

Article

The Grimus–Neufeld Model with FlexibleSUSY at One-Loop

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Abstract: The Grimus–Neufeld model can explain the smallness of measured neutrino masses by extending the Standard Model with a single heavy neutrino and a second Higgs doublet, using the seesaw mechanism and radiative mass generation. The Grimus–Lavoura approximation allows us to calculate the light neutrino masses analytically. By inverting these analytic expressions, we determine the neutrino Yukawa couplings from the measured neutrino mass differences and the neutrino mixing matrix. Short-cutting the full renormalization of the model, we implement the Grimus–Neufeld model in the spectrum calculator FlexibleSUSY and check the consistency of the implementation. These checks hint that FlexibleSUSY is able to do the job of numerical renormalization in a restricted parameter space. As a summary, we also comment on further steps of the implementation and the use of FlexibleSUSY for the model.

Keywords: neutrinos; seesaw mechanism; radiative masses; spectrum calculator

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1. Introduction

The last 30 years of collider physics showed an ever-increasing success for the predictions of the Standard Model (SM) [1]. A similar statement can be said about the experimental program in neutrino physics [2], but no unambiguous common treatment for both areas exists up to now. The masses and the mixing of neutrinos can easily be formulated in a Lagrangian picture; nevertheless, these terms in the Lagrangian are still considered to be “beyond the Standard Model” (BSM). Whereas the formulation of the SM and the accurate calculations for the experimental predictions require the framework of Quantum Field Theory (QFT), the analysis of neutrino measurements is still done in the framework of plain Quantum Mechanics (QM); an explanation of why QM is usually enough for the study of neutrino oscillations can be found in [3].

The usual two explanations of the smallness of neutrino masses in the Lagrangian context are the seesaw mechanism [4] or radiative mass generation for the light neutrinos [5–8]. In 1989, Walter Grimus and Helmut Neufeld pointed to the possibility that both mechanisms can be comparable for explaining the masses of the light neutrinos [9]. We call the minimal extension of the SM that allows this feature Grimus–Neufeld Model (GNM). This minimal extension adds to the SM only a single heavier Majorana singlet and one additional Higgs doublet. The qualitative behavior of the GNM is described in [10]. An extended description of our approach is presented in [11]. Here, we review shortly the features of the GNM and discuss its implementation in FlexibleSUSY [12,13],

which uses SARAH [14–17] and SOFTSUSY [18,19]. This implementation gives us a tool to check the renormalization and the consistency of the model numerically.

The main aim of this work is to discuss the plans for the checks of the model and the possible modifications that can be done to the code. The first trials of the implemented code hint that, to have only small loop corrections, the model has a natural preference for a small seesaw scale.

2. Results

2.1. Summary of Features of the Grimus–Neufeld Model

2.1.1. Lagrangian

Since the GNM is a minimal extension of the SM, we only need to give the additional parts of the Lagrangian. These are the Majorana mass term for the fermionic singlet N

$$-\mathcal{L}_{\text{Majorana-mass}} = \frac{1}{2} \bar{N}_R M_R \hat{N}_R + h.c. = -\frac{1}{2} \bar{N}_R M_R \mathbf{C} \bar{N}_R^\top + h.c. , \quad (1)$$

the Yukawa terms (ignoring quarks) among the lepton doublets ℓ_j , the Higgs doublets Φ_a , and either the charged lepton singlets E_k or the fermionic singlet N

$$-\mathcal{L}_{\text{F-H}} = \bar{\ell}_j \Phi_a (Y_E^{(a)})_{jk} E_k + \bar{\ell}_j \tilde{\Phi}_a (Y_N^{(a)})_j N + h.c. , \quad (2)$$

and the Higgs potential

$$-\mathcal{L}_H = V(\Phi_a) = Y_{ab} (\Phi_a^\dagger \Phi_b) + \frac{1}{2} Z_{abcd} (\Phi_a^\dagger \Phi_b) (\Phi_c^\dagger \Phi_d) , \quad (3)$$

which is just the generic two Higgs doublet potential, which we write in the Higgs basis [20], meaning that only Φ_1 has a vacuum expectation value (vev) v . $\hat{N}_R$ in Equation (1) is the Lorentz covariant conjugate [21] of $N_R = P_R N$, which is the right-chiral part of N .

2.1.2. Tree-Level Mass Matrices and Tree-Level Masses

As we are interested mainly in the neutrino sector, we deal only with the mass matrices of the leptons and the Higgs bosons. For the charged leptons, the relevant Yukawa coupling is $Y_E^{(1)}$, since we work in the Higgs basis. Then, diagonalization of the charged lepton mass matrix M_E ,

$$U_{eL} M_E U_{eR}^\dagger = U_{eL} \left(\frac{v}{\sqrt{2}} Y_E^{(1)} \right) U_{eR}^\dagger = \text{diag}(m_e, m_\mu, m_\tau) , \quad (4)$$

defines the mass eigenstates for the charged leptons and, consequently, the flavor basis for the SM neutrino fields $(\nu_e, \nu_\mu, \nu_\tau)$, which are the partners of the charged leptons in the weak lepton doublets.

The mass matrix for the neutrinos in the GNM in flavor basis at tree level is the symmetric 4×4 -matrix M_ν ,

$$M_\nu = \begin{pmatrix} M_L & M_D \\ M_D^\top & M_R \end{pmatrix} , \quad \text{with} \quad \begin{aligned} M_L &= 0_{3 \times 3} \text{ at tree level} \\ M_D &= \frac{v}{\sqrt{2}} Y_N^{(1)} \end{aligned} , \quad (5)$$

which is only rank 2 and therefore has only two non-vanishing singular values. That means we have the heavy mass $m_4^{(0)} \sim M_R$ and only a single light, massive neutrino with mass $m_s^{(0)}$ coming from the seesaw. The two other masses $m_o^{(0)} = m_r^{(0)} = 0$ are zero at tree level. The radiative mass will be $m_r > 0$ at one-loop, but the remaining other neutrino with mass $m_o = 0$ stays massless even at one-loop level.

Following the idea in [22,23] to formulate the 2HDM potential in terms of basis independent physical quantities, we skip the discussion of the mass matrix of the Higgs bosons and point the reader to the relevant literature [20,24–26]. For the tree-level masses of the Higgs bosons, we just

want to note that we take the lightest boson, h , to correspond to the boson observed at the LHC with the mass $m_h = 125.18 \text{ GeV}$ [1]. The other Higgs boson masses are free parameters subject to the experimental constraints.

2.1.3. Leading Order Loop-Level Masses

The observation that the predicted loop-level mass for one neutrino, m_r , can be of the same order as the seesaw generated mass $m_s^{(0)}$ was the main point of the paper of Grimus and Neufeld [9] in 1989. Since the remaining neutrino stays massless at one-loop order, $m_o^{(1)} = 0$, and all other particles already have a mass (unless they are protected by an unbroken gauge symmetry), this radiative neutrino mass is the main effect at one loop. This predicted mass m_r is finite and gauge invariant, as proven in [10,27,28] using different approaches. One loop radiative corrections affect the seesaw generated neutrino mass $m_s^{(0)}$, too. We denote the resulting mass as m_s .

2.2. The Grimus–Lavoura Approximation

The full calculation of the renormalized neutrino masses for the GNM in analytic form is not available yet. We adapt the proof of finiteness and gauge invariance of the one-loop corrections to the effective light neutrino mass matrix from [27] and formulate the Grimus–Lavoura approximation for calculating light neutrino masses:

- Staying in the interaction eigenstates, as defined by the charged leptons, calculate the neutrino mass matrix M_ν using Equation (5).
- The light neutrino block M_L does not have a counter-term $\delta^{\text{ct}} M_L = 0$ at one loop level, as $M_L^{(0)}$ vanishes at tree-level. The loop corrections δM_L , depicted by the diagrams in Figure 1, are finite and gauge invariant, as proven in [27].
- Reducing the problem to the light neutrinos, one arrives at the effective symmetric 3×3 neutrino mass matrix $\mathcal{M}_\nu$, which has the tree-level value

$$\mathcal{M}_\nu^{\text{tree}} = -M_D M_R^{-1} M_D^\top, \quad (6)$$

that has obviously rank 1 and will give only the neutrino mass $m_s^{(0)}$. However, at one-loop, this matrix becomes

$$\mathcal{M}_\nu^{1\text{-loop}} = \mathcal{M}_\nu^{\text{tree}} + \delta \mathcal{M}_\nu, \quad (7)$$

with

$$\delta \mathcal{M}_\nu = \delta M_L - \delta M_D M_R^{-1} M_D^\top - M_D M_R^{-1} \delta M_D^\top - M_D M_R^{-1} (\delta M_R) M_R^{-1} M_D^\top. \quad (8)$$

With this correction, $\mathcal{M}_\nu^{1\text{-loop}}$ can have rank > 1 .

- The approximation consists now of:
 1. Assuming δM_R to be irrelevant for the light neutrinos with the reasoning that M_R (or m_4) is not measured. It is still a free parameter of the theory that can be adjusted as needed.
 2. Observing that the corrections with δM_D are subdominant, because they are suppressed by the squares of small Yukawa or gauge couplings and additionally by the small charged lepton masses.
 3. Assuming that the loop correction δM_L is of the same order as the tree-level value $\mathcal{M}_\nu^{\text{tree}}$.

The result of this approximation is that we can derive analytic formulas that predict the masses of the light neutrinos, as depending on the tree-level input parameters:

$$m_{r,s} = m_{r,s}(m_{H_k}^2, \vartheta_{1j}; m_4^{(0)}, Y_N^{(a)}) , \quad (9)$$

where $m_4^{(0)} \sim m_4 \sim M_R$. ϑ_{1j} are the basis-independent mixing angles of the neutral Higgs fields [20]. Following Grzadkowski et al. [22], these angles can be expressed by the $H_j^0 W^+ W^-$ -couplings $\frac{1}{2}g^2 e_j$ as

$$e_1 = v c_{12} c_{13} \ , \quad e_2 = v s_{12} c_{13} \ , \quad e_3 = v s_{13} \ , \quad \text{where} \quad c_{1j} = \cos \vartheta_{1j} \quad \text{and} \quad s_{1j} = \sin \vartheta_{1j} \ . \quad (10)$$

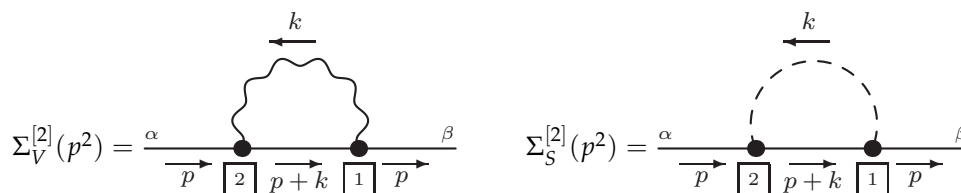


Figure 1. Feynman diagrams contributing to the self-energies of the light neutrinos. For the correction to the mass, the internal fermion line has to be a Majorana propagator with a mass insertion; hence, charged particles will not contribute to δM_L at this loop level.

2.3. Using the Grimus–Lavoura Approximation

Since our model predicts one neutrino mass to remain zero at one-loop level ($m_o^{(0)} = m_o^{(1)} = 0$), we can use the measured neutrino mass squared differences [2] to determine the values of the other light neutrino masses $m_{r,s}$.

Parameterizing the neutrino Yukawa couplings as

$$(Y_N^{(1)})_k = \frac{\sqrt{2}m_D}{v} u_{3k} \quad \text{and} \quad (Y_N^{(2)})_k := d u_{2k} + d' u_{3k} \ , \quad (11)$$

with three orthonormal three-vectors $u_{\alpha k} = (\vec{u}_\alpha)_k$, we can invert the analytic expressions of the masses, Equation (9), and determine the parameters d and $|d'|$. The explicit formulas and the discussion of the difficulties in finding solutions for $|d'|$ can be found in [11]. The values of d and $|d'|$ do not depend on the three orthonormal three-vectors $u_{\alpha k}$. At tree-level, these vectors $u_{\alpha k}$ can be understood as an approximate neutrino mixing matrix $\tilde{V}_{\text{PMNS}}$, as it diagonalizes the effective tree-level neutrino mass matrix with the Takagi decomposition:

$$u_{\alpha j}^* (\mathcal{M}_\nu^{\text{tree}})_{jk} u_{\beta k}^* = (\tilde{V}_{\text{PMNS}}^\top)_{\alpha j} (\mathcal{M}_\nu^{\text{tree}})_{jk} (\tilde{V}_{\text{PMNS}})_{k\beta} = -\text{diag}(m_o^{(0)} = 0, m_r^{(0)} = 0, m_s^{(0)} > 0) \ . \quad (12)$$

Applying these same vectors to the effective one-loop neutrino mass matrix

$$u_{\alpha j}^* (\mathcal{M}_\nu^{1\text{-loop}})_{jk} u_{\beta k}^* = \begin{pmatrix} 0 & 0 & 0 \\ 0 & A & B \\ 0 & B & C \end{pmatrix}_{\alpha\beta} \ , \quad (13)$$

we see that the effective one-loop neutrino mass matrix is rank 2, and hence provides two massive light neutrino states. The full neutrino mixing matrix V_{PMNS} should diagonalize the full one-loop mass matrix

$$(V_{\text{PMNS}}^\top)_{\alpha j} (\mathcal{M}_\nu^{1\text{-loop}})_{jk} (V_{\text{PMNS}})_{k\beta} = \text{diag}(m_o^{(1)} = 0, m_r^{(1)} > 0, m_s^{(1)} > 0) \ . \quad (14)$$

A reordering of the masses is possible, if $m_r^{(1)} > m_s^{(1)}$. For a more detailed discussion, see [11].

As a side note: The minus-sign on the right-hand side of Equation (12) comes from the convention of the seesaw, where the seesaw rotation is written with an orthogonal matrix and the minus sign kept to be absorbed in the phase of the light state. We do not write a minus-sign on the right-hand side of Equation (14) following the convention of the normal singular value decomposition and the positivity of the masses. The phase in this second case is part of the complex mixing matrix.

Diagonalizing the reduced effective one-loop neutrino mass matrix, Equation (13), we get an effective 2×2 mixing matrix that connects the full mixing matrix V_{PMNS} with the used vectors $u_{\alpha k}$. This finally allows us to determine the vectors $u_{\alpha k}$, Equation (11), from the measured PMNS matrix:

$$u_{\alpha k} = u_{\alpha k}(m_4, \Delta m_{\text{atm}}^2, \Delta m_{\text{sol}}^2, V_{\text{PMNS}}; m_{H_k^0}^2, \vartheta_{1j}; \frac{\sqrt{2}m_D}{v}, \phi' = \arg[d']) \quad (15)$$

2.4. The One-Loop Improved Lagrangian

Using the neutrino Yukawa couplings as defined in the previous subsection, the determination of the Higgs potential parameters following Grzadkowski et al. [22], and the identification of the singlet mass parameter with the heavy neutrino mass $M_R \sim m_4$, we get a new parametrization of the Lagrangian as

$$-\mathcal{L}'_{\text{Majorana-mass}} = \frac{1}{2} \bar{N}_R m_4 \hat{N}_R + h.c. = -\frac{1}{2} \bar{N}_R m_4 \mathbf{C} \bar{N}_R^\top + h.c. \quad (16)$$

the Yukawa terms

$$-\mathcal{L}'_{\text{F-H}} = \bar{\ell}_j \Phi_a (Y_E^{(a)})_{jk} E_k + (\frac{\sqrt{2}m_D}{v} \tilde{\Phi}_1 + d' \tilde{\Phi}_2) u_{3j} \bar{\ell}_j P_R N + d \tilde{\Phi}_2 u_{2j} \bar{\ell}_j P_R N + h.c. \quad (17)$$

where d and d' are functions depending on the same parameters as $u_{\alpha j}$, Equation (15), and the Higgs potential

$$-\mathcal{L}'_{\text{H}} = V(\Phi_a; Y, Z) \quad \text{with } \{Y, Z\} = \{Y, Z\}(m_{H^\pm}^2, m_{H_k^0}^2, e_j, q_j, q) \quad (18)$$

is expressed in terms of physical masses and couplings of the Higgs bosons.

As an advantage, more parameters of the one-loop improved Lagrangian $\mathcal{L}'$ correspond directly to measured quantities: instead of six complex parameters in two-neutrino Yukawa couplings in $\mathcal{L}$, $\mathcal{L}'$ contains two real selectable parameters ($\frac{\sqrt{2}m_D}{v}$ and ϕ'), two real parameters that we determine from the measured neutrino mass squared differences (d and $|d'|$), and six parameters in the vectors $u_{\alpha j}$ that we determine from the measured neutrino mixing matrix V_{PMNS} . One of the two “missing” parameters in $\mathcal{L}'$ is the vanishing one-loop level neutrino mass, and the other parameter is the undeterminable Majorana phase of this zero-mass neutrino.

2.5. Renormalizing the GNM

The final goal of our efforts is to fully renormalize the GNM. The full renormalization will also indicate the importance and validity of the one-loop improved Lagrangian. However, we are still far from that goal. Only the mass renormalization of light neutrinos has been tackled in detail [28,29]. Since the full renormalization is difficult, we plan to use a spectrum calculator that performs the renormalization numerically. A foreseeable difficulty lies in the hierarchy of the seesaw. It is hard to have a reliable numerical implementation of the mass hierarchies of more than 10 orders of magnitude. We found that FlexibleSUSY [13] is able to do the job if we limit the seesaw scale. Now, we can study the renormalized model, but only numerically.

2.6. FlexibleSUSY for the GNM in a Nutshell

The primary idea of FlexibleSUSY was to numerically implement the renormalization group running of a model between two scales and to be able to give boundary conditions on both scales. In the case of a SUSY-GUT, one can require the low energy measured masses and couplings as one boundary condition and the GUT constraints at the GUT scale as another boundary condition. Then, the program tries to find a numerical solution that interpolates between the two scales and fulfills both boundary conditions.

In our case, we do not have the high scale, and we are not interested in imposing conditions at a scale other than our low scale. However, with the accuracy needed for today's collider physics, one has

to interpolate between the various scales of the different measurements that determine the masses and couplings of the SM and also the masses and mixing parameters of the neutrino sector. That means that the capabilities of FlexibleSUSY are needed also for implementing models that “live” only at the low scale.

To implement a model in FlexibleSUSY, one has first to define the Lagrangian of the model with SARAH [14–17] and check the consistency and completeness of the Lagrangian. In the next step, FlexibleSUSY uses SARAH to produce the model code. Additionally, one has to define the boundary conditions in a separate steering file, using the convention for the names of fields implemented in SARAH. This step includes the definition of what is to be treated as input and what is the desired output for the spectrum to be calculated. When this is done, one has to compile the generated code to get the actual spectrum calculator for the implemented model.

This program can be used from the terminal with the help of input and output files in the SLHA [30,31] format. That means writing the values of the input parameters and additional optional arguments into the SLHA input file and the program writes from this input the SLHA output file, which contains the mass spectrum of the model and also the decay rates of the particles of the model.

Another option to use the program is from the provided Mathematica™ [32] interface. There, the playing around with parameters and the generation of plots become much easier, but the comparison of results with other scientists becomes less accurate or much more cumbersome.

2.6.1. Our Achievements with the GNM in FlexibleSUSY

We succeeded to implement the basic Lagrangian of the GNM, $\mathcal{L}$ in Equations (1)–(3), in FlexibleSUSY and to generate a working code. This code could qualitatively reproduce the effect of the seesaw mechanism. However, it was difficult to find a point in the Higgs potential that gives sensible results for the Higgs masses also at one-loop level. This might be due to our limited experience with spectrum calculators and numerics. However, we are learning a lot of physics while trying to figure out where the problems can come from.

2.6.2. Plans with the GNM in FlexibleSUSY

The first step is to check our implementation:

- Do we understand the tree-level correctly?
- Are the different formulations of defining the input parameters really equivalent?
- Does the GL approximation give correct neutrino masses not only at leading order, but also at full one-loop level?
- Is the limitation of the seesaw scale a numeric artifact from the finite precision or can we find a physical reason for the limitation?

The next step is to investigate the parameter space of the GNM at one-loop level, i.e., going beyond the analysis of Jurčiukonis et al. [11]. The real restrictions to the model come from comparing to measurements. We hope to recycle the FlexibleSUSY implementations of predictions of other models that are already implemented in FlexibleSUSY. One definite goal is to work out the connection between the low-energy observables such as $(g - 2)_\mu$, $\mu \rightarrow 3e$, and $\mu \rightarrow e\gamma$, which is caused by the GNM. Another goal for the future is to work out the implications of the GNM for cosmology, where the question might be, if the heavy singlet can be a candidate for the dark matter.

3. Discussion

The main result is the implementation and the analytic check of the Grimus–Lavoura approximation in the Grimus–Neufeld model (GNM) to replace the Yukawa couplings with the measured masses and oscillation parameters as the input for the model. The model itself [9] and the approximation [33] do not try to estimate the model parameters from measurements. In addition,

the renormalization group analysis of this model in the limit of heavy right-handed neutrinos and heavy Higgs doublets [10] does not provide the explicit analytical and numerical analysis that is given in [11] and is shortly recapitulated in this paper.

The additional content of this presentation compared to the one in [11] is to address the question of the full renormalization of the GNM. The first steps in that direction were done in [29], but the full and explicit formulation of the renormalization of the GNM is work in progress with its end still far away.

To shorten the time to some results, we propose the use of generic spectrum calculators and give an account on the progress achieved with FlexibleSUSY, which is such a spectrum calculator. The final goal from both approaches, of the analytic one with fully renormalizing the model and of the numeric one with FlexibleSUSY, is to test the GNM with measurements. To that end, we want to give predictions that can be tested, such as correlations of low energy observables or correlations between the decay rates of heavy particles, which are influenced by the neutrino Yukawa coupling.

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Abbreviations

The following abbreviations are used in this manuscript:

2HDM	two Higgs doublet model
BSM	beyond the standard model physics
GNM	Grimus–Neufeld model
GUT	grand unified theory
LHC	large hadron collider
PMNS	Pontecorvo–Maki–Nakagawa–Sakata matrix
QFT	quantum field theory
QM	quantum mechanics
SM	standard model

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