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A Stochastic Intelligent Computing with Neuro-Evolution Heuristics for Nonlinear Sitr System of Novel COVID-19 Dynamics

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Abstract: The present study aims to design stochastic intelligent computational heuristics for the numerical treatment of a nonlinear Sitr system representing the dynamics of novel coronavirus disease 2019 (COVID-19). The mathematical Sitr system using fractal parameters for COVID-19 dynamics is divided into four classes; that is, susceptible (S), infected (I), treatment (T), and recovered (R). The comprehensive details of each class along with the explanation of every parameter are provided, and the dynamics of novel COVID-19 are represented by calculating the solution of the mathematical Sitr system using feed-forward artificial neural networks (FF-ANNs) trained with global search genetic algorithms (GAs) and speedy fine tuning by sequential quadratic programming (SQP)—that is, an FF-ANN-GASQP scheme. In the proposed FF-ANN-GASQP method, the objective function is formulated in the mean squared error sense using the approximate differential mapping of FF-ANNs for the Sitr model, and learning of the networks is proficiently conducted with the integrated capabilities of GA and SQP. The correctness, stability, and potential of the proposed FF-ANN-GASQP scheme for the four different cases are established through comparative assessment study from the results of numerical computing with Adams solver for single as well as multiple autonomous trials. The results of statistical evaluations further authenticate the convergence and prospective accuracy of the FF-ANN-GASQP method.

Keywords: coronavirus; Sitr model; artificial neural networks; numerical Adams results; genetic algorithm; diseases; sequential quadratic programming; treatment

1. Introduction

Human life has faced various challenges and obstacles throughout time in the form of floods, earthquakes, and diseases, among others. We first mention some widespread diseases such as dengue fever, which is a mosquito-borne disease produced by the dengue virus. This disease spread extensively

in the Africa, South and Central America, Eastern Mediterranean, Caribbean, Southeast and South Asia, and Oceania regions. It is a very serious disease, and almost half a million people infected with dengue virus are hospitalized every year, although most of the patients recover after a few days. Some major symptoms of this disease are high fever, vomiting, headache, and joint and muscle pains. Ebola, HIV, and Lassa are other infectious diseases of paramount interest to the research community concerning safeguarding humanity. Therefore, several medical utilities have been unified by humans to prevent the propagation of these diseases.

Recently, the world has suffered the dangerous and deathly disease of coronavirus disease 2019 (COVID-19), which is a respiratory infection spreading in an almost uncontrollable manner. COVID-19 is a disease that is highly transferable from one individual to another through droplets [1]. Many people have been infected with COVID-19, with the rate of death as a result of this virus increasing daily. The deathly COVID-19 was reported for the first time on 31 December 2019 in Wuhan city, which is in the Hubei province of China [2]. Currently, there is no particular treatment or vaccine for COVID-19, and it has become a big task for all countries to control it. Many people were killed in Wuhan, China by COVID-19, which led the Chinese government to lock down the infected cities as well as put the diseased people under quarantine. The number of infected cases is in the thousands in developing countries, and the reported deaths in advanced countries such as the United States of America, Italy, Spain, England, and Russia have overtaken those in China. The key symptoms of this virus are dry cough, fever, and tiredness. COVID-19 badly affects older people (around the age of 60 or older), as well as individuals facing serious medical diseases including cardiovascular disease, diabetes, chronic respiratory, and cancer. Those who have fever, breathing difficulty, and dry cough should seek medical care immediately. The spread ratio of COVID-19 is very problematic and has become a worry for the whole world. To present the mathematical model is another task, while predicting the outcomes of this disease is very frustrating as well. One of the main challenges is how to control the disease if it spreads to animals or birds. To prevent this virus from causing further damage, the proper vaccine should be discovered; otherwise, the world should be ready to face many new challenges such as human loss, shortage of food, unemployment, poverty, and many more.

The aim of the present investigation is to find the solution to a nonlinear SITR system using parameters for the dynamics of novel COVID-19, along with numerical analysis for a better understanding of the spread using stochastic approaches through the exploitation of artificial intelligence algorithms. The substantial potential of neuro-evolution-based computing is exploited/explored for linear and nonlinear systems by the functioning of neural networks trained with the competences of local and global search schemes [3–7]. A few recent studies using soft computing schemes include the singular three-point boundary value model [8], financial market forecasting [9], the Thomas–Fermi model [10], summer precipitation prediction for meteorological stations [11], nonlinear prey–predator models [12], nonlinear models of fuel ignition models on Bratu’s differential equation [13], the nonlinear Troesch’s problem [14], magneto hydrodynamic studies [15], the nonlinear functional differential model [16,17], the model of heartbeat dynamics [18], fluid dynamics [19], the corneal shape model [20], nonlinear models arising in electric circuits [21], multi-singularity based nonlinear models [22], the nonlinear reactive transport model [23], the singular periodic nonlinear model [24], the heat distribution model of human head based on a nonlinear singular system [25], the atomic physics model [26], the nonlinear Riccati equation [27], and the singular Lane–Emden model [28]. Recently, fractional order systems have also been solved with artificial intelligence (AI) techniques [29–33]. These provide the motivation to investigate AI methodologies for the numerical solution of the mathematical model of SITR representing the dynamics of COVID-19.

The SITR model for novel COVID-19 dynamics is designed on the mechanism of four classes: susceptible class (S), infected class (I), treatment class (T), and recovered class (R). The S is further segmented into two sub-classes $S_1(\chi)$ and $S_2(\chi)$. The sub-class $S_1(\chi)$ specifies the individuals not yet affected from COVID-19 disease and sub-class $S_2(\chi)$ represents those not yet infected but who have some serious illnesses or older. $I(\chi)$ shows the class of those people who are infected by COVID-19

disease, $T(\chi)$ is the treatment class, and the last class $R(\chi)$ is the recovered class from COVID-19 disease. The Sitr model governing the novel COVID-19 dynamics is given as follows [34]:

$$\begin{cases} S_1'(\chi) = B - \beta I(\chi)S_1(\chi) - \delta\beta T(\chi) - \alpha S_1(\chi), & S_1(0) = A_1, \\ S_2'(\chi) = B - \beta I(\chi)S_2(\chi) - \delta\beta T(\chi) - \alpha S_2(\chi), & S_2(0) = A_2, \\ I'(\chi) = -\mu I(\chi) + \beta I(\chi)(S_1(\chi) + S_2(\chi)) - \alpha I(\chi) + \beta\delta T(\chi) + \sigma I(\chi), & I(0) = A_3, \\ T'(\chi) = \mu I(\chi) - \rho T(\chi) - \alpha T(\chi) + \psi T(\chi) + \varepsilon T(\chi), & T(0) = A_4, \\ R'(\chi) = -\alpha R(\chi) + \rho T(\chi), & R(0) = A_5, \end{cases} \quad (1)$$

The objective of the proposed study is to find the solution of the system of equations given in set (1) by integrating intelligent neuro-evolution computing heuristics via feed-forward artificial neural networks (FF-ANNs), and its adjustable parameters are tuned with an optimization procedure of genetic algorithms (GAs) integrated with sequential quadratic programming (SQP), i.e., FF-ANN-GASQP solver. The parameter of interest with appropriate settings for particular descriptions of the dynamics are provided in Tables 1 and 2, while the block diagram of the proposed study is given in Figure 1, which presents the system model, mathematical formulation, proposed methodology, evaluation criteria, and main results in pictorial form.

The major features of the suggested FF-ANN-GASQP scheme are briefly given as:

- The solution of the mathematical expression for the nonlinear Sitr model for novel COVID-19 dynamics is calculated viably by using the novel application of the intelligent neuro-evolution-based integrated computing paradigm, i.e., FF-ANN-GASQP.
- Closely matching of the results of the proposed FF-ANN-GASQP solver with the solutions of the reference state of the art numerical procedure of Adams methods for variants of the nonlinear Sitr-based mathematical model established the value and worth.
- Authentication and verification of the performance through statistical assessments studies is proven on multiple implementations of FF-ANN-GASQP in terms of Theil's inequality coefficient (TIC) as well as root mean square error (RMSE)-based indices.
- In addition to the precise and accurate solutions for the Sitr-based mathematical model of the COVID-19 pandemic, other valuable perks are that it is easy to understand the concepts, and it also has smooth operation, exhaustive applicability, consistency, and extendibility.

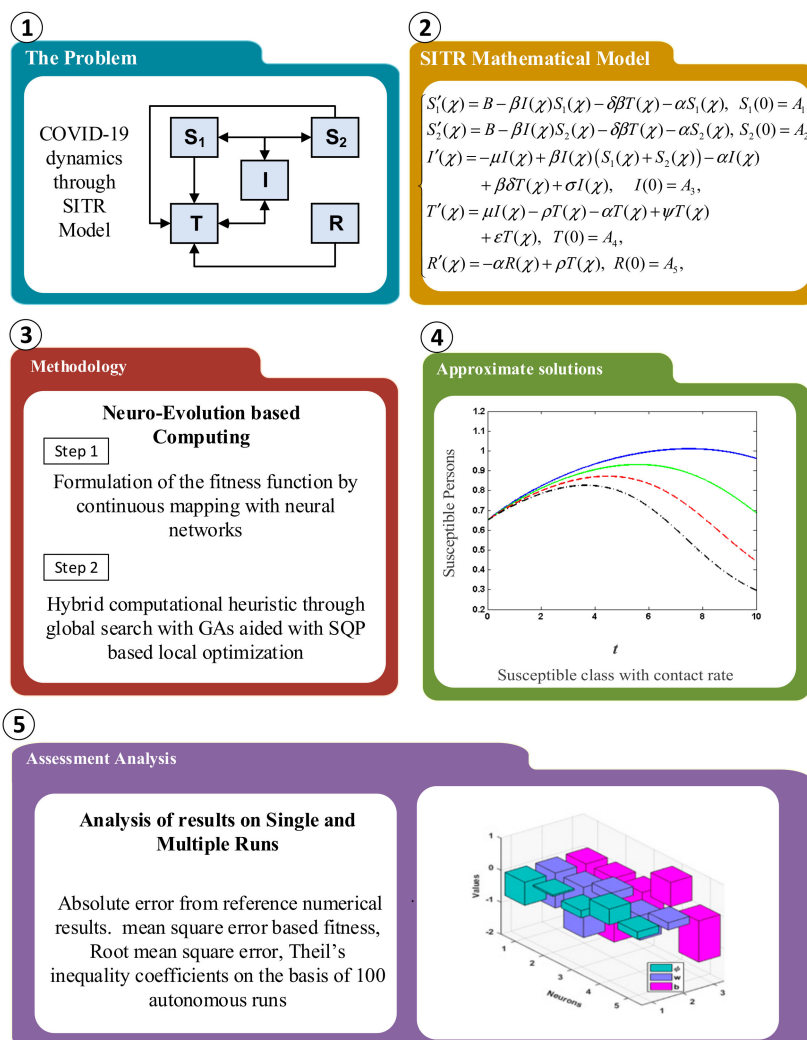
The remaining sections of the current study are organized as follows. The designed framework of FF-ANN-GASQP is given in Section 2, measures of the performance are listed in Section 3, a description of the results with necessary interpretations are provided in Section 4, and concluding remarks with related potential studies are given in the last section.

Table 1. State variables for the dynamics of the Sitr model.

Variable	Description
$S_1(\chi)$	Non-infected individuals
$S_2(\chi)$	Non-infected older or major diseased people
$I(\chi)$	Rate of infected from COVID-19
$R(\chi)$	Recovery rate from COVID-19
$T(\chi)$	Treatment

Table 2. Descriptions of the state variables based on the nonlinear Sitr model.

Parameter	Description
β	Contact rate
B	Rate of natural birth
δ	Reduce infection from treatment
σ	Fever, tiredness and dry cough rate
μ	Recovery rate
α	Death rate
ρ	Rate of infection from treatment
ψ	Healthy food rate
ε	Sleep rate
$A_j, j = 1, 2, 3, 4, 5$	Initial conditions

**Figure 1.** Process workflow of proposed FF-ANN-GASQP structure for Sitr model based on COVID-19. FF-ANN-GASQP: feed-forward artificial neural networks trained with global search genetic algorithms and speedy fine tuning by sequential quadratic programming.

2. Proposed Methodology

ANN is an artificial intelligence-based computing approach that is used widely to effectively solve a variety of challenging stiff linear/nonlinear problems in terms of reliability, stability, and robustness. In this study, the strength of feed-forward ANN is exploited to solve the Sitr model representing novel COVID-19. The weights of ANN are optimized with a genetic algorithms (GA)-based evolutionary metaheuristic along with sequential quadratic programming (SQP). The proposed FF-ANN-GASQP structure for getting the numerical solutions of the nonlinear Sitr model based on COVID-19 is described in two phases:

- To exploit the FF-ANN models, an error-based objective function is introduced.
- Optimize the objective function for system (1) using the hybrid GA-SQ programming approach.

2.1. ANN Modeling

The mathematical measures using system (1) are given by the differential mapping of FF-ANN for approximate solutions $\hat{S}_1(\chi)$, $\hat{S}_2(\chi)$, $\hat{I}(\chi)$, $\hat{T}(\chi)$ and $\hat{R}(\chi)$ along with their n th order derivatives, which are given as:

$$\begin{bmatrix} \hat{S}_1(\chi) & \hat{S}_2(\chi) \\ \hat{I}(\chi) & \hat{T}(\chi) \\ \hat{R}(\chi) & \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^m \phi_{S_1,i} P(w_{S_1,i}\chi + b_{S_1,i}) & \sum_{i=1}^m \phi_{S_2,i} P(w_{S_2,i}\chi + b_{S_2,i}) \\ \sum_{i=1}^m \phi_{I,i} P(w_{I,i}\chi + b_{I,i}) & \sum_{i=1}^m \phi_{T,i} P(w_{T,i}\chi + b_{T,i}) \\ \sum_{i=1}^m \phi_{R,i} P(w_{R,i}\chi + b_{R,i}) & \end{bmatrix}, \quad (2)$$

$$\begin{bmatrix} \hat{S}_1^{(n)}(\chi) & \hat{S}_2^{(n)}(\chi) \\ \hat{I}^{(n)}(\chi) & \hat{T}^{(n)}(\chi) \\ \hat{R}^{(n)}(\chi) & \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^m \phi_{S_1,i} P^{(n)}(w_{S_1,i}\chi + b_{S_1,i}) & \sum_{i=1}^m \phi_{S_2,i} P^{(n)}(w_{S_2,i}\chi + b_{S_2,i}) \\ \sum_{i=1}^m \phi_{I,i} P^{(n)}(w_{I,i}\chi + b_{I,i}) & \sum_{i=1}^m \phi_{T,i} P^{(n)}(w_{T,i}\chi + b_{T,i}) \\ \sum_{i=1}^m \phi_{R,i} P^{(n)}(w_{R,i}\chi + b_{R,i}) & \end{bmatrix},$$

where the unknown weight vector is W and given as:

$$W = [W_{S_1}, W_{S_2}, W_I, W_T, W_R], \text{ for } W_{S_1} = [\phi_{S_1}, w_{S_1}, b_{S_1}], W_{S_2} = [\phi_{S_2}, w_{S_2}, b_{S_2}], \\ W_I = [\phi_I, w_I, b_I], W_T = [\phi_T, w_T, b_T] \text{ and } W_R = [\phi_R, w_R, b_R]$$

where

$$\begin{aligned} \phi_{S_1} &= [\phi_{S_1,1}, \phi_{S_1,2}, \phi_{S_1,3}, \dots, \phi_{S_1,m}], & \phi_{S_2} &= [\phi_{S_2,1}, \phi_{S_2,2}, \phi_{S_2,3}, \dots, \phi_{S_2,m}], & \phi_I &= [\phi_{I,1}, \phi_{I,2}, \phi_{I,3}, \dots, \phi_{I,m}], \\ \phi_T &= [\phi_{T,1}, \phi_{T,2}, \phi_{T,3}, \dots, \phi_{T,m}], & \phi_R &= [\phi_{R,1}, \phi_{R,2}, \phi_{R,3}, \dots, \phi_{R,m}], & \omega_{S_1} &= [w_{S_1,1}, w_{S_1,2}, w_{S_1,3}, \dots, w_{S_1,m}], \\ \omega_{S_2} &= [w_{S_2,1}, w_{S_2,2}, w_{S_2,3}, \dots, w_{S_2,m}], & \omega_I &= [w_{I,1}, w_{I,2}, w_{I,3}, \dots, w_{I,m}], & \omega_T &= [w_{T,1}, w_{T,2}, w_{T,3}, \dots, w_{T,m}], \\ \phi_R &= [w_{R,1}, w_{R,2}, w_{R,3}, \dots, w_{R,m}], & b_{S_1} &= [b_{S_1,1}, b_{S_1,2}, b_{S_1,3}, \dots, b_{S_1,m}], & b_{S_2} &= [b_{S_2,1}, b_{S_2,2}, b_{S_2,3}, \dots, b_{S_2,m}], \\ b_I &= [b_{I,1}, b_{I,2}, b_{I,3}, \dots, b_{I,m}], & b_T &= [b_{T,1}, b_{T,2}, b_{T,3}, \dots, b_{T,m}], & b_R &= [b_{R,1}, b_{R,2}, b_{R,3}, \dots, b_{R,m}]. \end{aligned}$$

Using the log-sigmoid, i.e., an function activation $P(\chi) = (1 + e^{-\chi})^{-1}$, the updated form of system (2) becomes:

$$\begin{bmatrix} \hat{S}_1(\chi), \hat{S}_2(\chi), \hat{I}(\chi), \\ \hat{R}(\chi), \hat{T}(\chi) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^m \frac{\phi_{S1,i}}{1+e^{-(w_{S1,i}\chi+b_{S1,i})}}, \sum_{i=1}^m \frac{\phi_{S2,i}}{1+e^{-(w_{S2,i}\chi+b_{S2,i})}}, \sum_{i=1}^m \frac{\phi_{I,i}}{1+e^{-(w_{I,i}\chi+b_{I,i})}}, \\ \sum_{i=1}^m \frac{\phi_{T,i}}{1+e^{-(w_{T,i}\chi+b_{T,i})}}, \sum_{i=1}^m \frac{\phi_{R,i}}{1+e^{-(w_{R,i}\chi+b_{R,i})}} \end{bmatrix}, \quad (3)$$

$$\begin{bmatrix} \hat{S}'_1(\chi), \hat{S}'_2(\chi), \\ \hat{I}'(\chi), \hat{R}'(\chi), \\ \hat{T}'(\chi) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^m \frac{w_{S1,i}\phi_{E,i}e^{-(\rho_{S1,i}\chi+b_{S1,i})}}{(1+e^{-(w_{S1,i}\chi+b_{S1,i})})^2}, \sum_{i=1}^m \frac{w_{S2,i}\phi_{S2,i}e^{-(w_{S2,i}\chi+b_{S2,i})}}{(1+e^{-(w_{S2,i}\chi+b_{S2,i})})^2}, \sum_{i=1}^m \frac{w_{I,i}\phi_{I,i}e^{-(w_{I,i}\chi+b_{I,i})}}{(1+e^{-(w_{I,i}\chi+b_{I,i})})^2}, \\ \sum_{i=1}^m \frac{w_{T,i}\phi_{T,i}e^{-(w_{T,i}\chi+b_{T,i})}}{(1+e^{-(w_{T,i}\chi+b_{T,i})})^2}, \sum_{i=1}^m \frac{w_{R,i}\phi_{A,i}e^{-(w_{R,i}\chi+b_{R,i})}}{(1+e^{-(w_{R,i}\chi+b_{R,i})})^2} \end{bmatrix}.$$

Using the values of system (3), an error-based objective function e is given as:

$$e = e_1 + e_2 + e_3 + e_4 + e_5 + e_6. \quad (4)$$

$$e_1 = \frac{1}{N} \sum_{m=1}^N ((\hat{S}'_1)_m - B + \beta I_m (\hat{S}_1)_m + \delta \beta T_m + \alpha (\hat{S}_1)_m)^2, \quad (5)$$

$$e_2 = \frac{1}{N} \sum_{m=1}^N ((\hat{S}'_2)_m - B + \beta I_m (\hat{S}_2)_m + \delta \beta T_m + \alpha (\hat{S}_2)_m)^2, \quad (6)$$

$$e_3 = \frac{1}{N} \sum_{m=1}^N (\hat{I}'_m + \mu I_m - \beta I_m ((\hat{S}_1)_m + (\hat{S}_2)_m) + \alpha I_m - \beta \delta T_m - \sigma I_m)^2, \quad (7)$$

$$e_4 = \frac{1}{N} \sum_{m=1}^N (\hat{T}'_m - \mu I_m + \rho \hat{T}_m + \alpha \hat{T}_m - \psi \hat{T}_m - \varepsilon \hat{T}_m)^2, \quad (8)$$

$$e_5 = \frac{1}{N} \sum_{m=1}^N (\hat{R}'_m + \alpha \hat{R}_m - \rho \hat{T}_m)^2, \quad (9)$$

$$e_6 = \frac{1}{5} ((\hat{S}_1)_0 - A_1)^2 + ((\hat{S}_2)_0 - A_2)^2 + (\hat{I}_0 - A_3)^2 + (\hat{T}_0 - A_4)^2 + (\hat{R}_0 - I_5)^2, \quad (10)$$

where $Nh = 1$, $\chi_m = mh$, $(\hat{S}_1)_m = \hat{S}_1(\chi_m)$, $(\hat{S}_2)_m = \hat{S}_2(\chi_m)$, $\hat{I}_m = \hat{I}(\chi_m)$, $\hat{T}_m = \hat{T}(\chi_m)$ and $\hat{R}_m = \hat{R}(\chi_m)$. The approximate results for both sub-classes of susceptible S_1 and S_2 , infected class I , treatment class T , and recovered class R are denoted as $\hat{S}_1$, $\hat{S}_2$, $\hat{I}$, $\hat{T}$, and $\hat{R}$. Accordingly, e_1 , e_2 , e_3 , e_4 , e_5 , and e_6 are the error-based functions for system dynamics and initial conditions of the Sitr model representing COVID-19 as given in the set of Equation in (1). The proposed results are achieved from the adjustable weights for which the objective function as shown in system (4) tends to zero, i.e., mathematically $e \rightarrow 0$. Hence, approximate solutions $[\hat{S}_1(\chi), \hat{S}_2(\chi), \hat{I}(\chi), \hat{T}(\chi), \hat{R}(\chi)]$ are matched with the exact results, i.e., $[\hat{S}_1(\chi) \rightarrow S_1(\chi)]$, $[\hat{S}_2(\chi) \rightarrow S_2(\chi)]$, $[\hat{I}(\chi) \rightarrow I(\chi)]$, $[\hat{T}(\chi) \rightarrow T(\chi)]$ and $[\hat{R}(\chi) \rightarrow R(\chi)]$.

2.2. Optimization Technique: Hybrid of GA with SQP

In this section, a brief overview of GASQP is given to optimize networks for system (4) representing COVID-19 dynamics.

GAs is an efficient global search optimization approach, which was introduced by Holland [30,31]. GA is one of the mathematical models of human genetics, which is implemented as a global search tool for linear/nonlinear optimization processes in assorted domains of applied sciences. Population in GAs represents the set of possible solutions as per the obligation of the optimizing systems, while the populations are modified through the exploitation of selection, crossover, elitism, and mutation such that an appropriate outcome is achieved. Some well-known applications of GA in various fields include multilayer piezoelectric transducer [32–37], Hammerstein controlled autoregressive models [38],

nonlinear circuits [39], economic load dispatch [40], active noise control system [41], and power signal modeling [42].

The GA optimization procedure is further enhanced normally using the hybridization process with the speedy local search approach, so in this study, SQP is applied for a fine-tuning mechanism. The GA optimization procedure is further enhanced normally using the hybridization process with the speedy local search approach, so in this study, SQP is applied for nonlinear models in combustion theory [43], Bagley–Torvik systems [44], and system identification [45]. In the presented investigation, the integrated metaheuristics through GASQP is exploited to optimize the unknown weights of FF-ANN models, which are demonstrating the Sitr model based on COVID-19. The pseudocode detail of FF-ANN-GASQP for a nonlinear Sitr system is given in Table 3.

Table 3. Pseudocode of FF-ANN-GASQP for nonlinear Sitr system for COVID-19 dynamics.

Start the GA process Inputs: The individuals with genes equally representing the decision values of FF-ANN as: $W = [W_{S_1}, W_{S_2}, W_I, W_T, W_R]$, where $W_{S_1} = [\phi_{S_1}, w_{S_1}, b_{S_1}]$, $W_{S_2} = [\phi_{S_2}, w_{S_2}, b_{S_2}]$, $W_I = [\phi_I, w_I, b_I]$, $W_T = [\phi_T, w_T, b_T]$ and $W_R = [\phi_R, w_R, b_R]$ as per the details provided in the system (3). Population: Number of chromosomes in a set define a population as: $P = [W_1, W_2, W_3, \dots, W_n]^t$, for ith component $W_i = [W_{S_1,i}, W_{S_2,i}, W_{I,i}, W_{T,i}, W_{R,i}]$ with $W_{S_1,i} = [\phi_{S_1,i}, w_{S_1,i}, b_{S_1,i}]$, $W_{S_2,i} = [\phi_{S_2,i}, w_{S_2,i}, b_{S_2,i}]$, $W_{I,i} = [\phi_{I,i}, w_{I,i}, b_{I,i}]$, $W_{T,i} = [\phi_{T,i}, w_{T,i}, b_{T,i}]$, and $W_{R,i} = [\phi_{R,i}, w_{R,i}, b_{R,i}]$ Output: The best global decision variables/trained weights of the ANN-GASQ programming scheme denoted as $W_{GA-Best}$ Initialization: Generate chromosome W and P with pseudo random numbers. Initialization is performed for {GA} and {gaoptimset} routines with suitable declarations and settings. Fitness evaluated: Calculate the fitness and its parts shown in Equations (5)–(10) for each $\{W\}$ in P. Termination: Terminate the procedure, when any requirements meets • {Achieved Fitness = 10^{-20}}, {Generations = 60} • {Tolerances: {TolFun = 10^{-20} and TolCon = 10^{-21}}, • {StallGenLimit = 100}, {Population size = 300} • Others values: default. When the above conditions meet, go to storage Ranking: Rank is proficient for every 'W' of 'P' indicates the attained fitness. Reproduction: • {Selection: selectionuniform} • {Mutations: mutationadaptfeasible}. • {Crossover: crossoverheuristic}. • {Elitism: Transmit 5% individuals in P} Go to fitness assessment step. Storage: Store the $W_{GA-Best}$, i.e., the weight vector, fitness assessment, generations, time and count of functions for the present run of GAs. End of GA SQP Process Start Inputs: Start point is $W_{GA-Best}$ Output: GASQP best weights are denoted as W_{GASQ} Initialize: Set the limited constraints, iterations and other values of optimset. Terminate: The SQ programming process terminates when one the criteria meets • {Generations = 900}, {Fitness = 10^{-18}}, • (TolFun = TolCon = TolX = 10^{-22}) and {MaxFunEvals = 285000}. While (Terminate)
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Table 3. Cont.

Fitness Evaluate: Compute fitness value of every W of P by using system (4) to (10). Adjustments: Fine-tune {fmincon} with SQ programming scheme to tune W and adjust again the fitness value by using systems (4) to (10). Accumulate Store fitness, time, W_{GASQ}, function counts and generations for multiple trials of SQ programming. End of the SQ programming scheme Data Generations Repeat the procedure 30 times based on the GASQ programming to get a massive dataset of the optimization variables of ANNs for numerical solutions of the Sitr model based on COVID-19
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3. Performance Measures

The mathematical form of the performance grades using the root mean square error (RMSE) and Theil's inequality coefficient (TIC) for Sitr model based on COVID-19 is presented as:

$$\begin{bmatrix} \text{RMSE}_{S_1}, & \text{RMSE}_{S_2}, \\ \text{RMSE}_I, & \text{RMSE}_T, \\ \text{RMSE}_R \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{1}{m} \sum_{i=1}^m ((S_1)_i - (\hat{S}_1)_i)^2}, & \sqrt{\frac{1}{m} \sum_{i=1}^m ((S_2)_i - (\hat{S}_2)_i)^2}, \\ \sqrt{\frac{1}{m} \sum_{i=1}^m (I_i - \hat{I}_i)^2}, & \sqrt{\frac{1}{m} \sum_{i=1}^m (T_i - \hat{T}_i)^2}, \\ \sqrt{\frac{1}{m} \sum_{i=1}^m (R_i - \hat{R}_i)^2} \end{bmatrix}, \quad (11)$$

$$\begin{bmatrix} \text{TIC}_{S_1}, & \text{TIC}_{S_2}, \\ \text{TIC}_I, & \text{TIC}_T, \\ \text{TIC}_R \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{\frac{1}{m} \sum_{i=1}^m ((S_1)_i - (\hat{S}_1)_i)^2}}{\left(\sqrt{\frac{1}{m} \sum_{i=1}^m (S_1)_i^2} + \sqrt{\frac{1}{m} \sum_{i=1}^m (\hat{S}_1)_i^2} \right)}, & \frac{\sqrt{\frac{1}{m} \sum_{i=1}^m ((S_2)_i - (\hat{S}_2)_i)^2}}{\left(\sqrt{\frac{1}{m} \sum_{i=1}^m (S_2)_i^2} + \sqrt{\frac{1}{m} \sum_{i=1}^m (\hat{S}_2)_i^2} \right)}, \\ \frac{\sqrt{\frac{1}{m} \sum_{i=1}^m (I_i - \hat{I}_i)^2}}{\left(\sqrt{\frac{1}{m} \sum_{i=1}^m I_i^2} + \sqrt{\frac{1}{m} \sum_{i=1}^m \hat{I}_i^2} \right)}, & \frac{\sqrt{\frac{1}{m} \sum_{i=1}^m (T_i - \hat{T}_i)^2}}{\left(\sqrt{\frac{1}{m} \sum_{i=1}^m T_i^2} + \sqrt{\frac{1}{m} \sum_{i=1}^m \hat{T}_i^2} \right)}, \\ \frac{\sqrt{\frac{1}{m} \sum_{i=1}^m (R_i - \hat{R}_i)^2}}{\left(\sqrt{\frac{1}{m} \sum_{i=1}^m R_i^2} + \sqrt{\frac{1}{m} \sum_{i=1}^m \hat{R}_i^2} \right)} \end{bmatrix}, \quad (12)$$

4. Results and Discussion

The detailed description of the results with necessary interpretations are provided here to solve the Sitr system for COVID-19 dynamics, as given in the system of Equation (1). The numerical results are also determined with Adams method, which are used in this work as reference solutions for comparison with the designed FF-ANN-GASQP solver. Moreover, statistical observations are also listed to evaluate the performance. The stability and reliability are verified by the results obtained through 30 independent runs achieving a higher level of accuracy by means of statistical measures based on TIC and RMSE, which further validate the worth of the proposed FF-ANN-GASQP solver. The convergence analysis based on the performance measures of statistical operators TIC and RMSE is provided for reliability and robustness. The graphs of absolute error (AE) using the proposed results and numerical reference solutions are presented for the validity of the proposed scheme.

Nonlinear SITS Model Based on COVID-19

The different suitable values of the epidemic parameters have been given in Table 4, and the updated form of the SITS model based on COVID-19 given in system (1) using these Table 4 values is given as:

$$\begin{cases} S'_1(\chi) = 0.3 - 0.3I(\chi)S_1(\chi) - 0.9T(\chi) - 0.25S_1(\chi), & S_1(0) = 0.65, \\ S'_2(\chi) = 0.3 - 0.3I(\chi)S_2(\chi) - 0.9T(\chi) - 0.25S_2(\chi), & S_2(0) = 0.15, \\ I'(\chi) = -0.1I(\chi) + 0.3I(\chi)(S_1(\chi) + S_2(\chi)) - 0.25I(\chi) + 0.9T(\chi) + 0.005I(\chi), & I(0) = 0.75, \\ T'(\chi) = 0.1I(\chi) - 0.3T(\chi) - 0.25T(\chi) + 0.2T(\chi) + 0.1T(\chi), & T(0) = 0.35, \\ R'(\chi) = -0.25R(\chi) + 0.3T(\chi), & R(0) = 0.1, \end{cases} \quad (13)$$

The objective function of set (13) is formulated as follows:

$$e = \frac{1}{N} \sum_{m=1}^N \left(\begin{aligned} & \left[(\hat{S}'_1)_m - 0.3 + 0.3I_m(\hat{S}_1)_m + 0.9T_m + 0.25(\hat{S}_1)_m \right]^2 + \\ & \left[(\hat{S}'_2)_m - 0.3 + 0.3I_m(\hat{S}_2)_m + 0.9T_m + 0.25(\hat{S}_2)_m \right]^2 + \\ & \left[\hat{I}'_m + 0.1I_m - 0.3I_m((\hat{S}_1)_m + (\hat{S}_2)_m) + 0.25I_m - 0.9T_m - 0.005I_m \right]^2 + \\ & \left[\hat{T}'_m - 0.1I_m + 0.3\hat{T}_m + 0.25\hat{T}_m + 0.2\hat{T}_m - 0.1\hat{T}_m \right]^2 + \\ & \left[\hat{R}'_m + 0.25\hat{R}_m - 0.3\hat{T}_m \right]^2 \\ & + \frac{1}{5} \left(((\hat{S}_1)_0 - 0.65)^2 + ((\hat{S}_2)_0 - 0.15)^2 + (\hat{I}_0 - 0.75)^2 + (\hat{T}_0 - 0.35)^2 + (\hat{R}_0 - 0.1)^2 \right) \end{aligned} \right) \quad (14)$$

Optimization of the SITS system for COVID-19 dynamics is performed by the hybrid-computing structure of GA and SQP for 30 numbers of independent trials to achieve the parameters of FF-ANN having five neurons. The set of the weight vectors of FF-ANN by GASQP are provided in Figure 2 for the best fitness, and these weights are applied to achieve the numerical outcomes of the FF-ANN-GASQP solver for the SITS system. Values of different parameters against four cases are given in Table 5. The mathematical formulations of the approximate numerical outcomes of FF-ANN-GASQP is given as follows:

$$\begin{aligned} \hat{S}_1(\chi) &= \frac{-1.4704}{1 + e^{-(0.2502\chi - 1.1644)}} - \frac{0.2502}{1 + e^{-(1.9550\chi + 0.4603)}} - \frac{0.3483}{1 + e^{-(1.1161\chi - 2.8067)}} + \frac{1.5199}{1 + e^{-(0.2957\chi + 1.1530)}} \\ &+ \frac{0.1591}{1 + e^{-(0.7159\chi - 0.3006)}}, \end{aligned} \quad (15)$$

$$\begin{aligned} \hat{S}_2(\chi) &= \frac{1.5969}{1 + e^{-(0.2577\chi + 0.2176)}} - \frac{0.7521}{1 + e^{-(0.5993\chi + 0.8183)}} - \frac{0.5261}{1 + e^{-(0.7527\chi - 0.7915)}} - \frac{1.1453}{1 + e^{-(0.7548\chi - 2.4308)}} \\ &+ \frac{0.1292}{1 + e^{-(0.7124\chi - 0.6757)}}, \end{aligned} \quad (16)$$

$$\begin{aligned} \hat{I}(\chi) &= \frac{0.00001}{1 + e^{-(0.4977\chi - 1.0265)}} + \frac{0.0815}{1 + e^{-(0.9180\chi + 0.0571)}} + \frac{1.3974}{1 + e^{-(2.0394\chi + 1.0937)}} + \frac{0.1292}{1 + e^{-(0.2777\chi - 1.2619)}} \\ &- \frac{1.0806}{1 + e^{-(2.1174\chi - 0.6641)}}, \end{aligned} \quad (17)$$

$$\begin{aligned} \hat{T}(\chi) &= \frac{-2.0966}{1 + e^{-(0.9354\chi - 0.6124)}} - \frac{0.5464}{1 + e^{-(1.3223\chi - 0.8716)}} - \frac{0.2651}{1 + e^{-(1.4587\chi - 0.0180)}} + \frac{0.9895}{1 + e^{-(1.3325\chi + 1.0869)}} \\ &+ \frac{0.8115}{1 + e^{-(0.2659\chi + 1.3133)}}, \end{aligned} \quad (18)$$

$$\begin{aligned} \hat{R}(\chi) &= \frac{-0.7228}{1 + e^{-(1.0915\chi - 0.7499)}} + \frac{0.0451}{1 + e^{-(1.5696\chi - 1.9934)}} - \frac{0.2232}{1 + e^{-(0.2699\chi - 1.0638)}} + \frac{0.4927}{1 + e^{-(0.7613\chi + 0.7571)}} \\ &+ \frac{0.2232}{1 + e^{-(0.2270\chi - 1.2859)}}, \end{aligned} \quad (19)$$

Figure 2a depicts that the rate of contact is the source of a significant increase in the number of susceptible persons primarily, but with the passage of time, it starts declining. This happens because the higher contact rate causes most of the people to get infected and transfer to an infected class;

therefore, the number of people in the susceptible class is reduced. Figure 2b clearly reflects that the number of infected persons increases with the increase of contact rate. Reduction in the contact rate results in a slight rise in the number of infected persons, while an increase in the contact rate suddenly enhances the number of infected persons. Figure 2c shows the growing behavior of susceptible persons with higher recovery rate values. Figure 2d depicts that as the recovery rate upsurges, so does the number of persons in the infectious class. It can be noticed that when the recovery rate is low, then less persons are recovered from the virus, because of the death of infected persons.

Figure 3a demonstrates the increasing behavior of susceptible persons with various death rates. Figure 3b shows the variation of recovered persons with different death rate values. Figure 3 depicts that with a high death rate, the number of recovered persons are reduced abruptly. This is because when the death rate is high, then a large number of persons from the infected and recovered class die, which results in the reduction of persons in all of these classes. As the death rate becomes very high, the infectious and recovered persons are approximately wiped out.

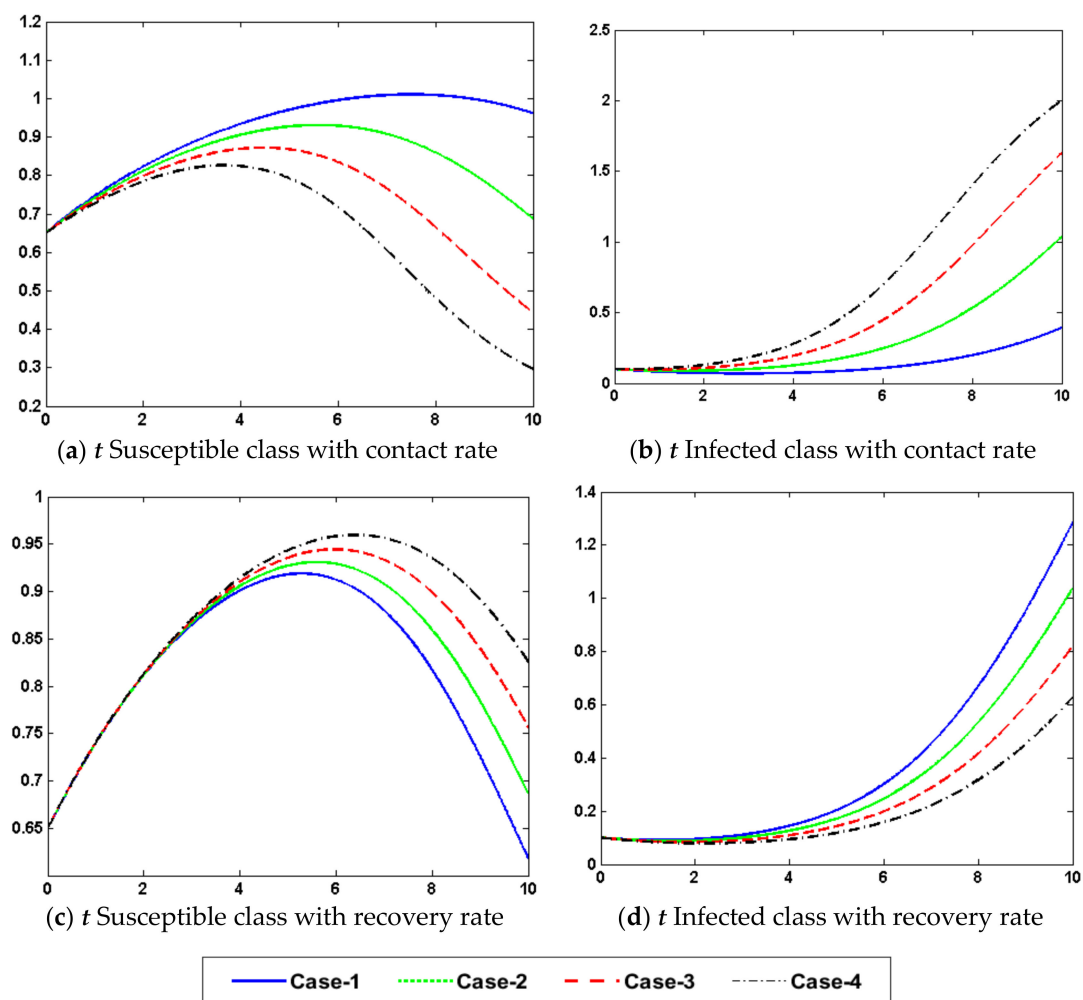


Figure 2. Impact of contact rate and recovery rate on susceptible and infected classes.

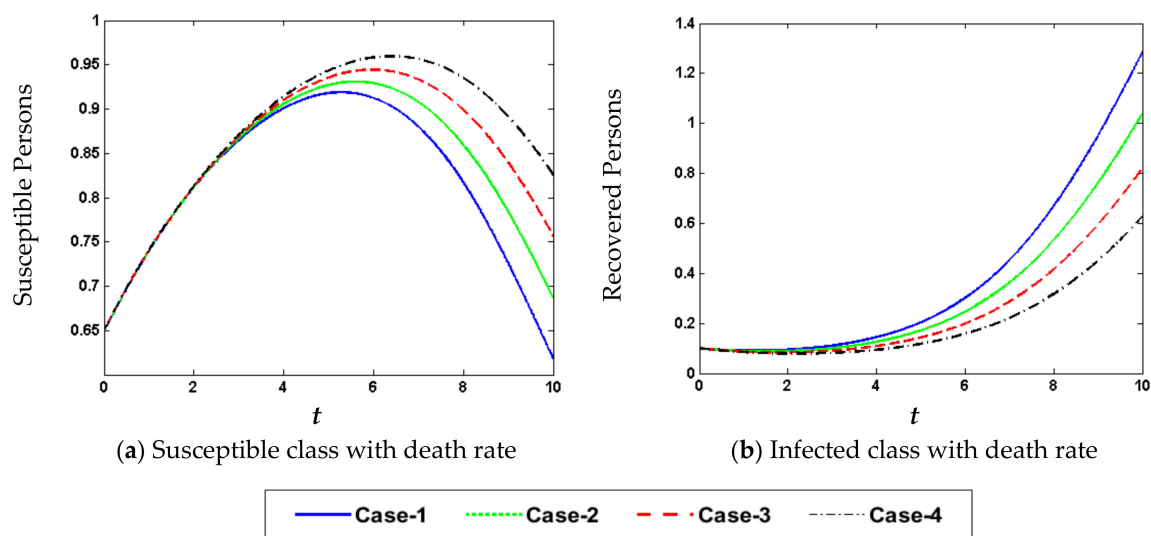


Figure 3. Impact of death rate on susceptible and recovered persons classes.

Table 4. Description and values for various involved parameters in Sitr model.

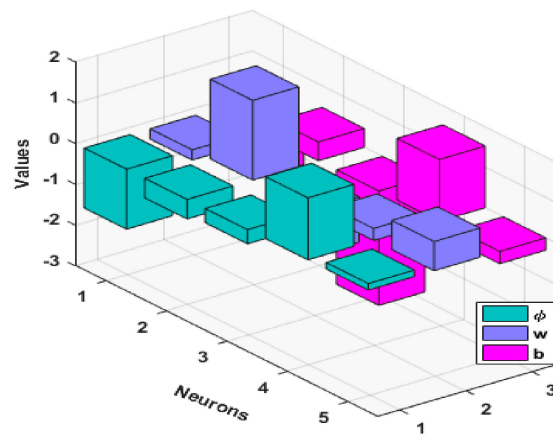
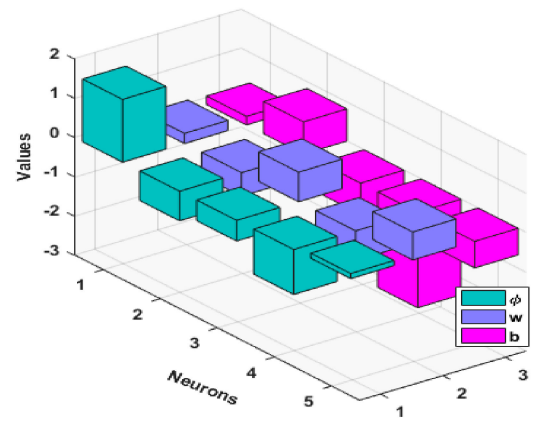
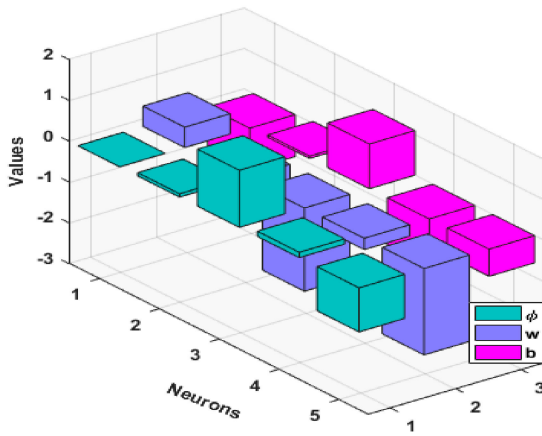
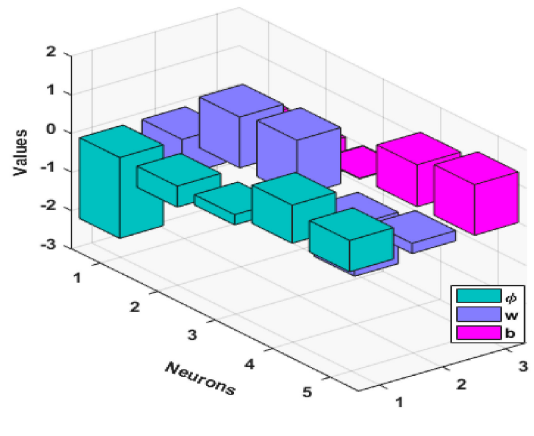
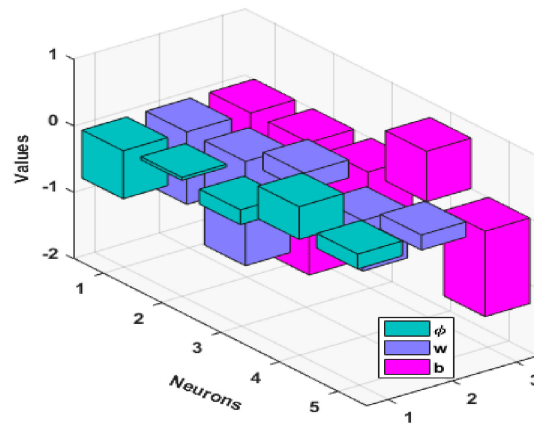
Symbol	Parameter Description	Assigned Value
β	Contact rate	0.3
B	Natural birth rate	0.3
δ	Reduce infection from the treatment	0.3
σ	Fever, tiredness, and dry cough rate	0.005
μ	Recovery rate	0.1
α	Death rate	0.25
ρ	Rate of infection from the treatment	0.3
ψ	Healthy food rate	0.2
ε	Sleep rate	0.1

Table 5. Variation of different involved parameters.

Scenarios	Variable Parameter	Case I	Case II	Case III	Case IV
1	Contact Rate	$\beta = 0.25$	$\beta = 0.30$	$\beta = 0.35$	$\beta = 0.40$
2	Recovery Rate	$\mu = 0.08$	$\mu = 0.10$	$\mu = 0.12$	$\mu = 0.14$
3	Death Rate	$\alpha = 0.20$	$\alpha = 0.25$	$\alpha = 0.30$	$\alpha = 0.35$

The set of Equations (15)–(19) are used to get the solution of the nonlinear Sitr model based on COVID-19 as given in system (13) using the FF-ANN-GASQP, and numerical results are provided in Figures 2–4 using five neurons. The trained weights set for the five neurons-based FF-ANN model is shown in Figure 4a–e for all the classes of a nonlinear Sitr system for COVID-19 dynamics. The graphs of absolute error (AE) using the proposed results and numerical Adams scheme are plotted in Figure 5. It is seen that most of the AE values lie around 10^{-4} to 10^{-5} for all the parameters of the nonlinear Sitr model using five numbers of neurons.

The convergence analysis based on the performance measures of the statistical operators TIC and RMSE values is drawn in Figure 6. The first part of Figure 6 shows the performance measures of the TIC values, while the second part of Figure 6 indicates the performance of the RMSE values for all the parameters of the nonlinear Sitr model of COVID-19. One may conclude from these obtainable results that most of independent runs achieved a higher level of accuracy.

(a) Tuned weights for $S_1(\chi)$.(b) Tuned weights for $S_2(\chi)$.(c) Tuned weights for $I(\chi)$.(d) Tuned weights for $T(\chi)$.(e) Tuned weights for $R(\chi)$.**Figure 4.** Set of best fitted trained weights/decision variables of FF-ANN-GASQP.

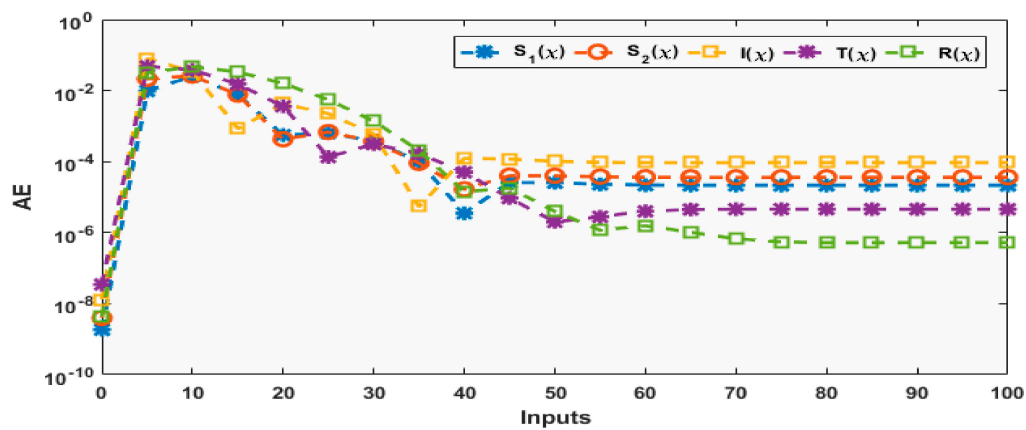
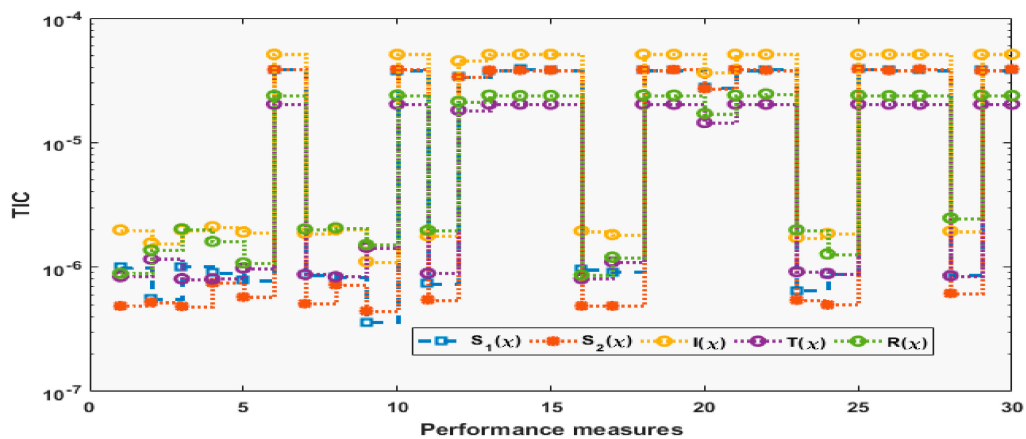
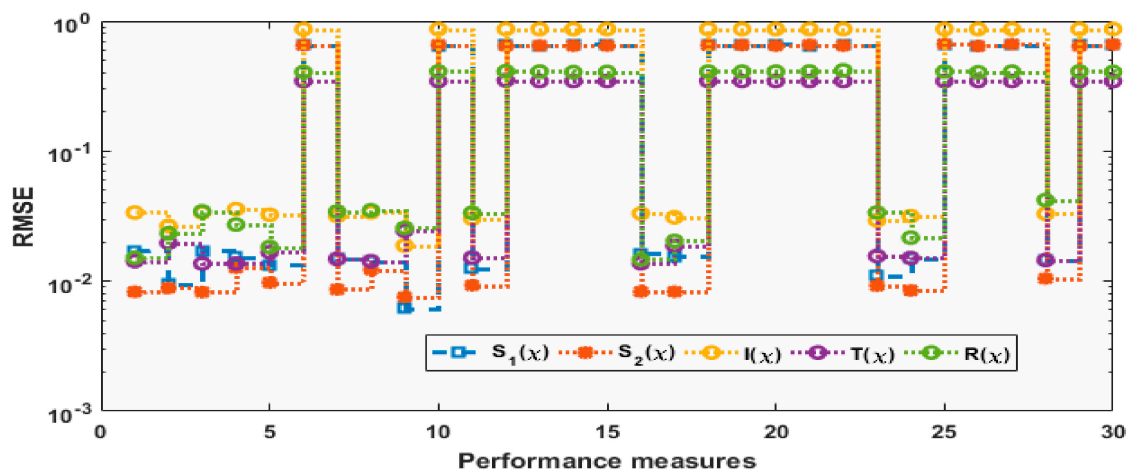


Figure 5. Absolute error (AE) results for all the variables of the nonlinear Sitr model based on COVID-19 using five neurons.



(a)



(b)

Figure 6. Convergence analysis based on the performance measures of Theil's inequality coefficient (TIC) and root mean square error (RMSE) values for all the variables of the nonlinear Sitr model.

5. Conclusions

A new computational intelligent solver FF-ANN-GASQP is designed for the solution of a nonlinear Sitr system for COVID-19 dynamics using the approximation capability of FF-ANN optimized globally

with GAs and locally with SQP. The dynamics of the nonlinear Sitr model of COVID-19 are proficiently assessed by the designed FF-ANN-GASQP approach with the single hidden layer formulation of FF-ANNs using five neurons. The exactness of the designed FF-ANN-GASQP technique is endorsed by matching of the outcomes with reasonable precision from the reference Adams numerical scheme to solve a nonlinear Sitr system based on COVID-19. Moreover, statistical clarifications through 30 autonomous runs by means of TIC and RMSE indices further authenticate the worth and value of the proposed FF-ANN-GASQP solver.

In future research, the presented FF-ANN-GASQP scheme algorithm can be implemented as an efficient/accurate stochastic numerical solver for the nonlinear dynamical measures of a computational fluid mechanics model [46–50], renewed fractional order processing systems [51–54], higher-order singular models [55], parameter estimation problems and electromagnetic waves [56,57], a prediction differential model [58], and bioinformatics [59–61].

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