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Optimal Design and Singularity Analysis of a Spatial Parallel Manipulator

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Abstract: Optimal design and singularity analysis are two important aspects of mechanism design, and they are discussed within a spatial parallel manipulator in this work. Resorting to matrix transformation, the parametric kinematic model is established, upon which the inverse position and Jacobian are analyzed. As for optimal design, dexterity and payload indices are taken into consideration. From the simulation results, two optimal configurations are obtained, namely, the star-shaped one and the T-shaped one, and they respectively own the best payload performance and the best dexterity performance. Moreover, the concept of shape singularity is introduced and generalized, which is a special type of singularity that will lead to the singularity in all configurations. The shape singularity of the proposed manipulator is indicated by dexterity index and identified by screw theory. A case study is presented to demonstrate the implication of the shape singularity. Both optimal and singular configurations are useful, and new devices can thus be envisaged for this type of application.

Keywords: parallel manipulator; optimal design; singularity; screw theory

1. Introduction

Parallel manipulators (PMs) are closed-loop mechanisms modeled as a set of serial kinematic chains connected in parallel to two rigid bodies, namely, the mobile platform (MP) and the base platform. Compared with conventional serial manipulators, PMs exhibit many favorable advantages, such as higher stiffness, smaller mobile mass, larger load-weight ratio, better orientation precision, and higher speed capability. On the other hand, PMs also suffer from some shortcomings, such as complex forward position problems, relatively small workspaces, limited manipulability and coupled dynamics [1,2].

In order to overcome the drawbacks above, PMs with fewer than six degrees of freedom (DOF) are considered [3]. For examples, Rezaei et al. [4] presented a 3-PSP (P, R, U and S represent prismatic, revolute, universal and spherical joints, and an underlined letter indicates an actuator.) robot with symmetric geometry, and the forward position problem has a unique solution. Zhao et al. [5] investigated the forward kinematics of the Gough-Stewart manipulator with natural coordinates. Bai et al. [6,7] proposed a U-shape 3-PPR planar PM with the purpose of maximizing the workspace. Tsai and Joshi [3] had undertaken a workspace optimization that considered the quality of the workspace of a spatial 3-UPU PM. Stock and Miller [8] focused on the optimal kinematic design of a linear Delta robot, aiming to reach a compromise between manipulability and workspace size. Xu and Li [9] converted the original 3-PRC translational PM into a non-overconstrained one in order to solve the movability problem. Liu et al. [10] studied a new family of spatial 3-DOF fully PMs, which can exhibit high rotational capability.

With the purpose of identifying the nature of the interdependence of the pose parameters and the geometric significance, three symmetric zero-torsion PMs were considered in Reference [11], among

which a spatial 3-PPS PM was introduced, while the closed-form solution to the direct kinematics was obtained. Singularity analysis of the 3-PPS PM was presented in Reference [12], with the consideration of the size of the workspace. A similar 3-PPS PM was reported in Reference [13], where the instantaneous mobility and constraint were analyzed. The 3-PPS PM was also proposed for the development of a dexterous 3-DOF force-controlled end-effector module which was available in Reference [14]. Moreover, the 3-PPS PM was chosen as a suitable candidate for nano-alignment applications with a research prototype developed in References [15,16]. The position and workspace analysis and optimal design of a 3-PPS were provided in Reference [17]. One can notice that the mechanisms mentioned above are only a special type of the 3-PPS PMs, which did not take the shape parameters into consideration.

One of the objectives of this work is to take shape parameters into consideration for optimal design. It is found that most manipulators studied are assumed as having a fixed shape of the platforms, while cases with different shapes of the platforms are rarely considered. It is thus desirable to study what will happen with a variety of shapes of the platforms. As for the generalized 3-PPS PM with shape parameters, two optimal configurations are obtained based on the performance evaluation.

Another objective of this work is to analyze the shape singularity of the 3-PPS PM. The shape singularity of the proposed manipulator is indicated by dexterity index and identified by screw theory. As reported in Reference [18], the shape singularity was found numerically within a planar PMs, while it is generalized from planar PMs to spatial PMs in this work.

The last objective is to explore how to utilize shape singularity in a novel mechanism design. While most singularity analyses were carried out with the purpose to identify the singular configurations and to eliminate singularity from the perspective of manipulator design and control, it would be of great significance if a singularity analysis could inspire novel mechanism design.

The rest of this paper is organized as follows. Section 2 explains the basic theory and background knowledge. Section 3 contains the kinematics analysis of the considered spatial 3-PPS PM. Optimal design and analysis of shape singularity are developed in Section 4. Discussion and conclusions are presented in the final section.

2. Materials and Methods

In the design process, dexterity, payload and compliance capability are very important for a machine, and these criteria are considered for the proposed 3-PPS PM. In terms of the optimal design of the proposed mechanism, two performance indices are considered, namely, dexterity and payload index. Moreover, the compliance index is applied to evaluate the compliance quality of shape singularity.

2.1. Dexterity Index

Dexterity index is an important performance index for mechanism design, which is defined as the mechanism's ability to change the pose arbitrarily, and it can be expressed in terms of condition number [19]. For a Jacobian matrix J of full rank, the corresponding condition number can be expressed as:

$$k = \|J\| \|J^{-1}\| \quad (1)$$

The value of condition number will range from one to infinity, without an upper bound. So the reciprocal of the condition number called a local conditioning index (LCI) is used more commonly.

$$LCI = \frac{1}{k} \quad (2)$$

LCI is pose-dependent, which means it is a local performance. In order to evaluate the global dexterity behavior of a mechanism over the entire workspace, the global conditioning index (GCI) is defined as:

$$\text{GCI} = \frac{\int_W \left(\frac{1}{k}\right) dW}{\int_W dW} \quad (3)$$

where W represents the entire workspace. Since the shape of the workspace is complex, it is not easy to obtain a closed form for the GCI, so the GCI is normally calculated numerically.

$$\text{GCI} \approx \frac{1}{N_W} \sum_{j=1}^{N_W} \frac{1}{k} \quad (4)$$

where N_W is equal to the number of the points uniformly distributed over the workspace. The larger the GCI is, the better the global dexterity performance of the mechanism is.

2.2. Payload Index

Payload capability is another important performance of PMs, which is applied to measure the manipulator's ability to stand the external force acted on the MP.

Referring to the virtual work principle [20], the relationship between virtual displacement $\delta \mathbf{q}$ of the joints and the virtual displacement $\delta \mathbf{x}$ of the MP can be expressed as:

$$\boldsymbol{\tau}^T \delta \mathbf{q} = \mathbf{f}^T \delta \mathbf{x} \quad (5)$$

where $\boldsymbol{\tau}$ is the external force, $\mathbf{f}$ is the driving force.

On the other hand, the relationship between $\delta \mathbf{q}$ and $\delta \mathbf{x}$ can be expressed in another form as:

$$\delta \mathbf{q} = \mathbf{J} \delta \mathbf{x} \quad (6)$$

which leads to:

$$\mathbf{f} = \mathbf{J}^T \boldsymbol{\tau} \quad (7)$$

Let $\mathbf{f}$ be a unit vector, the extremum of the norm of $\boldsymbol{\tau}$ can be obtained as:

$$P_{\min} = \sqrt{\min(|\lambda_{pi}|)} \quad (8)$$

where λ_{pi} are the eigenvalues of matrix $\mathbf{J}\mathbf{J}^T$, and $P_{\min}$ is the minimum external force that the mechanism can stand.

In order to measure the payload capability of a mechanism over the entire workspace, the global payload index (GPI) is defined as:

$$\text{GPI} \approx \frac{1}{N_W} \sum_{j=1}^{N_W} P_{\min} \quad (9)$$

The larger the GPI is, the better the payload performance of the mechanism is.

2.3. Compliance Index

As reported in [20,21], the stiffness matrix of PMs can be written as:

$$\mathbf{K} = \mathbf{J}^T \mathbf{K}_p \mathbf{J} \quad (10)$$

where $\mathbf{K}_p$ is a matrix describing the stiffness of the platform mounted with elastic elements. For the 3-DOF 3-PPS PM, $\mathbf{K}_p = \text{diag}(k_1, k_2, k_3)$.

The deflection of the MP which corresponds to the external force is defined as:

$$\Delta \mathbf{X} = \mathbf{K}^{-1} \mathbf{f} \quad (11)$$

It can be found that the extremum eigenvalue λ_{si} of the matrix $\mathbf{K}^{-T} \mathbf{K}^{-1}$ is the limit value of $\|\Delta \mathbf{X}\|^2$. Let $k_1 = k_2 = k_3 = 1$ and $\|\mathbf{f}\| = 1$, the maximum deformations can be expressed as:

$$S_{\max} = \sqrt{\max(|\lambda_{si}|)} \quad (12)$$

In order to evaluate the stiffness performance of a mechanism over the workspace, the global stiffness index (GSI) is defined as:

$$\text{GSI} \approx \frac{1}{N_W} \sum_{j=1}^{N_W} S_{\max} \quad (13)$$

For a mechanism, the reciprocal of the stiffness is referred to as compliance, so the larger the GSI is, the better the compliance performance of the mechanism is.

With the selected performance indices, the concrete flow for optimal design and shape singularity analysis is as shown in Figure 1. Firstly, the inverse position and Jacobian are analyzed based on the parametric kinematic model. Then, dexterity and payload performance are evaluated for optimal design. Moreover, shape singularity is identified by screw theory with its compliance performance analyzed. At last, the implication of shape singularity is discussed.

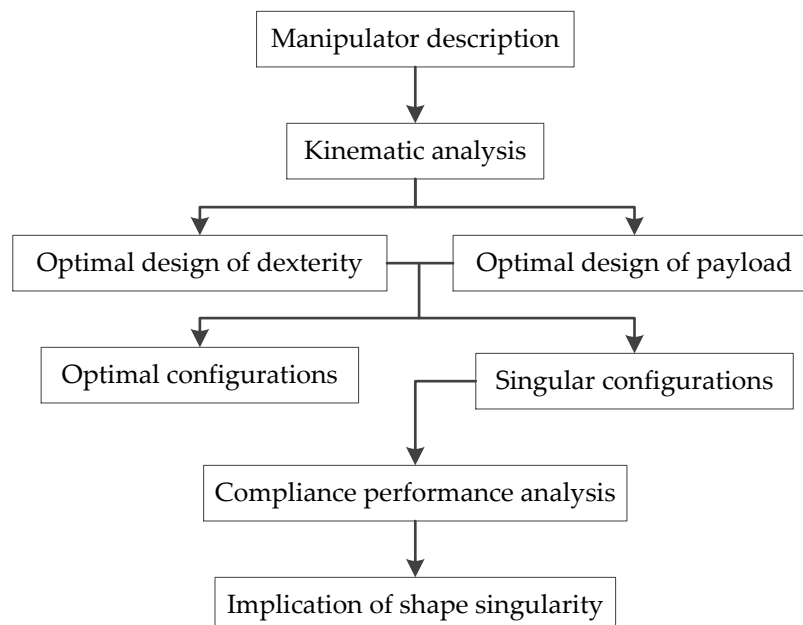


Figure 1. Flowchart for optimal design and shape singularity analysis.

3. Structure Description and Kinematic Analysis

The considered spatial 3-PPS PM is shown in Figure 2, and it consists of three identical PPS kinematic limbs, all connecting the MP to the base. The first prismatic joint in each limb is chosen as the active joint, and the axes of the passive prismatic joints g_i are perpendicular to the axes of the active prismatic joints f_i . Moreover, the shape of the base is parameterized, namely, the orientation of axes of the active prismatic joints is defined by pan angle δ ($0^\circ \leq \delta \leq 90^\circ$) and tilt angle γ ($0^\circ \leq \gamma \leq 90^\circ$). On the other hand, both the MP and base platform are modeled as equilateral triangles, with side lengths a and b , respectively.

3.1. Pose of the MP

The proposed 3-PPS PM is a 3-DOF mechanism, which means the pose (position and orientation) of the MP can be described by three independent kinematic parameters. For general 3-PPS PM ($\delta \neq 0^\circ$), $(x, y, z, \psi, \theta, \phi)$ is the pose of the MP with respect to the global frame. Here, z, ψ , and θ are chosen as the independent kinematic parameters, so the first step is to determine the pose of the MP with the three independent kinematic parameters.

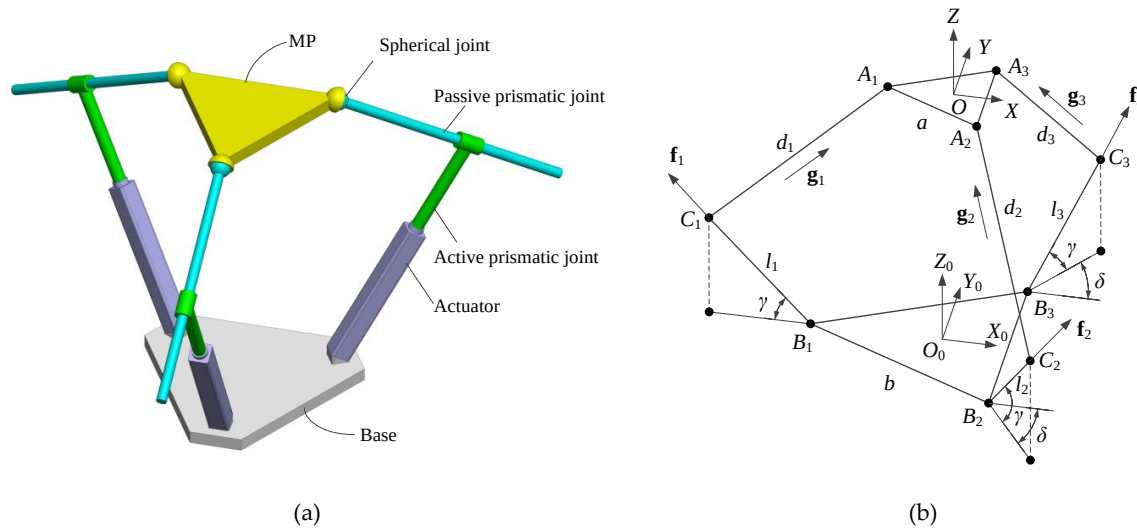


Figure 2. The 3-PPS parallel manipulator (PM): (a) The 3D model and (b) the kinematic description.

As depicted in Figure 2b, global frame $\{O_0\}$ is fixed on the base, with the origin coinciding with the centroid of the base, where Y_0 -axis is pointed along the direction of B_2B_3 , Z_0 -axis is perpendicular to the base. Local frame $\{O\}$ is attached to the centroid of the MP in a similar way.

By the given pose of the MP, the position vector of point A_i in the global frame results in:

$$\mathbf{a}_i = \mathbf{R}\mathbf{a}'_i + \mathbf{p} \quad (14)$$

where $\mathbf{a}'_i$ is the position vector of point A_i in the local frame, $\mathbf{R} = \mathbf{R}_z(\phi)\mathbf{R}_y(\theta)\mathbf{R}_x(\psi)$ is the rotation matrix, $\mathbf{p} = [x, y, z]^T$ is the position vector.

$$\mathbf{a}'_1 = \left[-\frac{\sqrt{3}}{3}a, 0, 0\right]^T, \mathbf{a}'_2 = \left[\frac{\sqrt{3}}{6}a, -\frac{1}{2}a, 0\right]^T, \mathbf{a}'_3 = \left[\frac{\sqrt{3}}{6}a, \frac{1}{2}a, 0\right]^T \quad (15)$$

Projection of A_i in the base plane satisfies following equations:

$$y_{A_1} = 0 \quad (16)$$

$$y_{A_2} - y_{B_2} = (x_{B_2} - x_{A_2}) \tan \delta, \delta \neq 90^\circ \quad (17)$$

$$y_{A_3} - y_{B_3} = (x_{A_3} - x_{B_3}) \tan \delta, \delta \neq 90^\circ \quad (18)$$

where $\mathbf{a}_i = [x_{A_i}, y_{A_i}, z_{A_i}]^T$, $\mathbf{b}_i = [x_{B_i}, y_{B_i}, z_{B_i}]^T$, and the position vector of point B_i in the global frame can be written as:

$$\mathbf{b}_1 = \left[-\frac{\sqrt{3}}{3}b, 0, 0\right]^T, \mathbf{b}_2 = \left[\frac{\sqrt{3}}{6}b, -\frac{1}{2}b, 0\right]^T, \mathbf{b}_3 = \left[\frac{\sqrt{3}}{6}b, \frac{1}{2}b, 0\right]^T \quad (19)$$

By virtue of Equations (16)–(18), the three unknown pose parameters ϕ , x , and y can be determined.

$$\phi = \arctan \frac{N}{M} \quad (20)$$

$$x = \frac{\sqrt{3}}{6 \tan \delta} (b \tan \delta - \sqrt{3}b + \sqrt{3}ac\phi c\psi - ac\phi c\theta \tan \delta + \sqrt{3}as\phi s\psi s\theta) \quad (21)$$

$$y = \frac{\sqrt{3}}{3} ac\theta s\phi \quad (22)$$

with

$$M = c\psi \tan \delta + \sqrt{3}c\theta \quad (23)$$

$$N = s\psi s\theta \tan \delta \quad (24)$$

where s and c stand for sine and cosine.

Note that when $\delta = 90^\circ$ we have

$$\phi = \arctan(s\theta \tan \psi) \quad (25)$$

$$x = \frac{\sqrt{3}}{6} (b - ac\phi c\theta) \quad (26)$$

$$y = \frac{\sqrt{3}}{3} ac\theta s\phi \quad (27)$$

Until now, the pose of the MP of general 3-PPS PM ($\delta \neq 0^\circ$) can be expressed by the three chosen independent kinematic parameters.

3.2. Inverse Position and Jacobian

The inverse position problem is to seek the input of each actuator which corresponds to the known pose of the MP. Under the global frame, the unit vectors of the axes of the active prismatic joints are denoted by:

$$\mathbf{f}_1 = -\mathbf{R}_y(\gamma)\mathbf{i} \quad (28)$$

$$\mathbf{f}_2 = \mathbf{R}_z(-\delta)\mathbf{R}_y(-\gamma)\mathbf{i} \quad (29)$$

$$\mathbf{f}_3 = \mathbf{R}_z(\delta)\mathbf{R}_y(-\gamma)\mathbf{i} \quad (30)$$

where $\mathbf{i}$ represents the unit vectors of X_0 -axis.

The unit vectors of the axes of the passive prismatic joints are derived as:

$$\mathbf{g}_1 = -\mathbf{R}_y(\gamma)\mathbf{R}_y(\frac{\pi}{2})\mathbf{i} \quad (31)$$

$$\mathbf{g}_2 = \mathbf{R}_z(-\delta)\mathbf{R}_y(-\gamma)\mathbf{R}_y(-\frac{\pi}{2})\mathbf{i} \quad (32)$$

$$\mathbf{g}_3 = \mathbf{R}_z(\delta)\mathbf{R}_y(-\gamma)\mathbf{R}_y(-\frac{\pi}{2})\mathbf{i} \quad (33)$$

On the other hand, the position vector of point A_i can be expressed in another form as:

$$\mathbf{a}_i = \mathbf{b}_i + l_i\mathbf{f}_i + d_i\mathbf{g}_i \quad (34)$$

where d_i represents the displacement of the passive prismatic joint, and l_i denotes the input of the active prismatic joint.

The inverse position problem can be solved by combining Equations (14) and (34). For $0^\circ < \delta < 90^\circ$, we have:

$$l_1 = z_{A_1} s\gamma - x_{A_1} c\gamma - \frac{\sqrt{3}}{3} b c\gamma \quad (35)$$

$$l_2 = z_{A_2} s\gamma c\delta + x_{A_2} c\gamma - \frac{\sqrt{3}}{6 c\delta} b c\gamma \quad (36)$$

$$l_3 = z_{A_3} s\gamma c\delta + x_{A_3} c\gamma - \frac{\sqrt{3}}{6 c\delta} b c\gamma \quad (37)$$

When $\delta=90^\circ$, the inverse position can be expressed as:

$$l_1 = z_{A_1} s\gamma - x_{A_1} c\gamma - \frac{\sqrt{3}}{3} b c\gamma \quad (38)$$

$$l_2 = z_{A_2} s\gamma - y_{A_2} c\gamma - \frac{\sqrt{3}}{2} b c\gamma \quad (39)$$

$$l_3 = z_{A_3} s\gamma + y_{A_3} c\gamma - \frac{\sqrt{3}}{2} b c\gamma \quad (40)$$

Note that it is a special configuration when $\delta = 0^\circ$, as shown in Figure 5a. Within this configuration, all the three kinematic parameters y, ψ, ϕ are equal to zero, so the pose of the MP can be described by x, z , and θ . Thus, the inverse position can be written as:

$$l_1 = (z + \frac{\sqrt{3}}{3} a s\theta) s\gamma - (x - \frac{\sqrt{3}}{3} a c\theta + \frac{\sqrt{3}}{3} b) c\gamma \quad (41)$$

$$l_2 = (z - \frac{\sqrt{3}}{6} a s\theta) s\gamma + (x + \frac{\sqrt{3}}{6} a c\theta - \frac{\sqrt{3}}{6} b) c\gamma \quad (42)$$

$$l_3 = (z - \frac{\sqrt{3}}{6} a s\theta) s\gamma + (x + \frac{\sqrt{3}}{6} a c\theta - \frac{\sqrt{3}}{6} b) c\gamma \quad (43)$$

For general PMs, the relationship between the velocity of the active joints $\dot{\mathbf{q}}$ and the velocity of the MP $\dot{\mathbf{x}}$ can be expressed as:

$$\dot{\mathbf{q}} = \mathbf{J} \dot{\mathbf{x}} \quad (44)$$

Differentiating both sides of inverse position equations with respect to time, the corresponding Jacobian matrices for the three cases discussed above can be obtained.

$$\mathbf{J}_1 = \begin{bmatrix} \frac{\partial z_{A_1}}{\partial z} s\gamma - \frac{\partial x_{A_1}}{\partial z} c\gamma & \frac{\partial z_{A_1}}{\partial \psi} s\gamma - \frac{\partial x_{A_1}}{\partial \psi} c\gamma & \frac{\partial z_{A_1}}{\partial \theta} s\gamma - \frac{\partial x_{A_1}}{\partial \theta} c\gamma \\ \frac{\partial z_{A_2}}{\partial z} s\gamma c\delta + \frac{\partial x_{A_2}}{\partial z} c\gamma & \frac{\partial z_{A_2}}{\partial \psi} s\gamma c\delta + \frac{\partial x_{A_2}}{\partial \psi} c\gamma & \frac{\partial z_{A_2}}{\partial \theta} s\gamma c\delta + \frac{\partial x_{A_2}}{\partial \theta} c\gamma \\ \frac{\partial z_{A_3}}{\partial z} s\gamma c\delta + \frac{\partial x_{A_3}}{\partial z} c\gamma & \frac{\partial z_{A_3}}{\partial \psi} s\gamma c\delta + \frac{\partial x_{A_3}}{\partial \psi} c\gamma & \frac{\partial z_{A_3}}{\partial \theta} s\gamma c\delta + \frac{\partial x_{A_3}}{\partial \theta} c\gamma \end{bmatrix} \quad (45)$$

$$\mathbf{J}_2 = \begin{bmatrix} \frac{\partial z_{A_1}}{\partial z} s\gamma - \frac{\partial x_{A_1}}{\partial z} c\gamma & \frac{\partial z_{A_1}}{\partial \psi} s\gamma - \frac{\partial x_{A_1}}{\partial \psi} c\gamma & \frac{\partial z_{A_1}}{\partial \theta} s\gamma - \frac{\partial x_{A_1}}{\partial \theta} c\gamma \\ \frac{\partial z_{A_2}}{\partial z} s\gamma - \frac{\partial y_{A_2}}{\partial z} c\gamma & \frac{\partial z_{A_2}}{\partial \psi} s\gamma - \frac{\partial y_{A_2}}{\partial \psi} c\gamma & \frac{\partial z_{A_2}}{\partial \theta} s\gamma - \frac{\partial y_{A_2}}{\partial \theta} c\gamma \\ \frac{\partial z_{A_3}}{\partial z} s\gamma + \frac{\partial y_{A_3}}{\partial z} c\gamma & \frac{\partial z_{A_3}}{\partial \psi} s\gamma + \frac{\partial y_{A_3}}{\partial \psi} c\gamma & \frac{\partial z_{A_3}}{\partial \theta} s\gamma + \frac{\partial y_{A_3}}{\partial \theta} c\gamma \end{bmatrix} \quad (46)$$

$$J_3 = \begin{bmatrix} -c\gamma & s\gamma & \frac{\sqrt{3}}{3}a(s\gamma c\theta - c\gamma s\theta) \\ c\gamma & s\gamma & -\frac{\sqrt{3}}{6}a(s\gamma c\theta + c\gamma s\theta) \\ c\gamma & s\gamma & -\frac{\sqrt{3}}{6}a(s\gamma c\theta + c\gamma s\theta) \end{bmatrix} \quad (47)$$

4. Results

Based on the kinematic analysis and the selected performance evaluations, optimal design and shape singularity analyses are conducted. We look at first at optimal design in terms of dexterity and payload performance. Afterward, shape singularity and its compliance quality are considered.

4.1. Optimal Design Based on Dexterity Performance

Generally, the units of different columns of the Jacobian matrix are not all the same. For example, the first two columns of the Jacobian matrix as mentioned in Equation (47) are without units while the third one has units of length. A simple method for rendering these units homogeneous is to divide the elements of the third column by a characteristic length as reported in Reference [20], this approach also can be applied to the considered 3-PPS PM.

As analyzed in Section 3.2, it can be found that only dimensional parameter a , shape parameters δ and γ , and motion parameters ψ and θ have an influence on the Jacobian matrix. Moreover, as presented in References [15,16], in order to satisfy the application needs, the tilting angle of the MP of the 3-PPS PM under study is defined as $\pm 2^\circ$, and the mechanism will obtain a relatively great performance when $a = 125$ mm. For this reason, the side length of the MP of the proposed mechanism in this work is defined as $a = 125$ mm, the workspace is predefined as $z = 300$ mm, $-2^\circ \leq \psi \leq 2^\circ$, and $-2^\circ \leq \theta \leq 2^\circ$.

The GCI distribution of the considered 3-PPS PM is depicted in Figure 3. It is found that the dexterity performance of the mechanism is great when $15^\circ \leq \delta \leq 90^\circ$ and $15^\circ \leq \gamma \leq 90^\circ$. Even the GCI distribution among this zone is nearly the same, there still exists an optimal configuration with the largest GCI = 0.01285 when $\delta = 90^\circ$ and $\gamma = 90^\circ$, which is presented by point D_1 in the plot. The corresponding mechanism is the T-shape 3-PPS PM, as shown in Figure 4. For the T-shape 3-PPS PM, it has a partially decoupled DOF when $\psi = 0^\circ$, namely, a rotation about the axis of A_2A_3 .

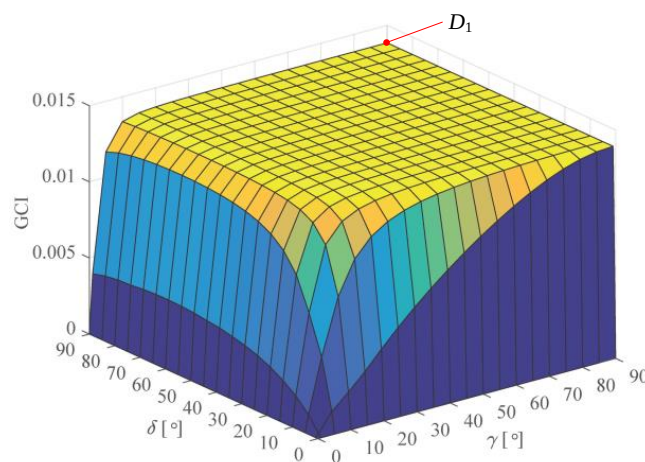


Figure 3. Global conditioning index (GCI) of the 3-PPS PM.

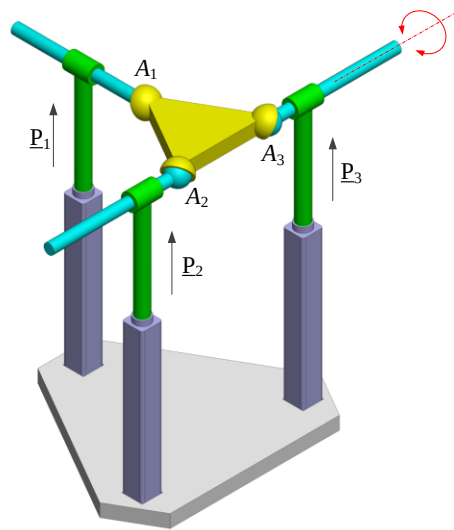


Figure 4. The T-shape 3-PPS PM.

One can notice that the GCI is equal to 0 when $\delta = 0^\circ$ or $\gamma = 0^\circ$, two special shapes pertain to this case, as depicted in Figure 5. A zero GCI implies zero LCI in the entire predefined workspace, which means that the mechanism will always be singular. Further analysis of these two shapes will be presented in Section 4.3.

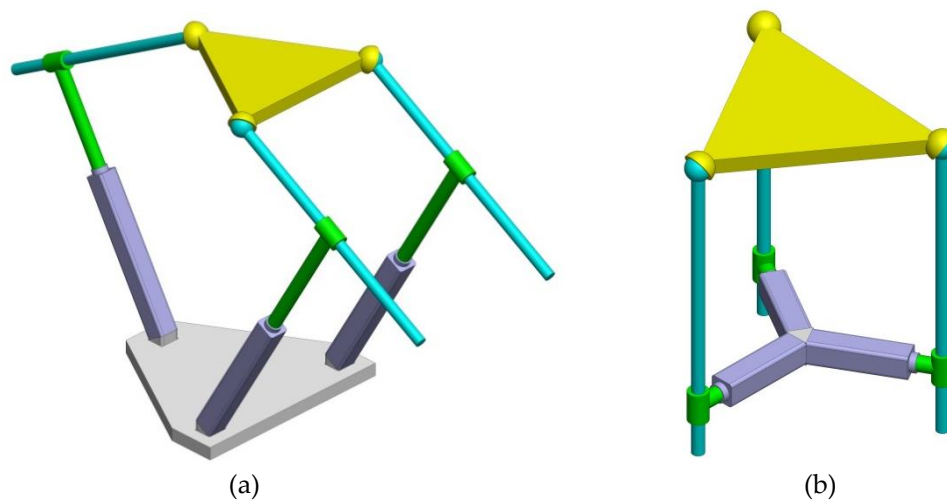


Figure 5. Two special shapes: (a) $\delta = 0^\circ$ and (b) $\gamma = 0^\circ$.

4.2. Optimal Design Based on Payload Performance

The GPI distribution of the proposed 3-PPS PM is shown in Figure 6. It is seen that the payload performance of the mechanism is great when $\gamma = 90^\circ$ and $0^\circ < \delta \leq 90^\circ$. Moreover, the maximum GPI is equal to 1.7321 at point D_2 ($\delta = 60^\circ$, $\gamma = 90^\circ$). As shown in Figure 7, the corresponding optimal mechanism is the star-shape 3-PPS PM, whose three PPS limbs are distributed symmetrically, thus the mechanism's payload capability is improved. Note that the GPI is equal to 0 when $\delta = 0^\circ$ or $\gamma = 0^\circ$, which means the mechanism can no longer stand external force at these configurations.

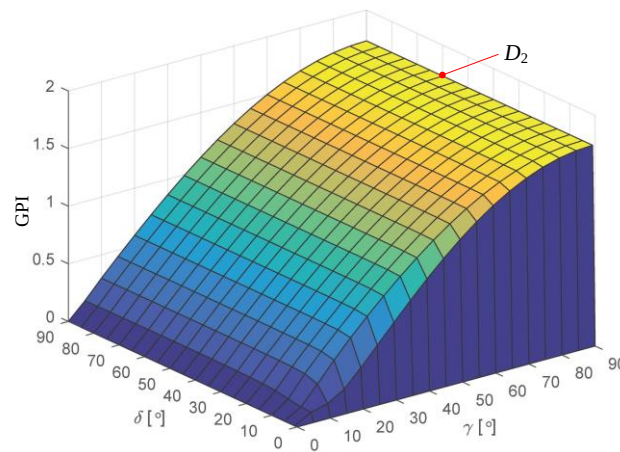


Figure 6. Payload performance of the 3-PPS PM.

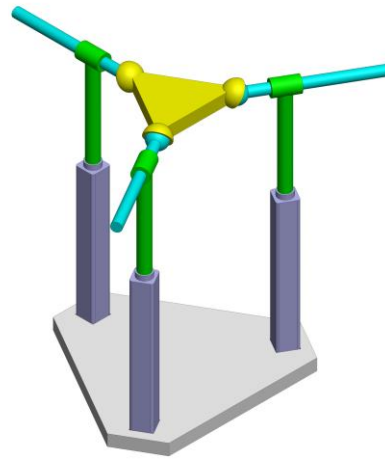


Figure 7. The star-shaped 3-PPS PM.

4.3. Analysis of Shape Singularity

Shape singularity is a special type of singularity associated with special designs of the base or mobile platforms that lead to a singularity in all configurations, which means it is configuration independent. The platform shape of the PM under shape singularity is called a singular shape. As reported in Reference [18], shape singularity of the planar PM under study is identified by a zero GCI. From the GCI analysis above, the two special shapes ($\delta = 0^\circ$, $\gamma = 0^\circ$) are possible singular shapes, due to the fact that the numerical calculation of GCI covers only a predefined workspace, not the entire one.

It is well known that screw theory [22] is a useful approach for singularity analysis. In this section, screw theory is applied to shape singularity analysis of the proposed 3-PPS PM. As shown in Figure 8, the considered mechanism is described by screws. Taking the first limb as an example, the twist screws can be written as:

$$\mathcal{S}_{11} = (0, 0, 0; \mathbf{f}_1) \quad (48)$$

$$\mathcal{S}_{12} = (0, 0, 0; \mathbf{g}_1) \quad (49)$$

$$\mathcal{S}_{13} = (\mathbf{i}; \mathbf{a}_1 \times \mathbf{i}) \quad (50)$$

$$\mathcal{S}_{14} = (\mathbf{j}; \mathbf{a}_1 \times \mathbf{j}) \quad (51)$$

$$\mathcal{S}_{15} = (\mathbf{k}; \mathbf{a}_1 \times \mathbf{k}) \quad (52)$$

where $\mathbf{j}$ and $\mathbf{k}$ represent the unit vectors of Y_0 - and Z_0 -axis, respectively.

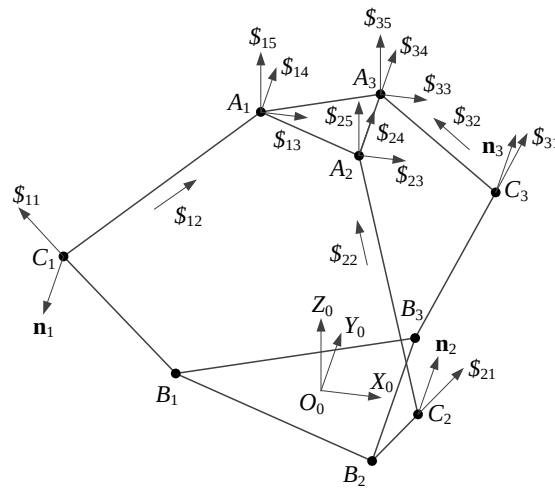


Figure 8. The general 3-PPS PM described by screws.

When the actuator is locked, the wrench screws of the first limb are:

$$s_{11}^r = (f_1; a_1 \times f_1) \quad (53)$$

$$s_{12}^r = (n_1; a_1 \times n_1) \quad (54)$$

where $n_i = g_i \times f_i$.

In a similar way, the wrench screws of the other two limbs can be expressed as:

$$s_{21}^r = (f_2; a_2 \times f_2) \quad (55)$$

$$s_{22}^r = (n_2; a_2 \times n_2) \quad (56)$$

$$s_{31}^r = (f_3; a_3 \times f_3) \quad (57)$$

$$s_{32}^r = (n_3; a_3 \times n_3) \quad (58)$$

Then the system of wrench screws of the 3-PPS PM can be written as:

$$s^r = [s_{11}^r; s_{12}^r; s_{21}^r; s_{22}^r; s_{31}^r; s_{32}^r] \quad (59)$$

which also can be expressed in matrix as:

$$Q = \begin{bmatrix} 0 & 1 & 0 & -z_{A1} & 0 & x_{A1} \\ -c\gamma & 0 & s\gamma & y_{A1}s\gamma & -z_{A1}c\gamma - x_{A1}s\gamma & y_{A1}c\gamma \\ -s\delta & -c\delta & 0 & z_{A2}c\delta & -z_{A2}s\delta & y_{A2}s\delta - x_{A2}c\delta \\ c\delta c\gamma & -c\gamma s\delta & s\gamma & y_{A2}s\gamma + z_{A2}c\gamma s\delta & z_{A2}c\delta c\gamma - x_{A2}s\gamma & -c\gamma(y_{A2}c\delta + x_{A2}s\delta) \\ s\delta & -c\delta & 0 & z_{A3}c\delta & z_{A3}s\delta & -x_{A3}c\delta - y_{A3}s\delta \\ c\delta c\gamma & c\gamma s\delta & s\gamma & y_{A3}s\gamma - z_{A3}c\gamma s\delta & z_{A3}c\delta c\gamma - x_{A3}s\gamma & -c\gamma(y_{A3}c\delta - x_{A3}s\delta) \end{bmatrix} \quad (60)$$

At ordinary poses, the rank of Q is equal to 6, which means the manipulator is fully constrained and has no singularity. On the other hand, if shape singularity exists, Q is singular.

The two possible singular shapes as discussed in Section 4.1 are considered, and they are described by screws as shown in Figure 9. With regard to $\delta = 0^\circ$, as shown in Figure 9a, the wrench screws can be obtained as:

$$\mathbf{Q}_1 = \begin{bmatrix} 0 & 1 & 0 & -z_{A_1} & 0 & x_{A_1} \\ -c\gamma & 0 & s\gamma & y_{A_1}s\gamma & -z_{A_1}c\gamma - x_{A_1}s\gamma & 0 \\ 0 & -1 & 0 & z_{A_2} & 0 & -x_{A_2} \\ c\gamma & 0 & s\gamma & y_{A_2}s\gamma & z_{A_2}c\gamma - x_{A_2}s\gamma & -y_{A_2}c\gamma \\ 0 & -1 & 0 & z_{A_2} & 0 & -x_{A_2} \\ c\gamma & 0 & s\gamma & -y_{A_2}s\gamma & z_{A_2}c\gamma - x_{A_2}s\gamma & y_{A_2}c\gamma \end{bmatrix} \quad (61)$$

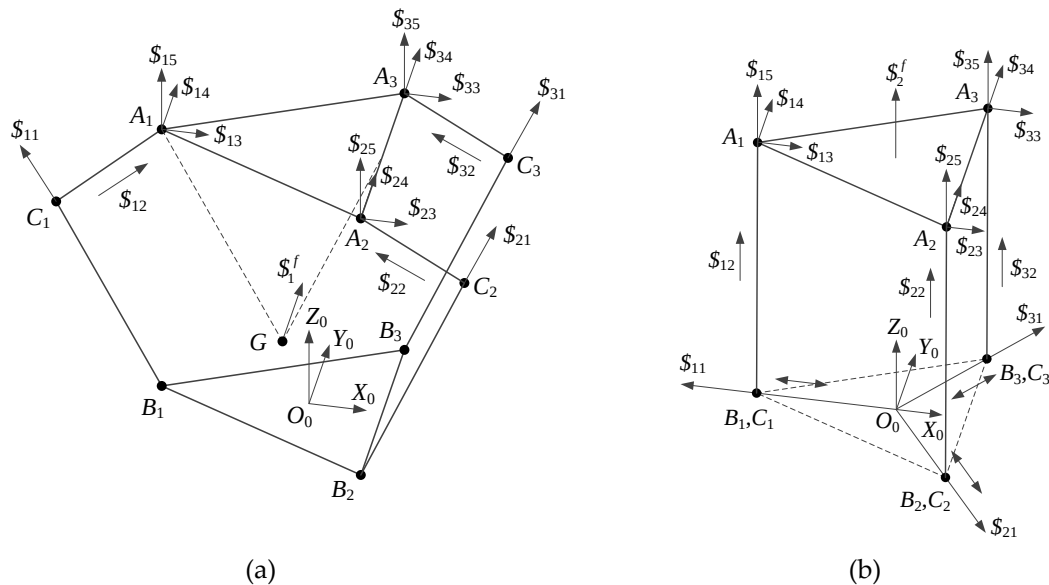


Figure 9. Two possible singular shapes of the 3-PPS PM: (a) $\delta = 0^\circ$ and (b) $\gamma = 0^\circ$

The reciprocal screw of the wrench screws is:

$$\mathbf{s}_1^f = (0, 1, 0; \frac{x_{A_2} - x_{A_1}}{2} \tan \gamma - \frac{z_{A_1} + z_{A_2}}{2}, 0, \frac{z_{A_1} - z_{A_2}}{2} \cot \gamma + \frac{x_{A_2} + x_{A_1}}{2}) \quad (62)$$

which means the MP can rotate about an axis parallel to Y_0 -axis which passing through point G, even all the actuators are locked. The position vector of point G is:

$$\mathbf{r}_G = \left[\frac{z_{A_1} - z_{A_2}}{2} \cot \gamma + \frac{x_{A_2} + x_{A_1}}{2}, 0, \frac{x_{A_1} - x_{A_2}}{2} \tan \gamma + \frac{z_{A_1} + z_{A_2}}{2} \right]^T \quad (63)$$

For $\gamma = 0^\circ$, as shown in Figure 9b, the wrench screws can be expressed as:

$$\mathbf{Q}_2 = \begin{bmatrix} 0 & 1 & 0 & -z_{A_1} & 0 & x_{A_1} \\ -1 & 0 & 0 & 0 & -z_{A_1} & y_{A_1} \\ -s\delta & -c\delta & 0 & z_{A_2}c\delta & -z_{A_2}s\delta & y_{A_2}s\delta - x_{A_2}c\delta \\ c\delta & -s\delta & 0 & z_{A_2}s\delta & z_{A_2}c\delta & -y_{A_2}c\delta - x_{A_2}s\delta \\ s\delta & -c\delta & 0 & z_{A_3}c\delta & z_{A_3}s\delta & -x_{A_3}c\delta - y_{A_3}s\delta \\ c\delta & s\delta & 0 & -z_{A_3}c\delta & z_{A_3}s\delta & x_{A_3}s\delta - y_{A_3}c\delta \end{bmatrix} \quad (64)$$

The reciprocal screw of the wrench screws is:

$$\$_2^f = (0, 0, 0; 0, 0, 1) \quad (65)$$

which means the MP can translate along Z_0 -axis without actuating all the actuators.

Based on the analysis above, the two considered special shapes are proved to be singular shapes.

4.4. Compliance Performance of Shape Singularity and Its Application

Shape singularity can have some practical implication in PM design. One is the adjustment of robot compliance. Robot compliance is important in automation applications which require a strong interaction between the robot and the environment, while passive and active compliance are two usually used compliance [23]. For passive compliance, it contains an elastic element, e.g., a spring which can store energy. On the other hand, active compliance is based on the control of stiff actuators to mimic the behavior of the spring [24]. It is known that a mechanism will obtain great compliance at the zones near singular configurations. With the shape singularity, it is possible to adjust the PMs' compliance by means of shape adjustment.

According to Equation (13), the distribution of compliance performance of the proposed 3-PPS PM is obtained and depicted in Figure 10. It is found that the GSI changes significantly for the PM close to shape singularity ($\delta = 0^\circ$, $\gamma = 0^\circ$), while the zones near the shape singularity own great compliance. On the other hand, the compliance performances of the zones far away from shape singularity are very poor.

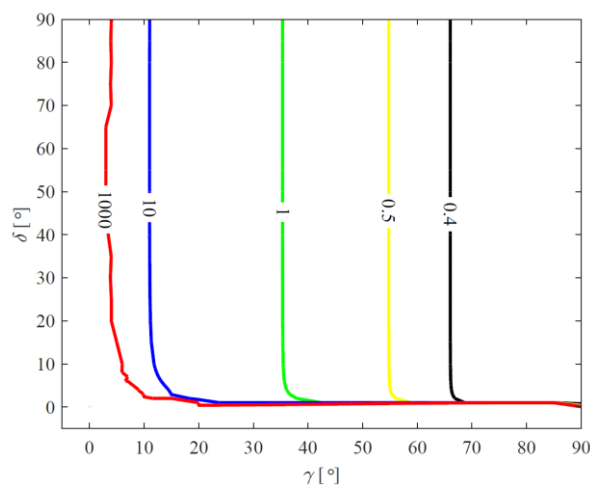


Figure 10. Compliance performance of the 3-PPS PM.

The inherent great compliance of shape singularity can be applied to mechanism design. Here, adjustable compliance is proposed, to be exact, the compliance is adjusted by changing the shape of the mechanism. As shown in Figure 11, the second and third PPS limbs of the considered PM are attached to the base by revolute joints, and the axis of the rotation is perpendicular to the base. $\underline{R}$ is the actuator of the shape adjusting mechanism, which is used to change the shape of the base. At the positioning stage, $\underline{R}$ is locked, namely, the base is fixed with the non-singular shape. The mechanism will possess a good stiffness performance, which makes precision positioning possible. At the stage of interaction with the environment, $\underline{R}$ will drive the two limbs to the singular shape ($\delta = 0^\circ$), then the mechanism could own a great compliance performance under the singular shape.

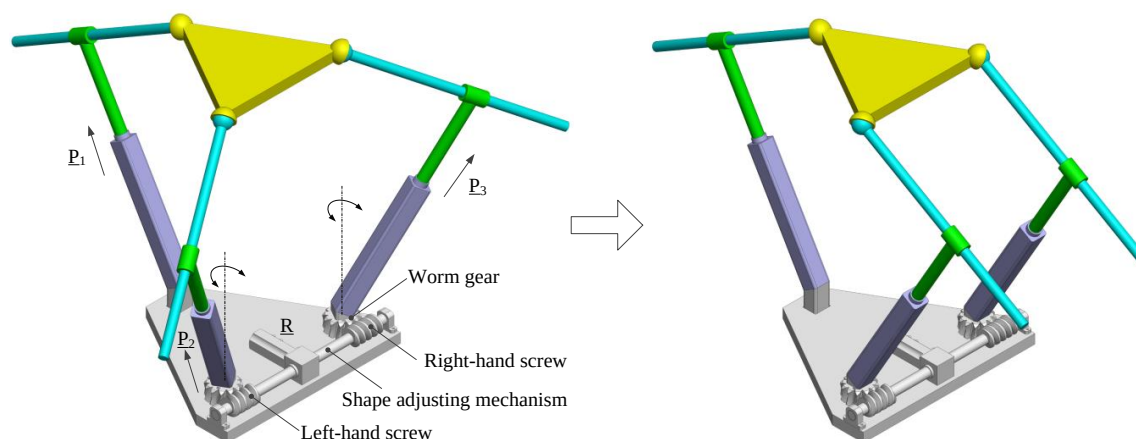


Figure 11. Compliance adjustment.

5. Discussion and Conclusions

In this work, aimed at optimal design and shape singularity analysis, a parametric kinematic model of the considered spatial 3-PPS PM is established. With this model, the inverse position and Jacobian analyses are carried out. In the optimal design, dexterity and payload performance are considered, from which two optimal configurations are obtained. Performance evaluation shows that the T-shaped one possesses the best dexterity performance, while the star-shaped one exhibits the best payload performance. Moreover, both the dexterity and payload performances of the PM with $\gamma = 90^\circ$ and $0^\circ < \delta \leq 90^\circ$ are great. On the basis of dexterity analysis, two possible singular shapes ($\delta = 0^\circ$, $\gamma = 0^\circ$) are pointed out. Since screw theory is an effective approach for singularity analysis, it is applied to the identification of the shape singularity, and the result coincides with the performance analysis.

A contribution of this work is the establishment of a parametric model of the 3-PPS PM. By including the shape parameters of the platforms, the model allows us to investigate the influence of the platforms' shape on the dexterity and payload performances, while shape parameters are rarely considered in other studies [13–16]. The optimal shape designs in this work show the significance of this model.

Another contribution is the identification and generalization of shape singularity. Here, we provide a method to identify shape singularity. Firstly, possible shape singularity is indicated by dexterity index. Then, screw theory is applied to identify shape singularity. Moreover, shape singularity is generalized from planar PMs to spatial PMs, while it is only reported in planar PMs before in Reference [18].

The last contribution is the application of shape singularity. In related studies, [2,12,22], the singularity is identified with the purpose to eliminate the singularity from workspace of the manipulators. In this work, we find that shape singularity could inspire novel mechanism design. A PM with shape singularity could find potential applications in assembly, as it can provide high compliance due to the presence of the instantaneous center of rotation, thus a remote center of compliance can be formed.

In this work, only two major performance indices are considered for optimal design; other indices should be selected according to the working conditions. On the other hand, the shape singularity of the other PMs should be verified. Needless to say, new compliant PMs with shape singularity and their applications will be topics of future research.

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