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# Paramagnetic versus Diamagnetic Interaction in the SU(2) Higgs Model

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**Abstract:** We present an analytic calculation of the paramagnetic and diamagnetic contributions to the one-loop effective action in the SU(2) Higgs model. The paramagnetic contribution is produced by the gauge boson, while the diamagnetic contribution is produced by the gauge boson and the ghost. In the limit, where these particles are massless, the standard result of $-12$ for the ratio of the paramagnetic to the diamagnetic contribution is reproduced. If the mass of the gauge boson and the ghost become much larger than the inverse vacuum correlation lengths of the Yang–Mills vacuum, the value of the ratio goes to $-8$. We also find that the same values of the ratio are achieved in the deconfinement phase of the model, up to the temperatures at which the dimensional reduction occurs.

**Keywords:** Higgs model; Yang–Mills vacuum; effective action; world-line formalism; Wilson loop

## 1. The Model

In this paper, we consider the SU(2) Higgs model, whose Euclidean Lagrangian has the form

$$\mathcal{L} = \frac{1}{4}(F^a_{\mu\nu})^2 + (D_\mu\Phi)^\dagger D_\mu\Phi - \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2, \tag{1}$$

where $\Phi$ is the doublet of complex-valued scalar fields $\Phi_1$ and $\Phi_2$, and the Yang-Mills field-strength tensor $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc}A^b_\mu A^c_\nu$ and the covariant derivative in the fundamental representation $D_\mu = \partial_\mu - ig\frac{\tau^a}{2}A^a_\mu$ contain the SU(2) gauge coupling $g$ and the Pauli matrices $\tau^a$. By choosing the vacuum state corresponding to $A^{a\,\mathrm{vac}}_\mu = \Phi^{\mathrm{vac}}_1 = 0$ and $\Phi^{\mathrm{vac}}_2 = \frac{\mu}{\sqrt{2\lambda}}$, one can break the SU(2) symmetry completely, which leads to the appearance of three vector bosons of the same mass $m = \frac{g\mu}{2\sqrt{\lambda}}$. It is worth emphasizing that this symmetry-breaking pattern of the group SU(2) (discussed, e.g., in [1]) is different from that of the Standard Model. In the latter case, the symmetry group SU(2)×U(1) is broken only partially, and the corresponding symmetry-breaking pattern results in different masses of $W^\pm$- and $Z$-bosons. Yet, some similarities can be drawn between the present case and the high-temperature phase of (the bosonic sector of) the Standard Model, as the latter admits an effective description by means of a three-dimensional SU(2) Higgs model [2,3], in which all the gauge bosons have equal masses. Techniques similar to those which are used in the present paper have been utilized, in [4,5], to obtain the ratio of the paramagnetic to the diamagnetic contribution to the one-loop effective action of that three-dimensional theory. Namely, in the limit of vanishing mass of the gauge bosons, the ratio has been found [4] to be equal to $-16$, while a generalization of this result to arbitrary values of the gauge-boson mass has been obtained in [5]; yielding, in particular, a finite value of $-24$ in the opposite limit, where this mass became much larger than the inverse correlation length of the high-temperature three-dimensional Yang–Mills theory.

Furthermore, by considering the electroweak phase transition as a vacuum instability resulting from the negative sign of the paramagnetic contribution to the vacuum-energy energy, it became possible—within the present approach—to correctly reproduce the known critical temperature of that phase transition [5]. In the present paper, we will apply the same techniques to the calculation of the paramagnetic and the diamagnetic contributions to the one-loop effective action of the four-dimensional Higgs model (1).

In the corresponding theory resulting from the full breaking of the SU(2) symmetry, the one-loop effective action of a ghost (which is a spinless adjointly charged particle of mass $m$) has the form

$$\langle\Gamma[A^a_\mu]\rangle = \int_0^\infty \frac{ds}{s}\,\mathrm{e}^{-m^2 s}\int_P \mathcal{D}z_\mu\,\mathrm{e}^{-\frac{1}{4}\int_0^s d\tau\,\dot{z}^2_\mu}\langle W(C)\rangle. \tag{2}$$

In this equation, the contour $C$ is parameterized by the vector function $z_\mu(\tau)$ and $P$ stands for the periodic boundary conditions (i.e., $\int_P \equiv \int_{z_\mu(s)=z_\mu(0)}$). In the world-line representation for the effective action of a gauge boson (which is a spinning particle), an additional term $\propto F^a_{\mu\nu}T^a$ appears [6,7], where $T^a$ is an SU(2)-generator in the adjoint representation: $(T^a)^{bc} = -i\varepsilon^{abc}$. This term can be recovered by acting on the Wilson loop with the area-derivative operator [8,9] $\frac{\delta}{\delta\sigma_{\mu\nu}(z)}$. For this reason, in the spinning case, the gauge-field dependence of the effective action can be reduced to that of the Wilson loop in the same way as in the spinless case (see, e.g., [10–12]).

The Yang–Mills vacuum of the theory (1) has two correlation lengths, $1/M$ and $1/\mathrm{M}$, where $M$ and M are the masses of the so-called 1- and 2-gluon gluelumps, respectively. These gluelumps are the bound states of one and two gluons in the field of a hypothetical infinitely heavy adjoint source [13–18]. An adjoint string interconnecting two heavy adjoint sources breaks upon the creation of a glueball and the subsequent recombination process. This leads to the appearance of two 1-gluon gluelumps and yields the perimeter-law exponential in the Wilson loop of the heavy adjoint source. This exponential has the form $\mathrm{e}^{-ML}$, where $L$ is the length of the contour $C$. The full adjoint Wilson loop reads [19–22]

$$\langle W(C)\rangle = \frac{\mathrm{e}^{-\sigma_a S} + \frac{1}{N^2}\,\mathrm{e}^{-ML}}{1+\frac{1}{N^2}}, \tag{3}$$

where the normalization condition $\langle W(0)\rangle = 1$ has been imposed and the number of colors $N$ should be set equal to 2. In Equation (3), $S$ denotes the area of the minimal surface $\Sigma_C$ bounded by $C$ and $\sigma_a$ is the string tension in the adjoint representation. Henceforth, we will be using the expression [18]

$$M \simeq \sqrt{6\sigma_a} = 3\sqrt{\sigma\big|_{N=3}} = 1.32\,\mathrm{GeV}, \tag{4}$$

where we have adopted Casimir scaling [23,24] to express the adjoint string tension $\sigma_a$ at $N=2$ by the known fundamental string tension at $N=3$, $\sigma\big|_{N=3} \simeq (0.44\,\mathrm{GeV})^2$, as $\sigma_a = \frac{3}{2}\sigma\big|_{N=3}$.

In what follows, for the effective action, we will use the known closed-form expression (That is, this expression is valid to all orders in $s$, being therefore suitable for the study of infra-red physics. It can be obtained by using either the standard covariant perturbation theory, or the world-line formalism [25,26].), which corresponds to two $F^a_{\mu\nu}$ terms standing in the pre-exponent [4,25,26]:

$$\langle\Gamma[A^a_\mu]\rangle = -\frac{g^2}{(4\pi)^2}\int_0^\infty \frac{ds}{s}\,\mathrm{e}^{-m^2 s}\int_x \mathrm{tr}_c\,\langle(F^a_{\mu\nu}T^a)\left[f-\frac{1}{2}\cdot\frac{f-1}{\xi}\right](F^a_{\mu\nu}T^a)\rangle, \tag{5}$$

where $\int_x \equiv \int d^4x$, $f \equiv f(\xi) = \int_0^1 du\, e^{u(1-u)\xi}$, and $\xi = sD_\mu^2$. This effective action represents the sum of contributions produced by the ghost and by the gauge boson, $\langle \Gamma[A_\mu^a] \rangle = \langle \Gamma[A_\mu^a] \rangle_{\rm gh} + \langle \Gamma[A_\mu^a] \rangle_{\rm g.b.}$, where

$$\langle \Gamma[A_\mu^a] \rangle_{\rm gh} = -\frac{g^2}{2(4\pi)^2} \int_0^\infty \frac{ds}{s}\, e^{-m^2 s} \int_x {\rm tr}_c\, \langle (F_{\mu\nu}^a T^a)\, \frac{f-1}{\xi}\, (F_{\mu\nu}^a T^a) \rangle$$

and

$$\langle \Gamma[A_\mu^a] \rangle_{\rm g.b.} = -\frac{g^2}{(4\pi)^2} \int_0^\infty \frac{ds}{s}\, e^{-m^2 s} \int_x {\rm tr}_c\, \langle (F_{\mu\nu}^a T^a)\, \left[ f - \frac{f-1}{\xi} \right]\, (F_{\mu\nu}^a T^a) \rangle .$$

Note that Equation (5) depends on the vacuum correlation lengths $1/M$ and $1/{\rm M}$ through the two-point correlation function of the $F_{\mu\nu}^a$, whose parts contributing to the perimeter and area laws of the Wilson loop fall off at the distances equal to these lengths.

## 2. Calculation

The general strategy of our calculation is based on the reduction of the effective action (5) to an equivalent effective action, corresponding to some auxiliary Abelian field with a Gaussian action. The possibility for such a reduction is visible already from the fact that the entire dependence on the non-Abelian gauge field is encoded in the effective action (2), in the form of the area- and perimeter-law terms in Equation (3). The auxiliary Abelian fields appear, then, in the course of regularization of the area $S$ of the minimal surface $\Sigma_C$ and the perimeter $L$ of the contour $C$. Furthermore, while such a regularization is straightforward for $L$ (owing to the one-dimensionality of the contour); for $S$, one can adopt various parametrizations in terms of $C$, so as to render the path integral in the effective action calculable [10–12,27,28]. From all such parametrizations, we found the one used in [12] to be mostly suitable for the present analysis, as it explicitly accounts for the finiteness of the vacuum correlation length $1/{\rm M}$. The regularized expression for $\sigma_a S$ can be obtained by using the Casimir-scaling formula $\sigma_a = \frac{8}{3}\sigma$, where $\sigma$ is the SU(2) fundamental string tension. This regularized expression has the form

$$\sigma_a S \simeq \frac{2}{3\pi}\, \sigma {\rm M}^2 \int_{\Sigma_C} d\sigma_{\mu\nu}(x) \int_{\Sigma_C} d\sigma_{\mu\nu}(x') e^{-{\rm M}|x-x'|}, \tag{6}$$

while the regularized expression for $ML$ can be straightforwardly written as

$$ML \simeq \frac{M^2}{2} \oint_C dx_\mu \oint_C dx'_\mu\, e^{-M|x-x'|}. \tag{7}$$

We will further follow the method of [12] to represent the so-regularized area- and perimeter-laws, in terms of the functional integrals over the auxiliary antisymmetric-tensor and vector fields, $B_{\mu\nu}$ and $h_\mu$, as

$$\exp\left[ -\frac{2}{3\pi}\, \sigma {\rm M}^2 \int_{\Sigma_C} d\sigma_{\mu\nu}(x) \int_{\Sigma_C} d\sigma_{\mu\nu}(x') e^{-{\rm M}|x-x'|} \right] = \langle e^{\frac{i}{2}\int_x B_{\mu\nu}\Sigma_{\mu\nu}} \rangle_B \tag{8}$$

and

$$\exp\left[ -\frac{M^2}{2} \oint_C dx_\mu \oint_C dx'_\mu\, e^{-M|x-x'|} \right] = \langle e^{i\int_x h_\mu j_\mu} \rangle_h, \tag{9}$$

where the averages are defined as follows:

$$\langle \cdots \rangle_B = \int \left[ \prod_{\mu<\nu} \mathcal{D}B_{\mu\nu}\, e^{-\frac{1}{64\pi\sigma {\rm M}^3}\int_x B_{\mu\nu}(-\partial^2+{\rm M}^2)^{5/2} B_{\mu\nu}} \right] (\cdots)$$

and

$$\langle\cdots\rangle_h = \int \mathcal{D}h_\mu \, \mathrm{e}^{-\frac{1}{24\pi^2 M^3}\int_x h_\mu(-\partial^2+M^2)^{5/2}h_\mu}(\cdots),$$

where $j_\mu \equiv j_\mu(x;C) = \oint_C dz_\mu \, \delta(x-z)$ is the Abelian current associated with the contour $C$ and $\Sigma_{\mu\nu} \equiv \Sigma_{\mu\nu}(x;C) = \int_{\Sigma_C} d\sigma_{\mu\nu}(z)\,\delta(x-z)$ is the surface tensor. Furthermore, similarly to [12], we choose the surface element $d\sigma_{\mu\nu}$ in the form of an oriented, infinitely thin triangle built up of the position vector $z_\mu(\tau)$ and the differential element $dz_\mu = \dot{z}_\mu d\tau$ as $d\sigma_{\mu\nu}(z) = \frac{1}{2}(z_\mu \dot{z}_\nu - z_\nu \dot{z}_\mu)d\tau$. Then, the surface tensor takes the form $\Sigma_{\mu\nu} = \frac{1}{2}\int_0^S d\tau\,(z_\mu \dot{z}_\nu - z_\nu \dot{z}_\mu)\delta(x - z(\tau))$. With this expression for $\Sigma_{\mu\nu}$, the exponential $\mathrm{e}^{\frac{i}{2}\int_x B_{\mu\nu}\Sigma_{\mu\nu}}$ can be written as $\mathrm{e}^{\frac{i}{2}\int_x B_{\mu\nu}\Sigma_{\mu\nu}} = \mathrm{e}^{i\int_x \mathcal{A}_\mu j_\mu}$. Here, instead of $B_{\mu\nu}$, we have introduced an auxiliary vector field $\mathcal{A}_\mu(x) = \frac{1}{2}x_\nu B_{\nu\mu}(x)$, whose strength tensor $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$ has the form $\mathcal{F}_{\mu\nu} = B_{\mu\nu} + C_{\mu\nu}$, where $C_{\mu\nu}(x) = \frac{1}{2}x_\lambda(\partial_\mu B_{\lambda\nu} - \partial_\nu B_{\lambda\mu})$. Thus, in the case of $N = 2$, the Wilson loop (3) can altogether be written as

$$\langle W(C)\rangle = \frac{4}{5}\left(\langle \mathrm{e}^{i\int_x \mathcal{A}_\mu j_\mu}\rangle_B + \frac{1}{4}\,\langle \mathrm{e}^{i\int_x h_\mu j_\mu}\rangle_h\right). \tag{10}$$

Accordingly, depending on whether we consider the area- or the perimeter-law term in $\langle W(C)\rangle$, we should replace each of the two $(F^a_{\mu\nu}T^a)$'s in Equation (5) either by $\mathcal{F}_{\mu\nu}$ or $H_{\mu\nu} \equiv \partial_\mu h_\nu - \partial_\nu h_\mu$, replacing also $A^a_\mu T^a$ in $D_\mu$ either by $\mathcal{A}_\mu$ or $h_\mu$, and remove $\mathrm{tr}_c$. For example, we can write down the following path-integral representation for the area-law contribution to the effective action (see, e.g., [12,29,30]):

$$\int_x \langle \mathcal{F}_{\mu\nu}(x)\left[f - \frac{1}{2}\cdot\frac{f-1}{\xi}\right]\mathcal{F}_{\mu\nu}(x)\rangle_B =$$

$$= \int_x \langle \mathcal{F}_{\mu\nu}(x)\int_0^1 du\left[\mathrm{e}^{u(1-u)\xi} - \frac{1}{2}\,u(1-u)\int_0^1 d\alpha\,\mathrm{e}^{\alpha u(1-u)\xi}\right]\mathcal{F}_{\mu\nu}(x)\rangle_B = V\int_0^1 du\times$$

$$\times\int_y\left[\int_{r_\mu(0)=0}^{r_\mu(u(1-u)s)=y_\mu}\mathcal{D}r_\mu\,\mathrm{e}^{-\frac{1}{4}\int_0^{u(1-u)s}d\tau\,\dot{r}_\mu^2} - \frac{1}{2}\,u(1-u)\int_0^1 d\alpha\int_{r_\mu(0)=0}^{r_\mu(\alpha u(1-u)s)=y_\mu}\mathcal{D}r_\mu\,\mathrm{e}^{-\frac{1}{4}\int_0^{\alpha u(1-u)s}d\tau\,\dot{r}_\mu^2}\right]\times$$

$$\times\,\langle \mathcal{F}_{\mu\nu}(0)\,\exp\left(i\int_0^y dr_\mu\mathcal{A}_\mu\right)\mathcal{F}_{\mu\nu}(y)\rangle_B, \tag{11}$$

where $V$ is the volume occupied by the system and $\int_y \equiv \int d^4y$. Some details of the derivation of the last equality in this formula are presented in Appendix A, below. Furthermore, for consistency, the phase factor $\exp\left(i\int_0^y dr_\mu\mathcal{A}_\mu\right)$ in Equation (11) should be approximated by unity, in accordance with the initial two-point approximation (see [12]). Indeed, the use of the form factor $f - \frac{1}{2}\cdot\frac{f-1}{\xi}$ corresponds to accounting only for two $\mathcal{F}_{\mu\nu}$ terms, while Taylor expansion of the phase factor $\exp\left(i\int_0^y dr_\mu\mathcal{A}_\mu\right)$ would yield correlation functions of more than two $\mathcal{F}_{\mu\nu}$ terms. The path integrals over $r_\mu(\tau)$ in Equation (11), then, are reduced to the Green's function of the heat equation. Furthermore, the resulting correlation function has the form $\langle \mathcal{F}_{\mu\nu}(0)\mathcal{F}_{\mu\nu}(y)\rangle_B = \langle B_{\mu\nu}(0)B_{\mu\nu}(y)\rangle_B + \langle B_{\mu\nu}(0)C_{\mu\nu}(y)\rangle_B$, where $C_{\mu\nu}(0) = 0$ has been used. A straightforward calculation of the latter correlation functions yields

$$\langle B_{\mu\nu}(0)B_{\mu\nu}(y)\rangle_B = \frac{32}{\pi}\,\sigma\mathrm{M}^2\,\mathrm{e}^{-\mathrm{M}|y|}, \quad \langle B_{\mu\nu}(0)C_{\mu\nu}(y)\rangle_B = -\frac{8}{\pi}\,\sigma\mathrm{M}^3|y|\,\mathrm{e}^{-\mathrm{M}|y|}. \tag{12}$$

In the same way, we can treat the contribution produced to $\langle\Gamma[A^a_\mu]\rangle$ by the perimeter law, which stems from the average $\langle e^{i\int_x h_\mu j_\mu}\rangle_h$ in Equation (10). The corresponding correlation function $\langle H_{\mu\nu}(0)H_{\mu\nu}(y)\rangle_h$ can be calculated using the average $\langle h_\mu(0)h_\nu(y)\rangle_h = M^2\delta_{\mu\nu}e^{-M|y|}$, and reads

$$\langle H_{\mu\nu}(0)H_{\mu\nu}(y)\rangle_h = 6M^3\left(\frac{3}{|y|}-M\right)e^{-M|y|}. \tag{13}$$

The paramagnetic and the diamagnetic contributions to the effective action (5) correspond, respectively, to the terms $f$ and $-\frac{1}{2}\cdot\frac{f-1}{\xi}$ in the form factor $f-\frac{1}{2}\cdot\frac{f-1}{\xi}$. Accordingly, the absolute value of the ratio of these contributions can be written in terms of the $B$- and $h$-averages, as

$$\left|\frac{\langle\Gamma[A^a_\mu]\rangle_{\rm para}}{\langle\Gamma[A^a_\mu]\rangle_{\rm dia}}\right| = 2\,\frac{\int_0^1 du(J_1+\frac{1}{4}J_2)}{\int_0^1 du(I_1+\frac{1}{4}I_2)}, \tag{14}$$

where

$$J_1=\int_0^\infty\frac{ds}{s}e^{-m^2s}\int_x\langle\mathcal{F}_{\mu\nu}(x)\,f\,\mathcal{F}_{\mu\nu}(x)\rangle_B,\;\; J_2=\int_0^\infty\frac{ds}{s}e^{-m^2s}\int_x\langle H_{\mu\nu}(x)\,f\,H_{\mu\nu}(x)\rangle_h,$$
$$I_1=\int_0^\infty\frac{ds}{s}e^{-m^2s}\int_x\langle\mathcal{F}_{\mu\nu}(x)\,\frac{f-1}{\xi}\,\mathcal{F}_{\mu\nu}(x)\rangle_B,\;\; I_2=\int_0^\infty\frac{ds}{s}e^{-m^2s}\int_x\langle H_{\mu\nu}(x)\,\frac{f-1}{\xi}\,H_{\mu\nu}(x)\rangle_h.$$

By using Equations (11)–(13), we obtain the following intermediate expressions for these quantities (see [12]):

$$J_1=\frac{2}{\pi^3}\,\frac{\sigma\mathrm{M}^2}{[u(1-u)]^2}\int_0^\infty\frac{ds}{s^3}e^{-m^2s}\int_y e^{-\frac{y^2}{4u(1-u)s}-\mathrm{M}|y|}\Big(1-\frac{\mathrm{M}|y|}{4}\Big),$$
$$J_2=-\frac{3}{8\pi^2}\,\frac{M^4}{[u(1-u)]^2}\int_0^\infty\frac{ds}{s^3}e^{-m^2s}\int_y e^{-\frac{y^2}{4u(1-u)s}-M|y|}\left(1-\frac{3}{M|y|}\right),$$
$$I_1=\frac{8}{\pi^3}\,\sigma\mathrm{M}^2\int_0^\infty\frac{ds}{s^2}e^{-m^2s}\int_y\frac{e^{-\frac{y^2}{4u(1-u)s}-\mathrm{M}|y|}}{y^2}\Big(1-\frac{\mathrm{M}|y|}{4}\Big),$$
$$I_2=-\frac{3}{2\pi^2}\,M^4\int_0^\infty\frac{ds}{s^2}e^{-m^2s}\int_y\frac{e^{-\frac{y^2}{4u(1-u)s}-M|y|}}{y^2}\left(1-\frac{3}{M|y|}\right).$$

The $s$-integrations in these expressions can be performed analytically, which yield

$$J_1=\frac{32}{\pi}\,\sigma m^2\frac{1}{u(1-u)}\int_0^\infty dz\,z\,K_2(az)\,e^{-z}\left(1-\frac{z}{4}\right), \tag{15}$$

$$J_2=-6\,\frac{M^4m^2}{\mathrm{M}^2}\,\frac{1}{u(1-u)}\int_0^\infty dz\,z\,K_2(az)\,e^{-\frac{M}{\mathrm{M}}z}\left(1-\frac{3\mathrm{M}}{Mz}\right), \tag{16}$$

$$I_1=\frac{64}{\pi}\,\sigma m\mathrm{M}\sqrt{u(1-u)}\int_0^\infty dz\,K_1(az)\,e^{-z}\left(1-\frac{z}{4}\right), \tag{17}$$

$$I_2=-12\,\frac{M^4m}{\mathrm{M}}\sqrt{u(1-u)}\int_0^\infty dz\,K_1(az)\,e^{-\frac{M}{\mathrm{M}}z}\left(1-\frac{3\mathrm{M}}{Mz}\right), \tag{18}$$

where $a = \frac{m/\mathrm{M}}{\sqrt{u(1-u)}}$, $z = \mathrm{M}|y|$, and $K_\nu(az)$ are the Macdonald functions. Using the relation $K_2(x) = K_0(x) + \frac{2}{x} K_1(x)$, we see that the leading contribution to Equation (14) stems from the $\frac{1}{z}$-parts of the integrals $J_2$ and $I_2$, for sufficiently small $z$. This contribution, thus, yields

$$\left| \frac{\langle \Gamma[A^a_\mu] \rangle_{\rm para}}{\langle \Gamma[A^a_\mu] \rangle_{\rm dia}} \right| \simeq 2 \, \frac{\int_0^1 du \frac{1}{\sqrt{u(1-u)}} \, I}{\int_0^1 du \sqrt{u(1-u)} \, I}, \;\; \text{where} \;\; I = \int_0^\infty \frac{dz}{z} \, K_1(az) \, \mathrm{e}^{-\frac{M}{\mathrm{M}} z}.$$

We notice that, as we consider the full exponentials in Equations (12) and (13), $|y|$ is larger than both $1/\mathrm{M}$ and $1/M$. As $\mathrm{M} > M$ [13–18], this means that we should restrict ourselves only to such $y$'s for which $|y| > 1/M$. Introducing, instead of $z$, a new integration variable $t = \frac{M}{m} az$, we thus have

$$I = \int\limits_{1/\sqrt{u(1-u)}}^{\infty} \frac{dt}{t} \, K_1\left(\frac{m}{M} t\right) \, \mathrm{e}^{-\sqrt{u(1-u)}\, t}.$$

Furthermore, noticing that $1/\sqrt{u(1-u)}$ varies in the interval $[2, \infty)$, we can consider two cases: $m \gg \frac{M}{2}$ and $m \ll \frac{M}{2}$. In the first case, we have

$$\left| \frac{\langle \Gamma[A^a_\mu] \rangle_{\rm para}}{\langle \Gamma[A^a_\mu] \rangle_{\rm dia}} \right| \simeq 2 \, \frac{\int_0^1 \frac{du}{\sqrt{u(1-u)}} \, \mathrm{e}^{-\frac{m/M}{\sqrt{u(1-u)}}}}{\int_0^1 du \sqrt{u(1-u)} \, \mathrm{e}^{-\frac{m/M}{\sqrt{u(1-u)}}}}. \tag{19}$$

As a function of $\frac{m}{M}$, this expression monotonically decreases towards the value of 8.0 at $\frac{m}{M} = \mathcal{O}(100)$, and stays at this value with any further increase of $m$. This result is the main finding of the present paper.

In the second case, the dominant contribution to $I$ appears at $\frac{m}{M} < \sqrt{u(1-u)}$. In the limit of $\frac{m}{M} \ll \frac{1}{2}$ of interest, we have

$$\left| \frac{\langle \Gamma[A^a_\mu] \rangle_{\rm para}}{\langle \Gamma[A^a_\mu] \rangle_{\rm dia}} \right| \simeq$$

$$\simeq 2 \, \frac{\int\limits_{(m/M)^2}^{1-(m/M)^2} \frac{du}{\sqrt{u(1-u)}} \int\limits_{1/\sqrt{u(1-u)}}^{M/m} \frac{dt}{t^2} \, \mathrm{e}^{-\sqrt{u(1-u)}t}}{\int\limits_{(m/M)^2}^{1-(m/M)^2} du \sqrt{u(1-u)} \int\limits_{1/\sqrt{u(1-u)}}^{M/m} \frac{dt}{t^2} \, \mathrm{e}^{-\sqrt{u(1-u)}t}} \simeq 2 \, \frac{\int\limits_{(m/M)^2}^{1-(m/M)^2} du \left( \frac{1}{\mathrm{e}} - \mathrm{e}^{-\frac{M}{m}\sqrt{u(1-u)}} \right)}{\int\limits_{(m/M)^2}^{1-(m/M)^2} du u(1-u) \left( \frac{1}{\mathrm{e}} - \mathrm{e}^{-\frac{M}{m}\sqrt{u(1-u)}} \right)}. \tag{20}$$

In particular, at $m \to 0$, we recover the standard result of the massless Yang–Mills theory:

$$\left| \frac{\langle \Gamma[A^a_\mu] \rangle_{\rm para}}{\langle \Gamma[A^a_\mu] \rangle_{\rm dia}} \right| \to \frac{2}{\int_0^1 du u(1-u)} = 12.$$

Finally, let us extrapolate the above considerations to the deconfinement phase. To this end, we notice that, at temperatures larger than the deconfinement critical temperature $T_c$, the chromo-electric condensate $\langle (E^a_i)^2 \rangle$ vanishes, while the chromo-magnetic condensate $\langle (H^a_i)^2 \rangle$ does not; which leads to the so-called spatial confinement (For references, see, e.g., Section 4 of [5].). This means that only the spatial components

of the surface tensor $\Sigma_{\mu\nu}$ and the current $j_\mu$ remain involved in the regularized expressions (6) and (7), so that Equations (8) and (9) take the form

$$\exp\left[-\frac{2}{3\pi}\,\sigma \mathrm{M}^2 \int_{\Sigma_C} d\sigma_{ij}(x) \int_{\Sigma_C} d\sigma_{ij}(x') \mathrm{e}^{-\mathrm{M}|x-x'|}\right] = \langle \mathrm{e}^{\frac{i}{2}\int_{\mathbf{x}} B_{ij}\Sigma_{ij}}\rangle_B$$

and

$$\exp\left[-\frac{M^2}{2} \oint_C dx_i \oint_C dx'_i\, \mathrm{e}^{-M|x-x'|}\right] = \langle \mathrm{e}^{i\int_{\mathbf{x}} h_i j_i}\rangle_h,$$

where

$$\langle\cdots\rangle_B = \int \left[\prod_{i<j} \mathcal{D}B_{ij}\, \mathrm{e}^{-\frac{1}{64\pi\sigma \mathrm{M}^3}\int_{\mathbf{x}} B_{ij}(-\partial^2+\mathrm{M}^2)^{5/2}B_{ij}}\right](\cdots)$$

and

$$\langle\cdots\rangle_h = \int \mathcal{D}h_i\, \mathrm{e}^{-\frac{1}{24\pi^2 M^3}\int_{\mathbf{x}} h_i(-\partial^2+M^2)^{5/2}h_i}(\cdots),$$

where $\sigma$ is the spatial string tension in the fundamental representation and $\int_{\mathbf{x}} \equiv \int d^3x$. The averages (12) can then be modified as

$$\langle B_{ij}(0)B_{ij}(y)\rangle_B = \frac{16}{\pi}\,\sigma \mathrm{M}^2\, \mathrm{e}^{-\mathrm{M}|y|}, \quad \langle B_{ij}(0)C_{ij}(y)\rangle_B = -\frac{4}{\pi}\,\sigma \mathrm{M}^3|y|\, \mathrm{e}^{-\mathrm{M}|y|},$$

while the average (13) is modified as

$$\langle H_{ij}(0)H_{ij}(y)\rangle_h = 4M^3\left(\frac{2}{|y|} - M\right)\mathrm{e}^{-M|y|}.$$

Accordingly, Equations (15)–(18) take the form

$$J_1 = \frac{16}{\pi}\,\sigma m^2 \frac{1}{u(1-u)} \int_0^\infty dz\, z\, K_2(az)\, \mathrm{e}^{-z}\left(1-\frac{z}{4}\right), \tag{21}$$

$$J_2 = -4\,\frac{M^4 m^2}{\mathrm{M}^2}\,\frac{1}{u(1-u)} \int_0^\infty dz\, z\, K_2(az)\, \mathrm{e}^{-\frac{M}{\mathrm{M}}z}\left(1-\frac{2\mathrm{M}}{Mz}\right), \tag{22}$$

$$I_1 = \frac{32}{\pi}\,\sigma m \mathrm{M}\sqrt{u(1-u)} \int_0^\infty dz\, K_1(az)\, \mathrm{e}^{-z}\left(1-\frac{z}{4}\right), \tag{23}$$

$$I_2 = -8\,\frac{M^4 m}{\mathrm{M}}\sqrt{u(1-u)} \int_0^\infty dz\, K_1(az)\, \mathrm{e}^{-\frac{M}{\mathrm{M}}z}\left(1-\frac{2\mathrm{M}}{Mz}\right). \tag{24}$$

We see that the ratio of the leading contributions to Equation (14), which stems from the $\frac{1}{z}$-parts of the integrals $J_2$ and $I_2$ at sufficiently small $z$, remains the same as at zero temperature. Consequently, the above-obtained zero-temperature results (19) and (20) remain valid up to temperatures at which the theory undergoes the dimensional reduction, becoming effectively three-dimensional.

## 3. Summary

In the standard Yang–Mills theory, the absolute value of the paramagnetic contribution to the one-loop effective action exceeds the diamagnetic contribution by a factor of 12, which is the origin of the factor of $11 = 12 - 1$ in the one-loop coefficient of the Yang–Mills $\beta$-function. In this paper, we have calculated both contributions in the SU(2) Yang–Mills–Higgs model, where the SU(2)-symmetry is broken completely, such that all three vector bosons have equal masses. We have found that the leading contributions to both

paramagnetic and diamagnetic interactions stem from the perimeter-law part of the corresponding adjoint Wilson loops of the vector boson and the ghost, while the contributions stemming from the area-law part are only subleading. By regularizing the perimeter law through the phenomenological Green's function of the so-called one-gluon gluelump, we have calculated the ratio of the paramagnetic and the diamagnetic contributions as a function of $m/M$, where $m$ is the mass of vector bosons and $M$ is the mass of the one-gluon gluelump. While, in the limit $m \to 0$, we have recovered the aforementioned factor of 12, in the opposite limit, with $m$ as large as about $\mathcal{O}(100\ M)$, we have found that this factor goes to 8. This finding suggests that, at such large values of $m$, the one-loop coefficient of the $\beta$-function in the SU(2) Yang–Mills–Higgs model can be $-\frac{7\cdot 2}{3}$, instead of the coefficient $-\frac{11\cdot 2}{3}$ in the massless Yang–Mills case. Finally, we have found that the obtained results hold also in the deconfinement phase, up to temperatures where the dimensional reduction of the model to its effective three-dimensional counterpart occurs.

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## Appendix A

Let us illustrate how the last equality in Equation (11) can be derived. For simplicity, we consider the expression which does not involve the $u$- and $\alpha$-integrations. This expression has the form

$$\int_x \langle \mathcal{F}_{\mu\nu}(x) \mathrm{e}^{sD_\mu^2} \mathcal{F}_{\mu\nu}(x) \rangle_B = \int_x \int_y \int\limits_{r_\mu(0)=x_\mu}^{r_\mu(s)=x_\mu+y_\mu} \mathcal{D}r_\mu \, \mathrm{e}^{-\frac{1}{4}\int_0^s d\tau \dot{r}_\mu^2} \langle \mathcal{F}_{\mu\nu}(x) \, \exp\left( i \int_x^{x+y} dr_\mu \mathcal{A}_\mu \right) \mathcal{F}_{\mu\nu}(x+y) \rangle_B,$$

where the path-integral representation for the operator $\mathrm{e}^{sD_\mu^2}$ has been used. Next, owing to the translation invariance of the $B$-average, we have

$$\langle \mathcal{F}_{\mu\nu}(x) \, \exp\left( i \int_x^{x+y} dr_\mu \mathcal{A}_\mu \right) \mathcal{F}_{\mu\nu}(x+y) \rangle_B = \langle \mathcal{F}_{\mu\nu}(0) \, \exp\left( i \int_0^{y} dr_\mu \mathcal{A}_\mu \right) \mathcal{F}_{\mu\nu}(y) \rangle_B.$$

Accordingly, shifting trajectories $r_\mu(\tau)$ by the vector $-x_\mu$, we obtain

$$\int_x \langle \mathcal{F}_{\mu\nu}(x) \mathrm{e}^{sD_\mu^2} \mathcal{F}_{\mu\nu}(x) \rangle_B = V \int_y \int\limits_{r_\mu(0)=0}^{r_\mu(s)=y_\mu} \mathcal{D}r_\mu \, \mathrm{e}^{-\frac{1}{4}\int_0^s d\tau \dot{r}_\mu^2} \langle \mathcal{F}_{\mu\nu}(0) \, \exp\left( i \int_0^{y} dr_\mu \mathcal{A}_\mu \right) \mathcal{F}_{\mu\nu}(y) \rangle_B,$$

which yields the last equality in Equation (11).

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