1,2}^2 \\
& y_1^2 y_2 y_{1,2} y_{2,2} K 4 s^2 t x_{1,1} x_{1,2}^2 y_1 y_2^2 y_{1,1} y_{2,2} C 4 s^2 t x_{1,1} x_{1,2} x_{2,2} y_1^3 y_{1,2} y_{2,2} \\
& K 4 s^2 t x_{1,1} x_{1,2} x_{2,2} y_1^2 y_2 y_{1,1} y_{2,2} C 8 s^2 t x_{1,1} x_{1,2} x_{2,2} y_1^2 y_2 y_{1,2}^2 \\
& C 4 s^2 t x_{1,1} x_{1,2} x_{2,2} y_1 y_2^2 y_{1,1} y_{1,2} C 2 s^2 t x_{1,1} x_{2,2}^2 y_1^3 y_{1,1} y_{2,2} K 4 s^2 t x_{1,1} x_{2,2}^2 y_1^3 \\
& y_{1,2}^2 K s^2 t x_{1,1} x_{2,2}^2 y_1 y_2^2 y_{1,1}^2 C 8 s^2 t x_{1,2}^3 y_1^2 y_2 y_{1,1} y_{2,2} K 4 s^2 t x_{1,2}^2 x_{2,2}^3 y_1 y_{1,2} y_{2,2} \\
& K 8 s^2 t x_{1,2}^2 x_{2,2}^2 y_1^2 y_2 y_{1,1} y_{1,2} C 4 s^2 t x_{1,2} x_{2,2}^2 y_1^3 y_{1,1} y_{1,2} C 2 s^2 t x_{1,2} x_{2,2}^2 y_1^2 y_2 y_{1,1}^2 \\
& K s^2 t x_{2,2}^3 y_1^3 y_{1,1}^2 C s^2 t x_1^2 x_2 x_{1,1}^2 y_{2,2}^3 K 4 s^2 t x_1^2 x_2 x_{1,1} x_{1,2} y_{1,2} y_{2,2}^2 K 2 s^2 t \\
& x_1^2 x_2 x_{1,1} x_{2,2} y_{1,1} y_{2,2}^2 C 4 s^2 t x_1^2 x_2 x_{1,1} x_{2,2} y_{1,2}^2 y_{2,2} C 4 s^2 t x_1^2 x_2 x_{1,2}^2 y_{1,1} y_{2,2}^2 \\
& K 4 s^2 t x_1^2 x_2 x_{1,2} x_{2,2} y_{1,1} y_{1,2} y_{2,2} C s^2 t x_1^2 x_2 x_{2,2}^2 y_{1,1}^2 y_{2,2} K s^2 t x_1^2 x_{1,1}^2 x_{2,2} y_2 y_{2,2}^2 \\
& C 2 s^2 t x_1^2 x_{1,1}^2 y_1 y_{2,2}^3 K 2 s^2 t x_1^2 x_{1,1}^2 y_2 y_{1,2} y_{2,2}^2 C 4 s^2 t x_1^2 x_{1,1} x_{1,2} x_{2,2} y_2 y_{1,2} y_{2,2} \\
& K 8 s^2 t x_1^2 x_{1,1} x_{1,2} y_1 y_{1,2} y_{2,2}^2 C 8 s^2 t x_1^2 x_{1,1} x_{1,2} y_2 y_{1,2}^2 y_{2,2} C 2 s^2 t x_1^2 x_{1,1} \\
& x_{2,2}^2 y_2 y_{1,1} y_{2,2} K 4 s^2 t x_1^2 x_{1,1} x_{2,2}^2 y_2 y_{1,2}^2 K 4 s^2 t x_1^2 x_{1,1} x_{2,2} y_1 y_{1,1} y_{2,2}^2 C 8 s^2 t \\
& x_1^2 x_{1,1} x_{2,2} y_1 y_{1,2} y_{2,2} C 4 s^2 t x_1^2 x_{1,1} x_{2,2} y_2 y_{1,1} y_{1,2} y_{2,2} K 8 s^2 t x_1^2 x_{1,1} x_{2,2} y_2 y_{1,2}^3 \\
& K 4 s^2 t x_1^2 x_{1,1}^2 x_{2,2} y_2 y_{1,1} y_{2,2} C 8 s^2 t x_1^2 x_{1,1}^2 y_1 y_{1,1} y_{2,2}^2 K 8 s^2 t x_1^2 \\
& x_{1,2}^2 y_2 y_{1,1} y_{1,2} y_{2,2} C 4 s^2 t x_1^2 x_{1,2} x_{2,2}^2 y_2 y_{1,1} y_{1,2} K 8 s^2 t x_1^2 x_{1,2} x_{2,2} y_1 y_{1,1} y_{1,2} y_{2,2} \\
& C 8 s^2 t x_1^2 x_{1,2} x_{2,2} y_2 y_{1,1} y_{1,2}^2 K s^2 t x_1^2 x_{2,2}^3 y_2 y_{1,1}^2 C 2 s^2 t x_1^2 x_{2,2}^2 y_1 y_{1,1}^2 y_{2,2} K 2 s^2 t \\
& x_1^2 x_{2,2}^2 y_2 y_{1,1} y_{1,2} K 2 s^2 t x_1 x_2^2 x_{1,1}^2 y_{1,2} y_{2,2}^2 C 8 s^2 t x_1 x_2^2 x_{1,1} x_{1,2} y_{1,2} y_{2,2} C 4 s^2 t x_1 \\
& x_2^2 x_{1,1} x_{2,2} y_{1,1} y_{1,2} y_{2,2} K 8 s^2 t x_1 x_2^2 x_{1,1} x_{2,2} y_{1,2}^3 K 8 s^2 t x_1 x_2^2 x_{1,2}^2 y_{1,1} y_{1,2} y_{2,2} \\
& C 8 s^2 t x_1 x_2^2 x_{1,2} x_{2,2} y_{1,1} y_{1,2}^2 K 2 s^2 t x_1 x_2^2 x_{2,2}^2 y_{1,1}^2 y_{1,2} C 2 s^2 t x_1 x_2 x_{1,1}^2 x_{1,2} y_2 y_{2,2}^2 \\
& K 2 s^2 t x_1 x_2 x_{1,1}^2 y_1 y_{1,2} y_{2,2}^2 C 2 s^2 t x_1 x_2 x_{1,1}^2 y_2 y_{1,1} y_{2,2}^2 K 8 s^2 t x_1 x_2 x_{1,1} \\
& x_{1,2}^2 y_2 y_{1,2} y_{2,2} K 4 s^2 t x_1 x_2 x_{1,1} x_{1,2} x_{2,2} y_2 y_{1,1} y_{2,2} C 8 s^2 t x_1 x_2 x_{1,1} x_{1,2} x_{2,2} y_2 y_{1,2}^2
\end{aligned}$$

$$\begin{aligned}
& \mathbb{C} 8 s t^2 x_1 x_2 x_{1,1} x_{1,2} y_1 y_{1,2}^2 y_{2,2} \mathbb{K} 8 s t^2 x_1 x_2 x_{1,1} x_{1,2} y_2 y_{1,1} y_{1,2} y_{2,2} \\
& \mathbb{C} 4 s t^2 x_1 x_2 x_{1,1} x_{2,2} y_1 y_{1,1} y_{1,2} y_{2,2} \mathbb{K} 8 s t^2 x_1 x_2 x_{1,1} x_{2,2} y_1 y_{1,2}^3 \\
& \mathbb{K} 4 s t^2 x_1 x_2 x_{1,1} x_{2,2} y_2 y_{1,1}^2 y_{2,2} \mathbb{C} 8 s t^2 x_1 x_2 x_{1,1} x_{2,2} y_2 y_{1,1} y_{1,2}^2 \mathbb{C} 8 s t^2 x_1 x_2 \\
& x_{1,2}^3 y_2 y_{1,1} y_{2,2} \mathbb{K} 8 s t^2 x_1 x_2 x_{1,2}^2 x_{2,2} y_2 y_{1,1} y_{1,2} \mathbb{K} 8 s t^2 x_1 x_2 x_{1,2}^2 y_1 y_{1,1} y_{1,2} y_{2,2} \\
& \mathbb{C} 8 s t^2 x_1 x_2 x_{1,2}^2 y_2 y_{1,1}^2 y_{2,2} \mathbb{C} 2 s t^2 x_1 x_2 x_{1,2} x_{2,2}^2 y_2 y_{1,1}^2 \\
& \mathbb{C} 8 s t^2 x_1 x_2 x_{1,2} x_{2,2} y_1 y_{1,1} y_{1,2}^2 \mathbb{K} 8 s t^2 x_1 x_2 x_{1,2} x_{2,2} y_2 y_{1,1}^2 y_{1,2} \mathbb{K} 2 s t^2 x_1 x_2 \\
& x_{2,2}^2 y_1 y_{1,1}^2 y_{1,2} \mathbb{C} 2 s t^2 x_1 x_2 x_{2,2}^2 y_2 y_{1,1}^3 \mathbb{C} 2 s t^2 x_1 x_{1,1}^2 x_{1,2} y_1 y_2 y_{2,2}^2 \mathbb{K} 2 s t^2 x_1 \\
& x_{1,1}^2 x_{2,2} y_1^2 y_{2,2}^2 \mathbb{K} 8 s t^2 x_1 x_{1,1} x_{1,2}^2 y_1 y_2 y_{1,2} y_{2,2} \mathbb{C} 8 s t^2 x_1 x_{1,1} x_{1,2} x_{2,2} y_1^2 y_{1,2} y_{2,2} \\
& \mathbb{K} 4 s t^2 x_1 x_{1,1} x_{1,2} x_{2,2} y_1 y_2 y_{1,1} y_{2,2} \mathbb{C} 8 s t^2 x_1 x_{1,1} x_{1,2} x_{2,2} y_1 y_2 y_{1,2}^2 \mathbb{C} 4 s t^2 x_1 x_{1,1} \\
& x_{2,2}^2 y_1^2 y_{1,1} y_{2,2} \mathbb{K} 8 s t^2 x_1 x_{1,1} x_{2,2}^2 y_1^2 y_{1,2}^2 \mathbb{C} 8 s t^2 x_1 x_{1,2}^3 y_1 y_2 y_{1,1} y_{2,2} \mathbb{K} 8 s t^2 x_1 \\
& x_{1,2}^2 x_{2,2} y_1^2 y_{1,1} y_{2,2} \mathbb{K} 8 s t^2 x_1 x_{1,2}^2 x_{2,2} y_1 y_2 y_{1,1} y_{1,2} \mathbb{C} 8 s t^2 x_1 x_{1,2} x_{2,2}^2 y_1^2 y_{1,1} y_{1,2} \\
& \mathbb{C} 2 s t^2 x_1 x_{1,2} x_{2,2}^2 y_1 y_2 y_{1,1}^2 \mathbb{K} 2 s t^2 x_1 x_{2,2}^2 y_1^2 y_{1,1}^2 \mathbb{C} s t^2 x_2^3 x_{1,1}^2 y_{1,1} y_{2,2}^2 \mathbb{K} 4 s t^2 \\
& x_2^3 x_{1,1} x_{1,2} y_{1,1} y_{1,2} y_{2,2} \mathbb{K} 2 s t^2 x_2^3 x_{1,1} x_{2,2} y_{1,1}^2 y_{2,2} \mathbb{C} 4 s t^2 x_2^3 x_{1,1} x_{2,2} y_{1,1} y_{1,2}^2 \\
& \mathbb{C} 4 s t^2 x_2^3 x_{1,2}^2 y_{1,1}^2 y_{2,2} \mathbb{K} 4 s t^2 x_2^3 x_{1,2} x_{2,2} y_{1,1}^2 y_{1,2} \mathbb{C} s t^2 x_2^3 x_{2,2}^2 y_{1,1}^3 \mathbb{K} s t^2 x_2^2 x_{1,1}^3 y_2 \\
& y_{2,2}^2 \mathbb{C} 4 s t^2 x_2^2 x_{1,1}^2 x_{1,2} y_2 y_{1,2} y_{2,2} \mathbb{C} 2 s t^2 x_2^2 x_{1,1}^2 x_{2,2} y_2 y_{1,1} y_{2,2} \mathbb{K} 4 s t^2 x_2^2 \\
& x_{1,1}^2 x_{2,2} y_2 y_{1,2}^2 \mathbb{K} 4 s t^2 x_2^2 x_{1,1} x_{1,2}^2 y_2 y_{1,1} y_{2,2} \mathbb{C} 4 s t^2 x_2^2 x_{1,1} x_{1,2} x_{2,2} y_2 y_{1,1} y_{1,2} \\
& \mathbb{K} s t^2 x_2^2 x_{1,1} x_{2,2}^2 y_2 y_{1,1}^2 \mathbb{K} 2 s t^2 x_2 x_{1,1}^3 y_1 y_2 y_{2,2}^2 \mathbb{C} 2 s t^2 x_2 x_{1,1}^2 x_{1,2} y_1^2 y_{2,2}^2 \mathbb{C} 8 s t^2 x_2 \\
& x_{1,1}^2 x_{1,2} y_1 y_2 y_{1,2} y_{2,2} \mathbb{C} 4 s t^2 x_2 x_{1,1}^2 x_{2,2} y_1 y_2 y_{1,1} y_{2,2} \mathbb{K} 8 s t^2 x_2 x_{1,1}^2 x_{2,2} y_1 y_2 y_{1,2}^2 \\
& \mathbb{K} 8 s t^2 x_2 x_{1,1} x_{1,2}^2 y_1^2 y_{1,2} y_{2,2} \mathbb{K} 8 s t^2 x_2 x_{1,1} x_{1,2}^2 y_1 y_2 y_{1,1} y_{2,2} \\
& \mathbb{K} 4 s t^2 x_2 x_{1,1} x_{1,2} x_{2,2} y_1^2 y_{1,1} y_{2,2} \mathbb{C} 8 s t^2 x_2 x_{1,1} x_{1,2} x_{2,2} y_1^2 y_{1,2}^2 \\
& \mathbb{C} 8 s t^2 x_2 x_{1,1} x_{1,2} x_{2,2} y_1 y_2 y_{1,1} y_{1,2} \mathbb{K} 2 s t^2 x_2 x_{1,1} x_{2,2}^2 y_1 y_2 y_{1,1}^2 \mathbb{C} 8 s t^2 x_2 x_{1,2}^3 \\
& y_{1,1}^2 y_{1,2} y_{2,2} \mathbb{K} 8 s t^2 x_2 x_{1,2}^2 x_{2,2}^2 y_1^2 y_{1,1} y_{1,2} \mathbb{C} 2 s t^2 x_2 x_{1,2} x_{2,2}^2 y_1^2 y_{1,1}^2 \mathbb{C} t^3 x_1^3 x_{1,1}^2 y_{2,2}^3 \\
& \mathbb{K} 4 t^3 x_1^3 x_{1,1} x_{1,2} y_{1,2} y_{2,2}^2 \mathbb{K} 2 t^3 x_1^3 x_{1,1} x_{2,2} y_{1,1} y_{2,2}^2 \mathbb{C} 4 t^3 x_1^3 x_{1,1} x_{2,2} y_{1,2} y_{2,2} \\
& \mathbb{C} 4 t^3 x_1^3 x_{1,2}^2 y_{1,1} y_{2,2}^2 \mathbb{K} 4 t^3 x_1^3 x_{1,2} x_{2,2} y_{1,1} y_{1,2} y_{2,2} \mathbb{C} t^3 x_1^3 x_{2,2}^2 y_{1,1}^2 y_{2,2} \mathbb{K} 2 t^3 x_1^2 x_2 \\
& x_{1,1}^2 y_{1,2} y_{2,2}^2 \mathbb{C} 8 t^3 x_1^2 x_2 x_{1,1} x_{1,2} y_{1,2} y_{2,2} \mathbb{C} 4 t^3 x_1^2 x_2 x_{1,1} x_{2,2} y_{1,1} y_{1,2} y_{2,2} \mathbb{K} 8 t^3 \\
& x_1^2 x_2 x_{1,1} x_{2,2} y_{1,2}^3 \mathbb{K} 8 t^3 x_1^2 x_2 x_{1,2}^2 y_{1,1} y_{1,2} y_{2,2} \mathbb{C} 8 t^3 x_1^2 x_2 x_{1,2} x_{2,2} y_{1,1} y_{1,2}^2 \mathbb{K} 2 t^3 \\
& x_1^2 x_2 x_{2,2}^2 y_{1,1} y_{1,2} \mathbb{K} t^3 x_1^2 x_{1,1}^2 x_{2,2} y_1 y_{2,2}^2 \mathbb{C} 4 t^3 x_1^2 x_{1,1} x_{1,2} x_{2,2} y_1 y_{1,2} y_{2,2} \mathbb{C} 2 t^3 \\
& x_1^2 x_{1,1} x_{2,2}^2 y_1 y_{1,1} y_{2,2} \mathbb{K} 4 t^3 x_1^2 x_{1,1} x_{2,2}^2 y_1 y_{1,2}^2 \mathbb{K} 4 t^3 x_1^2 x_{1,2}^2 x_{2,2} y_1 y_{1,1} y_{2,2} \mathbb{C} 4 t^3 \\
& x_1^2 x_{1,2} x_{2,2}^2 y_1 y_{1,1} y_{1,2} \mathbb{K} t^3 x_1^2 x_{2,2}^3 y_1 y_{1,1}^2 \mathbb{C} t^3 x_1 x_2^2 x_{1,1}^2 y_{1,1} y_{2,2}^2 \mathbb{K} 4 t^3 x_1 \\
& x_{2,2}^2 x_{1,1} x_{1,2} y_{1,1} y_{1,2} y_{2,2} \mathbb{K} 2 t^3 x_1 x_2^2 x_{1,1} x_{2,2} y_{1,1}^2 y_{2,2} \mathbb{C} 4 t^3 x_1 x_2^2 x_{1,1} x_{2,2} y_{1,1} y_{1,2}^2 \\
& \mathbb{C} 4 t^3 x_1 x_2^2 x_{1,2}^2 y_{1,1} y_{2,2} \mathbb{K} 4 t^3 x_1 x_2^2 x_{1,2} x_{2,2} y_{1,1} y_{1,2} \mathbb{C} t^3 x_1 x_2^2 x_{2,2}^2 y_{1,1}^3 \mathbb{C} 2 t^3 x_1 x_2 \\
& x_{1,1}^2 x_{1,2} y_1 y_{2,2}^2 \mathbb{K} 8 t^3 x_1 x_2 x_{1,1} x_{1,2}^2 y_1 y_{1,2} y_{2,2} \mathbb{K} 4 t^3 x_1 x_2 x_{1,1} x_{1,2} x_{2,2} y_1 y_{1,1} y_{2,2}
\end{aligned}$$

$$\begin{aligned}
& \mathbb{C} 8 t^3 x_1 x_2 x_{1,1} x_{1,2} x_{2,2} y_1 y_{1,2}^2 \mathbb{C} 8 t^3 x_1 x_2 x_{1,2}^3 y_1 y_{1,1} y_{2,2} \mathbb{K} 8 t^3 x_1 x_2 \\
& x_{1,2}^2 x_{2,2} y_1 y_{1,1} y_{1,2} \mathbb{C} 2 t^3 x_1 x_2 x_{1,2} x_{2,2}^2 y_1 y_{1,1}^2 \mathbb{K} t^3 x_2^2 x_{1,1}^3 y_1 y_{2,2}^2 \mathbb{C} 4 t^3 x_2^2 \\
& x_{1,1}^2 x_{1,2} x_{1,2} y_1 y_{1,2} y_{2,2} \mathbb{C} 2 t^3 x_2^2 x_{1,1}^2 x_{2,2} y_1 y_{1,1} y_{2,2} \mathbb{K} 4 t^3 x_2^2 x_{1,1}^2 x_{2,2} y_1 y_{1,2}^2 \mathbb{K} 4 t^3 \\
& x_2^2 x_{1,1} x_{1,2}^2 y_1 y_{1,1} y_{2,2} \mathbb{C} 4 t^3 x_2^2 x_{1,1} x_{1,2} x_{2,2} y_1 y_{1,1} y_{1,2} \mathbb{K} t^3 x_2^2 x_{1,1} x_{2,2}^2 y_1 y_{1,1}^2 \\
& \mathbb{K} 2 s^2 x[ ] x_2 x_{1,1}^2 y_1 y_{2,2}^3 \mathbb{C} 2 s^2 x[ ] x_2 x_{1,1}^2 y_2 y_{1,2} y_{2,2}^2 \mathbb{C} 8 s^2 x[ ] x_2 x_{1,1} x_{1,2} y_1 y_{1,2} \\
& y_{2,2}^2 \mathbb{K} 8 s^2 x[ ] x_2 x_{1,1} x_{1,2} y_2 y_{1,2}^2 y_{2,2} \mathbb{C} 4 s^2 x[ ] x_2 x_{1,1} x_{2,2} y_1 y_{1,1} y_{2,2}^2 \\
& \mathbb{K} 8 s^2 x[ ] x_2 x_{1,1} x_{2,2} y_1 y_{1,2}^2 y_{2,2} \mathbb{K} 4 s^2 x[ ] x_2 x_{1,1} x_{2,2} y_2 y_{1,1} y_{1,2} y_{2,2} \\
& \mathbb{C} 8 s^2 x[ ] x_2 x_{1,1} x_{2,2} y_2 y_{1,2}^3 \mathbb{K} 8 s^2 x[ ] x_2 x_{1,2}^2 y_1 y_{1,1} y_{2,2}^2 \mathbb{C} 8 s^2 x[ ] x_2 \\
& x_{1,2}^2 y_2 y_{1,1} y_{1,2} y_{2,2} \mathbb{C} 8 s^2 x[ ] x_2 x_{1,2} x_{2,2} y_1 y_{1,1} y_{1,2} y_{2,2} \\
& \mathbb{K} 8 s^2 x[ ] x_2 x_{1,2} x_{2,2} y_2 y_{1,1} y_{1,2}^2 \mathbb{K} 2 s^2 x[ ] x_2 x_{2,2}^2 y_1 y_{1,1} y_{2,2} \mathbb{C} 2 s^2 x[ ] x_2 x_{2,2}^2 y_2 \\
& y_{1,1} y_{1,2} \mathbb{K} 2 s^2 x[ ] x_{1,1}^2 x_{1,2} y_2^2 y_{2,2}^2 \mathbb{C} 2 s^2 x[ ] x_{1,1}^2 x_{2,2} y_1 y_2 y_{2,2}^2 \mathbb{K} s^2 x[ ] x_{1,1}^2 y_1^2 y_{2,2}^3 \\
& \mathbb{C} 2 s^2 x[ ] x_{1,1}^2 y_1 y_2 y_{1,2} y_{2,2}^2 \mathbb{K} s^2 x[ ] x_{1,1}^2 y_1^2 y_{1,1} y_{2,2}^2 \mathbb{C} 8 s^2 x[ ] x_{1,1} x_{1,2}^2 y_2^2 y_{1,2} y_{2,2} \\
& \mathbb{K} 8 s^2 x[ ] x_{1,1} x_{1,2} x_{2,2} y_1 y_2 y_{1,2} y_{2,2} \mathbb{C} 4 s^2 x[ ] x_{1,1} x_{1,2} x_{2,2} y_2^2 y_{1,1} y_{1,2} \\
& \mathbb{K} 8 s^2 x[ ] x_{1,1} x_{1,2} x_{2,2} y_2^2 y_{1,2}^2 \mathbb{C} 4 s^2 x[ ] x_{1,1} x_{1,2} y_1^2 y_{1,2} y_{2,2}^2 \\
& \mathbb{K} 8 s^2 x[ ] x_{1,1} x_{1,2} y_1 y_2 y_{1,2}^2 y_{2,2} \mathbb{C} 4 s^2 x[ ] x_{1,1} x_{1,2} y_2^2 y_{1,1} y_{1,2} y_{2,2} \mathbb{K} 4 s^2 x[ ] x_{1,1} \\
& x_{2,2}^2 y_1 y_2 y_{1,1} y_{2,2} \mathbb{C} 8 s^2 x[ ] x_{1,1} x_{2,2}^2 y_1 y_2 y_{1,2}^2 \mathbb{C} 2 s^2 x[ ] x_{1,1} x_{2,2} y_1^2 y_{1,1} y_{2,2}^2 \\
& \mathbb{K} 4 s^2 x[ ] x_{1,1} x_{2,2} y_1^2 y_{1,2}^2 y_{2,2} \mathbb{K} 4 s^2 x[ ] x_{1,1} x_{2,2} y_1 y_2 y_{1,1} y_{1,2} y_{2,2} \\
& \mathbb{C} 8 s^2 x[ ] x_{1,1} x_{2,2} y_1 y_2 y_{1,2}^3 \mathbb{C} 2 s^2 x[ ] x_{1,1} x_{2,2} y_2^2 y_{1,1} y_{2,2} \mathbb{K} 4 s^2 x[ ] x_{1,1} x_{2,2} \\
& y_2^2 y_{1,1} y_{1,2}^2 \mathbb{K} 8 s^2 x[ ] x_{1,2}^3 y_2^2 y_{1,1} y_{2,2} \mathbb{C} 8 s^2 x[ ] x_{1,2}^2 x_{2,2} y_1 y_2 y_{1,1} y_{2,2} \mathbb{C} 8 s^2 x[ ] \\
& x_{1,2}^2 x_{2,2} y_2^2 y_{1,1} y_{1,2} \mathbb{K} 4 s^2 x[ ] x_{1,2}^2 y_1^2 y_{1,1} y_{2,2}^2 \mathbb{C} 8 s^2 x[ ] x_{1,2}^2 y_1 y_2 y_{1,1} y_{1,2} y_{2,2} \\
& \mathbb{K} 4 s^2 x[ ] x_{1,2}^2 y_2^2 y_{1,1} y_{2,2} \mathbb{K} 8 s^2 x[ ] x_{1,2} x_{2,2}^2 y_1 y_2 y_{1,1} y_{1,2} \mathbb{K} 2 s^2 x[ ] x_{1,2} x_{2,2}^2 y_2^2 \\
& y_{1,1}^2 \mathbb{C} 4 s^2 x[ ] x_{1,2} x_{2,2} y_1^2 y_{1,1} y_{1,2} y_{2,2} \mathbb{K} 8 s^2 x[ ] x_{1,2} x_{2,2} y_1 y_2 y_{1,1} y_{1,2}^2 \\
& \mathbb{C} 4 s^2 x[ ] x_{1,2} x_{2,2} y_2^2 y_{1,1} y_{1,2} \mathbb{C} 2 s^2 x[ ] x_{2,2}^3 y_1 y_2 y_{1,1}^2 \mathbb{K} s^2 x[ ] x_{2,2}^2 y_1^2 y_{1,1} y_{2,2} \\
& \mathbb{C} 2 s^2 x[ ] x_{2,2}^2 y_1 y_2 y_{1,1} y_{1,2} \mathbb{K} s^2 x[ ] x_{2,2}^2 y_2^3 y_{1,1}^2 \mathbb{C} 2 s^2 x_2 x_{1,1}^2 y[ ] y_1 y_{1,2} y_{2,2}^2 \\
& \mathbb{K} 2 s^2 x_2 x_{1,1}^2 y[ ] y_2 y_{1,1} y_{2,2}^2 \mathbb{K} 8 s^2 x_2 x_{1,1} x_{1,2} y[ ] y_1 y_{1,2}^2 y_{2,2} \\
& \mathbb{C} 8 s^2 x_2 x_{1,1} x_{1,2} y[ ] y_2 y_{1,1} y_{1,2} y_{2,2} \mathbb{K} 4 s^2 x_2 x_{1,1} x_{2,2} y[ ] y_1 y_{1,1} y_{1,2} y_{2,2} \\
& \mathbb{C} 8 s^2 x_2 x_{1,1} x_{2,2} y[ ] y_1 y_{1,2}^3 \mathbb{C} 4 s^2 x_2 x_{1,1} x_{2,2} y[ ] y_2 y_{1,1}^2 y_{2,2} \\
& \mathbb{K} 8 s^2 x_2 x_{1,1} x_{2,2} y[ ] y_2 y_{1,1} y_{1,2}^2 \mathbb{C} 8 s^2 x_2 x_{1,2}^2 y[ ] y_1 y_{1,1} y_{1,2} y_{2,2} \mathbb{K} 8 s^2 x_2 \\
& x_{1,2}^2 y[ ] y_2 y_{1,1}^2 y_{2,2} \mathbb{K} 8 s^2 x_2 x_{1,2} x_{2,2} y[ ] y_1 y_{1,1} y_{1,2}^2 \mathbb{C} 8 s^2 x_2 x_{1,2} x_{2,2} y[ ] y_2 \\
& y_{1,1}^2 y_{1,2} \mathbb{C} 2 s^2 x_2 x_{2,2}^2 y[ ] y_1 y_{1,1}^2 y_{1,2} \mathbb{K} 2 s^2 x_2 x_{2,2}^2 y[ ] y_2 y_{1,1}^3 \mathbb{C} 3 s^2 x_{1,1}^3 y[ ] y_2^2 y_{2,2}^2 \\
& \mathbb{K} 4 s^2 x_{1,1}^2 x_{1,2} y[ ] y_1 y_2 y_{2,2}^2 \mathbb{K} 12 s^2 x_{1,1}^2 x_{1,2} y[ ] y_2^2 y_{1,2} y_{2,2} \mathbb{C} s^2 x_{1,1}^2 x_{2,2} y[ ] y_1^2 \\
& y_{2,2}^2 \mathbb{K} 6 s^2 x_{1,1}^2 x_{2,2} y[ ] y_2^2 y_{1,1} y_{2,2} \mathbb{C} 12 s^2 x_{1,1}^2 x_{2,2} y[ ] y_2^2 y_{1,2}^2 \mathbb{C} 16 s^2 x_{1,1}
\end{aligned}$$

$$\begin{aligned}
& x_{1,2}^2 y[ ] y_1 y_2 y_{1,2} y_{2,2} \mathbf{C} 12 s^2 x_{1,1} x_{1,2}^2 y[ ] y_2^2 y_{1,1} y_{2,2} \mathbf{K} 4 s^2 x_{1,1} x_{1,2} x_{2,2} y[ ] \\
& y_1^2 y_{1,2} y_{2,2} \mathbf{C} 8 s^2 x_{1,1} x_{1,2} x_{2,2} y[ ] y_1 y_2 y_{1,1} y_{2,2} \mathbf{K} 16 s^2 x_{1,1} x_{1,2} x_{2,2} y[ ] y_1 y_2 y_{1,2}^2 \\
& \mathbf{K} 12 s^2 x_{1,1} x_{1,2} x_{2,2} y[ ] y_2^2 y_{1,1} y_{1,2} \mathbf{K} 2 s^2 x_{1,1} x_{2,2}^2 y[ ] y_1^2 y_{1,1} y_{2,2} \mathbf{C} 4 s^2 x_{1,1} \\
& x_{2,2}^2 y[ ] y_1^2 y_{1,2}^2 \mathbf{C} 3 s^2 x_{1,1} x_{2,2}^2 y[ ] y_2^2 y_{1,1}^2 \mathbf{K} 16 s^2 x_{1,2}^3 y[ ] y_1 y_2 y_{1,1} y_{2,2} \mathbf{C} 4 s^2 \\
& x_{1,2}^2 x_{2,2} y[ ] y_1^2 y_{1,1} y_{2,2} \mathbf{C} 16 s^2 x_{1,2}^2 x_{2,2} y[ ] y_1 y_2 y_{1,1} y_{1,2} \mathbf{K} 4 s^2 x_{1,2} x_{2,2}^2 y[ ] \\
& y_1^2 y_{1,1} y_{1,2} \mathbf{K} 4 s^2 x_{1,2} x_{2,2}^2 y[ ] y_1 y_2 y_{1,1}^2 \mathbf{C} s^2 x_{2,2}^3 y[ ] y_1^2 y_{1,1}^2 \mathbf{K} 2 s t x[ ] x_1 x_2 x_{1,1}^2 y_{2,2}^3 \\
& \mathbf{C} 8 s t x[ ] x_1 x_2 x_{1,1} x_{1,2} y_{1,2} y_{2,2}^2 \mathbf{C} 4 s t x[ ] x_1 x_2 x_{1,1} x_{2,2} y_{1,1} y_{2,2}^2 \\
& \mathbf{K} 8 s t x[ ] x_1 x_2 x_{1,1} x_{2,2} y_{1,2}^2 y_{2,2} \mathbf{K} 8 s t x[ ] x_1 x_2 x_{1,2}^2 y_{1,1} y_{2,2}^2 \\
& \mathbf{C} 8 s t x[ ] x_1 x_2 x_{1,2} x_{2,2} y_{1,1} y_{1,2} y_{2,2} \mathbf{K} 2 s t x[ ] x_1 x_2 x_{2,2}^2 y_{1,1} y_{2,2} \mathbf{C} 2 s t x[ ] x_1 \\
& x_{1,1}^2 x_{2,2} y_2 y_{2,2}^2 \mathbf{K} 4 s t x[ ] x_1 x_{1,1}^2 y_1 y_{2,2}^3 \mathbf{C} 4 s t x[ ] x_1 x_{1,1}^2 y_2 y_{1,2} y_{2,2}^2 \\
& \mathbf{K} 8 s t x[ ] x_1 x_{1,1} x_{1,2} x_{2,2} y_2 y_{1,2} y_{2,2} \mathbf{C} 16 s t x[ ] x_1 x_{1,1} x_{1,2} y_1 y_{1,2} y_{2,2}^2 \\
& \mathbf{K} 16 s t x[ ] x_1 x_{1,1} x_{1,2} y_2 y_{1,2}^2 y_{2,2} \mathbf{K} 4 s t x[ ] x_1 x_{1,1} x_{2,2}^2 y_2 y_{1,1} y_{2,2} \\
& \mathbf{C} 8 s t x[ ] x_1 x_{1,1} x_{2,2}^2 y_2 y_{1,2}^2 \mathbf{C} 8 s t x[ ] x_1 x_{1,1} x_{2,2} y_1 y_{1,1} y_{2,2}^2 \\
& \mathbf{K} 16 s t x[ ] x_1 x_{1,1} x_{2,2} y_1 y_{1,2}^2 y_{2,2} \mathbf{K} 8 s t x[ ] x_1 x_{1,1} x_{2,2} y_2 y_{1,1} y_{1,2} y_{2,2} \\
& \mathbf{C} 16 s t x[ ] x_1 x_{1,1} x_{2,2} y_2 y_{1,2}^3 \mathbf{C} 8 s t x[ ] x_1 x_{1,2}^2 x_{2,2} y_2 y_{1,1} y_{2,2} \mathbf{K} 16 s t x[ ] x_1 \\
& x_{1,2}^2 y_1 y_{1,1} y_{2,2}^2 \mathbf{C} 16 s t x[ ] x_1 x_{1,2}^2 y_2 y_{1,1} y_{1,2} y_{2,2} \mathbf{K} 8 s t x[ ] x_1 x_{1,2} x_{2,2}^2 y_2 y_{1,1} y_{1,2} \\
& \mathbf{C} 16 s t x[ ] x_1 x_{1,2} x_{2,2} y_1 y_{1,1} y_{1,2} y_{2,2} \mathbf{K} 16 s t x[ ] x_1 x_{1,2} x_{2,2} y_2 y_{1,1} y_{1,2}^2 \\
& \mathbf{C} 2 s t x[ ] x_1 x_{2,2}^3 y_2 y_{1,1}^2 \mathbf{K} 4 s t x[ ] x_1 x_{2,2}^2 y_1 y_{1,1}^2 y_{2,2} \mathbf{C} 4 s t x[ ] x_1 x_{2,2}^2 y_2 y_{1,1}^2 y_{1,2} \\
& \mathbf{C} 2 s t x[ ] x_{2,2}^2 x_{1,1}^2 y_{1,2} y_{2,2}^2 \mathbf{K} 8 s t x[ ] x_{2,2}^2 x_{1,1} x_{1,2} y_{1,2}^2 y_{2,2} \mathbf{K} 4 s t x[ ] \\
& x_{2,2}^2 x_{1,1} x_{2,2} y_{1,1} y_{1,2} y_{2,2} \mathbf{C} 8 s t x[ ] x_{2,2}^2 x_{1,1} x_{2,2} y_{1,2}^3 \mathbf{C} 8 s t x[ ] x_{2,2}^2 x_{1,2}^2 y_{1,1} y_{1,2} y_{2,2} \\
& \mathbf{K} 8 s t x[ ] x_{2,2}^2 x_{1,2} x_{2,2} y_{1,1} y_{1,2}^2 \mathbf{C} 2 s t x[ ] x_{2,2}^2 x_{2,2}^2 y_{1,1}^2 y_{1,2} \mathbf{K} 2 s t x[ ] x_2 x_{1,1}^2 x_{1,2} y_2 \\
& y_{2,2}^2 \mathbf{C} 2 s t x[ ] x_2 x_{1,1}^2 y_1 y_{1,2} y_{2,2}^2 \mathbf{K} 2 s t x[ ] x_2 x_{1,1}^2 y_2 y_{1,1} y_{2,2}^2 \mathbf{C} 8 s t x[ ] x_2 x_{1,1} \\
& x_{1,2}^2 y_2 y_{1,2} y_{2,2} \mathbf{C} 4 s t x[ ] x_2 x_{1,1} x_{1,2} x_{2,2} y_2 y_{1,1} y_{2,2} \mathbf{K} 8 s t x[ ] x_2 x_{1,1} x_{1,2} x_{2,2} y_2 \\
& y_{1,2}^2 \mathbf{K} 8 s t x[ ] x_2 x_{1,1} x_{1,2} y_1 y_{1,2}^2 y_{2,2} \mathbf{C} 8 s t x[ ] x_2 x_{1,1} x_{1,2} y_2 y_{1,1} y_{1,2} y_{2,2} \\
& \mathbf{K} 4 s t x[ ] x_2 x_{1,1} x_{2,2} y_1 y_{1,1} y_{1,2} y_{2,2} \mathbf{C} 8 s t x[ ] x_2 x_{1,1} x_{2,2} y_1 y_{1,2}^3 \\
& \mathbf{C} 4 s t x[ ] x_2 x_{1,1} x_{2,2} y_2 y_{1,1}^2 y_{2,2} \mathbf{K} 8 s t x[ ] x_2 x_{1,1} x_{2,2} y_2 y_{1,1} y_{1,2}^2 \mathbf{K} 8 s t x[ ] x_2 \\
& x_{1,2}^3 y_2 y_{1,1} y_{2,2} \mathbf{C} 8 s t x[ ] x_2 x_{1,2}^2 x_{2,2} y_2 y_{1,1} y_{1,2} \mathbf{C} 8 s t x[ ] x_2 x_{1,2}^2 y_1 y_{1,1} y_{1,2} y_{2,2} \\
& \mathbf{K} 8 s t x[ ] x_2 x_{1,2}^2 y_2 y_{1,1}^2 y_{2,2} \mathbf{K} 2 s t x[ ] x_2 x_{1,2}^2 y_2 y_{1,1}^2 \\
& \mathbf{K} 8 s t x[ ] x_2 x_{1,2} x_{2,2} y_1 y_{1,1} y_{1,2}^2 \mathbf{C} 8 s t x[ ] x_2 x_{1,2} x_{2,2} y_2 y_{1,1}^2 y_{1,2} \mathbf{C} 2 s t x[ ] x_2 \\
& x_{2,2}^2 y_1 y_{1,1}^2 y_{1,2} \mathbf{K} 2 s t x[ ] x_2 x_{2,2}^2 y_2 y_{1,1}^3 \mathbf{K} 2 s t x[ ] x_{1,1}^2 x_{1,2} y_1 y_2 y_{2,2}^2 \mathbf{C} 2 s t x[ ] \\
& x_{1,1}^2 x_{2,2} y_1^2 y_{2,2}^2 \mathbf{C} 8 s t x[ ] x_{1,1} x_{1,2}^2 y_1 y_2 y_{1,2} y_{2,2} \mathbf{K} 8 s t x[ ] x_{1,1} x_{1,2} x_{2,2} y_1^2 y_{1,2} y_{2,2} \\
& \mathbf{C} 4 s t x[ ] x_{1,1} x_{1,2} x_{2,2} y_1 y_2 y_{1,1} y_{2,2} \mathbf{K} 8 s t x[ ] x_{1,1} x_{1,2} x_{2,2} y_1 y_2 y_{1,2}^2
\end{aligned}$$

$$\begin{aligned}
& \mathbf{K} 4 stx[ ]x_{1,1}x_{2,2}^2y_{1,1}^2y_{2,2} \mathbf{C} 8 stx[ ]x_{1,1}x_{2,2}^2y_{1,1}^2y_{1,2}^2 \mathbf{K} 8 stx[ ] \\
& x_{1,2}^3y_1y_2y_{1,1}y_{2,2} \mathbf{C} 8 stx[ ]x_{1,2}^2x_{2,2}y_{1,1}^2y_{1,2} \mathbf{C} 8 stx[ ]x_{1,2}^2x_{2,2}y_1y_2y_{1,1}y_{1,2} \\
& \mathbf{K} 8 stx[ ]x_{1,2}x_{2,2}^2y_{1,1}^2y_{1,2} \mathbf{K} 2 stx[ ]x_{1,2}x_{2,2}^2y_1y_2y_{1,1}^2 \mathbf{C} 2 stx[ ]x_{2,2}^3y_{1,1}^2y_{1,2}^2 \\
& \mathbf{C} 2 stx_1x_2x_{1,1}^2y[ ]y_{1,2}y_{2,2}^2 \mathbf{K} 8 stx_1x_2x_{1,1}x_{1,2}y[ ]y_{1,2}^2y_{2,2} \\
& \mathbf{K} 4 stx_1x_2x_{1,1}x_{2,2}y[ ]y_{1,1}y_{1,2}y_{2,2} \mathbf{C} 8 stx_1x_2x_{1,1}x_{2,2}y[ ]y_{1,2}^3 \mathbf{C} 8 stx_1x_2 \\
& x_{1,2}^2y[ ]y_{1,1}y_{1,2}y_{2,2} \mathbf{K} 8 stx_1x_2x_{1,2}x_{2,2}y[ ]y_{1,1}y_{1,2}^2 \mathbf{C} 2 stx_1x_2x_{2,2}^2y[ ]y_{1,1}^2y_{1,2} \\
& \mathbf{K} 4 stx_1x_{1,1}^2x_{1,2}y[ ]y_{2,2}^2 \mathbf{C} 2 stx_1x_{1,1}^2x_{2,2}y[ ]y_{1,2}^2 \mathbf{C} 2 stx_1x_{1,1}^2y[ ]y_1y_{1,2} \\
& y_{2,2}^2 \mathbf{K} 2 stx_1x_{1,1}^2y[ ]y_2y_{1,1}y_{2,2}^2 \mathbf{C} 16 stx_1x_{1,1}x_{1,2}^2y[ ]y_2y_{1,2}y_{2,2} \\
& \mathbf{K} 8 stx_1x_{1,1}x_{1,2}x_{2,2}y[ ]y_1y_{1,2}y_{2,2} \mathbf{C} 8 stx_1x_{1,1}x_{1,2}x_{2,2}y[ ]y_2y_{1,1}y_{2,2} \\
& \mathbf{K} 16 stx_1x_{1,1}x_{1,2}x_{2,2}y[ ]y_2y_{1,2}^2 \mathbf{K} 8 stx_1x_{1,1}x_{1,2}y[ ]y_1y_{1,2}^2y_{2,2} \\
& \mathbf{C} 8 stx_1x_{1,1}x_{1,2}y[ ]y_2y_{1,1}y_{1,2}y_{2,2} \mathbf{K} 4 stx_1x_{1,1}x_{2,2}^2y[ ]y_1y_{1,1}y_{2,2} \\
& \mathbf{C} 8 stx_1x_{1,1}x_{2,2}^2y[ ]y_1y_{1,2}^2 \mathbf{K} 4 stx_1x_{1,1}x_{2,2}y[ ]y_1y_{1,1}y_{1,2}y_{2,2} \\
& \mathbf{C} 8 stx_1x_{1,1}x_{2,2}y[ ]y_1y_{1,2}^3 \mathbf{C} 4 stx_1x_{1,1}x_{2,2}y[ ]y_2y_{1,1}y_{2,2} \\
& \mathbf{K} 8 stx_1x_{1,1}x_{2,2}y[ ]y_2y_{1,1}y_{1,2}^2 \mathbf{K} 16 stx_1x_{1,2}^3y[ ]y_2y_{1,1}y_{2,2} \mathbf{C} 8 stx_1 \\
& x_{1,2}^2x_{2,2}y[ ]y_1y_{1,1}y_{2,2} \mathbf{C} 16 stx_1x_{1,2}^2x_{2,2}y[ ]y_2y_{1,1}y_{1,2} \mathbf{C} 8 stx_1 \\
& x_{1,2}^2y[ ]y_1y_{1,1}y_{1,2}y_{2,2} \mathbf{K} 8 stx_1x_{1,2}^2y[ ]y_2y_{1,1}y_{2,2} \mathbf{K} 8 stx_1x_{1,2} \\
& x_{2,2}^2y[ ]y_1y_{1,1}y_{1,2} \mathbf{K} 4 stx_1x_{1,2}x_{2,2}^2y[ ]y_2y_{1,1}^2 \mathbf{K} 8 stx_1x_{1,2}x_{2,2}y[ ]y_1y_{1,1}y_{1,2}^2 \\
& \mathbf{C} 8 stx_1x_{1,2}x_{2,2}y[ ]y_2y_{1,1}^2y_{1,2} \mathbf{C} 2 stx_1x_{2,2}^3y[ ]y_1y_{1,1}^2 \mathbf{C} 2 stx_1x_{2,2}^2y[ ]y_1 \\
& y_{1,1}^2y_{1,2} \mathbf{K} 2 stx_1x_{2,2}^2y[ ]y_2y_{1,1}^3 \mathbf{K} 2 stx_2^2x_{1,1}^2y[ ]y_{1,1}y_{2,2}^2 \mathbf{C} 8 st \\
& x_2^2x_{1,1}x_{1,2}y[ ]y_{1,1}y_{1,2}y_{2,2} \mathbf{C} 4 stx_2^2x_{1,1}x_{2,2}y[ ]y_{1,1}^2y_{2,2} \mathbf{K} 8 st \\
& x_2^2x_{1,1}x_{2,2}y[ ]y_{1,1}y_{1,2}^2 \mathbf{K} 8 stx_2^2x_{1,2}^2y[ ]y_{1,1}^2y_{2,2} \mathbf{C} 8 stx_2^2x_{1,2}x_{2,2}y[ ]y_{1,1}^2y_{1,2} \\
& \mathbf{K} 2 stx_2^2x_{2,2}^2y[ ]y_{1,1}^3 \mathbf{C} 4 stx_2x_{1,1}^3y[ ]y_2y_{2,2}^2 \mathbf{K} 2 stx_2x_{1,1}^2x_{1,2}y[ ]y_{1,2}^2y_{2,2} \\
& \mathbf{K} 16 stx_2x_{1,1}^2x_{1,2}y[ ]y_2y_{1,2}y_{2,2} \mathbf{K} 8 stx_2x_{1,1}^2x_{2,2}y[ ]y_2y_{1,1}y_{2,2} \mathbf{C} 16 stx_2 \\
& x_{1,1}^2x_{2,2}y[ ]y_2y_{1,2}^2 \mathbf{C} 8 stx_2x_{1,1}x_{1,2}^2y[ ]y_1y_{1,2}y_{2,2} \mathbf{C} 16 stx_2x_{1,1} \\
& x_{1,2}^2y[ ]y_2y_{1,1}y_{2,2} \mathbf{C} 4 stx_2x_{1,1}x_{1,2}x_{2,2}y[ ]y_1y_{1,1}y_{2,2} \\
& \mathbf{K} 8 stx_2x_{1,1}x_{1,2}x_{2,2}y[ ]y_1y_{1,2}^2 \mathbf{K} 16 stx_2x_{1,1}x_{1,2}x_{2,2}y[ ]y_2y_{1,1}y_{1,2} \\
& \mathbf{C} 4 stx_2x_{1,1}x_{2,2}^2y[ ]y_2y_{1,1}^2 \mathbf{K} 8 stx_2x_{1,2}^3y[ ]y_1y_{1,1}y_{2,2} \mathbf{C} 8 stx_2 \\
& x_{1,2}^2x_{2,2}y[ ]y_1y_{1,1}y_{1,2} \mathbf{K} 2 stx_2x_{1,2}x_{2,2}^2y[ ]y_1y_{1,1}^2 \mathbf{C} 2 stx_{1,1}^3y[ ]y_1y_2y_{2,2}^2 \\
& \mathbf{K} 2 stx_{1,1}^2x_{1,2}y[ ]y_{1,2}^2y_{2,2}^2 \mathbf{K} 8 stx_{1,1}^2x_{1,2}y[ ]y_1y_2y_{1,2}y_{2,2} \mathbf{K} 4 st \\
& x_{1,1}^2x_{2,2}y[ ]y_1y_2y_{1,1}y_{2,2} \mathbf{C} 8 stx_{1,1}^2x_{2,2}y[ ]y_1y_2y_{1,2}^2 \mathbf{C} 8 stx_{1,1}x_{1,2}^2y[ ] \\
& y_{1,2}^2y_{1,2}y_{2,2} \mathbf{C} 8 stx_{1,1}x_{1,2}^2y[ ]y_1y_2y_{1,1}y_{2,2} \mathbf{C} 4 stx_{1,1}x_{1,2}x_{2,2}y[ ]y_{1,2}^2y_{1,1}y_{2,2} \\
& \mathbf{K} 8 stx_{1,1}x_{1,2}x_{2,2}y[ ]y_{1,2}^2y_{1,2}^2 \mathbf{K} 8 stx_{1,1}x_{1,2}x_{2,2}y[ ]y_1y_2y_{1,1}y_{1,2} \mathbf{C} 2 stx_{1,1}
\end{aligned}$$

$$\begin{aligned}
& x_{2,2}^2 y[ ] y_1 y_2 y_{1,1}^2 K 8 s t x_{1,2}^3 y[ ] y_1^2 y_{1,1} y_2 C 8 s t x_{1,2}^2 x_{2,2} y[ ] y_1^2 y_{1,1} y_1 y_2 \\
& K 2 s t x_{1,2} x_{2,2}^2 y[ ] y_1^2 y_{1,1}^2 K 3 t^2 x[ ] x_1^2 x_{1,1}^2 y_{2,2}^3 C 12 t^2 x[ ] x_1^2 x_{1,1} x_{1,2} y_{1,2} y_{2,2}^2 \\
& C 6 t^2 x[ ] x_1^2 x_{1,1} x_{2,2} y_{1,1} y_{2,2}^2 K 12 t^2 x[ ] x_1^2 x_{1,1} x_{2,2} y_{1,2} y_{2,2} K 12 t^2 x[ ] x_1^2 \\
& x_{1,2}^2 y_{1,1} y_{2,2}^2 C 12 t^2 x[ ] x_1^2 x_{1,2} x_{2,2} y_{1,1} y_{1,2} y_{2,2} K 3 t^2 x[ ] x_1^2 x_{2,2}^2 y_{1,1}^2 y_{2,2} \\
& C 4 t^2 x[ ] x_1 x_2 x_{1,1}^2 y_{1,2} y_{2,2}^2 K 16 t^2 x[ ] x_1 x_2 x_{1,1} x_{1,2} y_{1,2}^2 y_{2,2} \\
& K 8 t^2 x[ ] x_1 x_2 x_{1,1} x_{2,2} y_{1,1} y_{1,2} y_{2,2} C 16 t^2 x[ ] x_1 x_2 x_{1,1} x_{2,2} y_{1,2}^3 C 16 t^2 x[ ] x_1 x_2 \\
& x_{1,2}^2 y_{1,1} y_{1,2} y_{2,2} K 16 t^2 x[ ] x_1 x_2 x_{1,2} x_{2,2} y_{1,1} y_{1,2}^2 C 4 t^2 x[ ] x_1 x_2 x_{2,2}^2 y_{1,1}^2 y_{1,2} \\
& C 2 t^2 x[ ] x_1 x_{1,1}^2 x_{2,2} y_1 y_{2,2}^2 K 8 t^2 x[ ] x_1 x_{1,1} x_{1,2} x_{2,2} y_1 y_{1,2} y_{2,2} K 4 t^2 x[ ] x_1 x_{1,1} \\
& x_{2,2}^2 y_1 y_{1,1} y_{2,2} C 8 t^2 x[ ] x_1 x_{1,1} x_{2,2}^2 y_1 y_{1,2}^2 C 8 t^2 x[ ] x_1 x_{1,2}^2 x_{2,2} y_1 y_{1,1} y_{2,2} \\
& K 8 t^2 x[ ] x_1 x_{1,2} x_{2,2}^2 y_1 y_{1,1} y_{1,2} C 2 t^2 x[ ] x_1 x_{2,2}^3 y_1 y_{1,1}^2 K t^2 x[ ] x_{2,2}^2 x_{1,1}^2 y_{1,1} y_{2,2}^2 \\
& C 4 t^2 x[ ] x_{2,2}^2 x_{1,1} x_{1,2} y_{1,1} y_{1,2} y_{2,2} C 2 t^2 x[ ] x_{2,2}^2 x_{1,1} x_{2,2} y_{1,1}^2 y_{2,2} K 4 t^2 x[ ] \\
& x_{2,2}^2 x_{1,1} x_{2,2} y_{1,1} y_{1,2}^2 K 4 t^2 x[ ] x_{2,2}^2 x_{1,1}^2 y_{1,1}^2 y_{2,2} C 4 t^2 x[ ] x_{2,2}^2 x_{1,2} x_{2,2} y_{1,1}^2 y_{1,2} \\
& K t^2 x[ ] x_{2,2}^2 x_{2,2}^2 y_{1,1}^3 K 2 t^2 x[ ] x_{2,2}^2 x_{1,1}^2 x_{1,2} y_1 y_{2,2}^2 C 8 t^2 x[ ] x_{2,2} x_{1,1} x_{1,2}^2 y_1 y_{1,2} y_{2,2} \\
& C 4 t^2 x[ ] x_{2,2} x_{1,1} x_{1,2} x_{2,2} y_1 y_{1,1} y_{2,2} K 8 t^2 x[ ] x_{2,2} x_{1,1} x_{1,2} x_{2,2} y_1 y_{1,2}^2 K 8 t^2 x[ ] x_{2,2} \\
& x_{1,2}^3 y_1 y_{1,1} y_{2,2} C 8 t^2 x[ ] x_{2,2} x_{1,2}^2 x_{2,2} y_1 y_{1,1} y_{1,2} K 2 t^2 x[ ] x_{2,2} x_{1,2} x_{2,2}^2 y_1 y_{1,1}^2 C t^2 x_{1,1}^2 \\
& x_{1,1}^2 x_{2,2} y[ ] y_{2,2}^2 C 2 t^2 x_{1,1}^2 x_{1,1}^2 y[ ] y_{1,2} y_{2,2}^2 K 4 t^2 x_{1,1}^2 x_{1,1} x_{1,2} x_{2,2} y[ ] y_{1,2} y_{2,2} K 8 t^2 \\
& x_{1,1}^2 x_{1,1} x_{1,2} y[ ] y_{1,2} y_{2,2} K 2 t^2 x_{1,1}^2 x_{1,1} x_{2,2} y[ ] y_{1,1} y_{2,2} C 4 t^2 x_{1,1}^2 x_{1,1} x_{2,2}^2 y[ ] y_{1,2}^2 \\
& K 4 t^2 x_{1,1}^2 x_{1,1} x_{2,2} y[ ] y_{1,1} y_{1,2} y_{2,2} C 8 t^2 x_{1,1}^2 x_{1,1} x_{2,2} y[ ] y_{1,2}^3 C 4 t^2 x_{1,1}^2 \\
& x_{1,2}^2 x_{2,2} y[ ] y_{1,1} y_{2,2} C 8 t^2 x_{1,1}^2 x_{1,1}^2 y[ ] y_{1,1} y_{1,2} y_{2,2} K 4 t^2 x_{1,1}^2 x_{1,2} x_{2,2}^2 y[ ] y_{1,1} y_{1,2} \\
& K 8 t^2 x_{1,1}^2 x_{1,2} x_{2,2} y[ ] y_{1,1} y_{1,2}^2 C t^2 x_{1,1}^2 x_{2,2}^3 y[ ] y_{1,1}^2 C 2 t^2 x_{1,1}^2 x_{2,2}^2 y[ ] y_{1,1}^2 y_{1,2} \\
& K 2 t^2 x_1 x_2 x_{1,1}^2 x_{1,2} y[ ] y_{2,2}^2 K 2 t^2 x_1 x_2 x_{1,1}^2 y[ ] y_{1,1} y_{2,2}^2 C 8 t^2 x_1 x_2 x_{1,1} \\
& x_{1,2}^2 y[ ] y_{1,2} y_{2,2} C 4 t^2 x_1 x_2 x_{1,1} x_{1,2} x_{2,2} y[ ] y_{1,1} y_{2,2} K 8 t^2 x_1 x_2 x_{1,1} x_{1,2} x_{2,2} y[ ] \\
& y_{1,2}^2 C 8 t^2 x_1 x_2 x_{1,1} x_{1,2} y[ ] y_{1,1} y_{1,2} y_{2,2} C 4 t^2 x_1 x_2 x_{1,1} x_{2,2} y[ ] y_{1,1}^2 y_{2,2} \\
& K 8 t^2 x_1 x_2 x_{1,1} x_{2,2} y[ ] y_{1,1} y_{1,2}^2 K 8 t^2 x_1 x_2 x_{1,1}^2 y[ ] y_{1,1} y_{2,2} C 8 t^2 x_1 x_2 \\
& x_{1,2}^2 x_{2,2} y[ ] y_{1,1} y_{1,2} K 8 t^2 x_1 x_2 x_{1,1}^2 y[ ] y_{1,1}^2 y_{2,2} K 2 t^2 x_1 x_2 x_{1,2} x_{2,2}^2 y[ ] y_{1,1}^2 \\
& C 8 t^2 x_1 x_2 x_{1,2} x_{2,2} y[ ] y_{1,1}^2 y_{1,2} K 2 t^2 x_1 x_2 x_{2,2}^2 y[ ] y_{1,1}^3 K 2 t^2 x_1 x_{1,1}^2 x_{1,2} y[ ] y_{1,1} \\
& y_{2,2}^2 C 8 t^2 x_1 x_{1,1} x_{1,2}^2 y[ ] y_{1,1} y_{1,2} y_{2,2} C 4 t^2 x_1 x_{1,1} x_{1,2} x_{2,2} y[ ] y_{1,1} y_{1,1} y_{2,2} \\
& K 8 t^2 x_1 x_{1,1} x_{1,2} x_{2,2} y[ ] y_{1,1} y_{1,2}^2 K 8 t^2 x_1 x_{1,1}^3 y[ ] y_{1,1} y_{1,1} y_{2,2} C 8 t^2 x_1 \\
& x_{1,2}^2 x_{2,2} y[ ] y_{1,1} y_{1,1} y_{1,2} K 2 t^2 x_1 x_{1,2} x_{2,2}^2 y[ ] y_{1,1} y_{1,1}^2 C t^2 x_{2,2}^3 x_{1,1}^2 y[ ] y_{2,2}^2 K 4 t^2 x_{2,2}^2 \\
& x_{1,1}^2 x_{1,1} x_{1,2} y[ ] y_{1,2} y_{2,2} K 2 t^2 x_{2,2}^2 x_{1,1}^2 x_{2,2} y[ ] y_{1,1} y_{2,2} C 4 t^2 x_{2,2}^2 x_{1,1}^2 x_{2,2} y[ ] y_{1,2}^2 C 4 t^2 \\
& x_{2,2}^2 x_{1,1} x_{1,2}^2 y[ ] y_{1,1} y_{1,2} K 4 t^2 x_{2,2}^2 x_{1,1} x_{1,2} x_{2,2} y[ ] y_{1,1} y_{1,2} C t^2 x_{2,2}^2 x_{1,1} x_{2,2}^2 y[ ] y_{1,1}^2 \\
& C 2 t^2 x_{2,2}^3 x_{1,1} y[ ] y_{1,1} y_{2,2}^2 K 8 t^2 x_{2,2}^2 x_{1,1} x_{1,2} y[ ] y_{1,1} y_{1,2} y_{2,2} K 4 t^2 x_{2,2}^2
\end{aligned}$$

$$\begin{aligned}
& x_{1,1}^2 x_{2,2} y[ ] y_1 y_{1,1} y_{2,2} \mathbf{C} 8 t^2 x_2 x_{1,1}^2 x_{2,2} y[ ] y_1 y_{1,1}^2 \mathbf{C} 8 t^2 x_2 x_{1,1} x_{1,2}^2 y[ ] y_1 y_{1,1} y_{2,2} \\
& \mathbf{K} 8 t^2 x_2 x_{1,1} x_{1,2} x_{2,2} y[ ] y_1 y_{1,1} y_{1,2} \mathbf{C} 2 t^2 x_2 x_{1,1} x_{2,2}^2 y[ ] y_1 y_{1,1}^2 \mathbf{C} s x[ ]^2 x_2 x_{1,1}^2 \\
& y_{2,2}^3 \mathbf{K} 4 s x[ ]^2 x_2 x_{1,1} x_{1,2} y_{1,2} y_{2,2}^2 \mathbf{K} 2 s x[ ]^2 x_2 x_{1,1} x_{2,2} y_{1,1} y_{2,2}^2 \\
& \mathbf{C} 4 s x[ ]^2 x_2 x_{1,1} x_{2,2} y_{1,2}^2 y_{2,2} \mathbf{C} 4 s x[ ]^2 x_2 x_{1,2}^2 y_{1,1} y_{2,2}^2 \\
& \mathbf{K} 4 s x[ ]^2 x_2 x_{1,2} x_{2,2} y_{1,1} y_{1,2} y_{2,2} \mathbf{C} s x[ ]^2 x_2 x_{2,2}^2 y_{1,1} y_{2,2} \mathbf{K} s x[ ]^2 x_{1,1}^2 x_{2,2} y_2 y_{2,2}^2 \\
& \mathbf{C} 2 s x[ ]^2 x_{1,1}^2 y_1 y_{2,2}^3 \mathbf{K} 2 s x[ ]^2 x_{1,1}^2 y_2 y_{1,2} y_{2,2}^2 \mathbf{C} 4 s x[ ]^2 x_{1,1} x_{1,2} x_{2,2} y_2 y_{1,2} y_{2,2} \\
& \mathbf{K} 8 s x[ ]^2 x_{1,1} x_{1,2} y_1 y_{1,2} y_{2,2}^2 \mathbf{C} 8 s x[ ]^2 x_{1,1} x_{1,2} y_2 y_{1,2}^2 y_{2,2} \mathbf{C} 2 s x[ ]^2 x_{1,1} \\
& x_{2,2}^2 y_2 y_{1,1} y_{2,2} \mathbf{K} 4 s x[ ]^2 x_{1,1} x_{2,2}^2 y_2 y_{1,2}^2 \mathbf{K} 4 s x[ ]^2 x_{1,1} x_{2,2} y_1 y_{1,1} y_{2,2}^2 \\
& \mathbf{C} 8 s x[ ]^2 x_{1,1} x_{2,2} y_1 y_{1,2}^2 y_{2,2} \mathbf{C} 4 s x[ ]^2 x_{1,1} x_{2,2} y_2 y_{1,1} y_{1,2} y_{2,2} \\
& \mathbf{K} 8 s x[ ]^2 x_{1,1} x_{2,2} y_2 y_{1,2}^3 \mathbf{K} 4 s x[ ]^2 x_{1,2}^2 x_{2,2} y_2 y_{1,1} y_{2,2} \mathbf{C} 8 s x[ ]^2 x_{1,2}^2 y_1 y_{1,1} y_{2,2}^2 \\
& \mathbf{K} 8 s x[ ]^2 x_{1,2}^2 y_2 y_{1,1} y_{1,2} y_{2,2} \mathbf{C} 4 s x[ ]^2 x_{1,2} x_{2,2}^2 y_2 y_{1,1} y_{1,2} \\
& \mathbf{K} 8 s x[ ]^2 x_{1,2} x_{2,2} y_1 y_{1,1} y_{1,2} y_{2,2} \mathbf{C} 8 s x[ ]^2 x_{1,2} x_{2,2} y_2 y_{1,1} y_{1,2}^2 \mathbf{K} s x[ ]^2 x_{2,2}^3 y_2 \\
& y_{1,1}^2 \mathbf{C} 2 s x[ ]^2 x_{2,2}^2 y_1 y_{1,1}^2 y_{2,2} \mathbf{K} 2 s x[ ]^2 x_{2,2}^2 y_2 y_{1,1}^2 y_{1,2} \mathbf{K} 2 s x[ ] x_2 x_{1,1}^2 y[ ] y_{1,2} \\
& y_{2,2}^2 \mathbf{C} 8 s x[ ] x_2 x_{1,1} x_{1,2} y[ ] y_{1,2}^2 y_{2,2} \mathbf{C} 4 s x[ ] x_2 x_{1,1} x_{2,2} y[ ] y_{1,1} y_{1,2} y_{2,2} \\
& \mathbf{K} 8 s x[ ] x_2 x_{1,1} x_{2,2} y[ ] y_{1,2}^3 \mathbf{K} 8 s x[ ] x_2 x_{1,2}^2 y[ ] y_{1,1} y_{1,2} y_{2,2} \\
& \mathbf{C} 8 s x[ ] x_2 x_{1,2} x_{2,2} y[ ] y_{1,1} y_{1,2}^2 \mathbf{K} 2 s x[ ] x_2 x_{2,2}^2 y[ ] y_{1,1} y_{1,2} \mathbf{C} 4 s x[ ] \\
& x_{1,1}^2 x_{1,2} y[ ] y_2 y_{2,2}^2 \mathbf{K} 2 s x[ ] x_{1,1}^2 x_{2,2} y[ ] y_1 y_{2,2}^2 \mathbf{K} 2 s x[ ] x_{1,1}^2 y[ ] y_1 y_{1,2} y_{2,2}^2 \\
& \mathbf{C} 2 s x[ ] x_{1,1}^2 y[ ] y_2 y_{1,1} y_{2,2}^2 \mathbf{K} 16 s x[ ] x_{1,1} x_{1,2}^2 y[ ] y_2 y_{1,2} y_{2,2} \\
& \mathbf{C} 8 s x[ ] x_{1,1} x_{1,2} x_{2,2} y[ ] y_1 y_{1,2} y_{2,2} \mathbf{K} 8 s x[ ] x_{1,1} x_{1,2} x_{2,2} y[ ] y_2 y_{1,1} y_{2,2} \\
& \mathbf{C} 16 s x[ ] x_{1,1} x_{1,2} x_{2,2} y[ ] y_2 y_{1,2}^2 \mathbf{C} 8 s x[ ] x_{1,1} x_{1,2} y[ ] y_1 y_{1,2}^2 y_{2,2} \\
& \mathbf{K} 8 s x[ ] x_{1,1} x_{1,2} y[ ] y_2 y_{1,1} y_{1,2} y_{2,2} \mathbf{C} 4 s x[ ] x_{1,1} x_{2,2}^2 y[ ] y_1 y_{1,1} y_{2,2} \\
& \mathbf{K} 8 s x[ ] x_{1,1} x_{2,2}^2 y[ ] y_1 y_{1,2}^2 \mathbf{C} 4 s x[ ] x_{1,1} x_{2,2} y[ ] y_1 y_{1,1} y_{1,2} y_{2,2} \\
& \mathbf{K} 8 s x[ ] x_{1,1} x_{2,2} y[ ] y_1 y_{1,2}^3 \mathbf{K} 4 s x[ ] x_{1,1} x_{2,2} y[ ] y_2 y_{1,1}^2 y_{2,2} \\
& \mathbf{C} 8 s x[ ] x_{1,1} x_{2,2} y[ ] y_2 y_{1,1} y_{1,2}^2 \mathbf{C} 16 s x[ ] x_{1,2}^3 y[ ] y_2 y_{1,1} y_{2,2} \mathbf{K} 8 s x[ ] \\
& x_{1,2}^2 x_{2,2} y[ ] y_1 y_{1,1} y_{2,2} \mathbf{K} 16 s x[ ] x_{1,2}^2 x_{2,2} y[ ] y_2 y_{1,1} y_{1,2} \mathbf{K} 8 s x[ ] \\
& x_{1,2}^2 y[ ] y_1 y_{1,1} y_{1,2} y_{2,2} \mathbf{C} 8 s x[ ] x_{1,2}^2 y[ ] y_2 y_{1,1}^2 y_{2,2} \mathbf{C} 8 s x[ ] x_{1,2} \\
& x_{2,2}^2 y[ ] y_1 y_{1,1} y_{1,2} \mathbf{C} 4 s x[ ] x_{1,2} x_{2,2}^2 y[ ] y_2 y_{1,1}^2 \mathbf{C} 8 s x[ ] x_{1,2} x_{2,2} y[ ] y_1 y_{1,1} y_{1,2}^2 \\
& \mathbf{K} 8 s x[ ] x_{1,2} x_{2,2} y[ ] y_2 y_{1,1}^2 y_{1,2} \mathbf{K} 2 s x[ ] x_{2,2}^3 y[ ] y_1 y_{1,1}^2 \mathbf{K} 2 s x[ ] x_{2,2}^2 y[ ] y_1 \\
& y_{1,1}^2 y_{1,2} \mathbf{C} 2 s x[ ] x_{2,2}^2 y[ ] y_2 y_{1,1}^3 \mathbf{C} s x_2 x_{1,1}^2 y[ ]^2 y_{1,1} y_{2,2}^2 \\
& \mathbf{K} 4 s x_2 x_{1,1} x_{1,2} y[ ]^2 y_{1,1} y_{1,2} y_{2,2} \mathbf{K} 2 s x_2 x_{1,1} x_{2,2} y[ ]^2 y_{1,1}^2 y_{2,2} \\
& \mathbf{C} 4 s x_2 x_{1,1} x_{2,2} y[ ]^2 y_{1,1} y_{1,2}^2 \mathbf{C} 4 s x_2 x_{1,2}^2 y[ ]^2 y_{1,1}^2 y_{2,2} \mathbf{K} 4 s x_2 x_{1,2} x_{2,2} y[ ]^2 \\
& y_{1,1}^2 y_{1,2} \mathbf{C} s x_2 x_{2,2}^2 y[ ]^2 y_{1,1}^3 \mathbf{K} 3 s x_{1,1}^3 y[ ]^2 y_2 y_{2,2}^2 \mathbf{C} 2 s x_{1,1}^2 x_{1,2} y[ ]^2 y_1 y_{2,2}^2 \mathbf{C} 12 s
\end{aligned}$$

$$\begin{aligned}
& x_{1,1}^2 x_{1,2} y [ ]^2 y_2 y_{1,2} y_{2,2} \mathbf{C} 6 s x_{1,1}^2 x_{2,2} y [ ]^2 y_2 y_{1,1} y_{2,2} \mathbf{K} 12 s x_{1,1}^2 x_{2,2} y [ ]^2 y_2 y_{1,2}^2 \\
& \mathbf{K} 8 s x_{1,1} x_{1,2}^2 y [ ]^2 y_1 y_{1,2} y_{2,2} \mathbf{K} 12 s x_{1,1} x_{1,2}^2 y [ ]^2 y_2 y_{1,1} y_{2,2} \\
& \mathbf{K} 4 s x_{1,1} x_{1,2} x_{2,2} y [ ]^2 y_1 y_{1,1} y_{2,2} \mathbf{C} 8 s x_{1,1} x_{1,2} x_{2,2} y [ ]^2 y_1 y_{1,2}^2 \\
& \mathbf{C} 12 s x_{1,1} x_{1,2} x_{2,2} y [ ]^2 y_2 y_{1,1} y_{1,2} \mathbf{K} 3 s x_{1,1} x_{2,2}^2 y [ ]^2 y_2 y_{1,1}^2 \mathbf{C} 8 s \\
& x_{1,2}^3 y [ ]^2 y_1 y_{1,1} y_{2,2} \mathbf{K} 8 s x_{1,2}^2 x_{2,2} y [ ]^2 y_1 y_{1,1} y_{1,2} \mathbf{C} 2 s x_{1,2} x_{2,2}^2 y [ ]^2 y_1 y_{1,1}^2 \\
& \mathbf{C} 3 t x [ ]^2 x_1 x_{1,1}^2 y_{2,2}^3 \mathbf{K} 12 t x [ ]^2 x_1 x_{1,1} x_{1,2} y_{1,2} y_{2,2}^2 \mathbf{K} 6 t x [ ]^2 x_1 x_{1,1} x_{2,2} y_{1,1} y_{2,2}^2 \\
& \mathbf{C} 12 t x [ ]^2 x_1 x_{1,1} x_{2,2} y_{1,2}^2 y_{2,2} \mathbf{C} 12 t x [ ]^2 x_1 x_{1,2}^2 y_{1,1} y_{2,2}^2 \\
& \mathbf{K} 12 t x [ ]^2 x_1 x_{1,2} x_{2,2} y_{1,1} y_{1,2} y_{2,2} \mathbf{C} 3 t x [ ]^2 x_1 x_{2,2}^2 y_{1,1}^2 y_{2,2} \mathbf{K} 2 t x [ ]^2 x_2 x_{1,1}^2 y_{1,2} \\
& y_{2,2}^2 \mathbf{C} 8 t x [ ]^2 x_2 x_{1,1} x_{1,2} y_{1,2}^2 y_{2,2} \mathbf{C} 4 t x [ ]^2 x_2 x_{1,1} x_{2,2} y_{1,1} y_{1,2} y_{2,2} \\
& \mathbf{K} 8 t x [ ]^2 x_2 x_{1,1} x_{2,2} y_{1,2}^3 \mathbf{K} 8 t x [ ]^2 x_2 x_{1,2}^2 y_{1,1} y_{1,2} y_{2,2} \mathbf{C} 8 t x [ ]^2 x_2 x_{1,2} x_{2,2} y_{1,1} \\
& y_{1,2}^2 \mathbf{K} 2 t x [ ]^2 x_2 x_{2,2}^2 y_{1,1}^2 y_{1,2} \mathbf{K} t x [ ]^2 x_{1,1}^2 x_{2,2} y_1 y_{2,2}^2 \\
& \mathbf{C} 4 t x [ ]^2 x_{1,1} x_{1,2} x_{2,2} y_1 y_{1,2} y_{2,2} \mathbf{C} 2 t x [ ]^2 x_{1,1} x_{2,2}^2 y_1 y_{1,1} y_{2,2} \mathbf{K} 4 t x [ ]^2 x_{1,1} \\
& x_{2,2}^2 y_1 y_{1,2}^2 \mathbf{K} 4 t x [ ]^2 x_{1,2}^2 x_{2,2} y_1 y_{1,1} y_{2,2} \mathbf{C} 4 t x [ ]^2 x_{1,2} x_{2,2}^2 y_1 y_{1,1} y_{1,2} \mathbf{K} t x [ ]^2 \\
& x_{2,2}^3 y_1 y_{1,1}^2 \mathbf{K} 2 t x [ ] x_1 x_{1,1}^2 x_{2,2} y [ ] y_{2,2}^2 \mathbf{K} 4 t x [ ] x_1 x_{1,1}^2 y [ ] y_{1,2} y_{2,2}^2 \\
& \mathbf{C} 8 t x [ ] x_1 x_{1,1} x_{1,2} x_{2,2} y [ ] y_{1,2} y_{2,2} \mathbf{C} 16 t x [ ] x_1 x_{1,1} x_{1,2} y [ ] y_{1,2}^2 y_{2,2} \\
& \mathbf{C} 4 t x [ ] x_1 x_{1,1} x_{2,2}^2 y [ ] y_{1,1} y_{2,2} \mathbf{K} 8 t x [ ] x_1 x_{1,1} x_{2,2}^2 y [ ] y_{1,2}^2 \\
& \mathbf{C} 8 t x [ ] x_1 x_{1,1} x_{2,2} y [ ] y_{1,1} y_{1,2} y_{2,2} \mathbf{K} 16 t x [ ] x_1 x_{1,1} x_{2,2} y [ ] y_{1,2}^3 \mathbf{K} 8 t x [ ] x_1 \\
& x_{1,2}^2 x_{2,2} y [ ] y_{1,1} y_{2,2} \mathbf{K} 16 t x [ ] x_1 x_{1,2}^2 y [ ] y_{1,1} y_{1,2} y_{2,2} \mathbf{C} 8 t x [ ] x_1 x_{1,2} \\
& x_{2,2}^2 y [ ] y_{1,1} y_{1,2} \mathbf{C} 16 t x [ ] x_1 x_{1,2} x_{2,2} y [ ] y_{1,1} y_{1,2}^2 \mathbf{K} 2 t x [ ] x_1 x_{2,2}^3 y [ ] y_{1,1}^2 \\
& \mathbf{K} 4 t x [ ] x_1 x_{2,2}^2 y [ ] y_{1,1}^2 y_{1,2} \mathbf{C} 2 t x [ ] x_2 x_{1,1}^2 x_{1,2} y [ ] y_{2,2}^2 \mathbf{C} 2 t x [ ] x_2 \\
& x_{1,1}^2 y [ ] y_{1,1} y_{2,2}^2 \mathbf{K} 8 t x [ ] x_2 x_{1,1} x_{1,2}^2 y [ ] y_{1,2} y_{2,2} \\
& \mathbf{K} 4 t x [ ] x_2 x_{1,1} x_{1,2} x_{2,2} y [ ] y_{1,1} y_{2,2} \mathbf{C} 8 t x [ ] x_2 x_{1,1} x_{1,2} x_{2,2} y [ ] y_{1,2}^2 \\
& \mathbf{K} 8 t x [ ] x_2 x_{1,1} x_{1,2} y [ ] y_{1,1} y_{1,2} y_{2,2} \mathbf{K} 4 t x [ ] x_2 x_{1,1} x_{2,2} y [ ] y_{1,1}^2 y_{2,2} \\
& \mathbf{C} 8 t x [ ] x_2 x_{1,1} x_{2,2} y [ ] y_{1,1} y_{1,2}^2 \mathbf{C} 8 t x [ ] x_2 x_{1,2}^3 y [ ] y_{1,1} y_{2,2} \mathbf{K} 8 t x [ ] x_2 \\
& x_{1,2}^2 x_{2,2} y [ ] y_{1,1} y_{1,2} \mathbf{C} 8 t x [ ] x_2 x_{1,2}^2 y [ ] y_{1,1}^2 y_{2,2} \mathbf{C} 2 t x [ ] x_2 x_{1,2} x_{2,2}^2 y [ ] y_{1,1}^2 \\
& \mathbf{K} 8 t x [ ] x_2 x_{1,2} x_{2,2} y [ ] y_{1,1}^2 y_{1,2} \mathbf{C} 2 t x [ ] x_2 x_{2,2}^2 y [ ] y_{1,1}^3 \mathbf{C} 2 t x [ ] \\
& x_{1,1}^2 x_{1,2} y [ ] y_1 y_{2,2}^2 \mathbf{K} 8 t x [ ] x_{1,1} x_{1,2}^2 y [ ] y_1 y_{1,2} y_{2,2} \\
& \mathbf{K} 4 t x [ ] x_{1,1} x_{1,2} x_{2,2} y [ ] y_1 y_{1,1} y_{2,2} \mathbf{C} 8 t x [ ] x_{1,1} x_{1,2} x_{2,2} y [ ] y_1 y_{1,2}^2 \mathbf{C} 8 t x [ ] \\
& x_{1,2}^3 y [ ] y_1 y_{1,1} y_{2,2} \mathbf{K} 8 t x [ ] x_{1,2}^2 x_{2,2} y [ ] y_1 y_{1,1} y_{1,2} \mathbf{C} 2 t x [ ] x_{1,2} x_{2,2}^2 y [ ] y_1 y_{1,1}^2 \\
& \mathbf{C} 2 t x_1 x_{1,1}^2 x_{1,2} y [ ]^2 y_{2,2}^2 \mathbf{C} t x_1 x_{1,1}^2 y [ ]^2 y_{1,1} y_{2,2}^2 \mathbf{K} 8 t x_1 x_{1,1} x_{1,2}^2 y [ ]^2 y_{1,2} y_{2,2} \\
& \mathbf{K} 4 t x_1 x_{1,1} x_{1,2} x_{2,2} y [ ]^2 y_{1,1} y_{2,2} \mathbf{C} 8 t x_1 x_{1,1} x_{1,2} x_{2,2} y [ ]^2 y_{1,2}^2 \\
& \mathbf{K} 4 t x_1 x_{1,1} x_{1,2} y [ ]^2 y_{1,1} y_{1,2} y_{2,2} \mathbf{K} 2 t x_1 x_{1,1} x_{2,2} y [ ]^2 y_{1,1}^2 y_{2,2}
\end{aligned}$$

$$\begin{aligned}
& C 4 t x_1 x_{1,1} x_{2,2} y [ ]^2 y_{1,1} y_{1,2}^2 C 8 t x_1 x_{1,2}^3 y [ ]^2 y_{1,1} y_{2,2} K 8 t x_1 \\
& x_{1,2}^2 x_{2,2} y [ ]^2 y_{1,1} y_{1,2} C 4 t x_1 x_{1,2}^2 y [ ]^2 y_{1,1} y_{2,2} C 2 t x_1 x_{1,2} x_{2,2}^2 y [ ]^2 y_{1,1}^2 \\
& K 4 t x_1 x_{1,2} x_{2,2} y [ ]^2 y_{1,1}^2 y_{1,2} C t x_1 x_{2,2}^2 y [ ]^2 y_{1,1}^3 K 2 t x_2 x_{1,1}^3 y [ ]^2 y_{2,2}^2 C 8 t x_2 \\
& x_{1,1}^2 x_{1,2} y [ ]^2 y_{1,2} y_{2,2} C 4 t x_2 x_{1,1}^2 x_{2,2} y [ ]^2 y_{1,1} y_{2,2} K 8 t x_2 x_{1,1}^2 x_{2,2} y [ ]^2 y_{1,2}^2 \\
& K 8 t x_2 x_{1,1} x_{1,2}^2 y [ ]^2 y_{1,1} y_{2,2} C 8 t x_2 x_{1,1} x_{1,2} x_{2,2} y [ ]^2 y_{1,1} y_{1,2} K 2 t x_2 x_{1,1} \\
& x_{2,2}^2 y [ ]^2 y_{1,1}^2 K t x_{1,1}^3 y [ ]^2 y_{1,1} y_{2,2}^2 C 4 t x_{1,1}^2 x_{1,2} y [ ]^2 y_{1,1} y_{1,2} y_{2,2} C 2 t \\
& x_{1,1}^2 x_{2,2} y [ ]^2 y_{1,1} y_{1,2} y_{2,2} K 4 t x_{1,1}^2 x_{2,2} y [ ]^2 y_{1,1} y_{1,2}^2 K 4 t x_{1,1} x_{1,2}^2 y [ ]^2 y_{1,1} y_{1,2} y_{2,2} \\
& C 4 t x_{1,1} x_{1,2} x_{2,2} y [ ]^2 y_{1,1} y_{1,2} y_{2,2} K t x_{1,1} x_{2,2}^2 y [ ]^2 y_{1,1} y_{1,2}^2 K x [ ]^3 x_{1,1}^2 y_{2,2}^3 \\
& C 4 x [ ]^3 x_{1,1} x_{1,2} y_{1,2} y_{2,2}^2 C 2 x [ ]^3 x_{1,1} x_{2,2} y_{1,1} y_{2,2}^2 K 4 x [ ]^3 x_{1,1} x_{2,2} y_{1,2} y_{2,2}^2 \\
& K 4 x [ ]^3 x_{1,2}^2 y_{1,1} y_{2,2}^2 C 4 x [ ]^3 x_{1,2} x_{2,2} y_{1,1} y_{1,2} y_{2,2} K x [ ]^3 x_{2,2}^2 y_{1,1} y_{2,2} C x [ ]^2 \\
& x_{1,1}^2 x_{2,2} y [ ] y_{2,2}^2 C 2 x [ ]^2 x_{1,1}^2 y [ ] y_{1,2} y_{2,2}^2 K 4 x [ ]^2 x_{1,1} x_{1,2} x_{2,2} y [ ] y_{1,2} y_{2,2} \\
& K 8 x [ ]^2 x_{1,1} x_{1,2} y [ ] y_{1,2}^2 y_{2,2} K 2 x [ ]^2 x_{1,1} x_{2,2}^2 y [ ] y_{1,1} y_{2,2} C 4 x [ ]^2 x_{1,1} \\
& x_{2,2}^2 y [ ] y_{1,2}^2 K 4 x [ ]^2 x_{1,1} x_{2,2} y [ ] y_{1,1} y_{1,2} y_{2,2} C 8 x [ ]^2 x_{1,1} x_{2,2} y [ ] y_{1,2}^3 C 4 x [ ]^2 \\
& x_{1,2}^2 x_{2,2} y [ ] y_{1,1} y_{2,2} C 8 x [ ]^2 x_{1,2}^2 y [ ] y_{1,1} y_{1,2} y_{2,2} K 4 x [ ]^2 x_{1,2} x_{2,2}^2 y [ ] y_{1,1} y_{1,2} \\
& K 8 x [ ]^2 x_{1,2} x_{2,2} y [ ] y_{1,1} y_{1,2}^2 C x [ ]^2 x_{2,2}^3 y [ ] y_{1,1}^2 C 2 x [ ]^2 x_{2,2}^2 y [ ] y_{1,1}^2 y_{1,2} \\
& K 2 x [ ] x_{1,1}^2 x_{1,2} y [ ]^2 y_{2,2}^2 K x [ ] x_{1,1}^2 y [ ]^2 y_{1,1} y_{2,2}^2 C 8 x [ ] x_{1,1} x_{1,2}^2 y [ ]^2 y_{1,2} y_{2,2} \\
& C 4 x [ ] x_{1,1} x_{1,2} x_{2,2} y [ ]^2 y_{1,1} y_{2,2} K 8 x [ ] x_{1,1} x_{1,2} x_{2,2} y [ ]^2 y_{1,2}^2 \\
& C 4 x [ ] x_{1,1} x_{1,2} y [ ]^2 y_{1,1} y_{1,2} y_{2,2} C 2 x [ ] x_{1,1} x_{2,2} y [ ]^2 y_{1,1}^2 y_{2,2} \\
& K 4 x [ ] x_{1,1} x_{2,2} y [ ]^2 y_{1,1} y_{1,2}^2 K 8 x [ ] x_{1,2}^3 y [ ]^2 y_{1,1} y_{2,2} C 8 x [ ] \\
& x_{1,2}^2 x_{2,2} y [ ]^2 y_{1,1} y_{1,2} K 4 x [ ] x_{1,2}^2 y [ ]^2 y_{1,1}^2 y_{2,2} K 2 x [ ] x_{1,2} x_{2,2}^2 y [ ]^2 y_{1,1}^2 \\
& C 4 x [ ] x_{1,2} x_{2,2} y [ ]^2 y_{1,1}^2 y_{1,2} K x [ ] x_{2,2}^2 y [ ]^2 y_{1,1}^3 C x_{1,1}^3 y [ ]^3 y_{2,2}^2 K 4 \\
& x_{1,1}^2 x_{1,2} y [ ]^3 y_{1,2} y_{2,2} K 2 x_{1,1}^2 x_{2,2} y [ ]^3 y_{1,1} y_{2,2} C 4 x_{1,1}^2 x_{2,2} y [ ]^3 y_{1,2}^2 C 4 x_{1,1} \\
& x_{1,2}^2 y [ ]^3 y_{1,1} y_{2,2} K 4 x_{1,1} x_{1,2} x_{2,2} y [ ]^3 y_{1,1} y_{1,2} C x_{1,1} x_{2,2}^2 y [ ]^3 y_{1,1}^2 y_{1,2} ) / \\
& \left( 4 \left( s^2 x_{1,1} y_1 y_2 y_{2,2} K s^2 x_{1,1} y_{2,2}^2 y_{1,2} K s^2 x_{1,2} y_{1,2}^2 y_{2,2} C s^2 x_{1,2} y_{2,2}^2 y_{1,1} C s^2 x_{2,2} \right. \right. \\
& y_{1,2}^2 y_{1,2} K s^2 x_{2,2} y_1 y_2 y_{1,1} C s t x_1 x_{1,1} y_2 y_{2,2} K 2 s t x_1 x_{1,2} y_1 y_{2,2} C 2 s t x_1 x_{2,2} y_1 y_{1,2} \\
& K s t x_1 x_{2,2} y_2 y_{1,1} C s t x_2 x_{1,1} y_1 y_{2,2} K 2 s t x_2 x_{1,1} y_2 y_{1,2} C 2 s t x_2 x_{1,2} y_2 y_{1,1} \\
& K s t x_2 x_{2,2} y_1 y_{1,1} K t^2 x_1^2 x_{1,2} y_{2,2} C t^2 x_1^2 x_{2,2} y_{1,2} C t^2 x_1 x_2 x_{1,1} y_{2,2} \\
& K t^2 x_1 x_2 x_{2,2} y_{1,1} K t^2 x_2^2 x_{1,1} y_{1,2} C t^2 x_2^2 x_{1,2} y_{1,1} K s x [ ] x_{1,1} y_2 y_{2,2} \\
& C 2 s x [ ] x_{1,2} y_1 y_{2,2} K 2 s x [ ] x_{2,2} y_1 y_{1,2} C s x [ ] x_{2,2} y_2 y_{1,1} K s x_{1,1} y [ ] y_1 y_{2,2} \\
& C 2 s x_{1,1} y [ ] y_2 y_{1,2} K 2 s x_{1,2} y [ ] y_2 y_{1,1} C s x_{2,2} y [ ] y_1 y_{1,1} C 2 t x [ ] x_1 x_{1,2} y_{2,2}
\end{aligned}$$

$$\begin{aligned}
& \mathbb{K}^2 t x[ ] x_1 x_{2,2} y_{1,2} \mathbb{K} t x[ ] x_2 x_{1,1} y_{2,2} \mathbb{C} t x[ ] x_2 x_{2,2} y_{1,1} \mathbb{K} t x_1 x_{1,1} y[ ] y_{2,2} \\
& \mathbb{C} t x_1 x_{2,2} y[ ] y_{1,1} \mathbb{C}^2 t x_2 x_{1,1} y[ ] y_{1,2} \mathbb{K}^2 t x_2 x_{1,2} y[ ] y_{1,1} \mathbb{K} x[ ]^2 x_{1,2} y_{2,2} \\
& \mathbb{C} x[ ]^2 x_{2,2} y_{1,2} \mathbb{C} x[ ] x_{1,1} y[ ] y_{2,2} \mathbb{K} x[ ] x_{2,2} y[ ] y_{1,1} \mathbb{K} x_{1,1} y[ ]^2 y_{1,2} \\
& \mathbb{C} x_{1,2} y[ ]^2 y_{1,1} \Big)^2 (x_2 \mathbb{K} y_1) \Big)
\end{aligned}$$