Article

On Neutrosophic Extended Triplet LA-hypergroups and Strong Pure LA-semihypergroups

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Abstract: We introduce the notions of neutrosophic extended triplet LA-semihypergroup, neutrosophic extended triplet LA-hypergroup, which can reflect some symmetry of hyperoperation and discuss the relationships among them and regular LA-semihypergroups, LA-hypergroups, regular LA-hypergroups. In particular, we introduce the notion of strong pure neutrosophic extended triplet LA-semihypergroup, get some special properties of it and prove the construction theorem about it under the condition of asymmetry. The examples in this paper are all from Python programs.

Keywords: LA-semihypergroup; LA-hypergroup; neutrosophic extended triplet LA-semihypergroup; neutrosophic extended triplet LA-hypergroup

1. Introduction and Preliminaries

Left almost semigroup (abbreviated as LA-semigroup, some researchers also call it Abel Grassmann's groupoid), a non-associative and noncommutative algebraic structure, was first proposed by Kazim and Naseeruddin in Reference [1]. Hyperstructure theory was first introduced by Marty in Reference [2]. In the following decades and nowadays, various hyperstructures are widely studied and applied [3–6]. In Reference [7], Hila and Dine extended the concept of LA-semigroup to LA-semihypergroup and investigated several properties of LA-semihypergroups. Since then, many researchers have been done a lot of studies in this field [8–13].

In recent years, as an application of idea of neutrosophic set, the new notion of neutrosophic triplet group (NTG) was firstly introduced by F. Smarandache and M. Ali in Reference [14]. Soon after, M. Gulistan, S. Nawaz and N. Hassan applied the idea of NTG to LA-semihypergroup, proposed the concept of NTG-LA-semihypergroup and got some interesting results in Reference [15]. Meanwhile, F. Smarandache extended the concept of NTG to neutrosophic triplet extended group (NETG) in Reference [16]. Later, some research articles in this field are published. F. Smarandache, X.H. Zhang, X.G. An and Q.Q. Hu investigated properties and structures of NETG in Reference [17]; T.G. Jaíyéolá and F. Smarandache obtained some conclusions on neutrosophic triplet groups and discussed their applications in Reference [18]; The new concept of NET-Abel-Gassmann's Groupoid was introduced and the relationships of NETGs and regular semigroups were studied in Reference [19]; X.H. Zhang and X.Y. Wu prove that the construction theorem of NETG in Reference [20]; The concept of generalized neutrosophic extended group were proposed by Y.C. Ma and the relationships of NETGs and generalized groups were studied in References [21,22]. In particular, the notions of NET-semihypergroup and NET-hypergroup were introduced by X.H. Zhang, F. Smarandache and Y.C. Ma and the decomposition theorem of PWC-NET-semihypergroup was proved in Reference [23]. For the study of some related algebraic systems, please refer to Reference [24–26].

In this study, we apply the concept of NETG to LA-semihypergroup and introduce the new notions of NET-LA-semihypergroup, NET-LA-hypergroup, SPNET-LA-semihypergroup; Further, we discuss their properties, relations and so forth.

First of all, recall some conclusions and definitions on LA-semihypergroups.

Definition 1. [7] We say that a mapping

$$\circ : H \times H \rightarrow P^*(H)$$

is a binary hyperoperation, if H is a nonempty set, $P(H)$ is power set of H and $P^*(H) = P(H)/\phi$.

Definition 2. [7] $(H, \circ)$ is a binary hypergroupoid, if H is a nonempty set and $\circ$ is a binary hyperoperation. In addition, we write

$$X \circ Y = \bigcup_{a \in X, b \in Y} (a \circ b), X \circ a = X \circ \{a\}, a \circ Y = \{a\} \circ Y,$$

where $a \in H, X \subseteq H, Y \subseteq H$ and $X \neq \phi, Y \neq \phi$.

Definition 3. [7] A binary hypergroupoid $(H, \circ)$ is an LA-semihypergroup, if

$$(a \circ b) \circ c = (c \circ b) \circ a \quad (1)$$

for all $a, b, c \in H$, that is

$$\bigcup_{s \in (a \circ b)} (s \circ c) = \bigcup_{t \in (c \circ b)} (t \circ a). \quad (2)$$

By Equation (1), we know that every LA-semihypergroup $(H, \circ)$ satisfies

$$(a \circ b) \circ (c \circ d) = (a \circ c) \circ (b \circ d) \quad (3)$$

for all $a, b, c, d \in H$.

Note that, the Equations (1) and (3) are all set equations. If we replace all the elements in the equations (1) and (3) with nonempty subsets of H , these equations still hold.

Definition 4. [7] $(T, \circ)$ is a sub LA-semihypergroup of $(H, \circ)$, if the following conditions hold:

- (a) $T \subseteq H, T \neq \phi$;
- (b) $m \circ n \subseteq T$ for all $m, n \in T$;
- (c) $(H, \circ)$ is an LA-semihypergroup.

Definition 5. [8] Suppose $(H, \circ)$ is an LA-semihypergroup. An element $a \in H$ is regular if there is an element $t \in H$ such that

$$a \in a \circ t \circ a.$$

Furthermore, $(H, \circ)$ is a regular LA-semihypergroup if each element of H is regular.

Definition 6. [7] Suppose $(H, \circ)$ is an LA-semihypergroup. $(H, \circ)$ is an LA-hypergroup if it satisfies

$$t \circ H = H \circ t = H$$

for all $t \in H$.

Definition 7. [8] Suppose $(H, \circ)$ is an LA-semihypergroup. An element $e \in H$ is

- (a) a left identity, if $a \in e \circ a$ for each $a \in H$;
- (b) a right identity, if $a \in a \circ e$ for each $a \in H$;

- (c) an identity, if $a \in (e \circ a) \cap (a \circ e)$ for each $a \in H$;
- (d) a pure left identity, if $a = e \circ a$ for each $a \in H$;
- (e) a pure right identity, if $a = a \circ e$ for each $a \in H$;
- (f) a pure identity, if $a = (e \circ a) \cap (a \circ e)$ for each $a \in H$;
- (g) a scalar identity, if $a = e \circ a = a \circ e$ for each $a \in H$.

In addition, we say that $x \in H$ is an inverse of $a \in H$ if x satisfies

$$e \in (a \circ x) \cap (x \circ a),$$

where e is an identity of $(H, \circ)$.

Definition 8. $(H, \circ)$ is a regular LA-hypergroup, if it satisfies the following conditions:

- (a) $(H, \circ)$ is an LA-hypergroup;
- (b) There exists $e \in H$ such that e is identity of $(H, \circ)$;
- (c) Every element $a \in H$ has at least one inverse.

Definition 9. [16] A nonempty set M is said to be a neutrosophic extended triplet set if to any given $a \in M$, there are $s \in M$ and $t \in M$, in such a way that

$$a \circ s = s \circ a = a \tag{4}$$

$$a \circ t = t \circ a = s, \tag{5}$$

where $\circ$ is a binary operation on M , s is an extend neutral of ' a ', t is an opposite of ' a ' about s , (a, s, t) is a neutrosophic extend triplet.

Definition 10. [14,16] A semihypergroup $(H, \circ)$ is said to be an NET-semihypergroup if to any given $a \in H$, there are $s \in H$ and $t \in H$, in such a way that

$$a \in (s \circ a) \cap (a \circ s), \tag{6}$$

$$s \in (t \circ a) \cap (a \circ t). \tag{7}$$

In addition, for a certain $a \in H$, we say that (a, s, t) is a hyper-neutrosophic-triplet and use $\{\}_{neut(a)}$ for the set of all s that satisfy Formula (6) and (7). For a certain $s \in \{\}_{neut(a)}$, we use $\{\}_{anti(a)_s}$ for the set of all t that satisfy Formula (7).

2. Neutrosophic Extended Triplet LA-Semihypergroups and Neutrosophic Extended Triplet LA-Hypergroups

Definition 11. An LA-semihypergroup $(L, *)$ is said to be

- (a) a left neutrosophic extended triplet LA-semihypergroup (LNET-LA-semihypergroup) if to any given $a \in L$, there are $p \in L$ and $q \in L$, in such a way that

$$a \in p * a \tag{8}$$

$$p \in q * a. \tag{9}$$

Furthermore, for a certain $a \in L$, p, q and (a, p, q) are called left neutral of a , left opposite of a and left hyper-neutrosophic-triplet respectively. $\{\}_{lneut(a)}$ is used to represent the set of all p that satisfy Formula (8), (9) and for a certain $p \in \{\}_{lneut(a)}$, $\{\}_{lanti(a)_p}$ is used to represent the set of all q that satisfy Formula (9).

- (b) a right neutrosophic extended triplet LA-semihypergroup (RNET-LA-semihypergroup), if to any given $a \in L$, there are $s \in L$ and $t \in L$, in such a way that

$$a \in a * s \quad (10)$$

$$s \in a * t. \quad (11)$$

Furthermore, for a certain $a \in H$, (a, s, t) is called *right-hyper-neutrosophic-triplet*. $\{s\}_{neut(a)}$ is used to represent the set of all s that satisfy Formula (10), (11) and for a certain $s \in \{s\}_{neut(a)}$, $\{t\}_{anti(a)_s}$ is used to represent the set of all t that satisfy Formula (11).

- (c) a neutrosophic extended triplet LA-semihypergroup (NET-LA-semihypergroup), if to any given $a \in L$, there are $m \in L$ and $n \in L$, in such a way that

$$a \in (m * a) \cap (a * m) \quad (12)$$

$$m \in (n * a) \cap (a * n). \quad (13)$$

Furthermore, for a certain $a \in L$, (a, m, n) is called a *hyper-neutrosophic-triplet*, $\{m\}_{neut(a)}$ is used to represent the set of all m that satisfy Formula (12), (13) and for a certain $m \in \{m\}_{neut(a)}$, $\{n\}_{anti(a)_m}$ is used to represent the set of all n that satisfy Formula (13).

Example 1. Put $L = \{0, 1, 2\}$, the binary hypergroupoid $(L, *)$ is as follows (see Table 1).

Table 1. The binary hypergroupoid $(L, *)$.

*	0	1	2
0	0	0	0
1	0	1	0
2	0	0	{0, 2}

By Python program 1, $(L, *)$ is an LA-semihypergroup (please see Figure 1).

Python program 1 Verification of LA-semihypergroup 1

```

1: T = [ [[0],[0],[0]], [[0], [1], [0]], [[0], [0], [0,2]] ]
2: count = 0
3: for x in range(3):
4: for y in range(3):
5: for z in range(3):
6: T1 = T[x][y]
7: T2 = set()
8: k1 = len(T1)
9: for m in range(k1):
10: T2 = set(T1[m][z]).union(T2)
11: T3 = T[z][y]
12: T4 = set()
13: k2 = len(T3)
14: for n in range(k2):
15: T4 = set(T3[n][x]).union(T4)
16: if T2 == T4:
17: count += 1
18: while count == 3**3:
19: print('{} is an LA-semihypergroup'.format(T))
20: break

```

Run: program 1

C:\Users\Think\Anaconda3\python.exe C:/Users/Think/PycharmProjects/1/program1.py

[[[0],[0],[0]], [[0], [1], [0]], [[0], [0], [0,2]]] is an LA-semihypergroup.

Process finished with exit code 0

Figure 1. The result of Python program 1.

Furthermore, we get

$$0 \in (0 * 0) \cap (0 * 0), 0 \in (0 * 0) \cap (0 * 0)$$

$$0 \in (0 * 0) \cap (0 * 0), 0 \in (1 * 0) \cap (0 * 1)$$

$$0 \in (0 * 0) \cap (0 * 0), 0 \in (2 * 0) \cap (0 * 2)$$

$$1 \in (1 * 1) \cap (1 * 1), 1 \in (1 * 1) \cap (1 * 1)$$

$$2 \in (2 * 2) \cap (2 * 2), 2 \in (2 * 2) \cap (2 * 2).$$

By Definition 11, $(0, 0, 0), (0, 0, 1), (0, 0, 2), (1, 1, 1), (2, 2, 2)$ are all hyper neutrosophic-triplets and $(L, *)$ is an NET-LA-semihypergroup. These results can also be verified by Python program 2 (please see Figure 2).

Python program 2 Verification of NET-LA-semihypergroup 1

```
1: T = [ [[0],[0],[0]], [[0], [1], [0]], [[0], [0], [0,2]] ]
2: test = []
3: for t in range(3):
4: for neut_t in range(3):
5: for anti_t in range(3):
6: S1 = set(T[t][neut_t])
7: S2 = set(T[t][anti_t])
8: S3 = set(T[neut_t][t])
9: S4 = set(T[anti_t][t])
10: S5 = set(list([t]))
11: S6 = set(list([neut_t]))
12: if S5.issubset(S1 & S3) and S6.issubset(S2 & S4):
13: test.append([t, neut_t, anti_t])
14: test2 = test
15: test1 = set([test2[i][0] for i in range(len(test2))])
16: if test1 == set([x for x in range(3)]):
17: print('{0} is an Net-LA-semihypergroup and hyper neutrosophic-triplet are {1}'.format(T, test2))
```

Run: programm 2

C:\Users\Think\Anaconda3\python.exe C:/Users/Think/PycharmProjects/1/program2.py

[[[0],[0],[0]], [[0], [1], [0]], [[0], [0], [0,2]]] is an Net-LA-semihypergroup and hyper neutrosophic-triplet are [[0,0,0], [0,0,1], [0,0,2], [1,1,1], [2,2,2]]

Process finished with exit code 0

Figure 2. The result of Python program 2.

Example 2. Suppose R is the set of real numbers, the binary hypergroupoid $(R, *)$ is as follows.

$$x * y = \begin{cases} (x, y) & x < y, \\ (y, x) & y < x, \\ x & x = y. \end{cases}$$

for all $x, y \in R$, where (x, y) is the open interval.

When $z < x < y$,

$$(x * y) * z = \bigcup_{s \in (x * y)} (s * z) \cup \bigcup_{s \in (x, y)} (z, s) = (z, y)$$

$$\begin{aligned} (z * y) * x &= \bigcup_{t \in (z * y)} (t * x) = \bigcup_{t \in (z, y)} (t * x) = \left[\bigcup_{t \in (z, x)} (t * x) \right] \cup \left[\bigcup_{t=x} (t * x) \right] \cup \left[\bigcup_{t \in (x, y)} (t * x) \right] \\ &= \left[\bigcup_{t \in (z, x)} (t, x) \right] \cup (\{x\}) \cup \left[\bigcup_{t \in (x, y)} (x, t) \right] = (z, x) \cup (\{x\}) \cup (x, y) = (z, y) = (x * y) * z. \end{aligned}$$

In the same way, we have

$$(x * y) * z = (z * y) * x,$$

for all $x, y, z \in R$. Hence $(R, *)$ is an LA-semihypergroup. On the other hand, Since

$$x \in (x * x) \cap (x * x), x \in (x * x) \cap (x * x),$$

for any given $x \in R$, $x \in \{\}_{neut(x)}$, $x \in \{\}_{anti(x)_x}$. By Definition 11, $(R, *)$ is an NET-LA-semihypergroup.

Example 3. Put $L = \{0, 1, 2\}$, the binary hypergroupoid $(L, *)$ is as follows (see Table 2).

Table 2. The binary hypergroupoid $(L, *)$.

$*$	0	1	2
0	0	0	0
1	0	0	0
2	0	0	$\{0, 1\}$

By Python program, $(L, *)$ is an LA-semihypergroup. In addition, we get

$$1 \notin (0 * 1) \cap (1 * 0), 1 \notin (1 * 1) \cap (1 * 1), 1 \notin (2 * 1) \cap (1 * 2).$$

This shows that $\{\}_{neut(1)} = \phi$. By Definition 11, $(L, *)$ is not an NET-LA-semihypergroup.

Remark 1. Every NET-LA-semihypergroup is an LA-semihypergroup but not vice versa.

Example 4. Put $L = \{0, 1, 2, 3\}$, the binary hypergroupoid $(L, *)$ is as follows (see Table 3).

Table 3. The binary hypergroupoid $(L, *)$.

$*$	0	1	2	3
0	0	0	0	$\{0, 1, 2, 3\}$
1	0	0	0	$\{0, 1, 2, 3\}$
2	0	0	$\{0, 1\}$	$\{2, 3\}$
3	$\{1, 2, 3\}$	$\{0, 1, 2, 3\}$	$\{2, 3\}$	$\{0, 3\}$

By Python program 3 and Python program 4, $(L, *)$ is both an LA-semihypergroup (please see Figure 3) and an NET-LA-semihypergroup (please see Figure 4). In addition,

$$(0, 0, 0), (0, 0, 1), (0, 0, 2), (0, 1, 3), (0, 2, 3), (1, 3, 3), (2, 3, 3)$$

$(3, 0, 1), (3, 0, 3), (3, 1, 0), (3, 1, 1), (3, 2, 0), (3, 2, 1), (3, 2, 2), (3, 3, 0), (3, 3, 1), (3, 3, 2), (3, 3, 3)$ are all hyper neutrosophic-triplets (please see Figure 4). Let $M = \{0, 1, 2\} \subseteq L$, then $(M, *)$ is a sub LA-semihypergroup of $(L, *)$. From Example 3, $(M, *)$ is not an NET-LA-semihypergroup.

Python program 3 Verification of LA-semihypergroup 2

```

1: T = [ [[0],[0],[0],[0,1,2,3]], [[0], [0], [0],[0,1,2,3]], [[0], [0], [0,1],[2,3]], [[1,2,3],[0,1,2,3],[2,3],[0,3]] ]
2: count = 0
3: for x in range(4):
4:   for y in range(4):
5:     for z in range(4):
6:       T1 = T[x][y]
7:       T2 = set()
8:       k1 = len(T1)
9:       S3 = set(T[neut_t][t])
10:      for m in range(k1):
11:        T2 = set(T[T1[m]][z]).union(T2)
12:      T3 = T[z][y]
13:      T4 = set()
14:      k2 = len(T3)
15:      for n in range(k2):
16:        T4 = set(T[T3[n]][x]).union(T4)
17:      if T2 == T4:
18:        count += 1
19:      while count == 4**3:
20:        print('(T,*) is an LA-semihypergroup.')
21:        break

```

Run: program 3

C:\Users\Think\Anaconda3\python.exe C:/Users/Think/PycharmProjects/1/program3.py

$(T, *)$ is an LA-semihypergroup.

Process finished with exit code 0

Figure 3. The result of Python program 3.

Python program 4 Verification of NET-LA-semihypergroup 2

```

1: T = [ [[0],[0],[0],[0,1,2,3]], [[0], [0], [0],[0,1,2,3]], [[0], [0], [0,1],[2,3]], [[1,2,3],[0,1,2,3],[2,3],[0,3]] ]
2: test = []
3: for t in range(4):
4:   for neut_t in range(4):
5:     for anti_t in range(4):
6:       S1 = set(T[t][neut_t])
7:       S2 = set(T[t][anti_t])
8:       S3 = set(T[neut_t][t])
9:       S4 = set(T[anti_t][t])
10:      S5 = set(list([t]))
11:      S6 = set(list([neut_t]))
12:      if S5.issubset(S1 & S3) and S6.issubset(S2 & S4):
13:        test.append([t, neut_t, anti_t])
14:      test2 = test
15:      test1 = set([test2[i][0] for i in range(len(test2))])
16:      if test1 == set([x for x in range(3)]):
17:        print('(T,*) is an NET-LA-semihypergroup and hyper neutrosophic-triplet are {}'.format(test2)).

```

Run: program 4

C:\Users\Think\Anaconda 3\python.exe C:/Users/Think/PycharmProjects/1/proram4.py
 (T,*) is an NET-LA-semihypergroup and hyper neutrosopic-triplet are [[0,0,0], [0,0,1], [0,0,2],
 [0,2,3], [1,3,3], [2,3,3], [3,0,1], [3,0,3], [3,1,0], [3,1,1], [3,2,0], [3,2,1], [3,2,2], [3,3,0], [3,3,1],
 [3,3,2], [3,3,3]]
 Process finished with exit code 0

Figure 4. The result of Python program 4.

Remark 2. From Example 4, we know that for a certain t in an NET-LA-semihypergroup, $|\{neut(x)\}|$ may be greater than or equal to one and for a certain $p \in \{neut(x)\}$, $|\{anti(x)_p\}|$ may be greater than or equal to one. According to the results of Example 4, we have

$$\begin{aligned} \{neut(0)\} &= \{0, 1, 2\}, \{anti(0)_0\} = \{0, 1, 2\}, \{anti(0)_1\} = \{3\}, \{anti(0)_2\} = \{3\} \\ \{neut(1)\} &= \{3\}, \{anti(1)_3\} = \{3\}; \{neut(2)\} = \{3\}, \{anti(2)_3\} = \{3\} \\ \{neut(3)\} &= \{0, 1, 2, 3\}, \{anti(3)_0\} = \{3\}, \{anti(3)_1\} = \{0, 1\}, \{anti(3)_2\} = \{0, 1, 2\}; \{anti(3)_3\} = \{0, 1, 2, 3\} \end{aligned}$$

Definition 12. $(L, *)$ is said to be an NET-LA-hypergroup if it is both an LA-hypergroup (see Definition 6) and an NET-LA-semihypergroup.

Proposition 1. Every LA-hypergroup is a regular LA-semihypergroup.

Proof. Since $(L, *)$ is an LA-hypergroup, to every $t \in L$, $t * L = L * t = L$. Thus

$$t \in L = L * t = t * L * t$$

By Definition 5, $(L, *)$ is a regular LA-semihypergroup. $\square$

Example 5. Put $L = \{0, 1, 2\}$, the binary hypergroupoid $(L, *)$ is as follows (see Table 4).

Table 4. The binary hypergroupoid $(L, *)$.

*	0	1	2
0	0	0	0
1	0	1	{0, 1}
2	0	{0, 1}	{2}

By Python program, $(L, *)$ is an LA-semihypergroup. Furthermore, we have

$$0 \in 0 * 0 * 0, 1 \in 1 * 2 * 1, 2 \in 2 * 2 * 2$$

By Definition 5, $(L, *)$ is a regular LA-semihypergroup. But

$$0 * L = 0 \neq L$$

By Definition 6, $(L, *)$ is not an LA-hypergroup.

Remark 3. From Example 5, a regular LA-semihypergroup is not necessarily an LA-hypergroup.

Proposition 2. Every NET-LA-semihypergroup is a regular LA-semihypergroup.

Proof. Suppose $(L, *)$ is an NET-LA-semihypergroup, then to any given $a \in L$, there are $p \in \{_{neut(a)} \subseteq L$ and $q \in \{_{anti(a)_p} \subseteq L$ such that

$$a \in (p * a) \cap (a * p)$$

$$p \in (q * a) \cap (a * q)$$

Hence

$$a \in (p * a) \text{ and } p \in (a * q)$$

that is

$$a \in p * a \in (a * q) * a$$

By Definition 5, $(L, *)$ is a regular LA-semihypergroup. $\square$

Example 6. Put $L = \{0, 1, 2\}$, the binary hypergroupoid $(L, *)$ is as follows (see Table 5).

Table 5. The binary hypergroupoid $(L, *)$.

$*$	0	1	2
0	0	0	0
1	0	2	2
2	0	$\{0,1,2\}$	$\{0,1,2\}$

By Python program, $(L, *)$ is an LA-semihypergroup. Furthermore, we have

$$0 \in 0 * 0 * 0, 1 \in 1 * 2 * 1, 2 \in 2 * 1 * 2$$

By Definition 5, $(L, *)$ is a regular LA-semihypergroup. But

$$1 \notin (0 * 1) \cap (1 * 0), 1 \notin (1 * 1) \cap (1 * 1), 1 \notin (2 * 1) \cap (1 * 2)$$

This shows that $\{_{neut(1)} = \emptyset$. By Definition 11, $(L, *)$ is not an NET-LA-semihypergroup.

Remark 4. From Example 6, a regular LA-semihypergroup is not necessarily an NET-LA-semihypergroup.

Example 7. Put $L = \{0, 1, 2\}$, the binary hypergroupoid $(L, *)$ is as follows (see Table 6).

Table 6. The binary hypergroupoid $(L, *)$.

$*$	0	1	2
0	0	0	0
1	0	1	0
2	0	0	$\{0,2\}$

By Python program, $(L, *)$ is an LA-semihypergroup. Furthermore, we get

$$(0, 0, 0), (0, 0, 1), (0, 0, 2), (1, 1, 1), (2, 2, 2)$$

are all hyper neuromorphic-triplets. By Definition 11, $(L, *)$ is an NET-LA-semihypergroup. But

$$0 * L = 0 \neq L$$

By Definition 6, $(L, *)$ is not an LA-hypergroup.

Example 8. Put $L = \{0, 1, 2\}$, the binary hypergroupoid $(L, *)$ is as follows (see Table 7).

Table 7. The binary hypergroupoid $(L, *)$.

$*$	0	1	2
0	0	$\{0,1,2\}$	$\{0,1,2\}$
1	0	$\{0,2\}$	$\{1,2\}$
2	$\{0,1,2\}$	$\{0,2\}$	$\{0,1,2\}$

By Python program, $(L, *)$ is an LA-semi hypergroup. Furthermore, we get

$$0 * L = L * 0 = L, 1 * L = L * 1 = L, 2 * L = L * 2 = L$$

By Definition 6, $(L, *)$ is an LA-hypergroup. But

$$1 \notin (0 * 1) \cap (1 * 0), 1 \notin (1 * 1) \cap (1 * 1), 1 \notin (2 * 1) \cap (1 * 2)$$

This shows that $\{1\}_{\text{neut}(1)} = \emptyset$. By Definition 11, $(L, *)$ is not an NET-LA-semihypergroup.

Proposition 3. Every regular LA-hypergroup is an NET-LA-hypergroup.

Example 9. Put $L = \{0, 1, 2\}$, the binary hypergroupoid $(L, *)$ is as follows (see Table 8).

Table 8. The binary hypergroupoid $(L, *)$.

$*$	0	1	2
0	$\{1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{0,1,2\}$	$\{0,2\}$	$\{0,2\}$
2	$\{0,1\}$	$\{1,2\}$	$\{0,1\}$

By Python program, $(L, *)$ is an LA-semihypergroup. Furthermore, we get

$$(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (1, 0, 0), (1, 0, 1), (2, 1, 0), (2, 1, 2)$$

are all hyper neutrosophic-triplets, and

$$0 * L = L * 0 = L, 1 * L = L * 1 = L, 2 * L = L * 2 = L$$

by Definition 12, $(L, *)$ is an NET-LA-hypergroup. But

$$0 \notin (0 * 0) \cap (0 * 0), 1 \notin (1 * 1) \cap (1 * 1), 2 \notin (2 * 2) \cap (2 * 2)$$

This shows that the identity of $(L, *)$ does not exist. By Definition 8, $(L, *)$ is not a regular LA-hypergroup.

Based on the above, the relationships of LA-semihypergroup, regular LA-semihypergroup, LA-hypergroup, NET-LA-semihypergroup, NET-LA-hypergroup and regular LA-hypergroup, can be represented by the flowing Figure 5.

Figure 5. The relationships of various LA-semihypergroups.

Proposition 4. An NET-LA-semihypergroup $(L, *)$ is both an LNET-LA-semihypergroup and a RNET-LA-semihypergroup.

Proof. Since $(L, *)$ is an NET-LA-semihypergroup, to any given $a \in L$, there are $s \in \{neut(a)\}$ and $t \in \{anti(a)_s\}$ such that

$$a \in (s * a) \cap (a * s) \text{ and } s \in (t * a) \cap (a * t).$$

Hence $a \in (s * a)$ and $s \in (t * a)$, This shows

$$s \in \{neut(a)\} \in \{anti(a)_s\}.$$

Thus $(L, *)$ is an LNET-LA-semihypergroup. In the same way, we can prove that $(L, *)$ is also a RNET-LA-semihypergroup. $\square$

Example 10. Put $L = \{0, 1, 2\}$, the binary hypergroupoid $(L, *)$ is as follows (see Table 9).

Table 9. The binary hypergroupoid $(L, *)$.

*	0	1	2
0	0	0	0
1	0	2	2
2	0	$\{1, 2\}$	$\{1, 2\}$

By Python program, $(L, *)$ is an LA-semihypergroup and

$$(0, 0, 0), (0, 0, 1), (0, 0, 2), (1, 2, 1), (1, 2, 2), (2, 1, 2), (2, 2, 1), (2, 2, 2)$$

are all left-hyper neutrosophic-triplets;

$$(0, 0, 0), (0, 0, 1), (0, 0, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)$$

are all right-hyper neutrosophic-triplets;

$$(0, 0, 0), (0, 0, 1), (0, 0, 2), (2, 1, 2), (2, 2, 1), (2, 2, 2)$$

are all hyper neutrosophic-triplets. By Definition 11, $(L, *)$ is an LNET-LA-semihypergroup but it is neither a RNET-LA-semihypergroup nor an NET-LA-semihypergroup.

Example 11. Put $L = \{0, 1, 2\}$, the binary hypergroupoid $(L, *)$ is as follows (see Table 10).

Table 10. The binary hypergroupoid $(L, *)$.

$*$	0	1	2
0	0	$\{0, 1, 2\}$	$\{0, 1, 2\}$
1	2	2	$\{1, 2\}$
2	$\{0, 1, 2\}$	$\{0, 2\}$	$\{0, 1, 2\}$

By Python program, $(L, *)$ is an LA-semihypergroup and

$$(0, 0, 0), (0, 0, 2), (0, 2, 1), (0, 2, 2), (1, 0, 0), (1, 0, 2), (2, 0, 0)$$

$$(2, 0, 2), (2, 1, 0), (2, 1, 1), (2, 1, 2), (2, 2, 0), (2, 2, 1), (2, 2, 2)$$

are all left-hyper neutrosophic-triplets;

$$(0, 0, 0), (0, 0, 1), (0, 0, 2), (0, 1, 1), (0, 1, 2), (0, 2, 1), (0, 2, 2), (1, 2, 0), (1, 2, 1)$$

$$(1, 2, 2), (2, 0, 0), (2, 0, 1), (2, 0, 2), (2, 1, 0), (2, 1, 2), (2, 2, 0), (2, 2, 1), (2, 2, 2)$$

are all right-hyper neutrosophic-triplets; But

$$1 \notin (0 * 1) \cap (1 * 0), 1 \notin (1 * 1) \cap (1 * 1), 1 \notin (2 * 1) \cap (1 * 2)$$

This shows that $\{_{\text{neut}(1)} = \emptyset$. By Definition 11, $(L, *)$ is both an LNET-LA-semihypergroup and a RNET-LA-semihypergroup but not an NET-LA-semihypergroup. Moreover, from Example 11, we know that

$$\{_{\text{Ineut}(0)} = \{0, 2\}, \{_{\text{lanti}(0)_0} = \{0, 2\}, \{_{\text{lanti}(0)_2} = \{1, 2\}$$

$$\{_{\text{Ineut}(1)} = \{0\}, \{_{\text{lanti}(1)_0} = \{0, 2\}$$

$$\{_{\text{Ineut}(2)} = \{0, 1, 2\}, \{_{\text{lanti}(2)_0} = \{0, 2\}, \{_{\text{lanti}(2)_1} = \{0, 1, 2\}, \{_{\text{lanti}(2)_2} = \{0, 1, 2\}$$

These means that for a certain x in an LNET-LA-semihypergroup, $|\{_{\text{Ineut}(x)}|$ may be greater than or equal to one and for a certain $p \in \{_{\text{Ineut}(x)}$, $|\{_{\text{lanti}(x)_p}|$ may be greater than or equal to one. There are similar conclusions in RNET-LA-semihypergroup. In addition, for a certain x in an LA-semihypergroup, if $s \in \{_{\text{Ineut}(x)}$ (or $s \in \{_{\text{rneut}(x)}$), then s may be not in $\{_{\text{rneut}(x)}$ (or $\{_{\text{Ineut}(x)}$). By Example 11, we have $1 \in \{_{\text{rneut}(0)}$ but $1 \notin \{_{\text{Ineut}(0)}$.

Remark 5. Non-LNET-LA-semihypergroup (or Non-RNET-LA-semihypergroup) is not an NET-LA-semihypergroup. $(L, *)$ is both an LNET-LA-semihypergroup and a RNET-LA-semihypergroup but it is not necessarily an NET-LA-semihypergroup.

Based on the above, the relationships of NET-LA-semihypergroup, RNET-LA-semihypergroup and LNET-LA-hypergroup, can be represented by Figure 6.

Figure 6. The relationships of various LA-semihypergroups.

3. Strong Pure Neutrosophic Extended Triplet LA-Semihypergroups (SPNET-LA-Semihypergroups)

Definition 13. An LA-semihypergroup $(L, *)$ is said to be

- (a) a pure left neutrosophic extended triplet LA-semihypergroup (PLNET-LA-semihypergroup), if to any given $a \in L$, there are $p \in L$ and $q \in L$, in such a way that

$$a = p * a \text{ and } p = q * a$$

- (b) a pure right neutrosophic extended triplet LA-semihypergroup (PRNET-LA-semihypergroup), if to any given $a \in L$, there are $s \in L$ and $t \in L$, in such a way that

$$a = a * s \text{ and } s = a * t$$

- (c) a pure neutrosophic extended triplet LA-semihypergroup (PNET-LA-semihypergroup), if to any given $a \in L$, there are $m \in L$ and $n \in L$, in such a way that

$$a = (m * a) \cap (a * m) \text{ and } m = (n * a) \cap (a * n)$$

- (d) a strong pure neutrosophic extended triplet LA-semihypergroup (SPNET-LA-semihypergroup), if to any given $a \in L$, there are $m \in L$ and $n \in L$, in such a way that

$$a = m * a = a * m \text{ and } m = n * a = a * n$$

Proposition 5. Every SPNET-LA-semihypergroup is a PNET-LA-semihypergroup; Every PNET-LA-semihypergroup is an NET-LA-semihypergroup. Every PLNET-LA-semihypergroup is an LNET-LA-semihypergroup; Every PRNET-LA-semihypergroup is a RNET-LA-semihypergroup.

Remark 6. From Proposition 5, we know that the signs in the Definition 11 can still be used, such as $\{_{lneut(a)}$, $\{_{rneut(a)}$, $\{_{neut(a)}$, $\{_{lantia(a)}_p$, $\{_{rantia(a)}_p$, etc.

Proposition 6. Every commutative PNET-LA-semihypergroup is an SPNET-LA-semihypergroup; Every commutative PLNET-LA-semihypergroup(or PRNET-LA-semihypergroup) is an SPNET-LA-semihypergroup.

Proposition 7. Suppose $(L, *)$ is an SPNET-LA-semihypergroup, for any $a, b, c \in L$,

- (1) if $s \in \{_{neut(a)}$, then s is unique and $s * s = s$;
- (2) if $s = neut(a)$, then $neut(s) = s$ and $s \in \{_{anti(s)}_s$;

- (3) if $s = neut(a)$, $t \in \{\}_{anti(a)_s}$, $r \in \{\}_{anti(s)_s}$, then $r * t \subseteq \{\}_{lanti(a)_s}$;
 (4) if $s = neut(a)$, $t \in \{\}_{anti(a)_s}$, then $s * t \subseteq \{\}_{lanti(a)_s}$;
 (5) if $p = neut(a)$, $s = neut(b)$, $q \in \{\}_{anti(a)_p}$, $t \in \{\}_{anti(b)_s}$ and $|a * b| = |p * s| = 1$, then

$$neut(a * b) = p * sandq * t \subseteq \{\}_{anti(a*b)_{p*s}}$$

- (6) if $s = neut(a) = neut(b)$, $q \in \{\}_{anti(a)_s}$, $t \in \{\}_{anti(b)_s}$ and $|a * b| = 1$, then

$$neut(a * b) = sandq * t \subseteq \{\}_{anti(a*b)_s}$$

- (7) if $neut(a) = neut(b)$, then $a * b = b * a$;
 (8) then $s * b = s * c$ if $b * a = c * a$, where $s = neut(a)$;
 (9) if $s = neut(a)$, $q, t \in \{\}_{anti(a)_s}$, then $s * q = s * t$.

Proof. (1) Suppose there are $s, p \in \{\}_{neut(a)}$, $t \in \{\}_{anti(a)_s}$, $q \in \{\}_{anti(a)_p}$. $(L, *)$ is an SPNET-LA-semihypergroup, hence

$$a = s * a = a * s, s = t * a = a * t$$

$$a = a * p = p * a, p = a * q = q * a$$

we get

$$s * p = (t * a) * p = (p * a) * t = a * t = s$$

$$p * s = (q * a) * s = (s * a) * q = a * q = p$$

$$s * p = (a * t) * (q * a) = (a * q) * (t * a) = p * s$$

Thus $p = s$, it implies s is unique and $s * s = s$.

(2) From (1), if $s = neut(a) \in L$, then $s * s = s * s = s$, This implies $neut(s) = s$ and $s \in \{\}_{anti(s)_s}$.

(3) For any given $a \in L$, if $s = neut(a)$, $t \in \{\}_{anti(a)_s}$, then

$$a = a * s = s * a, s = a * t = t * a$$

On the other hand, from $neut(s) = s$ and $r \in \{\}_{anti(s)_s}$, we get

$$s = s * s = s * s, s = r * s = s * r$$

Thus

$$\bigcup_{m \in r * t} (m * a) = (r * t) * a = (a * t) * r = s * r = s$$

where $m * a$ is a nonempty set, hence for any $m \in r * t$, $m * a = s$. This implies $m \in \{\}_{lanti(a)_s}$. In other words, $r * t \subseteq \{\}_{lanti(a)_s}$.

(4) By (2), (3), we can get (4).

(5) if $p = neut(a)$, $s = neut(b)$, $q \in \{\}_{anti(a)_p}$, $t \in \{\}_{anti(b)_s}$, then

$$(p * s) * (a * b) = (p * a) * (s * b) = a * b$$

$$(a * b) * (p * s) = (a * p) * (b * s) = a * b.$$

That is,

$$(p * s) * (a * b) = (a * b) * (p * s) = a * b. \quad (14)$$

On the other hand,

$$\bigcup_{l \in q * t} [(a * b) * l] = (a * b) * (q * t) = (a * q) * (b * t) = p * s,$$

where $(a * b) * l$ is a nonempty set, $|a * b| = 1$ and $|p * s| = 1$. Hence for any $l \in q * t$, $(a * b) * l = p * s$. In the same way, we can prove that for any $l \in q * t$, $l * (a * b) = p * s$. Thus for any $l \in q * t$,

$$l * (a * b) = (a * b) * l = p * s. \quad (15)$$

From (14), (15) and $|a * b| = |p * s| = 1$, we get $neut(a * b) = p * s$ and $q * t \subseteq \{anti(a * b)_{p * s}\}$.

(6) Let $p = s$ in Proposition 7 (5), we can get the conclusion.

(7) $(L, *)$ is an SPNET-LA-semihypergroup, hence for any given $a, b \in L$, there are $neut(a) = s$, $neut(b) = p$, $t \in \{anti(a)_s\}$, $q \in \{anti(b)_p\}$ such that

$$a = a * s = s * a, s = a * t = t * a$$

$$b = b * p = p * b, p = b * q = q * b.$$

If $s = p$, then we have

$$a * b = (a * s) * (b * p) = (a * b) * (s * p) = (a * b) * (s * s) = (a * b) * s = (s * b) * a = (p * b) * a = b * a.$$

(8) Suppose that $b * a = c * a$ for $a, b, c \in L$. There are $s = neut(a) \in L$ and $t \in \{anti(a)_s\}$. Multiply $b * a = c * a$ by t , we have

$$(b * a) * t = (c * a) * t$$

$$(t * a) * b = (t * a) * c$$

$$s * b = s * c$$

(9) For any given $a \in L$, there is $s = neut(a) \in L$, if $q, t \in \{anti(a)_s\}$, then

$$s * q = (t * a) * q = (q * a) * t = s * t.$$

□

Theorem 1. Suppose $(L, *)$ is a PRNET-LA-semihypergroup, for any $x \in L$,

(a) if $p \in \{rneut(x)\}$, $q \in \{ranti(x)_p\}$ and $|p * p| = 1$, then

$$p * \subseteq \{lneut(x) * \} \subseteq \{lanti(x)_{p * p}\}$$

and $(L, *)$ is an PLNET-LA-semihypergroup.

(b) if $p \in \{rneut(x)\}$, $q \in \{ranti(x)_p\}$, $p * p = p$ and $q \in p * q$, then

$$p = neutx \in \{anti(x)_p\}$$

and $(L, *)$ is an SPNET-LA-semihypergroup.

Proof. (1) Since $(L, *)$ is a PRNET-LA-semihypergroup, for any given $x \in L$, there are $p \in \{rneut(x)\}$ and $q \in \{ranti(x)_p\}$ such that

$$x = x * p, p = x * q$$

multiply $x = x * p$ by p , we have

$$x = x * p = (x * p) * p = (p * p) * x$$

In addition,

$$\bigcup_{s \in p * q} (s * x) = (p * q) * x = (x * q) * p = p * p$$

where $s * x$ is a nonempty set and $|p * p| = 1$. Thus for any $s \in p * q$, $s * x = p * p$. It means that for any $x \in L$, there are $p * p, s \in p * q$ such that

$$(p * p) * x = x, s * x = p * p$$

It shows that

$$p * p \subseteq \{ \}_{lneut(x)}, s \in p * q \subseteq \{ \}_{lanti(x)_{p * p}}$$

By Definition 11, $(L, *)$ is an LNET-LA-semihypergroup.

(2) By Theorem 1 (a),

$$p = p * p \in \{ \}_{lneut(x)} \\ q \in p * q \subseteq \{ \}_{lanti(x)_{p * p}} = \{ \}_{lanti(x)_p}$$

It shows that for any given $x \in L$, there is $p \in L$ such that

$$p * x = x \text{ and } q * x = p$$

On the other hand, $p \in \{ \}_{rneut(x)}, q \in \{ \}_{ranti(x)_p}$, we get

$$x = x * p \text{ and } x * q = p$$

Based on the above, for any given $x \in L$, there are p and q such that

$$x = x * p = p * x$$

$$p = x * q = q * x$$

That is,

$$p \in \{ \}_{neut(x)} \text{ and } q \in \{ \}_{anti(x)_p}$$

By Definition 11, $(L, *)$ is an SPNET-LA-semihypergroup. Applying Proposition 7 (1), we get $p = neut(x)$. $\square$

Example 12. Put $L = \{0, 1, 2\}$, the binary hypergroupoid $(L, *)$ is as follows (see Table 11).

Table 11. The binary hypergroupoid $(L, *)$.

$*$	0	1	2
0	0	1	$\{0,1,2\}$
1	1	0	$\{0,1,2\}$
2	$\{0,1,2\}$	$\{0,1,2\}$	2

By Python program, $(L, *)$ is an LA-semihypergroup. Furthermore, we have

$$rneut(0) = 0, rneut(1) = 0, rneut(2) = 2$$

$$ranti(0)_{rneut(0)=0}, ranti(1)_{rneut(1)=0}, ranti(2)_{rneut(2)=2} = 2$$

$$0 * 0 = 0, 0 * 1 = 0, 1 * 1 = 0$$

$$0 \in 0 * 0, 1 \in 0 * 1, 2 \in 2 * 2$$

By Theorem 1 (b), we know that $(L, *)$ is an SPNET-LA-semihypergroup.

Corollary 1. A PRNET-LA-semihypergroup $(L, *)$, which satisfies conditions of Theorem 1 (b), then $neut(p * s) = neut(p) * neut(s)$ if $|p * s| = |neut(p) * neut(s)| = 1$, where $p, s \in L$.

Proof. It follows from Theorem 1 (b) and Proposition 7 (5). $\square$

Corollary 2. *An idempotent PRNET-LA-semihypergroup is a PLNET-LA-semihypergroup.*

Proof. It follows from Theorem 1 (a). $\square$

Proposition 8. *An idempotent PRNET-LA-semihypergroup with pure left identity is a commutative SPNET-LA-semihypergroup and its pure left identity is pure right identity.*

Proof. Put e is a pure left identity of $(L, *)$. Then for any $t \in L$,

$$e * t = t,$$

by idempotent law, we get

$$t * e = (t * t) * e = (e * t) * t = t * t = t.$$

It shows that e is pure right identity of $(L, *)$. Furthermore, for any $m, n \in L$,

$$m * n = (m * e) * n = (n * e) * m = n * m.$$

It follows that $(L, *)$ satisfies commutative law.

On the other hand, $(L, *)$ is a PRNET-LA-semihypergroup. Hence for any given $a \in L$, there are $s \in \{\}_{rneut(a)}$ and $t \in \{\}_{ranti(a)_s}$ such that

$$a = a * s, s = a * t.$$

Applying commutative law, we get

$$a = a * s = s * a, s = a * t = t * a.$$

Thus $(L, *)$ a commutative SPNET-LA-semihypergroup. $\square$

Proposition 9. *Suppose $(L, *)$ is a PRNET-LA-semihypergroup (or a PLNET-LA-semihypergroup) with pure right identity, then pure right identity is pure left identity and $(L, *)$ is a commutative Net-semihypergroup.*

Proof. Put e is a pure right identity of $(L, *)$, Then for any given $t \in L$,

$$t * e = t,$$

we have

$$t = t * e = (t * e) * e = (e * e) * t = e * t.$$

This shows that e is pure left identity of $(L, *)$. Furthermore, for any $l, m, n \in L$,

$$m * n = (m * e) * n = (n * e) * m = n * m$$

$$(l * m) * n = (l * m) * (e * n) = (l * e) * (m * n) = l * (m * n).$$

It follows that $(L, *)$ satisfies commutative law and associative law. In addition, $(L, *)$ is a PRNET-LA-semihypergroup. Hence for any given $s \in L$, there are $p \in \{\}_{rneut(s)}$ and $q \in \{\}_{ranti(s)_p}$ such that

$$s = s * p, p = s * q.$$

Applying commutative law, we get

$$s = s * p = p * s, p = s * q = q * s.$$

By Definition 10, $(L, *)$ is a commutative NET-semihypergroup. $\square$

Theorem 2. Let $(L, *)$ be a PRNET-LA-semihypergroup, which satisfies the following conditions:

(1) for any $t \in L$, there are $p \in \{\}_{neut(t)}$, $q \in \{\}_{ranti(t)_p}$ such that

$$p * p = p, q = p * q; \quad (16)$$

By condition (1), for a certain q in (1), there are $r \in \{\}_{rneut(q)}$, $l \in \{\}_{ranti(q)_r}$ such that

$$r * r = r, l = r * l \quad (17)$$

(2) $|p * r| = 1$, where p in (16) and r in (17);

(3) for any $m, n \in L$, if $neut(m) = neut(n)$, then $|m * n| = 1$.

Define an equivalent relation φ on L ,

$$m \varphi n \text{ if and only if } neut(m) = neut(n)$$

Then

(a) (To every $t \in L$, $([t], *)$ is a sub NET-LA-semihypergroup of $(L, *)$, in which $[t]$ is the equivalent class of t based on equivalent relation φ ;

(b) To every $t \in L$, $([t], *)$ is a regular LA-hypergroup.

Proof. (a) Firstly, by Theorem 1 (b) and Theorem 2's condition (1), we know that $(L, *)$ is an SPNET-LA-semihypergroup. Suppose $m, n \in [t]$, by Theorem 2's condition (3), we have

$$neut(m) = neut(n) = neut(t) \text{ and } |m * n| = 1$$

Applying Proposition 7 (6), we get $neut(m * n) = neut(t)$. It shows that $m * n \in [t]$.

Secondly, applying Proposition 7 (2), we have

$$neut(neut(m)) = neut(neut(t)) = neut(t)$$

It means that for any $m \in [t]$, $neut(m) \in [t]$.

Lastly, by Theorem 2's condition (1) and Theorem 1 (b), for any $m \in [t] \subseteq L$, there is $q \in L$ such that

$$q = neut(m) * q \in \{\}_{anti(m)_{neut(m)}} \quad (18)$$

and for the q in (18), there are $r \in \{\}_{rneut(q)}$, $l \in \{\}_{ranti(q)_r}$ such that

$$r * r = r, l = r * l$$

$$\text{and } r = neut(q). \quad (19)$$

By Theorem 2's condition (2) and (19), we get

$$|neut(m) * r| = |neut(m) * neut(q)| = |neut(neut(m)) * neut(q)| = 1.$$

Applying Proposition 7 (5), we get

$$neut(neut(m) * q) = neut(neut(m)) * neut(q) = neut(m) * neut(q)$$

$$= neut(m * q) = neut(neut(m)) = neut(m) = neut(t).$$

This implies $q = neut(m) * q \in \{\}_{anti(m)_{neut(m)}} \in [t]$. Thus $([t], *)$ is a sub SPNET-LA-semihypergroup.

(b) Firstly, from (a), for any given $t \in L$, $([t], *)$ is a sub-SPNET-LA-semihypergroup of $(L, *)$. By the definition of ϕ , if $m \in [t]$, then for any $n \in [t]$, $\text{neut}(m) = \text{neut}(n) = \text{neut}(t)$. Applying Proposition 7 (7), we get

$$m * n = n * m.$$

That is $m * [t] = [t] * m$.

Secondly, for any $s \in [t]$, $s * m \in [t]$, hence $[t] * m \subseteq [t]$; On the other hand, by proof of (a), we know that for any $m \in [t]$, there is $q \in [t]$ such that

$$q = \text{neut}(m) * q \setminus \{\text{anti}(m)_{\text{neut}(m)}\}$$

hence for any $s \in [t]$, $s * q \in [t]$. Thus

$$s = \text{neut}(s) * s = \text{neut}(m) * s = (m * q) * s = (s * q) * m \subseteq [t] * m.$$

That is, $[t] \subseteq [t] * m$. Thus $[t] = [t] * m = m * [t]$. It implies that $([t], *)$ is a LA-hypergroup.

Lastly, it can be easily proved that $\text{neut}(t)$ is a scalar identity of $([t], *)$ and for every $l \in [t]$ has at least one inverse. By Definition 8, $([t], *)$ is a regular LA-hypergroup. $\square$

Corollary 3. If a PRNET-LA-semihypergroup $(L, *)$ which satisfies conditions of Theorem 2 and ϕ is the equivalence relation on L defined in Theorem 2, then L/ϕ is the partition of set L .

Example 13. Put $L = \{0, 1, 2, 3, 4\}$, the binary hypergroupoid $(L, *)$ is as follows (see Table 12).

Table 12. The binary hypergroupoid $(L, *)$.

*	0	1	2	3	4
0	0	1	$\{0, 1, 2\}$	0	4
1	1	0	$\{0, 1, 2\}$	1	4
2	$\{0, 1, 2\}$	$\{0, 1, 2\}$	2	$\{0, 1, 2\}$	4
3	0	1	$\{0, 1, 2\}$	3	4
4	4	4	4	4	4

By Python program, $(L, *)$ is LA-semihypergroup. Firstly, we have

$$\text{rneut}(0) = 0, \text{rneut}(1) = 0, \text{rneut}(2) = 2, \text{rneut}(3) = 3, \text{rneut}(4) = 4$$

$$\text{ranti}(0)_{\text{rneut}(0)=0} = 0, \text{ranti}(1)_{\text{rneut}(1)=0} = 1, \text{ranti}(2)_{\text{rneut}(2)=2} = 2, \text{ranti}(3)_{\text{rneut}(3)=3} = 3, \text{ranti}(4)_{\text{rneut}(4)=4} = 4$$

$$0 * 0 = 0, 0 * 1 = 0, 1 * 0 = 0, 1 * 1 = 0, 2 * 2 = 2, 3 * 3 = 3, 4 * 4 = 4$$

$$0 = 0 * 0, 1 = 0 * 1, 2 = 2 * 2, 3 = 3 * 3, 4 = 4 * 4.$$

These means that Theorem 2's condition 1) hold; Secondly, we get

$$\left| \text{rneut}(0) * \text{rneut}(\text{ranti}(0)_{\text{rneut}(0)=0}) \right| = \left| \text{rneut}(0) * \text{rneut}(0) \right| = |0| = 1$$

$$\left| () * \text{rneut}(\text{ranti}(1)_{\text{rneut}(1)=0}) \right| = \left| \text{rneut}(1) * \text{rneut}(1) \right| = |0| = 1$$

$$\left| () * \text{rneut}(\text{ranti}(2)_{\text{rneut}(2)=2}) \right| = \left| \text{rneut}(2) * \text{rneut}(2) \right| = |2| = 1$$

$$\left| () * \text{rneut}(\text{ranti}(3)_{\text{rneut}(3)=3}) \right| = \left| \text{rneut}(3) * \text{rneut}(3) \right| = |3| = 1$$

$$\left| () * \text{rneut}(\text{ranti}(4)_{\text{rneut}(4)=4}) \right| = \left| \text{rneut}(4) * \text{rneut}(4) \right| = |4| = 1$$

These means that Theorem 2's condition (2) hold. Lastly,

$$rneut(0) = rneut(1) = 0, |0 * 1| = 1$$

These means that Theorem 2's condition (3) hold. By Theorem 1 and Theorem 2, we know that $(L, *)$ is an SPNET-LA-semihypergroup and

$$L_1 = \{0, 1\} = [0] = [1], L_2 = \{2\} = [2], L_3 = \{3\} = [3], L_4 = \{4\} = [4]$$

$$L = L_1 \cup L_2 \cup L_3 \cup L_4$$

where $(L_1, *), (L_2, *), (L_3, *), (L_4, *)$ are all regular LA-hypergroups.

Definition 14. An NET-LA-semihypergroup $(L, *)$ satisfies weak commutative law, if for any $y \in L$,

$$p * y = y * p, q * x = x * q$$

where x is any element of set L , $p \in \{neut(x)\}$, $q \in \{anti(x)_p\}$.

Proposition 10. An SPNET-LA-semihypergroup $(L, *)$ satisfies weak commutative law if and only if it is a commutative.

Proof. If $(L, *)$ is a weak commutative, then for any $x, y \in L$, $l \in \{neut(x)\}$, $m \in \{neut(y)\}$, we have

$$x * y = (x * l) * (y * m) = (l * x) * (y * m) = (l * y) * (x * m) = (y * l) * (m * x) = (y * m) * (l * x) = y * x$$

That is, $(L, *)$ is commutative. $\square$

4. Conclusions

In this study, we give the new notions of NET-LA-semihypergroup, NET-LA-hypergroup, LNET-LA-semihypergroup, RNET-LA-semihypergroup, PLNET-LA-semihypergroup, PRNET-LA-semihypergroup, PNET-LA-semihypergroup, SPNET-LA-semihypergroup, discuss the relationships of them (see Figures 5 and 6), get some special properties of SPNET-LA-semihypergroup (see Proposition 7). In particular, we prove that a RNET-LA-semihypergroup which satisfies certain conditions (the condition of asymmetry) be an SPNET-LA-semihypergroup and this SPNET-LA-semihypergroup is the union of some disjoint regular hypergroups, where every regular hypergroup is its subhypergroup (see Theorem 2). At last, we discuss the relationships of various NET-LA-semihypergroups (see Figure 7).

Figure 7. The relationships of various NET-LA-semihypergroups.

These studies help us to enhance the understanding of this hyperalgebraic structure about NET and tell us this hyper algebraic structure is a complex and unique structure. There is still a lot of unknown knowledge in this field to explore. In the future, we will discuss properties of NET-CA-semihypergroup.

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