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Complex Patterns to the (3+1)-Dimensional B-type Kadomtsev-Petviashvili-Boussinesq Equation

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Abstract: This paper presents many new complex combined dark-bright soliton solutions obtained with the help of the accurate sine-Gordon expansion method to the B-type Kadomtsev-Petviashvili-Boussinesq equation with binary power order nonlinearity. With the use of some computational programs, we plot many new surfaces of the results obtained in this paper. In addition, we present the interactions between complex travelling wave patterns and their solitons.

Keywords: B-type Kadomtsev-Petviashvili-Boussinesq equation; sine-Gordon expansion method; complex mixed dark-bright soliton solutions; dark soliton solution

1. Introduction

Mathematical models named nonlinear evaluation equations (NEEs) arise in different areas of nonlinear science such as plasma physics, quantum mechanics, hydro-dynamics molecular biology, nonlinear optics, stratum water wave, optics fibers, biological science, chemistry, etc. Investigations of NEEs render possible the better understanding the complex phenomena. Recently, many new mathematical models used to describe today's real-world problems have attracted the attention of experts from all over the world. In this sense, to observe these models some important methods such as the trial equation method, extended tanh method, modified simple equation method, extended simplest equation method, modified extended tanh method, complex method, generalized hyperbolic-function method, the homogeneous balance method, the improve F-expansion method with a Riccati equation, the improved Bernoulli sub-equation function method, the modified exponential function method and many more methods [1–49]. One of such models named as (3 + 1)-dimensional B-type Kadomtsev-Petviashvili-Boussinesq equation (B-type KPB) defined by

$$u_{ty} - u_{xxxy} - 3\left(u_x u_y\right)_x + 3u_{xz} + u_{tt} = 0, \tag{1}$$

has been investigated [50,51]. Physically, the term u_{tt} has been added and used to investigate the effect of dispersion relation and phase shift properties of the generalized B-type Kadomtsev–Petviashvili equation [51].

Y.S. Deng and his team have observed the Equation (1) via the breather-type kind soliton solutions with the help of the breather-soliton mixture [51].

In Section 2, SGEM based on fundamental equation being sine-Gordon equation Equation (1) will be defined in a detailed manner. In Section 3, some new complex combined dark-bright travelling wave solutions, which have not been studied so far to the Equation (1) will be obtained. Considering the suitable values of parameters, some graphical simulations will be also discussed. In the last section of this paper, conclusions will be presented.

2. The SGEM

Let's consider the following sine-Gordon equation [52,53];

$$u_{xx} - u_{tt} = m^2 \sin(u), \tag{2}$$

where $u = u(x,t)$, and m is a real constant. When we apply the wave transform $u(x,t) = U(\xi)$, $\xi = \mu(x - ct)$ to Equation (2), we obtain a nonlinear ordinary differential equation (NODE) in the form:

$$U'' = \frac{m^2}{\mu^2(1-c^2)} \sin(U), \tag{3}$$

where $U = U(\xi)$, and, ξ is the amplitude of the travelling wave, c is the velocity of the travelling wave. If we reconsider Equation (3), we can write in the full simplify version as following:

$$\left[\left(\frac{U}{2}\right)'\right]^2 = \frac{m^2}{\mu^2(1-c^2)} \sin^2\left(\frac{U}{2}\right) + K, \tag{4}$$

where K is the integration constant. When we resubmit as $K = 0$, $w(\xi) = \frac{U}{2}$, and $a^2 = \frac{m^2}{\mu^2(1-c^2)}$ in Equation (4), we can obtain following equation:

$$w' = a\sin(w). \tag{5}$$

If we put as $a = 1$ in Equation (5), we can obtain following equation:

$$w' = \sin(w). \tag{6}$$

If we solve Equation (6) by using separation of variables, we find the following two significant equations:

$$\sin(w) = \sin(w(\xi)) = \left.\frac{2pe^{\xi}}{p^2e^{2\xi}+1}\right|_{p=1} = \sec h(\xi), \tag{7}$$

$$\cos(w) = \cos(w(\xi)) = \left.\frac{p^2e^{2\xi}-1}{p^2e^{2\xi}+1}\right|_{p=1} = \tanh(\xi), \tag{8}$$

where p is the integral constant and non-zero. For the solution of following nonlinear partial differential equation;

$$P(u, u_x, u_t, \cdots) = 0, \tag{9}$$

let's consider as

$$U(\xi) = \sum_{i=1}^{\delta} \tanh^{i-1}(\xi)[B_i \sec h(\xi) + A_i \tanh(\xi)] + A_0. \tag{10}$$

We can rewrite Equation (10) according to Equations (7) and (8) in the form:

$$U(w) = \sum_{i=1}^{\delta} \cos^{i-1}(w)[B_i \sin(w) + A_i \cos(w)] + A_0. \tag{11}$$

Under the terms of homogenous balance technique, we can determine the values of n under the terms of *NODE*. Let the coefficients of $\sin^i(w)\cos^j(w)$ all be zero, it yields a system of equations.

Solving this system, the values of A_i, B_i, μ, c can be found. Finally, substituting the values of A_i, B_i, μ, c into Equation (10), we can find the new analytical solutions to the Equation (9).

3. Application

In present part of the paper, we apply SGEM to obtain new mixed dark-bright soliton solutions to B-type KPB. Consider the following wave transformation for B-type KPB equation:

$$\boldsymbol{u}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t},\boldsymbol{z}) = \boldsymbol{U}(\xi),\ \xi = \boldsymbol{kx} + \boldsymbol{wy} + \boldsymbol{rz} - \boldsymbol{ct}. \tag{12}$$

This can be obtained the following differential equation:

$$\left(3\boldsymbol{rk} + \boldsymbol{c}^2 - \boldsymbol{cw}\right)\boldsymbol{U}'' - 3\boldsymbol{w}\boldsymbol{k}^2(\boldsymbol{UU}')' - \boldsymbol{w}\boldsymbol{k}^3\boldsymbol{U}^4 = 0. \tag{13}$$

Integrating Equation (12) and the constants of integrate to zero yield:

$$\left(3\boldsymbol{rk} + \boldsymbol{c}^2 - \boldsymbol{cw}\right)\boldsymbol{U} - \frac{3\boldsymbol{w}\boldsymbol{k}^2}{2}\boldsymbol{U}^2 - \boldsymbol{w}\boldsymbol{k}^3\boldsymbol{U}'' = 0. \tag{14}$$

Applying some simplifications, we find the following nonlinear ordinary differential equation (NODE) for B-type KPB equation:

$$2\left(3\boldsymbol{rk} + \boldsymbol{c}^2 - \boldsymbol{cw}\right)\boldsymbol{U} - 3\boldsymbol{w}\boldsymbol{k}^2\boldsymbol{U}^2 - 2\boldsymbol{w}\boldsymbol{k}^3\boldsymbol{U}'' = 0. \tag{15}$$

The homogeneous balance principle produces $n = 2$. If we consider this into Equation (11), we get the following:

$$\boldsymbol{U}(\boldsymbol{w}) = \boldsymbol{B}_1\mathbf{sin}(\boldsymbol{w}) + \boldsymbol{A}_1\mathbf{cos}(\boldsymbol{w}) + \boldsymbol{B}_2\mathbf{cos}(\boldsymbol{w})\mathbf{sin}(\boldsymbol{w}) + \boldsymbol{A}_2\mathbf{cos}^2\boldsymbol{w} + \boldsymbol{A}_0, \tag{16}$$

$$\begin{aligned}\boldsymbol{U}''(\boldsymbol{w}) = \boldsymbol{B}_1\mathbf{cos}^2\ &(\boldsymbol{w})\mathbf{sin}(\boldsymbol{w}) - \boldsymbol{B}_1\mathbf{sin}^3(\boldsymbol{w}) - 2\boldsymbol{A}_1\mathbf{sin}^2(\boldsymbol{w})\mathbf{cos}(\boldsymbol{w})\\ &+\boldsymbol{B}_2\mathbf{cos}^3(\boldsymbol{w})\mathbf{sin}(\boldsymbol{w}) - 5\boldsymbol{B}_2\mathbf{sin}^3(\boldsymbol{w})\mathbf{cos}(\boldsymbol{w})\\ &- 4\boldsymbol{A}_2\mathbf{cos}^2(\boldsymbol{w})\mathbf{sin}^2(\boldsymbol{w}) + 2\boldsymbol{A}_2\mathbf{sin}^4(\boldsymbol{w}),\end{aligned} \tag{17}$$

Substituting Equations (15) and (16) into Equation (14) produces an equation including many trigonometric terms. When we take a set of algebraic equations to zero, we find a system. Solving this system, we find the following coefficients:

Case 1 If $A_0 = 2k;\ A_1 = 0;\ A_2 = -2k;\ B_1 = 0; B_2 = 2ik;\ r = \frac{-c^2+\left(c+k^3\right)w}{3k}$; inserting this value into Equation (10), yields the following complex combined dark-bright soliton solution:

$$\begin{aligned}&u_1(x,y,z,t) = 2k - 2ik\sec h\left[ct - kx - wy - \frac{-c^2+w\left(c+k^3\right)}{3k}z\right]\\ &\times\tanh\left[ct - kx - wy - \frac{-c^2+w\left(c+k^3\right)}{3k}z\right] - 2k\tanh\left[ct - kx - wy - \frac{-c^2+w\left(c+k^3\right)}{3k}z\right]^2,\end{aligned} \tag{18}$$

in which c, k, w are real constants and non-zero. See Figures 1–3 to illustrate.

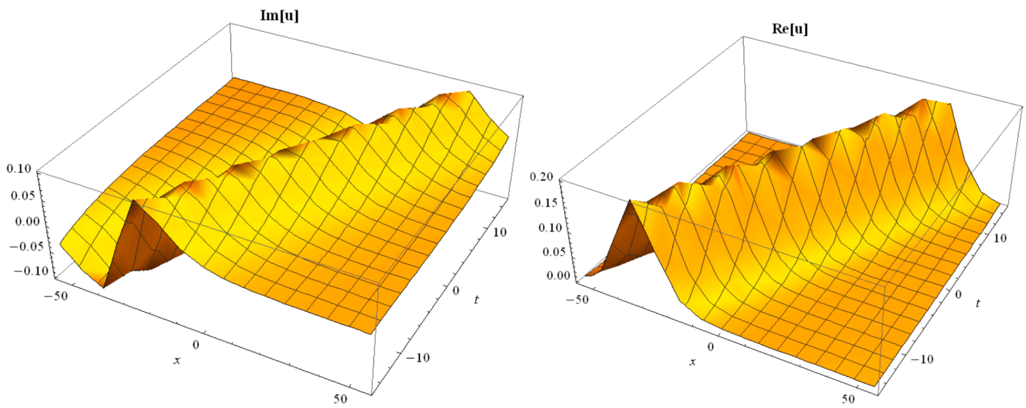


Figure 1. The 3D-dimensional surfaces of imaginary and real parts of Equation (17).

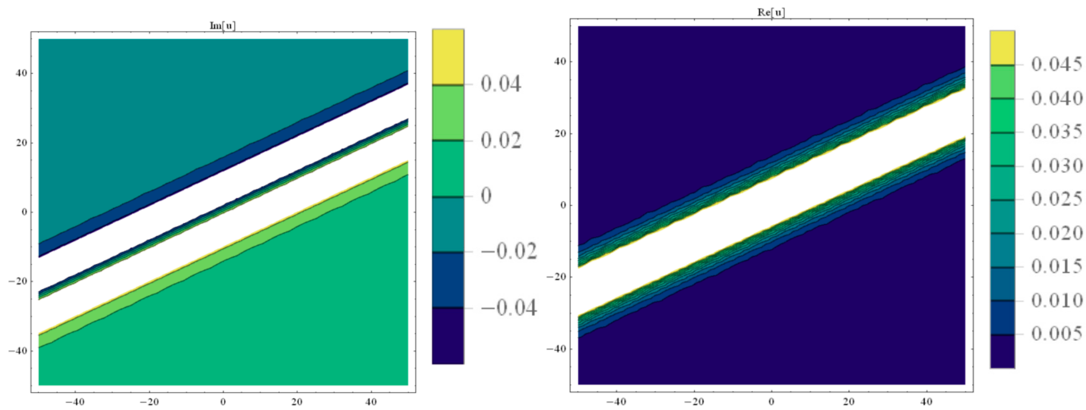


Figure 2. Contour graphs of imaginary and real parts of Equation (17).

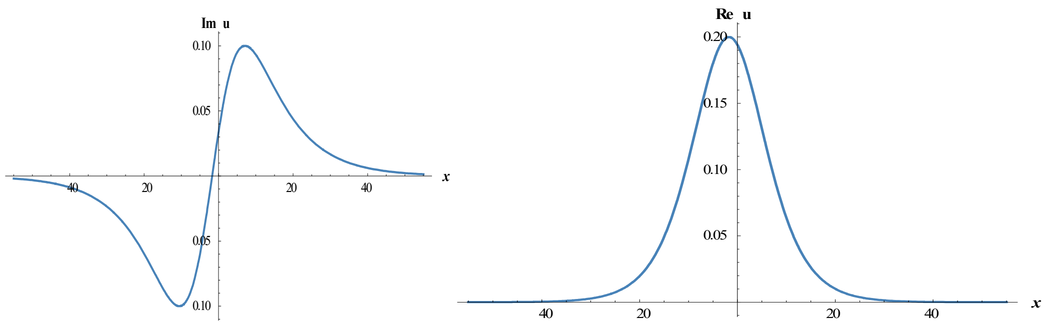


Figure 3. The 2D-dimensional surfaces of imaginary and real parts of Equation (17).

Case-2: when we choose, $A_0 = \frac{4k}{3}$; $A_1 = 0$; $A_2 = -2k$; $B_1 = 0; B_2 = 2ik$; $r = -\frac{\left(c^2 - cw + k^3 w\right)z}{3k}$; taking these values into Equation (10) produces another complex combined dark-bright solution to Equation (1):

$$\begin{aligned} u_2(x,y,z,t) &= \tfrac{4k}{3} - 2k\tanh\left[ct - kx - wy + \tfrac{c^2 - cw + wk^3}{3k}z\right]^2 \\ &+ 2ik\sec h\left[ct - kx - wy + \tfrac{c^2 - cw + wk^3}{3k}z\right]\tanh\left[ct - kx - wy + \tfrac{c^2 - cw + wk^3}{3k}z\right], \end{aligned} \tag{19}$$

where k, c, w are real constants with non-zero values. Choosing the suitable values of parameters, some figures may be found as follows (see Figures 4–11).

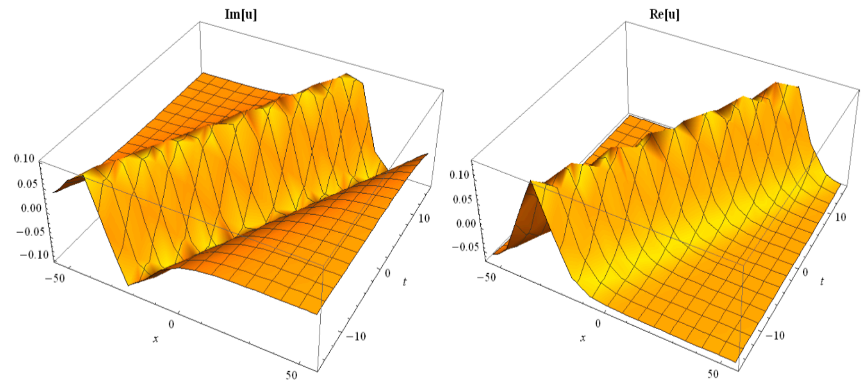


Figure 4. The 3D-dimensional surfaces of imaginary and real parts of Equation (18).

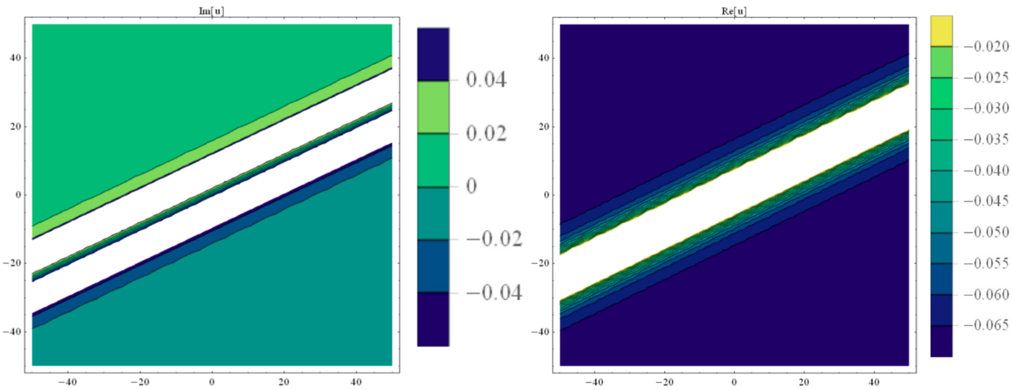


Figure 5. Contour surfaces of imaginary and real parts of Equation (18).

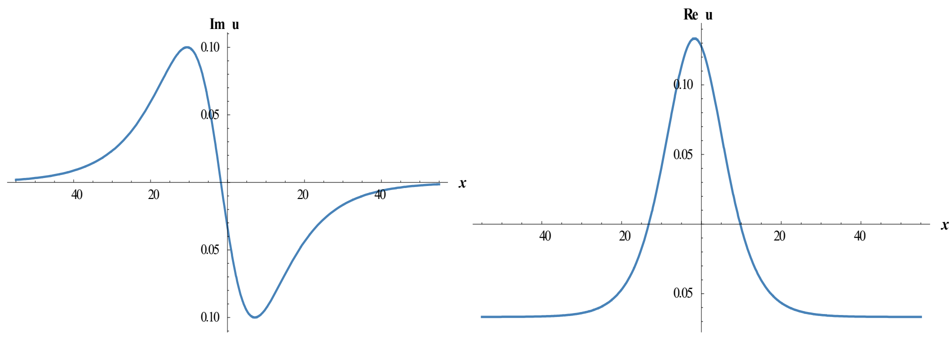
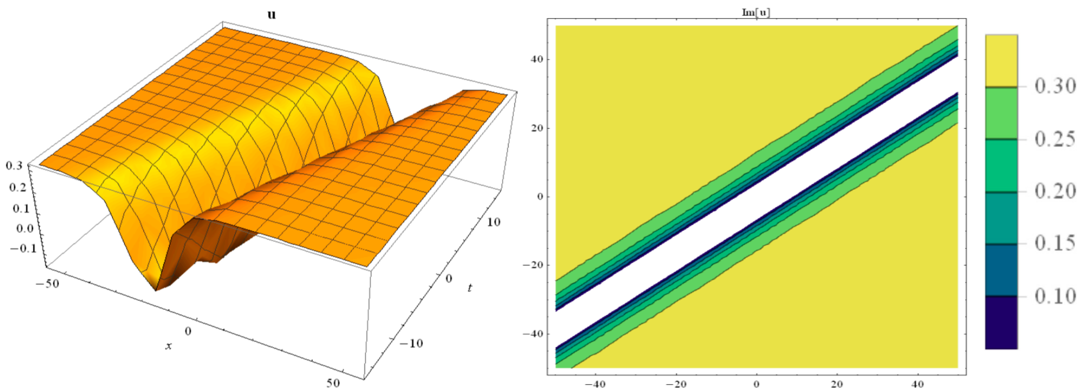


Figure 6. The 2D-dimensional surfaces of imaginary and real parts of Equation (18).



(**a**) (**b**)

Figure 7. (**a**) The 3D-dimensional (left side) (**b**). contour surfaces of Equation (19) (right side).

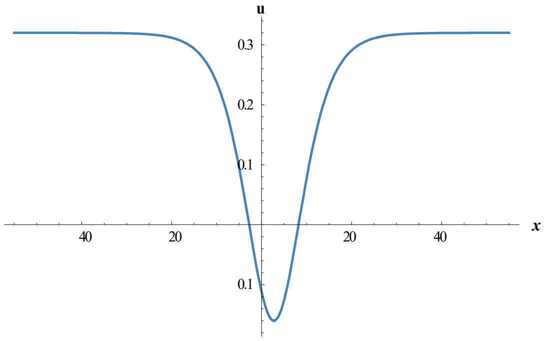


Figure 8. The 2D-dimensional surfaces of Equation (19).

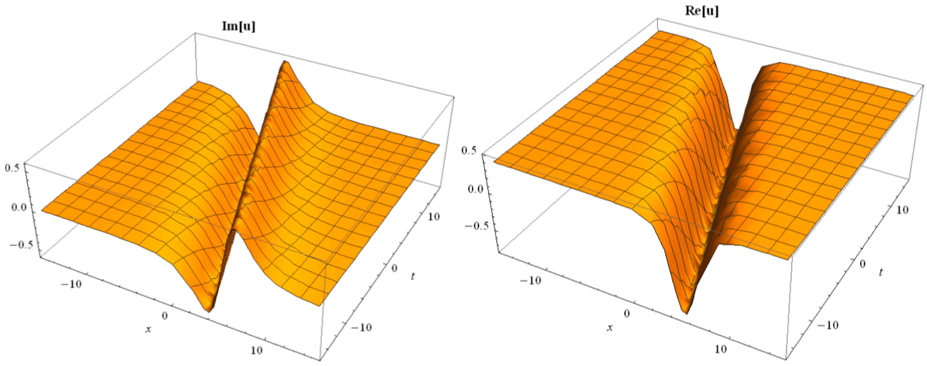


Figure 9. The 3D-dimensional surfaces of imaginary and real parts of Equation (20).

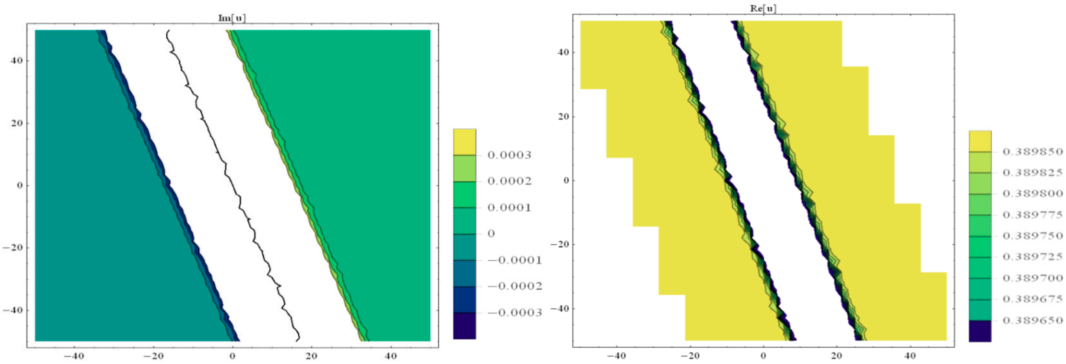


Figure 10. The contour plots of imaginary and real parts of Equation (20).

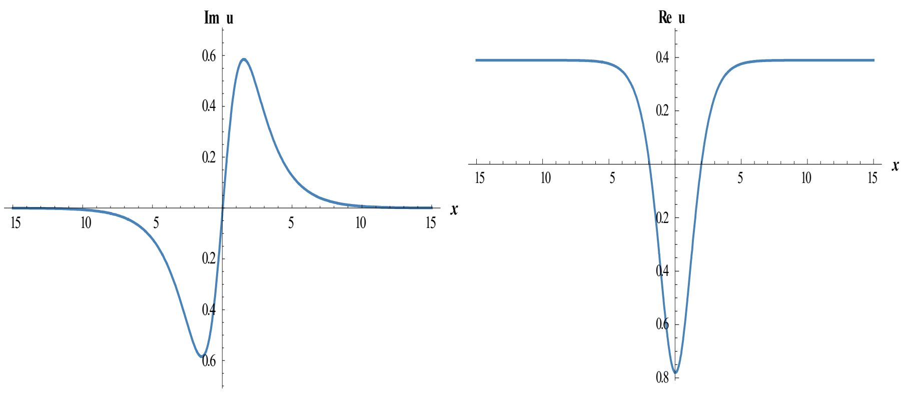


Figure 11. The 3D-dimensional surfaces of imaginary and real parts of Equation (20).

Case-3: if it is taken as for Equation (10), $A_0 = \frac{4k}{3}$; $A_1 = 0$; $A_2 = -4k$; $B_1 = 0; B_2 = 0$ and $c = \frac{1}{2}\left(w - \sqrt{-12kr - 16k^3w + w^2}\right)$ results in the following dark soliton solution:

$$u_3(x,y,t,z) = \frac{4k}{3} - 4k\tanh\left[-kx - wy - rz + \frac{t}{2}\left(w - \sqrt{-12kr - 16k^3w + w^2}\right)\right]^2, \tag{20}$$

in which k, w, r are constants and non-zero. Some suitable values of parameters, we plot as follows:

Case-4: considering other coefficients such as $A_0 = \frac{-4}{3}c^{\frac{1}{3}}; A_1 = 0; A_2 = 2c^{\frac{1}{3}}; B_1 = 0; B_2 = -2c^{\frac{1}{3}}$; and $\boldsymbol{k} = -\boldsymbol{c}^{\frac{1}{3}}$; $\boldsymbol{w} = \frac{c}{2} - \frac{3r}{2}\boldsymbol{c}^{\frac{-2}{3}}$, for Equation (10), we find another new complex combined dark-bright solution to Equation (1) as follows:

$$\begin{aligned} u_4(x,y,z,t) &= \tfrac{-4}{3}c^{1/3} + 2ic^{1/3}\sec h\left[ct + c^{1/3}x - rz - \left(\tfrac{c}{2} - \tfrac{3r}{2}c^{-2/3}\right)y\right] \\ &\times\tanh\left[ct + c^{1/3}x - rz - \left(\tfrac{c}{2} - \tfrac{3r}{2}c^{-2/3}\right)y\right] + 2c^{1/3}\tanh\left[ct + c^{1/3}x - rz - \left(\tfrac{c}{2} - \tfrac{3r}{2}c^{-2/3}\right)y\right]^2, \end{aligned} \tag{21}$$

with c, r being real constants with non-zero values.

4. Conclusions

In this paper, we have successfully employed the SGEM to the (3 + 1)-dimensional B-type KPB equation. New dark, complex combined dark-bright soliton solutions to the governing equation have been constructed. We have observed that all solutions found in this paper have satisfied the (3 + 1)-dimensional B-type KPB equation with the help of some computational programs. To gain a better understanding of complex wave patterns, we have plotted two- and three-dimensional surfaces of the results in Figure 1, Figure 3, Figure 4, Figure 6, Figure 7a, Figure 8, Figure 9, and Figure 11 along with contour simulations given more detailed information about the low and high points of waves in a selected area Figure 2, Figure 5, Figure 7b, and Figure 10. In this sense contour simulations have become an alternative observing managing system for the results in terms of depth and height observations. Physically, dark solution, which is the third solution for Equation (19) was used to explain gravitational potential of gravity [54]. With this sense, it is estimated that these results are of such gravitational physical meanings. When we compared these results with the existing works in the literature [51], it can be observed that these solutions were of entirely new dark and complex combined dark-bright soliton solutions to the (3 + 1)-dimensional B-type KPB equation. It is estimated that the sine-Gordon equation expansion method is an efficient and powerful computational tool that can be used for studying complex nonlinear models.

Moreover, as a future work, we will investigate the stability properties of the negative solitary solutions obtained by using sine-Gordon equation expansion method in terms of orbital dynamical stability [55]. N.T. Nguyen et al. [55] have observed the orbital stability of negative solitary waves via numerical simulation by using a spectral discretization.

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